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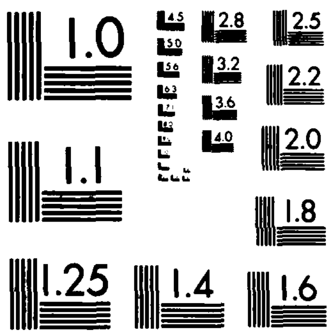

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PRESEARCH

FLEET MOORING LEG
DESIGN PROGRAM DOCUMENTATION

Volume 1

EQUILIBRIUM EQUATIONS REPORT

FPO-1-82-(32)

December 1982

General Distribution

PRESEARCH INCORPORATED

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Performed for
Ocean Engineering and Construction Project Office
Chesapeake Division
Naval Facilities Engineering Command
Washington, D.C. 20374
Under
Contract N62477-81-C-0025

Presearch Incorporated
2361 South Jefferson Davis Highway
Arlington, Virginia 22202

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FLEET MOORING LEG DESIGN PROGRAM DOCUMENTATION

Volume 1

EQUILIBRIUM EQUATIONS REPORT

I. EQUILIBRIUM EQUATIONS REPORT

OCEAN FLOOR VARIABLES

The standard coordinate system used for a mooring leg is a right-handed xzy coordinate system, where the x and z axes are horizontal. The origin is taken to be at the anchor of a simple leg, or at the midpoint of the line joining the anchors of a compound leg. Load directions are measured counterclockwise from the positive x-axis.

The ocean floor is a plane surface defined by the x, z, and D coordinates of three points, where D denotes depth under water. These point coordinates may be denoted (x_1, z_1, D_1), (x_2, z_2, D_2), (x_3, z_3, D_3). Because the ocean floor includes the origin of the standard coordinate system, its equation in that system must have the form $y = C_x x + C_z z$, where C_x and C_z are constant coefficients.

Variables used by the solution procedure are computed as follows. First, C_x and C_z are found by solving the linear system:

$$\begin{cases} (x_1 - x_3)\,C_x + (z_1 - z_3)\,C_z = D_3 - D_1 \\ (x_2 - x_3)\,C_x + (z_2 - z_3)\,C_z = D_3 - D_2 \end{cases}$$

The explicit solution of this system of equations is

$$C_x = \frac{\begin{vmatrix} (D_3 - D_1) & (z_1 - z_3) \\ (D_3 - D_2) & (z_2 - z_3) \end{vmatrix}}{\begin{vmatrix} (x_1 - x_3) & (z_1 - z_3) \\ (x_2 - x_3) & (z_2 - z_3) \end{vmatrix}} \qquad C_z = \frac{\begin{vmatrix} (x_1 - x_3) & (D_3 - D_1) \\ (x_2 - x_3) & (D_3 - D_2) \end{vmatrix}}{\begin{vmatrix} (x_1 - x_3) & (z_1 - z_3) \\ (x_2 - x_3) & (z_2 - z_3) \end{vmatrix}}$$

where $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ is the matrix determinant.

The gradient vector for ocean floor elevation is (c_x, c_z). The length of that vector is the maximum floor slope. Therefore $\theta_F = \tan^{-1}\left(\sqrt{c_x^2 + c_z^2}\right)$.

The direction in which the floor has its maximum slope is given by

$$\varphi_F = \begin{cases} \tan^{-1}\left(\frac{c_z}{c_x}\right) & \text{if } c_x > 0 \\ \tan^{-1}\left(\frac{c_z}{c_x}\right) + \pi & \text{if } c_x < 0 \\ \frac{\pi}{2} & \text{if } c_x = 0 \text{ and } c_z > 0 \\ -\frac{\pi}{2} & \text{if } c_x = 0 \text{ and } c_z < 0 \end{cases}$$

SIMPLE LEG - TAUT LINE SOLUTION

In the taut line solution, it is assumed that the leg lies entirely above the ocean floor. This restriction is made to simplify the equilibrium equations, thus permitting an arbitrary selection of unknown input parameters.

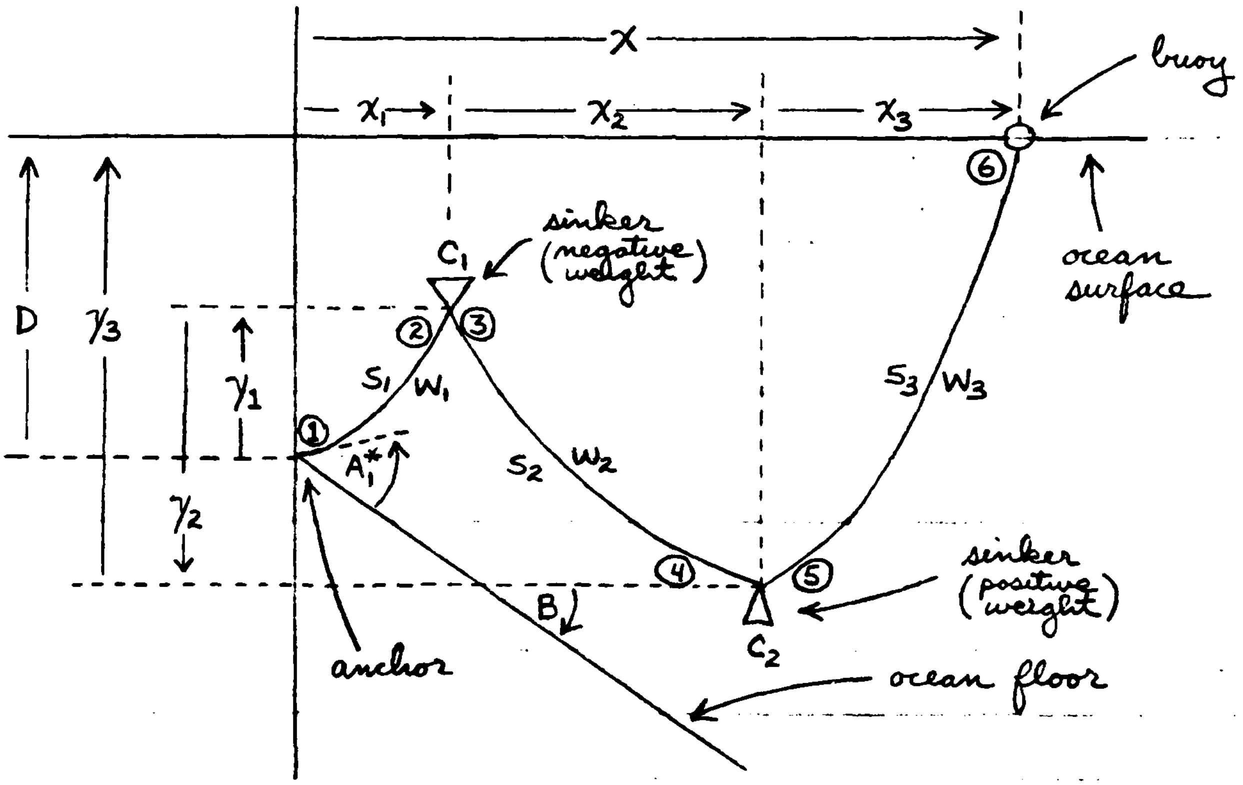
Input Parameters

D Depth at anchor.
B Angle from horizontal to ocean floor.
A_1^* Angle from ocean floor to chain at anchor.
S_1 Scope of first chain segment.
W_1 Linear weight of first chain segment.
C_1 Weight of first sinker.
S_2 Scope of second chain segment.
W_2 Linear weight of second chain segment.
C_2 Weight of second sinker.
S_3 Scope of third chain segment.
W_3 Linear weight of third chain segment.
X Horizontal displacement of buoy from anchor.
H Magnitude of horizontal component of load on buoy.

Parameters D and B are not input directly; they are computed from the coordinates of three points, selected by the user, that determine the ocean floor as a plane surface. The user also selects a load direction. Angle B is determined by the intersection of the ocean floor with a vertical plane in the load direction; it is the effective ocean floor angle for the two-dimensional situation represented by the simple leg program. B is defined to be negative if the ocean floor slopes downward in the direction from the anchor to the buoy.

Parameters D and B are always known. Exactly two of the remaining input parameters are input as unknown.

Simple Leg Schematic Diagram (FIGURE 1)



NOTE 1: Each sinker may have positive, negative or zero weight. Each chain segment may have positive or negative (but not zero) linear weight.

NOTE 2: ① - ⑥ denote points at the ends of chain segments at which angles and tensions are computed. These points are referred to as nodes.

Output Parameters

A_i	Angle at node i	$1 \le i \le 6$
V_i	Vertical component of tension at node i	
T_i	Magnitude of tension at node i	
x_K	Horizontal displacement of segment K	$1 \le K \le 3$
y_K	Vertical displacement of segment K	

NOTE: x_K is always positive; y_K is positive if the right end of segment K (as shown in figure 1) is higher than the left end, and is zero or negative otherwise.

Equilibrium Equations

(1) $A_1 = A_1^* + B$

(2) $A_2 = \tan^{-1}\left(\tan A_1 + \frac{S_1 W_1}{H}\right)$

(3) $x_1 = \frac{H}{W_1} \ln\left(\frac{\tan A_2 + \sec A_2}{\tan A_1 + \sec A_1}\right)$

(4) $y_1 = \frac{H}{W_1}(\sec A_2 - \sec A_1)$

(5) $A_3 = \tan^{-1}\left(\tan A_2 + \frac{C_1}{H}\right)$

(6) $A_4 = \tan^{-1}\left(\tan A_3 + \frac{S_2 W_2}{H}\right)$

(7) $x_2 = \frac{H}{W_2} \ln\left(\frac{\tan A_4 + \sec A_4}{\tan A_3 + \sec A_3}\right)$

(8) $y_2 = \frac{H}{W_2}(\sec A_4 - \sec A_3)$

(9) $A_5 = \tan^{-1}\left(\tan A_4 + \frac{C_2}{H}\right)$

(10) $A_6 = \tan^{-1}\left(\tan A_5 + \frac{S_3 W_3}{H}\right)$

(11) $x_3 = \frac{H}{W_3} \ln\left(\frac{\tan A_6 + \sec A_6}{\tan A_5 + \sec A_5}\right)$

(12) $y_3 = \frac{H}{W_3}(\sec A_6 - \sec A_5)$

(13) $x_1 + x_2 + x_3 = X$

(14) $y_1 + y_2 + y_3 = D$

Tension Equations

(15)–(20) $T_i = \frac{H}{\cos A_i}$

(21)–(26) $V_i = T_i \sin A_i$

$1 \le i \le 6$

Solution Procedure

Equations ①-⑭ may be used to compute X and D, given values for the parameters $B, A_1^*, S_1, W_1, C_1, S_2, W_2, C_2, S_3, W_3$ and H. This computation also produces values for the angles, tensions and displacements listed as output parameters. The true value of D is always known; that of X may or may not be known. Two cases may be distinguished:

1. X is unknown. Then D may be regarded as a function of the other unknown parameter U. That is, if a test value is specified for U, then equations ①-⑭ may be applied to compute a value for D, which can be compared to the known value D_{KNOWN}. To find the true value of U, the program first queries the user for two initial guesses $U^{(1)}, U^{(2)}$ for U. It then generates a sequence of values $U^{(3)}, U^{(4)}, \ldots$ by the one-dimensional secant method. This sequence, if continued indefinitely, would converge to the true value of U, provided that the initial guesses are sufficiently close to that value. The iteration is terminated when $|D(U^{(K)}) - D_{KNOWN}| < D_{KNOWN} * 10^{-10}$.
2. X is known. Then X and D are regarded as functions of the unknown parameters U_1 and U_2. That is, if test values are specified for U_1 and U_2, then equations ①-⑭ may be used to compute values for X and D, which can be compared to the known values X_{KNOWN} and D_{KNOWN}. The program first queries the user for initial values $U_1^{(1)}, U_2^{(1)}$ for the unknown parameters. It then generates a sequence of values $(U_1^{(2)}, U_2^{(2)}), (U_1^{(3)}, U_2^{(3)}), \ldots$ by the two-dimensional Steffensen method. The iteration is terminated when $|X(U_1^{(K)}, U_2^{(K)}) - X_{KNOWN}| < X_{KNOWN} * 10^{-10}$ and $|D(U_1^{(K)}, U_2^{(K)}) - D_{KNOWN}| < D_{KNOWN} * 10^{-10}$.

SIMPLE LEG - SLACK LINE SOLUTION

In the slack line solution, part of the leg may rest on the floor. This "slack" portion may include one or both sinkers. Thus cases may be considered that the taut line solution cannot handle. The choice of unknown parameters, however, is more restrictive than for the taut line solution, as explained below.

Input Parameters

The parameter A_1^* of the taut leg solution is replaced by a parameter denoted V, giving the vertical component of tension at the buoy. V is an implicit parameter; the user is not queried for its value, and it is always to be regarded as an unknown. Except for the replacement of A_1^* by V, the taut and slack solutions have the same set of input parameters. Either X or H is unknown, as determined by the following three user choices:

1. Specify load direction and magnitude (X unknown)
2. Specify load direction and buoy displacement (H unknown)
3. Specify buoy coordinates (H unknown)

(NOTE: The buoy has two coordinates, denoted "X" and "Z", that are user-specified in choice 3. Unless the load direction is zero, the "X" coordinate does not have the same value as the parameter named "X".)

Output Parameters

Output parameters for the slack line solution consist of:

- All output parameters given by the taut line solution.
- A parameter denoted L, giving the length of chain along the ocean floor. This is called the "slack length" of the leg.

Critical Tensions

The form of the equilibrium and tension equations changes at each of six "critical values" for the vertical component V of tension at the buoy:

$$VC_1 = S_1W_1 + C_1 + S_2W_2 + C_2 + S_3W_3 + H\tan B$$
$$VC_2 = C_1 + S_2W_2 + C_2 + S_3W_3 + H\tan B$$
$$VC_3 = S_2W_2 + C_2 + S_3W_3 + H\tan B$$
$$VC_4 = C_2 + S_3W_3 + H\tan B$$
$$VC_5 = S_3W_3 + H\tan B$$
$$VC_6 = H\tan B$$

In general, VC_n is the value that V has when the chain lifts off the ocean floor at point n, as defined in diagram 1.

If all components are of positive weight, then $VC_1 > VC_2 > \cdots > VC_6$. In this case the range of validity for V can be divided into six intervals:

1.	$V \geq VC_1$	Chain leaves floor at anchor.
2.	$V < VC_1$ and $V \geq VC_2$	Chain leaves floor along first segment.
3.	$V < VC_2$ and $V \geq VC_3$	Chain leaves floor at first sinker.
4.	$V < VC_3$ and $V \geq VC_4$	Chain leaves floor along second segment.
5.	$V < VC_4$ and $V \geq VC_5$	Chain leaves floor at second sinker.
6.	$V < VC_5$ and $V \geq VC_6$	Chain leaves floor along third segment.

We assume that any negative weight component, and any other component between it and the buoy, lies entirely above the ocean floor. This means that if $VC_n < VC_{n+1}$ then intervals $n+1$ through 6 are excluded from the range of validity.

Equilibrium Equations

$V \geq VC_1$

(100) $L = 0$

(101) $A_1 = \tan^{-1}\left(\tan B + \frac{V - VC_1}{H}\right)$

(102) – (114) identical to (2) – (14) in taut solution.

$V < VC_1$ and $V \geq VC_2$

(200) $L = \frac{VC_1 - V}{W_1}$

(201) $A_1 = B$

(202) $A_2 = \tan^{-1}\left(\tan B + \frac{(S_1 - L)W_1}{H}\right)$

(203) $x_1 = L\cos B + \frac{H}{W_1}\ln\left(\frac{\tan A_2 + \sec A_2}{\tan A_1 + \sec A_1}\right)$

(204) $y_1 = L\sin B + \frac{H}{W_1}(\sec A_2 - \sec A_1)$

(205) – (214) identical to (5) – (14) in taut solution.

$V < VC_2$ and $V \geq VC_3$

(300) $L = S_1$

(301) identical to (201)

$$\left.\begin{array}{ll} (302) & A_2 = B \\ (303) & x_1 = S_1\cos B \\ (304) & y_1 = S_1\sin B \end{array}\right\} \text{equivalent to (202) – (204)}$$

(305) $A_3 = \tan^{-1}\left(\tan B + \frac{V - VC_3}{H}\right)$

(306) – (314) identical to (6) – (14) in taut solution

$V < VC_3$ and $V \geq VC_4$

400 $L = S_1 + \frac{VC_3 - V}{W_2}$

401 - 404 identical to 301 - 304

405 $A_3 = B$

406 $A_4 = \tan^{-1}\left(\tan B + \frac{(S_1 + S_2 - L)\,W_2}{H}\right)$

407 $x_2 = (L - S_1)\cos B + \frac{H}{W_2}\ln\left(\frac{\tan A_4 + \sec A_4}{\tan A_3 + \sec A_3}\right)$

408 $y_2 = (L - S_1)\sin B + \frac{H}{W_2}(\sec A_4 - \sec A_3)$

409 - 414 identical to 9 - 14 in taut solution

$V < VC_4$ and $V \geq VC_5$

500 $L = S_1 + S_2$

501 - 505 identical to 401 - 405

$$\left.\begin{array}{ll} 506 & A_4 = B \\ 507 & x_2 = S_2 \cos B \\ 508 & y_2 = S_2 \sin B \end{array}\right\} \text{equivalent to } 406 - 408$$

509 $A_5 = \tan^{-1}\left(\tan B + \frac{V - VC_5}{H}\right)$

510 - 514 same as 10 - 14 in taut solution

$V < VC_5$ and $V \geq VC_6$

600 $L = S_1 + S_2 + \frac{VC_5 - V}{W_3}$

601 - 608 identical to 501 - 508

609 $A_5 = B$

610 $A_6 = \tan^{-1}\left(\tan B + \frac{(S_1 + S_2 + S_3 - L)\,W_3}{H}\right)$

(611) $x_3 = (L - S_1 - S_2)\cos B + \frac{H}{W_3}\ln\left(\frac{\tan A_6 + \sec A_6}{\tan A_5 + \sec A_5}\right)$

(612) $y_3 = (L - S_1 - S_2)\sin B + \frac{H}{W_3}(\sec A_6 - \sec A_5)$

(613) - (614) identical to (13) - (14) in taut solution

Tension Equations

Equations for the vertical components of tension are identical to the corresponding equations in the taut solution: (21) - (26) $V_i = T_i \sin A_i$, $1 \le i \le 6$

Equations for the magnitudes T_i of tension are given below.

All assume zero friction between the leg and the ocean floor.

$V \geq VC_1$

(115) - (120) $T_i = \frac{H}{\cos A_i}$ (identical to (15) - (20) in taut solution)

$V < VC_1$ and $V \geq VC_2$

(215) $T_1 = \frac{H}{\cos B} - W_1 L \sin B$

(216) - (220) identical to (16) - (20) in taut solution

$V < VC_2$ and $V \geq VC_3$

(315) $T_1 = \frac{H\cos(A_3 - B)}{\cos A_3} - (W_1 S_1 + C_1)\sin B$

(316) $T_2 = \frac{H\cos(A_3 - B)}{\cos A_3} - C_1 \sin B$

(317) - (320) identical to (17) - (20) in taut solution

$V < VC_3$ and $V \geq VC_4$

415 $$T_1 = \frac{H}{\cos B} - (\omega_1 S_1 + C_1 + \omega_2 (L - S_1)) \sin B$$

416 $$T_2 = \frac{H}{\cos B} - (C_1 + \omega_2 (L - S_1)) \sin B$$

417 $$T_3 = \frac{H}{\cos B} - \omega_2 (L - S_1) \sin B$$

418 - 420 identical to 18 - 20 in taut case

$V < VC_4$ and $V \geq VC_5$

515 $$T_1 = \frac{H \cos(A_5 - B)}{\cos A_5} - (\omega_1 S_1 + C_1 + \omega_2 S_2 + C_2) \sin B$$

516 $$T_2 = \frac{H \cos(A_5 - B)}{\cos A_5} - (C_1 + \omega_2 S_2 + C_2) \sin B$$

517 $$T_3 = \frac{H \cos(A_5 - B)}{\cos A_5} - (\omega_2 S_2 + C_2) \sin B$$

518 $$T_4 = \frac{H \cos(A_5 - B)}{\cos A_5} - C_2 \sin B$$

519 - 520 identical to 19 - 20 in taut case

$V < VC_5$ and $V \geq VC_6$

615 $$T_1 = \frac{H}{\cos B} - (\omega_1 S_1 + C_1 + \omega_2 S_2 + C_2 + (L - S_1 - S_2)\omega_3) \sin B$$

616 $$T_2 = \frac{H}{\cos B} - (C_1 + \omega_2 S_2 + C_2 + (L - S_1 - S_2)\omega_3) \sin B$$

617 $$T_3 = \frac{H}{\cos B} - (\omega_2 S_2 + C_2 + (L - S_1 - S_2) W_3) \sin B$$

618 $$T_4 = \frac{H}{\cos B} - (C_2 + (L - S_1 - S_2)\omega_3) \sin B$$

619 $$T_5 = \frac{H}{\cos B} - (L - S_1 - S_2)\omega_3 \sin B$$

620 identical to 20 in taut case.

Solution Procedure

The slack line equilibrium equations may be used to compute X and D, given values for the parameters B, S_1, W_1, C_1, S_2, W_2, C_2, S_3, W_3, H and V. This computation also produces values for the slack length, angles, tensions and displacements listed as output parameters. The true value of D is always known; that of X may or may not be known. There are two cases.

1. X is unknown, H is known. Then D is regarded as a function of V. The program solves the equation $D(V) = D_{KNOWN}$ by the one-dimensional secant method.
2. X is known, H is unknown. Then X and D are regarded as function of H and V. The program solves the system of equations

$$\begin{cases} X(V,H) = X_{KNOWN} \\ D(V,H) = D_{KNOWN} \end{cases}$$

by the two-dimensional Steffensen method.

In both cases, the program generates its own initial guesses for the variables V and H. Limits on the relative error in X and D are 10^{-10}, as for the taut line solution.

COMPOUND LEG

A compound leg consists of two simple legs joined together at a spider plate or equalizer, which is connected by a riser to a buoy.

<u>Input Parameters</u>

d	Horizontal anchor separation	ocean floor
D	Depth at origin	ocean floor
C_x	x-coefficient in ocean floor equation	ocean floor
C_z	z-coefficient in ocean floor equation	ocean floor
S_{1A}	Scope of first chain segment	branch A
W_{1A}	Linear weight of first chain segment	branch A
C_{1A}	Weight of first sinker	branch A
S_{2A}	Scope of second chain segment	branch A
W_{2A}	Linear weight of second chain segment	branch A
C_{2A}	Weight of second sinker	branch A
S_{3A}	Scope of third chain segment	branch A
W_{3A}	Linear weight of third chain segment	branch A
S_{1B}	Scope of first chain segment	branch B
W_{1B}	Linear weight of first chain segment	branch B
C_{1B}	Weight of first sinker	branch B
S_{2B}	Scope of second chain segment	branch B
W_{2B}	Linear weight of second chain segment	branch B
C_{2B}	Weight of second sinker	branch B
S_{3B}	Scope of third chain segment	branch B
W_{3B}	Linear weight of third chain segment	branch B
C_3	Weight of spider plate or equalizer	junction and riser
λ_F	Friction coefficient for equalizer	junction and riser
S_4	Scope of riser	junction and riser
W_4	Linear weight of riser	junction and riser

H ... Magnitude of horizontal component of load
φ_H ... Direction of load
R_{TOT} ... Buoy displacement in direction of load
φ_R ... Direction of riser (equals direction of load)
x_{TOT} ... Buoy x-coordinate
z_{TOT} Buoy z-coordinate

potential unknowns

The parameters c_x, c_z and D are not input directly; they are computed from the coordinates of three points that determine the ocean floor as a plane surface defined by the equation $y = c_x x + c_z z$.

Parameters φ_H and φ_R both denote load direction; they are kept separate to simplify user selection of parameters whose values are to be specified. Choices are the same as for the slack line solution of the simple leg:

1. Specify load direction and magnitude: (H, φ_H)
2. Specify load direction and buoy displacement: (R_{TOT}, φ_R)
3. Specify buoy x and z coordinates: (x_{TOT}, z_{TOT})

Thus four of the last six input parameters listed above are input as unknown; all of the other parameters are known. Each choice of known parameters employs a different solution procedure, as will be described later. (In the slack line solution of the simple leg, choices 2 and 3 employ the same solution procedure.)

Output Parameters

L_A	Slack length of branch		branch A
A_{iA}	Angle at node i	$1 \le i \le 6$	branch A
V_{iA}	Vertical component of tension at node i	$1 \le i \le 6$	branch A
T_{iA}	Magnitude of tension at node i	$1 \le i \le 6$	branch A
x_{KA}	Horizontal displacement of segment K	$1 \le K \le 3$	branch A
y_{KA}	Vertical displacement of segment K	$1 \le K \le 3$	branch A
φ_A	Branch direction		branch A
θ_A	Effective ocean floor angle		branch A
L_B	Slack length of branch		branch B
A_{iB}	Angle at node i	$1 \le i \le 6$	branch B
V_{iB}	Vertical component of tension at node i	$1 \le i \le 6$	branch B
T_{iB}	Magnitude of tension at node i	$1 \le i \le 6$	branch B
x_{KB}	Horizontal displacement of segment K	$1 \le K \le 3$	branch B
y_{KB}	Vertical displacement of segment K	$1 \le K \le 3$	branch B
φ_B	Branch direction		branch B
θ_B	Effective ocean floor angle		branch B
L	Slack length of riser		riser
A_j	Angle at node j	$j = 7, 8$	riser
V_j	Vertical component of tension at node j	$j = 7, 8$	riser
T_j	Magnitude of tension at node j	$j = 7, 8$	riser
x_4	Horizontal displacement of riser		riser
y_4	Vertical displacement of riser		riser
φ_F	Direction of maximum ocean floor slope		
θ_F	Effective ocean floor angle in direction of maximum slope		
H_A	Horizontal component of tension on branch A		
H_B	Horizontal component of tension on branch B		
$\hat{L}_c$	Length of coiled portion of coil branch, configurations 2 and 3		

Internal Variables

$\hat{d}$ Distance between anchors along ocean floor.

S_A Scope of branch A.

S_B Scope of branch B.

$X_{E\phi}$ X-coord of junction

$Z_{E\phi}$ Z-coord of junction

$Y_{E\phi}$ Y-coord of junction

$X_{A\phi}$ Horizontal displacement of branch A

$X_{B\phi}$ Horizontal displacement of branch B

$\varphi_{A\phi}$ Direction of branch A

$\varphi_{B\phi}$ Direction of branch B

For configuration having junction on ocean floor and both branches under tension.

θ_H Ocean floor angle in load direction

X_E X-coord of junction

Z_E Z-coord of junction

Y_E Y-coord of junction

X_A Horizontal displacement of branch A

X_B Horizontal displacement of branch B

φ'_A Angle between branch A and riser

φ'_B Angle between branch B and riser

plan view

Y_A Vertical displacement of branch A

Y_B Vertical displacement of branch B

Y_{BUOY} Y-coord of buoy

V_A Vertical component of tension at end of branch A

V_B Vertical component of tension at end of branch B

T_A Magnitude of tension at end of branch A

T_B Magnitude of tension at end of branch B

γ Contact angle for equalizer

Internal Variables (continued)

u_T Unit index for tension branch (configurations 2 and 3)

y_{KA} y-coord of anchor A

y_{KB} y-coord of anchor B

z_{KA} z-coord of anchor A

z_{KB} z-coord of anchor B

L_H Length of hanging portion of coil branch } configuration 3

C_E Weight of hanging portion of coil branch } configuration 3

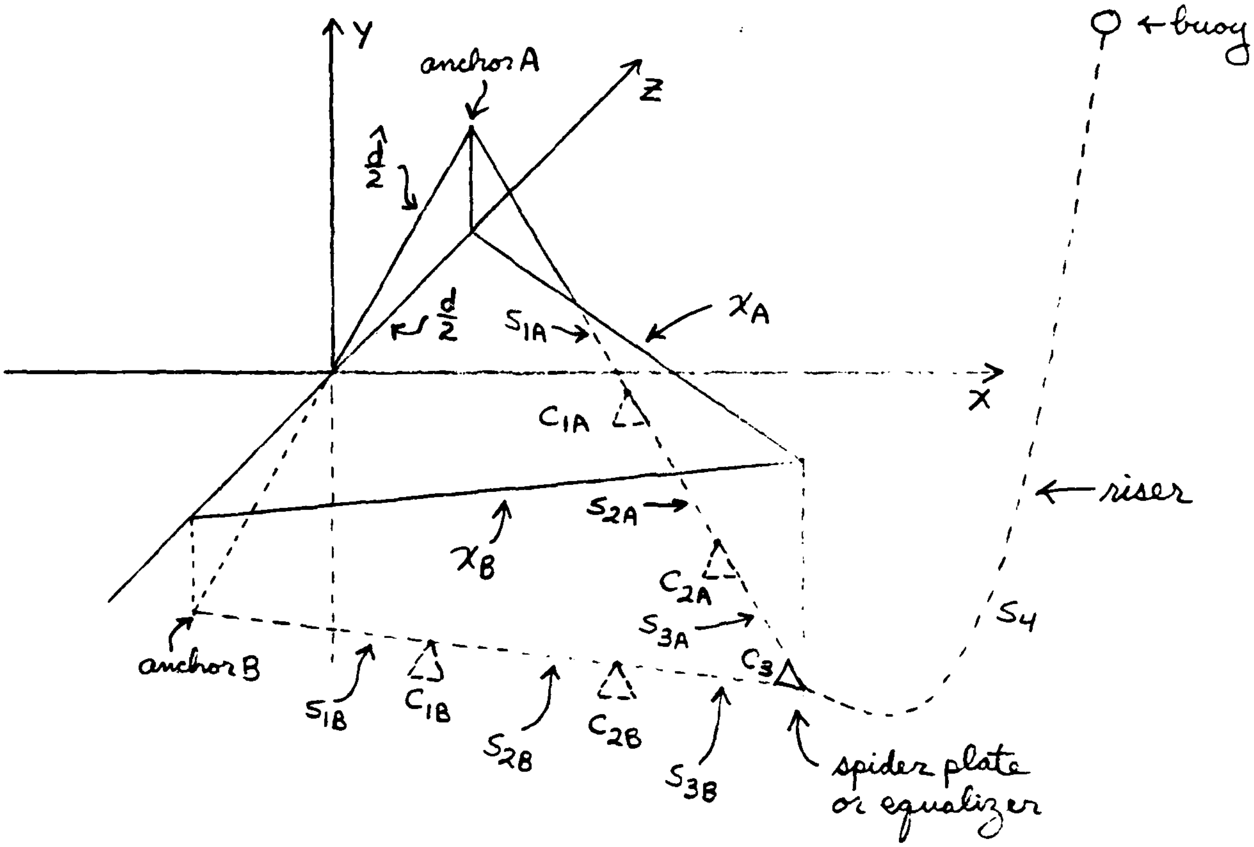
ΔSEC Secant of A_8 minus secant of A_7

ΔTAN Tangent of A_8 minus tangent of A_7

$$y_i^* \begin{cases} \text{y-coord of right end of segment } i, \; i=1,2,3 \\ \text{y-coord of anchor, } i=0 \end{cases}$$

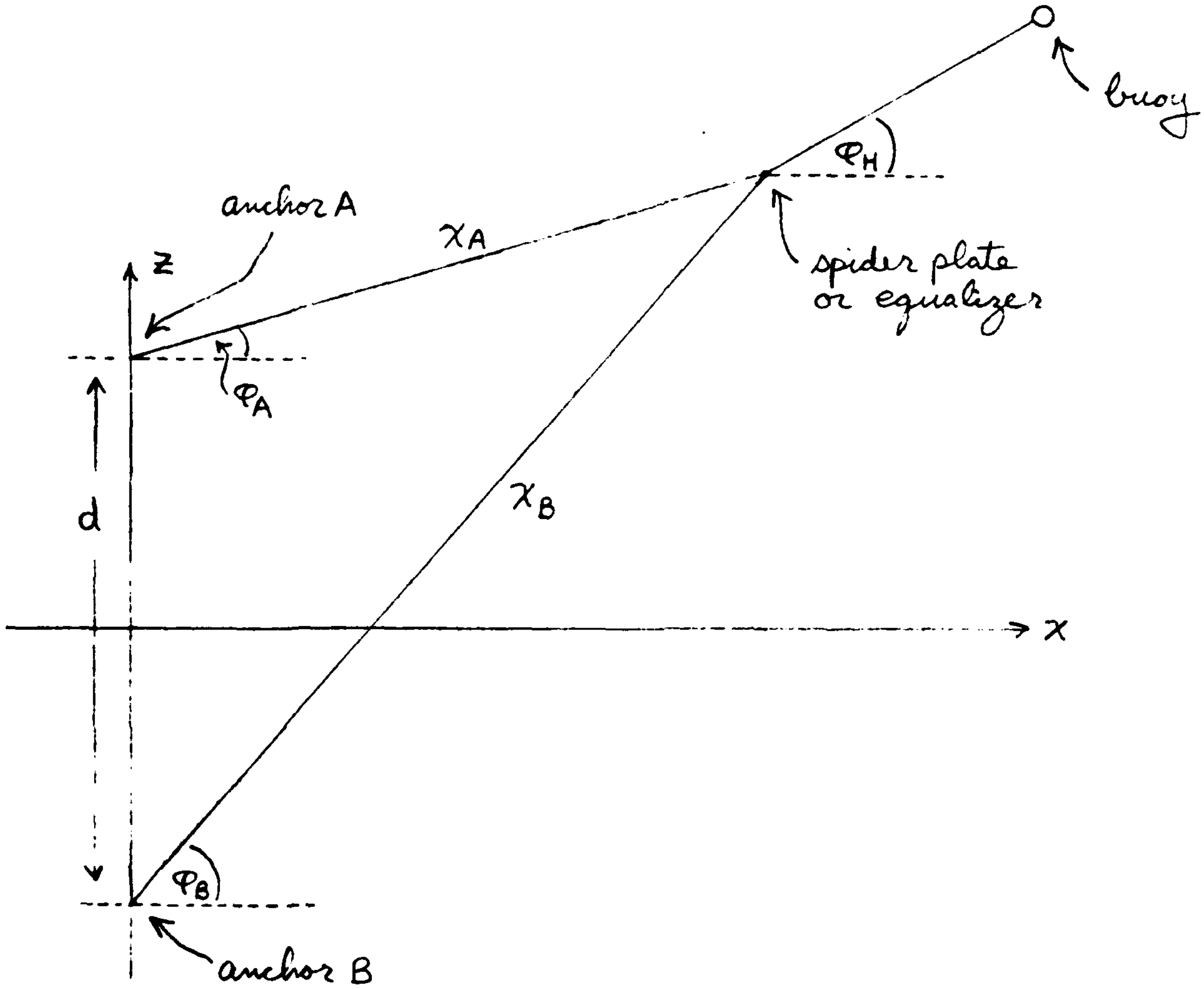
NOTE: For configurations 2 and 3 we will generate additional variables of indeterminate branch by replacing branch symbols (A or B) with "T" (for tension branch) or "C" (for coil branch).

Compound Leg Schematic Diagram (FIGURE 2)

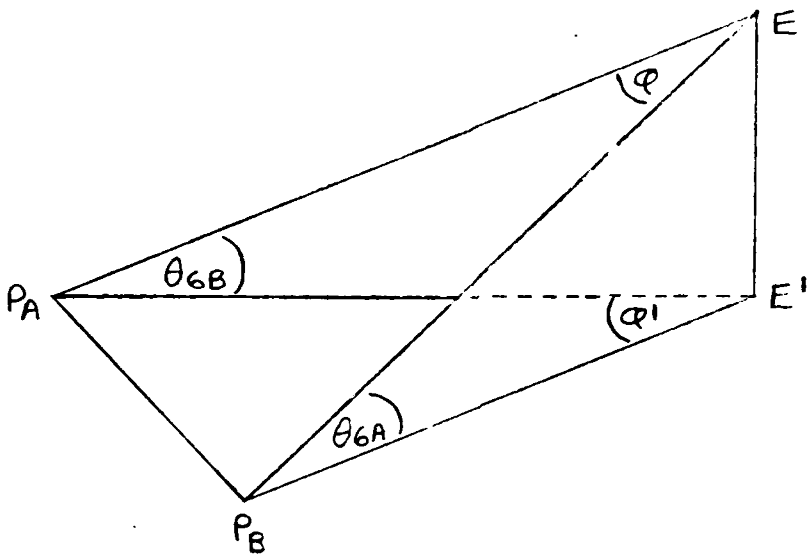


This diagram depicts a leg in configuration type 1: junction on ocean floor, both legs under tension.

Compound Leg Plan View (FIGURE 3)



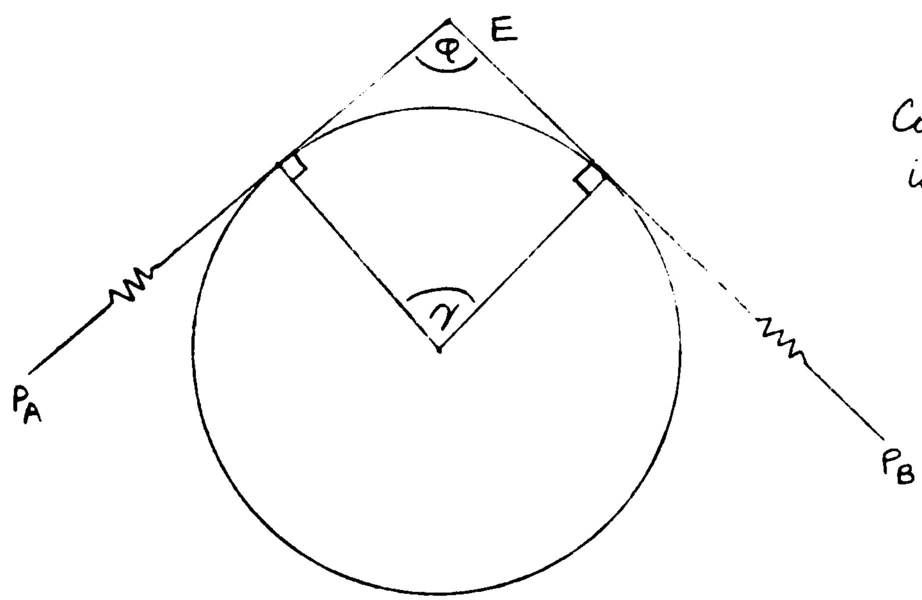
Equalizer Contact Angle (FIGURE 4A)



EP_A and EP_B are lines tangent to the leg branches at the equalizer.
$E'P_A$ and $E'P_B$ are horizontal lines in the branch directions.
Get angle φ by the formula:

$$\cos\varphi = \sin\theta_{6A}\sin\theta_{6B} + \cos\theta_{6A}\cos\theta_{6B}\cos\varphi'$$

Equalizer Contact Angle (FIGURE 4B)



Contact angle γ is given by

$$\gamma = \pi - \varphi$$

Solution Procedure and Equilibrium Equations

There are six general solution procedures, depending on the type of junction (spider plate or equalizer) and the set of known input parameters chosen from the last six in the list ($\{H, \varphi_H\}$ or $\{R_{TOT}, \varphi_R\}$ or $\{x_{TOT}, z_{TOT}\}$). The spider plate solution with $\{H, \varphi_H\}$ given serves as basis for the equalizer solution with $\{H, \varphi_H\}$ given. For each type of junction, the solution with $\{H, \varphi_H\}$ given serves as basis for the other two solutions. These dependency relations are diagrammed below:

$$
\begin{tikzcd}
\text{spider plate } \{H, \varphi_H\} \text{ given} \arrow[r] \arrow[d] \arrow[dd, bend right] & \text{equalizer } \{H, \varphi_H\} \text{ given} \arrow[d] \arrow[dd, bend right] \\
\text{spider plate } \{R_{TOT}, \varphi_R\} \text{ given} & \text{equalizer } \{R_{TOT}, \varphi_R\} \text{ given} \\
\text{spider plate } \{x_{TOT}, z_{TOT}\} \text{ given} & \text{equalizer } \{x_{TOT}, z_{TOT}\} \text{ given}
\end{tikzcd}
$$

All equilibrium equations will be presented in the context of the solution procedure given $\{H, \varphi_H\}$, dealing first with the spider plate solution. We will refer to the spider plate solution given $\{H, \varphi_H\}$ as the <u>basic solution</u>.

__Preliminaries__

Certain variables remain fixed throughout the basic solution procedure. They are computed at the beginning of that procedure, as follows:

(900) $$S_A = S_{1A} + S_{2A} + S_{3A}$$

(901) $$S_B = S_{1B} + S_{2B} + S_{3B}$$

(902) $$\hat{d} = d\sqrt{1 + C_Z^2}$$

(903) $$x_{E\phi} = \frac{1}{2d}\sqrt{\frac{4S_A^2\hat{d}^2 - (S_A^2 + \hat{d}^2 - S_B^2)^2}{1 + C_X^2 + C_Z^2}}$$

(904) $$z_{E\phi} = \frac{1}{2d(1+C_Z^2)}\left(S_B^2 - S_A^2 - C_X C_Z\sqrt{\frac{{}_A^2\hat{d}^2 - (S_A^2 + \hat{d}^2 - S_B^2)^2}{1 + C_X^2 + C_Z^2}}\right)$$

(905) $$y_{E\phi} = C_X x_{E\phi} + C_Z z_{E\phi}$$

(906) $$x_{A\phi} = \sqrt{x_{E\phi}^2 + (z_{E\phi}^2 - \tfrac{d}{2})^2}$$

(907) $$x_{B\phi} = \sqrt{x_{E\phi}^2 + (z_{E\phi}^2 + \tfrac{d}{2})^2}$$

(908) $$\varphi_{A\phi} = \cos^{-1}\left(\frac{x_{A\phi}^2 + d^2 - x_{B\phi}^2}{2d\,x_{A\phi}}\right) - \frac{\pi}{2}$$

(909) $$\varphi_{B\phi} = \frac{\pi}{2} - \cos^{-1}\left(\frac{x_{B\phi}^2 + d^2 - x_{A\phi}^2}{2d\,x_{B\phi}}\right)$$

(910) $$\theta_H = \tan^{-1}\left(\cos(\varphi_H - \varphi_F)\tan\theta_F\right)$$

NOTE: (903) – (909) are applied only if $|S_A - S_B| \le \hat{d}$.

Configuration Types

A compound leg can assume one of four types of configuration, each having a different set of equilibrium equations. The types of configuration are:

1. Junction on ocean floor, both branches under tension
2. Junction on ocean floor, one branch under tension
3. Junction above ocean floor, one branch under tension
4. Junction above ocean floor, both branches under tension

When we say that a branch is "under tension", we mean that the force it exerts on the junction has a nonzero horizontal component. (A branch not under tension will in general exert a nonzero vertical force on the junction.) The non-tense branch of configuration types 2 and 3 will in general have a portion of its length coiled into a point. This coil lies either at the junction (configuration type 2) or directly underneath it (configuration type 3), and its weight is assumed to be entirely supported by the anchor. For configuration type 3, the portion of the branch between the coil and the junction hangs vertically, and is entirely supported by the junction. This part of the branch may include a fractional portion of one of the sinkers.

As for the simple leg, we assume that any negative weight component of a branch, and all components between that component and the end of the branch, must lie entirely above the ocean floor. In addition, we assume that any branch having negative-weight components must be under tension. This implies the following:

1. If either branch has negative-weight components, then configuration is of type 3 or 4.

2. If both branches have negative-weight components, then configuration is of type 4.

General Solution Procedure

In this section we describe in general terms the procedure by which the configuration type is determined and the solution obtained. The equilibrium equations will be presented in subsequent sections.

First, the possible types of configuration are determined according to the presence of negative-weight components. If both branches include negative-weight components, the program goes directly to the solution procedure for configuration type 4. If exactly one branch includes negative-weight components, the program goes directly to the solution procedure for configuration type 3. Assume that neither branch includes negative-weight components. We may rule out configuration type 1 if either of the following is true:

(1) $|S_A - S_B| > \hat{d}$ (Configuration 1 is geometrically impossible.)

(2) $|S_A - S_B| \le \hat{d}$ and ($\varphi_H < \varphi_{A\Phi}$ or $\varphi_H > \varphi_{B\Phi}$)

(Configuration 1 would imply that one of the branches is under compression.)

If either (1) or (2) holds, the program goes directly to the solution procedure for configuration 2. Otherwise, the program attempts a solution of configuration 1.

Configuration 1. Junction and anchors form the vertices of a triangle on the ocean floor having sides S_A, S_B and $\hat{d}$. The junction coordinates ($X_{E\Phi}$, $Z_{E\Phi}$, $Y_{E\Phi}$) were obtained from the preliminary calculations. First the buoy coordinate Y_{BUOY} is computed assuming that the ocean floor supports none of the weight of the junction; this gives the maximum value of Y_{BUOY} possible for configuration 1.

If $Y_{BUOY} < D$ then we have ruled out the possibility of a solution having the junction on the ocean floor. (D is the depth at the origin.) In this case, the program goes to the solution procedure for configuration 3. If $Y_{BUOY} \geq D$, then only the riser parameters (slack length, angles, displacements) remain to be computed. This is done directly, using a formula applicable to a single leg having no sinkers and one weight of chain.

Configuration 2. The following rules are applied to determine which branch is under tension:

1) If $|S_A - S_B| > \hat{d}$ then the shorter branch is under tension.
2) If $|S_A - S_B| \leq \hat{d}$ then the branch under tension is A if $\varphi_H < \varphi_{A\phi}$, B if $\varphi_H > \varphi_{B\phi}$. (Both cannot be true, as we always have $\varphi_{A\phi} < \varphi_{B\phi}$.)

Then the buoy elevation Y_{BUOY} is computed assuming that the ocean floor supports none of the weight of the junction. If $Y_{BUOY} < D$ then we have eliminated the possibility of a solution having the junction on the ocean floor, and the program goes to the solution procedure for configuration 3. If $Y_{BUOY} \geq D$ then the riser parameters are computed by the same method used for configuration 1.

Configuration 3. The program will have selected a branch to be assumed under tension before invoking this procedure. This branch is designated the tension branch; the other branch is designated the coil branch. Buoy elevation Y_{BUOY} is computed assuming that the ocean floor supports none of the weight of the junction. The resulting value of Y_{BUOY} is the minimum possible value of Y_{BUOY}; if $Y_{BUOY} > D$ then we have eliminated the possibility of configuration 3 for the current tension branch. If $Y_{BUOY} \leq D$ then the equilibrium position for the tension branch is found by the one dimensional secant method, treating Y_{BUOY} as a function of the vertical tension V_T at the end of the tension branch. The weight C_E of the hanging portion of the coil branch is added to the weight of the junction when riser parameters are calculated. In some cases, this additional weight includes a fractional part of a sinker; the value of C_E is then computed from the known value of vertical riser displacement. Once the solution of the tension branch is obtained the coil length $\hat{L}_C$ is computed geometrically. If $\hat{L}_C \geq 0$ then we are done; otherwise, there is no configuration 3 solution in the current tension branch. Recall that coil branches may not have negative weight components. If the first tension branch to be attempted fails and is of positive weight, then the program attempts to find a solution having the other branch as tension branch. If all valid choices of tension and coil branch have failed, then the program goes to the procedure for configuration 4.

<u>Configuration 4</u>. Solution is obtained by a two-dimensional Steffensen iteration. Given test values for x_A and x_B, the branch horizontal tensions H_A and H_B can be computed. Solutions for each branch are then obtained by the one-dimensional secant method, treating x_A and x_B as functions of V_A and V_B respectively. This gives a value Δy_{JUNCT} equal to the difference between the elevations of the two branch ends. Riser parameters are then computed, taking the junction elevation as the average of the two branch elevations. This gives a second value Δy_{BUOY} equal to the difference between the buoy elevation and the water level. The two-dimensional Steffensen iteration is applied to solve the system of equations

$$\begin{cases} \Delta y_{JUNCT}(x_A, x_B) = 0 \\ \Delta y_{BUOY}(x_A, x_B) = 0 \end{cases}$$

with the program generating its own initial approximations for x_A and x_B.

Equilibrium Equations

Configuration 1

(1000) $\varphi_A = \varphi_{A\phi}$

(1001) $\varphi_B = \varphi_{E\phi}$

(1002) $\varphi'_A = \varphi_H - \varphi_A$

(1003) $\varphi'_B = \varphi_B - \varphi_H$

(1004) $H_A = \dfrac{H \sin \varphi'_B}{\sin(\varphi'_A + \varphi'_B)}$

(1005) $H_B = \dfrac{H \sin \varphi'_A}{\sin(\varphi'_A + \varphi'_B)}$

(1006) $\theta_A = \tan^{-1}\left(\cos(\varphi_A - \varphi_F)\tan\theta_F\right)$

(1007) $\theta_B = \tan^{-1}\left(\cos(\varphi_B - \varphi_F)\tan\theta_F\right)$

(1008) $x_E = x_{E\phi}$

(1009) $z_E = z_{E\phi}$

(1010) $y_E = y_{E\phi}$

(1011) $\gamma = \pi - \cos^{-1}\left(\sin A_{6A} \sin A_{6B} + \cos A_{6A} \cos A_{6B} \cos(\varphi'_A + \varphi'_B)\right)$

(1012) $A_7 = \tan^{-1}\left(\dfrac{H_A \tan A_{6A} + H_B \tan A_{6B} + C_3}{H}\right)$

(1013) $A_8 = \tan^{-1}\left(\tan A_7 + \dfrac{S_4 W_4}{H}\right)$

(1014) $x_4 = \dfrac{H}{W_4} \ln\left(\dfrac{\tan A_8 + \sec A_8}{\tan A_7 + \sec A_7}\right)$

(1015) $y_4 = \dfrac{H}{W_4}\left(\sec A_8 - \sec A_7\right)$

(1016) $y_{BUOY} = y_E + y_4$

Equation (1011) is used only for equalizer solutions, as discussed in a later section.

Equations (1012)–(1016) are applied only for the determination of minimum buoy elevation. If $Y_{BUOY} \geq D$ then the following equations are applied:

(1100A)	$L_A = S_A$	(1100B)	$L_B = S_B$
(1101A)	$A_{1A} = \theta_A$	(1101B)	$A_{1B} = \theta_B$
(1102A)	$A_{2A} = \theta_A$	(1102B)	$A_{2B} = \theta_B$
(1103A)	$X_{1A} = S_{1A} \cos\theta_A$	(1103B)	$X_{1B} = S_{1B} \cos\theta_B$
(1104A)	$Y_{1A} = S_{1A} \sin\theta_A$	(1104B)	$Y_{1B} = S_{1B} \sin\theta_B$
(1105A)	$A_{3A} = \theta_A$	(1105B)	$A_{3B} = \theta_B$
(1106A)	$A_{4A} = \theta_A$	(1106B)	$A_{4B} = \theta_B$
(1107A)	$X_{2A} = S_{2A} \cos\theta_A$	(1107B)	$X_{2B} = S_{2B} \cos\theta_B$
(1108A)	$Y_{2A} = S_{2A} \sin\theta_A$	(1108B)	$Y_{2B} = S_{2B} \sin\theta_B$
(1109A)	$A_{5A} = \theta_A$	(1109B)	$A_{5B} = \theta_B$
(1110A)	$A_{6A} = \theta_A$	(1110B)	$A_{6B} = \theta_B$
(1111A)	$X_{3A} = S_{3A} \cos\theta_A$	(1111B)	$X_{3B} = S_{3B} \cos\theta_B$
(1112A)	$Y_{3A} = S_{3A} \sin\theta_A$	(1112B)	$Y_{3B} = S_{3B} \sin\theta_B$
(1113A)	$X_A = X_{1A} + X_{2A} + X_{3A}$	(1113B)	$X_B = X_{1B} + X_{2B} + X_{3B}$
(1114A)	$Y_A = Y_{1A} + Y_{2A} + Y_{3A}$	(1114B)	$Y_B = Y_{1B} + Y_{2B} + Y_{3B}$

(1115) $Y_4 = D - Y_E$

(1116) $$L = \max\left\{0, \frac{S_4 - Y_4 \sin\theta_H - \sqrt{(Y_4 - S_4 \sin\theta_H)^2 + 2\frac{H}{W_4}\cos\theta_H (Y_4 - S_4 \sin\theta_H)}}{(\cos\theta_H)^2}\right.$$

(1117) $$A_7 = \begin{cases} \tan^{-1}\left[\frac{1}{2}\left(-\frac{S_4 W_4}{H} + \frac{Y_4 W_4}{H}\sqrt{1 + 4H^2/[W_4^2(S_4^2 - Y_4^2)]}\right)\right] & \text{if } L = 0 \\ \theta_H \text{ if } L > 0 \end{cases}$$

(1118) $$A_8 = \tan^{-1}\left[\tan A_7 + \frac{(S_4 - L)W_4}{H}\right]$$

(1119) $$X_4 = L\cos\theta_H + \frac{H}{W_4}\ln\left(\frac{\tan A_8 + \sec A_8}{\tan A_7 + \sec A_7}\right)$$

Configuration 2

(2000) $$u_T = \begin{cases} 1 & \text{if branch A is under tension} \\ -1 & \text{if branch B is under tension} \end{cases}$$

(2001) $$S_T = \begin{cases} S_A & \text{if branch A is under tension} \\ S_B & \text{if branch B is under tension} \end{cases}$$

(2002) $$S_C = S_A + S_B - S_T$$

(2003) $$z_{KT} = u_T \frac{d}{2}$$

(2004) $$y_{KT} = C_z z_{KT}$$

(2005) $$y_E = y_{KT} + S_T \sin\theta_H$$

Determine maximum buoy elevation by applying

(2006) $$A_7 = \tan^{-1}\left(\tan\theta_H + \frac{C_3}{H}\right)$$

and equations (1013) – (1016).

If $y_{BUOY} \geq D$ then the following equations are applied:

(2020) $$x_T = S_T \cos\theta_H$$

(2021) $$x_E = x_T \cos\varphi_H$$

(2022) $$z_E = z_{KT} + x_T \sin\varphi_H$$

(2023) $$x_C = \sqrt{x_E^2 + (z_{KT} + z_E)^2}$$

(2024) $$L_C = \sqrt{x_C^2 + (2y_{KT} + x_T \tan\theta_H)^2}$$

(2025) $$\varphi_C = u_T\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{d^2 + x_C^2 - x_T^2}{2 d x_C}\right)\right]$$

Assume that branch A is under tension.

Then the remaining equilibrium equations are as follows:

For branch A:

(2030A) $\varphi_A = \varphi_H$

(2031A) $\theta_A = \theta_H$

(2032A) $H_A = H$

(1100A) – (1114A)

For junction and riser:

(2040) $\gamma = \pi$

(1115) – (1119)

For coil:

(2050) $\hat{L}_C = S_C - L_C$

For branch B:

(2030B) $\varphi_B = \varphi_C$

(2032B) $H_B = 0$

(2031B) $\theta_B = \tan^{-1}(\cos(\varphi_B - \varphi_F)\tan\theta_F)$

(2100B) $L_B = L_C$

(2101B) $A_{1B} = \theta_B$

(2102B) $A_{2B} = \theta_B$

(2103B) $x_{1B} = \min(S_{1B}, L_B)\cos\theta_B$

(2104B) $y_{1B} = \min(S_{1B}, L_B)\sin\theta_B$

(2105B) $A_{3B} = \theta_B$

(2106B) $A_{4B} = \theta_B$

(2107B) $x_{2B} = \min(S_{2B}, L_B - S_{1B})\cos\theta_B$

(2108B) $y_{2B} = \min(S_{2B}, L_B - S_{1B})\sin\theta_B$

(2109B) $A_{5B} = \theta_B$

(2110B) $A_{6B} = \theta_B$

(2111B) $x_{3B} = \min(S_{3B}, L_B - S_{1B} - S_{2B})\cos\theta_B$

(2112B) $y_{3B} = \min(S_{3B}, L_B - S_{1B} - S_{2B})\sin\theta_B$

(2113B) $x_B = x_C$

(2114B) $y_B = x_B \tan\theta_B$

If branch B is under tension, then only the branch equations change. They are obtained from the branch equations on the previous page by reversing the roles of the symbols "A" and "B".

Configuration 3

Assume that branch A is under tension. Equations are as follows:

(2000) - (2004) for $u_T, S_T, S_C, Z_{KT}, \gamma_{KT}$.

For branch A, apply (2030A) - (2032A) to get φ_A, θ_A, H_A. Then use equations for slack line solution of simple leg, with these substitutions:

$D \leftarrow \gamma_A$	$C_1 \leftarrow C_{1A}$	$C_2 \leftarrow C_{2A}$	$x \leftarrow x_A$
$B \leftarrow \theta_A$	$S_2 \leftarrow S_{2A}$	$S_3 \leftarrow S_{3A}$	$H \leftarrow H_A$
$S_1 \leftarrow S_{1A}$	$W_2 \leftarrow W_{2A}$	$W_3 \leftarrow W_{3A}$	$V \leftarrow V_A$
$W_1 \leftarrow W_{1A}$			$L \leftarrow L_A$

The simple leg equations used here are those numbered (n00) - (n14) with $1 \le n \le 6$.

Additional parameters of indeterminate branch are determined as follows:

(2021) - (2025) for $x_E, z_E, x_C, L_C, \varphi_C$.

(3020) $$x_T = \begin{cases} x_A & \text{if branch A is under tension} \\ x_B & \text{if branch B is under tension} \end{cases}$$

(3031) $$\gamma_T = \begin{cases} \gamma_A & \text{if branch A is under tension} \\ \gamma_B & \text{if branch B is under tension} \end{cases}$$

(3032) $$L_H = \gamma_T - x_T \tan\theta_H$$

(3033) $$\hat{L}_C = S_C - L_C - L_H$$

For branch B, apply (2030B) – (2032B) to get φ_B, θ_B, H_B.
Then apply (2100B) – (2114B) to get slack length, node angles, and displacements. Vertical displacements are adjusted as follows.

1) Compute node y-coords from vertical displacements:

(3120B) $y_0^* = -y_{KT}$

(3121B) $y_1^* = y_0^* + y_{1B}$

(3122B) $y_2^* = y_1^* + y_{2B}$

(3123B) $y_3^* = y_2^* + y_{3B}$

2) Adjust node y-coords on hanging portion of branch:

(3130B) $y_3^* = y_3^* + \max(0, L_H)$

(3131B) $y_2^* = y_2^* + \max(0, L_H - S_{3B})$

(3132B) $y_1^* = y_1^* + \max(0, L_H - S_{3B} - S_{2B})$

3) Recompute vertical displacements from adjusted node y-coords:

(3140B) $y_{1B} = y_1^* - y_0^*$

(3141B) $y_{2B} = y_2^* - y_1^*$

(3142B) $y_{3B} = y_3^* - y_2^*$

For junction and riser:
Compute weight of hanging portion of coil branch as follows:

(3035)
$$C_E = \begin{cases} L_H W_{3C} & \text{if } L_H < S_{3C} \\ \langle\text{computed from } y_4\rangle & \text{if } L_H = S_{3C} \\ S_{3C} W_{3C} + C_{2C} + (L_H - S_{3C}) W_{2C} & \text{if } S_{3C} < L_H < S_{3C} + S_{2C} \\ \langle\text{computed from } y_4\rangle & \text{if } L_H = S_{3C} + S_{2C} \\ S_{3C} W_{3C} + C_{2C} + S_{2C} W_{2C} + C_{1C} + (L_H - S_{3C} - S_{2C}) W_{1C} & \text{if } S_{3C} + S_{2C} < L_H \end{cases}$$

Contact angle and riser slack length are given by

(3040) $\gamma = A_{6T} + \frac{\pi}{2}$

(3041) $L = 0$

If $L_H \neq S_{3C}$ and $L_H \neq S_{3C} + S_{2C}$ then riser displacements and angles are given by

(3050) $A_7 = \tan^{-1}\left(\tan A_{6T} + \frac{C_E + C_3}{H}\right)$

(1013) – (1015)

If $L_H = S_{3C}$ or $L_H = S_{3C} + S_{2C}$ then riser displacements and angles, and the weight of the hanging portion of the coil branch, are given by

(3060) $\gamma_4 = D - (\gamma_{KT} + \gamma_T)$

(3061) $\Delta SEC = \frac{w_4 \gamma_4}{H}$

(3062) $\Delta TAN = \frac{w_4 S_4}{H}$

(3063) $A_7 = \tan^{-1}\left[\frac{1}{2}\left(-\Delta TAN + \Delta SEC\sqrt{1 + \frac{4}{(\Delta TAN)^2 - (\Delta SEC)^2}}\right)\right]$

(3064) $C_E = H(\tan A_7 - \tan A_{6T}) - C_3$

(1013) – (1015)

Configuration 4

(4000) $\varphi_A = \cos^{-1}\left(\frac{x_A^2 + d^2 - x_B^2}{2 d x_A}\right) - \frac{\pi}{2}$

(4001) $\varphi_B = \frac{\pi}{2} - \cos^{-1}\left(\frac{x_B^2 + d^2 - x_A^2}{2 d x_B}\right)$

(1002) - (1007)

Branch A equations are identical to those used for configuration 3 where it was assumed that branch A is under tension.

To obtain branch B equations, make the substitution $A \leftarrow B$ in the branch A equations.

The remaining equations are as follows:

(4010) $y_B = y_A + C_z d$

(1011) - (1015)

(4020) $D = \frac{1}{2}(y_A + y_B) + y_4$

(4030) $x_E = x_A \cos\varphi_A$

(4032) $z_E = \frac{d}{2} + x_A \sin\varphi_A$

All Configurations

(5000) $x_{TOT} = x_E + x_4 \cos\varphi_H$

(5001) $z_{TOT} = z_E + x_4 \sin\varphi_H$

(5002) $R_{TOT} = x_{TOT} \cos\varphi_H + z_{TOT} \sin\varphi_H$

Equalizer Solution

In this section we describe the solution procedure when the junction is an equalizer, rather than a spider plate, and the direction and magnitude of the load are known. The values input for lengths of the last segment on each branch are regarded as starting lengths. A solution is obtained for the starting position, using the procedure described earlier for the spider plate. The function $F_{TEN} = \begin{cases} T_A - e^{\lambda_F \gamma} T_B & \text{if } T_A \geq T_B \\ T_B - e^{\lambda_F \gamma} T_A & \text{if } T_A < T_B \end{cases}$ is computed. The particular expression that defines F_{TEN} does not change during the solution. If $F_{TEN} \leq 0$ then the equalizer does not move, for the difference in tension is not great enough to overcome friction in the starting position. Otherwise, the equalizer moves toward the beginning of the branch having the lower tension (T_A or T_B) at the junction. Call this branch the "slack branch". The true position of the equalizer is found by a one-dimensional secant iteration, treating F_{TEN} as a function of the length S_S of the slack branch. This iteration gives one of two results:

1. $F_{TEN} > 0$ at the beginning of the slack branch. Then the beginning of the slack branch is the final position of the equalizer.
2. $F_{TEN} = 0$ at some intermediate point along the slack branch. Then the equation

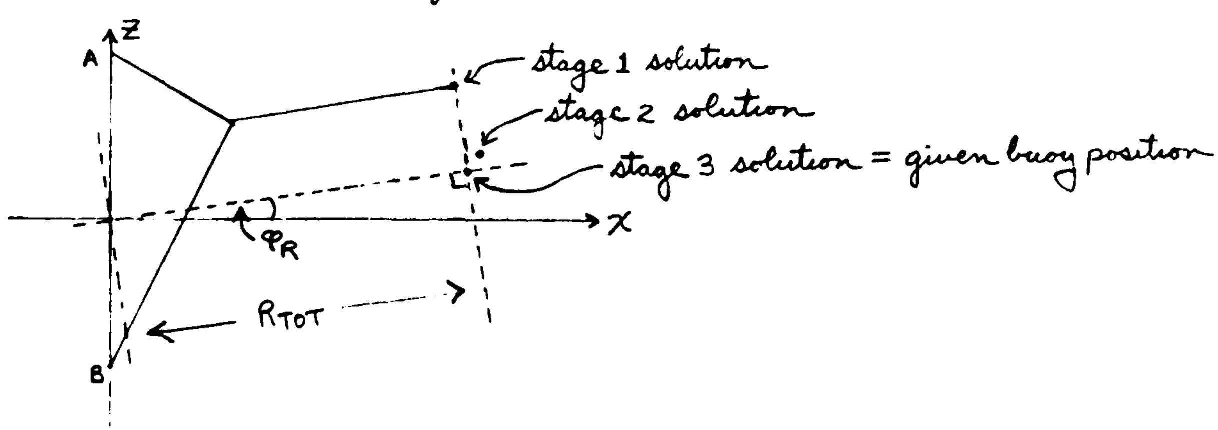
$$\frac{T_A}{T_B} = e^{\pm \lambda_F \gamma}$$ is satisfied in the final position.

Solution Procedures when Load Vector is not Known

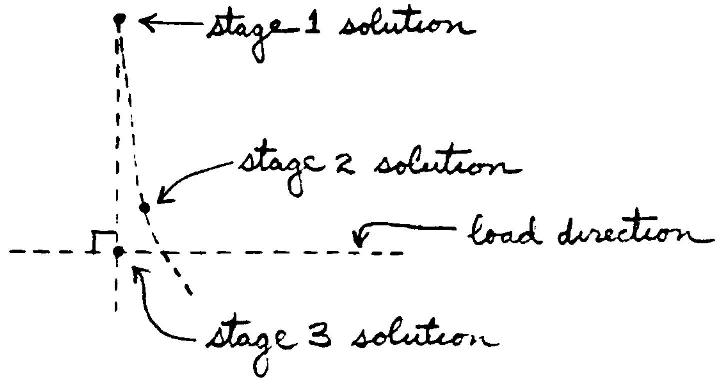
The compound leg solution procedure described in preceeding sections assumes that the vector horizontal load (H and φ_H) is known. Let us call this procedure the basic solution procedure. If the choice of known input parameters is $\{R_{TOT}, \varphi_R\}$ or $\{x_{TOT}, z_{TOT}\}$, then the solution is obtained by testing different values of $\{H, \varphi_H\}$ with the basic solution procedure, comparing the computed values of R_{TOT} or $\{x_{TOT}, z_{TOT}\}$ to the known values.

Case 1. $\{R_{TOT}, \varphi_R\}$ given. Since φ_R is just another name for φ_H, the problem is to find the correct value of H. The one-dimensional secant method is applied to the equation $R_{TOT}(H) = R_{TOT}^{(KNOWN)}$. The initial value of H for the iteration is obtained by modeling the leg by a simple leg of the same total weight and approximately the same length. The iteration is terminated when the relative error in R_{TOT} is no greater than 10^{-10}.

Case 2. $\{x_{TOT}, z_{TOT}\}$ given. The solution is obtained in three stages. In the first stage, R_{TOT} and φ_R are assigned the values $R_{TOT} = \sqrt{x_{TOT}^2 + z_{TOT}^2}$, $\varphi_R = \tan^{-1}\left(\frac{z_{TOT}}{x_{TOT}}\right)$, and the solution procedure of Case 1 is applied. As a rule, the buoy coordinates of the stage 1 solution are close to the known values of x_{TOT} and z_{TOT}, but not equal to them (see figure below).



In the second stage, the load vector is resolved into components parallel and normal to the riser direction φ_R. The parallel component is held fixed while the normal component is varied in an iterative procedure (one-dimensional secant method) to bring the computed buoy position closer to its true value. Usually only two or three iterations are carried out during this stage. The curved dotted line in the figure below depicts the path travelled by the buoy as the normal component of load is varied. Upon completion of this stage, the load vector is decomposed into its components H_X and H_Z with respect to the coordinate axes.



In the third stage, the exact solution is obtained by applying the two-dimensional Steffensen's method to the system of equations

$$\begin{cases} X_{TOT}(H_X, H_Z) = X_{TOT}^{(KNOWN)} \\ Z_{TOT}(H_X, H_Z) = Z_{TOT}^{(KNOWN)} \end{cases}$$

The point (H_X, H_Z) returned by the second stage solution serves as the initial test point for this iteration. The third stage iteration is terminated when the relative error in the buoy displacement vector is no greater than 10^{-6}.

The initial slippage for an equalizer is reset to its original value each time the basic solution procedure is called. This is done in the Case 1 solution procedure, and in all three stages of the Case 2 solution procedure.

Compound Leg Tension Equations

Tensions at the branch nodes are computed as follows.

Case 1. Junction on floor. The first step is to compute branch tensions at the junctions. The force T_J that the junction exerts on the branches is given by

(6000) $$T_J = \frac{H}{\cos A_7} \cos(A_7 - \theta_H) - (C_3 + W_4 L) \sin\theta_H$$

Suppose that both branches are under tension.
The force T_J must be split between the branches. This force is assumed to act in the direction obtained by projecting the horizontal load vector $\vec{H}$ onto the ocean floor. A vector $\vec{V}_T$ in that direction is given by

(6001) $$\vec{V}_T = (\cos\varphi_H, \sin\varphi_H, \tan\theta_H)$$

Coordinate axes within the ocean floor are $\hat{z}$ (pointing from anchor B to anchor A) and $\hat{x}$ (normal to $\hat{z}$). Their unit vectors are given by

(6002) $$\vec{u}_{\hat{x}} = \frac{1}{\sqrt{(1+C_z^2)(1+C_x^2+C_z^2)}} (1+C_z^2, -C_x C_z, C_x)$$

(6003) $$\vec{u}_{\hat{z}} = \frac{1}{\sqrt{1+C_z^2}} (0, 1, C_z)$$

To determine branch tensions at the junction, the direction $\hat{\varphi}_H$ of $\vec{V}_T$ with respect to $\hat{x}$ is computed (counterclockwise from $\hat{x}$):

(6004) $$\hat{\varphi}_H = \begin{cases} \tan^{-1}\left(\dfrac{\vec{V}_T \cdot \vec{u}_{\hat{z}}}{\vec{V}_T \cdot \vec{u}_{\hat{x}}}\right) & \text{if } \vec{V}_T \cdot \vec{u}_{\hat{x}} \neq 0 \\ \dfrac{\pi}{2} & \text{if } \vec{V}_T \cdot \vec{u}_{\hat{x}} = 0 \text{ and } \vec{V}_T \cdot \vec{u}_{\hat{z}} > 0 \\ -\dfrac{\pi}{2} & \text{if } \vec{V}_T \cdot \vec{u}_{\hat{x}} = 0 \text{ and } \vec{V}_T \cdot \vec{u}_{\hat{z}} < 0 \end{cases}$$

(6005) $$\frac{\vec{V}_T \cdot \vec{u}_{\hat{z}}}{\vec{V}_T \cdot \vec{u}_{\hat{x}}} = \sqrt{1+C_x^2+C_z^2} \; \frac{\sin\varphi_H + C_z \tan\theta_H}{(1+C_z^2)\cos\varphi_H - C_x C_z \sin\varphi_H + C_x \tan}$$

Branch directions $\hat{\varphi}_A$ and $\hat{\varphi}_B$ within the ocean floor plane, counterclockwise from $\hat{x}$, are computed by

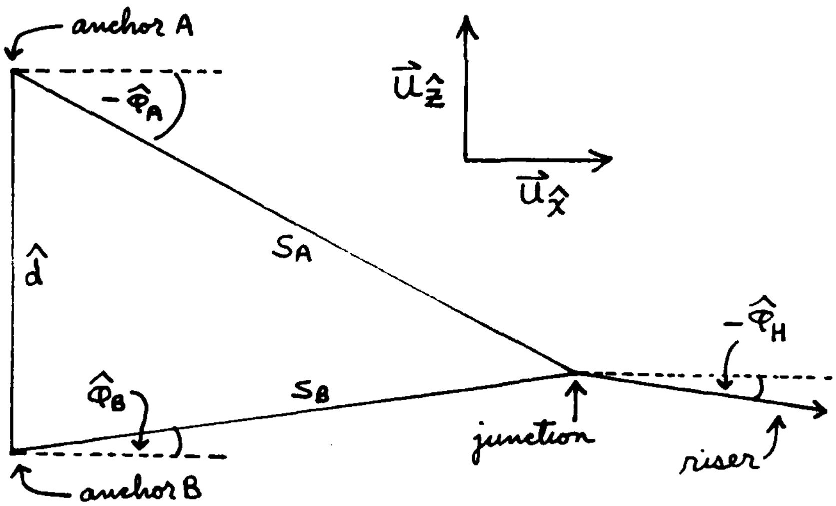
(6006) $$\hat{\varphi}_A = \cos^{-1}\left(\frac{\hat{d}^2 + S_A^2 - S_B^2}{2\hat{d}\, S_A}\right) - \frac{\pi}{2}$$

(6007) $$\hat{\varphi}_B = \frac{\pi}{2} - \cos^{-1}\left(\frac{\hat{d}^2 + S_B^2 - S_A^2}{2\hat{d}\, S_B}\right)$$

Branch tensions at the junction are then given by

(6008) $$T_A = \frac{T_J \sin(\hat{\varphi}_B - \hat{\varphi}_H)}{\sin(\hat{\varphi}_B - \hat{\varphi}_A)}$$

(6009) $$T_B = \frac{T_J \sin(\hat{\varphi}_H - \hat{\varphi}_A)}{\sin(\hat{\varphi}_B - \hat{\varphi}_A)}$$



If only one of the branches is under tension, then branch tensions at the junction are given by

(6010) $T_T = T_J$ (tension branch)

(6011) $T_C = 0$ (coil branch)

After the branch tensions at the junction have been computed, tensions at the branch nodes are given as follows.

6115A $\quad T_{1A} = T_A - (w_{1A}S_{1A} + C_{1A} + w_{2A}S_{2A} + C_{2A} + w_{3A}S_{3A})\sin\theta_A$

6116A $\quad T_{2A} = T_A - (C_{1A} + w_{2A}S_{2A} + C_{2A} + w_{3A}S_{3A})\sin\theta_A$

6117A $\quad T_{3A} = T_A - (w_{2A}S_{2A} + C_{2A} + w_{3A}S_{3A})\sin\theta_A$

6119A $\quad T_{4A} = T_A - (C_{2A} + w_{3A}S_{3A})\sin\theta_A$

6120A $\quad T_{5A} = T_A - w_{3A}S_{3A}\sin\theta_A$

6121A $\quad T_{6A} = T_A$

for branch A, and 6115B – 6121B for branch B.
Equations 6115B – 6121B are obtained from 6115A – 6121A by replacing the subscript "A" with the subscript "B".

Vertical tensions are given by

$$\left.\begin{array}{lll} \text{6121A} - \text{6126A} & & V_{iA} = T_{iA}\sin\theta_A \\ \text{6121B} - \text{6126B} & & V_{iB} = T_{iB}\sin\theta_B \end{array}\right\} \quad 1 \le i \le 6$$

Case 2. Junction above floor, both branches under tension.
Then the simple leg tension equations $n15$ – $n20$, $1 \le n \le 6$, for tension, and 21 – 26 for vertical tension, are applied with the following substitutions.

For branch A:

$$\begin{array}{llll} B \leftarrow \theta_A & V \leftarrow V_A & T_i \leftarrow T_{iA} & S_K \leftarrow S_{KA} \\ H \leftarrow H_A & VC_i \leftarrow VC_{iA} & V_i \leftarrow V_{iA} & C_j \leftarrow C_{jA} \\ L \leftarrow L_A & & A_i \leftarrow A_{iA} & \end{array}$$

where $1 \le i \le 6$; $1 \le K \le 3$; $j = 1, 2$.

For branch B: Same substitutions as for branch A, using subscript "B" rather than subscript "A".

Case 3. Junction above floor, one branch under tension.

For the tension branch, the tension equations of Case 2 apply. For the coil branch, tensions are computed as follows. First, let $VC\phi_i$ denote the weight between node i and the junction:

(6301) $VC\phi_1 = S_{1c}W_{1c} + C_{1c} + S_{2c}W_{2c} + C_{2c} + S_{3c}W_{3c}$

(6302) $VC\phi_2 = C_{1c} + S_{2c}W_{2c} + C_{2c} + S_{3c}W_{3c}$

(6303) $VC\phi_3 = S_{2c}W_{2c} + C_{2c} + S_{3c}W_{3c}$

(6304) $VC\phi_4 = C_{2c} + S_{3c}W_{3c}$

(6305) $VC\phi_5 = S_{3c}W_{3c}$

(6306) $VC\phi_6 = 0$

Recall that C_E denotes that part of the coil branch weight supported by the junction. For each node i we get T_i and V_i as follows.

$$\left.\begin{aligned} &(6310)\quad T_i = -(VC\phi_i - C_E)\sin\theta_H \\ &(6311)\quad V_i = T_i \sin\theta_H \end{aligned}\right\} \quad \text{if } VC\phi_i \geq C_E \quad \text{(node on floor)}$$

$$\left.\begin{aligned} &(6320)\quad T_i = C_E - VC\phi_i \\ &(6321)\quad V_i = T_i \end{aligned}\right\} \quad \text{if } VC\phi_i < C_E \quad \text{(node above floor)}$$

In all three cases, tensions for the riser are given by

(6501) $T_7 = \dfrac{H}{\cos A_7} - W_4 L \sin\theta_H$ (6511) $V_7 = T_7 \sin A_7$

(6502) $T_8 = \dfrac{H}{\cos A_8}$ (6512) $V_8 = T_8 \sin A_8$

END

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