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VESSEL NAVIGATION SYSTEM SIMULATION VOLUME I TECHNICAL

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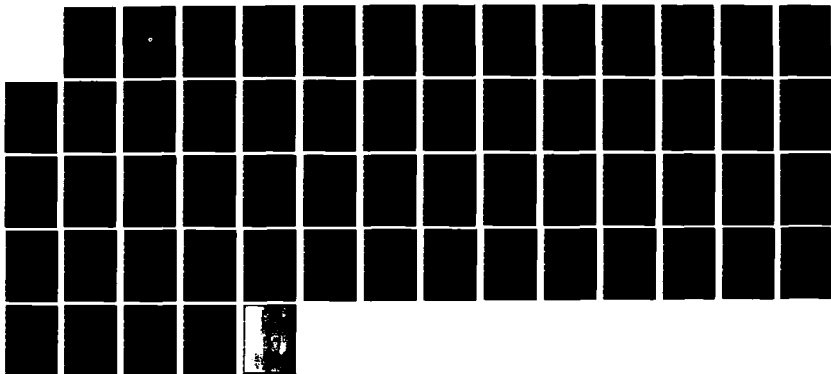
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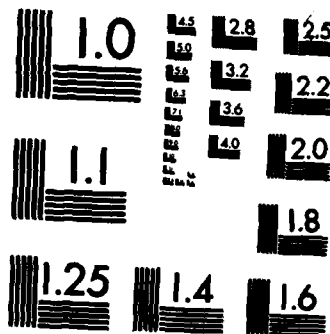
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VESSEL NAVIGATION SYSTEM SIMULATION

VOLUME I: TECHNICAL DESCRIPTION

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16. Abstract This Final Report discusses the mathematical formulation, computer program implementation, and examples of the Vessel Navigation System Simulation (VNSS). The VNSS simulates piloting a vessel in a restricted waterway, with channel banks, current, and obstacles. The techniques of modern optimal control theory is used to derive the control history and resultant vessel trackline, using USCG supplied code definition for vehicle dynamics.					
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1. INTRODUCTION

Delta Research, Inc., recognizes the need for and the objectives of the continuing coordinated program established by the United States Coast Guard (USCG) to investigate basic incident causes and to derive analysis techniques in order to assess the risk of Collision, Ramming, and Grounding (CRG) in restricted waterways, with the ultimate goal of increased safety in harbor areas and restricted inland waterways. Previous studies¹ in the Coast Guard program have conducted exhaustive analysis on incident records to determine basic causes of and contributing factors for incidents. Other studies² have generalized these causes into 14 generic classes and have initiated human factors research in merchant vessels casualty through task analysis of bridge personnel. Additional early work^{3,4,5} was done in analytic modelling of ship collisions including human response. In addition, extensive data collection into pilot decision-making⁶ has been sponsored. Analysis of critical incidents⁷⁻¹⁰ also sheds light on pilot decision processes.

Based on these past elements of the Coast Guard program, a sufficient data base exists to extend the previous analysis by developing and implementing a computer model to simulate the vessel navigation system, including vessel hydrodynamics, navigation information systems, environmental disturbances, and human operator behaviors. This facility will allow the Coast Guard to extend the scope of their analysis beyond existing accident records without the time and cost of utilizing the Kings Point (CAORF), or other ship maneuvering simulator.^{11,12}

This report documents the activities toward this goal performed by Delta Research, Inc., under Contract No. DTICG23-80-C-20036 for the contract duration expiring February 3, 1982. This activity has culminated in delivery

of the Vessel Navigation System Simulation (VNSS) to the USCG, installed on the USCG PDP-11/34 computer. This simulation generated the vessel control history and resultant trackline for navigation of a defined waterway with a given vessel.

The problem of generating this control history and trackline has been formulated, solved, and implemented in the VNSS using the techniques of modern control theory as a basis. Modern control theoretic approaches have been utilized previously to derive autopilot models based on the so-called linear optimum regulator approach, wherein the optimal control for a linearized system is formulated to minimize a penalty function, defined as the distance squared, from a preassigned or nominal trackline. The solution form of this problem definition is a linear feedback law where the control function at any time is a linear combination of the instantaneous vessel state. This linear feedback control is implemented through a feedback "gain" matrix, wherein the control is derived as the product of the gain matrix and the vessel state vector.¹³ The gain matrix is found as the solution of the matrix Riccati equation, solved backwards in time. Due to the feedback nature of the solution, there is no anticipatory characteristics to the solution, i.e., the control is dependent only on the current vessel state and preassigned trackline at that time. The actual navigation problem is then relegated to the creation of the nominal trackline to take into account the time lag in vessel response, with any anticipatory behavior built into the nominal trackline definition.

The optimal control formulation utilized to construct the VNSS differs from this approach in that a true optimal trajectory is derived rather than a linearized optimal trackline following trajectory. The problem has thus been defined as a nonlinear constrained optimization problem where the control

function is derived such that the resultant trackline is optimal in the sense that it minimizes an objective function related to the channel definition rather than to a preconceived trackline. This form of solution does not lead to a feedback control law, but rather to a control law that essentially assesses the future behavior of the system and derives the control policy accordingly. This property gives the control definition the proper anticipatory behavior wherein control action, viz., rudder angle, is applied prior to a channel bend to allow the vessel to acquire a yaw rate and drift angle to properly execute the turn.

The particulars of the VNSS that implement this control solution to the defining channel boundary are discussed in detail in the appropriate Programmers' and Users' manuals; this report discusses the underlying theory and the general capabilities of the VNSS. Section 2 presents the mathematical formulation of the navigation scenario as an optimal control problem in both a general mathematical discussion and a discussion relating to this particular problem with appropriate equations and variable definitions. Section 3 discusses the capabilities of the implemented VNSS simulation, along with the scenario limitations and program restrictions.

Section 4 presents several examples of problem solutions using the VNSS as installed on the USCG PDP-11/34 computer including a real segment of the Mississippi River, an artificially defined river segment, and a solution to an arbitrary geometric problem definition. Finally, Section 5 presents recommendations for future activities in areas of program efficiency, program capability extension, and further applications of the general problem methodology.

Delta Research, Inc., believes that this application of modern control theoretical techniques to the problem of vessel navigation in a restricted

waterway introduces a powerful mathematical technique with a potentially wide range of applications to USCG and other maritime problems. The implementation in the VNSS is a first step to accommodate a general channel and vessel scenario description with which to formulate and solve the vessel control problem. Delta Research, Inc., believes that the high degree of success achieved in modelling vessel passage through the restricted channels, as shown in the examples presented, is indicative of the power and general utility of this solution methodology.

2. MATHEMATICAL REPRESENTATION

2.1 Optimal Control Problem Formulation

The general optimal control problem is formulated as a constrained optimization problem; specifically, a control function for a dynamic system is to be found such that the resultant trajectory minimizes some objective function. It should be pointed out that the control function and the trajectory are time functions, and the objective function to be minimized is the time integral of a scalar function of the trajectory and of the control.

Throughout this report, a state variable definition of the trajectory and control function is assumed. The system equations can then be written as:

$$\dot{x}(t) = a[x(t), u(t)] \quad (1)$$

where $x(t)$ is the state of the system, $u(t)$ is the control input, and the function $a[x(t), u(t)]$ represents the dynamic differential equations of motion of the system. For the vessel control problem, $a[x(t), u(t)]$ represents the differential equations of motion of the appropriate vessel.

The objective function is the time integral of a scalar function of the state and control and can be written as:

$$J(u) = h[x(t_f)] + \int_{t_0}^{t_f} g[x(t), u(t)] dt \quad (2)$$

In addition to the time integral, a contribution to the objective function due to the final system state is considered. It is assumed that the initial state $x(t_0)$ is specified.

In general, the final time t_f need not be specified; a final state $x(t_f)$, a terminal manifold defined by $m[x(t)] = 0$, or a target point $x(t_f) = \theta(t)$ can be specified. These alternative stopping criteria give rise to the so-called transversality conditions which, while mathematically precise, do not readily yield to the numerical techniques required to solve the optimal control problem

for the realistic equations of motion used. Thus, for this application, the final time t_f is considered to be a fixed value specified in the problem specifications.

The optimal control formulation of the vessel control problem is then to find the optimal control $u^*(t)$ that causes the system:

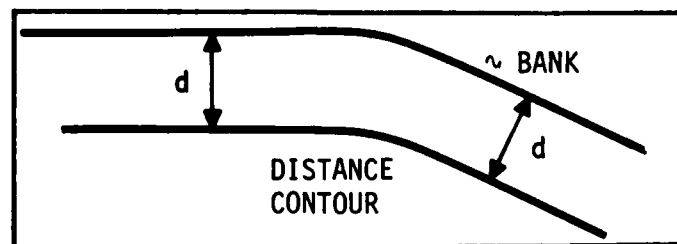
$$\dot{x}(t) = a[x(t), u(t)] \quad (3)$$

to follow an optimal trajectory $x^*(t)$ that minimizes the objective function

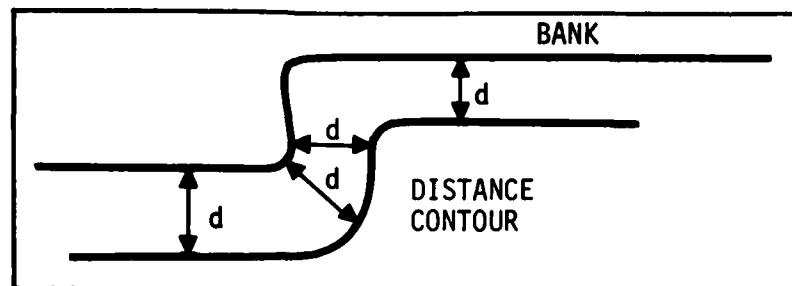
$$J(u) = h[x(t_f)] + \int_{t_0}^{t_f} g[x(t), u(t)] dt \quad (4)$$

where the initial state $x(t_0)$, and the initial and final times, t_0 and t_f are specified.

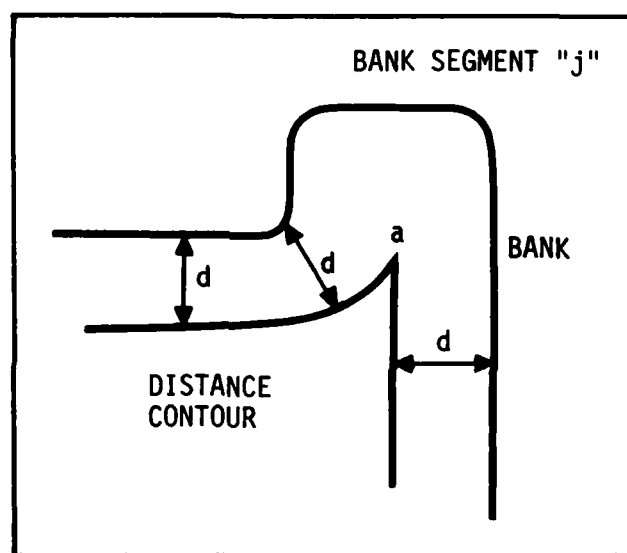
The choice of objective function is central to the problem formulation. For the problem of navigation within a confined waterway, the objective function should penalize trajectories that deviate from the channel. The general measure of position within the channel is characterized by the distance d_1 and d_2 representing the distances from the opposing boundary channels. These generalized distance functions, while conceptually clear, can be less than mathematically well-defined for an arbitrary, convoluted channel with sharp bends, coves, and bank indentations. The ensuing discussion assumes, however, that these distances from the banks, d_1 and d_2 , can be and are well defined. The term well defined refers to the distance to the bank as being a function of the local bank position only. For the simple channel bend, the distance contour of distance "d" is shown below:



Such a bank configuration is called well defined. A different bend, still well defined, is shown below:



Addition of another bank segment to create a "cove" is shown below:



Here, the contour at point "a" is independent of the next local segment, "j", and depends instead on the next segment. Mathematically, a "well defined" channel is one in which a smooth bank, i.e., a continuous bank with continuous directional derivatives, yields smooth distance contours. A "non-well defined" channel is one in which smooth bank yields distance contours that are not smooth. This extension of the requirement for well-defined distances in a linear spline definition leads to some of the channel definition constraints in the implemented VNSS, as defined in Section 3.

In any case, the penalty function forming the kernel of the integral for $J(u)$, $g[x(t), u(t)]$, can be constructed using these distance functions $d_1[x(t)]$, $d_2[x(t)]$. For the problem as formulated herein, it is assumed that the penalty function $g[x(t), u(t)]$ is a function of the state $x(t)$ only; i.e., it is not explicitly a function of the control $u(t)$ and is written $g[x(t)]$.

Various penalty functions were investigated for use in this problem formulation. Specific forms included an inverse square formulation penalizing severely for approaching the channel boundaries; i.e.:

$$g[x(t)] = \frac{1}{d_1[x(t)]^2} + \frac{1}{d_2[x(t)]^2} \quad (5)$$

An alternative form minimizes the sum of the squares of the distance from the boundaries, balancing the effect of the channel definition; i.e.:

$$g[x(t)] = d_1[x(t)]^2 + d_2[x(t)]^2 \quad (6)$$

The solution methodology utilizes a discrete time approximation to the continuous system with the vessel state, $x(t)$, represented by a series of points, $x(t_i)$, $i = 1, \dots, N$, and the penalty functions represented by discrete summations:

$$\sum_{i=1}^N g[x(t_i), u(t_i)]$$

The inverse square formulation, therefore, can be dominated by the value $x(t_i)$ that is closest to the bank. Additionally, a minor change in position of this point can alter the penalty function by orders of magnitude under some conditions. In the iterative solution utilized, these wide variations in penalty function, due to small changes in position from iteration to iteration, create serious problems with control of convergence rate of the solution and can, in fact, cause instabilities in the numerical technique.

Also, the inverse square formulation requires that the trackline never cross the channel boundaries, resulting in a problem of properly selecting the initial trajectory in the iterative process so as to be totally contained within the channel. Alternatively, complex methodologies for moving channel boundaries to encompass the trajectory and to finally converge to the desired channel are required. Based on these implementational considerations, and based on experiments and investigations yielding closely equivalent behaviors with the quadratic distance from the boundaries penalty function, the latter was chosen for inclusion in the VNSS.

2.2 General Problem Solution

The general problem as defined above, e.g., find $u^*(t)$ to minimize:

$$J(u) = h[x(t_f)] + \int_{t_0}^{t_f} g[x(t), u(t)] dt \quad (7)$$

with $x(t)$ related to $u(t)$ through:

$$\dot{x}(t) = a[x(t), u(t)] \quad (8)$$

is solved through application of generalized calculus of variations. This technique introduces a set of functional Lagrange multipliers called co-state variables or adjoint variables, donated herein by $p(t)$. The adjoint variables $p(t)$ have the same dimensionality as the state equations $x(t)$; viz., if (as in VNSS case) there are 6 state variables $x(t)$, there are 6 adjoint variables $p(t)$.

The calculus of variations formulation then considers an augmented penalty function called the Hamiltonian which is defined as:

$$H[x(t), u(t), p(t)] = g[x(t), u(t)] + p^T(t) \{a[x(t), u(t)]\} \quad (9)$$

The necessary conditions for a control function $u^*(t)$ to yield an optimal trajectory $x^*(t)$ are then:

$$\dot{x}^*(t) = \frac{\partial H}{\partial p} [x^*(t), u^*(t), p^*(t)] \quad (10)$$

$$\dot{p}^*(t) = -\frac{\partial H}{\partial x} [x^*(t), u^*(t), p^*(t)] \quad (11)$$

$$0 = \frac{\partial H}{\partial u} [x^*(t), u^*(t), p^*(t)] \quad (12)$$

It can be noted that the first set of equations for $\dot{x}^*(t)$ represents an identity with respect to the state equations of motion, the second set defines the adjoint variables, and the third represents a gradient stationarity condition.

This set of coupled differential equations have the boundary conditions, for the fixed final time problem as previously defined, with the initial state conditions:

$$x^*(t_0) = x(t_0) \quad (13)$$

and final adjoint conditions:

$$p^*(t_f) = \frac{\partial h}{\partial x} [x^*(t_f)] \quad (14)$$

This set of coupled differential equations, with part of the boundary conditions specified at the initial time and the rest of the boundary conditions specified at the final time, is referred to as a two-point boundary value problem (TPBVP) and, with only certain exceptions, is not amenable to analytic solution.

For any specific case, except for the particular exceptions which are inapplicable to this problem, the TPBVP must be solved using a numerical technique.

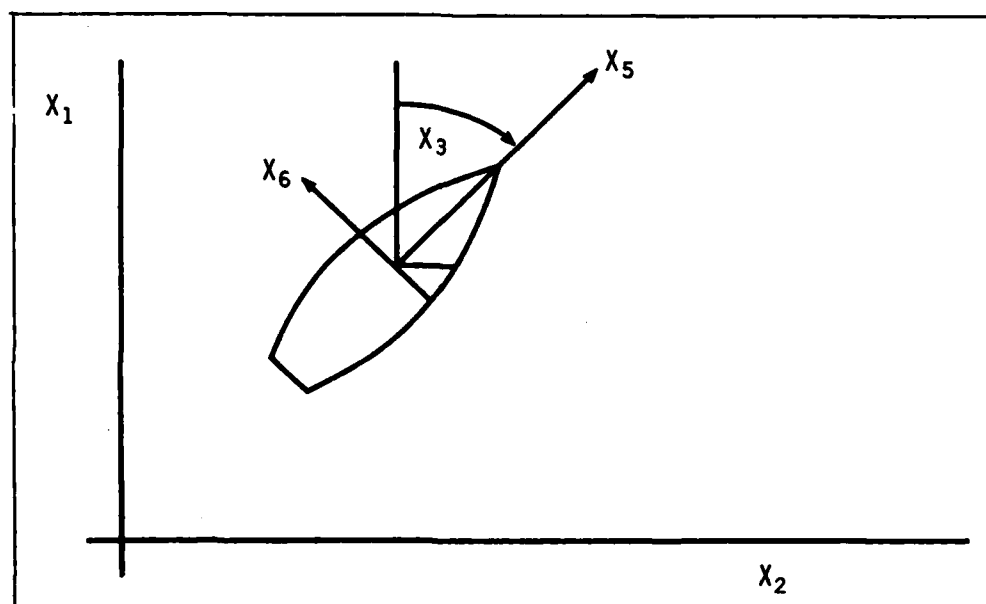
2.3 VNSS Problem Specifics

The general problem formulation and solution defined above will be applied to the specific problem of vessel navigation within a restrictive waterway. The system is represented by a 6-state variable set of differential equations. The

states are defined with respect to an x-y scene definition shown in Figure 2-1 with:

- x_1 - "y" component of vessel position
- x_2 - "x" component of vessel position
- x_3 - vessel heading angle (ofttimes called ψ)
- x_4 - yaw rate (ofttimes called r)
- x_5 - axial or surge velocity (ofttimes called u)
- x_6 - lateral or sway velocity (ofttimes called v)

FIGURE 2-1. STATE VARIABLE DEFINITION



With this definition, the state differential equations are given by:

$$\dot{x}_1 = x_5 \cos x_3 + x_6 \sin x_3 \quad (15)$$

$$\dot{x}_2 = x_5 \sin x_3 - x_6 \cos x_3 \quad (16)$$

$$\dot{x}_3 = x_4 \quad (17)$$

$$\dot{x}_4 = f_4(x_4, x_5, x_6, \delta, s) \quad (18)$$

$$\dot{x}_5 = f_5(x_4, x_5, x_6, \delta, s) \quad (19)$$

$$\dot{x}_6 = f_6(x_4, x_5, x_6, \delta, s) \quad (20)$$

The first pair of equations, $\dot{x}_1$ and $\dot{x}_2$, represent the velocity resolution of x_5 and x_6 , the $\dot{x}_3$ equation is a definition, and the last three equations, $\dot{x}_4$, $\dot{x}_5$, and $\dot{x}_6$, represent the vessel torque and force equations. These equations are defined by the USCG supplied code modules¹⁴ and are considered as "black box" definitions of the vessel motion. The control variables, δ and s , are the rudder angle and propeller RPM, respectively. These functions are evaluated by exercising these code modules and derivatives, where required, are evaluated by finite difference approximations using these code modules.

The Hamiltonian, then, is given by:

$$F(x_1, x_2) + p_1 \dot{x}_1 + p_2 \dot{x}_2 + p_3 \dot{x}_3 + p_4 \dot{x}_4 + p_5 \dot{x}_5 + p_6 \dot{x}_6 \quad (21)$$

The penalty function is chosen as the sum of the squared distances from the banks, scaled by the width of the channel; i.e.:

$$F(x_1, x_2) = \frac{d_1(x_1, x_2)^2 + d_2(x_1, x_2)^2}{w} \quad (22)$$

where w is the channel width at the appropriate point (x_1, x_2) as determined from the channel definition. The width scaling is included to force the trajectory or trackline closer to the channel center where the channel is narrower. The function $F(x_1, x_2)$ and its derivatives are calculated as discussed in the VNSS Program Manual, Volume II, Programmers' Guide. The form of the calculation is related to the specific implementation of channel boundaries and of the algorithm defining the "nearest boundary."

With these definitions, the Hamiltonian becomes:

$$H = F(x_1, x_2) + p_1(x_5 \cos x_3 + x_6 \sin x_3) + p_2(x_5 \sin x_3 - x_6 \cos x_3) + p_3 x_4 + p_4 f_4(x_4, x_5, x_6, \delta, s) + p_5 f_5(x_4, x_5, x_6, \delta, s) + p_6 f_6(x_4, x_5, x_6, \delta, s) \quad (23)$$

The adjoint equations, $\dot{p} = -\partial H / \partial x$, become:

$$\dot{p}_1 = -\partial F / \partial x_1 \quad (24)$$

$$\dot{p}_2 = -\partial F / \partial x_2 \quad (25)$$

$$\begin{aligned}\dot{p}_3 &= -\partial H / \partial x_3 = -p_1(x_6 \cos x_3 - x_5 \sin x_3) \\ &\quad -p_2(x_5 \cos x_3 + x_6 \sin x_3)\end{aligned}\quad (26)$$

$$\dot{p}_4 = -\frac{\partial H}{\partial x_4} = -p_3 - p_4 \frac{\partial f_4}{\partial x_4} - p_5 \frac{\partial f_5}{\partial x_4} - p_6 \frac{\partial f_6}{\partial x_4} \quad (27)$$

$$\dot{p}_5 = -\frac{\partial H}{\partial x_5} = -p_4 \frac{\partial f_4}{\partial x_5} - p_5 \frac{\partial f_5}{\partial x_5} - p_6 \frac{\partial f_6}{\partial x_5} - p_1 \cos x_3 - p_2 \sin x_3 \quad (28)$$

$$\dot{p}_6 = -\frac{\partial H}{\partial x_6} = -p_4 \frac{\partial f_4}{\partial x_6} - p_5 \frac{\partial f_5}{\partial x_6} - p_6 \frac{\partial f_6}{\partial x_6} - p_1 \sin x_3 + p_2 \cos x_3 \quad (29)$$

The final condition is

$$\frac{\partial H}{\partial \delta} = p_4 \frac{\partial f_4}{\partial \delta} - p_5 \frac{\partial f_5}{\partial \delta} - p_6 \frac{\partial f_6}{\partial \delta} = 0 \quad (30)$$

$$\frac{\partial H}{\partial s} = p_4 \frac{\partial f_4}{\partial s} - p_5 \frac{\partial f_5}{\partial s} - p_6 \frac{\partial f_6}{\partial s} = 0 \quad (31)$$

The boundary values are the 6 initial conditions:

$$x^*(t_0) = x(t_0) \quad (32)$$

which usually reduce to:

$$x_1(t_0) = x \quad (33)$$

$$x_2(t_0) = y \quad (34)$$

$$x_3(t_0) = \psi \quad (35)$$

$$x_4(t_0) = 0 \quad (36)$$

$$x_5(t_0) = v \quad (37)$$

$$x_6(t_0) = 0 \quad (38)$$

where the vessel is at some point (X,Y) with heading ψ , with zero yaw rate, zero drift angle, and speed v . The boundary values for the adjoint equations are as defined in the mathematical discussion, with $h x(t_f)$ not present in the object function:

$$p_i(t_f) = 0 \quad i = 1, 6 \quad (39)$$

The solution of this TPBVP with 6 state equations, 6 adjoint equations, and 2 gradient stationarity conditions then will yield the optimal control functions $\delta^*(t)$ and $s^*(t)$, and the associated optimal trackline definition.

2.4 Solution Methodology

As previously indicated, the complexity of the f_4 , f_5 , and f_6 equations precludes an analytic solution of the above TPBVP for δ^* and s^* ; thus, a numerical solution methodology is required. The method of steepest descent has been selected and implemented in the VNSS.

The method of steepest descent, as applied herein, is an iterative procedure related to the steepest descent or gradient search applied to a scalar function of several variables.¹⁵ In this technique, an initial guess to the solution is selected, indicated by $u^{(0)}(t)$. While the technique is defined as a time continuous process, the implementation is with finite step discrete approximations to the continuous function, with Taylor series integration of the differential equations defining $\dot{x}$ and $\dot{p}$.

With this initial guess of control, the state equations ($\dot{x}$) are integrated forward from t_0 to t_f yielding the state solution with initial condition $x(t_0)$. Then, with the vessel states known, the adjoint equations are integrated backward from t_f to t_0 with "starting" final state $p(t_f) = 0$. Then having both the states $x(t)$ and adjoint variables $p(t)$, the gradient components $\delta H/\delta u$ can be calculated.

To find a minimum of the objective function J , a step is taken in the direction of decreasing J or in the negative gradient direction, i.e.:

$$u^{(1)}(t) = u^{(0)}(t) - \tau \frac{\delta H^{(0)}}{\delta u} \quad (40)$$

The step τ is selected through a single variable search, i.e., τ is chosen for any iteration to minimize the value of the objective function J . The process is then repeated with control history $u^{(1)}$. The method of steepest descent can be summarized by the following procedure:

1. Select an initial guess $u^{(1)}(t)$ for iteration $i = 0$
2. With control $u^{(1)}(t)$, integrate the state equations forward with initial conditions $x(t_0)$ to obtain $x^{(1)}(t)$
3. With this control, $u^{(1)}(t)$ and state $x^{(1)}(t)$, integrate the adjoint equations backward with final condition $p(t_f) = 0$ to obtain $p^{(1)}(t)$
4. With control $u^{(1)}(t)$, state $x^{(1)}(t)$, and adjoints $p^{(1)}(t)$, calculate the gradient $\frac{\delta H^{(1)}}{\delta u}(t)$

5. Find τ through a search procedure such that:

$$u(t) = u^{(1)}(t) - \tau \frac{\delta H^{(1)}}{\delta u}(t) \quad (41)$$

minimizes $J[u(t)]$

6. Let:

$$u^{(i+1)}(t) = u^{(1)}(t) - \tau \frac{\delta H^{(1)}}{\delta u}(t) \quad (42)$$

7. Go to step 2.

This procedure is numerically implemented in the VNSS, with the initial choice;

$$\delta^{(0)}(t) = 0 \quad (43)$$

$$s^{(0)}(t) = \text{initial RPM} \quad (44)$$

The techniques and variables to control the convergence, step size search, and number of iterations performed are discussed in the Programmers Manual.

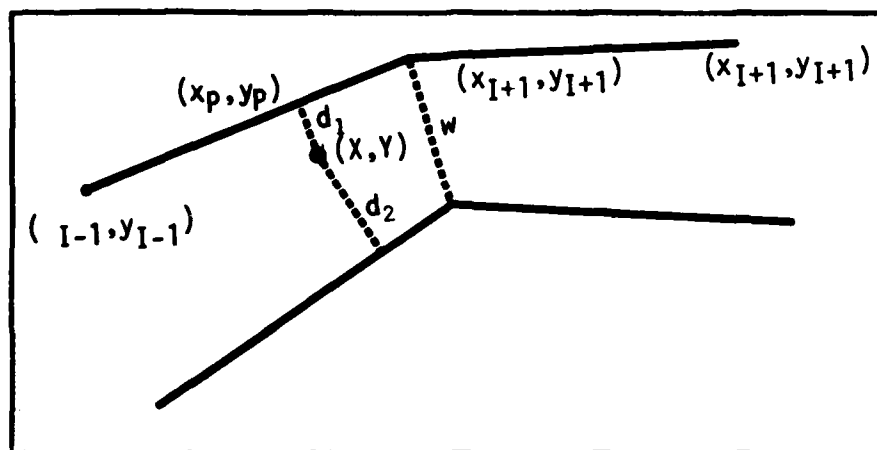
2.5 Objective Function in Line Segment Channel

The objective function minimized by the VNSS program is quadratic scaled by the channel width and summed at each point where the trajectory is evaluated. A typical point (X,Y) is shown within the channel boundaries in Figure 2-2.

From the geometry of the figure, the objective function for the point is

$$f = \frac{d_1^2 + d_2^2}{w} \quad (45)$$

FIGURE 2-2. OBJECTIVE FUNCTION DEFINITION - TYPICAL POINT



A more convenient mathematical representation is the so-called slope/intercept formulation. This formulation describes a straight line as:

$$y = mx + b \quad (46)$$

where, $m = \frac{y_I - y_{I-1}}{x_I - x_{I-1}}$; $b = y_I - mx_I$

(In the unlikely event that the denominator of m is sufficiently small as to cause numerical trouble, an appropriate fix is as follows:

$$\text{If } |x_I - x_{I-1}| < 10^{-2}, \text{ then } x_I = x_I + 2 \cdot 10^{-2}$$

Although this safeguard is included in the code, there is a stipulation in the Users' Manual which precludes vertical segments.)

The perpendicular from the line, $y = mx + b$, to the point (X, Y) intersects the line at a point (x_p, y_p) which is given by:

$$x_p = \frac{X + m(Y-b)}{1 + m^2} \quad y_p = \frac{m^2 Y + mX + b}{1 + m^2} \quad (47)$$

By assumption, the point (X, Y) has already passed the point (x_{I-1}, y_{I-1}) .

Therefore, to determine whether or not it has passed the point (x_I, y_I) , it is only necessary to check the following:

If $|x_p - x_{I-1}| > |x_I - x_{I-1}|$ then, it has passed (x_I, y_I) ;
otherwise, it has not.*

When it has not passed the point (x_I, y_I) , then the appropriate penalty function (for the upper bank) is:

$$f = \frac{(X - x_p)^2 + (Y - y_p)^2}{w} \quad (48)$$

Substitution of (47) into (48), after simplification, leads to:

$$f = \frac{(mX + b - Y)^2}{(1 + m^2)w} \quad (49)$$

The rate of change of (49), with respect to the vessel position (X, Y) is given by:

$$\frac{\partial f}{\partial x} = \frac{2m(mX + b - Y)}{(1 + m^2)w} \quad \frac{\partial f}{\partial y} = \frac{-2(mX + b - Y)}{(1 + m^2)w} \quad (50)$$

For computer generation purposes, this can be simplified by the sequential calculation of (49) followed by:

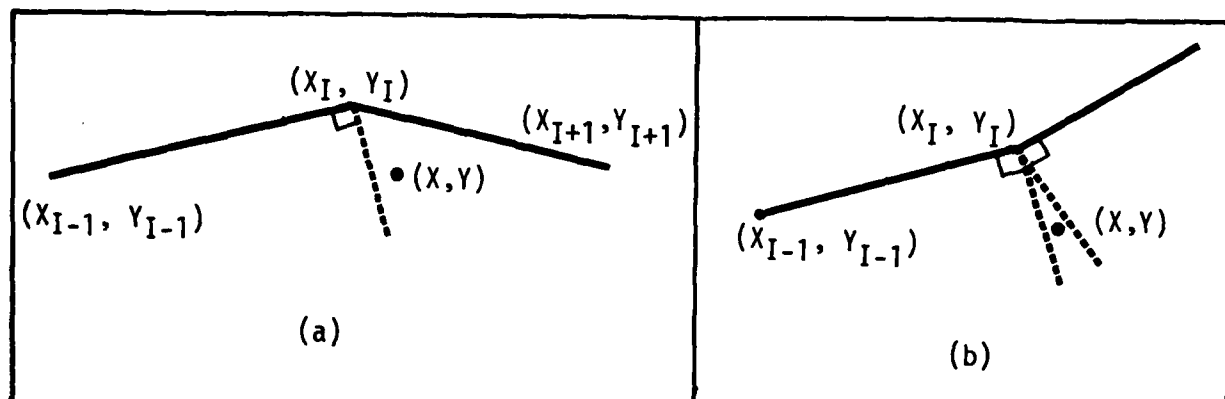
$$\frac{\partial f}{\partial y} = \frac{-2f}{(mX + b - Y)} \quad (51)$$

$$\frac{\partial f}{\partial x} = -m \frac{\partial f}{\partial y} \quad (52)$$

In the event that the point (x_I, y_I) has been passed by (X, Y) , then two alternatives arise which are depicted in Figure 2-3.

*Note: y_p need never be calculated.

FIGURE 2-3. OBJECTIVE FUNCTION DEFINITION - CORNER POINT



In order to make the appropriate determination, the procedure is to first, increment I by 1 and recalculate and store the appropriate m and b which are given by (46). Secondly, calculate x_p with the m and b as given by (47). Then perform the check:

IF $|x_p - x_I| > |x_{I-1} - x_I|$, then, Figure 2-3b applies.

Otherwise, Figure 2-3a applies.

In the latter situation, f is computed by (49), and the partial derivatives $\partial f / \partial y$ and $\partial f / \partial x$ are given by (51) and (52). In the situation shown in Figure 2-3b, that is, when the previous test is passed, then different calculations need to be implemented.

The appropriate formulas* are:

$$f = \frac{(X - x_{I-1})^2 + (Y - y_{I-1})^2}{w} \quad (53)$$

$$\frac{\partial f}{\partial x} = \frac{\partial (X - x_{I-1})}{w} \quad (54)$$

$$\frac{\partial f}{\partial y} = \frac{\partial (Y - y_{I-1})}{w} \quad (55)$$

*Note: The subscripts referral in (53), (54), and (55) pertain to the incremented subscripts in Figure 2-3.

3. SIMULATION CAPABILITIES AND LIMITATIONS

3.1 Simulation Capabilities

The optimal control techniques for determining a vessel control history and resultant trackline, as defined in the previous section, have been embedded in the Vessel Navigation System Simulation (VNSS), delivered and installed on the USCG PDP-11/34 computer system. The detailed discussion of the program structure, particular algorithmic formulation of certain design features, and input/output formats is relegated to the VNSS Program Documentation, Volume I, Users' Manual and Volume II, Programmers' Manual; the overall VNSS capabilities, features and limitations will be discussed herein.

The VNSS is constructed to accept definition of a scenario consisting of an arbitrary (within limits to be discussed) channel definition including obstacles, current and limited pilot visibility or field-of-view, along with a detailed vessel definition, and to construct the control history and resultant trackline, using the technique previously described. Two primary goals were observed during the derivation and construction of the algorithms and simulation; firstly, the scenario definition should be as general as possible or practical and, secondly, the formulation and solution of the optimal control problem should be transparent to the user. In addition, a convenient form of user input for maximal utility and flexibility was considered as mandatory. Design trades between global scenario generality and computer program design considerations including problem execution time yielded the final form of the simulation specifications discussed herein.

The major problem scenario definition elements of the VNSS are indicated in Table 3-1 and discussed in detail below.

TABLE 3-1. SCENARIO DEFINITION ELEMENTS

Vessel Definition (Dynamics) Channel Definition Current Definition Obstacle Inclusion Scene Analysis
--

The VNSS is constructed based on the vessel and dynamics as defined by the USCG supplied "black box" code. Within this code, the vessel is defined by a set of hydrodynamic coefficients residing in the "DTM10" subroutine. The VNSS is therefore based on the same set of coefficients and requires linking of the proper "DTN10" subroutine through the taskbuild process. This vessel definition is the only mechanism provided, no alternative vessel definition process is incorporated.

Similarly, the only vessel dynamics is through the USCG supplied subroutines "FW10", "CURT", "DRV10", and "DRU10". These subroutines derive the forces and moments due to vessel hydrodynamic interaction, winds, currents, rudder application, and propeller RPM. These forces are then converted to accelerations in the state variables, accounting for the fact the $x_5(u)$ and $x_6(v)$ are in a rotating coordinate system with rotation rate $x_4(r)$. These accelerations are then integrated using a Taylor series type of integration. These USCG supplied subroutines have been included in the VNSS with no alterations in the operational code sequence. Additionally, these subroutines are used in a finite difference approximation for the various derivatives of accelerations (f_4 , f_5 , and f_6 of Section 2) with respect to the respective variables (x_4 , x_5 , x_6 , δ , and s of Section 2). Again, no alternatives or options to this definition of the vessel dynamics is provided.

The channel definition is patterned after and is compatible with the channel specification procedure in the USCG supplied subroutines. This compatibility was enforced to allow use of the channel scenario as defined through the "DTN10" subroutine as a default option. Again, the proper "DTN10" subroutine corresponding to a particular predefined channel must be linked at taskbuild. In addition, capability is included to create a new channel definition, modify the default (DTN10) option, or modify a predefined channel definition. Implementation and execution of these options is discussed in the VNSS Program Documents.

The channel is defined by a series of straight line segments, or "splines," with the segments specified by the coordinates of the breakpoints of the splines. The breakpoints must be specified in pairs, with one on each bank. This requirement is necessary to enforce compatibility with the USCG supplied "CURT" and the "DTN10" options. The intent of the breakpoint is to roughly span the channel, with minor variations being non-catastrophic in nature. To again enforce compatibility with the USCG supplied "CURT" subroutine, a maximum of 30 breakpoints are allowed.

The current is defined by specifying the current speed and heading at eight stations across the channel, at each pair of breakpoints. These stations are equally spaced at positions $\left\{ (2i+1)/16 \quad i = 0,7 \right\}$ along the line joining the pair of breakpoints. This definition is again consistent with the USCG supplied subroutine "CURT" which interrogates the scene to find the current value at any (X,Y) point by associating the point with the geometrically closest current definition point. This algorithm defined by "CURT", while less accurate than a true interpolation scheme, was adopted through the use of "CURT" due to the interface structure already defined between "CURT" and the other USCG supplied subroutines. Again, the default values of current are

input through the linking of a proper "DTN10" subroutine. Also, options to input a new current definition related to a new scene, or to alter the current definition in the default or in an existing scenario are included.

Finally, the current can be deactivated; i.e., all current values are ignored (equivalently zero) by setting a single switch. This option, by eliminating all calls to subroutine "CURT", can reduce program run time by up to 2/3 as the program spends 1/2 to 2/3 of the execution time in "CURT" when it is activated, as established through timing experiments with the code.

The VNSS can include up to five discrete obstacles in the channel. Each obstacle is specified by a position (X,Y) and by an indication as to with which bank the obstacle is associated. This association forces the trackline to pass between the obstacle and the opposite bank; logic for automatically selecting the best bank association for a given obstacle is not currently included. The obstacles are included in the internal optimal control problem solution only when they occur within the pilots field-of-view, i.e., are within the specified visibility range or "window" from the current vessel position.

The VNSS is designed to simulate a limited field-of-view or visibility range for a pilot. From any given point, only those bank segments and obstacles within this field-of-view are included in the internal objective function calculation and thus affect the solution. This limited field-of-view is referred to as a "window" and the solution proceeds along the channel using a so-called "sliding window" solution.

The best solution methodology would therefore be to calculate the optimal control and trackline for the next window and use this control to advance one time step (nominally hard-coded as 2 seconds). At this new position, a new window is defined, the problem again solved for this new window, slightly

offset down-trajectory from the previous window, hence the term "sliding window." In a practical environment, the execution time of this solution methodology is quite excessive with the optimal control problem resolved at each and every time point. Thus, the implemented solution is to specify a restart position at which a new window is constructed as an approximation to the continuous sliding window. The observation window length and restart length are specified independently; experiments performed by Delta Research, Inc., indicate, however, that restart lengths approximately $1/3$ of a window length, i.e., the next window overlaps the previous window by $2/3$, represent a good compromise between trackline performance and execution time. Additionally, if the restart length is equal to the window length, and both are equivalent to the scene length (also user specified) the resultant trackline is generated in one single window with all down-trajectory information available to formulate the solution. The scene analysis is then controlled by three variables, viz., the window length, the restart length, and the overall scene length.

3.2 Simulation Restrictions

During the construction of the VNSS, the goal of total generality of the scene definition was tempered with the objective of keeping simulation run time minimal. This trade results in algorithms for calculating the nearest bank segment, the appropriate objective functions, and the derivatives of the objective function that place minor restrictions on the global generality of the channel definitions and on the relationship of the obstacles with respect to the bank spline breakpoints.

The primary restriction is with respect to the angular relationship between adjacent splines in the channel definition as sharp bends between adjacent splines, particularly convex bends where the bank bends into the channel, can have a deleterious effect on the algorithm that calculates the objective

function and its derivatives. This restriction can be quantified as no more than a 35° bend between adjacent spline segments. Thusly, for example, a right angle bend in the channel should be modelled as three breakpoints with 35° between splines. The 35° limit is not a "hard" limit, as there is no catastrophic program failure at, for example, 36° between splines. However, as the angle increases beyond 35° , the probability of anomolous simulation behavior increases.

The breakpoints to define the splines must be sequential in the direction of vessel travel with convention that the scene be roughly horizontal with the vessel travelling from left to right. This restriction is generated by the need to keep the subroutine that establishes the vessel location in the channel simple, to avoid the execution time overhead imposed by an algorithm such as "CURT". Additionally, spline segments should be a minimum of 300 feet in length and in no case can a spline segment be vertical. Since channel width is determined by the geometric distance between a pair of breakpoints, the breakpoints should nominally span the channel.

The obstacle definition is by location and association with a particular channel boundary. As such, the location must be a point within the nominal channel to avoid catastrophic program breakage. The VNSS incorporates the obstacle into the appropriate channel boundary by moving the adjacent spline breakpoints such that the appropriate bank segment passes through the obstacle. Following this, the spline breakpoints further from the obstacle are altered to maintain the maximum angular change between adjacent splines as discussed previously. In order to accomplish this, there must be breakpoints "near" the obstacle to allow the algorithms maximum flexibility in fairing the obstacle into the bank. While general in nature, the algorithm as implemented in the VNSS cannot anticipate every potentially possible combination of

obstacles and breakpoints. Thus, a general set of rules for defining auxiliary or artificial breakpoints "near" an obstacle are given. Since under some particular circumstances, the obstacle fairing can adversely affect the channel definition, the channel as redefined with the obstacle faired in is output to the user to allow examination and modification of the artificial breakpoints if required. Every attempt has been made to keep the algorithm robust within the general set of rules below, although the user can always find a particular input to break the algorithms.

The artificial breakpoints must, of course, be specified in pairs across the channel. The artificial break points should be closely spaced, at about the 300 feet minimum, and there should be several pairs of points on each side of the obstacle. Since obstacles extending well within the channel require a greater fairing, three pairs of breakpoints on each side should be specified. For obstacles near the bank, two pairs on each side of the obstacle should suffice. Finally, for obstacles in a general curve in the channel, the break points should fill in the curve and also extend around the curve into the straight segments, if required, to get the requisite to two or three artificial pairs. Alternatively, the user can himself fair the obstacles into the banks and enter the resultant modified channel definition as if it contained no obstacles. The user can also "failsafe" the problem by specifying the entire channel in 300 foot segments. The VNSS should be robust to problem specifications within the above conventions and restrictions.

The scene is specified through the window length, restart or "slide" length, and total scene length. Certain "rationality" conditions must be met, e.g., the scenario length must not exceed the channel specifications and the restart length must be within the window length. The total scenario length is limited to 500 time steps, nominally chosen at 2 seconds for 1,000 seconds of travel. For

a typical tow flotilla with velocity in the order of 10 ft/sec, this allows about 10,000 feet of scene definition. Finally, the distance specifications must not be "microscopic" with respect to the problem definition, i.e., distance specifications should be at least in the order of hundreds of feet.

The final comments are with respect to the convergence of the solution. As discussed in Section 2, the optimal control problem results in an iterative solution of the TPBVP. The "quality" of the solution found by the steepest descent method increases with increasing iterations at the expense of computer execution time, although the adaptive step size algorithm implemented tends to mitigate this to some degree. The stopping criterion is automated, although to a rudimentary degree. The iteration stopping criterion is user specified as a number of "fine tuning" iterations after the first iteration that yields a trajectory totally within the channel specifications. There is an automatic override of this stopping criterion if the iterative procedure is still in a rapidly converging situation, which allows further iterations until the convergence rate slows or a maximum of twice the specified number of fine tuning iterations occurs. Additionally, there are automatic limiting criteria in the adaptive step size algorithm.

It is axiomatic to the mathematical formulation of the problem that short, simple scenes converge faster than long, complex scenes containing reverse ("S") curves; therefore, execution time considerations favor "sliding window" solutions for long, complex scenarios rather than single pass solutions. Experiments have demonstrated that under relatively general conditions, the control histories generated by the two problem specifications (sliding window versus single pass) are closely aligned with virtually identical tracklines generated.

4. NUMERICAL EXAMPLES

The VNSS as delivered to the USCG has been exercised on sample scenarios. These scenarios all utilized the USCG supplied model of the Exxon Tennessee towboat floatilla. This model is defined, as discussed previously, by the USCG "DTM10" subroutine in terms of physical parameters and hydronamic coefficients. The towboat floatilla is roughly 745 feet long by 54 feet wide. In all cases, a maximum rudder command limit of .7 radians (40.1°) is observed; also, the vessel nominal (zero drift angle) steady state velocity is roughly 10 ft/sec at 100 RPM. The examples examined include an arbitrary "S" turn channel, an arbitrary channel with a more gentle turn, a segment of the Mississippi River (Berwick Bay), and a problem defined to indicate vessel response in a simple scenario.

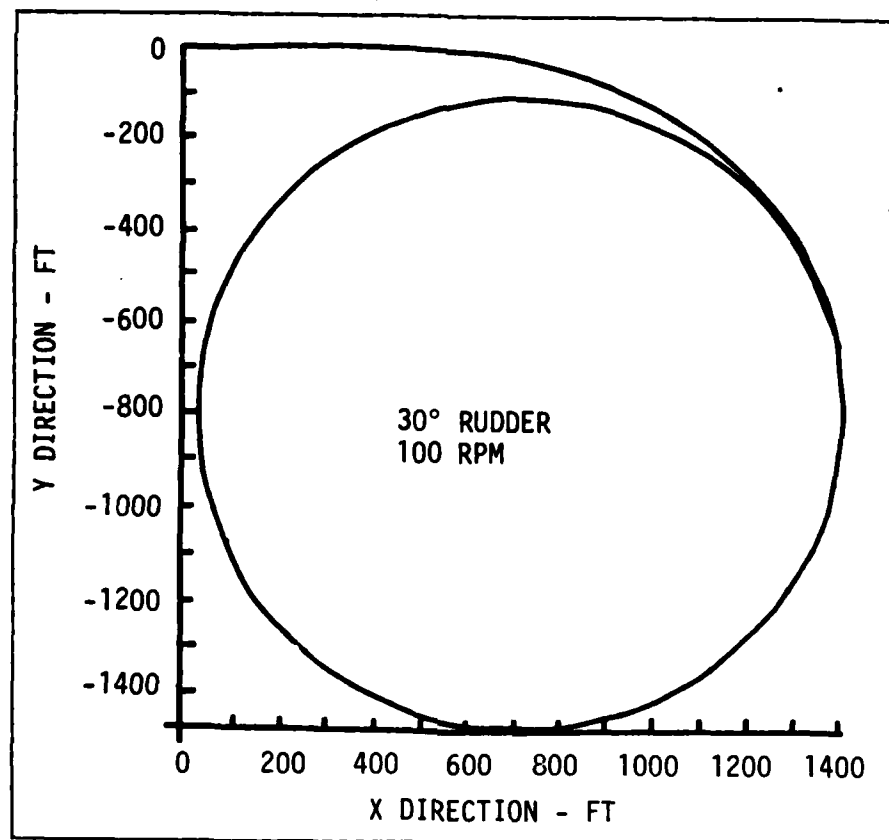
The response of the towboat floatilla is very sluggish, requiring the anticipatory nature of the solution as previously discussed. The characteristic response is shown in Figure 4-1, which shows the vessel response to a 30° rudder command, at initial velocity of 10 ft/sec at 100 RPM. The vessel is headed due east, along the "X" axis, when the rudder is applied at (0,0). The distance to achieve various lateral offsets are shown in Table 4-1.

TABLE 4-1. LATERAL OFFSET DISTANCES

TIME (SEC)	DISTANCE (FT)	LATERAL OFFSET (FT)
0	0	0
38	365	-8
58	531	0
95	920	100
115	1080	200

The initial motion is due to the force on the rudder before a drift angle is achieved, and is 8 feet to the left for the right rudder case shown. It requires 531 feet to compensate for this and achieve zero offset, with the offset then increasing ever more rapidly. The final steady state turn radius is 690 feet.

FIGURE 4-1. EXXON TENNESSEE RESPONSE



Example 1

This example consisted of a channel defined by 16 pairs of spline break points, indicated in Table 4-2 below.

TABLE 4-2. SPLINE BREAKPOINTS

"Lower" Bank		"Upper" Bank	
X	Y	X	Y
800.	2200.	800.	3200.
1600.	1400.	1600.	2400.
2000.	1000.	2400.	1600.
2700.	750.	2700.	1500.
3000.	600.	3000.	1400.
3300.	700.	3300.	1500.
3600.	800.	3600.	1600.
3900.	900.	3900.	1900.
4200.	1000.	4100.	2100.
4400.	1200.	4300.	2300.
4600.	1400.	4600.	2400.
5000.	1500.	5000.	2400.
5400.	1400.	5400.	2300.
6000.	800.	6000.	2200.
6400.	500.	6400.	2000.
6800.	200.	7000.	1400.

There are two defined obstacles as shown in Table 4-3.

TABLE 4-3. OBSTACLE DEFINITION

Position (X,Y)	Bank Association
(2900., 1200.)	Upper
(4000., 1200.)	Lower

Figure 4-2 shows the channel definition. The points indicated with circles were added as "auxiliary" points for purposes of fairing in the obstacles, also indicated.

FIGURE 4-2. ORIGINAL CHANNEL

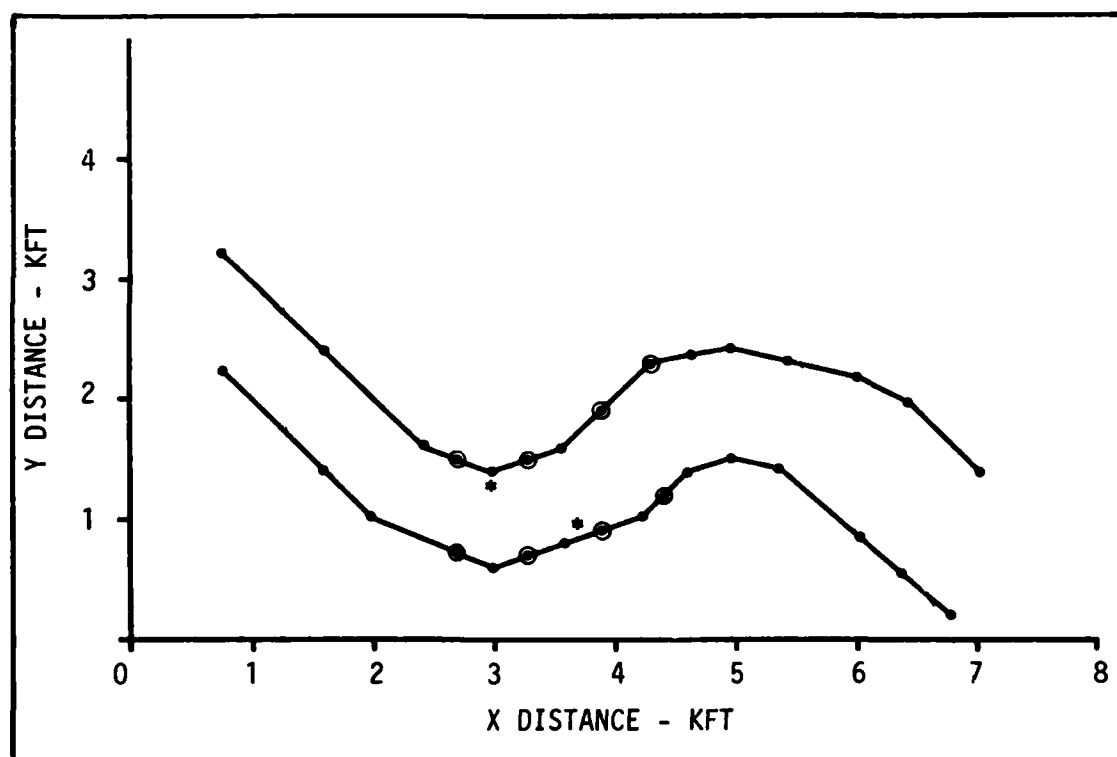
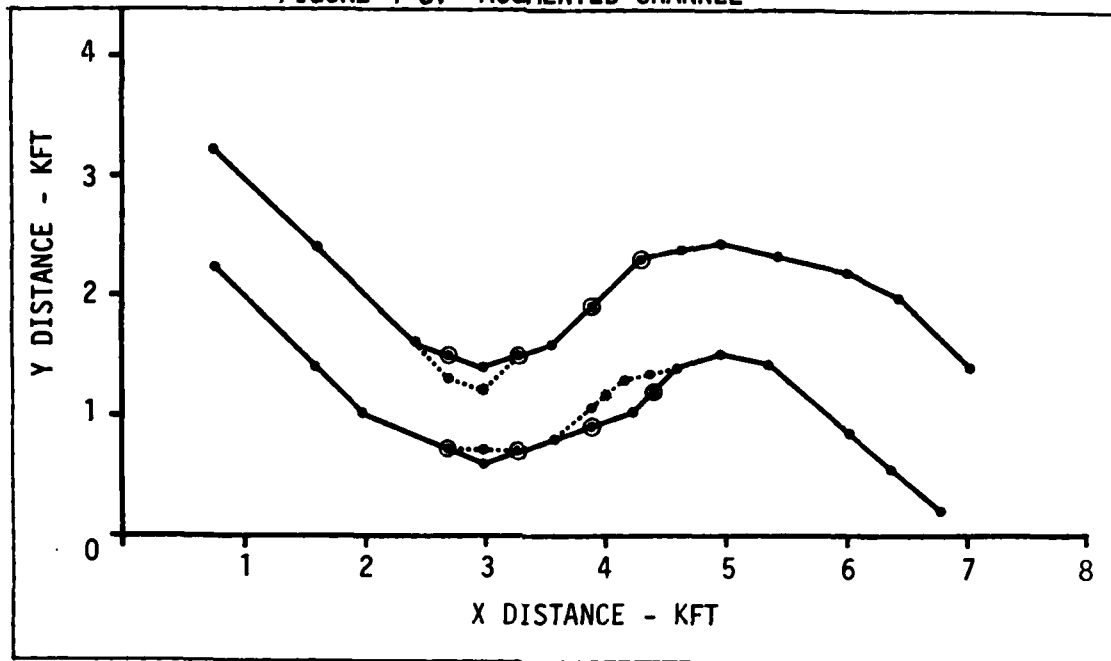


Figure 4-3 indicates the channel as defined with the obstacles faired into the bank definition. The use of the auxiliary spline breakpoints is clearly indicated in the figure. In addition, the lower corner at (3000, 600) has also been "smoothed" to conform to the maximum segment-to-segment bending criteria.

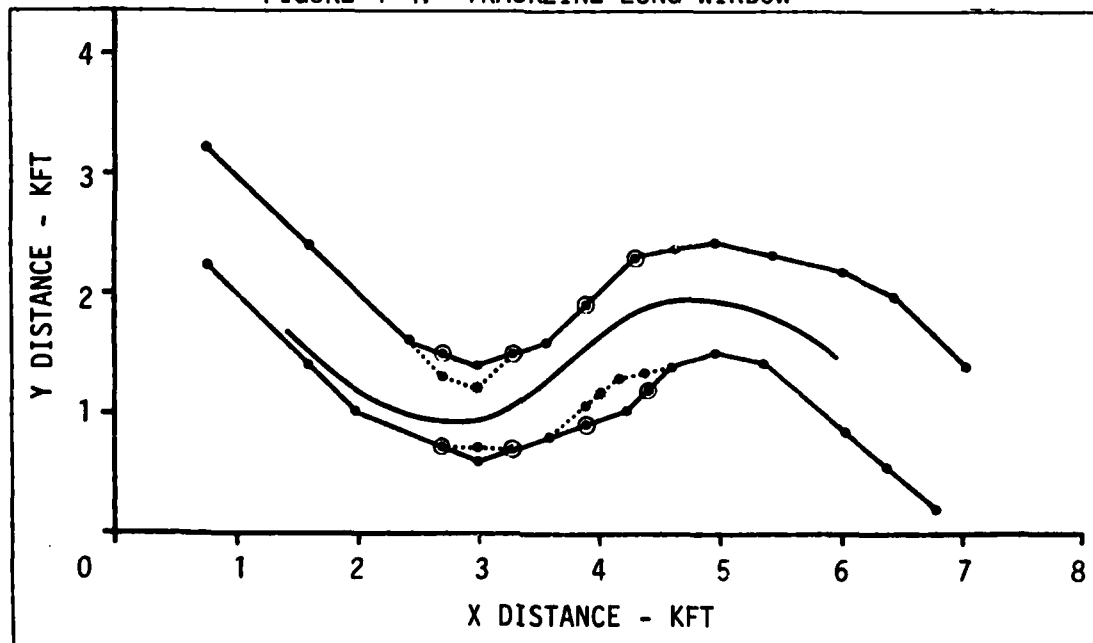
FIGURE 4-3. AUGMENTED CHANNEL



The initial vessel position is parallel to the right bank at the location (1400., 1700.). The vessel's initial speed is 10. ft/sec with RPM setting of 150 RPM.

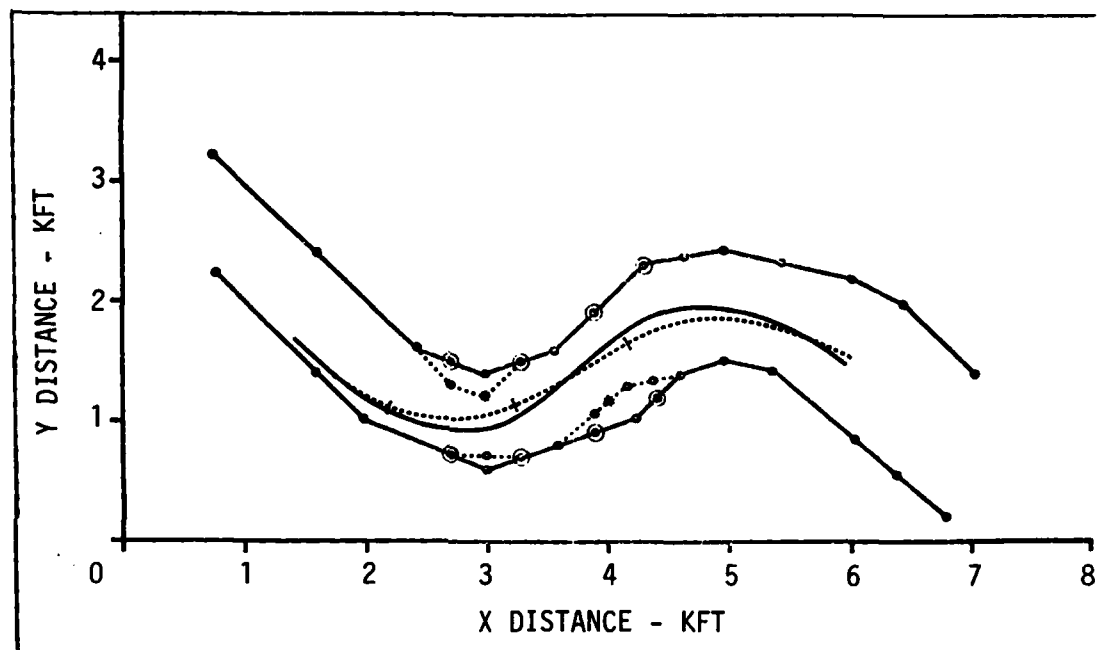
The resultant trackline for a full passage of the channel in one window is shown in Figure 4-4 after 50 iterations.

FIGURE 4-4. TRACKLINE-LONG WINDOW



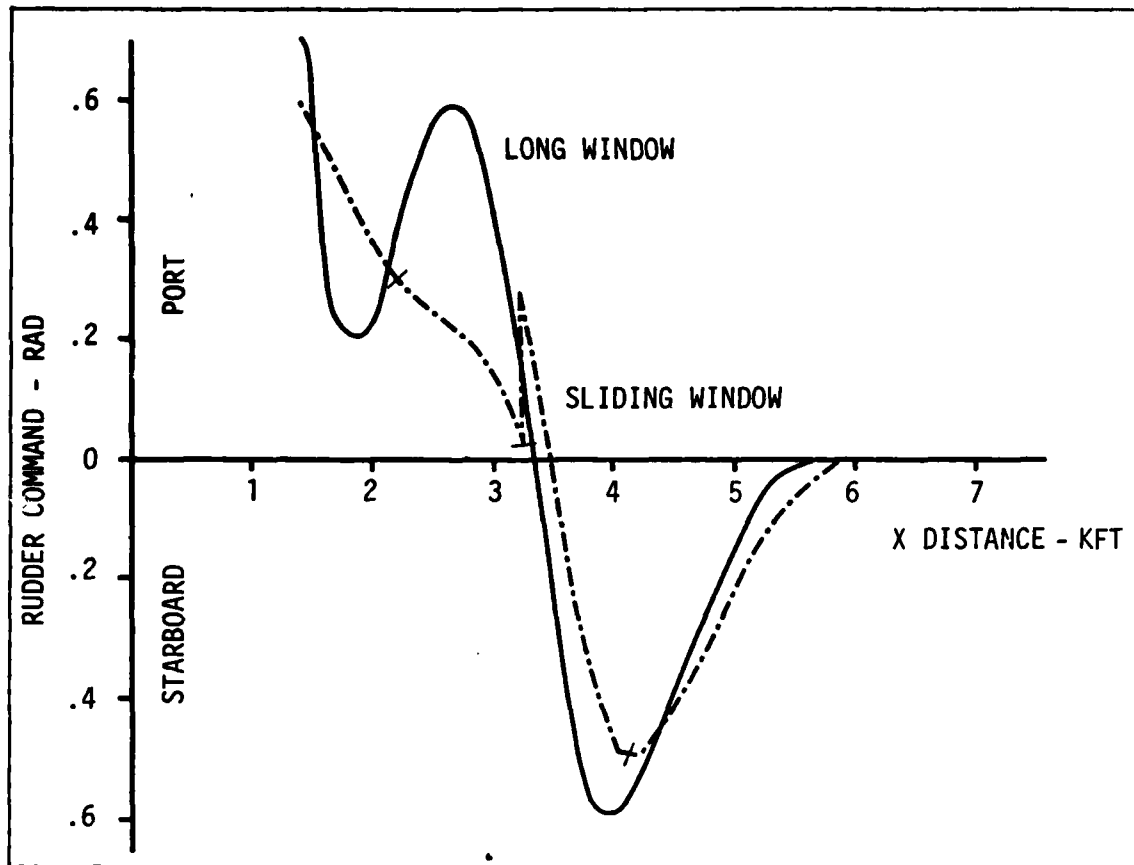
The scenario was also executed with a sliding window of 3000 feet, with restart after 1000 feet. The convergence criteria was set to five fine tuning iterations. This trackline is shown overlaid on the previous in Figure 4-5. Differences are attributed to the use of only 5 fine tuning iterations; 10 or even 20 is preferable.

FIGURE 4-5. TRACKLINE-SLIDING WINDOW



The rudder control functions are shown in Figure 4-6. Both cases show the hard transition from left rudder to right rudder at the switchback point, just past the apex of the left bend. The sliding window case would probably have maintained more left rudder in the initial portion with more fine tuning iterations. Also, the jump in rudder between the second and third window is characteristic of the form of solution, as the sliding window reinitialization does not enforce mathematical continuity of solution.

FIGURE 4-6. RUDDER HISTORY



In addition, the rudder control must always go to zero at the end of the scene due to the inherent formulation of the problem as implemented in the VNSS. Alternative formulations could enforce other rudder or vessel terminal conditions.

This example demonstrates the basic simulation capabilities, including channel definition, obstacle inclusion, and limited observation (sliding window) mode of operation.

Example 2

This scenario consists of a channel with 15 break points, shown in Table 4-3.

TABLE 4-4. SPLINE BREAK POINTS

"Lower" Bank		"Upper" Bank	
X	Y	X	Y
400.	-400.	0.	0.
1000.	200.	600.	600.
1500.	700.	1200.	1200.
1800.	1000.	1800.	1800.
2100.	1100.	2100.	1900.
2500.	1000.	2600.	1800.
2750.	750.	2750.	1650.
2900.	600.	3000.	1400.
3250.	250.	3300.	1100.
3700.	-200.	4000.	300.
4000.	-500.	4300.	0.
4500.	-1000.	4500.	-200.
5200.	-1300.	5600.	-700.
6200.	-1300.	6200.	-700.
9000.	-1300.	9000.	-700.

Three obstacles are included, defined in Table 4-5.

TABLE 4-5. OBSTACLE DEFINITION

Position (X,Y)	Bank Association
(2800, 1400)	Upper
(3900, -300)	Lower
(6000, -800)	Upper

Figures 4-7 and 4-8 show the channel, obstacles, and resultant "faired" banks. The points shown with circles are auxiliary spline breakpoints defined for obstacle smoothing.

FIGURE 4-7. ORIGINAL CHANNEL

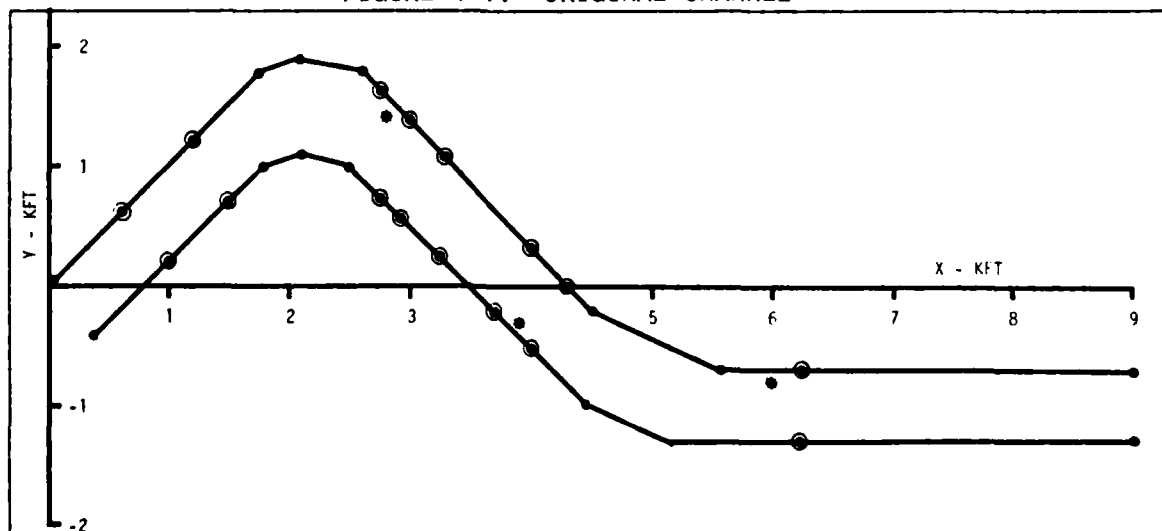
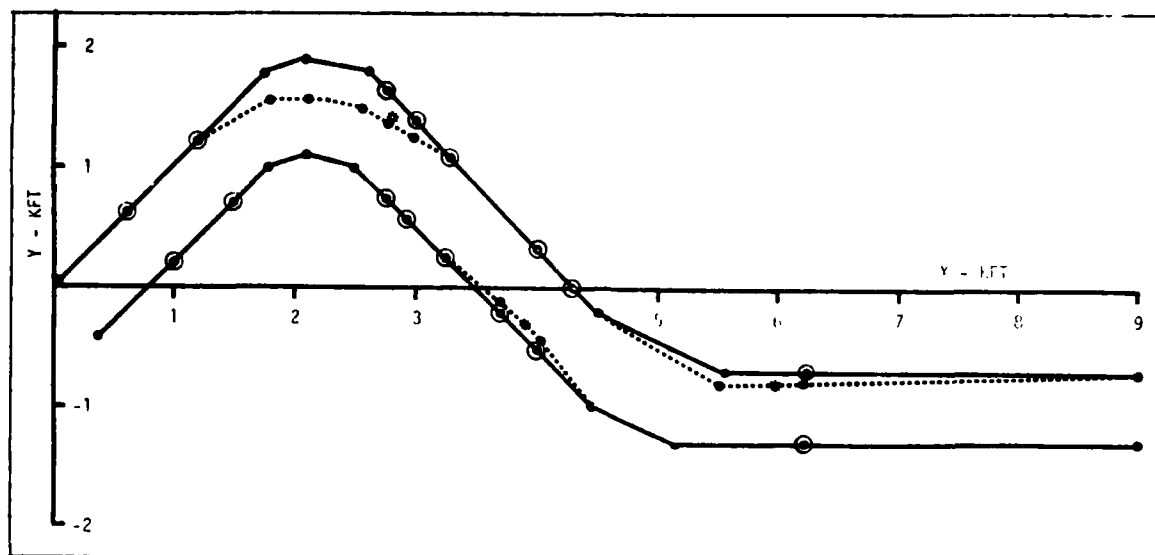
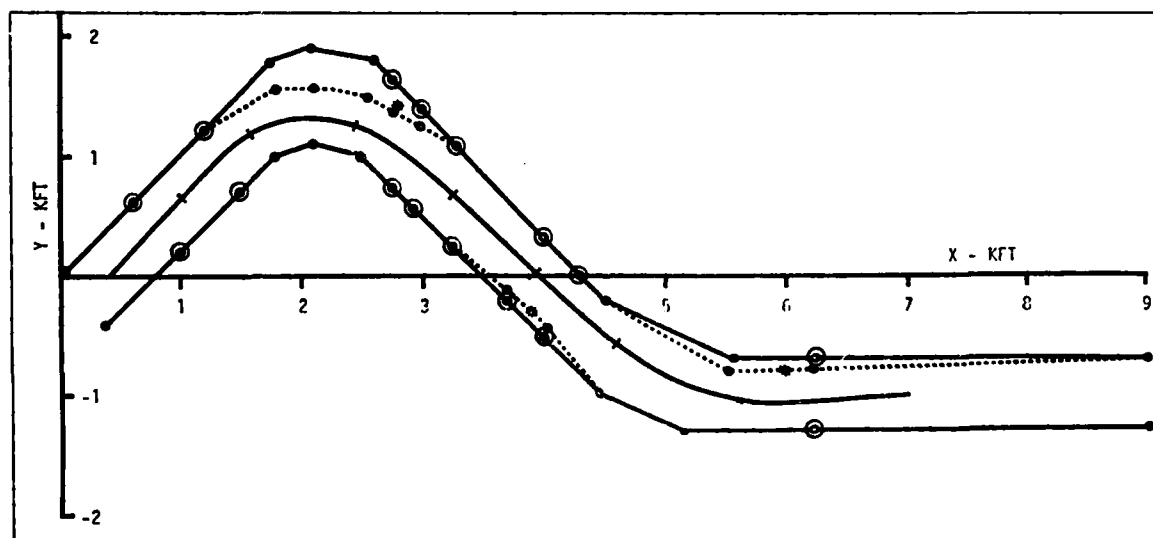


FIGURE 4-8. AUGMENTED CHANNEL



The vessel's initial position was at (400,0), i.e., in the channel center, with speed 10 ft/sec at 100 RPM. The trackline and control history are given in Figures 4-9 and 4-10, respectively, for a sliding window with 3000 feet window, with 1000 feet restart length, and 10 fine tuning iterations.

FIGURE 4-9. TRACKLINE-SLIDING WINDOW



With the rudder angle δ in radians and the propeller shaft speed in RPM, these (for a given point in the trajectory) have values shown in Table 4-6.

TABLE 4-6. DERIVATIVE VALUES

Adjoint	Partial Derivatives	
$p_4 = 3. \times 10^5$	$\frac{\partial \dot{x}_4}{\partial \text{RPM}} = .4 \times 10^{-6}$	$\frac{\partial \dot{x}_4}{\partial \delta} = .5 \times 10^{-3}$
$p_5 = 3. \times 10^3$	$\frac{\partial \dot{x}_5}{\partial \text{RPM}} = .7 \times 10^{-3}$	$\frac{\partial \dot{x}_5}{\partial \delta} = .1 \times 10^{-1}$
$p_6 = .7 \times 10^2$	$\frac{\partial \dot{x}_6}{\partial \text{RPM}} = .5 \times 10^{-4}$	$\frac{\partial \dot{x}_6}{\partial \delta} = .5 \times 10^{-}$
$\frac{\partial H}{\partial \text{RPM}} = 2.2 \text{ ft/RPM}$		$\frac{\partial H}{\partial \delta} = 184 \text{ ft/rad}$

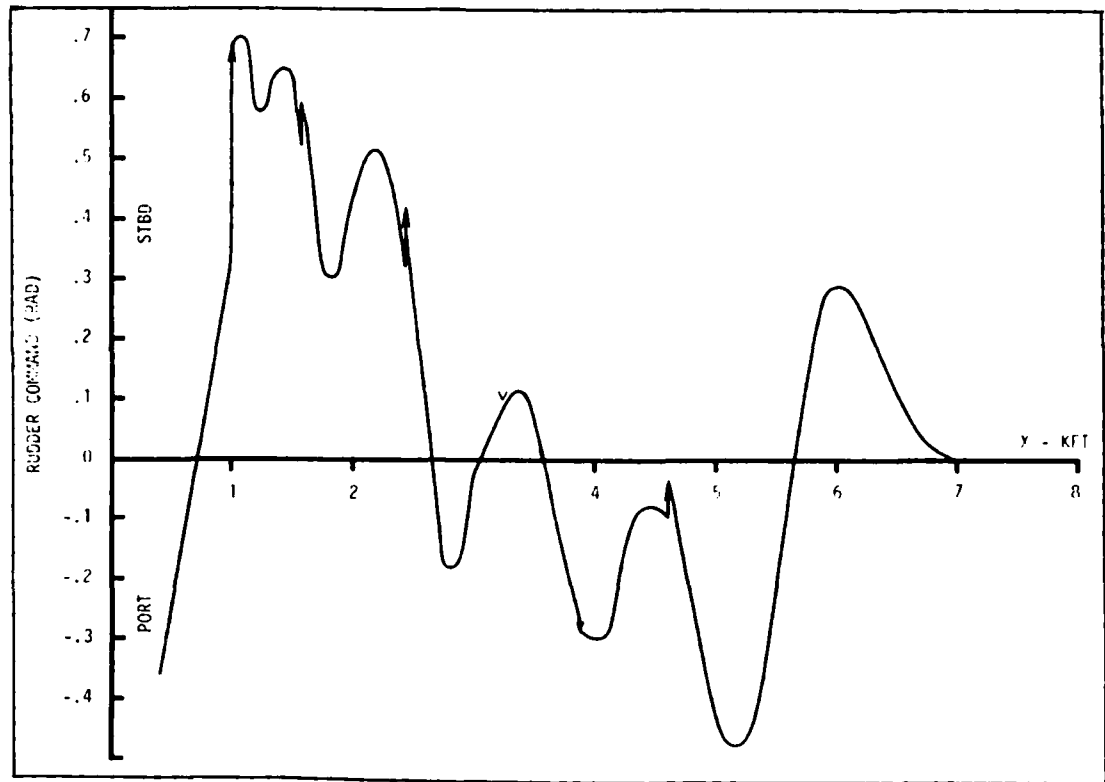
The difference in the gradient values of two orders of magnitude results in the solution to converge strongly in the rudder direction without significant change in RPM. Theoretically, as $\frac{\partial H}{\partial \delta}$ approaches zero, the RPM will converge. However, the two order of magnitude difference precludes this from occurring in the numerical method utilized. This scaling was investigated by artificially multiplying the RPM gradient by a constant value or, equivalently, treating τ as a matrix rather than a scalar. Table 4-7 shows the results of these experiments.

TABLE 4-7. RPM SCALING

Scaling Factor	Initial RPM	Final Objective Function
50	99	38315
100	98	38248
200	97	38493
500	80	37427
1000	85	37603
5000	50	36272

The RPM, in all cases, increased smoothly from the initial value to 100 RPM final value, as dictated by the mathematics. The primary reason for the decrease

FIGURE 4-10. RUDDER HISTORY



Once again, the discrete jump in rudder command at window restart is indicated. The vessel servomechanism response delay time, limiting rudder swing to 5 deg/sec would damp this discrete jump. This servomechanism response delay is high bandwidth with respect to the vessel response time and, thusly, is not modeled in the VNSS as it has inperceptible effect on the resultant trackline.

A detailed investigation was performed with regard to the change in RPM corresponding to the optimal solution. The RPM is changed along with the rudder command, with the changes as shown in Section 2 to be:

$$\Delta \text{RPM} = -\tau \frac{\partial H}{\partial \text{RPM}} \quad \Delta \delta = -\tau \frac{\partial H}{\partial \delta} \quad (1)$$

These become, as shown in Section 3:

$$\frac{\partial H}{\partial \text{RPM}} = p_4 \frac{\partial x_4}{\partial \text{RPM}} + p_5 \frac{\partial x_5}{\partial \text{RPM}} + p_6 \frac{\partial x_6}{\partial \text{RPM}} \quad (2)$$

$$\frac{\partial H}{\partial \delta} = p_4 \frac{\partial x_4}{\partial \delta} + p_5 \frac{\partial x_5}{\partial \delta} + p_6 \frac{\partial x_6}{\partial \delta} \quad (3)$$

in the objective function is the shortening of the trajectory placing more emphasis in the narrow part of the channel where the objective function is naturally lower. This cross-coupling between length of trajectory and objective function makes RPM scaling difficult to generalize, and Delta Research, Inc., recommends utilizing the unscaled gradient as currently implemented in the VNSS.

Example 3

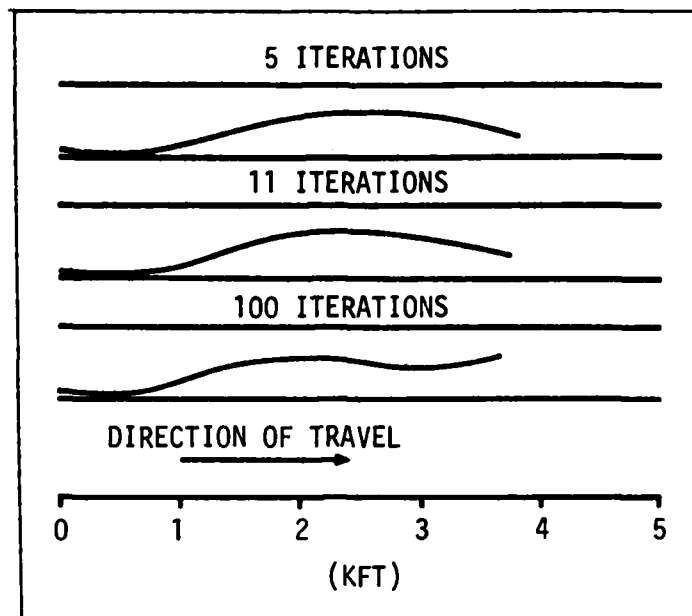
This example demonstrated the response of the algorithm to a simplistic case. The vessel starts near to the bank in a long, straight channel. The channel is defined to be 600 feet wide and 5000 feet long. The break points are as shown in Table 4-8.

TABLE 4-8. SPLINE BREAKPOINTS

LOWER BANK (X,Y)	UPPER BANK (X,Y)
(0,0)	(0,600.)
(5000.,0)	(5000.,600.)

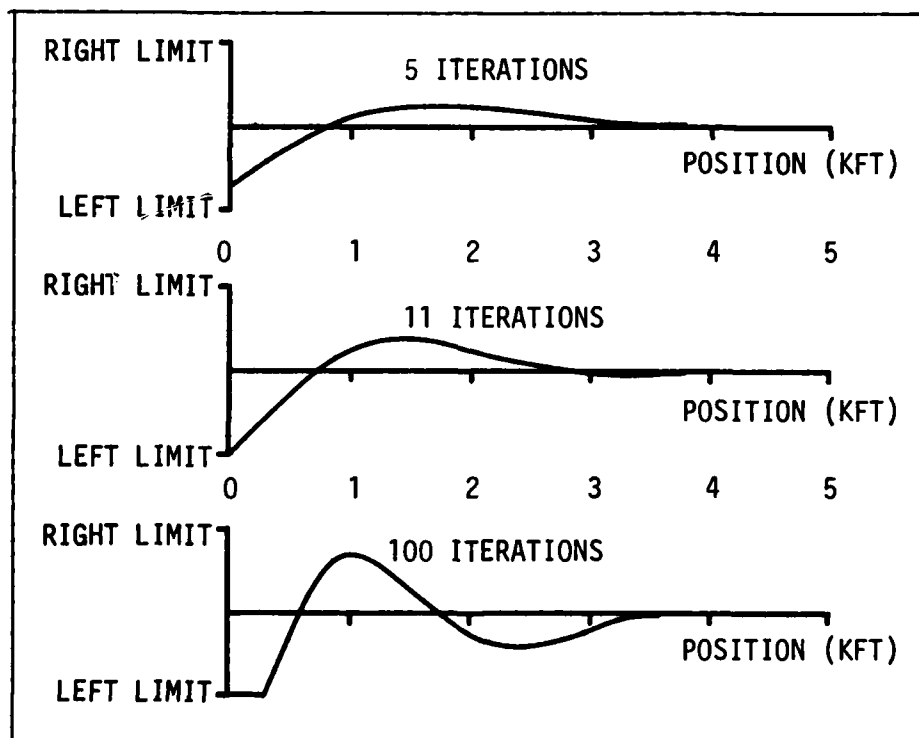
The vessel starting condition is (50.,50.), i.e., 50 feet off the starboard bank. There are no obstacles, and the solution was generated in one pass. Initial velocity and RPM were 10 ft/sec and 100 RPM. Figure 4-11 shows the trajectories after 5, 11, and 100 iterations, although the trajectory is stationary after, roughly, 30 iterations.

FIGURE 4-11. TRAJECTORIES



The control histories are shown in Figure 4-12.

FIGURE 4-12. CONTROL HISTORIES

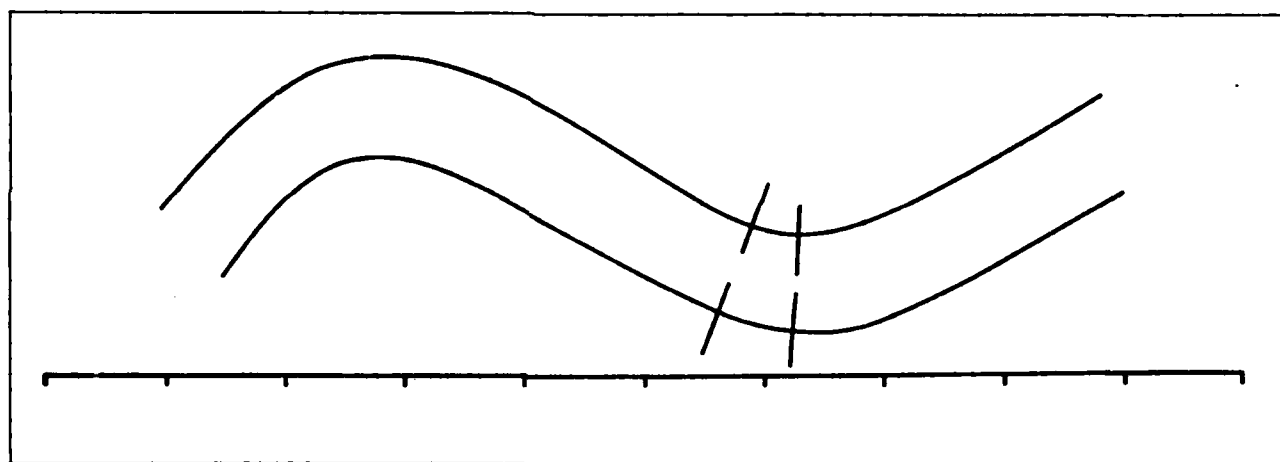


The solution converges to a hard (max rudder limited) turn off of the bank, followed by a "damping" process to minimize overshoot of the channel center. The vessel response can be noted to display an initial motion toward the right bank, to about 42 feet, due to the initial left rudder before a significant drift angle can be set.

Example 4

This example corresponds to the passage of the Berwick Bay section of the Mississippi River. It is the scene defined by the USCG supplied "DTM10" subroutine with the current suppressed. The basic channel is shown in Figure 4-13.

FIGURE 4-13. BERWICK BAY DEFINITION



The two railroad bridges are modelled by fairing the channel banks to include the bridges. This was done in this case by using the option to create and input new spline breakpoints to define the augmented channel. The solution shown is without current and corresponds to 10 ft/sec, 100 RPM initial condition. The solution is shown in Figure 4-14 with rudder history shown in Figure 4-15.

FIGURE 4-14. BERWICK BAY PASSAGE

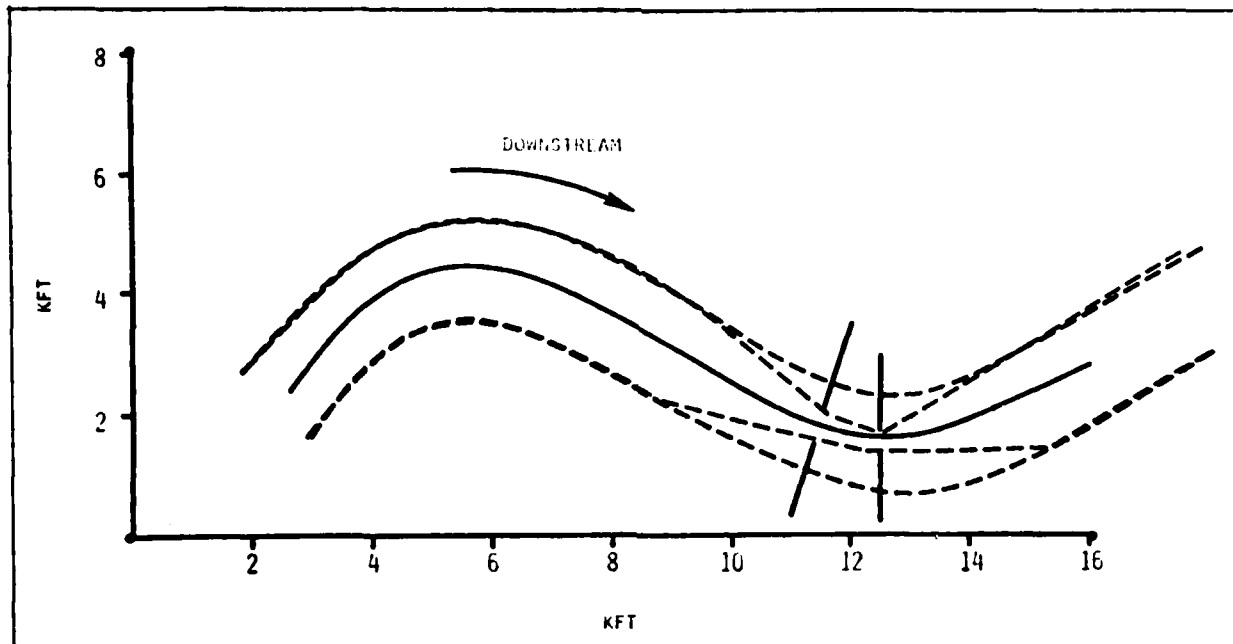
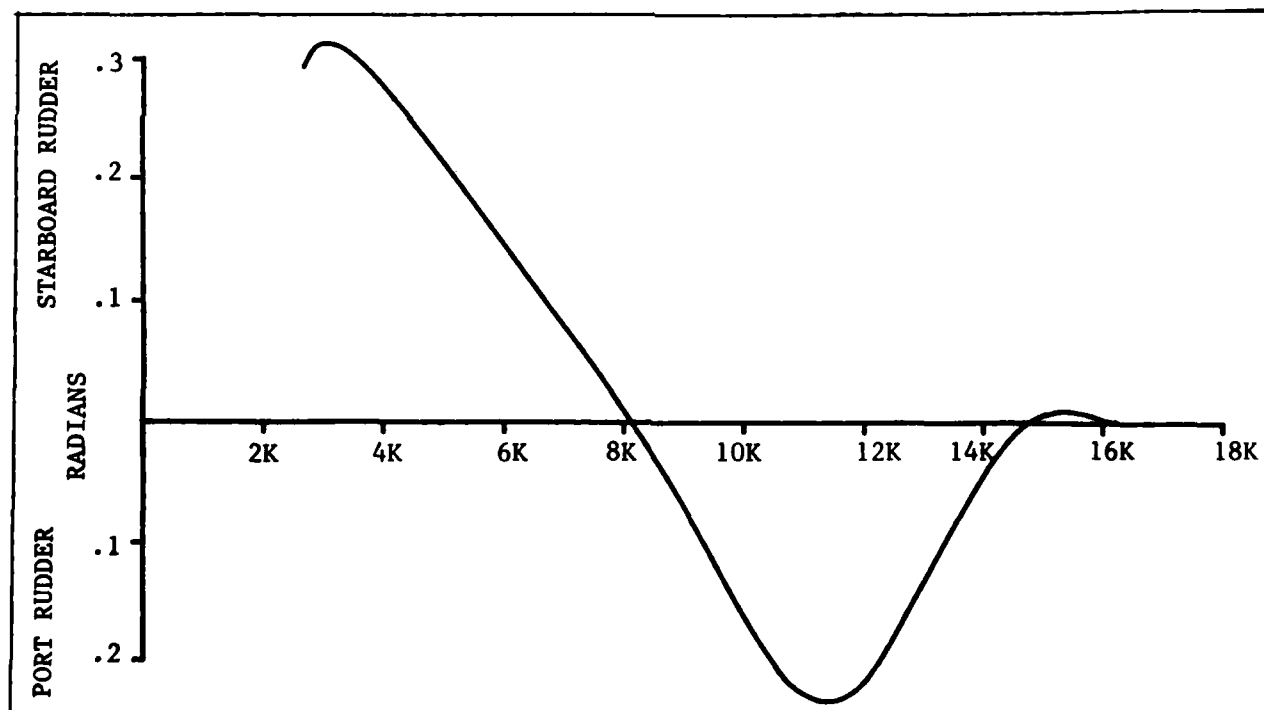


FIGURE 4-15. RUDDER COMMAND



The rudder command for this case peaks at .3 rad or, roughly, 20° rudder for the initial turn, followed by .22 rad for the reverse turn through the bridges.

5. FUTURE ACTIVITIES

Delta Research, Inc., believes that the application of optimal control theory as a tool for vessel navigation and control studies is a powerful technique and is applicable over a wide variety of problems and analyses. This section will discuss, out of these general applications, only those obvious extensions and applications of the currently developed VNSS. These extensions and applications fall in the three categories shown in Table 5-1, viz., efficiency enhancement of the current VNSS, validation of the current VNSS, and potential extensions of the VNSS capabilities

TABLE 5-1. FUTURE ACTIVITIES

- | |
|--|
| <ul style="list-style-type: none">• VNSS EFFICIENCY ENHANCEMENT<ul style="list-style-type: none">- Run Time Reduction- Scene Capability• VNSS VALIDATION<ul style="list-style-type: none">- Validation Plan- Tuning• VNSS UTILIZATION AND EXTENSIONS<ul style="list-style-type: none">- Capability Extension- Sensitivity Studies |
|--|

5.1 VNSS Efficiency Enhancement

The VNSS, as currently configured, executes on the USCG PDP-11/34 slower than real time with the actual ratio dependent on scenario. Reduction of this run time would enhance the utility of the simulation for some applications. A primary technique for execution time is through accelerated convergence techniques. The gradient search technique is a basic numerical technique used for this class of TPBVP solutions and is generally characterized by slow convergence. Alternative convergence techniques such as PARTAN¹⁶ offer quadratic convergence, in that the process will find the precise maximum of a

quadratic function on the third iteration. Although the objective function is significantly more convoluted than quadratic (in function space) as inferred through detail analysis of the converging solution for a sample problem, there is still great promise for this or other convergence acceleration schemes.

An alternative source of run time reduction is through the use of simpler dynamics than implemented in the USCG supplied subroutines. This trade of execution time for vessel dynamic model fidelity must, of course, be made based on the ultimate utility of the VNSS. As an intermediate step in the VNSS development sequence, Delta Research, Inc., utilized the so-called Nomoto equations for vessel dynamics simulation. These equations, involving significantly fewer terms in the Taylor series expansion for accelerations, offer a potential sevenfold reduction in run time. Finally, for scenarios including current specification, significant run time reduction can be accrued by simplifying and streamlining the USCG supplied "CURT" subroutine for interpolating current value for a given point in the channel.

The scene is currently limited to 500 time steps or, roughly, 10,000 feet total length. This limit is imposed by the amount of core available for variable storage. This can be increased through enhanced use of dynamic memory, or, for a sliding window case, by accumulating the control input only in large scale memory and recreating the trackline through a post-processor. This would allow virtually unlimited sliding window problems to be executed. Finally, the use of direct memory can be enhanced through general array overwriting when appropriate, i.e., destroying an array as it is used and utilizing the released memory for storing alternate arrays. These are pure programming issues rather than fundamental mathematical developments.

5.2 VNSS Validation

The test results detailed in Section 4 present the capabilities of the VNSS and indicate the nature of the solution form. The general control histories and resultant tracklines are, in the large, representative of the class of results expected. In this sense, the VNSS model has undergone some degree of "tuning" to give generally acceptable results. The level of acceptability, however, is based on a qualitative examination of the trajectory rather than a quantitative evaluation of the problem solution. This quantitative evaluation of the VNSS solution methodology is referred to as the model validation.

The validation of the model for a complex, human interactive process, such as the closed loop vessel navigation and control process, is generally by comparison of the model results with the real world implementation of the process. The real world implementation of the process, however, for the case in hand, represents passage of a vessel through a complex channel. The problem with obtaining this real world implementation is therefore twofold, e.g., it does not exist for arbitrary channel definition, and controlled experiments with real vessels in existing channels are prohibitively expensive.

Another potential source of data is observation and data collection on routine passages of commercial vessels. The detailed data requirements for full validation, i.e., numerical comparison of model trajectory with the real vessel trajectory are listed in Table 5-2.

TABLE 5-2. VALIDATION DATA REQUIREMENTS

- SCENARIO DATA
 - Channel Definition
 - Local Obstacles
 - Vessel Definition
- ENVIRONMENT DATA
 - Current Definition in Channel
 - Wind History
- CONTROL DATA
 - Rudder Command History
 - RPM Command History
- TRACKLINE DATA
 - Position History
 - Yaw and Yaw Rate History
 - Speed History

Data collection of lesser fidelity has already been undertaken by the USCG.¹⁷ Acquisition of the data elements above, with the necessary fidelity, represents a significant undertaking in all data elements.

The scenario data is the most readily obtainable from charts, visual observations, and photographic data. However, the vessel must either be one for which a representation exists in terms of parameters in the USCG supplied vessel dynamics model, or the vessel dynamic parameters must be derived and validated. Additionally, the environmental data, current and winds, must be available or derivable in a form compatible with the vessel dynamic model, to the degree of accuracy required to support the quantitative validation. The control data and trackline data must be recorded with time, granularity, and accuracy commensurate with the quantitative validation goals. Full utilization of this data will require a mechanical data gathering device, particularly to obtain vessel absolute position within the channel to a

valid degree of accuracy. This position can be established accurately by such means as LORAN, NAVSTAR, conventional range and bearing observations, triangulated bearings, or aerial photography.

Based on the above discussion, the collection of a comprehensive data base of detailed quantitative data to allow point-by-point comparison of model results with real vessel passage appears prohibitively expensive. There is, however, a possibility of obtaining data on a limited number of passages of previously modeled vessels in previously modeled channels with known current/wind characteristics. While this data is extremely useful for validation and tuning of the VNSS for these particular cases, it does not generally allow the validation of the model against a wider class of vessel/scenario combinations. Similar analysis concludes that the use of a scale model basin, such as the CAORF, David Taylor Basin, or Netherlands Model Basin, are prohibitively expensive except as sources of data on limited vessel/scenario combinations. Therefore, detailed data from real passages or simulated basin passages is of cost-effective utility for point-by-point quantitative model validation, if it can be obtained either "piggy-back" on existing experiments or by inexpensive on-board observations and measurements. Such data is most useful in the final phases of the validation process, for model certification after the model is fine tuned through the validation process.

With the unavailability of the above detailed, accurate data at non-prohibitive expense, the VNSS validation must rely on a process other than point-by-point comparison of the problem solution to a real passage either in a channel or in a model basin.

An alternate source of trackline data for comparison purposes is the USCG man-in-the-loop hybrid "Towboat Maneuvering Simulator,"¹⁸ which can be

utilized to generate a valid passage, or set of allowable passages, of a given channel with a specific vessel. This facility is extremely cost-effective to generate realistic passages in a short time. The "quality" of the passage is, of course, a function of the person exercising the control panel of the simulation. Therefore, trackline data for comparative analysis must be generated by a knowledgeable non-pilot or by an expert pilot. Validation of the results of the hybrid simulator, to be used as a reference for VNSS validation, therefore, becomes an issue. However, even with validation of the hybrid simulation results required, it remains a more viable source of tracklines and control histories than real or model vessel passages.

Alternatively, validation can be performed through the use of an expert pilot, to both define a range of realistic tracklines for given problem scenarios and to review the resultant VNSS control history and trackline for adequacy as response to the problem scenario. This may include the use of the hybrid simulator as discussed above. This source of data and evaluation is extremely cost effective for both model tuning and absolute validation. As a caveat, however, such validation is a good-bad objective indicator rather than a detailed quantitative evaluation of the quality of the solution. Additionally, subjective biases may enter the pilots judgement when transferred from a real vessel scenario to a conceptual problem, i.e., replacing "seat of the pants" navigation with "intellectualizing" of the problem.

Finally, quantification of solution effectiveness in model validation can be accomplished through the construction of a figure of merit (FOM) as a quantitative representation of the attributes of the solution. The important components of the FOM may be identified with the aid of expert pilots, experts in pilot behavioral studies, and USCG and other maritime agencies.

Components of this FOM may include attributes of the trackline, such as minimum and average distances from channel banks and obstacles, as well as characteristics of the control history such as average or total integrated control utilization, and minimum or average reserve control authority where reserve control authority is the difference between the maximum achievable control effort (rudder deflection) and the actual, applied control effort. This latter element has been suggested as one of the considerations of pilot behavioral analysis. The FOM, as a quantification of the solution quality, must be derived and developed from the qualitative judgments of solution quality as discussed above. Once a valid FOM is derived, the VNSS can be validated against a broad spectrum of scenario and vessel characteristics quickly, inexpensively, and effectively.

Table 5-3 summarizes the various validation techniques discussed above.

TABLE 5-3. VALIDATION TECHNIQUE SUMMARY

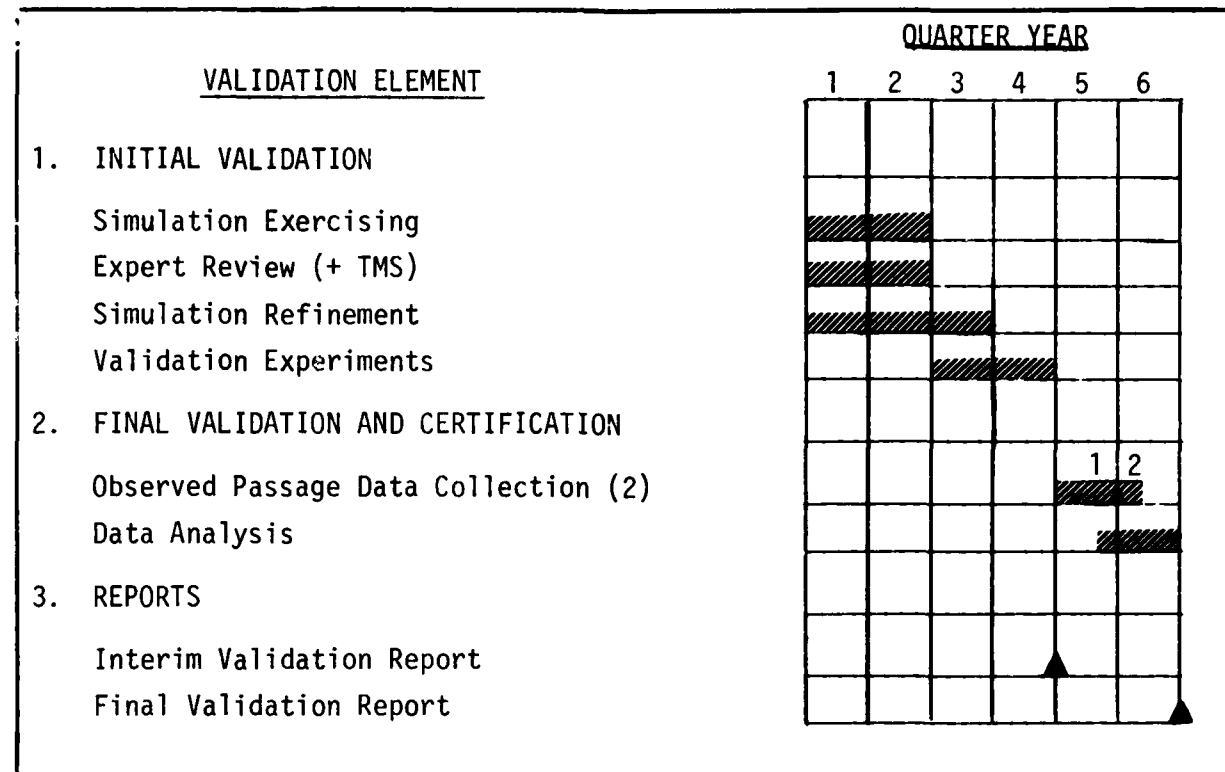
<u>DATA REFERENCE SOURCE</u>	<u>COST/EFFECTIVENESS</u>
• Experimental Real Passage	Very Poor
• Model Basin Passage	Very Poor
• Fully Instrumented Real Passage	Poor
• "Piggy-Back Data Gathering Passage	Good for Isolated Cases
• Towboat Maneuver Simulation (TMS) and/or Expert Consultation	Very Good
• Figure of Merit Analysis	Very Good

From the above validation techniques, the last three are most applicable to the validation of the VNSS. The validation process would start with an in-depth simulation exercise, where an extensive series of VNSS scenarios would be constructed, the cost and tracklines generated, and the solutions analyzed in detail. Expert pilot consultation and, potentially, the TMS as an adjunct to the expert analysis, would be utilized to subjectively evaluate the

the solutions and to "tune" the VNSS in terms of the obstacle modeling, channel boundary effects, and objective functions. Such "tuning" considerations may include obstacle buffer zones, switching obstacles out of the scenario under appropriate conditions, and inclusion of the bow and stern or the vessel corner points in the objective function rather than the vessel center of gravity as currently modeled.

Based on these experiments, tuning and evaluation, a figure of merit (FOM), would be constructed and validation would proceed with expert consultation, the TMS, and FOM analysis. When simulation has been extensively validated through this experimental program of VNSS simulation exercise, and is in a final state of refinement, a limited series of observed passage data collection would be performed on an as available, commercial, "piggy-back" basis. VNSS models of the same passage, bracketing the passage conditions in terms of the variable (current and wind) elements of the scenario, would be extensively analyzed and compared to the observed passage data as a final certification of the VNSS. The schedule for this Validation Plan, spanning 1½ years, is shown in Figure 5-1.

FIGURE 5-1. VALIDATION PLAN SCHEDULE



5.3 VNSS Utilization and Extension

The techniques used herein can be extended in many directions. Important examples include the modelling of imperfect pilot knowledge of either vessel dynamics or of the scene, including channel boundaries, current, or winds. The current VNSS uses the same dynamics and scene models for construction of the optimal control history and the generation of instantaneous vessel position. Use of different models for these two functions would accurately represent the mismatch of pilot knowledge to reality. Further extension can include the effect of pilot detection lag with respect to obstacles, to differentiate between "expected" conditions such as bank locations and "unexpected" conditions such as previously non-existing obstacles. Such unexpected obstacles would require a significantly longer recognition time in a real scenario.

An interesting extension of the techniques applied herein is collision avoidance, where the obstacle is the time-varying trajectory of another vessel. Mismatch between the pilot expectation of the opposing vessel trajectory and the actual trajectory would model pilot decision error in the collision avoidance problem.

Finally, the VNSS provides a base for performing parametric sensitivity studies to determine underlying conditional causes for vessel accidents including effects of observation length (visibility condition), current conditions, channel widths, or obstacle placement.

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¹⁷Huffner, op. cit.

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