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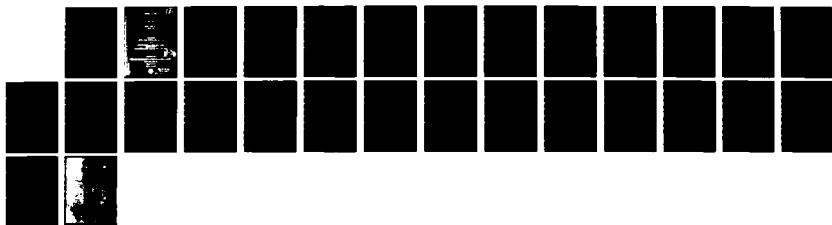
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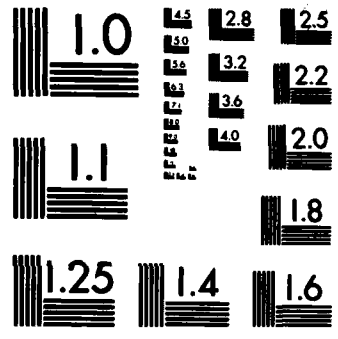
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DOUBLE AUCTIONS

by

Robert Wilson

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1. Introduction

In practice, markets are organized in many different ways. They differ in the trading rules that regulate the exchange process, and in the special roles played by intermediaries such as brokers and specialists. In every case, however, the terms of trade ultimately depend upon the actions chosen by the participants. Each agent therefore has an incentive to take account of this dependence in deciding on the trading strategy to use.

Much of modern economic theory in the Walrasian mold, nevertheless, has been based on a model that assumes that buyers and sellers respond to prices that are known beforehand to clear the market. Though implausible, this model is at least consistent: if the agents' preferences and endowments are common knowledge then (under suitable regularity assumptions) there exist prices that, if calculated and fixed beforehand, would achieve equality between the buyers' demands and the sellers' supplies. Or, if the agents have enough experience to anticipate fairly accurately the clearing price then this model may be a good predictor, as indeed it has been in many of the experiments that have been conducted (Smith [1981]).

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A fundamental theory of markets requires a better approximation than the Walrasian model provides. There are at least two desiderata. One is that an agent's preferences and perhaps endowment are not likely to be known to others. A second is that the numbers of buyers and sellers need not be so large as to yield "perfect competition." That is, the theory provides a better approximation to a wider range of phenomena if it encompasses trading among a few participants with incomplete information. This agenda is followed in the recent work that models the trading process as a noncooperative game with incomplete information. In this approach the criterion of Nash equilibrium among the agents' trading strategies is used to predict the outcome. That is, each agent's strategy is supposed to be an optimal response to the others' strategies.

Studying every possible set of trading rules in this way poses an enormous task, however. It is best to study first those trading rules that are important in practice or that have salient theoretical properties.

In this paper I study an example of a sealed-bid double auction. An auction of this kind institutionalizes the Marshallian conception of intersecting demand and supply curves. The rules specify that each buyer or seller submits a sealed bid or offer. These bids and offers are then arrayed into aggregate demand and supply curves and a clearing price is selected from their intersection. Those bids (offers) above (below) the clearing price are executed at the clearing price.

I choose sealed-bid double auctions to study in part because they are amenable to quite simple analysis. As we shall see, though, many of the results are applicable to a wider class of trading rules: the theory of incentive compatibility is used to establish that the main properties depend on the agents' chances of executing a trade. For the example studied, a sealed-bid double auction is an efficient trading process. Other trading processes that give the agents the same chances of executing a trade are also efficient. The results derived here might, therefore, have applications to trading rules found more commonly in practice. The prime candidate is the oral double auction used in experimental studies (Smith, Williams, Bratton, and Vannoni [1982], Plott and Smith [1978], Easley and Ledyard [1981]).

Our analysis is derived from the insightful studies of the one-buyer, one-seller case by Myerson and Satterthwaite [1981] and by Chatterjee and Samuelson [1979]. Section 2 formulates the model; Section 3 reviews the main results established in earlier studies, appropriately generalized to the case of multiple buyers and sellers, and develops the sufficient condition for a double auction to be an efficient trading process. Section 4 analyzes an example for which it is established that a double auction is indeed an efficient trading process. The case of many buyers and sellers is examined in Section 5. Some concluding remarks are included in Section 6.

2. Formulation

We consider a market in which two homogenous goods are traded. One is finely divisible and interpreted as money; the other occurs in

indivisible units, or items, any one of which is a perfect substitute for any other. There are $m + n$ agents, of whom m are buyers indexed by $i \in M$ and n are sellers indexed by $j \in N$. Each buyer i has an inelastic demand for one item at a valuation $u_i \in U$, and if he receives x_i items and pays p_i then his net payoff is $x_i u_i - p_i$. Similarly, each seller j has an inelastic supply of one item at a valuation $v_j \in V$, and if he gives up y_j items and receives q_j then his net payoff is $q_j - y_j v_j$. The agents' preferences are devoid of risk aversion and income effects, and each buyer has an ample endowment of money. A feasible allocation requires that each $x_i, y_j \in \{0,1\}$ and

$$\sum_i x_i = \sum_j y_j, \quad \sum_i p_i = \sum_j q_j.$$

Each agent knows whether he is a buyer or a seller and he knows his valuation; any other agent knows only the probability distribution of this valuation. The agents' valuations are distributed independently. The buyers' valuations are distributed identically on an interval $U = [u^0, u^1]$ with a distribution function F having a positive density f . Similarly, the sellers' valuations are distributed identically on an interval $V = [v^0, v^1]$ with a distribution function G having a positive density g .

All of the foregoing, as well as the trading rules to be specified subsequently, are common knowledge among the agents. That is, they define the rules of the game.

In general, a trading rule specifies a set R_i or S_j of feasible actions for each buyer i or seller j , and a function

$$\tau: (x_i \in R_i) \times (x_j \in S_j) \rightarrow A \subseteq M(\{0,1\} \times R^M),$$

that maps combinations of actions into the collection A of probability distributions over the set of feasible allocations. With such a trading rule a strategy for a buyer i or a seller j is a function

$$\rho_i: U \rightarrow R_i, \text{ or } \sigma_j: V \rightarrow S_j,$$

specifying his action contingent on his valuation. Here we are mainly concerned with revelation games in which $R_i = U$ and $S_j = V$. A direct revelation game has the further property that one obtains a Nash equilibrium in which the strategies are identity functions. It is easily verified that if (ρ, σ) is a strategy combination that is a Nash equilibrium for the game with trading rule τ then the composition $\hat{\tau} = \tau \circ (\rho, \sigma)$ yields a direct revelation game.

A (sealed-bid) double auction is a revelation game with the following trading rule. Let $r[k]$ be the k -th largest of the buyers' bids and $s[k]$ the k -th smallest of the sellers' offers, where $r[k] = -\infty$ if $k > m$ and $s[k] = \infty$ if $k > n$. Specify

$$p = [r[K] + s[K]]/2, \text{ where } K = \max\{k | r[k] > s[k]\}.$$

Then

$$(x_i, p_i) = (1, p) \text{ if } r_i > p \text{ and } (0, 0) \text{ otherwise,}$$

and

$$(y_j, q_j) = (1, p) \text{ if } s_j < p \text{ and } (0, 0) \text{ otherwise.}$$

That is, all trades are executed at the price that is the midpoint of the interval of prices that will clear the market.

3. Direct Revelation Games

In this section we review some of the properties of direct revelation games that can be found in Myerson and Satterthwaite [1981] for example.

To ensure that the problem is nontrivial assume that $u^1 > v^0$ so that ex ante there is some prospect of gains from trade.

Let $U_i(u)$ be the expected payoff of buyer i conditional on his valuation being $u_i = u$. This can be written as $U_i(u) = uP_i(u) - R_i(u)$ where, with the same conditioning, $P_i(u)$ is his probability of acquiring an item and $R_i(u)$ is his expected payment. Similarly, let $V_j(v)$ be the expected payoff of seller j conditional on his valuation being $v_j = v$, written as $V_j(v) = S_j(v) - vQ_j(v)$.

Lemma 1: In a direct revelation game U_i is convex and nondecreasing, $U_i' = P_i$ almost everywhere on U , and

$$(3.1) \quad R_i(u) - R_i(u^0) = \int_{u^0}^u t \, dP_i(t) \quad .$$

The sellers' expected payoffs have analogous properties.

Proof: In a direct revelation game buyer i 's strategy has $\rho_i(\hat{u}) = \hat{u}$, and for this to be optimal requires that $U_i(\hat{u}) > uP_i(\hat{u}) - R_i(u)$ for every $u, \hat{u} \in U$. Collecting terms appropriately yields

$$U_i(\hat{u}) > U_i(u) + [\hat{u} - u] \cdot P_i(u) ,$$

implying that U_i has a supporting hyperplane at u with slope $P_i(u) > 0$. Thus U_i is convex and nondecreasing with derivative $U_i'(u) = P_i(u)$ almost everywhere. This in turn implies that $R_i'(u) = uP_i'(u)$, which yields (3.1).

It is valuable to note that the relationship (1) between the agent's expected payment and his probability of trading has implications for more general games than direct revelation games. As noted in Section 2, a Nash equilibrium (ρ, σ) for a general trading rule τ induces a direct revelation game. Consequently, the properties in Lemma 1 are in general necessary conditions for a Nash equilibrium.

The trading rules encountered in practice have the property that each agent has a feasible strategy that ensures a nonnegative payoff. In a double auction, for instance, the identity function is a strategy that has this property; or, one could allow the possibility that an agent does not participate. In the corresponding induced direct revelation game this property implies that $U_i(u^0) > 0$ for each buyer i , and similarly $V_j(v^1) > 0$ for each seller j . Myerson and Satterthwaite [1981] obtain a remarkably simple characterization of the trading rules that satisfy this requirement. Denote an allocation's assignment of items by a subset $A \subseteq M \cup N$, indicating that those buyers $i \in A$ receive items from those sellers $j \in A$. Recall that a trading rule τ specifies a probability distribution over such assignments contingent on each combination (r, s) of the agents' actions.

Lemma 2: In a direct revelation game

$$\begin{aligned} & E\left\{ \sum_{i \in A} [u_i - [1 - F(u_i)]/f(u_i)] - \sum_{j \in A} [v_j + G(v_j)/g(v_j)] \right\} \\ &= \sum_i U_i(u_i^0) + \sum_j V_j(v_j^1) \quad , \end{aligned}$$

where E here denotes expectation with respect to u, v, A .

Proof: Observe first that:

$$E\left\{ \sum_{i \in A} u_i - \sum_{j \in A} v_j \right\} = \sum_i E\{U_i(u_i)\} + \sum_j E\{V_j(v_j)\}$$

Consequently, it suffices to construct the buyers' terms:

$$\begin{aligned} \sum_i E\{U_i(u_i)\} &= \sum_i E\left\{U_i(u_i^0) + \int_u^{u_i} P_i(t) dt\right\} \\ &= \sum_i U_i(u_i^0) + \sum_i E\left\{[1 - F(u_i)]/f(u_i) \cdot P_i(u_i)\right\} \\ &= \sum_i U_i(u_i^0) + E\left\{\sum_{i \in A} [1 - F(u_i)]/f(u_i)\right\} \quad . \end{aligned}$$

Here, the first equality uses Lemma 1. The sellers' terms are constructed similarly.

The property that each agent obtains a nonnegative expected payoff is called individual rationality. In the sequel we require that a trading rule has this property. In view of Lemmas 1 and 2, a direct revelation game satisfies individual rationality only if the inequality

$$(3.2) \quad E\left\{ \sum_{i \in A} [u_i - [1 - F(u_i)]/f(u_i)] - \sum_{j \in A} [v_j + G(v_j)/g(v_j)] \right\} > 0$$

is satisfied. Since a Nash equilibrium for any trading rule induces a direct revelation game, this inequality is also necessary for individual rationality in the general case. In the sequel we consider only cases in which (3.2) is an equality.

The study of efficient trading rules is facilitated considerably by Lemma 2. Consider the problem of designing a trading rule for which the expected gains from trade are maximized. Of course in comparing two trading rules one evaluates the agents' expected payoffs using their Nash equilibrium strategies.

Lemma 3: A direct revelation game with trading rule τ is efficient (and maximizes the expected gains from trade) if there exists a number $\lambda > 0$ such that $\tau(r,s)[A] > 0$ only if

$$(3.3) \quad A \in \arg \max_{A'} \sum_{i \in A'} [(1 + \lambda) \cdot r_i - \lambda \cdot [1 - F(r_i)]/f(r_i)] \\ - \sum_{j \in A'} [(1 + \lambda) \cdot s_j + \lambda \cdot G(s_j)/g(s_j)] .$$

Proof: Such a trading rule maximizes the expected gains from trade,

$$E\left\{ \sum_{i \in A} u_i - \sum_{j \in A} v_j \right\} ,$$

subject to the constraint (3.2), for which λ is a Lagrange multiplier.

The interpretation of Lemma 3 in the context of more general revelation games provides the main tool used in the analysis of double auctions in the next section.

Theorem 1: A double auction for which the Nash equilibrium strategies are

$$p_i(u) = \phi(u - \alpha \cdot [1 - F(u)]/f(u))$$

and

$$s_j(v) = \phi(v + \alpha \cdot G(v)/g(v)) ,$$

for some $\alpha \in [0,1]$ and some increasing function ϕ , is an efficient trading rule (and maximizes the expected gains from trade).

Proof: Recall that a double auction chooses

$$A \in \arg \max_{A'} \sum_{i \in A'} r_i - \sum_{j \in A'} s_j .$$

If the agents' strategies have the indicated form, and $\alpha = \lambda/[1 + \lambda]$, then this assignment is precisely the same as the one chosen in (3.3); that is, the selection of the assignment depends only on ordinal comparisons that are unaffected by the transformation ϕ .

4. An Example of a Double Auction

In this section we study an especially simple example of a double auction. The analysis of this example is guided by Theorem 1. Our aim is to verify that for this example a double auction is an efficient trading process. To do this we confirm that there exists a Nash equilibrium satisfying the sufficient condition specified in Theorem 1.

The special features of the example are that the numbers of buyers and sellers are the same ($m = n$), and the agents' valuations are all uniformly distributed on the same interval. Thus we can take $U = V = [0,1]$ and then $F(u) = u$ and $G(v) = v$. In this case Theorem 1 can be interpreted as saying that if there is a Nash equilibrium of the form

$$\rho_i(u) = \psi(u - \delta) \quad \text{and} \quad \sigma_j(v) = \psi(v + \delta)$$

for some number $\delta \in [0, 1/4]$ and some increasing function ψ , then the double auction is an efficient trading rule for this example. (Here, $\delta = .5\alpha/[1 + \alpha]$ and $\psi(x) = \phi([x + .5\alpha]/[1 + \alpha])$.) Initially we shall also adopt the hypothesis that ψ has the property that $\psi(x) + \psi(1 - x) = 1$ and then verify later that this property is satisfied. Further, as a technical matter it is well to realize that there is more than one function ψ that will provide the requisite Nash equilibrium. We will seek one that is differentiable and characterize it via a differential equation. However, typically this solution will have the unfortunate property that $\rho_i(u) > u$ if $u < 2\delta$, and analogously for the sellers. This is inconsequential for the Nash equilibrium, since no trade occurs in this situation, but in practice it is better to take the so-called perfect Nash equilibrium in which $\rho_i(u) = \min\{u, \psi(u - \delta)\}$ so that a buyer is not upset to find that a seller has used some strategy other than the one hypothesized in the equilibrium.

Our method is as follows. We first analyze the decision problem of a typical buyer i , and then indicate briefly the results of a

corresponding analysis of the decision problem of a typical seller. For the typical buyer we first suppose that each other buyer and the sellers are all using strategies of the form indicated in the hypothesized Nash equilibrium, and then determine which strategy is his optimal response. This best-response strategy will indeed be the same one specified in the Nash equilibrium if ψ satisfies a certain differential equation. As it turns out, this differential equation is the same as the one implied by the analysis of a typical seller's decision problem, and the solution ψ has the requisite properties. We then deal with a few loose ends, such as the determination of the right value of δ .

The following notation is helpful. Let $r(k)$ be the k -th largest bid among the buyers except buyer i , using the conventions that $r(0) = \infty$ and $r(m) = -\infty$. Defining

$$Y_k = \min \{r(k-1), s[k+1]\}$$

and

$$Z_k = \min \{r(k-1), s[k]\} ,$$

let $K = \arg \max_k Z_k$ and set

$$Y = \max \{Y_{K-1}, Y_K\} , \quad Z = Z_K .$$

One can check that $Y > Z$ and that $Y = Z$ only if $s[K] = r(K-1)$ or $s[K+1]$.

We will use the property of a double auction that buyer i trades if and only if $r_i > Z$. That is, if i is to trade it must be for

every k that if $r_i < r(k-1)$ then $r_i > s[k]$. Furthermore, though it is somewhat tedious to explain, if $Y > r_i > Z$ then he trades at the price $p = [r_i + Z]/2$, and if $r_i > Y$ then he trades at the price $p = [Y + Z]/2$. For example, if $Z = s[K]$ then $Y = Y_K$ and $Z = Y_{K-1}$, and the configuration is necessarily

$$r(K) < s[K] < r_i < \min \{r(K-1), s[K+1]\} \quad ;$$

hence, there are K trades and i is the marginal buyer. Similarly, if $Z = r(K-1)$ then $Z = Y_K$ and $Y = Y_{K-1}$, and the configuration is

$$s[K-1] < r(K-1) < r_i < \min \{r(K-2), s[K]\} \quad ;$$

hence, there are $K-1$ trades and again i is the marginal buyer. Now due to the hypothesized form of the Nash equilibrium strategies this amounts to saying that he trades at the first of these prices if $\psi(y) > r_i > \psi(z)$, and at the second if $r_i > \psi(y)$, where

$$\psi^{-1}(Z) \equiv z = \max_k \min \{u(k-1) - \delta, v[k] + \delta\}$$

and analogously for y , using corresponding definitions in terms of the rank ordered values of the other buyers' valuations- δ and the sellers' valuations+ δ . Thus, if we let $\hat{H}(y, z; \delta)$ be the joint distribution function of the pair (y, z) (on the support $\{y > z\}$) then buyer i 's expected payoff when his valuation is u and he bids r is

$$\begin{aligned} \int_{z < \psi^{-1}(r)} u \, d\hat{H}(y, z; \delta) - \int_{y < \psi^{-1}(r)} [\psi(y) + \psi(z)]/2 \, d\hat{H}(y, z; \delta) \\ - \int_{z < \psi^{-1}(r) \cap y} [r + \psi(z)]/2 \, d\hat{H}(y, z; \delta) \quad . \end{aligned}$$

To verify the Nash equilibrium we must show that $\rho_1(u)$ is the value of r that maximizes this expected payoff.

To simplify notation, let $H(x; \delta) = \Pr\{z < x < y\}$ and $h(x; \delta) = \Pr\{z = x < y\}$; that is

$$H(x; \delta) = \int_{z < x < y} d\hat{H}(y, z; \delta)$$

and

$$h(x; \delta) = \int_{x < y} d \frac{\partial \hat{H}}{\partial z}(y, x; \delta) .$$

Note too that $\partial \psi^{-1}(r) / \partial r = 1 / \psi'(x)$ if $r = \psi(x)$. Consequently, the first-order necessary condition for the optimal choice of r is

$$[u - r] \cdot h(\psi^{-1}(r); \delta) \cdot \frac{\partial \psi^{-1}(r)}{\partial r} = \frac{1}{2} \cdot H(\psi^{-1}(r); \delta) .$$

At the required value $r = \psi(u - \delta)$ this is just the differential equation

$$(4.1) \quad [u - \psi(u - \delta)] \cdot h(u - \delta; \delta) = \frac{1}{2} \cdot H(u - \delta; \delta) \cdot \psi'(u - \delta) .$$

which characterizes what the function ψ must be in order to sustain the hypothesized Nash equilibrium.

Turning to the decision problem of a typical seller, a similar analysis invokes an analogous set of definitions of Y, Z , et cetera, except that the roles of the buyers and sellers are reversed, as are the max and min operators. The symmetry of the formulation and the fact that the valuations are all uniformly distributed imply, however, that

the corresponding random variables "y" and "z" defined for a seller (denote them by \hat{y} and \hat{z}) have the property that the pair $(1 - \hat{y}, 1 - \hat{z})$ has precisely the same probability distribution as does the pair (y, z) defined for a buyer. Consequently, the first-order necessary condition for the typical seller's optimal offer yields the following differential equation that ψ must satisfy in order to sustain the Nash equilibrium:

$$(4.2) \quad [\psi(v + \delta) - v] \cdot h(1 - (v + \delta); \delta) \\ = \frac{1}{2} \cdot H(1 - (v + \delta); \delta) \cdot \psi'(v + \delta) .$$

If $\psi(x) + \psi(1 - x) = 1$ as we supposed initially, then (4.1) and (4.2) are identical, since u and $1 - v$ have identical roles. With this proviso, therefore, we have confirmed that there exists a Nash equilibrium of the required form, and that ψ is given by a particular solution to the differential equation (4.1).

We can now resolve the loose ends in the argument. First, the particular solution that is to be chosen is determined by the condition that $\psi(\delta) = 2\delta$. That is, considering a buyer 1, there is no chance of trading if $u_1 < 2\delta$ so $U_1(2\delta) = U_1(u^0) = 0$, and to ensure that $\lim_{u \rightarrow 2\delta} U_1(u) = 0$ requires that $\rho_1(2\delta) = 2\delta$. The symmetric condition for a seller, $\psi(1 - \delta) = 1 - 2\delta$, then determines the appropriate value of δ . If $n = 1$ then (as we shall see below) $\delta = 1/8$, and presumably δ is a declining function of n , so we will take it for granted here that $\delta \in [0, 1/4)$ as required. The proviso that $\psi(x) + \psi(1 - x) = 1$

when $x \neq \delta$ or $1 - \delta$ is resolved purely on the grounds of symmetry between the buyers and the sellers, since throughout the roles of u and $1 - v$ are symmetric with appropriate reversals of the max and min operators. Lastly we mention that the first-order necessary condition for an optimal bid that was used in the construction can be shown to be a sufficient condition: one employs the properties of affiliated random variables developed by Milgrom and Weber [1982] to show that H and h have the required properties, and that ψ is increasing as required.

To illustrate the preceding analysis we apply it to the case $n = 1$ studied by Chatterjee and Samuelson [1979]. In this case $Y_0 = s_1$, $Y_1 = \infty$, and $Z_1 = s_1$, so $Y = \infty$ and $Z = s_1$. Similarly, $y = \infty$ and $z = v_1 + \delta$, so $H(x; \delta) = x - \delta$ and $h(x; \delta) = 1$ if $x < 1 - \delta$. Thus, with $x = u - \delta$ the differential equation (3.4) is

$$x + \delta - \psi = \frac{1}{2}(x - \delta)\psi' ,$$

for which the solution is

$$\psi(x) = \frac{2[x^3/3 - \delta^2 x] + C}{[x - \delta]^2} ,$$

for any particular choice of the constant C . Requiring that $\psi(\delta) = 2\delta$ determines the choice $C = 4\delta^3/3$ and yields $\psi(x) = [2x + 4\delta]/3$. Requiring further that $\psi(1 - \delta) = 1 - 2\delta$ fixes $\delta = 1/8$. Thus, the final form is $\psi(x) = [4x + 1]/6$ and the equilibrium strategies are $\rho_1(u) = [8u + 1]/12$ and $\sigma_j(v) = [8v + 3]/12$.

There is an alternative interpretation of the construction that emphasizes the role of Lemma 1. Let β and γ be the functions $\beta(u) = u - \delta$ and $\gamma(v) = v + \delta$. We have constructed the buyers' and sellers' strategies from the composition $(\rho, \sigma) = \psi \circ (\beta, \gamma)$, having derived ψ from an analysis of the requirements for a Nash equilibrium. Another route is to apply the trading rule τ directly to get the outcome given by the composition $\tau \circ (\beta, \gamma)$, with which there is associated an imputed price π . However, this imputed price will not satisfy the requirements of incentive compatibility imposed by (3.1). One seeks, therefore, a transformation μ yielding the actual price $p = \mu(\pi)$ that conforms to (3.1). Then ψ can be identified from the commutative property $\tau \circ \psi = \mu \circ \tau$. The figure below depicts the commutative diagram, using i to represent the identity map:

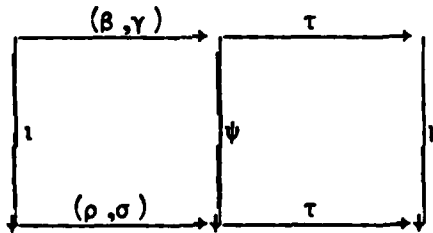


Figure 1: Commutative Diagram

In the case $n = 1$ considered above, $\mu = \psi$ since they are linear functions.

5. Many Buyers and Sellers

The connection between the Walrasian equilibrium and the Nash equilibrium of the double auction appears clearly in the example studied

in Section 4. The Walrasian equilibrium supposes, in effect, that each buyer or seller bids or offers his true valuation. Let π be the Walrasian clearing price, and let p be the clearing price in the Nash equilibrium. Then one would have $p \approx \psi(\pi)$ if the quantity traded were to be the same (and ψ is nearly linear), but in general the fact that $\delta > 0$ will entail fewer trades in the Nash equilibrium than is supposed in the Walrasian equilibrium.

The plausible conjecture is that the Walrasian and Nash equilibria are approximately the same if there are many buyers and sellers. We shall show, in fact, that for the example in Section 4 one gets $\delta \rightarrow 0$ and $\psi(x) \rightarrow x$ as $n \rightarrow \infty$.

The differential equation (4.1) admits the integrating factor

$$\Theta(x; \delta) = e^{2 \int_0^x \theta(t; \delta) dt} ,$$

where $x = u - \delta$ and $\theta = h/H$. Consequently, the solution has the form

$$\psi(x) = \delta + x - \int_{\delta}^x \frac{\Theta(t; \delta)}{\Theta(x; \delta)} dt ,$$

and one determines δ from the condition that $\psi(1 - \delta) = 1 - 2\delta$:

$$\delta = \frac{1}{2} \int_{\delta}^{1-\delta} \frac{\Theta(t; \delta)}{\Theta(1 - \delta; \delta)} dt .$$

One sees, therefore, that if $n \rightarrow \infty$ implies $\Theta(t; \delta)/\Theta(x; \delta) \rightarrow 0$ for $t < x$, then $\delta \rightarrow 0$ and $\psi(x) \rightarrow x$, as required. It suffices, therefore, to show that $\Theta(x; \delta) \rightarrow \infty$. In the present case

$$H(x; \delta) = \sum_{k=1}^n \binom{n}{k} \binom{n-1}{k-1} [x - \delta]^k [1 - (x - \delta)]^{n-k} [x + \delta]^{n-k} [1 - (x + \delta)]^{k-1}$$

and

$$h(x; \delta) = \sum_{k=1}^n \binom{n}{k} \binom{n-1}{k-1} \left[\frac{k}{x - \delta} + \frac{n-k}{x + \delta} \right] [x - \delta]^k \cdot [1 - (x - \delta)]^{n-k} [x + \delta]^{n-k} [1 - (x + \delta)]^{k-1} .$$

Thus, $\theta > n$, which is sufficient to prove the result. Of course p and π converge to $1/2$ almost surely.

For this example, therefore, the intuition that perfect competition, in the sense of many buyers and sellers, leads to the Walrasian equilibrium is confirmed.

6. Remarks

It remains an open question whether a double auction is an efficient trading process in more general circumstances than the example studied in Sections 4 and 5.^{1/} It is at least clear that in contexts in which the buyers and sellers are asymmetrically situated, either because there are differing numbers of agents on the two sides of the market or because their valuations are distributed differently, the method of analysis will need to be altered. Here we have relied on Lemma 3 which treats the agents symmetrically by taking the gains from trade as the measure of welfare. In the general case the welfare function imputed from a double auction is likely to assign differing weights to the

various agents. Moreover, adopting ex ante welfare maximization as the efficiency criterion may be unduly strong. If one adopts the weaker criterion of interim efficiency developed in the work of Holmström and Myerson [1981] then the appropriate criterion is to maximize

$$E\left\{\sum_i w_i(u_i)U_i(u_i) + \sum_j w_j(v_j)V_j(v_j)\right\} ,$$

for some system $w_i(\cdot)$ and $w_j(\cdot)$ of contingent weights. Using (3.1) this criterion can be put in a tractable form to produce analogues of Lemma 3 and Theorem 1, but we do not pursue this route any further here.

The sort of exercise pursued in Section 5 lies at the heart of the problem of unravelling the foundations of the Walrasian model of markets. Surely the justification for the Walrasian model lies in the hidden conjecture that when there are many agents on both sides of the market each agent's optimal strategy is very nearly to bid or offer his privately known valuation. Only if this conjecture is true can one suppose that aggregate demand and supply curves constructed from agents' bids and offers reflect truly their preferences.

Footnotes

- 1/ An easy generalization assumes that $m = n$ and $F(x) + G(1 - x) = 1$, but I have not succeeded in solving any cases that are genuinely asymmetric.

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