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UNIV PITTSBURGH PA DEPT OF MATHEMATICS R A NICOLAIDES

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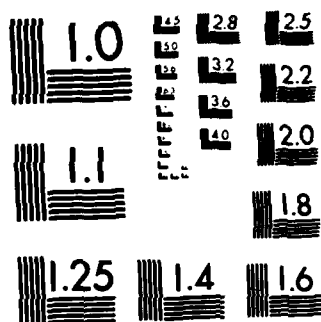
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The major thrust of the research undertaken was to find the most efficient methods for solving viscous flow problems governed by the Navier-Stokes equations. Both external flows past obstacles and internal flows in tanks and channels were investigated. Two aspects in particular have been studied most thoroughly. These are the finite-element discretization of the flow equations and the implementation aspects involving most effective use of hardware in order to obtain small solution times.		

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ITEM #20, CONTINUED:

The first topic involved a major mathematical investigation, and the problems have been solved essentially in their entirety. Specifically, simple rigorous tests have been developed for testing the stability of any potential finite element scheme. This problem which has hitherto resisted numerous attempts at a general solution is now reduced to a simple calculation with a small number of variables which is carried out a priori without the need for extensive computer calculations.

The second topic, which reduces to finding effective algorithms for solving the huge systems of nonlinear algebraic equations yielded by the discretization methods has been subdivided into two parts, associated with low and high Reynolds number flows respectively. For both of these cases new methods have been proposed and shown to be effective both theoretically and computationally.

Effort was also devoted to a number of other problems of interest. First, the question of periodic acoustic wave propagation, solved by mixed finite element methods was investigated. An extension of the theory for viscous flows was required and has been obtained. Second, the stability of Gaussian elimination applied to discrete Helmholtz problems, without pivoting, was studied via model problems. It was shown that contrary to various conjectures made in the literature the technique is unstable even under 'stabilizing' perturbations of the wave number. Third, it was realized that close connections exist between the finite element theory for low Reynolds number and that for the problems of linear elastomechanics posed in terms of stresses. The results obtained for the fluid case were then applied to the solution of some outstanding problems of the elasticity case.



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FINAL REPORT

Air Force Office of Scientific Research

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Period ending 5/31/1982

Proposal Title

Discretization and Solution Techniques
for Navier-Stokes and Transonic Flow Problems

Principal Investigator:

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A B S T R A C T

The major thrust of the research undertaken was to find the most efficient methods for solving viscous flow problems governed by the Navier-Stokes equations. Both external flows past obstacles and internal flows in tanks and channels were investigated. Two aspects in particular have been studied most thoroughly. These are the finite-element discretization of the flow equations and the implementation aspects involving most effective use of hardware in order to obtain small solution times.

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MATTHEW J. KEMNER

Chief, Technical Information Division

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Effort was also devoted to a number of other problems of interest. First, the question of periodic acoustic wave propagation, solved by mixed finite element methods was investigated. An extension of the theory for viscous flows was required and has been obtained. Second, the stability of Gaussian elimination applied to discrete Helmholtz problems, without pivoting, was studied via model problems. It was shown that contrary to various conjectures made in the literature the technique is unstable even under 'stabilizing' perturbations of the wave number. Third, it was realized that close connections exist between the finite element theory for low Reynolds number and that for the problems of linear elastomechanics posed in terms of stresses. The results obtained for the fluid case were then applied to the solution of some outstanding problems of the elasticity case.

1. Navier-Stokes flows

1.1 New finite element classes. [2] [4] [5] [6] [13]

Several new classes of finite element pairs have been introduced, which are of optimal accuracy in the $H_0^1(\dots) \times L_0^2(\dots)$ setting. Some of these pairs have been implemented in the Stokes, Oseen and Navier-Stokes cases. The results of the computer programs are as predicted and show in some cases an order of magnitude improvement in accuracy for the same amount of work as with conventional schemes.

1.2 Stability of Navier-Stokes discretization [7], [9], [10], [14]

The investigations revealed a remarkable dichotomy of behaviour between low order and high order finite element approximations. When the investigation began, the subject consisted of a number of special, often difficult proofs that a small number of discretizations indeed yielded stable approximations in the sense that as some mesh parameter h tended to zero, the computed solutions approached the exact solution of the Navier-Stokes equations. The finite elements covered by these proofs by no means included those used in practical engineering computations. The first problem solved consists of giving a test whereby any finite element pair of higher than quadratic order in velocities can be relatively simply processed and its stability or otherwise verified. The test was applied to verify the stability of numerous existing

computations and then to devise better and stable higher order schemes. The existence of this test completely disposes of the stability question for higher order element pairs. Thus, an outstanding problem was resolved.

Many engineering calculations in the aerospace and other fields are performed using element pairs of lower order than quadratic. One such pair, previously a candidate for the 'work-horse' of viscous flow calculations, is the bilinear/constant combination. Many calculations have been carried out with these elements. A major result of our work on the lower order elements has been that the pressure field for this element does not converge to the true pressure field of the Navier-Stokes equations as the meshsize h approaches zero. Although we showed how the resulting field could be reprocessed to obtain a useful pressure field in simple cases, it is not at all clear that this can be done in general. Thus, these popular elements have a basic flaw.

To circumvent this difficulty, several element pairs of comparable simplicity have been introduced and proved to be stable and hence useful for reliable computation. Nevertheless, the fact remains that low order elements are to be suspected of inherent instability from now on, in the absence of rigorous proofs to the contrary.

The work on low order elements was thought to be of sufficient interest to warrant an abstract theory, since

similar problems arise in other fields of computational mechanics. This work continues.

1.3 Solution algorithms. [7], [11], [12], [13]

For the low Reynolds number case, a method called the "semi-direct" solution algorithm has been introduced. To explain it, consider the system of equations

$$\begin{aligned} \underline{A}\underline{u} + Bp &= f & A &= A(\underline{u}) \\ B^T \underline{u} &= g \end{aligned}$$

which is typical of the form assumed by the stationary Navier-Stokes after discretization by finite element methods. Adding $-\epsilon p(|\epsilon| \ll 1)$ to both sides of the second equation and iterating according to

$$\begin{aligned} \underline{A}\underline{u}^{n+1} + B p^{n+1} &= f \\ B^T \underline{u}^{n+1} - \epsilon p^{n+1} &= g - \epsilon p^n \end{aligned}$$

leads to the inner iteration

$$(\underline{A} + \epsilon B B^T) \underline{u}^{n+1} = f - \frac{1}{\epsilon} B(g - \epsilon p^n).$$

The tremendous advantage of this problem over the original one is that the pressure p^{n+1} has been eliminated from the calculation. Because of this, computer resources required

are reduced by a factor of 10 or more over the original system. The inner iteration is implemented by a fast conjugate gradient algorithm. This technique has been thoroughly analysed and used by us in many different settings, always performing well, provided Re is not too large. It has in fact become a standard technique for these flows, offering the advantage over the more usual penalty method of computing the exact solution of the system, rather than the exact solution of a perturbed system.

For the higher Reynolds numbers, the semi-direct method is not needed, because the calculation can now be treated explicitly (convection dominated rather than diffusion dominated). Here, another problem, also associated with the pressure is encountered. The problem is that of enforcing the constraint of incompressibility which takes the form of the second of the above equations, namely $B \underline{u} = g = 0$. A new method for handling this has been introduced, based on the observation that for finite element discretizations the transpose of the matrix B occurs in the momentum equation. The technique is to observe that the velocity computation requires the value of Bp only, and not p itself. But Bp is simply the least squares solution of B times the momentum equation. This least squares computation can be done very cheaply (one or two iterations) in a time marching regime and thus the pressure difficulty may be overcome.

2.1 Periodic Acoustic Wave Propagation. [1], [3]

An extension of the analysis applicable to the viscous flow problems is required in order for the results to be valid for the acoustics case. This analysis has been successfully carried out, with the result that a significant improvement in accuracy for essentially the same amount of work over the least squares method previously used, is obtained.

A related study considered the solution of periodic acoustic phenomena governed by the Helmholtz equation. The objective here was to prove or disprove the proposition that Gaussian elimination could be applied directly without partial pivoting, provided the wave number was not an eigenvalue of the Laplacian operator. If true, this would considerably lessen the computer resources required for solution. Unfortunately, as we showed, the proposition is false. A rigorous analysis showed that even after making perturbations of the meshsize order to move the eigenvalues away from singular points the use of pivoting is essential in general. Otherwise, unbounded growth of the elimination multipliers can occur, and the ensuing growth of the roundoff errors will render the computations meaningless.

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Progress report on graduate student J.M. Boland

Mr. Boland will graduate Ph.D. Carnegie-Mellon University in January 1983. His work has been funded through AFOSR 80-0091.

Mr. Boland has been a truly outstanding student, maybe one of the two or three best of the last few years. Many people agree with this assessment.

Both he and I wish to offer our thanks to AFOSR (Mathematical Sciences Division) for the fine training facilities made possible through the support of AFOSR of the research projects reported herein.