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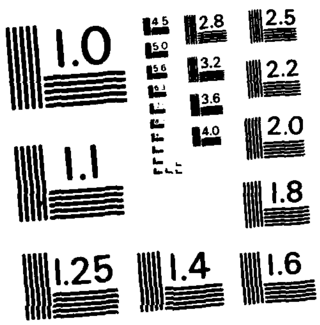

MICROCOPY RESOLUTION TEST CHART
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Final Report

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D.V. and G.V. Chudnovsky have successfully continued their studies of internal symmetries of important physical continuous and lattice (discrete) models, and the construction of exact lattice equivalents of continuous systems. These studies were based on the investigation of canonical (Bäcklund) transformations of multidimensional dynamical systems and properties of the symplectic structure of completely integrable systems in the classical and quantum cases.

During the years of the grant, the main subject of the investigation was an analysis of the intrinsic properties of two dimensional classical and quantum, and three dimensional classical physically relevant systems with (hidden) symmetries. The methods of Bäcklund transformations (BTs for short) were shown to play a crucial role in the determination of possible symplectic structure, the determination of their conservation laws and their actual solution [1], [2], [3], [4], [5], [6]. The concept of Bäcklund transformations (BTs) in its most general form was described by the authors as the isomonodromy deformation associated with the Riemann boundary value problem. This description of BTs can be traced back to Riemann, Lappo-Danilevsky, Garnier, et al. (see the introduction to the investigators' book [7]).

Using the method of BTs the investigators studied in detail classical and quantum two dimensional completely integrable systems, and lattice models equivalent to and approximating these continuous systems were constructed and examined. Among the continuous systems thoroughly examined in this way are the following important models of quantum field theory and statistical mechanics: the matrix (quantum and classical) nonlinear Schrodinger equation, the sin-Gordon equation, the matrix (non-Abelian) Toda lattice, the generalized XXX-and XXZ-quantum models and models of statistical mechanics, and the quantum generalized XYZ-lattice system (whose classical counterpart is the Landau-Lifshitz equation of superconductivity and whose statistical mechanics reductions include the eight-vertex model of Baxter), [1], [4], [5], [6], [8], [9], [10], [11].

Important properties of Bächlund transformations, that allow the construction of completely integrable systems and the determination on various symplectic structures associated with them, were described using the language of category theory and algebraic topology [12], [13], [10]. In this way classical and quantum systems were studied simultaneously using the algebra of BTs and the language of factorization axioms for the quantum S-matrices [10], [12], [5], [14].

The investigators applied the algebra of BTs to the three dimensional classical systems as well. Various identities between BTs produce the uniform description of all known matrix two dimensional completely integrable systems as subsystems of scalar three-dimensional equations of Zakhorov-Shabat type (equivalent to plasma physics equations of Kadomtzev-Petviashvili form) [10], [15]. These studies supplemented the earlier studies of the investigators on the new classes of infinite-dimensional Lie algebras [6], [8], [10].

Methods of BTs were used by the investigators to study Padė approximations of functions satisfying linear differential equations This was achieved using the interpretation of BTs as isomonodromy transformations and using the monodromy representation of Padė approximations. These connections were discovered for the first time by the investigators [3], [16], [17], [18]. Explicit Padė approximations were studied for algebraic functions and generalizations of Gauss hypergeometric functions. This provides us with new applications to the study of the diophantine approximations of algebraic numbers and values of the logarithmic and inverse trigonometric functions, including cubic irrationalities and numbers such as $\pi/\sqrt{3}$, cf. [17].

In 1981 the investigators visited and lectured at the Mittag-Leffler Institute in Stokholm (Sweden) during the special year of Mathematical Physics, the Laboratory of Theoretical Physics at Ecole Normale Superieure, Paris (France) and the Departmentof Applied Mathematics Weizmann Institute at Rehovoth (Israel). During the year 1981-82 the investigators were on Guggenheim fellowships.

The support of ONR was important for many of the investigators' projects. In 1982 the volume of proceedings of the investigators' seminar of 1979-80 on the Riemann problem was published [7]. The support of ONR is explicitly acknowledged with gratitude in the introduction.

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