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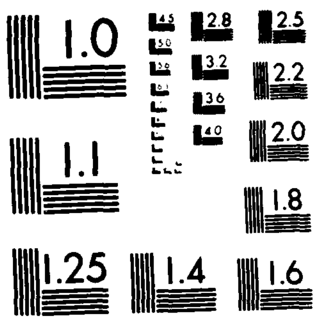

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TITLE

AN OUTPUT MATCHING APPROACH TO MULTIVARIABLE LINEAR DIGITAL CONTROL

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ABSTRACT

A simple and direct state space approach to the digital control of multivariable linear systems is discussed. Control is provided by minimizing the mean square error between controlled plant outputs and specified desired output trajectories at sampling instants. Systems of linear equations for digital control inputs result with solutions assuming a natural constant forward and feedback gain form. Optimal gains are determined using elementary results from linear systems theory and standard techniques from linear algebra. Numerical applications to examples in simple model following, digital redesign, and direct digital design are described. Partial state observability and the effects of sampling rate upon system performance are considered. Control smoothing through matching at multiple sampling instants is discussed. This strategy provides improved performance without additional computational expense or increased sampling rate.

I. RESEARCH OBJECTIVES

A general technique for the state space digital control of multivariable linear systems was presented in [1]. This technique is very similar in flavor to the Model Algorithmic control of Mehra [2] and the Model Reference Adaptive techniques surveyed by Landau [3]. Its basic idea is to provide system control by minimizing, at sampling instants, the mean square error between controlled plant outputs and specified reference trajectories. Digital control inputs are generated sequentially by solving systems of linear equations. Simple and direct control algorithms result which avoid the technical difficulties often associated with more classical optimal control techniques (e.g., nonlinear numerical optimization problems, Ricatti equation solutions, etc.).

Two specific problems were discussed in some detail in [1]: (1) the digital redesign problem and (2) the direct digital design problem. In the digital redesign problem, outputs of a linear plant with continuous controller are matched to the outputs of the same plant driven by piecewise constant (digital) control inputs. The objective is to determine inputs for the digitalized system which preserve, as closely as possible, the performance characteristics of the original continuous system. By contrast, the direct design problem is concerned with the direct determination of digital control inputs which force plant outputs to closely tract specified trajectories. These trajectories may be the outputs of existing baseline systems, dynamically generated paths to set points, or other appropriately constructed functions representing desired system evolutions.

The principle objectives of this research effort have been the further refinement of the basic control technique discussed in [1], and the application of this technique, through computer simulation, to realistic problems in digital redesign and direct design in order to evaluate its feasibility and effectiveness.

II. DESCRIPTION OF RESEARCH EFFORT

A detailed exposition of the major results of this research effort is presented in [4]. A descriptive summary of these results and other significant aspects of this effort is given here.

Further consideration of the ideas in [1] revealed that the solutions of the linear equations defining optimal control values at sampling instants assume a natural forward and feedback gain form. Specifically, each digital control input is simply the difference of the product of a feedback gain matrix and the current state vector, and the product of a forward gain matrix and the known or projected value of the reference trajectory one sampling period ahead. Since these gains are independent of time for time invariant plant equations, the digital control system assumes a more standard constant gain configuration. (See [4], figures 1 and 2). This configuration permits a concise formulation of the output matching digital control problem. Its solution is straightforward since optimal gains are easily determined from given system parameters using standard linear algebraic techniques. The formulation also underscores the simplicity of control implementation, and simplifies the inclusion of the digital observer in the case when not all system states are observable.

Having derived a precise problem formulation and solution procedure, in order to demonstrate the usefulness and flexibility of the output matching technique, its application to numerical examples was considered. Three examples were examined in detail:

1) a simple model following example,
2) the digital redesign problem of Kuo, Singh, and Yackel [5], and
3) the missile attitude control system direct design problem considered by Mehra [2]. (See [4] for detailed discussions and simulation graphics.)

Example (1) was chosen to illustrate the salient features of the output matching technique. Although exact output matching at sampling instants was realized in this example, simulations clearly showed the distinct inter-sample behavior of the exponential plant and the second order reference trajectory. Other simulations demonstrated the effects of reference system partial observability and one dimensional control constraints upon system performance.

Example (2) was selected to illustrate the application of output matching to a problem in digital redesign. The technique was used to digitalize the continuous control system of the simplified one-axis Skylab satellite of [5]. It was shown that the exact control objectives of [5] could be achieved using output matching, and furthermore, that a more general redesign problem could be considered, one in which the number of controlled variables does not have to equal the number of control variables. An application of this generalization to the average matching of states of the model in [5] resulted in better simultaneous matching of states than that provided by single state matching. The incorporation of the digital plant observer into the control scheme was also illustrated.

Example (3) was selected to illustrate the direct design technique. The problem of controlling the angle of attack and pitch and sideslip angles of the three axis hypothetical missile of [2] was studied. Since direct design appears to be the most important application of the output matching technique, this example was examined in considerable detail. It was shown that the basic control objectives of [2] could easily be achieved. Furthermore, a comparison of the computational requirements of the output matching technique and the impulse response technique of [2] showed that output matching required far fewer system constants to define control, and hence, correspondingly fewer algebraic operations for each control computation.

Attitude control simulations at the baseline conditions of [2] revealed marked intersample oscillations of outputs and rapid fluctuations of control inputs. (The fluctuations were observed in [2], but the oscillations of outputs were not, presumably being concealed by linear interpolation between computed data points.) Strategies for smoothing system control without having to increase the sampling rate were examined. The minimization of the integrated mean square error between plant outputs and reference trajectories as suggested in [1] was considered initially. Surprisingly, this performance criterion led to control gains which produced severe system instabilities. The introduction of matching at multiple sampling instants successfully permitted effective control smoothing at no additional computational expense. This was a particularly significant result since in [2], in order to smooth control fluctuations, the solution of a nonlinear numerical optimization problem using a gradient search algorithm was required for each control computation.

Theoretical considerations of the dependence of the effectiveness of the output matching technique on sampling rate failed to yield any conclusive, analytically precise relationships. Simulations illustrated only the obvious fact that closer matching of controlled system outputs and reference trajectories is provided by increased sampling rates. Since in many practical situations, exact output-reference trajectory matching at sampling instants is possible, intersample matching is often the most important factor in determining acceptable system performance. Consequently, effort was directed toward the intersample smoothing described in conjunction with example (3). Such smoothing provides the improved performance associated with an increased sampling rate, without requiring an actual increase.

Finally, the accommodation of control constraints by the output matching technique was given consideration. It was shown that the addition of a quadratic constraint term (as in linear regulator problems) to the mean square tracking performance index leaves the technique basically unchanged, introducing only a slight technical complication into the determination of optimal system gains. Alternatively, simulations showed that in cases when the general nature of the reference trajectories may be freely specified, as in paths to set points, controls may in practice be effectively constrained by limiting the growth rates of these trajectories.

III. CONCLUSIONS

A quite general output matching approach to digital multivariable linear control has been developed. It is based upon a problem formulated in constant forward and feedback gain form. It relies upon only the most elementary facts from linear systems theory, and hence is characterized by its conceptual simplicity. Optimal control gains are determined using only standard linear algebraic techniques.

Simulations of output matching as applied to examples in model following, digital redesign, and direct digital design have demonstrated its flexibility and ease of application. These simulations have also revealed that in cases when exact output matching is possible, intersample behavior of system outputs is of prime interest. A control smoothing strategy promises to provide improved system performance at no additional computational expense. In a missile control example, this strategy has produced impressive continuous tracking of exponential paths to set points at a relatively low sampling rate (10 samples per second).

This study has shown that the output matching technique is straightforward in application and surprisingly effective. This effectiveness, of course, depends upon sample rate and desired output trajectory selection. Since each linear plant has its own specific dynamical characteristics, it is felt that in any particular application, only experimentation will ultimately reveal those best choices of sampling rate and reference trajectories for given standards of acceptable system performance.

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18. SUPPLEMENTARY NOTES

The technical results of this research effort are presented in "Multivariable Linear Digital Control via State Space Output Matching" to be submitted to Optimal Control-Applications and Methods. (Wiley).

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multivariable Linear Systems, State space approach, digital control, output matching

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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