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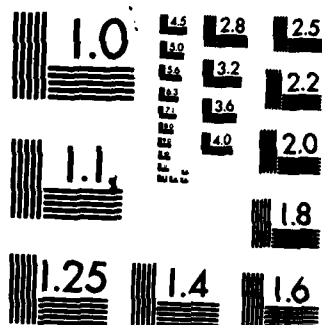
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A Technical Report

OPTIMAL CONSTRAINED REPRESENTATION AND
FILTERING OF SIGNALS

Submitted to:

Air Force Office of Scientific Research
Bolling Air Force Base
Washington, D. C. 20332

Attention: Dr. Robert Smythe

Submitted by:

Dimitri Kazakos
Associate Professor

Report No. UVA/525634/EE83/108

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ITEM #20, CONTINUED: continuous time. For large observation time and stationary signals, it is shown that the constrained optimal estimate is determined by the two spectral densities and a weighted Fourier Transform of the noise observations. The above result applies to both discrete time and continuous time signals.

The Wiener filter emerges as a special case of the constrained filtering estimate, when the linear subspace is enlarged to coincide with the total measurement space.

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Optimal Constrained Representation and
Filtering of Signals

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Abstract

A random signal is observed in independent random noise. We are addressing the problem of finding the optimum signal estimate that is constrained to lie in a given linear subspace. The optimality is defined in the sense of weighted mean square error. In the second step, we find the optimum linear subspace of given dimensionality. It is shown to be the linear manifold spanned by the eigenvectors of the simultaneous diagonalization of the signal and noise covariance, that correspond to the largest eigenvalues. The result is valid for both discrete and continuous time. For large observation time and stationary signals, it is shown that the constrained optimal estimate is determined by the two spectral densities and a weighted Fourier Transform of the noise observations. The above result applies to both discrete time and continuous time signals.

The Wiener filter emerges as a special case of the constrained filtering estimate, when the linear subspace is enlarged to coincide with the total measurement space.

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Key Words: Signal Filtering, Data Compression, and Data Representation.

I. General theory for discrete time signals

Consider a deterministic signal $x \in \mathbb{R}^n$ = n-dimensional Euclidean space. Let $V^m \triangleq \{V_1 \dots V_m; m \leq n\}$ be a collection of orthonormal column vectors $V_1 \in \mathbb{R}^n$. The minimum square error norm representation of x in the linear manifold of V^m is defined as the solution $(a_1 \dots a_m)$ that minimizes $\|x - \sum_{k=1}^m a_k V_k\|^2$. It is well known from linear algebra that the best approximation $\hat{x}$ on V^m is:

$$\hat{x} = Px \triangleq \left(\sum_{k=1}^m V_k V_k^T \right) x; \quad P = \text{projection operator} \quad (1)$$

If x is a random signal with zero mean and covariance matrix $R_x \triangleq Exx^T$, then it is easy to demonstrate that: [1]

$$S_m(V^n) \triangleq E\|x - \hat{x}\|^2 = \sum_{k=m+1}^n V_k^T R_x V_k \quad (2)$$

where $\hat{x}$ is the best approximation given by equation (1), and $S_m(V^n)$ is the minimum mean square error achievable for representation of x , using as basis the set V^n .

Suppose now that we wish to choose V^n so as to minimize $S_m(V^n)$. The minimization is achieved if V^n is chosen as the set of eigenvectors of R_x [1], and the resulting minimum value of $S_m(V^n)$ is: [1]

$$S_m^*(V^n) = \sum_{k=m+1}^n \lambda_k$$

where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ are the eigenvalues of R_x , ordered in decreasing magnitude.

Consider now the case of estimating x , on the basis of noisy data.

The available observation is:

$$\begin{aligned} y &= x + z, & Ex &= Ez = 0, & Exz^T &= 0 \\ Exx^T &= R_x, & Ezz^T &= R_z, & |R_z| &\neq 0 \end{aligned} \quad (4)$$

Let $\hat{x}$ be an estimate of x . The weighted mean square error $S_m(V^n)$ is defined as:

$$S_m(V^n) = E \left\| \hat{x} - x \right\|_{R_z^{-1}}^2 = E \left\| x_1 - \hat{x}_1 \right\|^2; \quad (5)$$

$$x_1 = R_z^{-1/2} x, \quad \hat{x}_1 = R_z^{-1/2} \hat{x}$$

and $R_z^{1/2}$ is the unique symmetric positive definite square root of R_z , which exists because $|R_z| \neq 0$. Let $y_1 = R_z^{-1/2} y$, $z_1 = R_z^{-1/2} z$. Then, multiplying (4) by $R_z^{-1/2}$ we achieve whitening of the noise: $y_1 = x_1 + z_1$, $Ez_1 z_1^T = I$.

We consider now estimates of x_1 of the form:

$$\hat{x}_1 = \sum_{k=1}^m c_k V_k V_k^T y_1 \quad (6)$$

From equation (5), the mean square error is found to be:

$$\begin{aligned} S_m(V^n) &\triangleq E \left\| x_1 - \hat{x}_1 \right\|^2 = \sum_{k=m+1}^n V_k^T R V_k + \sum_{k=1}^m c_k^2 [V_k^T R V_k + 1] \\ &\quad - 2c_k V_k^T R V_k + V_k^T R V_k \end{aligned} \quad (7)$$

where $R = R_z^{-1/2} R_x R_z^{-1/2}$.

If we minimize $S_m(V^n)$ over the choice of the constants $c_1 \dots c_m$, we find that the minimizing values are:

$$c_k = (V_k^T R V_k) (V_k^T R V_k + 1)^{-1} \quad (8)$$

with a resulting minimum of $S_m(V^n)$:

$$S_m^*(V^n) = \sum_{k=1}^m g(w_k) + \sum_{k=m+1}^n w_k \quad (9)$$

where:

$$w_k \triangleq V_k^T R V_k, \quad g(z) \triangleq 1 - (z + 1)^{-1} \leq z \quad (10)$$

Our next effort is to choose V^n so that $S_m^*(V^n)$ is minimized. From (9), (10) we observe that $\{w_k\}$ have to be minimized, and in such a manner that the m smallest achievable values are assigned to the second partial sum in equation (9), while the other $n-m$ larger ones are assigned to the first sum. So, we first minimize w_n , then $w_{n-1}, \dots, w_{m+1}, w_m, \dots, w_1$. At each step of minimizing w_j over V_j such that $\|V_j\|^2 = 1$, the previous vectors $V_{j+1}, V_{j+2}, \dots, V_n$ have been found and are fixed. Hence V_j has to be orthogonal to the previously found vectors $\{V_{j+1}, V_{j+2}, \dots, V_n\}$.

From matrix theory [2], we observe that the above procedure for finding the optimum set $\{V_j\}$ produces the eigenvectors $\{Q_j\}$ of R . Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be the corresponding eigenvalues. Let us denote by S_m^{**} the minimum over $\{V^n\}$ value of $S_m^*(V^n)$. Then,

$$S_m^{**} = \sum_{k=1}^m [1 - (1 + \lambda_k)^{-1}] + \sum_{k=m+1}^n \lambda_k \quad (11)$$

The corresponding optimal c_k 's are:

$$c_k = \lambda_k (1 + \lambda_k)^{-1} \quad (12)$$

and the optimal estimate $\hat{x}$ is:

$$\hat{x} = R_z^{1/2} \hat{x}_1 = \sum_{k=1}^m \lambda_k (1 + \lambda_k)^{-1} R_z^{1/2} Q_k Q_k^T R_z^{-1/2} y. \quad (13)$$

It is well known that the eigenvalues λ_k , of R satisfy the equation $|wR_z - R_x| = 0$, and Q_k are the eigenvectors of R_x with respect to R_z . [1] For the case $m=n$, the optimum estimator of x is identical to the Wiener filter, as can be easily verified. Thus, the new optimal representation or estimation of the noisy signal x can be viewed as a constrained linear estimate of prespecified dimensionality.

The main computational difficulty of the representation is the evaluation of the eigenvalues and eigenvectors $\{\lambda_k, Q_k; k=1, \dots, n\}$ of the matrix R_x with respect to the matrix R_n . There are some special cases in which the eigenvectors and eigenvalues can be evaluated in closed form, described next.

Example 1. Suppose that x is produced by uniform sampling every h sec. of a periodic process with period nh . Then R_x is a circulant: ($q_s = q_{-s}$)

$$R_x = \begin{bmatrix} q_0 & q_1 & q_2 & \dots & q_{n-1} \\ q_{n-1} & q_0 & q_1 & \dots & q_{n-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ q_1 & & & & q_0 \end{bmatrix} \quad (14)$$

i.e., each row is a cyclic permutation of the first one. Let, also z be white noise with $R_z = \sigma^2 I_n$. The matrix $R = R_z^{-1/2} R_x R_z^{-1/2} = \sigma^{-2} R_x$ has real eigenvalues [3], [4]:

$$\lambda_k = \sigma^{-2} \sum_{s=1}^n q_{s-1} \exp [-2\pi j(s-1)(k-1)n^{-1}]; k = 1, \dots, n. \quad (15)$$

The corresponding eigenvectors are:

$$Q_k = \{n^{-1/2} \exp [-2\pi j(k-1)(s-1)n^{-1}]; s = 1, \dots, n\}. \quad (16)$$

Hence, it is an easy matter to pick the m largest eigenvalues from (15), the corresponding eigenvectors from (16), and then formulate the optimum estimate according to equation (13).

Example 2. Tridiagonal correlation.

Let:

$$R_x = \begin{bmatrix} 1 & q & & 0 \\ q & 1 & & \\ & & \ddots & q \\ 0 & & q & 1 \end{bmatrix}; R_z = \sigma^2 I_n. \quad (17)$$

$$R = \sigma^{-2} R_x.$$

The eigenvalues of (17) are [5]:

$$\lambda_k = \sigma^{-2} (1 - 2|q| \cos[k\pi(n+1)^{-1}]); k = 1, \dots, n \quad (18)$$

The corresponding eigenvectors are [5]:

$$Q_k = \{[2/(n+1)]^{1/2} \sin[ks\pi(n+1)^{-1}]; s = 1, 2, \dots, n\} \quad (19)$$

We will consider next the case of stationary observations and large n . Then, the matrices R_x, R_z are Toeplitz. Let $s_x(\lambda), s_z(\lambda); \lambda \in [0, 2\pi]$ be the spectral densities of the processes x, z , assumed to be strictly positive for all $\lambda \in [0, 2\pi]$. We also assume that the autocorrelation sequences for x, z are square summable. Then, according to the theory developed in [4], we can determine the asymptotic distribution of the eigenvalues of the matrix R and the corresponding eigenvectors. The eigenvalues are for $k = 1, \dots, n$:

$$\lambda_k \approx s_x(2\pi kn^{-1}) s_z^{-1}(2\pi kn^{-1}) \quad (20)$$

$$Q_k \approx \{n^{-1/2} \exp[-2\pi j(k-1)(s-1)n^{-1}]; s = 1, \dots, n\} \quad (21)$$

The matrix R_z has the following approximate expression for large n :

$$R_z \approx \sum_{k=1}^n s_z(2\pi kn^{-1}) Q_k Q_k^T \quad (22)$$

For the optimal representation of x by an m dimensional subspace, the m indices corresponding to the m largest eigenvalues λ_k according to equation (20) will provide the solution. The corresponding eigenvector Q_k given by eq. (21) will determine the estimate $\hat{x}$, according to eq. (13). From equations (11), (20) we can derive the resulting mean square error:

$$\begin{aligned} S_m^{**} = & \sum_{k=1}^m [1 - [s_x(2\pi kn^{-1}) s_z^{-1}(2\pi kn^{-1}) + 1]^{-1}] + \\ & + \sum_{k=m+1}^n s_x(2\pi kn^{-1}) s_z^{-1}(2\pi kn^{-1}) \end{aligned} \quad (23)$$

We now let m be a fixed fraction of n , $m = np$, $0 < p < 1$. The question we consider now is: If a fraction p of the signal coordinates are allowed, what is the best achievable mean square error when $n \rightarrow \infty$? We are specifically interested in the asymptotic per sample mean square error, defined as:

$$S^{**}(p) \triangleq \lim_{n \rightarrow \infty} n^{-1} S_{np}^* \quad (\text{under the condition } m = np) \quad (24)$$

From (23), (24) we find:

$$\begin{aligned} S^{**}(p) = & (2\pi)^{-1} \int_{I_p} [1 - [s_x(\lambda) s_z^{-1}(\lambda) + 1]^{-1}] d\lambda + \\ & + (2\pi)^{-1} \int_{I_p} s_x(\lambda) s_z^{-1}(\lambda) d\lambda \end{aligned} \quad (25)$$

where I_p is a subset of the interval $[0, 2\pi]$, of Lebesgue measure $2\pi p$, and such that the largest values of $s_x(\lambda) s_z^{-1}(\lambda)$ are concentrated in I_p . In other words, for any $\lambda_1 \in I_p$, $\lambda_2 \notin I_p$, we will have $s_x(\lambda_1) s_z^{-1}(\lambda_1) \geq s_x(\lambda_2) s_z^{-1}(\lambda_2)$.

Our conclusions are useful for suboptimal filtering and compression of noisy data. The compression ratio when the subspace of dimensionality $m = np$ is used, is equal to p^{-1} .

From the theory of asymptotic approximation of Toeplitz matrices by circulant ones, ([3], [4]) we find that using (22), for large n , we have:

$$R_z^{\frac{1}{2}} \approx \sum_{k=1}^n s_z^{\frac{1}{2}} (2\pi k n^{-1}) Q_k Q_k^T; \quad R_z^{-\frac{1}{2}} \approx \sum_{k=1}^n s_z^{-\frac{1}{2}} (2\pi k n^{-1}) Q_k Q_k^T \quad (26)$$

where $\{Q_k\}$ are the Finite Fourier Transform vectors, defined by eq. (21).

Substituting the approximations (26), (20), (21) into (13), and using the orthogonality of $\{Q_k\}$, after some algebra we find the following expression for $\hat{x}$:

$$\hat{x} \approx \sum_{k=1}^m Q_k c_k y_k \quad (27)$$

where:

$$c_k \triangleq [1 - [s_x(2\pi kn^{-1}) s_z^{-1}(2\pi kn^{-1}) + 1]^{-1}] \quad (28)$$

and

$$y_k = Q_k^T y \quad (29)$$

are the Discrete Fourier Transform (DFT) coefficients of the vector y . They can be evaluated by Fast Fourier Transform methods.

Equation (27) provides the following conclusion. If one wishes to represent x using m of the DFT coefficients, and a noisy version of x is only available, the mean square optimal representation will be achieved through (27) - (29), assuming that s_x , s_z are known a-priori. The weighting coefficients c_k essentially act as optimal filtering coefficients in the orthogonal directions of the representation subspace. The directions are chosen so that the signal to noise ratio $s_x(2\pi kn^{-1}) \cdot s_z^{-1}(2\pi kn^{-1})$ is maximal.

It should be stressed that the estimate $\hat{x}$ is given by the approximate expression (27). The question of how well the approximate estimate performs as compared to the exact one, is of interest but not pursued here. For the unconstrained filtering problem, Pearl has derived interesting bounds for the variance of the approximate filtering estimate based on the Discrete Fourier Transform [6]. We believe that his work on asymptotic equivalence of spectral representations [7] is applicable in deriving bounds for the mean square error performance of the approximate constrained estimate (27).

II. Continuous time signals

We will consider now extension of the previous theory to continuous time random signals observed in the presence of noise. Let

$$y(t) = x(t) + z(t); \quad 0 \leq t \leq T$$

where $x(t)$, $z(t)$ are zero mean processes of finite mean square value, with correlation functions $R_x(t, s)$, $R_z(t, s)$ correspondingly.

Instead of developing a new theory parallel to the development of Section I, we will proceed stating the continuous time analogous results. All of the continuous time development can be made rigorous by using Kadota's simultaneous orthogonal expansion of two operators [8], [9].

Consider the operator:

$$R \triangleq R(t, s) = R_z^{-1/2} R_x R_z^{-1/2} \quad (30)$$

where $R_z^{-1/2}$ is the inverse of the square root of $R_z(t, s)$, appropriately defined. The assumption that R is a bounded and densely defined operator has to be made [9]. Then, R has a set of orthonormal eigenfunctions $\{\phi_k(t); k = 1, 2, \dots\}$ and corresponding eigenvalues $\{\lambda_1 \geq \lambda_2 \geq \dots\}$, satisfying the integral equation:

$$\int_0^T R(t, s) \phi_k(s) ds = \lambda_k \phi_k(t) \quad (31)$$

Let

$$z_k(t) = \int_0^T R_z^{-1/2}(t, s) \phi_k(s) ds \quad (32)$$

Then $\{z_k(t)\}$ are the eigenfunctions of R_x with respect to R_z with corresponding eigenvalues $\{\lambda_k\}$, in analogy to the time discrete case. The mean square error of estimating $x(t)$ using m coordinates only, will be minimized if

the eigenfunctions corresponding to the m largest eigenvalues are used. The eigenfunctions $z_k(t)$ satisfy the integral equation:

$$\int_0^T R_x(t, s) z_k(s) ds = \lambda_k \int_0^T R_z(t, s) z_k(s) ds \quad (33)$$

and the orthogonality condition with respect to R_z :

$$\int_0^T \int_0^T z_k(t) R_z(t, s) z_m^*(s) dt ds = \delta_{k, m} \quad (34)$$

The estimate $\hat{x}(t)$, after some manipulations, is found to have the expression:

$$\hat{x}(t) = \sum_{k=1}^m \lambda_k (1 + \lambda_k)^{-1} \int_0^T R_z(t, s) z_k(s) ds \cdot \int_0^T z_k(u) y(u) du \quad (35)$$

The resulting mean square error is:

$$S_m^{**} = \sum_{k=1}^m [1 - (\lambda_k + 1)^{-1}] + \sum_{k=m+1}^{\infty} \lambda_k \quad (36)$$

With the exception of some special cases mentioned in [8], the evaluation of eigenvalues and eigenfunctions cannot be achieved in closed form. Numerical solutions are required, in general. Hence, equations (33)-(36) are of theoretical interest but limited practical usefulness.

There is a special case of practical importance, in which simplifications are possible. Suppose that $x(t)$, $z(t)$ are wide sense stationary processes of finite average power, and are observed on $[-T/2, T/2]$. Suppose also that the corresponding spectral densities $s_x(f)$, $s_z(f)$ are positive on $[-W, W]$ and zero otherwise.

For large T , Van Trees develops in [10] approximations to the eigenvalues and eigenvectors of an integral equation of the type (33), but for the special case $R_z(t, s) = \delta(t - s)$. Straightforward extension of the approach of Van Trees yields the approximation:

$$z_k(t) \approx T^{-1/2} \exp(j2\pi kT^{-1}t); \quad |t| \leq T/2$$

$$\lambda_k \approx s_x(kT^{-1}) \cdot s_z^{-1}(kT^{-1}); \quad -WT \leq k \leq WT$$

for large T . Note that the approximate numbers of eigenvalues or degrees of freedom of the signals involved are $2WT$, equal to the number mandated by the sampling theorem for bandlimited random processes.

Suppose now that a fraction p of the $2WT$ degrees of freedom is used. The resulting approximation to the mean square error is:

$$S_p^{**}(T) \approx \sum_{k=1}^{2TWp} [1 - (1 + \lambda_k)^{-1}] + \sum_{k=2WTp+1}^{2TW} \lambda_k$$

where the ordering of eigenvalues is:

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{2WTp} \geq \dots \geq \lambda_{2TW}$$

We can evaluate now the asymptotic representation error per unit time as $T \rightarrow \infty$:

$$S_p^{**} \triangleq \lim_{T \rightarrow \infty} T^{-1} S_p^{**}(T) = \int_{I_p} s_x(f) [s_x(f) + s_z(f)]^{-1} df + \\ + \int_{I_p} s_x(f) s_z^{-1}(f) df.$$

where $I_p \subset [-W, W]$ is a subset of Lebesgue measure $2Wp$, such that the ratio $s_x(f) s_z^{-1}(f)$ is maximal, i.e. for all $f_1 \in I_p$, $f_2 \notin I_p$ we have $s_x(f_1) s_z^{-1}(f_1) \geq s_x(f_2) s_z^{-1}(f_2)$.

Conclusions

We have developed some intuitively sensible conclusions on the constrained representation of noisy signals, when the signal and noise covariances are known. We have shown that the eigenvalue-eigenvector expansion provides the minimum mean square error subspace of fixed dimensionality. The use of eigenvectors and associated subspaces has been proven useful in various signal and image processing applications, assuming no noise ([10] - [13]). Problems in which the developed techniques are applicable are the simultaneous compression and filtering of noisy speech and images, and the processing of sensor array data. (see, for example, [14]).

References

- (1) K. Fukunaga. Introduction to Statistical Pattern Recognition. Academic Press, 1972.
- (2) R. Bellman. Introduction to matrix analysis. McGraw Hill, 1960.
- (3) R. M. Gray, "Toëplitz and Circulant matrices, II." Stanford University Inform. Systems Lab, Tech. Report no. 6504-1, April 1977.
- (4) R. M. Gray, "On the asymptotic eigenvalue distribution of Toëplitz matrices", IEEE Trans. on IT, vol. 18, pp. 725-730, Nov. 1972.
- (5) B. L. Buzbee, "A fast Poisson solver amenable to parallel computation", IEEE Trans. on Computers, vol. C-22, no. 8, August 1973, pp. 793-796.
- (6) J. Pearl, "On coding and filtering stationary signals by discrete Fourier Transforms", IEEE Trans. on Infor. Theory, vol. IT-19, no. 2, pp. 229-232, March 1973.
- (7) J. Pearl, "Asymptotic equivalences of spectral representations", IEEE Trans. on ASSP, Vol. ASSP-23, no. 6, pp. 547-551, December 1975.
- (8) T. Kadota, "Simultaneously orthogonal expansion of two stationary Gaussian processes - examples." Bell System Tech. Journal, vol. 45, no. 7, pp. 1071-1096, 1966.
- (9) T. Kadota and L. A. Shepp, "On the best finite set of linear observables for discriminating two Gaussian signals", IEEE Trans. on Infor. Theory, vol. IT-13, no. 2, April 1967, pp. 278-284.
- (10) H. L. Van Trees. Detection, Estimation and Modulation Theory. Part I. John Wiley and Sons, 1968.
- (11) C. M. Walter, "Application of canonical coordinate methods to the characterization of a family of error minimizing signal compression techniques", Record of the 1978 IEEE Conf. on Acoustics, Speech and Signal Processing, Tulsa, OK, 1978, pp. 480-482.
- (12) C. M. Walter, "Geometrical characterization of the canonical coordinate basis underlying a family of error minimizing signal compression techniques", Record of the 1979 IEEE International Conference on Acoustics, Speech, and Signal Processing, Washington, D.C., April 1979, pp. 388-391.
- (13) H. C. Andrews and C. L. Patterson, "Singular value decompositions and digital image processing", IEEE Trans. on ASSP, vol. ASSP-24, no. 1, pp. 26-52.
- (14) F. C. Schwegge, "Sensor Array Data Processing for Multiple-Signal Sources", IEEE Trans. on Infor. Theory, vol. IT-14, no. 2, pp. 294-305, March 1968.

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School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,400 students with a graduate enrollment of approximately 600. There are 125 faculty members, a majority of whom conduct research in addition to teaching.

Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical and Aerospace to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering and Engineering Physics, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,530 full-time faculty with a total enrollment of about 16,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, Business Administration, and Education. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.