ROYAL AIRCRAFT ESTABLISHMENT

TEN ORBIT DETERMINATIONS FOR COSMOS 472 ROCKET

by

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SUMMARY

The orbit of Cosmos 472 rocket, 1972-04B, has been determined from 461 observations at ten epochs during the last two months of its life. The first eight orbits were satisfactory, with average standard deviations equivalent to 220 m in cross-track distance and 75 m in radial distance.

The variation of perigee height has been analysed to obtain four values of density scale height. The first three showed good agreement with *CIRA 1972*.

Unfortunately the geometry of the orbit was such that the decrease in inclination did not yield an exact value of atmospheric rotation, contrary to expectations.

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1 INTRODUCTION

Cosmos 472 rocket, 1972-04B, was the rocket which fired the satellite Cosmos 472 into orbit on 1972 January 25.47. After separation, 1972-04B had an orbital period[1] of 102 minutes, perigee height 193 km, apogee height 1526 km and inclination 82.0°. The rocket remained in orbit for 101.58 days, decaying on 1972 May 6.05.

With this initial orbit, 1972-04B promised to be useful for determining the atmospheric rotation rate from the decrease in orbital inclination. The orbit of 1972-04A had been used previously[2] to obtain a value of atmospheric rotation rate, Λ, with values of inclination from US Navy orbits; but the value of Λ obtained was not considered to be of a very high standard.

The orbit of 1972-04B has been determined from 461 observations with the aid of the RAE orbit refinement program[3] PROP 6, at ten epochs during the last two months of its life. Unfortunately however, the first six values of inclination obtained were at a time when ω, the argument of perigee, was near 270° and consequently there was little change in inclination, so that Λ could not be accurately determined. As ω moved towards 180° and a change in inclination became apparent, the orbit also passed through 15th-order resonance. As the satellite was rapidly decaying by this time, it was not possible to calculate the perturbation due to resonance from the sparse data available and also accurately evaluate a value of Λ.

The main reason for determining the orbit having proved unfruitful, the perigee height was analysed to determine the atmospheric density scale height. Four values of scale height were determined and the results are presented in this Memorandum.

2 THE OBSERVATIONS AND ORBITS

The orbit of 1972-04B has been determined at ten epochs using 517 observations. The type and number of observations used on each orbit are given in Table 1. The

Table 1

Sources of observations used on each orbit

Orbit No.	Source of observations				Total
	Visual	US Navy	Finland	British radar	
1	53	20		8	81
2	25	50	12	8	95
3	5	32			37
4	7	54			61
5		41			41
6		46			46
7	10	32		8	50
8	1	19		25	45
9	4	24			28
10	5	20	9		34
Total	110	338	21	49	518

observations fall into four groups; the visual observations made by volunteer observers now reporting to the Earth Satellite Research Unit at the University of Aston, the

US Navy observations supplied by the US Naval Research Laboratory, the Finnish observations made by the theodolite at Jokioinen, Finland, and observations made by British radars.

The orbital elements at each of the ten epochs are listed in Table 2 on page 5 with the standard deviations below each value; the epoch for each orbit is at 00 hours on the day indicated. The PROP 6 model fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \qquad (1)$$

where t is the time measured from epoch and the number of M-coefficients used depends on the drag. Best results were obtained by using M_0 to M_5, the full complement of coefficients in the model, on two of the ten orbits, two required only M_0 to M_3, and the remaining six required M_0 to M_4.

The orbits fit the observations in a satisfactory manner on the first eight orbits, with ϵ, the parameter indicating the measure of fit ranging between 0.45 and 1.0. The last two orbits were more difficult, as the satellite was decaying rapidly and there were less observations available.

The PROP program rejects observations which do not fit well, and 461 of the original 518 observations were accepted in the final orbits. The residuals of the observations have been obtained using the ORES computer program[4] and sent to the observers.

The accuracies of selected observing stations, with five or more observations accepted in the orbit determination, are listed in Table 3 along with the number of accepted and rejected observations. Three of the visual stations have had their rms residuals revised because during the orbit determination a few of their observations had their standard deviations increased in order to keep them in, rather than allowing them to be rejected. The revised set, omitting these observations (never more than 15%), gives a better impression of the observers' accuracy.

3   INCLINATION

For a high-drag satellite, atmospheric rotation is the most important force perturbing the inclination i , so if the change in i is measured accurately, and other perturbations are removed, the rotation rate of the upper atmosphere in the region near the satellite's perigee can be determined.

The values of inclination from Table 2 are plotted in Fig 1 after removal of the zonal harmonic, $J_{2,2}$ and lunisolar perturbations. The zonal harmonic and lunisolar perturbations were removed by using the PROD[5] computer program with 1-day integration steps, and the $J_{2,2}$ tesseral harmonic perturbation by using the value recorded on the PROP printout. Perturbations due to earth and ocean tides were expected to be smaller than the standard deviations of the values of i , so were ignored.

The theoretical variation of i due to atmospheric rotation can be calculated for a series of values of atmospheric rotation rate, Λ , using the computer program ROTATM

Table 2

Values of orbital parameters at ten epochs, with standard deviations

	MJD	Date 1972	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	ε	N	D	a(1-e)
1	41381	5 March	7084.794 2	0.07220 1	81.9812 14	281.503 1	317.88 1	297.54 1	5242.651 2	2.280 3	-0.0488 8	0.0056 6	-	0.58	68	3.7	6573.27
2	41386	10 March	7065.406 2	0.06946 1	81.9735 14	276.621 1	301.83 2	286.27 2	5264.252 2	1.889 2	-0.0429 3	-0.0024 2	-	0.79	81	5.6	6574.67
3	41393	17 March	7044.533 1	0.06664 1	81.9704 19	269.716 1	278.94 2	139.70 2	5287.676 3	1.884 2	0.0260 4	-0.0168 4	-	0.54	34	4.9	6575.08
4	41401	25 March	7019.992 2	0.06351 1	81.9702 22	261.740 1	252.67 2	73.26 3	5315.439 3	1.838 4	-0.0203 15	-0.0156 13	-	0.76	59	3.6	6574.15
5	41408	1 April	6995.585 2	0.06068 1	81.9697 14	254.682 1	229.60 1	294.77 1	5343.291 2	2.189 2	0.0256 6	0.0114 6	-	0.45	39	4.0	6571.08
6	41415	8 April	6961.450 2	0.05662 2	81.9685 20	247.528 2	206.55 1	23.24 1	5382.656 2	3.608 1	0.0760 6	-	-	0.82	43	3.9	6567.28
7	41422	15 April	6906.459 2	0.05007 2	81.9603 12	240.219 1	183.10 1	109.34 1	5447.097 3	5.607 3	0.0535 29	0.0060 7	0.0043 6	0.70	47	4.0	6560.65
8	41429	21 April	6841.028 3	0.04189 3	81.9518 29	233.766 3	162.47 4	255.75 4	5525.464 7	7.303 5	0.220 4	0.0072 7	-0.0165 5	1.00	41	4.7	6554.43
9	41436	29 April	6734.417 10	0.02828 5	81.9484 84	224.799 6	132.42 14	327.81 13	5657.248 12	10.713 7	0.429 11	-	-	1.56	22	2.5	6543.94
10	41441	4 May	6635.444 5	0.01522 3	81.9388 33	218.851 3	109.81 25	113.14 24	5784.349 6	17.765 10	2.116 3	0.453 4	-	2.12	27	3.7	6534.48

Key: MJD = modified Julian day

a = semi major axis (km)

e = eccentricity

i = inclination (deg)

Ω = right ascension of ascending node (deg)

ω = argument of perigee (deg)

M_0 = mean anomaly at epoch (deg)

M_1 = mean motion n (deg/day)

M_2-M_5 = later coefficients in the polynomial for M

ε = measure of fit

N = number of observations used

D = time covered by the observations (days)

Table 3

Residuals for observing stations with five or more observations accepted in the orbit determination

Station		Number of observations		Range km	Rms residuals Minutes of arc			Revised rms residuals minutes of arc		
Number	Name	Accepted	Rejected		RA	Dec	Total	RA	Dec	Total
1	US Navy	43	3		2.2	3.2	3.9			
2	US Navy	33	5		5.3	4.6	7.0			
3	US Navy	34	3		4.2	3.5	5.5			
4	US Navy	37	5		3.6	3.2	4.8			
5	US Navy	35	0		3.9	4.3	5.8			
6	US Navy	36	3		3.6	3.0	4.7			
29	US Navy	90	11	1.0	0.6	0.9				
1963	Jokioinen	21	0		4.5	9.2	10.3	4.1	4.8	6.3
2265	Farnham	6	0		1.3	1.4	1.9			
2304	Malvern radar	46	3	0.7	2.7	2.7				
2414	Bournemouth	8	0		2.3	5.2	5.7	1.8	3.2	3.7
2420	Willowbrae	32	1		6.6	5.8	8.7	1.7	2.5	3.0
2421	Malvern 4	5	1		1.8	1.3	2.2			
2550	Masira	7	2		3.4	1.4	3.7			
8591	Adelaide 4	5	0		1.1	2.7	2.9			

based on equation (32) of Ref 6. Unfortunately the value of argument of perigee ω passes through 270° just after orbit 3, with the result that the decrease in i over the first six values is very small and there is no chance of determining an accurate value of Λ. Over the last month, however, the inclination falls by 0.025°; but by a further misfortune the satellite passes through 15th-order and 31:2 resonances during this time. The theoretical curve with $\Lambda = 0.7$ gives the best fit to the values of inclination in Fig 1.

This value of Λ is unexpectedly low and so the effect of a meridional wind was investigated. The ROTATM program also allows for the effect of meridional winds but for a satellite with an inclination of 82° the effect is small, being $<0.001^\circ$ for $\mu = 0.1$ rev/day where μ is the south-to-north rotation rate. It would seem therefore that the values of i being fitted may have been perturbed by one or both of the resonances, and, as the data at the times of the resonances is so sparse, the perturbations cannot be reliably calculated.

The height for which the value of Λ applies is $\frac{3}{2}H$ above perigee, where H is the density scale height. This gives a value of 220 km, and from a recent review of upper atmosphere winds[7], the lowest value of Λ is 0.78 at this height; and that is only obtained when the satellite is experiencing morning and summer conditions. This satellite, 1972-04B, was not in summer conditions, the last four orbits being determined between 15 April and 4 May, so at best could only be described as having a tendency to summer. The local time over this final stage varied from 14 h to 10 h, so two of the orbits were determined in morning conditions. From Ref 7 a low value of Λ (near 0.9) would be expected for a satellite with the local time and seasonal conditions of 1972-04B, but not as low as 0.7.

Another explanation could be that before the last epoch the satellite was perturbed in its orbit by the escape of some residual propellant, thus causing an increase in the value of inclination. There is some support for this theory in the analysis of the perigee height in section 5.

4   PERIGEE HEIGHT

The values of perigee distance $a(1 - e)$ are plotted as circles in Fig 2, using the values of a and e from Table 2. If the odd zonal harmonic and lunisolar perturbations are removed from $a(1 - e)$, using PROD[5], the remaining variation should be a steady decrease as a result of air drag alone. These corrected values of perigee distance, Q, for the ten epochs of Table 2 are plotted as crosses in Fig 2, with a smooth curve drawn through the crosses.

The actual perigee height, y_p, above the Earth's surface is found by first restoring the zonal harmonic and lunisolar perturbations to the smoothed values of Q, giving smoothed values of perigee distance, $a(1 - e)_{smoothed}$; then from these values subtracting the local Earth's radius, R_p, at latitude ϕ_p, given by

$$R_p = R - 21.38 \sin^2 \phi_p$$

where R is the Earth's equatorial radius, 6378.14 km; and finally adding the small amount by which the actual path of the satellite departs from an exact ellipse[8], dr_p, where

$$dr_p = 1.388\left[1 - \frac{2}{1 + e}\sin^2\omega\right] \text{ km} .$$

Thus, since $\sin^2\phi_p = \sin^2 i \sin^2\omega$,

$$y_p = a(1 - e)_{smoothed} - 6376.75 + \left(20.96 - \frac{2.78}{1 + e}\right)\sin^2\omega . \tag{2}$$

The values of y_p calculated at the ten epochs, using the above equation, are plotted in Fig 3 and a smooth curve has been drawn through the points.

5 DENSITY SCALE HEIGHT

The perigee distance cleared of zonal harmonic and lunisolar perturbations, as given by Q, gradually decreases under the influence of air drag, and the decrease is proportional to the density scale height, H. The values of Q, given by the crosses in Fig 2, show this steady decrease and values of H can be found from the changes in Q.

The theoretical equation for the variation of Q is[9]

$$\dot{Q} = -\frac{2H_1 M_2}{3M_1 e}\left\{1 - 2e + \frac{H_1}{4ae} - \frac{2\epsilon'}{e}\sin^2 i \cos 2\omega + 0\left(e^2, \frac{H}{a}, \frac{H^2}{a^2 e^2}, \frac{\epsilon'^2}{e^2}\right)\right\} \tag{3}$$

where H is the density scale height, H_p is the value of H at perigee, H_1 is the value of H at a height $1.5H_p$ above perigee, and ϵ' is the ellipticity of the atmosphere, taken as the Earth's ellipticity, 0.00335. This equation for $\dot{Q}$ is valid for $ae/H_p > 3$.

Average values of $\dot{Q}$ were calculated over a time interval, Δt, long enough to ensure that an accurately measurable change, ΔQ, in Q has occurred. The values of $\Delta Q/\Delta t$ serve as values of $\dot{Q}$ in equation (3), and averaged values of M_1, M_2, e, a, i and $\cos 2\omega$ over the corresponding time Δt were used to calculate values of H_1 from equation (3).

Four values of H_1 were calculated and they are plotted as circles in Fig 4, and the time over which they are averaged is indicated. The corresponding average values of height $y_1 (= y_p + 1.5H_p)$ are given at the top of Fig 4.

The values of H_1 plotted as crosses in Fig 4 are the values of density scale height obtained from the COSPAR International Reference Atmosphere 1972[10] (CIRA 1972) for heights y_1 and appropriate exospheric temperature.

The agreement between the first three values of H_1 evaluated here and the CIRA values is very good, as the CIRA values would be expected to have an error of up to 10%. The last value of H_1 determined from the orbital data appears to be too low. No geophysical reason can be found for this, as agreement with the CIRA value would require

the perigee distance to be 5 km lower. This difference seems to be far too large to be due to orbital errors, which are of order 300 m, so again the possibility that some unburnt propellant escaped from the rocket, thus changing its orbit, must be given as the most likely explanation.

6   CONCLUSIONS

Ten orbits of 1972-04B have been computed, eight being of satisfactory accuracy. They fail to give a good value of Λ , as had been hoped, because the values of ω were unfavourable; but three good estimates of atmospheric scale height have been derived.

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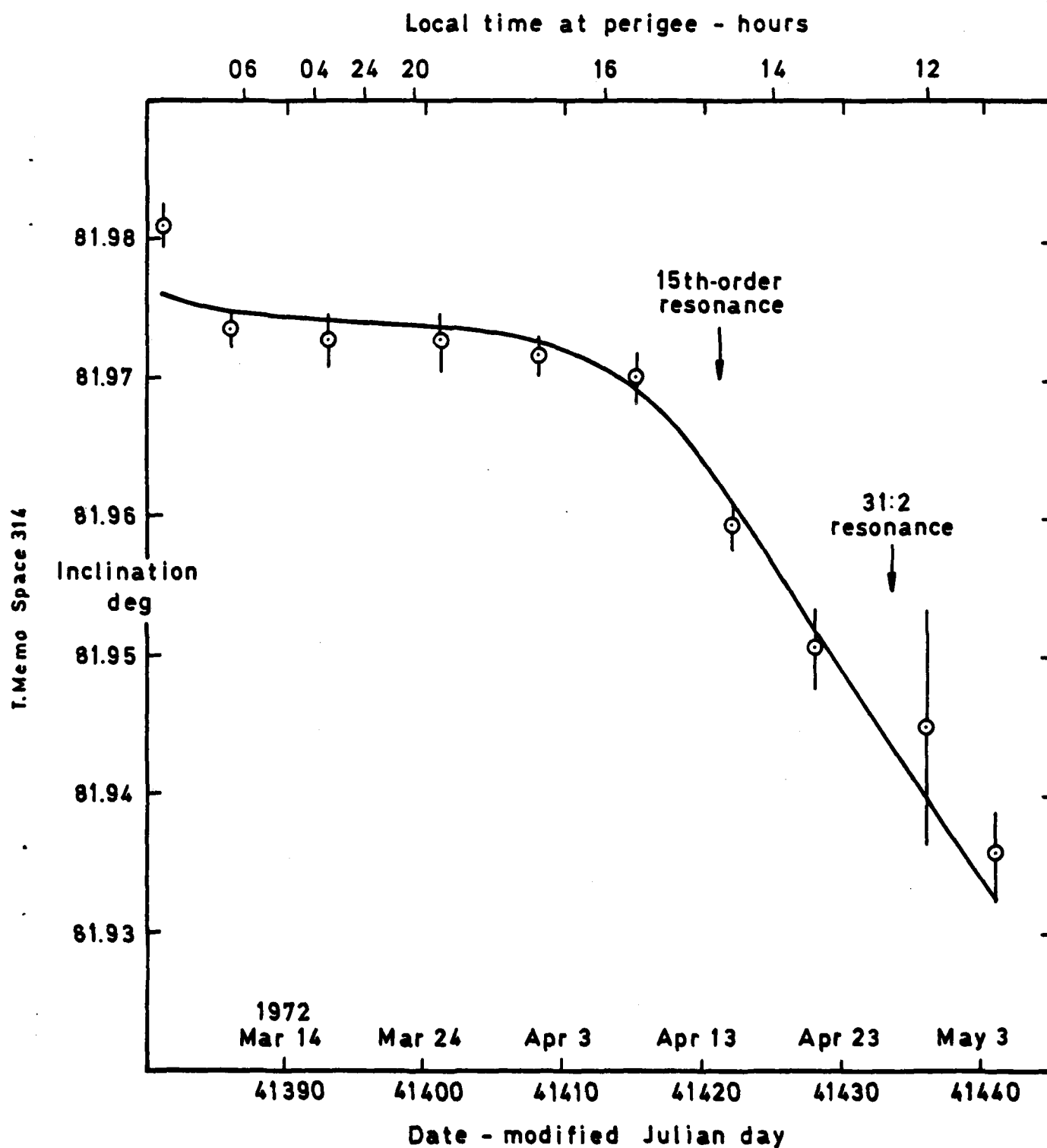
Fig 1   Values of inclination cleared of perturbations,
with theoretical curve for $\Lambda = 0.7$

Fig 2
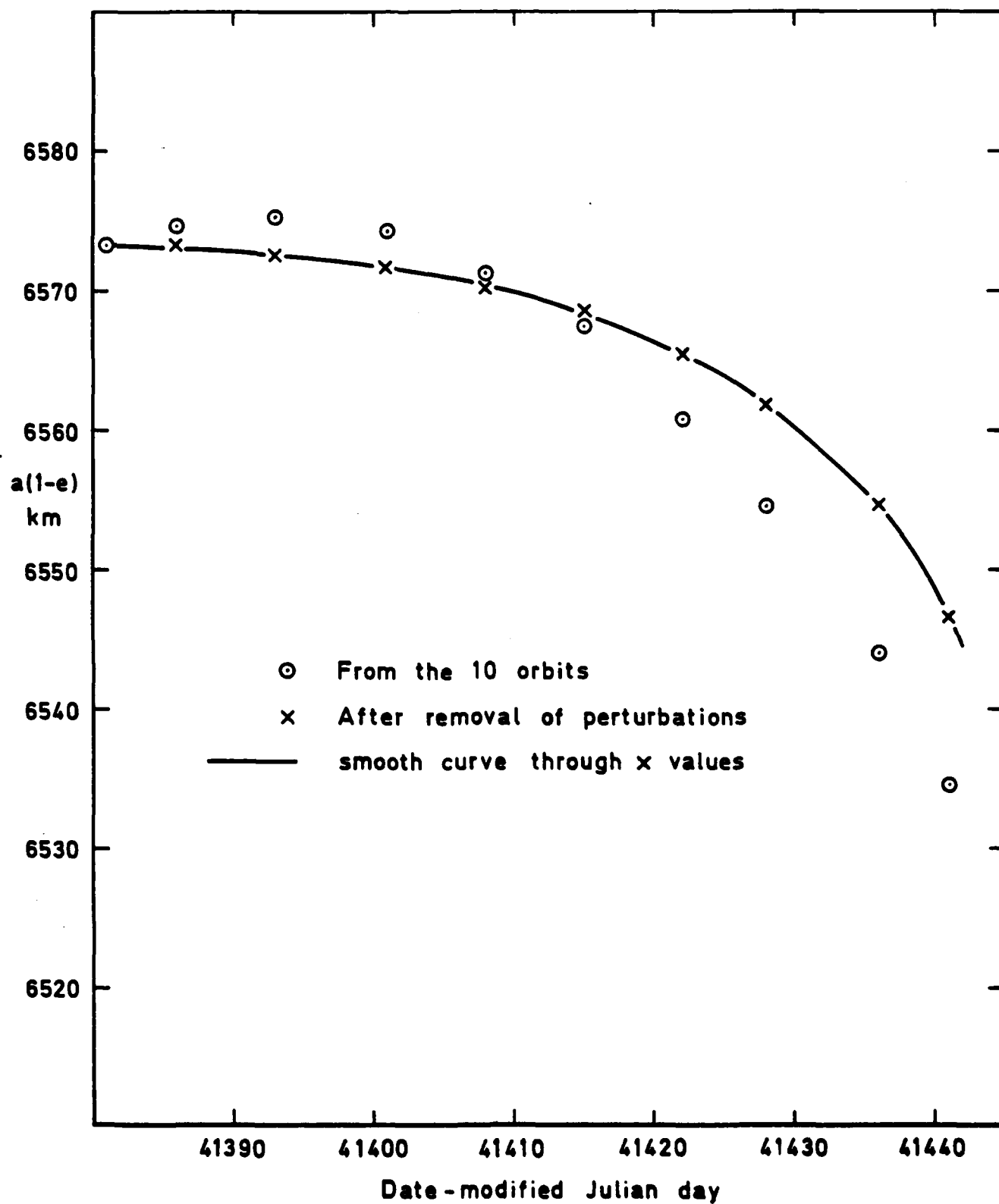
Fig 2   Values of a(1-e)
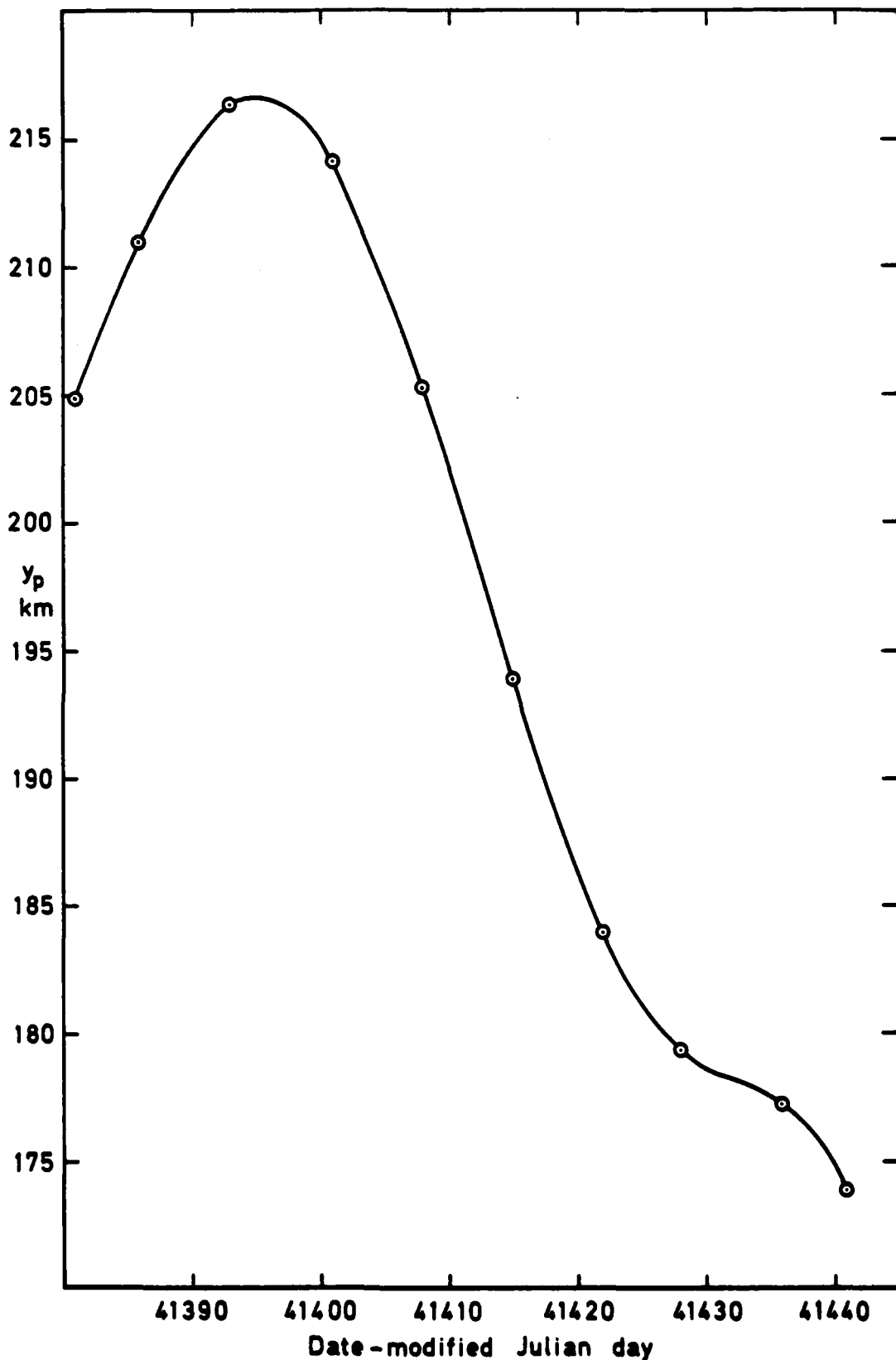
Fig 3

Fig 3 Variation of perigee height, y_p

Fig 4
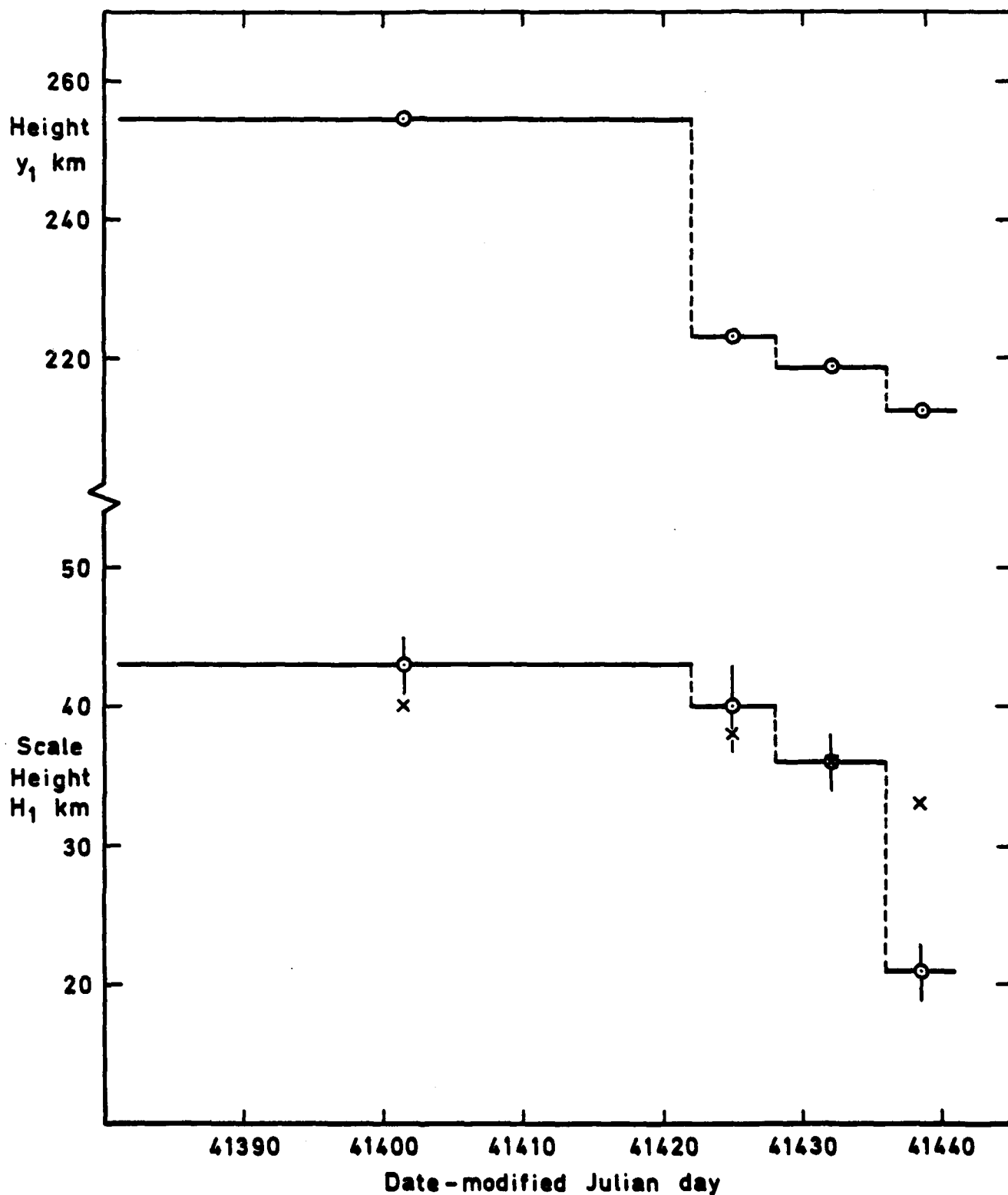
Fig 4   Values of density scale height H_1 at height y_1

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