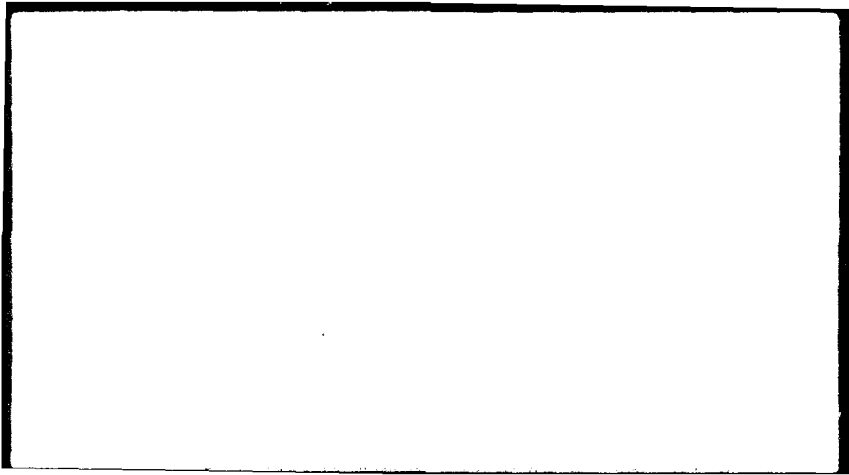


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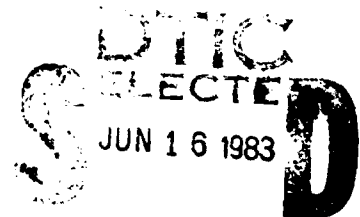
**ONE-STEP DIAGNOSIS ALGORITHMS FOR  
THE BGM SYSTEM LEVEL FAULT MODEL**

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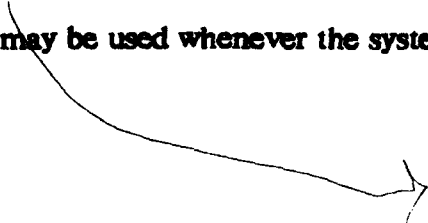


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ABSTRACT

A one-step  $\tau$ -fault diagnosable system is a system in which all faults may be identified from the test results, provided that the number of faults does not exceed  $\tau$ . In this paper we present two algorithms that may be used for the one-step diagnosability of the system level fault model proposed by Barsi, Grandoni and Maestrini. The first algorithm may be used when the system is one-step  $\tau$ -fault diagnosable and no two units test each other, and the second algorithm may be used whenever the system is one-step  $\tau$ -fault diagnosable.



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## 1. INTRODUCTION

In the past few years, a great deal of effort has been devoted to the presentation and analysis of system fault models. These models are characterized by the fact that they do not emphasize test generation, but rather emphasize the use of test results for the purpose of fault location and fault detection. The problems associated with system level fault models are (i) presentation and description of the fault models themselves; (ii) analysis of the diagnosability properties of the fault model under various assumptions concerning the fault-test relationships and the system testing interconnection network; and (iii) synthesis of diagnosis algorithms for fault detection, one-step diagnosis and sequential diagnosis.

Many fault models presently exist [2], [7], [8], [9], [14], and their diagnosability properties are essentially well understood [4], [5], [15-17]. The state of the art, as far as one-step diagnosis algorithms are concerned, is less satisfactory. The two most widely analyzed system level fault models are the PMC model, proposed by Preparata, Metze and Chien [14], and the BGM model proposed by Barsi, Grandoni and Maestrini [2]. There exist algorithms for the one-step diagnosis of the PMC model; unfortunately, they suffer from some serious drawbacks. The algorithms proposed by the author and Masson [10], [12] are only applicable to regular testing interconnection networks, and the ones proposed by the author [11], [13] have been shown to work only when the first Hakimi-Amin hypothesis [5, Theorem 1] is satisfied and the number of faults is limited. The algorithm proposed by Corluhan and Hakimi [3] depends on an unproved conjecture. The algorithms proposed by Allan, Kameda and Toida [1], [6] involve tree searching and backtracking.

The situation for the one-step diagnosis of the BGM model is even less satisfactory: no algorithm has been proposed to date. To remedy this situation, this paper presents two algorithms for the one-step diagnosis of the BGM model. The first identifies the set of faulty devices, provided that the first Hakimi-Amin condition is satisfied and the set of faulty modules is no greater than  $\tau$ , where  $\tau$  is the minimum number of system units that test each system unit. The second identifies the set of faulty modules whenever the system is one-step fault diagnosable. These two algorithms are easy to implement and their very existence is an *a posteriori* indication of the strength of the assumptions of the BGM model.

In the first part of the paper, the BGM model is described. Then the basic tenet of our approach, i.e., the partition of the system units into four subsets is presented, together with the properties of that partition. The first algorithm makes direct use of the basic partition properties, and its decoding properties are exhibited. Finally we perform an analysis of the failure mode of the first algorithm and present a remedy -- the second algorithm. We then show that this algorithm may be used to decode syndromes for the BGM model, provided that the model is one-step  $\tau$ -fault diagnosable, and that the number of faulty modules is no greater than  $\tau$ .

## 2. THE BGM SYSTEM LEVEL FAULT MODEL

The BGM model proposed by Barsi, Grandoni and Maestrini [2] is a system level fault model closely related to the PMC model proposed by Preparata, Metze and Chien [14]. In the BGM model, a system  $S$  consists of  $n$  modules  $U_0, U_1, \dots, U_{n-1}$  and a testing interconnection design  $TID$ , where

$$TID = \{ (i,j) \mid U_i \text{ tests } U_j \}.$$

It is assumed that no module tests itself, i.e., the diagonal does not belong to  $TID$ . A module is assumed to be either faulty or nonfaulty, and the state of each module in  $S$  is assumed to be constant during the application of the testing procedures. If  $(i,j)$  is in  $TID$ , then  $U_i$  tests  $U_j$ , and the test outcome  $a_{i,j}$  is assumed to be either "0" ( $U_j$  passes the test) or "1" ( $U_j$  fails the test). The set of test outcomes  $\{ a_{i,j} \mid (i,j) \in TID \}$  is the syndrome of the system. In the BGM model, the following relationships between fault and test outcomes are assumed:

- (i) if  $(i,j)$  is in  $TID$  and  $U_i$  and  $U_j$  are nonfaulty, then  $a_{i,j} = 0$ ;
- (ii) if  $(i,j)$  is in  $TID$ ,  $U_i$  is nonfaulty and  $U_j$  is faulty, then  $a_{i,j} = 1$ ;
- (iii) if  $(i,j)$  is in  $TID$  and both  $U_i$  and  $U_j$  are faulty, then  $a_{i,j} = 1$ ;
- (iv) if  $(i,j)$  is in  $TID$ ,  $U_i$  is faulty and  $U_j$  is nonfaulty, then  $a_{i,j}$  may take either the value 0 or 1.

Thus, the main difference between the PMC model and the BGM model is that if a module  $U_i$  is tested by a module  $U_j$ , and if  $a_{j,i} = 0$ , then the module  $U_i$  is nonfaulty.

A fault situation of the system  $S$  is described by the set  $F_S$  of the faulty modules in  $S$ . A set of possible syndromes corresponds to each fault situation. Given a fault situation, the computation of the corresponding syndromes is not difficult, but to compute the fault situations that are consistent with a given syndrome is not as easy. In this paper, we address the latter problem -- namely, syndrome decoding -- and we restrict ourselves to one-step  $\tau$ -fault diagnosability in the sense of Preparata, Metze and Chien [14].

**Definition 1:** A system  $S$  is one-step  $\tau$ -fault diagnosable if all faulty modules

within the system can be identified without replacement, provided that the number of faulty modules does not exceed  $\tau$ .

In the remainder of this work,  $s$  will be the index set that contains the indices of all the modules in  $S$ , i.e.,

$$s = \{0, 1, 2, \dots, n-1\};$$

$f_S$  will be the index set that contains the indices of all the faulty modules in  $S$ , i.e.,

$$f_S = \{i \in s \mid U_i \in F_S\};$$

and given an index set  $a$ ,  $|a|$  will be used to denote the number of elements in  $a$ .

### 3. IMPERFECT ONE-STEP DIAGNOSIS

Our approach to system diagnosis consists in partitioning the set  $s$  into four subsets ( $v$ ,  $h_1$ ,  $h_2$  and  $h_3$ ) that are easy to compute, and then to relate those four subsets to the set  $f_S$  which contains the indices of all the faulty modules in  $S$ .

The index set  $v$  contains the indices of all the modules in  $S$  that are tested by at least one other module in  $S$  and found to be nonfaulty by that module, i.e.,

$$v = \{i \in s \mid j \text{ in } s \text{ exists so that } (j,i) \in TID \text{ and } a_{ji} = 0\}. \quad (1)$$

Thus, if  $S$  is a BGM model, i.e., a fault model that satisfies the assumption of section 2, the module  $U_i$  is nonfaulty whenever the index  $i$  is in  $v$ .

The index set  $h_1$  contains the indices of all the modules in  $S$  that are tested by at least one module  $U_j$ ,  $j$  in  $v$  and found faulty, and the indices of all

the modules in  $S$  that test at least one module  $U_j$ ,  $j$  in  $v$ , and find it faulty, i.e.,

$$h_1 = \{ i \in s \mid j \text{ in } v \text{ exists so that } (j,i) \in TID \text{ and } a_{j,i} = 1 \} \\ \cup \{ i \in s \mid j \text{ in } v \text{ exists so that } (i,j) \in TID \text{ and } a_{i,j} = 1 \}. \quad (2)$$

One should note that if  $S$  is a BGM model, then the index sets  $v$  and  $h_1$  are disjoint, and  $U_i$  is faulty whenever the index  $i$  is in  $h_1$ .

The index set  $h_2$  depends on the cardinality of the sets  $L(i)$ , where, for every index  $i$  in  $s - (v \cup h_1)$ , the sets  $L(i)$  are defined by

$$L(i) = \{ j \in s - (v \cup h_1) \mid (i,j) \in TID \text{ and } a_{i,j} = 1 \} \\ \cup \{ j \in s - (v \cup h_1) \mid (j,i) \in TID \text{ and } a_{j,i} = 1 \}.$$

Given an index  $i$ , it is possible that an index  $j$  exists so that the pairs  $(i,j)$  and  $(j,i)$  are both in  $TID$ , and  $a_{i,j} = a_{j,i} = 1$ . Obviously, in such a case the index  $j$  appears in  $L(i)$  only once. The set  $L(i)$  contains the indices of all the modules adjacent to the module  $U_i$  that must be faulty if the module  $U_i$  is actually non-faulty. Given a scalar  $\tau$ , the set  $h_2$  consists of all the indices in  $s$ , but not in  $v \cup h_1$  such that the cardinality of  $L(i)$  is strictly greater than  $\tau$ , i.e.,

$$h_2 = \{ i \in s - (v \cup h_1) \mid \|L(i)\| \geq \tau + 1 \}. \quad (3)$$

It is clear that if  $S$  is a BGM model and if at most  $\tau$  modules in  $S$  are faulty, then  $U_i$  is faulty whenever  $i$  is in  $h_2$ .

The index set  $h_3$  contains the indices of the remaining modules in  $S$ , i.e.,

$$h_3 = s - (v \cup h_1 \cup h_2).$$

In this section we will examine the properties of the sets  $h_1$ ,  $h_2$  and  $h_3$  when every module is tested by at least  $\tau$  other modules. We will then use those properties to synthesize a decoding algorithm that produces an index set  $f_A$  containing the indices of all the faulty modules and some nonfaulty ones,

provided that the number of faulty modules  $\|f_S\|$  is at most  $\tau$ . We shall assume that the following assumption holds.

*Hypothesis 1:* Every module in  $S$  is tested by at least  $\tau$  other modules in  $S$ .

The next lemma is a direct consequence of the properties of the BGM model and is given without proof.

*Lemma 1:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied and if

$\|f_S\| \leq \tau$ , then the index sets  $h_1, h_2, h_3$  and  $f_S$  satisfy

$$h_1 \cup h_2 \subseteq f_S \subseteq h_1 \cup h_2 \cup h_3.$$

*Lemma 2:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied and if

$\|f_S\| \leq \tau - 1$ , then  $\|h_1\| \leq \tau - 1$ ,  $h_2 = \phi$ ,  $h_3 = \phi$ , and

$$f_S = h_1.$$

*Proof:* Let  $U_i$  be a nonfaulty module. By assumption, every module is tested by at least  $\tau$  other modules and the fact that  $\|f_S\| \leq \tau - 1$  implies that  $U_i$  is tested by at least one nonfaulty module, say  $U_j$ . Thus, if  $U_i$  is nonfaulty, an index  $j$  exists so that  $(j,i)$  is in  $TID$  and  $a_{j,i} = 0$ , and it follows that  $v$  contains the indices of all the nonfaulty modules in  $S$ .

Now let  $U_i$  be a faulty module. Hypothesis 1 and the fact that  $\|f_S\| \leq \tau - 1$  imply that  $U_i$  is tested by at least one nonfaulty module, say  $U_j$ . Thus, if  $U_i$  is faulty, an index  $j$  in  $v$  exists so that  $(j,i)$  is in  $TID$  and  $a_{j,i} = 1$ , and it follows that  $h_1$  contains the indices of all the faulty modules in  $S$ . Clearly,  $v$  and  $h_1$  form a partition for  $S$ , and we may conclude that  $h_2$  and  $h_3$  are empty.  $\square$

*Lemma 3:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied and if

$\|h_1 \cup h_2 \cup h_3\| \leq \tau$ , then

$$f_S = h_1 \cup h_2 \cup h_3.$$

Proof: If  $|h_1 \cup h_2 \cup h_3| \leq \tau$ , then either  $|f_S| \leq \tau - 1$  or  $|f_S| = \tau$ . If  $|f_S| \leq \tau - 1$ , then Lemma 2 implies that  $f_S = h_1 \cup h_2 \cup h_3$ . If  $|f_S| = \tau$ , the fact that  $f_S \subseteq h_1 \cup h_2 \cup h_3$  implies immediately that  $f_S$  must be equal to  $h_1 \cup h_2 \cup h_3$ .  $\square$

*Lemma 4:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied, if  $|f_S| \leq \tau$ , and if  $|h_1 \cup h_2| = \tau$ , then

$$f_S = h_1 \cup h_2.$$

Proof: If  $|f_S| \leq \tau$  and if  $|h_1 \cup h_2| = \tau$ , then Lemma 1 implies immediately that  $f_S$  must be equal to  $h_1 \cup h_2$ .  $\square$

We will now show that if  $|f_S| = \tau$  and  $|h_1 \cup h_2| \leq \tau - 1$ , the number of nonfaulty modules in  $h_3$  is at most  $\tau$ .

*Lemma 5:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied, if  $|f_S| = \tau$ , and if  $|h_1 \cup h_2| \leq \tau - 1$ , then  $h_3$  contains the indices of at most  $\tau$  nonfaulty modules and

$$|h_1 \cup h_2 \cup h_3| \leq 2\tau.$$

Proof: If an index  $i$  in  $h_3$  corresponds to a nonfaulty module  $U_i$ , then  $U_i$  is tested only by faulty modules. By assumption,  $|f_S| = \tau$  and  $|h_1 \cup h_2| \leq \tau - 1$  and therefore, the index  $j$  of at least one faulty module is in  $h_3$ . The module  $U_j$  cannot test more than  $\tau$  other modules in  $h_3$  (otherwise  $j$  would be in  $h_1 \cup h_2$ ), and it follows that the number of nonfaulty modules in  $h_3$  is at most  $\tau$ . The index set  $h_1 \cup h_2 \cup h_3$  contains the indices of all the faulty modules and since  $h_3$  contains at most  $\tau$  nonfaulty modules,

$$|h_1 \cup h_2 \cup h_3| \leq 2\tau. \square$$

Lemmas 3 and 4 suggest the following decoding algorithm.

*Algorithm 1:*

Step 0: Compute the set  $v$  as in Equation (1).

Step 1: If  $\|s - v\| \leq \tau$ , let  $f_A = s - v$  and stop; otherwise, go to Step 2.

Step 2: Compute the sets  $h_1$  and  $h_2$  as in Equations (2) and (3).

Step 3: If  $\|h_1 \cup h_2\| = \tau$ , let  $f_A = h_1 \cup h_2$  and stop; otherwise, go to Step 4.

Step 4: Let  $f_A = s - v$  and stop.

The properties of Algorithm 1 are easily obtained from Lemmas 3, 4 and 5, and are summarized below.

*Theorem 1:* Let  $S$  be a BGM model, let Hypothesis 1 be satisfied, let  $\|f_S\| \leq \tau$ , and let  $f_A$  be the set generated by Algorithm 1. If  $\|f_A\| \leq \tau$ , then  $f_S = f_A$ , and if  $\|f_A\| \geq \tau + 1$ , then  $\|f_S\| = \tau$ ,  $f_S \subseteq f_A$ , and  $\|f_A - f_S\| \leq \tau$ .

In 1974, Hakimi and Amin [5] proposed two characterizations of the testing interconnection design that insure one-step  $\tau$ -fault diagnosability for the PMC model. We will now examine the implications of the first of these assumptions on the sets  $h_1$ ,  $h_2$ ,  $h_3$  and Algorithm 1. We begin our investigation by repeating the first Hakimi-Amin assumption.

*Hypothesis 2:* No two modules in  $S$  test each other.

*Lemma 6:* If  $S$  is a BGM model, if Hypotheses 1 and 2 are satisfied, and if  $\|f_S\| \leq \tau$ , then either  $\|s - v\| \leq \tau - 1$  or  $\|h_1 \cup h_2\| = \tau$ .

Proof: If  $\|f_S\| \leq \tau - 1$ , then Lemma 2 implies that  $f_S = s - v$ , and therefore,  $\|s - v\| \leq \tau - 1$ .

If  $\|f_S\| = \tau$ , then  $h_3$  is either empty or not. If  $h_3$  is empty, then  $f_S = h_1 \cup h_2$  and  $\|h_1 \cup h_2\| = \tau$ . If  $h_3$  is nonempty, then let  $l$  be in  $h_3$ . The fact that no two modules test each other implies that no module in  $h_3$  may test

any other module in  $h_3$ . It follows that  $U_i$  is tested by exactly  $\tau$  other modules that must be in  $h_1 \cup h_2$ , and we may conclude that  $|h_1 \cup h_2|$  must be equal to  $\tau$ .  $\square$

The result of Lemma 6 shows that Algorithm 1 generates the set of faulty modules in  $S$  whenever Hypotheses 1 and 2 are satisfied and the number of faulty modules is no greater than  $\tau$ .

*Theorem 2:* Let  $S$  be a BGM model, and let Hypotheses 1 and 2 be satisfied. If  $|f_S| \leq \tau$ , then Algorithm 1 stops either in Step 1 or Step 3 and the set  $f_A$  that it generates satisfies

$$f_S = f_A.$$

It is clear from Theorem 2 that if  $S$  is a BGM model and if Hypotheses 1 and 2 are satisfied, then  $S$  is one-step  $\tau$ -fault diagnosable.

#### 4. ONE-STEP DIAGNOSIS

At this stage, we have a decoding algorithm, Algorithm 1, that produces an index set  $f_A$  that is equal to  $f_S$  whenever  $|f_A| \leq \tau$ , and that contains the index of some nonfaulty modules whenever  $|f_A| > \tau + 1$ . If, in addition to Hypothesis 1, we assume that either  $|f_S| \leq \tau - 1$  or that the first Hakimi-Amin hypothesis is satisfied and  $|f_S| \leq \tau$ , then  $f_A = f_S$ . In this section, we will modify Algorithm 1 so that it produces the set of indices of all the faulty modules when  $|f_S| \leq \tau$  and the appropriate assumptions on  $TID$  are verified.

When Hypothesis 1 is satisfied and  $|f_S| \leq \tau$ , Algorithm 1 fails to produce a set  $f_A$  that is equal to  $f_S$  when  $|f_S| = \tau$  and  $|h_1 \cup h_2| \leq \tau - 1$ . The only index set that presents a problem in that case is the set  $h_3$ . It is clear that if the index  $i$  is in  $h_3$ , the module  $U_i$  is tested by exactly  $\tau$  other modules

(otherwise  $i$  would be in  $v \cup h_1 \cup h_2$ ). Thus, since  $|h_1 \cup h_2| \leq \tau - 1$ , at least one index  $j(i)$  in  $h_3$  exists so that the module  $U_{j(i)}$  tests the module  $U_i$ . The fact that the index  $j(i)$  is also in  $h_3$  implies immediately that the modules  $U_{j(i)}$  and  $U_i$  test each other. We are led to define the following index set  $w$  that depends only on the interconnection design.

*Definition 2:* Let  $w$  be the subset of  $s$  that contains all the indices  $i$  in  $s$  such that:

- (i) the module  $U_i$  is tested by exactly  $\tau$  other modules, and
- (ii) an index  $j(i)$  in  $s$  exists such that  $U_{j(i)}$  is tested by exactly  $\tau$  other modules in  $S$ , and  $U_i$  and  $U_{j(i)}$  test each other.

In view of this discussion, we arrive at the result below.

*Lemma 7:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied, if  $|S| = \tau$  and if  $|h_1 \cup h_2| \leq \tau - 1$ , then

$$h_3 \subseteq w.$$

The set  $w$  given in Definition 2 may contain indices that correspond to nonfaulty modules. We must now define a subset  $x$  of  $w$  that possesses the desired property: a module  $U_i$  is faulty whenever  $i$  is in  $h_3 \cap x$ .

*Definition 3:* Let  $x$  be the set of all indices  $i$  in  $s$  such that:

- (i)  $U_i$  is tested by exactly  $\tau$  other modules;
- (ii) an index  $j(i)$  exists such that  $U_{j(i)}$  is tested by exactly  $\tau$  modules in  $s$  and  $U_i$  and  $U_{j(i)}$  test each other; and
- (iii) an index  $k(i)$  in  $s$  exists such that the module  $U_{k(i)}$  tests  $U_{j(i)}$  but not  $U_i$ , and  $U_{k(i)}$  is tested by at least one module that does not test  $U_i$ .

The set  $x$  given in Definition 3 depends only on the testing interconnection

design; it may be computed once and stored. Its importance lies in that it may be used directly to find faulty modules in  $h_3$ .

*Lemma 8:* Let  $S$  be a BGM model, let Hypothesis 1 be satisfied, let

$|f_S| = \tau$  and let  $|h_1 \cup h_2| \leq \tau - 1$ . If the index  $i$  is in  $h_3 \cap x$ , then  $i$  is in  $f_S$ .

*Proof:* Let  $i$  be in  $h_3 \cap x$  and let  $j(i)$  and  $k(i)$  be indices that satisfy the assumptions of Definition 3. Suppose that  $U_i$  and  $U_{k(i)}$  are both nonfaulty. All the modules that test  $U_i$  and those that test  $U_{k(i)}$  are then faulty. The module  $U_{i(k)}$  is tested by at least one module that does not test  $U_i$  and therefore, the assumption that both  $U_i$  and  $U_{k(i)}$  are nonfaulty implies that at least  $\tau + 1$  modules are faulty. This is impossible, and thus, if  $U_i$  is nonfaulty,  $U_{k(i)}$  must be faulty. The module  $U_{k(i)}$  does not test the module  $U_i$ , and the fact that  $i$  is in  $h_3$  implies that  $U_i$  does not test  $U_{k(i)}$ . Thus, if  $U_i$  is assumed to be nonfaulty, we must conclude that at least  $\tau + 1$  modules are faulty. Once again, this is impossible; Therefore, the module  $U_i$  must be faulty -- i.e.,  $i$  is in  $f_S$ .  $\square$

Using the result of Lemma 8, we may modify Algorithm 1 to incorporate the fact that under the appropriate conditions, the set  $h_3 \cap x$  is a subset of  $f_S$ .

*Algorithm 2:*

Step 0: Compute the set  $v$  as in Equation (1).

Step 1: If  $|s - v| \leq \tau$ , let  $f_B = s - v$  and stop; otherwise, go to Step 2.

Step 2: Compute the sets  $h_1$  and  $h_2$  as in Equations (2) and (3).

Step 3: If  $|h_1 \cup h_2| = \tau$ , let  $f_B = h_1 \cup h_2$  and stop; otherwise, go to Step 4.

Step 4: Let  $f_B = h_1 \cup h_2 \cup (h_3 \cap x)$  and stop.

At this point, it is clear that when the appropriate assumptions are

satisfied, Algorithm 2 generates a set  $f_B$  that is a lower bound for  $f_S$ .

*Lemma 9:* If  $S$  is a BGM model, if Hypothesis 1 is satisfied and if  $|f_S| \leq \tau$ , then the set  $f_B$  generated by Algorithm 2 satisfies

$$f_B \subseteq f_S.$$

In their 1976 paper, Barsi, Grandoni and Maestrini [2] proposed a condition on *TID* that insures one-step  $\tau$ -fault diagnosability. Using our notation, we will now repeat that assumption and show that it may be used to insure that  $f_S$  is found.

*Hypothesis 3:* If the indices  $i$  and  $j$  are in  $w$  and if  $U_i$  and  $U_j$  test each other, then  $i$  or  $j$  or both are in  $x$ .

*Lemma 10:* If  $S$  is a BGM model, if Hypotheses 1 and 3 are satisfied, if  $|f_S| = \tau$  and if  $|h_1 \cup h_2| \leq \tau - 1$ , then

$$|h_3 \cap x| = \tau - |h_1 \cup h_2|.$$

*Proof:* The set  $h_3$  contains at least  $\tau - |h_1 \cup h_2|$  indices and therefore, if all the indices in  $h_3$  are also in  $x$ , then

$$|h_3 \cap x| \geq \tau - |h_1 \cup h_2|.$$

If at least one index, say  $i$ , is in  $h_3$  but not in  $x$ , then  $U_i$  is tested by at least  $\tau - |h_1 \cup h_2|$  other modules with indices in  $h_3$ . All the modules with indices in  $h_3$  that test  $U_i$  are also tested by  $U_i$ . Hypothesis 3 implies that they are all in  $x$  and therefore,

$$|h_3 \cap x| \geq \tau - |h_1 \cup h_2|.$$

Lemma 9 implies that all indices in  $h_1 \cup h_2 \cup (h_3 \cap x)$  are in  $f_S$ , and using the assumption that  $|f_S| \leq \tau$ , we obtain

$$|h_3 \cap x| = \tau - |h_1 \cup h_2|. \square$$

The results of Lemmas 3, 4 and 10 show that, when the appropriate conditions are satisfied, Algorithm 2 generates the set of indices of all the faulty modules in  $S$ .

**Lemma 11:** If  $S$  is a BGM model, if Hypotheses 1 and 3 are satisfied and if  $|f_S| \leq \tau$ , then the index set  $f_B$  produced by Algorithm 2 satisfies

$$f_S = f_B.$$

Note that Lemma 11 implies that if  $S$  is a BGM model that satisfies Hypotheses 1 and 3, then  $S$  is one-step  $\tau$ -fault diagnosable, i.e., the result given in [2, Theorem 2, p. 585] is retrieved.

It is known that if a BGM model is one-step  $\tau$ -fault diagnosable, then Hypotheses 1 and 3 are satisfied [2]. Hence, we obtain the main result of the paper.

**Theorem 3:** Let  $S$  be a one-step  $\tau$ -fault diagnosable BGM fault model and let  $F_S$  be the set of faulty modules in  $S$ . If  $|F_S| \leq \tau$ , then the index set  $f_B$  generated by Algorithm 2 satisfies

$$F_S = \{U_i \mid i \in f_B\}.$$

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