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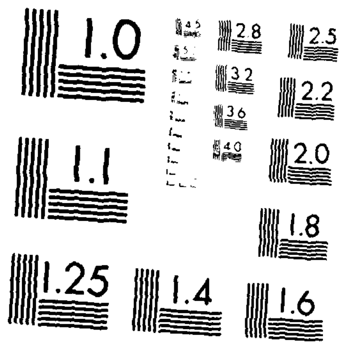

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TECHNICAL REPORT ARBRL-TR-02497

A NONSIMILAR SOLUTION FOR BLAST WAVES DRIVEN BY AN ASYMPTOTIC PISTON EXPANSION

Mark L. Bundy

June 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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An analytical technique is outlined for predicting the blast wave from a rapid but asymptotic expansion of an inner piston-like surface. The method more accurately describes real blasts compared to conventional point source models and agrees favorably with the exact method of characteristic solution for wave Mach numbers exceeding approximately 1.1. The method makes two a priori assumptions: 1) that the blast wave density can be described by a power law, and 2) that an equation exists for the motion of the driving piston. The present treatment extends the work of Bach and Lee, who assume a zero radius piston (i.e., a point source).

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I. INTRODUCTION

When the contact surface of an expanding gas moves faster than the speed of sound in the stagnant medium ahead of it, a blast wave is formed. Solutions to this type of problem are of interest in a variety of practical problems such as the rupture of pressurized vessels or the blast from high energy explosives. Ignoring chemical reactions within the blast wave, the rate at which work is done in pushing aside the gas ahead of the contact surface is the prime factor in determining which analytical model will most accurately describe the real blast wave. For instance, a self-similar point source model such as that used by Taylor[1] assumes that all the work of expansion is done initially, in an infinitesimal amount of time, and is valid only for strong blast waves (i.e., high shock Mach numbers). A more realistic self-similar model such as that used by Freeman[2] assumes continuous energy input to the blast wave but, except for the case of a constant wave Mach number, is also valid only in the strong blast regime. Clearly what is needed is an analytical technique which is valid for all blast wave strengths and uses the realistic assumption that the energy input exists for the finite amount of time during which the piston (contact surface) expands. Such a technique is presented here; it is nonsimilar and assumes an asymptotic energy input to the blast wave. The method is applied to the specific case of the rupturing of a sphere of compressed gas and the results compared with the numerical -- method of characteristics -- solution to the identical problem.

II. PROCEDURE

Self-similar solutions assume that the shock normalized blast wave variables do not have explicit time dependence. Though this assumption may be useful when the wave is strong, it must be inaccurate for the entire blast wave history. For example, the minimum in the blast wave pressure will be located at its innermost boundary when the wave is strong but moves forward as the wave overexpands. Self-similar models correctly predict the strong wave behavior but without intrinsic time dependence are unable to predict a change in the minimum location as the wave weakens. The method used here is nonsimilar and is a modification of the procedure used by Bach and Lee[3] which is not used intact because of its assumption of an instantaneous point source of energy. The Bach and Lee analysis is modified here to treat the more realistic case of an asymptotic energy supply.

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1. *G. I. Taylor, "The Formation of a Blast Wave by a Very Intense Explosion, I, Theoretical Discussion," Proceedings of the Royal Society (London), Series A, Vol. 201, 1950, pp. 159-174.*

2. *R. A. Freeman, "Variable-Energy Blast Waves," Brit. J. Appl. Phys. (J. Phys. D), Ser. 2, Vol. 1, 1968, pp. 1697-1710.*

3. *G. G. Bach, and J. H. S. Lee, "An Analytical Solution for Blast Waves," AIAA Journal, Vol. 8, No. 2, February 1970, pp. 271-275.*

Convenient coordinates for a spherically symmetric blast are

$$y = r/R_s \quad , \tag{1}$$

where R_s is the shock front radius, and

$$n = (c_o/\dot{R}_s)^2 \quad , \tag{2}$$

where c_o is the stagnant medium sound speed, with the dot symbolizing time differentiation. Let the density, pressure, and fluid velocity within the blast wave be defined by, respectively,

$$\rho(r,t) = \rho_o \, A(y,n) \quad , \tag{3}$$

$$p(r,t) = \rho_o(\dot{R}_s)^2 \, B(y,n) \quad , \tag{4}$$

$$u(r,t) = \dot{R}_s \, C(y,n) \quad . \tag{5}$$

One of two a priori assumptions associated with this analytical technique is to assume that the density in the blast wave conforms to

$$\rho(r,t) = \rho(R_s,t) \, y^q \quad , \tag{6}$$

or equivalently

$$A(y,n) = A(1,n) \, y^q \quad , \tag{7}$$

where q is a function of n to be determined. Assuming (7) for A makes it possible to solve for B and C from the equations for mass and momentum conservation alone, viz.

$$(C-y)A_y + AC_y + 2CA/y = 2hnA_n \quad , \tag{8}$$

$$(C-y)C_y + hC + B_y/A = 2hnC_n \quad , \tag{9}$$

where

$$h = R_s\ddot{R}_s/(\dot{R}_s)^2 \tag{10}$$

with the subscripts y and n indicating partial differentiation with respect to y or n accordingly. The Rankine-Hugoniot boundary relations across the shock front are, for a polytropic gas,

$$A_1 = A(1,n) = (\gamma+1)/(\gamma-1+2n) \quad , \tag{11}$$

$$B_1 = B(1,n) = (2\gamma + n(1-\gamma))/(\gamma(\gamma+1)) \quad , \tag{12}$$

$$C_1 = C(1,n) = 2(1-n)/(\gamma+1) \quad , \tag{13}$$

where γ is the specific heat ratio in the blast wave region. Substitution of (7) for A into the continuity Equation (8) yields upon integration the following closed form expression for C:

$$C(y,n) = y(T + (C_1-T)y^{-(q+3)} + Y\ln y) \quad , \tag{14}$$

where

$$T(q,n,h) = G(q) + hH(n,q) \quad , \tag{15}$$

$$Y(q,n,h) = hJ(n,q) \quad , \tag{16}$$

$$G(q) = (q)/(q+3) \quad , \tag{17}$$

$$H(n,q) = (2nA'_1)/(A_1(q+3)) + J/(q+3) \quad , \tag{18}$$

$$J(n,q) = (2nq')/(q+3) \quad , \tag{19}$$

with the prime signifying total differentiation with respect to n. Substitution of (14) for C and (7) for A into the momentum Equation (9) yields upon integration a similarly closed form expression for B,

$$B(y,n) = B_1 + A_1 \; (2hnT'-hT-(T-1)(T+Y))(y^{q+2}-1)/(q+2)$$

$$+ A_1(2hn(C'_1-T')-h(C_1-T)+(T-1)(C_1-T)(q+2)-(C_1-T)(T+Y))(1-1/y)$$

$$+ A_1(2hnY'-hY-Y(2T+Y-1))((\ln y-1/(q+2))(y^{q+2}/(q+2))+1/(q+2)^2)$$

$$+ A_1(2hn(T-C_1)q'-Y(T-C_1)(q+1))(1-(\ln y+1)(1/y))$$

$$+ A_1(Y^2)(2/(q+2)^3-(\ln^2 y-(2/(q+2))(\ln y-1/(q+2))(y^{q+2}/(q+2))$$

$$+ A_1((C_1-T)^2(q+2))(1-y^{-(q+4)})/(q+4) \quad . \tag{20}$$

The right-hand side of the closed form fluid variable expressions, (7), (14), and (20), are completely known at any given time provided the quantities q(t), h(t), and n(t) are known at that time. The variables q, h, and n are determined by invoking global mass and energy conservation across the blast wave region as follows. Conservation of mass within the blast wave requires that

$$\int_{R_c}^{R_s} 4\pi\rho r^2 \, dr = \int_{R_o}^{R_s} 4\pi\rho_o r^2 dr \quad , \tag{21}$$

where R_o and $R_c(t)$ are the contact surface radius at time t=0 and time t, respectively. Figure 1 shows the blast wave region, defined between R_c and R_s, as it evolves in time. Substitution of (6) into (21) yields upon integration an algebraic equation for q:

$$q = 3A_1(1-w^{q+3})/(1-w_o^{\,3}) - 3 \ , \tag{22}$$

where

$$w = R_c/R_s \quad , \tag{23}$$

$$w_o = R_o/R_s \quad . \tag{24}$$

Using (22) for q introduces the unknown $R_c(t)$; note R_o will be given as an initial condition of the problem. To obtain $R_c(t)$ a second a priori assumption is provided (the first being (6)); viz., it is assumed in advance that an expression for $R_c(t)$ exists. The following expression for $R_c(t)$ was found useful for modeling the contact surface of propellant plumes from guns and will be used here (though the theory remains valid for any contact surface expression):

$$R_c(t) = R_\infty(1-1/\sqrt{1+t/\tau}) + R_o \quad , \tag{25}$$

where R_∞ and τ are two constants which for the present can be assumed known. Note that expression (25) for the contact surface radius is asymptotic in time. With $R_c(t)$ known, the expression (22) for q depends solely on n and w. The three quantities h, n, and w are all related through the time derivatives of $R_s(t)$. By definition w satisfies

$$\dot{w} = w(\dot{R}_c - wc_o/\sqrt{n})/R_c \quad . \tag{26}$$

Similarly, the inverse Mach number squared, n, obeys

$$\dot{n} = -2c_o hw\sqrt{n}/R_c \quad . \tag{27}$$

Finally, an equation for h is developed from global energy conservation as follows. For a perfect gas, let the internal energy per unit mass be given by

$$e = p/(\rho(\gamma-1)) \quad . \tag{28}$$

Using (28), global energy conservation is written

$$E_s(t) = \int_{R_c(t)}^{R_s(t)} (p/(\gamma-1)+\rho u^2/2)4\pi r^2 dr$$

$$- 4\pi R_s^3(1-w_o{}^3)(\rho_o c_o{}^2/(3\gamma(\gamma-1))) \quad , \tag{29}$$

where $E_s(t)$ is the work input to the blast wave from the pushing action of the piston-like contact surface and is therefore given by

$$E_s(t) = \int_{R_o}^{R_s(t)} p(w)4\pi r^2 dr \quad . \tag{30}$$

Integration and rearrangement of (29) using (30) yields for dh/dt

$$\dot{h} = \dot{n}(L-W)/V \quad , \tag{31}$$

where

$$W(\dot{h},n,q,w) = E_s(t)/((4\pi\rho_o c_o{}^2 R_s{}^3)(1-w_o{}^3)) \quad , \tag{32}$$

$$L(h,n,q,w) = -(1-w_o{}^3)/(3k(k-1)) + ((1-w^2)/2)(-a_4-(a_8/2)+b_2-(b_5/2))$$

$$+ ((1-w^3)/3)(a_1-a_2+a_4+a_6+a_8+2a_9+a_{10}) + ((1-w^{-(q+1)})/(q+1))(a_{10}+b_4)$$

$$- ((w^{(q+5)}\ln^2 w)/(q+5))(b_6-(q+2)^2 a_9) + ((1-w^{(q+5)})/(q+5))(a_2-a_6(2q+7)/(q+5)$$

$$- 2a_9((2q+7)/(q+5)+(q+2)^2/(q+5)^2) + b_1-(b_3/(q+5))+(2b_6/(q+5)^2))$$

$$+ (w^2\ln w/2)(a_8-b_5) - (w^{(q+5)}\ln w/(q+5))((q+2)a_6$$

$$+2a_9 ((q+2)(2q+7)/(q+5))+b_3-(2b_6/(q+5))) \quad , \tag{33}$$

$$V(h,n,q,w) = a_5((1-w^2)/2) + ((1-w^3)/3)(a_3-a_5-a_7)$$

$$+ ((1-w^{(q+5)})/(q+5))(a_7(2q+7)/(q+5)-a_3) + ((w^{(q+5)}\ln w)/(q+5))(a_7(q+2)), \quad (34)$$

with the a_i's, b_i's and c_i's defined in the appendix. The $\dot{h}$ dependence in (32) for w can be factored out and moved to the left-hand side in (31) but is left undone here to shorten the expressions. It is convenient for numerical solution to put (22) for q into the form

$$\dot{q}=\dot{n}(((-2A_1(1-w^{(q+3)})/(\gamma-1+2n))-A_1(q+3)(w^{(q+2)}\dot{w}/\dot{n}))/(1-w_o{}^3)$$

$$+ 3w_o{}^3A_1(1-w^{(q+3)})/((2hn)(1-w_o{}^3)^2))/((A_1w^{(q+3)}\ln w)/(1-w_o{}^3) + 1/3) \quad . \quad (35)$$

The system of four first order ordinary differential equations for q, h, n, and w (viz. (35), (31), (27), and (26)) can be solved numerically using a Runge-Kutta procedure provided initial values for q, h, n, and w are available. To obtain these it is assumed that the initial expansion of the contact surface is analogous to the initial expansion after rupture of a shock tube diaphram. The theory for the shock tube case has a well-known analytic solution. It will thus be assumed for simplicity that the gas expands for an initial time interval Δt_o as if it were a one-dimensional planar problem. Values for $R_s(\Delta t_o)$, $R_c(\Delta t_o)$, $\dot{R}_s(\Delta t_o)$, and $\ddot{R}_s(\Delta t_o)$ are thus obtained and used in (22), (23), (2), and (10) to provide initial values for q, w, n, and h. Having obtained the solution for q(t), h(t), and n(t), their substitution into (7), (14), and (20) yields the blast wave density, velocity, and pressure, respectively, at that time as a function of y alone.

This treatment is fundamentally different from References 1-3 in that it is not a point blast theory. Unlike References 1-3, $R_c(t)$ and R_o are not assumed zero. In fact, replacing $R_c(t)$ and R_o by zero in the above expressions reduces them to their point blast counterparts in Reference 3. The present method is now demonstrated on a hypothetical test case.

III. RESULTS

Using (25) for $R_c(t)$, assume that an ideal gas, with

$$\begin{aligned} \tilde{\gamma} &\text{ (specific heat)} = 1.25 \quad , \\ \tilde{T} &\text{ (temperature)} = 2831\ ^\circ K \ , \\ \tilde{v} &\text{ (moles)} = 1.63 \text{ moles} \ , \\ \tilde{m} &\text{ (mass/mole)} = 0.024 \text{ kg/mol} \quad , \end{aligned} \quad (36)$$

is confined within a spherical volume, of radius

$$R_o = 0.1\ \text{m} \ , \tag{37}$$

which ruptures at time t = 0. The solution for the ensuing blast wave is predicted using both the present analytical technique and a more exact non-isentropic method of characteristics technique. For comparison, the results are also nondimensionalized and compared with the point blast model of Bach and Lee.

For the purpose of evaluating R_∞, used in (25), let it be assumed that the compressed gas undergoes an irreversible and adiabatic expansion against a background pressure of one atmosphere. Under these conditions, conservation of energy will lead to the following equation for the final radius

$$R_\infty = (3\tilde{v}k\tilde{T}/(4\pi\tilde{\gamma}p_o))^{(1/3)} - R_o \ , \tag{3}$$

where k is the universal gas constant and p_o is one atmosphere of pressur both in the SI system. Furthermore, to evaluate τ, in (25), use again the assumption that the initial expansion of the compressed gas in the radial direction can be approximated by a one-dimensional planar expansion in a shock tube. In which case the contact surface velocity at t = 0 is given by

$$\dot{R}_c(t=o) = R_\infty/(2\tau) = c_o Z/(\gamma(1+(\gamma+1)Z/(2\gamma))^{\frac{1}{2}}) \ , \tag{39}$$

where the first equality in (39) is obtained from the time derivative of (25) at t = 0, and Z in the last equality must satisfy

$$Z/(\gamma(1+(\gamma+1)Z/(2\gamma))^{\frac{1}{2}}) = (2\tilde{c}/(c_o(\tilde{\gamma}-1)))(1-(p_o(1+Z)/\tilde{p})^{(\tilde{\gamma}-1)/2\tilde{\gamma}}) \ , \tag{40}$$

where

$$\tilde{p} = \tilde{v}k\tilde{T}/(4\pi R_o^{\ 3}/3) \ , \tag{41}$$

$$\tilde{c} = (\tilde{\gamma}\tilde{p}/(\tilde{v}\tilde{m}/(4\pi R_o^{\ 3}/3)))^{\frac{1}{2}} \ . \tag{42}$$

From (39)

$$\tau = R_\infty(\gamma(1+(\gamma+1)Z/(2\gamma))^{\frac{1}{2}}/(2c_o Z)) \ . \tag{43}$$

Though the analysis leading to (38) and (43) is oversimplified, the expressions thus obtained were useful for the intended purpose of (25), viz., to predict the contact surface radius of the exhaust gases expelled after shot ejection from guns. In any case, the expression used for the contact surface radius is not at issue in the present treatment; it is merely assumed a priori that such an expression exists. Obtaining (38) and (43) for use in (25) gives the appropriate $R_c(t)$ for the example under investigation. The complete results for this test case are shown in Figures 2-4. Figure 2 is a composite of the time variation of: (a) the exponent q used in (6) for the power law form of the density; (b) the Mach number of the shock front, $M = n^{-\frac{1}{2}}$; and (c) the fraction of energy input at time t to the total energy input at time $t = \infty$.(The scale of time is chosen to show the most dramatic change in the variables, which in this problem is from t = 0 to t = 4500μsec.) Figure 2(c) shows that the energy input to the blast wave is indeed asymptotic with time as opposed to the instantaneous input characteristic of point blast theories. A comparison between the results predicted by the point blast theory of Bach and Lee and the present treatment is summarized in non-dimensional format in Figure 3. Perhaps surprising in view of their inherent differences is the closeness in the curves of Figure 3(a) for the shock trajectory R_s/R_e vs n, where

$$R_e = (E_{total}/(\rho_o c_o^2 4\pi))^{1/3} \tag{44}$$

It appears from this plot that approximating a timed release of energy E_{total} by an instantaneous release of E_{total} has essentially no effect on the predicted shock trajectory. On the other hand, Figures 3(b) - 3(d) show that there are differences in the predicted profiles of the other fluid variables. (The subscript 1 refers to the value at the shock.) Most noticeable of these is the difference in the inner boundary of the blast wave which is r = 0 for the point blast model and $r = R_c(t)$ for the piston blast model. In Figures 3(c) and 3(d), the piston blast solution falls within the limits predicted by the point blast solution. But Figure 3(b) shows a significant difference between the piston and point blast solutions. In the latter case, the fluid velocity has a substantial negative phase while in the former case the velocity is only beginning to show a region of reversed flow. In order that Figure 3 may be taken with confidence to illustrate the inherent limitations in the point blast approximation, a comparison is made between the piston solution used in Figure 3 and a more exact numerical solution. To serve this purpose, a nonisentropic, numerical method of characteristic (M.O.C.) solution was obtained for this problem. A comparison between solutions is shown at four different times in Figure 4. The particular times chosen illustrate collectively the full range of variable profiles. The time spanned in Figure 4 correlates with the time scale in Figure 2. In addition, the n = 0.82 solution of Figure 3 corresponds to the analytical solution shown in Figure 4(c). Shown at each time in Figure 4 are four subplots. The time of Figure 4(a), for example, is indicated by the dashed horizontal line in the top subplot. Following this dashed line across to the piston and shock curves and then down to the horizontal axis (distance axis) defines the blast wave region at this time. The blast wave region is marked through each of the lower subplots by vertical dashed lines. Each of the lower subplots - pressure, density, and velocity - shows the profile of the corresponding fluid variable as it changes from the piston to the shock. The analytical solution shows good agreement in all four subplots when the blast is strong, Figures 4(a) and 4(b). The favorable comparison has diminished, however, at the time of Figure 4(c). Figure 4(d) shows that the disagreement which began in the density variable, cf. Figure 4(c), now appears in all the fluid variables except the shock trajectory, which as noted in Figure 3 is a relatively insensitive quantity.

IV. CONCLUSION

The analytical point blast solution of Bach and Lee, which is based on a power law form for the density, is extended here to describe the more realistic case of blast waves driven by an asymptotic piston-like motion of an expanding inner gas. For the same total blast energy, it was found that the piston and point blast solutions predict virtually the same shock trajectory. However, the point blast model predicts overexpansion in the velocity profile significantly earlier than the piston blast solution. The analytical piston blast solution compared well with the more exact M.O.C. solution at Mach numbers greater than approximately 1.1. Theoretical disagreement appears first in the density profile but, as the wave weakens, is observed in all the fluid variables, with the exception of the shock trajectory. Thus, the analytic technique presented here for blast waves driven by a finite piston expansion supersedes its point blast predecessor, Reference 3, by giving a more accurate description of real blasts. The technique is also a viable alternative to numerical solutions provided the Mach number of the blast wave remains above approximately 1.1.

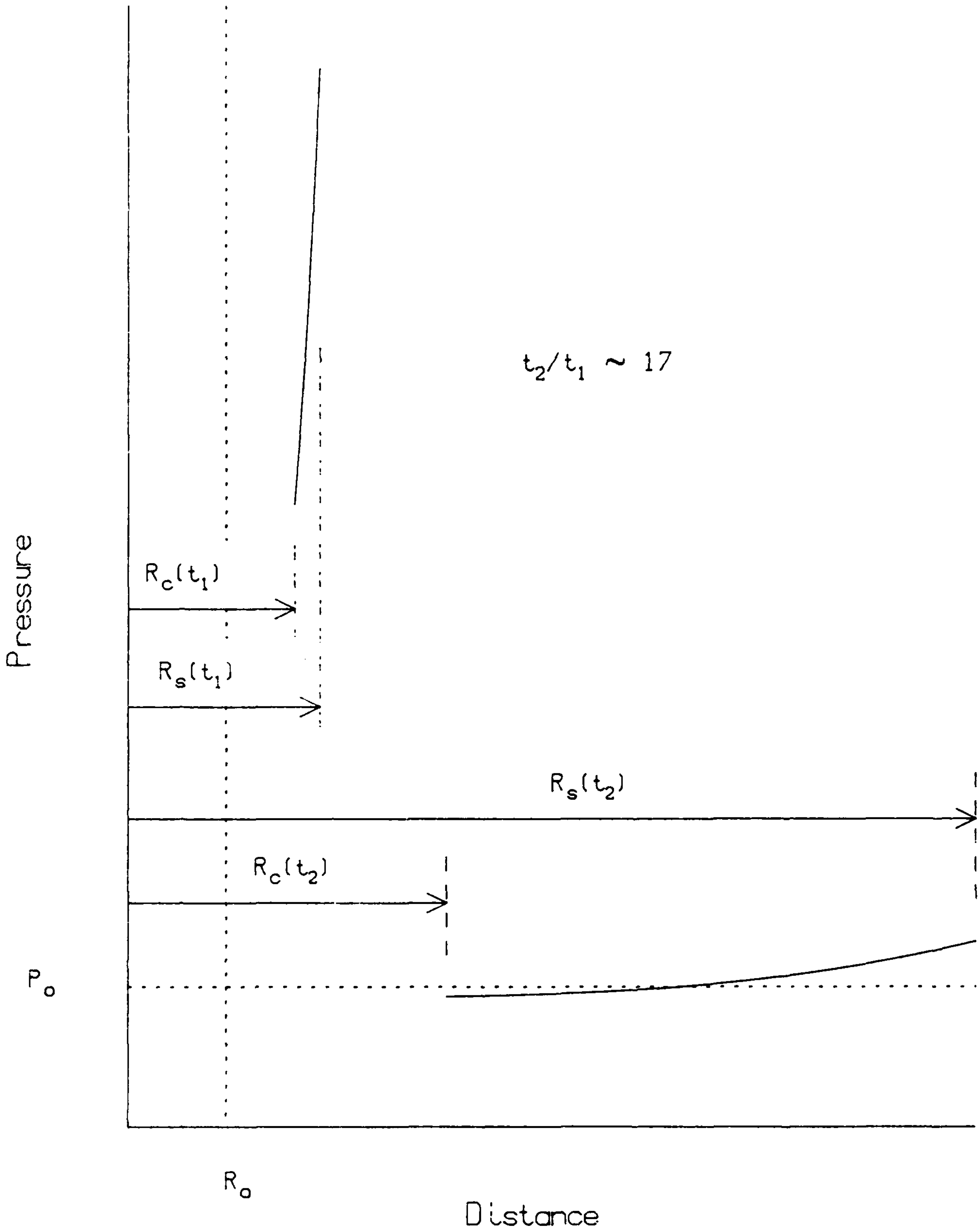

Fig. 1 Blast wave pressure contours at two widely separated times, where $R_c(t)$ is the contact surface radius, $R_s(t)$ is the shock front radius, R_o is the initial (preburst) sphere radius, and P_o is the ambient background pressure

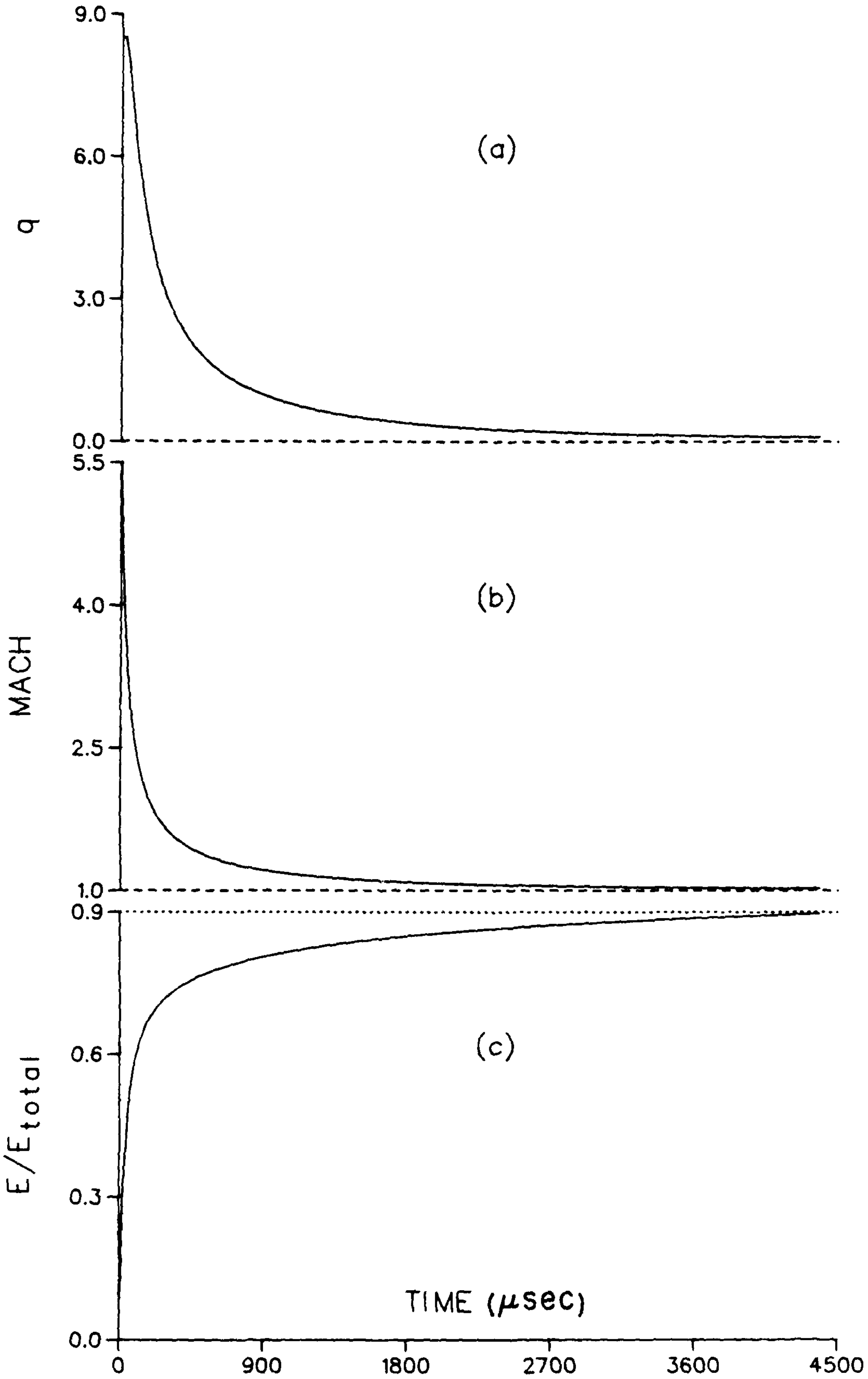

Fig. 2 Time variation of: a) the density's power law exponent q, b) the shock Mach number, and c) the fractional energy input to the blast wave.

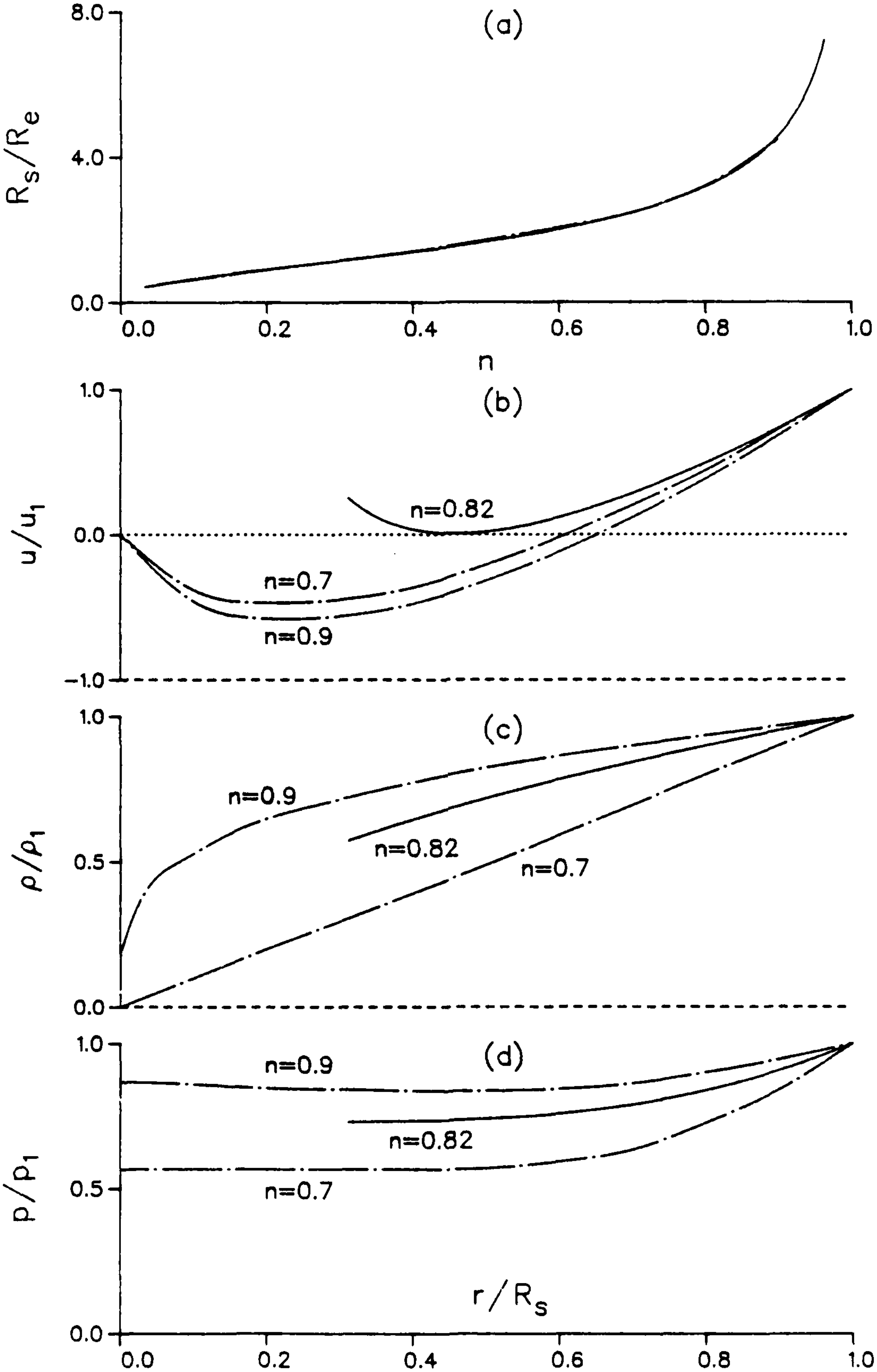

Fig. 3 Present piston blast theory (——) vs point blast theory of Bach and Lee (— · —) for: a) shock trajectory vs shock strength; and b) – d), variation of velocity, density, and pressure within the blast wave.

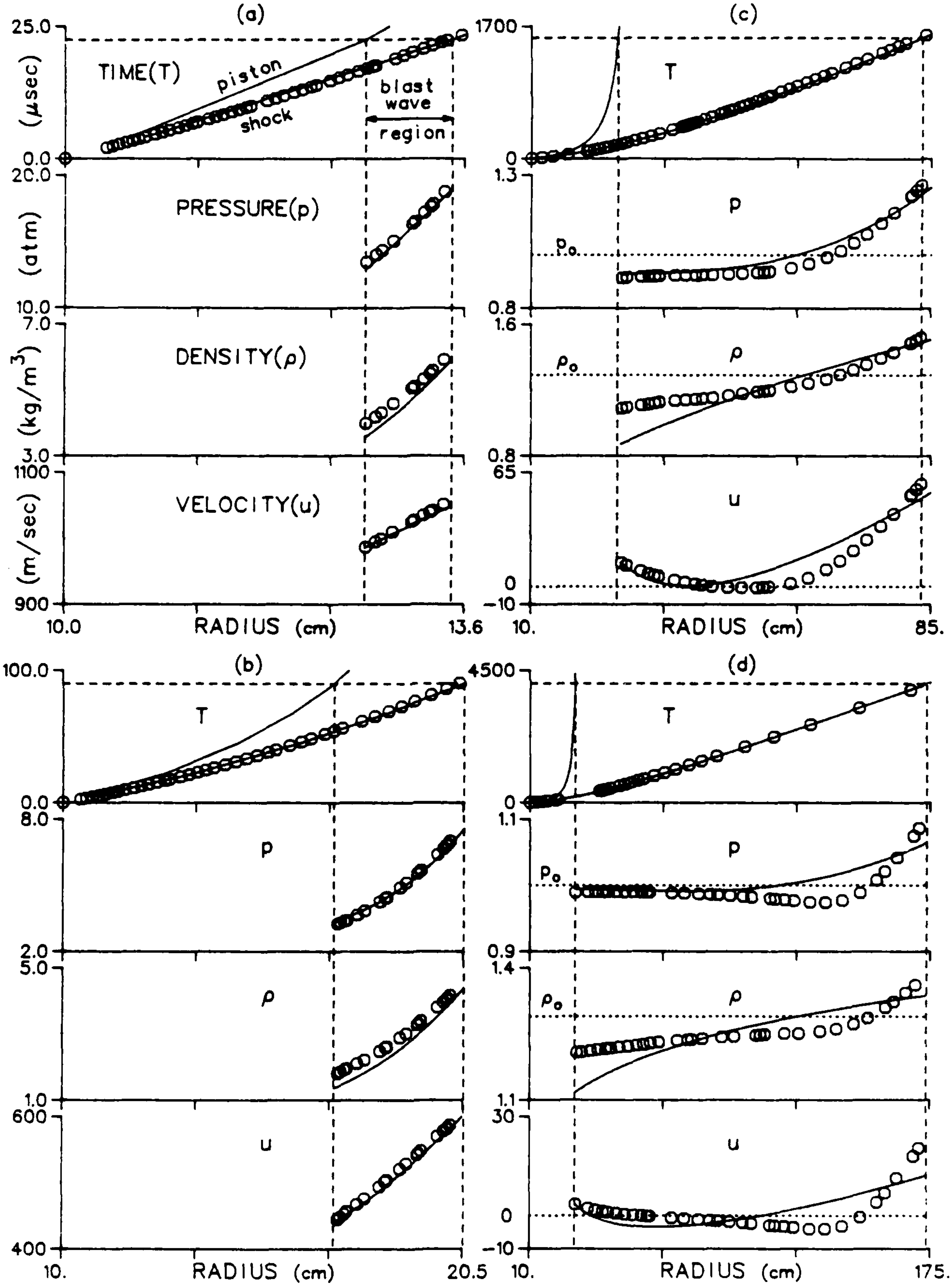

Fig. 4 Analytical (——) and numerical (ooo) predictions for the velocity(u), density(ρ), pressure(p), and shock trajectory of the blast wave for the times: a) 22.5μsec,Mach(M)=4.0; b) 89μsec,M=2.6; c) 1552μsec,M=1.1; and d) 4050μsec,M=1.02.

APPENDIX A

APPENDIX A

The a_i's, b_i's and c_i's used in (32) - (34) are as follows:

$$a_1 = B_1/(n(\gamma-1))$$

$$a_2 = A_1((2hn)(G'+hH')-hT+(1-T)(T+Y))/(n(q+2)(\gamma-1))$$

$$a_3 = A_1 2hH/((q+2)(\gamma-1))$$

$$a_4 = A_1((2hn)(C_1' -G'-hH')+h(T-C_1)+(T-1)(C_1-T)(q+2)+(T-C_1)(T+Y))/(n(\gamma-1))$$

$$a_5 = -A_1 2hH/(\gamma-1)$$

$$a_6 = A_1(2h^2nJ'-hY+Y(1-2T-Y))/(n(\gamma-1)(q+2)^2)$$

$$a_7 = A_1 2hJ/((\gamma-1)(q+2)^2)$$

$$a_8 = A_1(2hn)(T-C_1)(\dot{q}/\dot{n})+Y(C_1-T)(q+1))/(n(\gamma-1))$$

$$a_9 = A_1 Y^2/(n(\gamma-1)(q+2)^3)$$

$$a_{10} = A_1((q+2)(C_1-T)^2)/(n(\gamma-1)(q+4))$$

$$b_1 = A_1 T^2/2n$$

$$b_2 = A_1 T(C_1-T)/n$$

$$b_3 = A_1 TY/n$$

$$b_4 = -A_1(C_1-T)^2/2n$$

$$b_5 = A_1 Y(C_1-T)/n$$

$$b_6 = A_1 Y^2/2n$$

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