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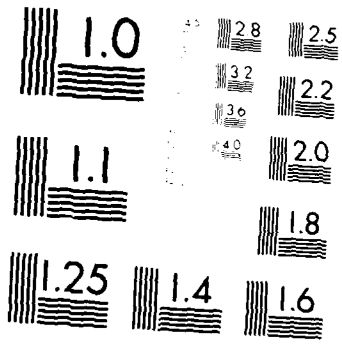

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TECHNICAL REPORT ARBRL-TR-02494

(Supersedes IMR No. 683)

AN APPROXIMATION OF THE EXPECTED STOWED KILLS IN SEQUENTIAL 1 X M ENGAGEMENTS

Lawrence D. Johnson

June 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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An equation is developed which approximates the expected number of red tanks killed per blue tank stowed load in sequential 1 X M tank engagements.

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TABLE OF CONTENTS

A

INTRODUCTION

Contained herein is a derivation of an equation which has been used to convert 1 X M duel simulation outputs into a measure of performance which integrates lethality, vulnerability and ammunition depletion rate.

Section I discusses the impetus for the derivation; Section II derives the equation; Section III discusses example applications, and Section IV summarizes the results.

I. BACKGROUND

The Ballistic Research Laboratory (BRL), working with the Army Materiel Systems Analysis Activity (AMSAA), has developed a computer model which simulates "N X M" tank engagements. Among the various outputs of this model are the probability that each of all possible terminal states can occur and the average ammunition depletion of each element associated with each state.

During a dry-run exercise of the program, it was observed that several systems had similar probable final engagement outcomes, but dissimilar burdens (in terms of ammunition depletion). It became evident that some method was required to integrate the burden directly* into the performance measure. It was decided that a measure which indicated the probable number of tanks killed per unresupplied system in a sequence of 1 X M engagements would be of interest.

This report discusses the derivation of such a measure.

II. THE DERIVATION AND EQUATION

The problem that faced us was that the expected number of tanks killed by blue in a sequence of 1 X M engagements had to be approximated with only the following information at hand:

P_{ij} = probability that exactly (i-1) blue tanks and (j-1) red tanks are killed at the termination** of a 1 X M engagement

N_{ij} = the average number of rounds expended per blue tank given the final state indicated by P_{ij}

L = original ammunition load.

Before leaping into the derivation, let's define a few terms which will help keep the equations relatively compact.

---

*The model does [illegible] allow for the burden by [illegible] element's ammunition load.

**The game ends when [illegible] completely [illegible] element can fire, e.g., [illegible] out of ammunition.

Term	Example	Definition
m	$= M + 1$	The maximum dimensions of the matrix (P_{ij}) associated with a 1 X M engagement.
P_S	$= \sum_{j=1}^{m} P_{1j}$	Probability of blue survival in a 1 X M engagement. Note: It is assumed that $P_S < 1.0$.
E_n	$= \sum_{i=1}^{2} \sum_{j=1}^{m} N_{ij} * P_{ij}$	Expected number of blue rounds expended per engagement
$E_{n/S}$	$= \sum_{j=1}^{m} N_{1j} * P_{1j}/P_S$	Expected number of blue rounds expended in a survived engagement
$E_{n/\bar{S}}$	$= \sum_{j=1}^{m} N_{2j} * P_{2j}/(1-P_S)$	Expected number of blue rounds expended in an engagement not survived
E_K	$= \sum_{i=1}^{2} \sum_{j=1}^{m} (j-1) * P_{ij}$	Expected number of red tanks destroyed in an engagement
$E_{K/S}$	$= \sum_{j=1}^{m} (j-1) * P_{1j}/P_S$	Expected number of red tanks destroyed in an engagement survived by blue
$E_{K/\bar{S}}$	$= \sum_{j=1}^{m} (j-1) * P_{2j}/(1-P_S)$	Expected number of red tanks destroyed in a blue attrited engagement
E_R	$= E_{K/S}/E_{n/S}$	An indicator of the number of red tanks killed per blue round fired in an engagement survived‡ by blue
N	$= \text{integer} \left\{\frac{L}{E_{n/S}}\right\}$	The average number of successful engagements implied by an ammunition load consisting of L rounds
N_R	$= L - N * E_{n/S}$	The average number of rounds remaining after N engagements have been survived‡
$E_{K/R}$	$= N_R * E_R$	An approximate number of red tanks killed by N_R rounds.

‡ *Is capable of engaging future targets.*

With the preceding information at hand, we approximate the expected value equation by using engagement averages for the possible values of the number of red tanks killed, and by limiting the possible number of successful engagements to the average number of successful engagements implied by the original ammunition complement, i.e.,

$$E_T = \left\{ \sum_{i=0}^{N-1} (i * E_{K/S} + E_{K/\bar{S}}) P_S^i (1 - P_S) \right\} + (N * E_{K/S} + E_{K/R}) * P_S^N \tag{1}$$

where E_T is approximate expected kills per ammunition load. Note that the term $(i * E_{K/S} + E_{K/\bar{S}})$ is the average number of red tanks killed given that blue was attrited during the (i+1) engagement. Also note that $P_S^i(1 - P_S)$ is the probability that blue was attrited on specifically the (i+1) engagement.

The term on the far right refers to the situation where blue survives all N engagements. Notice that the term $E_{K/R}$ is a type of "fudge factor" to indicate the potential kills of the residual rounds stowed after N successful engagements.

The series contained in Equation (1) can be solved in closed form by remembering that

$$\sum_{i=0}^{N-1} a^i = \frac{1 - a^N}{1 - a}$$

$$\sum_{i=0}^{N-1} ia^i = a \frac{\partial}{\partial a} \left\{ \sum_{i=0}^{N-1} a^i \right\} .$$

With these relations, Equation (1) can first be reduced to

$$E_T = E_{K/\bar{S}} (1 - P_S^N) + E_{K/S} \frac{P_S}{(1 - P_S)} (1 - P_S^N) + E_{K/R} P_S^N \tag{2}$$

which can further be contracted to

$$E_T = \frac{(1 - P_S^N)}{(1 - P_S)} \left\{ E_{K/\bar{S}} (1 - P_S) + E_{K/S} P_S \right\} + E_{K/R} P_S^N \tag{3}$$

which then is seen to be

$$E_T = \frac{(1 - P_S^N)}{(1 - P_S)} E_K + P_S^N E_{K/R} . \tag{4}$$

At this point we can only hope that the equation cannot be reduced further or it might completely disappear!

One item which must be verified is that Equation (4) is continuous with respect to N_R, i.e., does the example "fudge factor" work at the extremes of $N_R = 0$ and $N_R = E_{n/S}$. When $N_R = 0$ there are no residual rounds and since $E_{K/R} = 0$ under this condition, the factor is correct for this extreme. However, as $N_R \to E_{n/S}$ the E_T equation should approach the next order value. This can be shown by observing that

$$\lim_{N_R \to E_{n/S}} \frac{(1 - P_S^N)}{1 - P_S} E_K + P_S^N E_{K/R} = \frac{(1 - P_S^N)}{1 - P_S} E_K + \lim_{N_R \to E_{n/S}} (P_S^N E_{K/R})$$

$$= \frac{(1 - P_S^N)}{1 - P_S} E_K + P_S^N * (E_{n/S} * E_{K/S} E_{n/S})$$

$$= \left(\frac{1 - P_S^{N+1}}{1 - P_S}\right) E_K .$$

Since the factor is consistent at the extremes, it appears reasonable. The implicit assumption concerning $E_{K/R}$ which will remain challengeable is that $E_{K/R}$ is linear with respect to N_R.

Another item of interest concerns the degenerate cases; namely, $N = 1$ and $N_R = 0$ and $N \to \infty$. Since $N = 1$ and $N_R = 0$ imply that there is, on average, insufficient ammunition to entertain subsequent engagements, the answer of $E_T = E_K$ of Equation (4) is intuitively palatable.

Finally, taking $N \to \infty$ implies an unlimited load and the result of

$$E_T = \frac{E_K}{(1 - P_S)}$$

is again consistent since it represents the exchange ratio for the engagement, i.e., the ratio of the expected number of red tanks killed to the expected number of blue tanks killed.

Bolstered by the absence of inconsistencies, the next section will discuss a few examples.

III. EXAMPLES

1 Blue Versus 1 Red

Assume that L = 50, and that

$$P_{ij} = \begin{bmatrix} .1 & .5 \\ .2 & .2 \end{bmatrix} ; \quad N_{ij} = \begin{bmatrix} 50 & 4 \\ 2 & 1 \end{bmatrix}$$

then
$$E_K = (.5 + .2) * 1 = .7$$

$$P_S = .1 + .5 = .6$$

$$E_{n/S} = (50 * .1 + 4 * .5)/.6 = 11.67$$

$$N = \text{integer } (50/11.67) = \text{integer } (4.29) = 4$$

$$N_R = 50 - 4 * 11.67 = 3.32$$

$$E_{K/R} = 3.32 * (.83/11.67) = .24$$

$$\lim_{N \to \infty} E_T = .7/(1 - .6) = 1.75$$

$$E_T = \frac{(1 - .6^4)}{(1 - .6)} .7 + .6^4(.24) = 1.55 .$$

Thus, for this example the exchange ratio = 1.75
expected kills per engagement = .7
expected kills per ammunition load = 1.55

1 Blue Versus 3 Red

Assume that L = 50, and

$$P_{ij} = \begin{bmatrix} .0 & .2 & .15 & .05 \\ .05 & .25 & .2 & .1 \end{bmatrix}$$

$$N_{ij} = \begin{bmatrix} 0 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

then
$$E_K = (.2 + .25) * 1 + (.15 + .2) * 2 + (.05 + .1) * 3 = 1.6$$

$$P_S = .0 + .2 + .15 + .05 = .4$$

$$E_{n/S} = (0 * .0 + 3 * .2 + 4 * .15 + 5 * .05)/.4 = 3.625$$

$$N = \text{integer } (50/3.625) = \text{integer } (13.793) = 13$$

$$N_R = 50 - (13 * 3.625) = 2.875$$

$$E_{K/R} = 2.875 * (1.625/3.625) = 1.29$$

$$\lim_{N \to \infty} E_T = 1.6/(1 - .4) = 2.67$$

$$E_T = \frac{1 - .4^{13}}{(1 - .4)} 1.6 + .4^{13} (1.29) = 2.67$$

i.e.,

exchange rate = 2.67
expected kills per engagement = 1.6
expected kills per load = 2.67

IV. SUMMARY

An heuristic equation has been developed which allows the transformation of 1 X M tank engagement output data into a measure of performance which integrates the effects of lethality, vulnerability and ammunition depletion. The equation:

$$E_T = \frac{(1 - P_S^N)}{1 - P_S} E_K + P_S^N E_{K/R}$$

is an *approximation* of the expected number of red tanks killed per blue tank (given a finite ammunition load). An assumption, implicit to the equation, is that E_K remain constant as long as the residual ammunition load is greater than (or equal to) the average ammunition expended in a successful engagement. Although the assumption will tend to provide higher than the true answer at small values of N, the equation appears to be reasonable in terms of providing a good "feel" of the interplaying benefits and burdens of system performance.

It is a rather trival exercise to generalize the terms of the equation such that the approximation would be relevant to N X M engagements. However, the cumulation of assumptions* supporting such an exercise makes the author uncomfortable and by nature of the generalization, the resultant equation may attain more credence than it warrants. Thus, temporarily at least, the development terminates at this point.

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*For example, the current TANKWARS output does not differentiate between number of rounds fired by attrited blue tanks versus surviving blue tanks. Thus the average number of rounds expended [illegible]

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