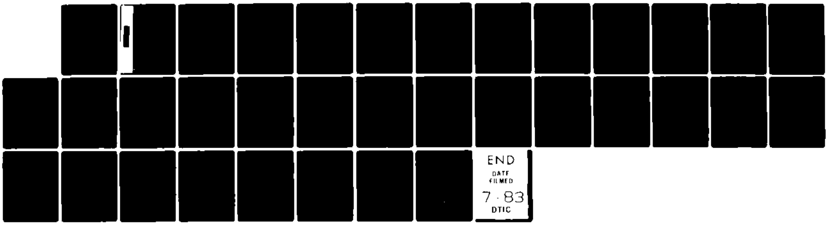
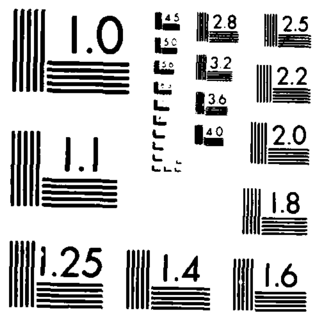
AD-A130 149 BEAM BREAKUP INSTABILITIES IN HIGH CURRENT ELECTRON BEAM RACETRACK INDUCT..(U) MISSION RESEARCH CORP ALBUQUERQUE NM B B GODFREY ET AL. 22 APR 83 AMRC-R-469 1/1

UNCLASSIFIED N00014-81-C-0647 F/G 20/7 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

ADA130149

AMRC-R-469
Copy 16

BEAM BREAKUP INSTABILITIES IN HIGH CURRENT ELECTRON BEAM RACETRACK INDUCTION ACCELERATORS

Brendan B. Godfrey
Thomas P. Hughes

Presented at: Particle Accelerator Conference
Santa Fe, New Mexico
21-23 March 1983

(Revised 22 April 1983)

Prepared for: Office of Naval Research
Physical Sciences Division
800 North Quincy Street
Arlington, Virginia 22207

DTIC
S JUL 6 1983
A

Under Contract: N00014-81-C-0647

Prepared by: MISSION RESEARCH CORPORATION
1720 Randolph Road, S.E.
Albuquerque, New Mexico 87106

DISTRIBUTION IN WHOLE OR IN PART IS PERMITTED FOR ANY PURPOSE OF THE UNITED STATES GOVERNMENT. APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

DTIC FILE COPY 83 07 6 017

SUMMARY

Beam breakup and negative mass instability growth rates for a 1 kA, 40 MeV electron beam racetrack induction accelerator are computed. The device is taken to have four acceleration gaps, each with 0.2 MeV applied voltage and 15 ohm transverse impedence; the guide field is 2 kg. We find that the total amplification of the beam breakup mode is limited to five e-foldings provided that the cavity mode quality factor Q is 6. Thus, the negative mass instability, which grows several times faster, is the dominant consideration. However, we also find that the energy range over which the negative mass instability occurs can be narrowed substantially by reducing the guide field strength after the beam has been accelerated to about 12 MeV. This approach, coupled with beam thermal effects, not considered here, probably is sufficient to limit negative mass growth to acceptable levels in the racetrack accelerator.

INTRODUCTION

High current racetrack induction accelerators and modified betatrons are a subject of increasing interest as sources of high power electron beams for free electron lasers, flash radiography, and other applications. The racetrack induction accelerator geometry is illustrated schematically in Figure 1. The beam is injected from a conventional pulsed diode beam generator into the drifttube, is progressively accelerated as it repetitively passes one or more induction modules, and then is extracted from the accelerator for its intended use. Extraction may even be unnecessary for microwave applications, because a slow-wave or rippled-magnetic-field cavity can be inserted in a straight section of the drifttube.[1]

Most beam stability studies for high current recirculating devices have dealt with the negative mass and resistive wall instabilities.[2-5] However, experience with linear induction accelerators suggests that beam

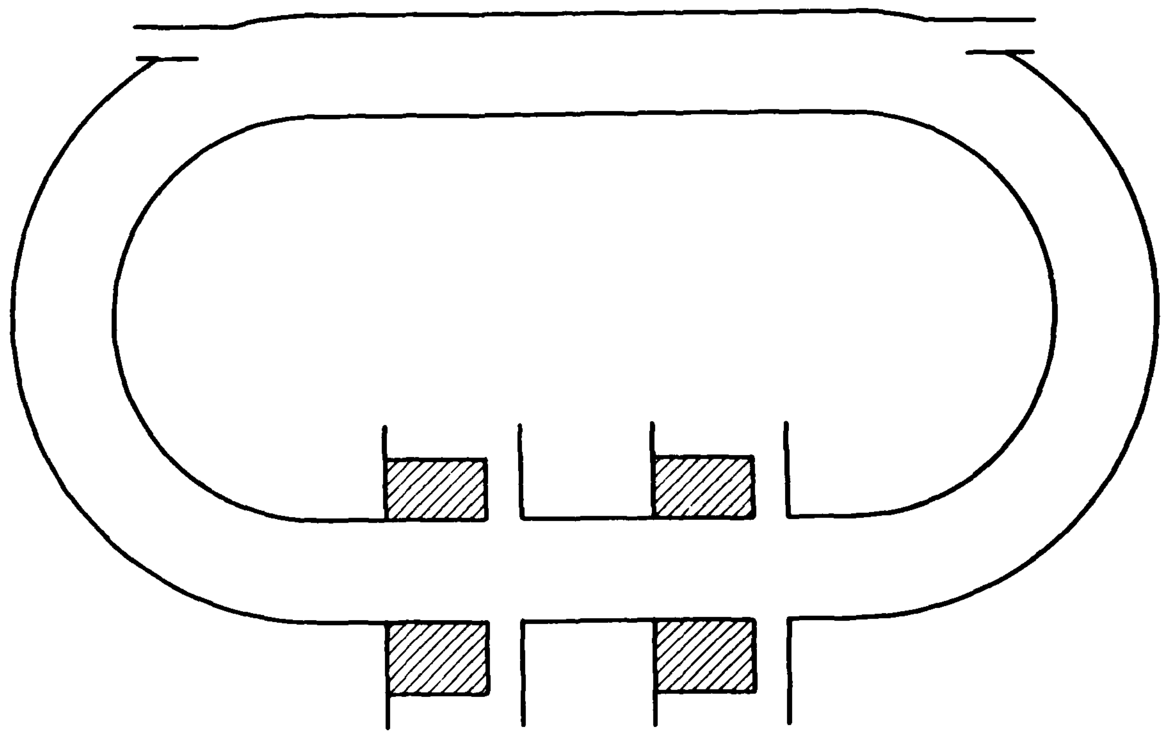


Figure 1. Simplified representation of cyclic induction accelerator with racetrack drift tube, acceleration gaps, and injection and extraction ports.

DTIC COPY INSPECTED 2

breakup due to interaction with the induction modules and other discontinuities in the drifttube may be significant.[6,7] The beam breakup instability arises from a resonant coupling between beam transverse oscillations and m=1 electromagnetic cavity modes localized to the acceleration gaps, resulting in large lateral displacements of the beam.[8,9] In this paper we present a linear dispersion relation describing both beam breakup and negative mass instabilities, including their possible interaction, and evaluate it for parameters of the proposed racetrack induction accelerator designed by the Naval Research Laboratory.[1]

The NRL device is based on the four module linear induction accelerator developed by the National Bureau of Standards.[10] It is expected to accelerate a 1 kA electron beam from 1 to 40 MeV in fifty cycles. The beam and drifttube radii are 1 and 7 cm, respectively. The principle m=1 resonance of the gaps has a frequency of 880 MHz, an impedence of 15 ohms, and a quality factor (Q) of 60. Experience with the ETA linear induction accelerator at Lawrence Livermore National Laboratory indicates that Q can be greatly reduced, however, and we shall take Q=6 in our numerical work.[11] The NRL design includes a 2 kg axial magnetic field to maintain the beam equilibrium and improve beam stability at low energies. Reducing or eliminating this guide field at higher energies is nonetheless an interesting possibility. The stability analyses below consider both options.

DISPERSION RELATION

For simplicity we represent the racetrack accelerator as a torus with a single gap. These two approximations are conservative in that omitting the straight sections of the racetrack and lumping the several gaps into one overestimate negative mass and beam breakup growth, respectively. The desired dispersion relation is

$$(\Omega^2-\omega_r^{\ 2} + F_r/\gamma L + \omega_\theta^{\ 2}\gamma^2\chi)\ (\Omega^2-\omega_z^{\ 2} + F_z/\gamma L) \tag{1}$$

$$- (B_\theta/\gamma)^2\ \Omega^2 = 0$$

with

$$\chi = \tilde{\nu}\ \frac{\omega}{v_\theta}\ (\omega v_\theta - \frac{\ell}{R})/[\Omega^2 + \tilde{\nu}\ (\omega^2 - \frac{\ell^2}{R^2})] \tag{2}$$

$$\tilde{\nu} = (1 + 2\ell n\ a/r_b)\ \nu/\gamma^3 \tag{3}$$

Here, $\Omega = \omega - \ell\omega_\theta$ is the Doppler-shifted wave frequency, ℓ is the toroidal mode number, and $\omega_\theta = v_\theta/R$ is the toroidal rotation frequency of the beam. (The poloidal rotation frequency is assumed negligible.) The radial and vertical betatron frequencies are

$$\omega_r^{\ 2} = (1 - n - n_s\ r_b^{\ 2}/a^2)\ \omega_\theta^{\ 2}$$

$$\omega_z^{\ 2} = (n - n_s\ r_b^{\ 2}/a^2)\ \omega_\theta^{\ 2}$$

$$n_s \equiv n_b/(2\omega_\theta^{\ 2}\gamma^3) \tag{4}$$

with n the betatron index, n_b the beam density, $\nu = n_b r_b^{\ 2}/2$ Budker's parameter, and γ the beam energy. The drifttube major radius is R, the drifttube minor radius is a, and the beam minor radius is r_b. $L=2\pi R$. The toroidal guide field strength is B_θ; the betatron field strength enters as $B_z = -\omega_\theta\gamma$.

In a high current betatron ω_r^2 and ω_z^2 can be of either sign. The beam is unstable, however, whenever

$$\omega_B^2 = \omega_z^2 \, \omega_r^2/(B_\theta/\gamma)^2 \qquad (5)$$

is negative. To avoid this situation, as well as for simplicity, we take n=1/2. The energy at which $\omega_B^2 = 0$ typically is labeled the transition energy,

$$\gamma_{tr} \approx (4 \, \nu \, R^2/a^2)^{1/3} \qquad (6)$$

The gap response function F is defined as[9]

$$\frac{F}{\gamma} = - \frac{Z_\perp}{Q} \; \frac{\omega_0^3}{\omega^2 + i\omega_0\omega/Q - \omega_0^2} \; \frac{\nu}{\gamma} \qquad (7)$$

where ω_0 is the resonant frequency, $Z_\perp/Q$ is the transverse impedence, and Q is the quality factor. Setting F=0 in (1) recovers the high current beam negative mass dispersion relation. The negative mass instability occurs for all ℓ over a broad range of energies when $\gamma > \gamma_{tr}$. For low ℓ only, one or two instability bands (often overlapping) also may exist when $\gamma < \gamma_{tr}$. Three of the six beam modes (m=0 spacecharge, m=1 spacecharge, and m=1 cyclotron; m is the poloidal mode number) have negative energy and so can couple unstably to the gap fields. Note that coupling in the m=0 spacecharge mode occurs only due to toroidal curvature. Choosing R=70 cm, we find maximum coupling at $\ell \approx 13$.

LARGE B ANALYSIS

For the parameters considered here and toroidal mode numbers in the vicinity of 13, the negative mass instability exists only beyond the transition energy $\gamma_{tr} \approx 2.9$. Just above the transition energy the instability is due solely to the interaction between the positive and negative energy m=1 spacecharge modes, while at still higher energies the m=0 spacecharge modes also are involved. This change is readily visible in the negative mass instability growth rate, the dashed curve in Figure 2. Which portion grows faster depends on circumstances.

Although the peak growth rate at lower energy is not readily determined analytically, the higher energy peak is easily shown to be

$$\Gamma \approx \frac{\sqrt{3}}{2} \left[2 \ell \omega_\theta \omega_B^2 \tilde{\nu} \gamma^2\right]^{1/3} \qquad (8)$$

Instability ceases for

$$\gamma > \left[6\sqrt{3} \ell R B_\theta \nu (1 + 2 \ln a/r_b)\right]^{1/2} \qquad (9)$$

here about 62.

In the absense of curvature, the beam breakup growth rate also is easily estimated. For Q not too large,[9]

$$\Gamma \approx \omega_0 Q \frac{\nu Z_\perp / Q}{B_\theta L} \qquad (10)$$

Both m=1 negative energy modes grow at this rate when their frequencies roughly match ω_0.

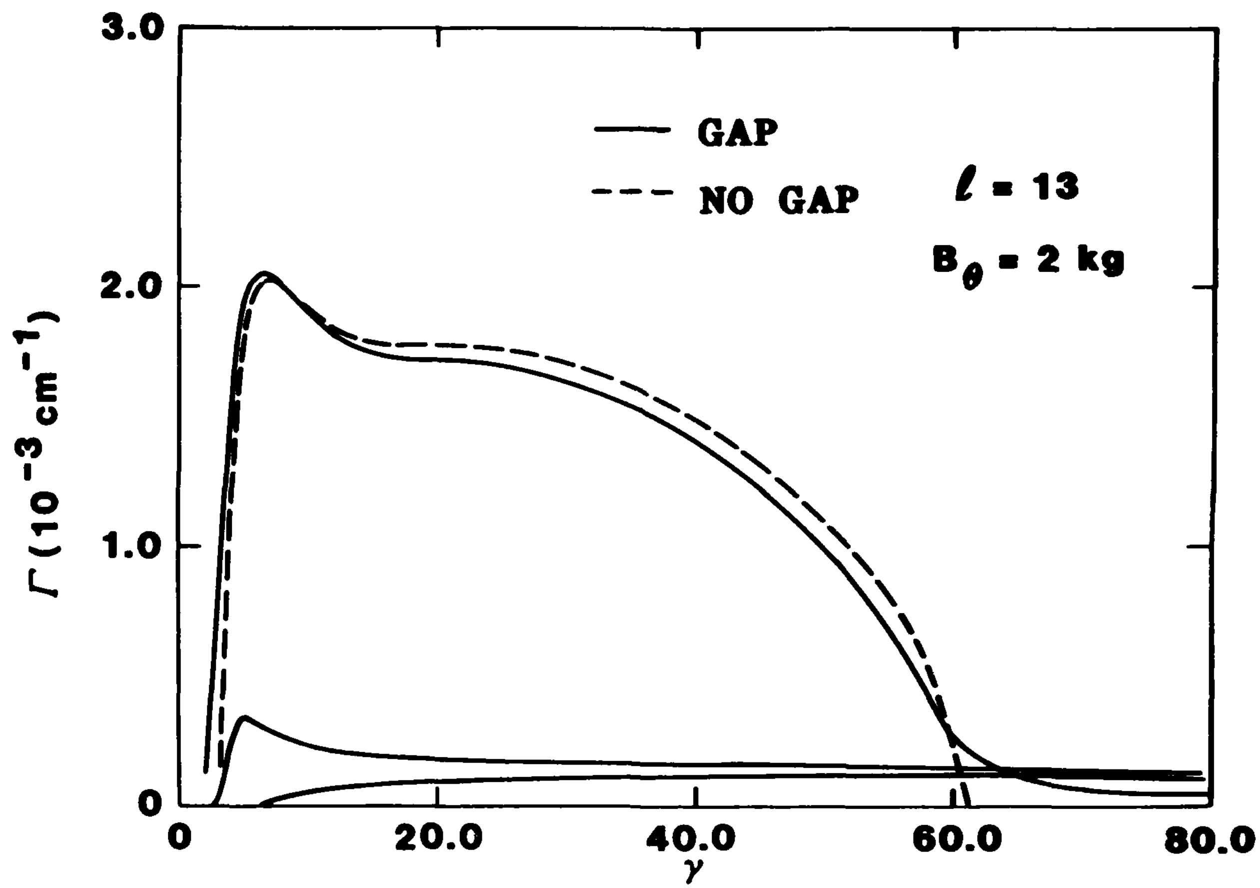


Figure 2. Combined negative mass and beam breakup instability growth rates (solid curves) for ℓ=13 and B_θ=2 kg. Growth of the negative mass instability alone (dashed curve) is included for comparison.

The solid curves in Figure 2 show growth rates of the negative mass and beam breakup instabilities combined. The negative mass results are seen to be only weakly affected by the gap resonance. The m=1 cyclotron and hybrid m=0/1 spacecharge modes have become unstable, however, with a growth rate agreeing with (10) to within a factor of 1.5. These findings are insensitive to small changes in the resonant frequency.

SMALL B ANALYSIS

Although modified betatron and racetrack induction accelerator studies usually assume a large toroidal guide field, large B_θ is in fact needed to provide a beam equilibrium only for γ small. Reducing or perhaps eliminating B_θ after the beam has been accelerated sufficiently has certain advantages for stability, as we see below.

For B_θ=0, the negative mass growth rate is approximately

$$\Gamma \approx \frac{\sqrt{3}}{2} [\ell \, \omega_\theta^3 \, \tilde{\nu} \, \gamma^2]^{1/3} \qquad (11)$$

This expression exceeds (8) whenever $B_\theta/\gamma > \omega_\theta$. However, the corresponding high energy cutoff,

$$\gamma > 3\sqrt{6} \, \ell\nu \, (1 + 2 \, \ell n \, a/r_b) \qquad (12)$$

here 27.5, typically is much lower than (9). See the dashed curve in Figure 3.

The beam breakup instability maximum growth rate is again readily estimated, this time giving

$$\Gamma \approx \omega_0 \, Q \, \frac{\nu \, Z_\perp/Q}{2\omega_z \, L \, \gamma} \qquad (13)$$

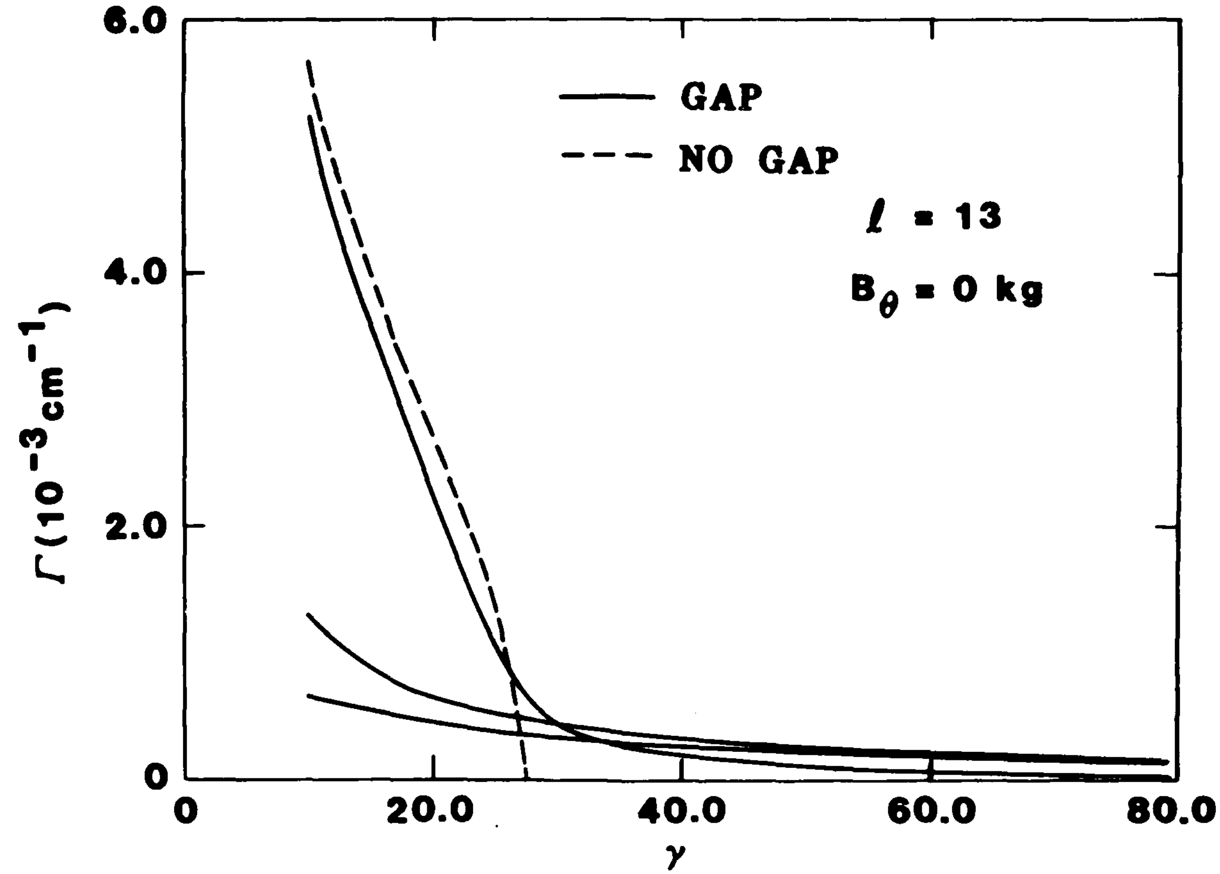


Figure 3. Combined negative mass and beam breakup instability growth rates (solid curves) for ℓ=13 and B_θ=0 kg. Growth of the negative mass instability alone (dashed curve) is included for comparison.

Equation (13) exceeds (10) for $B_\theta/\gamma > 2\omega_z$. The solid curves in Figure 3 show the effects of $F \neq 0$. As in Figure 2, the negative mass instability is only slightly modified by the gap; the beam breakup instability is described reasonably well by (13).

A comparison of the two figures suggests that some reduction in total instability growth during acceleration can be achieved by rapidly decreasing the guide field as the beam energy exceeds about 12 MeV.

RECOVERING "CONVENTIONAL" NEGATIVE MASS INSTABILITY BEHAVIOR FROM THE HIGH-CURRENT-BETATRON DISPERSION RELATION

The preceding small B_θ analysis predicts a high energy cutoff for the negative mass instability in a high current betatron, while standard derivations of negative mass growth, performed for a low current betatron, lead to no such cutoff.[12,13] The source of this apparent discrepancy, namely the failure of the usual approximation

$$1 - \omega^2 R^2/\ell^2 \approx \gamma^{-2} \qquad (14)$$

when $\tilde{\nu}\gamma^4$ becomes large, has been noted previously.[4] To emphasize this point, we here recover the accepted low current growth rate and show that it too exhibits a high energy cutoff, where (14) breaks down. Interestingly, a comparison of the new cutoff with (12), the high current limit, indicates that practical betatron parameters exist for which the negative mass instability does not occur at all at moderate toroidal mode numbers.

We begin by setting B_θ (and F) to zero in (1).

$$\Omega^2 - \omega_r^2 + \omega_o^2 \gamma^2 \chi = 0 \qquad (15)$$

Next, we drop Ω^2 as small compared to ω_r^2, valid for m=0 spacecharge waves in low current beams, and rearrange terms in the expression for χ.

$$1 = \left(\frac{\omega_\theta}{\omega_r}\right)^2 \gamma^2 \tilde{\nu} \; \frac{\ell}{R} \left(\Omega v_\theta - \frac{1}{\gamma^2} \frac{\ell}{R}\right) /$$

$$\left[\Omega^2 (1 + \tilde{\nu}) + 2\Omega \tilde{\nu} \frac{\ell}{R} v_\theta - \tilde{\nu} \frac{1}{\gamma^2} \frac{\ell^2}{R^2}\right] \qquad (16)$$

The conventional negative mass growth rate is obtained immediately by omitting terms linear in Ω from (16), an approximation equivalent to invoking (14).

Alternatively, (16) can be solved exactly.

$$2 (1 + \tilde{\nu}) \Omega = - \tilde{\nu} \frac{\ell}{R} v_\theta \left(2 - \frac{\omega_\theta^2}{\omega_r^2} \gamma^2\right) \pm$$

$$\left[\tilde{\nu} \frac{\ell}{R} v_\theta \left(2 - \frac{\omega_\theta^2}{\omega_r^2} \gamma^2\right)^2 + 4 (1 + \tilde{\nu}) \tilde{\nu} \frac{1}{\gamma^2} \frac{\ell^2}{R^2} \left(1 - \frac{\omega_\theta^2}{\omega_r^2} v^2\right)\right]^{1/2} \qquad (17)$$

Instability occurs whenever the argument of the square root is negative, approximately

$$\frac{\tilde{\nu} \gamma^4}{4} \left(\frac{\omega_\theta}{\omega_r}\right)^2 < 1 \qquad (18)$$

When (18) is well satisfied, the criterion for (14) to be valid, the desired growth rate is recovered.

$$\Gamma = \tilde{\nu}^{1/2} \frac{\ell}{R} \frac{\omega_\theta}{\omega_r} \qquad (19)$$

This derivation shows clearly the the negative mass instability at low currents is associated only with the m=0 spacecharge waves. At higher currents, for which Ω and ω_r become comparable, the m=1 spacecharge waves also are involved, and the complete quartic equation must be solved to give (11).

For the parameters of Table 1, the low current negative mass instability has a cutoff at γ = 7.0. Since the equilibrium fails below γ = 10.5, we see that the low current instability does not arise. Figure 4, a numerical solution of (1) for 350 A, illustrates growth in the low current limit. The analytical result from (19) at γ = 10.5 exceeds the numerical by about 30%.

Rewriting inequality (18) as

$$\gamma > \frac{1}{\nu} \frac{4(1 - n)}{1 + 2\ln a/r_b} \tag{20}$$

simplifies comparison with (12). Note that (20) predicts a narrow energy bandwidth at high current, while (12) gives the opposite. A window, therefore, exists at moderate currents,

$$\nu\,(1 + 2\ln a/r_b) \sim 0.5\text{-}1.0 \tag{21}$$

for which the energy range of the negative mass instability is minimal. For instance, a 750 A, 4.5 MeV electron beam injected into a betatron or racetrack accelerator with dimensions as in Table 1 exhibits no negative mass instability whatsoever for $\ell < 6$. Higher toroidal modes are likely to be stabilized by energy spread[2,4] or nonlinear effects.[14]

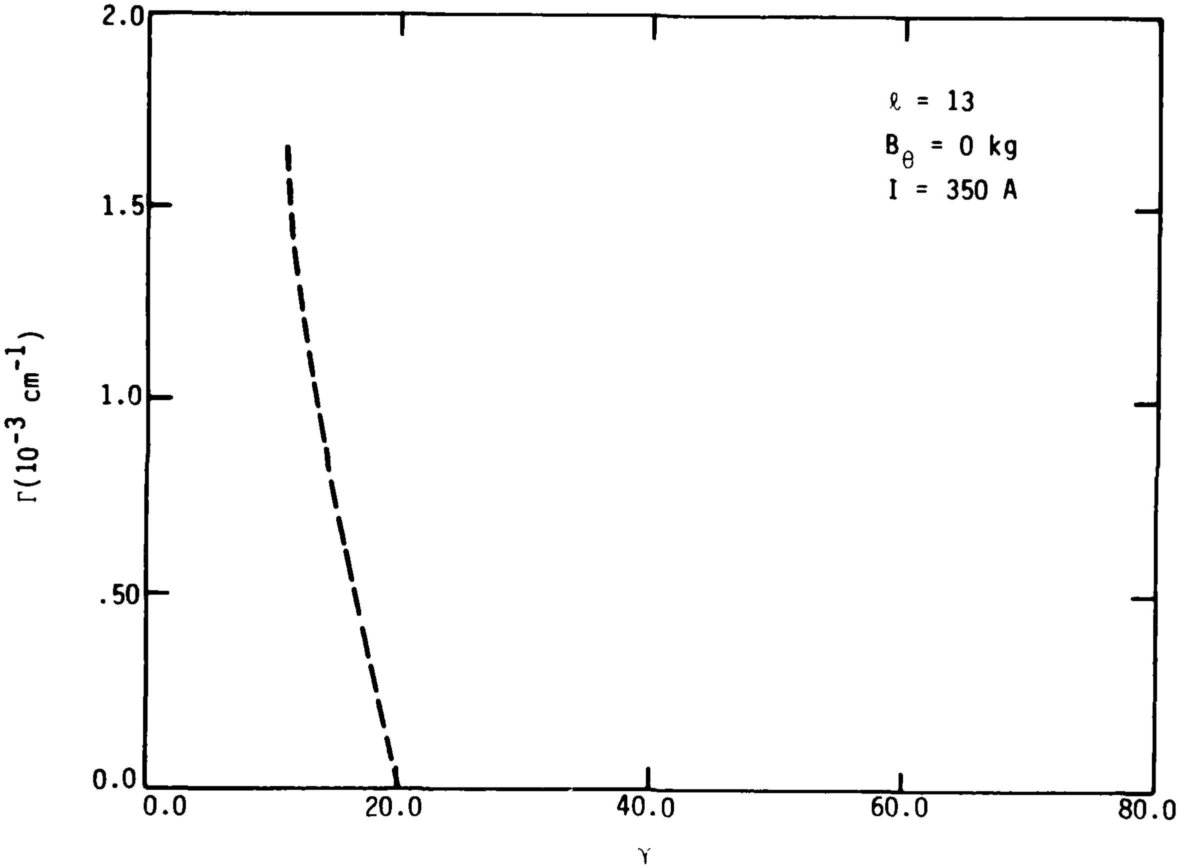


Figure 4. Growth rate of the low current negative mass instability for $\ell=13$ and $B_\theta=0$ kg at I=350 A.

TABLE 1. NOMINAL RACETRACK INDUCTION ACCELERATOR PARAMETERS

Path Lengths	$L = 460$ cm	
Drifttube Radius	$R = 7$ cm	
Beam Radius	$a = 1$ cm	
Guide Field	$B_\theta = 2$ kg	($\omega_c = 1.173\ cm^{-1}$)
Beam Current	$I = 1$ kA	($\nu = 0.0588$)
Beam Energy	$U = 0.4 - 40$ MeV	($\gamma = 1.5 - 80$)
Number of Revolutions	50	
Number of Gaps	$N = 4$	
Acceleration per Gap	$\Delta U = 0.2$ MeV	($\Delta\gamma = 0.4$)
Gap Resonant Frequency	880 MHz	
Mode Quality Factor	$Q = 60, 6$	
Gap Transverse Impedance	15 ohms	($Z_\perp/Q = 0.5$)
Gap Width	$\ell = 5$ cm	

REFERENCES

1. C. W. Roberson, IEEE Nuc. Sci. NS-28, 3433 (1981).

2. P. Sprangle and J. L. Vomvoridis, NRL-4688 (Naval Research Laboratory, Washington, 1981).

3. B. B. Godfrey and T. P. Hughes, AMRC-R-332 (Mission Research Corporation, Albuquerque, 1981).

4. T. P. Hughes and B. B. Godfrey, AMRC-R-354 (Mission Research Corporation, Albuquerque, 1982).

5. B. B. Godfrey and T. P. Hughes, AMRC-N-207 (Mission Research Corporation, Albuquerque, 1982).

6. T. J. Fessenden, W. A. Atchison, D. L. Birx, R. J. Briggs, J. C. Clark, R. E. Hester, V. K. Neil, A. C. Paul, D. Rogers, and K. W. Struve, Proc. 4th Conf. Electron and Ion Beam Research Technology (Palaiseu, 1981), p. 813.

7. R. B. Miller, J. W. Poukey, B. G. Epstein, S. L. Shope, T. C. Genoni, M. Franz, B. B. Godfrey, R. J. Adler, and A. Mondelli, IEEE Nuc. Sci. NS-28, 3343 (1981).

8. V. K. Neil, L. S. Hall, and R. K. Cooper, Part. Accel. 9, 213 (1979).

9. B. B. Godfrey and T. P. Hughes, AMRC-R-410 (Mission Research Corporation, Albuquerque, 1982).

10. J. E. Leiss, N. J. Norris, and M. A. Wilson, Part. Accel. 10, 223 (1980).

11. D. L. Brix, UCID-18630 (Lawrence Livermore National Laboratory, Livermore, 1980).

12. V. K. Neil and A. M. Sessler, Rev. Sci. Instr. 36, 429 (1965).

13. R. W. Landau and V. K. Neil, Phys. Fluids 9, 2412 (1966).

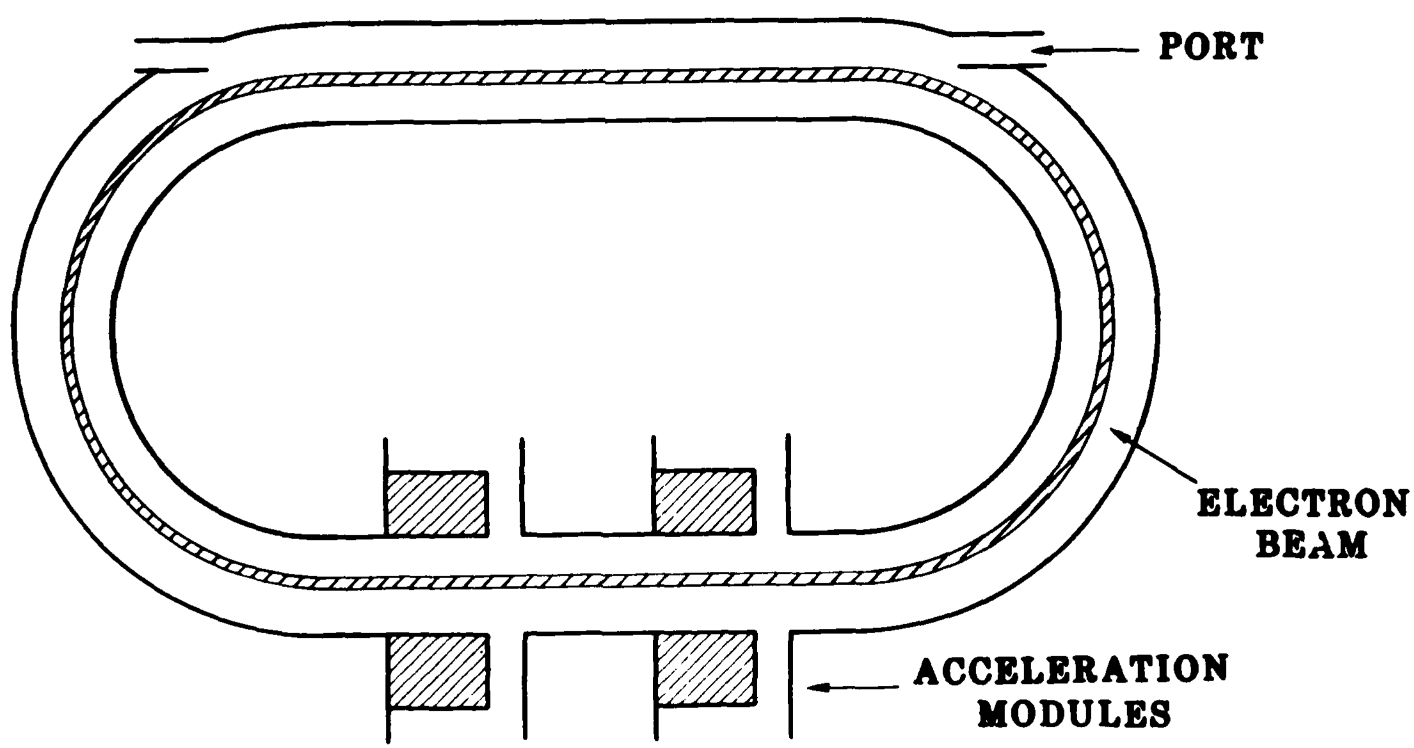
14. T. P. Hughes, M. M. Campbell, and B. B. Godfrey, IEEE Trans. Nuc. Sci., to be published.

PARTICLE ACCELERATOR CONFERENCE
SANTA FE, NEW MEXICO
21-23 MARCH 1983

BEAM BREAKUP INSTABILITIES IN HIGH CURRENT ELECTRON BEAM RACETRACK INDUCTION ACCELERATORS

BRENDAN B. GODFREY AND THOMAS P. HUGHES
MISSION RESEARCH CORPORATION
ALBUQUERQUE, NEW MEXICO

RACETRACK INDUCTION ACCELERATOR COMBINES HIGH FIELD GRADIENTS, HIGH ELECTRON CURRENTS OF LINEAR INDUCTION ACCELERATOR WITH COMPACTNESS OF RECIRCULATING DEVICE.



BEAM BREAKUP INSTABILITY A CONCERN IN RACETRACK ACCELERATIOR.

- ACCELERATION GAPS, OTHER DRIFTTUBE DISCONTINUITIES SUPPORT LOCALIZED ELECTROMAGNETIC RESONANCES
- INTERACTION OF NEGATIVE ENERGY BEAM WAVES, $M = 1$ RESONANCES DRIVE LARGE LATERAL BEAM OSCILLATIONS
- SERIOUS BEAM BREAKUP INSTABILITIES OBSERVED IN LINEAR INDUCTION, OTHER LINEAR ACCELERATORS
- CURVATURE EFFECTS ON BEAM BREAKUP, INCLUDING COUPLING TO NEGATIVE MASS INSTABILITY, UNKNOWN

GENERAL DISPERSION RELATION FOR COUPLED BEAM BREAKUP, NEGATIVE MASS INSTABILITIES DERIVED BY MRC, SELECTED RESULTS PRESENTED HERE.

CALCULATIONS BASED ON NAVAL RESEARCH LABORATORY RACETRACK ACCELERATOR DESIGN BY C.W. ROBERSON.

BEAM CURRENT (ν)	1 kA
BEAM ENERGY (γ)	1-40 MeV
BEAM RADIUS (r_b)	1 cm
DRIFTTUBE RADIUS (a)	7 cm
NUMBER OF CYCLES	50
NUMBER OF GAPS	4
GAP VOLTAGE	200 KeV
GAP RESONANCE (ω_0)	880 MHz
TRANSVERSE IMPEDANCE ($Z_\perp/Q$)	15 OHMS
QUALITY FACTOR (Q)	60, 6

PRESENT RESONANCE Q IS 60, BUT CAN BE REDUCED TO 6. LATTER VALUE USED IN COMPUTATIONS.

EXPLORATORY STUDY PERFORMED OF INSTABILITY GROWTH.

- ANALYTICALLY ESTIMATE NEGATIVE MASS GROWTH WITH NO ACCELERATION GAPS
- ANALYTICALLY ESTIMATE BEAM BREAKUP GROWTH WITH NO TORIDAL CURVATURE
- NUMERICALLY DETERMINE GROWTH RATES OF COUPLED INSTABILITIES FROM LINEAR DISPERSION RELATION
- VARY AXIAL MAGNETIC FIELD TO MINIMIZE GROWTH

IMPORTANT EFFECT OF BEAM ENERGY SPREAD NOT INCLUDED, IS KNOWN TO AMELIORATE NEGATIVE MASS INSTABILITY.

MULTIPLE GAP RACETRACK GEOMETRY SIMPLIFIED TO SINGLE GAP TOROIDAL GEOMETRY TO OBTAIN DISPERSION RELATION:

$$\left(\Omega^2 - \omega_r^2 + F_r/\gamma L + \omega_\theta^2 \gamma^2 \chi\right)\left(\Omega^2 - \omega_z^2 + F_z/\gamma L\right)$$

$$- \left(B_\theta/\gamma\right)^2 \Omega^2 = 0$$

$$\chi = \tilde{\nu} \frac{\omega}{v_\theta}\left(\omega v_\theta - \frac{\ell}{R}\right) / \left[\Omega^2 + \tilde{\nu}\left(\omega^2 - \frac{\ell^2}{R^2}\right)\right]$$

$$\tilde{\nu} = \left(1 + 2\ \mathrm{Ln}\left(a/r_b\right)\right) \cdot \nu/\gamma^3 \qquad \Omega = \omega - \ell\omega_\theta$$

$$\frac{F}{\gamma} = -\frac{Z_\perp}{Q} \cdot \frac{\omega_0^3}{\omega^2 + i\,\omega\,\omega_0/Q - \omega_0^2} \cdot \frac{\nu}{\gamma}$$

OMITTING STRAIGHT DRIFTTUBE SECTIONS, LUMPING SEVERAL GAPS INTO ONE LEAD TO SMALL OVERESTIMATES OF GROWTH.

DISPERSION RELATION DESCRIBES THREE PAIRS OF BEAM NORMAL MODES.

- M = 1 CYCLOTRON WAVES

$$\Omega = \pm B_\theta / \gamma$$

- M = 1 SPACECHARGE WAVES

$$\Omega = \pm \omega_B$$

- M = 0 SPACECHARGE WAVES

$$\Omega = \pm \tilde{\nu}^{1/2} \ell / R\gamma$$

BEAM BREAKUP INSTABILITY OCCURS AT RESONANCE BETWEEN M = 1 BEAM, GAP MODES. RESONANCE AT $\ell \approx 13$ FOR R = 70 cm. (ℓ, M — TOROIDAL, POLOIDAL MODE NUMBERS)

TRANSVERSE OSCILLATION FREQUENCIES STRONGLY AFFECTED BY HIGH BEAM CURRENTS.

- RADIAL FREQUENCY $\omega_r^2 = (1 - n - n_s r_b^2/a^2) \ \omega_\theta^2$
- VERTICAL FREQUENCY $\omega_2^2 = (n - n_s r_b^2/a^2) \ \omega_\theta^2$
- BOUNCE FREQUENCY $\omega_B^2 = \omega_r^2 \omega_2^2 / (B_\theta/\gamma)^2$
- SELF-FIELD INDEX $n_s = n_b/(2\omega_\theta^2 \gamma^3)$

CHOOSE EXTERNAL FIELD INDEX $n = 1/2$ TO AVOID STRONG INSTABILITY FOR $\omega_B^2 < 0$.

BEHAVIOR OF NEGATIVE MASS INSTABILITY DEPENDS ON SIGN OF ω_r^2, ω_Z^2.

- DEFINE TRANSITION ENERGY $(\omega_r^2, \omega_Z^2 = 0)$

$$\gamma_{tr} \approx \left(4 \nu R^2/a^2\right)^{1/3}$$

- FOR ENERGIES $\gamma < \gamma_{tr}$ $(\omega_r^2, \omega_z^2 < 0)$

  ONE OR TWO, POSSIBLY OVERLAPPING, NARROW INSTABILITY BANDS AT LOW ℓ ONLY

- FOR ENERGIES $\gamma > \gamma_{tr}$ $(\omega_r^2, \omega_z^2 > 0)$

  SINGLE, BROAD INSTABILITY BAND AT ALL ℓ

HIGH ENERGY ($\gamma > \gamma_{tr}$) NEGATIVE MASS GROWTH RATE DESCRIBABLE ANALYTICALLY.

- **BROAD GROWTH PEAK**

$$\Gamma = \frac{\sqrt{3}}{2}\left[2\,\ell\,\omega_\theta\,\omega_B^2\,\tilde{\nu}\,\gamma^2\right]^{1/3}$$

- **INSTABILITY CUTS OFF AT**

$$\gamma > \left[6\sqrt{3}\,\ell R\;B_\theta\;\nu\left(1+2\;\mathrm{Ln}\left(a/r_b\right)\right)\right]^{1/2}$$

SECOND GROWTH PEAK SOMETIMES OBSERVED JUST ABOVE γ_{tr}.

BEAM BREAKUP GROWTH RATE DESCRIBABLE ANALYTICALLY IF CURVATURE IGNORED.

- GENERAL EXPRESSION ($\Delta\omega$ – FREQUENCY MISMATCH)

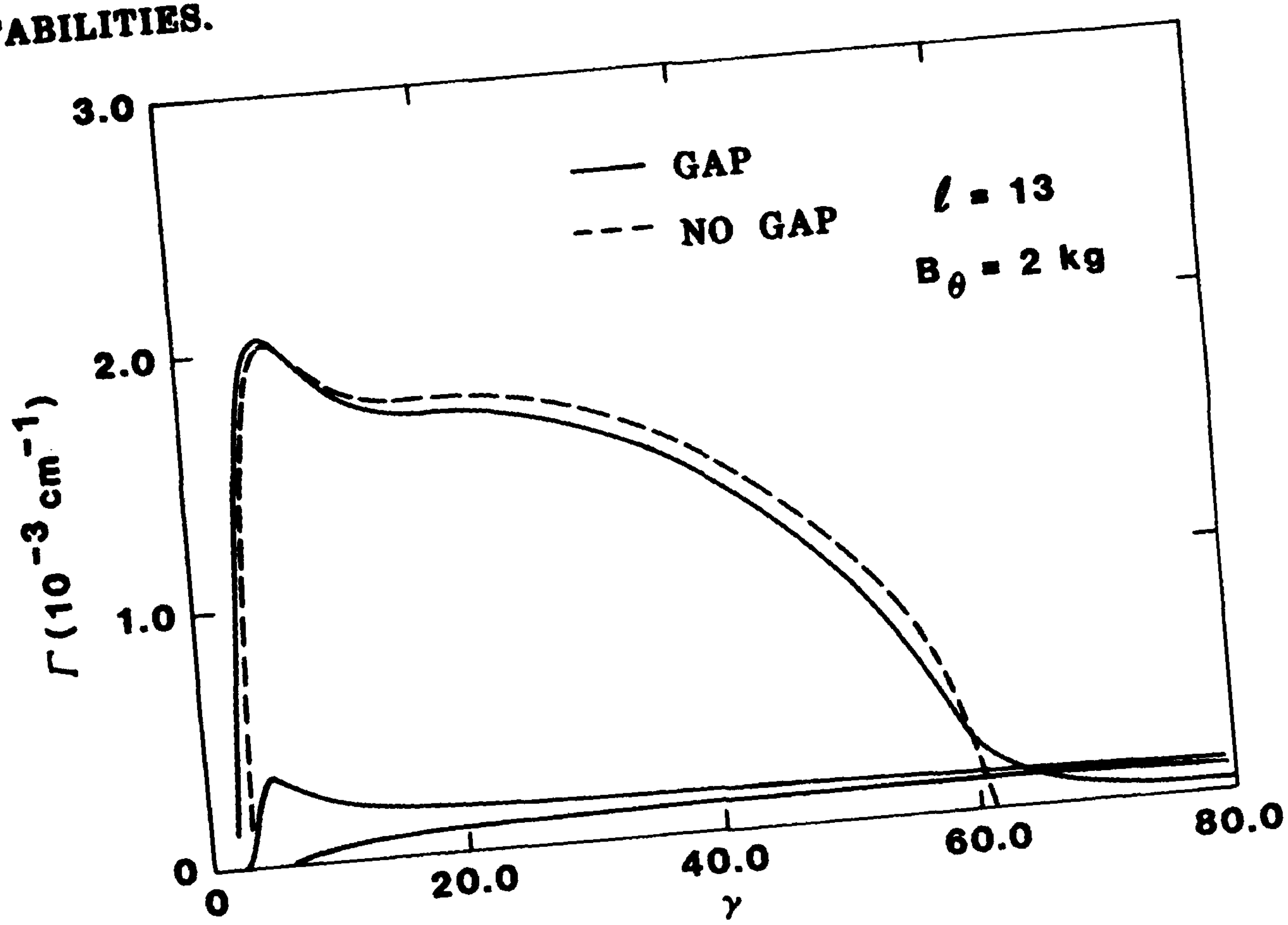
$$\omega - \omega_o \approx i\,\frac{\omega_o}{2}\left\{\left[\frac{2\,\nu\,Z_\perp/Q}{B_\theta L} + \left(\frac{1}{2Q} - i\,\frac{\Delta\omega}{\omega_o}\right)^2\right]^{1/2} - \left(\frac{1}{2Q} - i\,\frac{\Delta\omega}{\omega_o}\right)\right\}$$

- LOW Q LIMIT APPROPRIATE HERE

$$\Gamma \approx \omega_o\,Q\,\frac{\nu\,Z_\perp/Q}{B_\theta L}$$

($L = 2\pi R$ – RACETRACK CIRCUMFERENCE)

NUMERICAL SOLUTION OF COMPLETE DISPERSION RELATION SHOWS MINIMAL COUPLING BETWEEN NEGATIVE MASS, BEAM BREAKUP INSTABILITIES.



BEAM NORMAL MODES CHANGE SIGNIFICANTLY FOR SMALL GUIDE FIELD ($B_\theta < B_z$).

- **M = 1 SPACECHARGE (RADIAL)**

  $\Omega \approx \pm\, \omega_r$

- **M = 1 SPACECHARGE (VERTICAL)**

  $\Omega \approx \pm\, \omega_z$

- **M = 0 SPACECHARGE**

  $\Omega \approx \pm\, \tilde{\nu}\; 1/2\, \ell / R \gamma$

EQUILIBRIUM FAILS FOR $n_s > n$ WHEN $B_\theta = 0$.

NEGATIVE MASS GROWTH FASTER BUT OVER NARROWER ENERGY RANGE FOR B_θ SMALL.

APPROXIMATE GROWTH RATE

$$\Gamma \approx \frac{\sqrt{3}}{2}\left[\ell\, \omega_\theta{}^3\, \tilde{\nu}\, \gamma^2\right]^{1/3}$$

INSTABILITY CUTS OFF AT

$$\gamma > 3\sqrt{6}\, \ell\, \nu\, (1+2\, \mathrm{Ln}\,(a/r_b))$$

GROWTH RATES WITH, WITHOUT B_θ COMPARABLE FOR $B_\theta/\gamma \sim \omega_\theta$.

BEAM BREAKUP GROWTH ESTIMATES (CURVATURE OMITTED)

ALSO STRONGLY MODIFIED FOR SMALL B_θ.

- GENERAL EXPRESSION ($\Delta\omega$ – FREQUENCY MISMATCH)

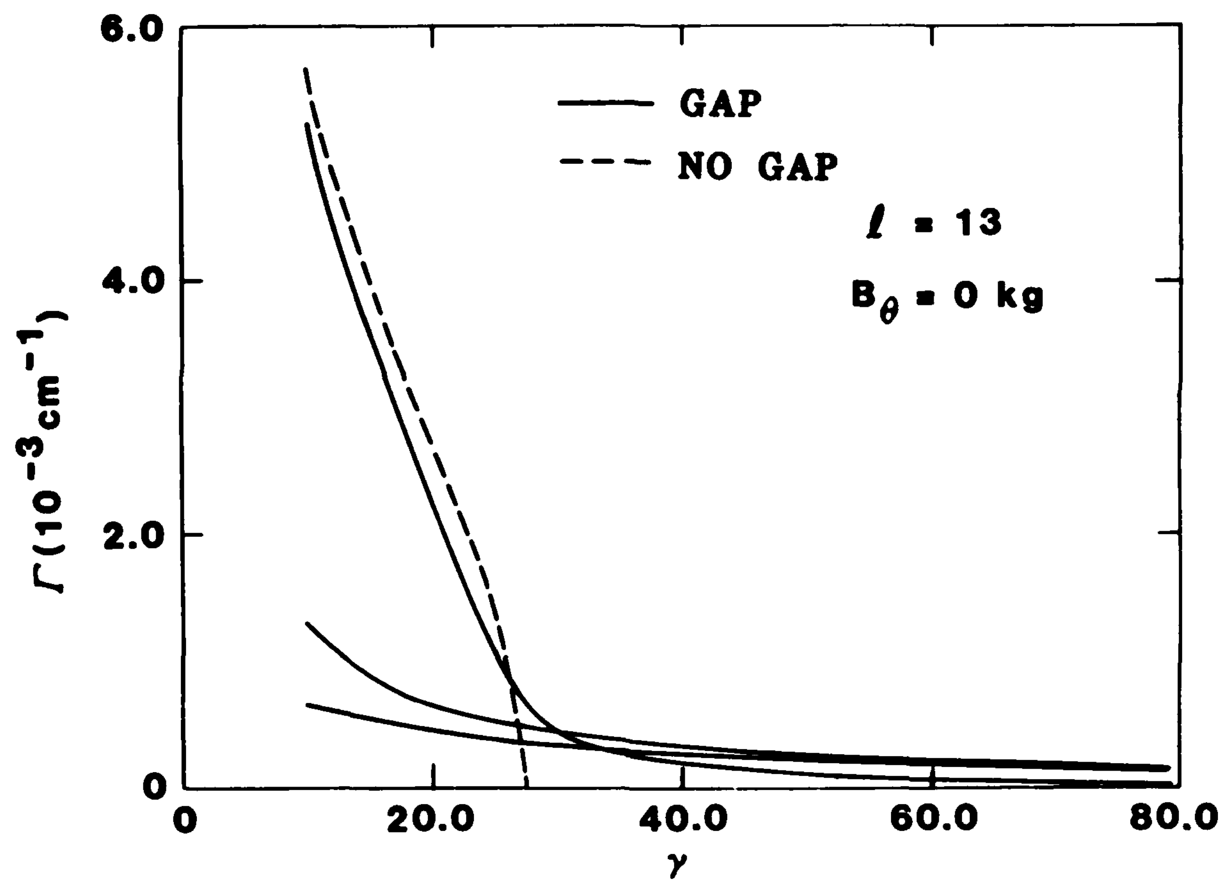
$$\omega - \omega_o \approx i \frac{\omega_o}{2} \left\{ \left[\frac{\nu Z_\perp / Q}{\omega_z L \gamma} + \left(\frac{1}{2Q} - i \frac{\Delta\omega}{\omega_o} \right)^2 \right]^{1/2} - \left(\frac{1}{2Q} - i \frac{\Delta\omega}{\omega_o} \right) \right\}$$

- LOW Q LIMIT APPROPRIATE HERE

$$\Gamma \approx \omega_o Q \frac{\nu Z_\perp / Q}{2 \omega_z L \gamma}$$

NOTE γ^{-1} DEPENDENCE.

NUMERICAL SOLUTION OF COMPLETE DISPERSION RELATION SHOWS MINIMAL COUPLING BETWEEN NEGATIVE MASS, BEAM BREAKUP INSTABILITIES.



ELECTRON BEAM STABILITY IN HIGH CURRENT RACETRACK INDUCTION ACCELERATOR SEEMS PROMISING.

- NO STRONG INTERACTION AMONG INSTABILITIES
- BEAM BREAKUP GROWTH LIMITED TO FIVE E-FOLDINGS OR LESS
- NEGATIVE MASS GROWTH, 20 E-FOLDINGS IN COLD BEAM MODEL, CUT SUBSTANTIALLY BY ELECTRON ENERGY SPREAD
- RESISTIVE WALL, IMAGE DISPLACMENT INSTABILITIES (NOT CONSIDERED HERE) ARE FAR SLOWER

POSTER R15 ADDRESSES NONLINEAR EVOLUTION OF NEGATIVE MASS INSTABILITY.

ACKNOWLEDGMENTS

THIS RESEARCH SUPPORTED BY OFFICE OF NAVAL RESEARCH UNDER CONTRACT N00014-81-C-0647.

AUTHORS INDEBTED TO CHARLES ROBERSON FOR SUGGESTING THIS PROBLEM AND PROVIDING NRL RACETRACK INDUCTION ACCELERATOR DESIGN PARAMETERS.

FRUITFUL DISCUSSIONS WITH NRL HIGH POWER ACCELERATOR TEAM ALSO APPRECIATED.

ED

8