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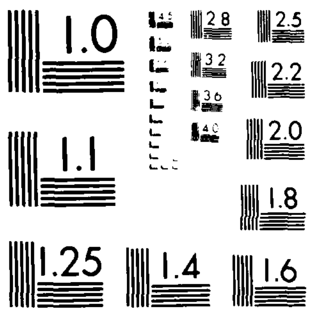

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FINAL REPORT - 1983

SOURCES OF SURFACE MAGNETIC FIELD VARIABILITY

APRIL 1983

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FINAL REPORT - 1983

SOURCES OF SURFACE MAGNETIC FIELD VARIABILITY

APRIL 1983

F49620-81-C-0001

Prepared by

W. P. Olson

Prepared for
Air Force Office of Scientific Research
Bolling Air Force Base
Washington, D.C.

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The contribution of non-ionospheric currents to the quiet daily variation in the earth's surface magnetic field (the S_q variation) is being reexamined. It has been found that the direct use of magnetospheric magnetic field models to represent the contribution of these currents to the surface field produces large errors. Thus direct integration over these currents was used. The induction problem was addressed and the contribution to S_q from currents induced in the earth's crust was determined. The total day to night

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) contribution to S_q was found to be a minimum of 12 nanotesla. This contrasts with a measured variation of from 20 to 45 nanotesla. Thus the magnetospheric currents produce from about 1/4 to over 1/2 of the observed pattern. The Birkeland currents also contribute to S_q at sub-auroral latitudes. A study was initiated to examine the day to day variability in S_q using ground based magnetometer data and direct (satellite observations) of the solar wind. A means for experimentally determining the baseline for S_q (and the main field) was developed.

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INTRODUCTION

It has been known for several hundred years that the earth possesses a planetary magnetic field. It has also been known for over 200 years that there is a daily variation in the surface strength of that field. The earth's surface field also exhibits changes associated with solar flares. Even when the sun is "quiet", however, there is a variation in the earth's surface magnetic field. This daily variation at mid latitude is quite regular and is typically represented by "S_q", which stands for solar quiet daily variation. Typically at mid latitude this daily variation ranges from 20 to 45 nanoteslas (nT). This is compared with a mean surface magnetic field of about 50,000 nT. Near the turn of the century, the existence of the ionosphere was inferred from these magnetic variations. It was suggested that the S_q pattern was caused by ionization in the upper regions of the atmosphere being moved by neutral particles through the earth's magnetic field. This eventually became known as the ionospheric dynamo theory.

When the existence of the ionosphere was proven by reflecting radio waves off of it, the ionospheric dynamo theory became firmly entrenched. With the discovery of the magnetosphere, however, it was realized that other currents flowing above the earth should also contribute to the surface magnetic field variations. In early work on these magnetospheric currents, it was suggested that indeed they do contribute to the earth's surface magnetic field, but produce an inconsequential portion of the observed S_q pattern. Since those reports, much has been learned concerning the nature of the magnetospheric current systems. The purpose of this work is to quantitatively reexamine the question of non-ionospheric currents and their contribution to S_q. (It was expected that somewhat different results might be obtained because of our increased understanding of the magnetospheric currents.) It is important to understand what this contribution is since it will put an upper bound on the neutral winds in the upper atmosphere required for the dynamo theory. It is also important to quantitatively determine the magnitude of the magnetospheric currents at the earth's surface in order to accurately determine the earth's main magnetic field. The accurate determination of the main field (to within several nanotesla's) is needed for the study of the secular variation in the main field, and for the accurate description of magnetic anomalies in the earth's crust. In addition, the development of the capability for representing the contribution of magnetospheric currents in the earth's surface magnetic field should also be helpful in the study of magnetospheric dynamics, since ground based magnetograms are routinely used to qualitatively describe magnetospheric events.

MAGNETOSPHERIC CONTRIBUTION TO SURFACE FIELD - DIRECT INTEGRATION

One of the problems with early work on the magnetospheric magnetic field was that models representing it were meant to be global - (i.e., useful over the entire magnetosphere) and little care was given to the accuracy of the field from the particular magnetospheric current at the earth's surface. Therefore it is expected that significant errors persist in the surface magnetic field predicted by these early models. Therefore the Biot-Savart law was used to integrate over the magnetospheric current systems to directly determine the magnetic field at the earth's surface. This was done individually for magnetopause, ring and tail current systems. The results are shown in Figure 1. It is seen when the magnetospheric magnetic field models are used, that the total contribution from the magnetospheric currents at the earth's surface produced a day to night variation of about 4 nT. However, when a direct integration is performed over the current systems, this variation is actually about 7 nT. Note that these numbers refer to the magnetic field produced in the region of the earth but assume that a zero conductivity earth is present, i.e., the field is given in free space. The S_q pattern for the ring, magnetopause, and tail currents, and their sum is shown in Figures 2, 3 and 4 for the north, east and vertical components of the total magnetic variation field employing direct integration over the currents. In each figure, the daily variation is shown at 0 $\pm$ 30^o, and $\pm$ 60^o magnetic latitude and the vertical tick marks are in 5 nT intervals. Note that in these figures a free space (zero conductivity earth) was assumed.

However, several problems remain. First, although there is some uncertainty in the representation of all three major magnetospheric current systems (magnetopause, ring, and tail), it is the ring current that causes the largest qualitative uncertainties. As shown in Figure 5, it is seen that different portions of the inner part of the ring current can be set to zero and have little effect on the total ring current contribution in those regions of the magnetosphere where it is most routinely observed. However, these changes in

MAGNETIC FIELD

SUN-EARTH LINE

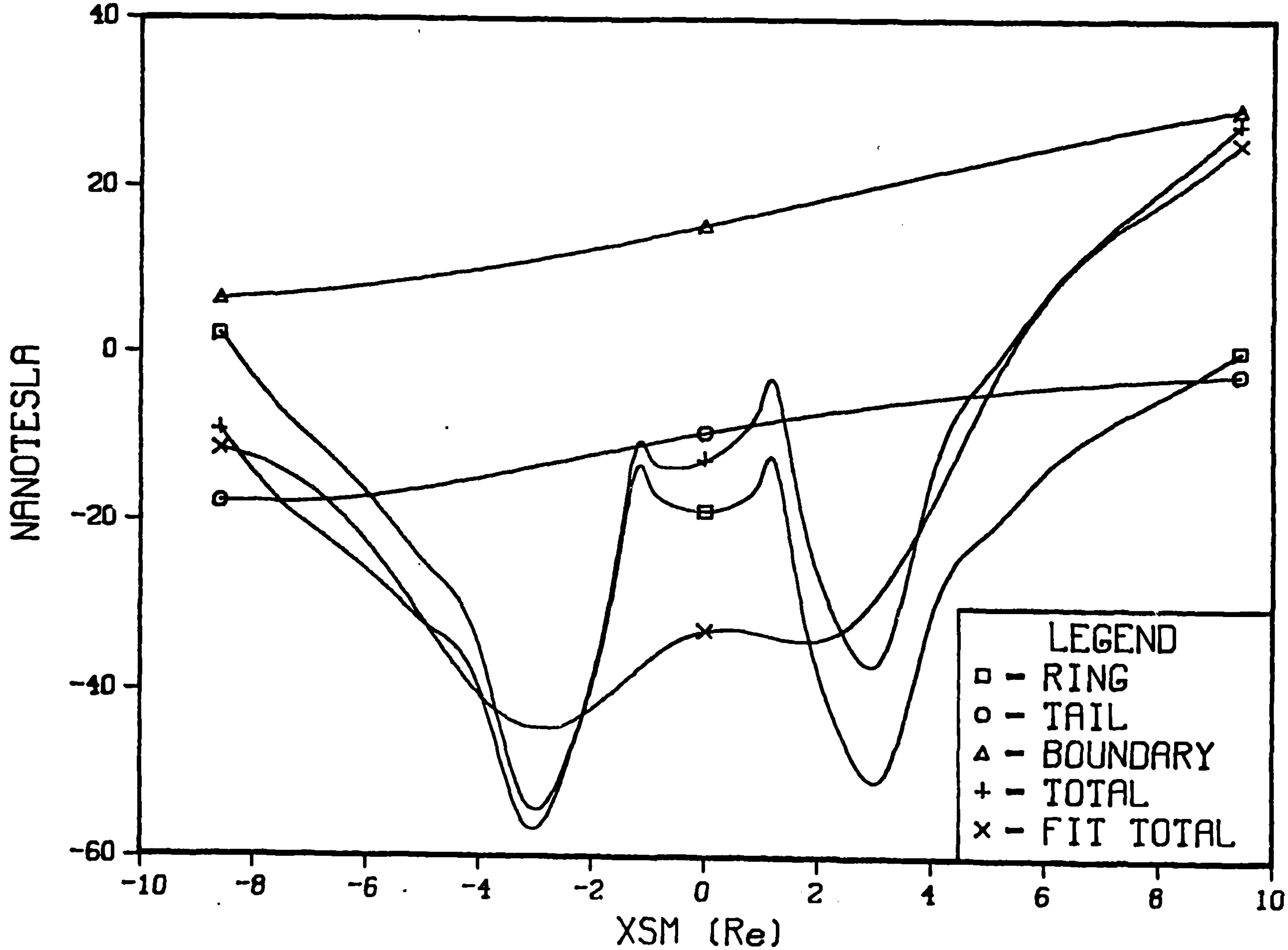


FIGURE 1

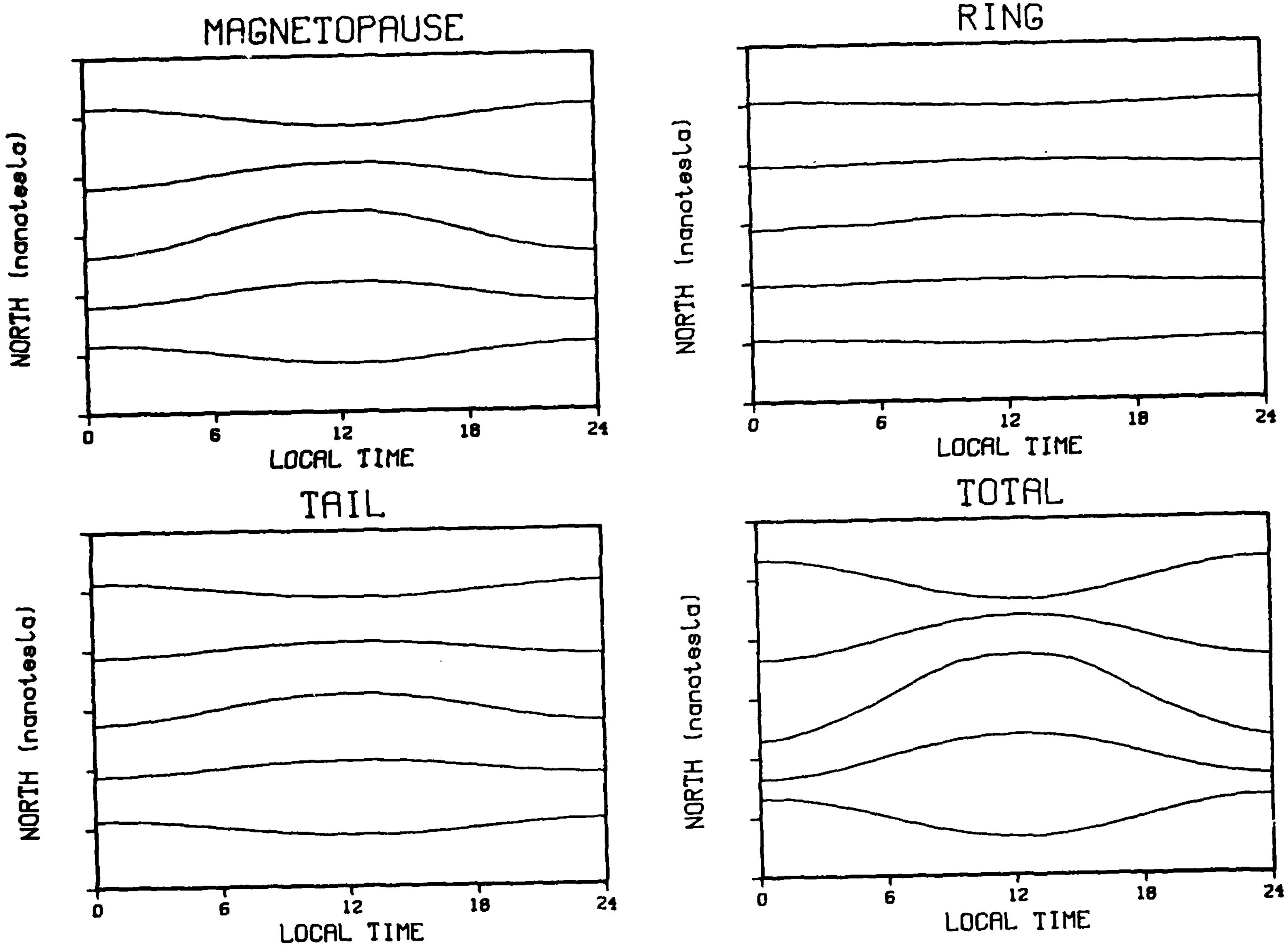

MAGNETOPAUSE
RING
TAIL
TOTAL
NORTH (nanotesla)
LOCAL TIME
0
6
12
18
24

FIGURE 2

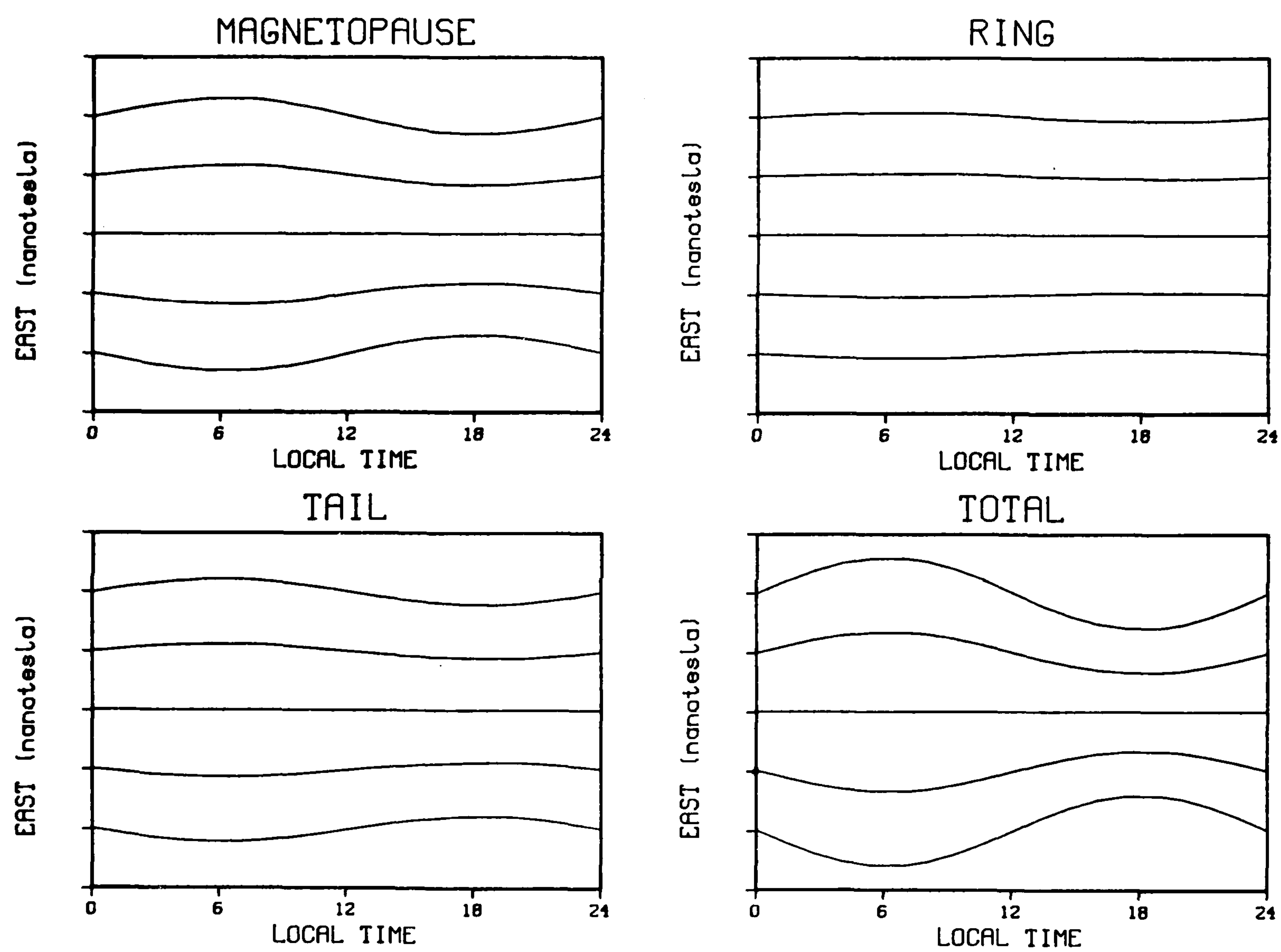

MAGNETOPAUSE
EAST (nanotesla)
0
6
12
18
24
LOCAL TIME
RING
EAST (nanotesla)
0
6
12
18
24
LOCAL TIME
TAIL
EAST (nanotesla)
0
6
12
18
24
LOCAL TIME
TOTAL
EAST (nanotesla)
0
6
12
18
24
LOCAL TIME

FIGURE 3

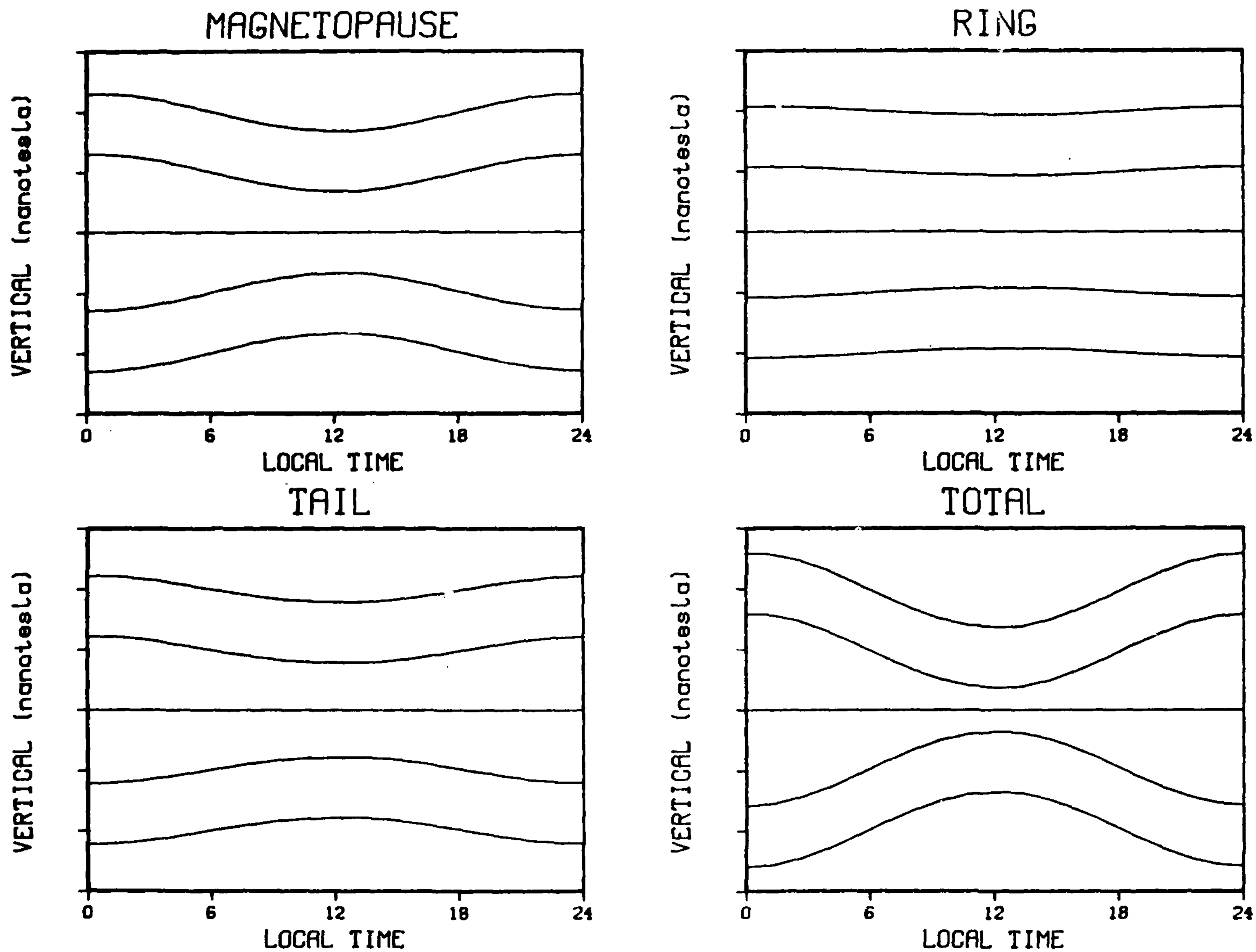
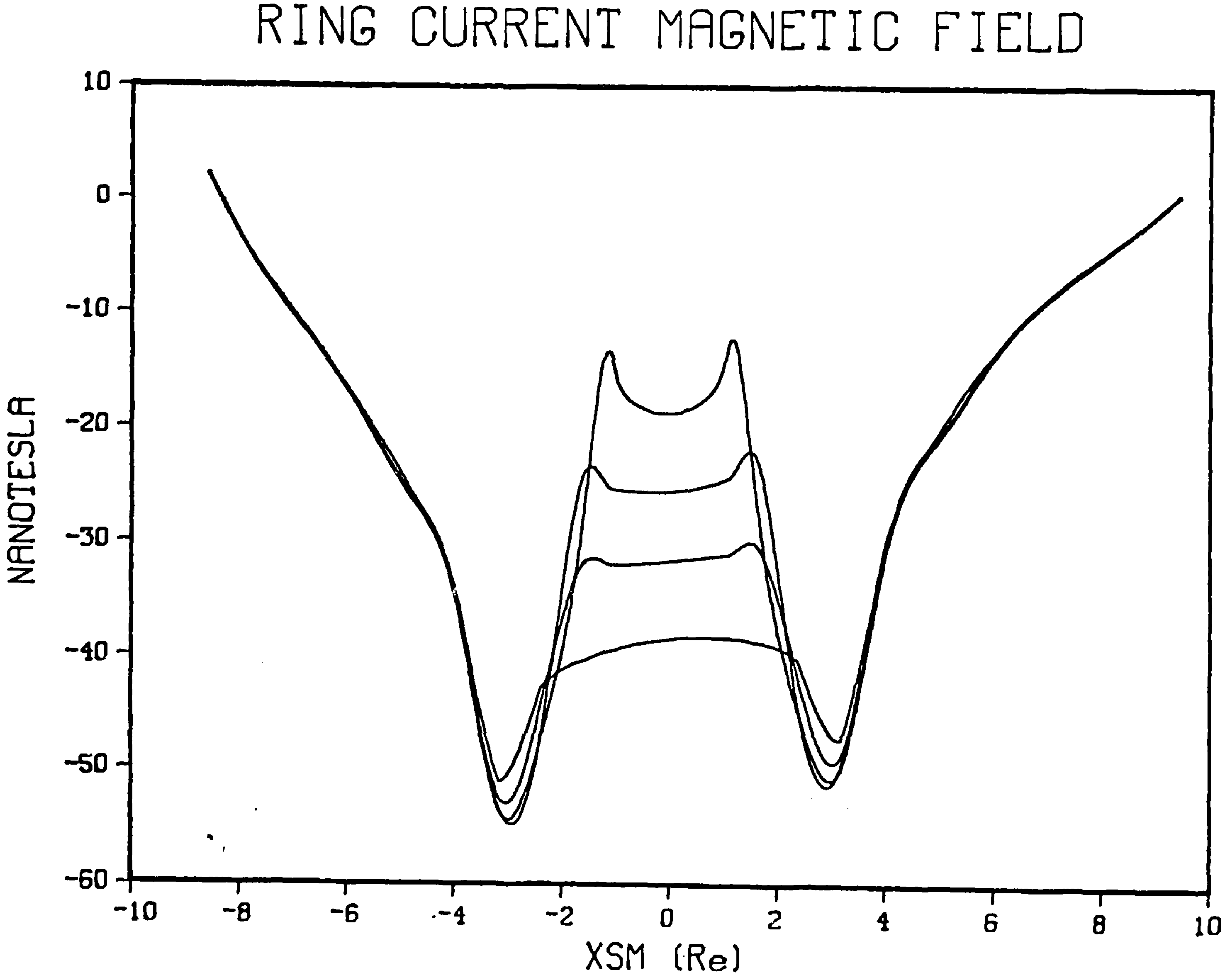

MAGNETOPAUSE
RING
TAIL
TOTAL
VERTICAL (nanotesla)
LOCAL TIME
0
6
12
18
24

FIGURE 4


RING CURRENT MAGNETIC FIELD
NANOTESLA
10
0
-10
-20
-30
-40
-50
-60
-10
-8
-6
-4
-2
0
2
4
6
8
10
XSM (Re)
FIGURE 5

the total ring current strength dramatically influence the value of the ring current in the vicinity of the earth. Another shortcoming of work to date is that the Birkeland, or field aligned currents, have not been considered. It is clear that they make an important contribution to the surface magnetic field near auroral latitudes.Finally, the results in Figures 2 through 4 assume a zero conductivity and zero permeability earth.

THE QUANTITATIVE DETERMINATION OF EARTH INDUCTION CURRENTS AND THEIR CONTRIBUTION TO THE SURFACE MAGNETIC FIELD

It has been known through this century that the earth's electromagnetic properties cause the magnetic field from external sources to be significantly modified in its vicinity. Thus it is expected that the surface magnetic field produced by the magnetospheric currents will be modified considerably by the real earth. It was therefore necessary to quantitatively study the problem of the currents electromagnetically induced in the earth as it rotates under these magnetospheric current systems. Although work on induced currents and their associated magnetic fields was not a part of the original work statement, it was felt that because of the impact of induction currents on the total magnetospheric S_q value, it was appropriate to quantitatively determine their contribution. Typically the earth has been considered as free space or as having infinite conductivity. A finite conductivity earth has been used typically only in induction problems where earth currents are important (e.g., in the study of crustal anomalies). Using the magnetospheric magnetic fields (as shown in Figures 2-4) as the forcing function, the daily variation in the S_q pattern was found for various values of conductivity for the earth. There is no question about either the existence or the magnitude of these magnetospheric currents nor their contribution to the S_q pattern and that the absolute minimum they contribute to the day to night amplitude is 12 nT.

We have solved the three-dimensional problem of the magnetic fields induced internal and external to a sphere having constant conductivity which is rotating in the presence of a static external magnetic field. The theory for determining these induced fields is developed in the Appendix.

In order to solve for the induced magnetic fields, it is necessary to express the forcing external fields as an internal spherical harmonic expansion. This has been done and the coefficients of the expansion are given in Table 1. The magnetopause expansion is taken from Mead (1964), assuming a 10 R_e magnetopause standoff distance. The ring and tail expansions come from magnetic field models developed at MDAC for zero "tilt" (Olson and Pfitzer, 1974).

The induced magnetic field found outside the sphere is represented by coefficients of an external spherical harmonic expansion. The coefficients on Table 2 give these coefficients assuming the sphere has a conductivity of .0065 mho/meter. This is the "preferred" surface conductivity given in Alldredge (1977), and since conductivity increases dramatically with depth below the surface, this model probably underestimates the actual induced magnetic fields. The conductivity dependence of the total surface field is examined in Figures 6 through 9. Figure 9 shows the north-south field component at the equator. Notice that the field variation is increased using the "preferred" conductivity by 50% over the non-conducting value. The same holds true at 30° latitude for both the north-south component (Figure 8) and the east-west component (Figure 9). The radial component variation (shown in Figure 7) is reduced to one-third of its non-conducting value assuming the same conductivity.

The value .0065 mho is most commonly used to represent the conductivity of the earth's crust. Since the earth's conductivity increases with depth, this value somewhat underestimates the ability of the earth to induce currents and shield its interior from the magnetospheric magnetic field. In absolute terms, the day-night variation of the forcing magnetic field is 7-3/4 nT at the equator. With the "preferred" conductivity of Alldredge, an induced field is produced to increase this variation to 11-3/4 nT, implying that magnetospheric sources contribute a large part of the ~ 30 nT S_q day-night variation.

The S_q pattern produced by the 3 magnetospheric current systems and their associated induced currents is shown for the north east and vertical components of the surface field at 0, $\pm$ 30^o and $\pm$ 60^o latitude in

Table 1

EXTERNAL MAGNETIC FIELD'S INTERNAL HARMONIC EXPANSION FORCING FUNCTION*

		Magnetopause**		Ring***		Tail***	
n	m	g_n^m	h_n^m	g_n^m	h_n^m	g_n^m	h_n^m
1	0	-25.111	0	+25.585	0	9.4248	0
2	1	-1.2424	0	-.26236	0	-.73475	0
3	0	-.00716	0	+.34294	0	-.013315	0
3	2	-.02333	0	+.002246	0	+.017078	0
4	1	-.002397	0	+.004280	0	-.001024	0
4	3	-.000163	0	+.0000252	0	+.0004479	0
5	0	+.0000569	0	-.012366	0	-.000130	0
5	2	-.0001078	0	-.000107	0	+.000105	0
5	4	-.0000103	0	+.0000355	0	-.0000446	0
6	1	+.00000126	0	-.000024	0	+.0000677	0
6	3	-.00000187	0	-.0000053	0	+.00000233	0
6	5	-.00000041	0	-.0000080	0	+.00000356	0

*Schmidt normalized coefficients. Units are nano Tesla.

**From Mead (1964) with 10 R_e magnetopause standoff.

***Fit of zero tilt current wire model magnetic field.

Table 2

INDUCED MAGNETIC FIELD'S EXTERNAL HARMONIC EXPANSION RESPONSE FUNCTION*

n	m	Magnetopause g_n^m	Magnetopause h_n^m	Ring g_n^m	Ring h_n^m	Tail g_n^m	Tail h_n^m
2	1	-.63506	.24036	-.13411	5.0758E-2	-.37557	.14215
3	2	-1.7466E-2	8.1466E-5	1.6815E-3	-7.8428E-6	1.2786E-2	-5.9635E-5
4	1	-1.91510E-3	1.4747E-5	3.4195E-3	-2.6332E-5	-8.1813E-4	6.3000E-6
4	3	-1.3026E-4	3.7092E-7	2.0138E-5	-5.7344E-8	3.5793E-4	-1.0193E-6
5	2	-8.9629E-5	8.6884E-7	-8.8964E-5	8.6239E-7	8.7301E-5	-8.4627E-7
5	4	-8.5665E-6	4.4993E-8	2.9525E-5	-1.5507E-7	-3.7094E-5	1.9482E-7
6	1	1.07476E-6	-4.0018E-8	-2.0472E-5	7.6225E-7	5.7747E-5	-2.1502E-6
6	3	-1.5974E-6	2.0758E-8	-4.5273E-6	5.8833E-8	1.9903E-8	-2.5864E-8
6	5	-3.5037E-7	2.9073E-9	-6.8364E-6	5.6728E-8	3.0422E-6	-2.5244E-8

*Schmidt normalized coefficient
Units are nano Tesla
Assumed conductivity is .0065 mho/meters

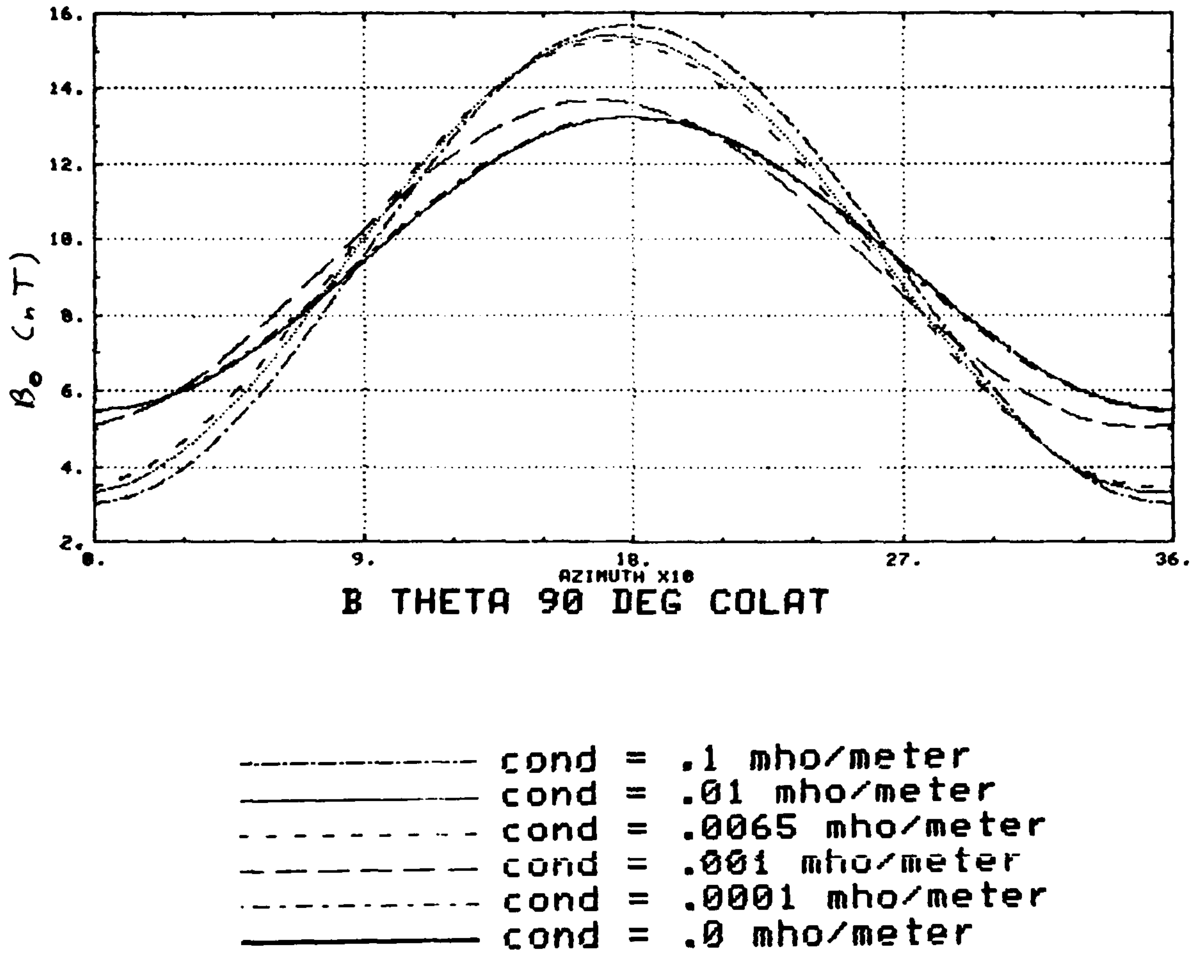
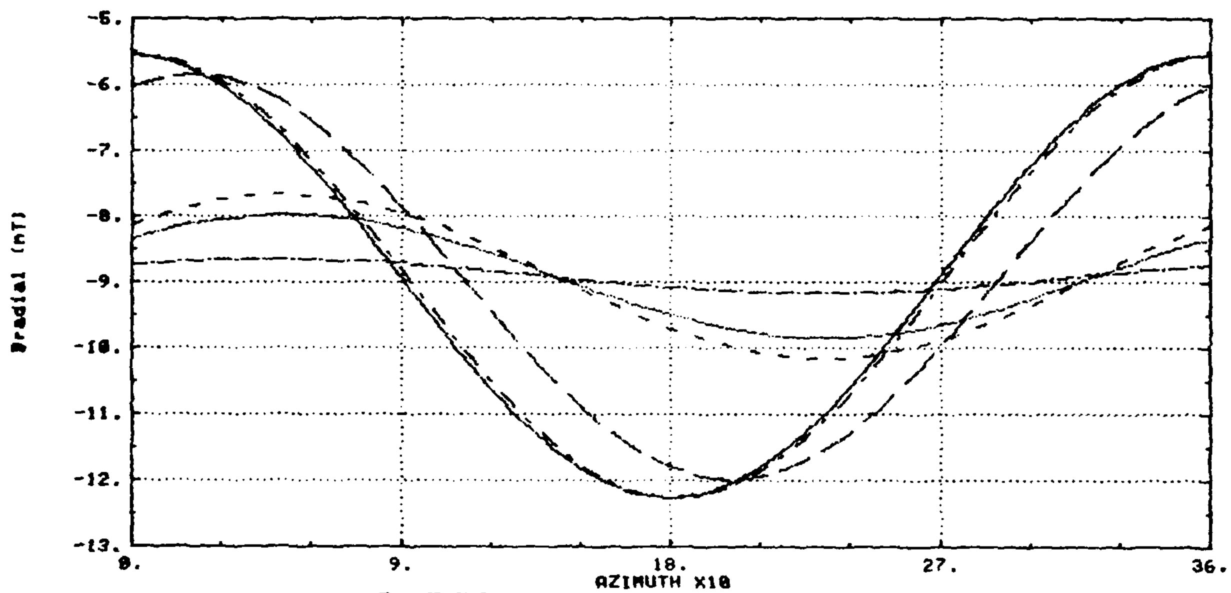

16.
14.
12.
10.
8.
6.
4.
2.
0.
9.
18.
27.
36.
AZIMUTH X10
B_o (nT)
B THETA 90 DEG COLAT
cond = .1 mho/meter
cond = .01 mho/meter
cond = .0065 mho/meter
cond = .001 mho/meter
cond = .0001 mho/meter
cond = .0 mho/meter

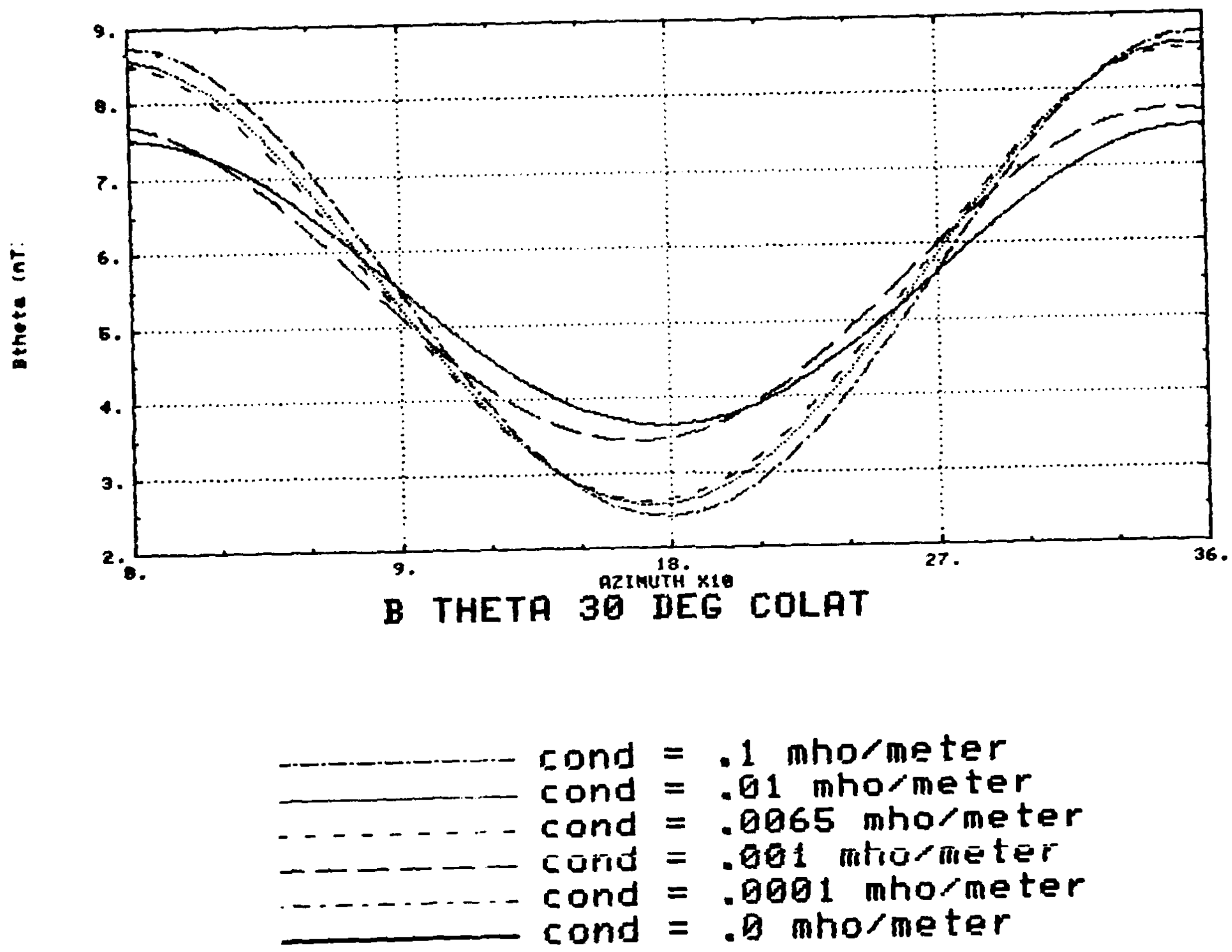
Figure 6



B RADIAL 30 DEG COLAT

cond = .1 mho/meter
cond = .01 mho/meter
cond = .0065 mho/meter
cond = .001 mho/meter
cond = .0001 mho/meter
cond = .0 mho/meter

Figure 7



Btheta (nT)
9.
8.
7.
6.
5.
4.
3.
2.
0.
9.
18.
27.
36.
AZIMUTH X10
B THETA 30 DEG COLAT
cond = .1 mho/meter
cond = .01 mho/meter
cond = .0065 mho/meter
cond = .001 mho/meter
cond = .0001 mho/meter
cond = .0 mho/meter

Figure 8

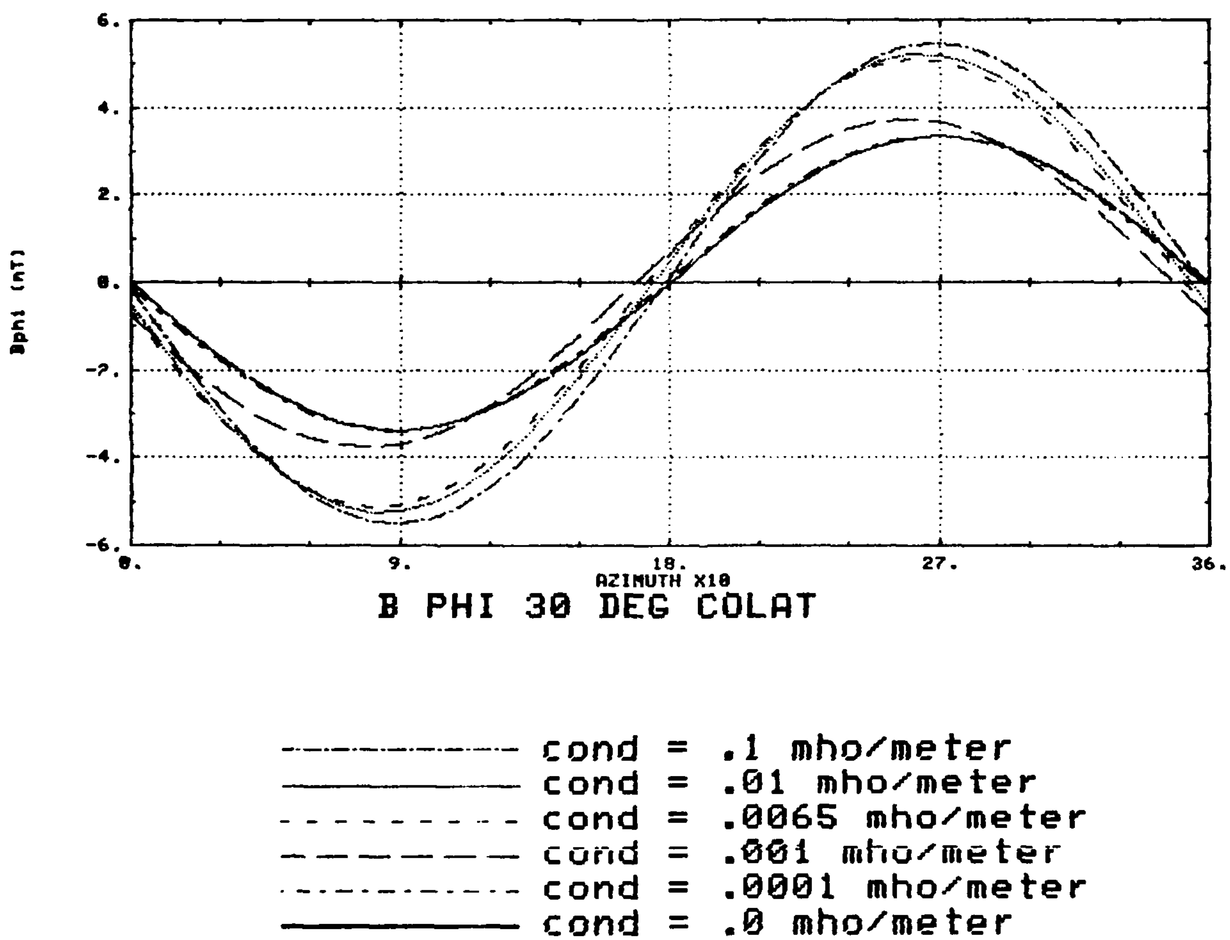

6.
4.
2.
0.
-2.
-4.
-6.
Bphi (nT)
0.
9.
18.
27.
36.
AZIMUTH X10
B PHI 30 DEG COLAT
cond = .1 mho/meter
cond = .01 mho/meter
cond = .0065 mho/meter
cond = .001 mho/meter
cond = .0001 mho/meter
cond = .0 mho/meter

Figure 9

Figures 10-12 with the earth's conductivity set at .0065 mho. It is seen that the amplitude of the variation is increased to about 12 nT when the earth's electrical conductivity is taken into account (as represented by the dashed lines). The "free space earth" values are represented by the solid lines. (Mathematical considerations for the representation of the induced earth current systems are given in the Appendix).

Note that as the conductivity goes from zero to infinite, the phasing of the variation pattern changes such that the maximum contribution occurs before noon for all finite conductivities, but occurs exactly at noon for both zero and infinite conductivity. It is also worth noting that the phasing problem has been used to suggest the magnetospheric currents do not make an appreciable contribution to S_q. The argument goes that because the maximum in S_q typically occurs after noon, and also typically occurs after hemispheric solstice, only lags in thermospheric heating associated with the ionospheric dynamo currents can explain the observations. However, phasing caused by the earth's finite electrical conductivity and also phasing in the annual variation pattern caused by the fact that the solar wind is not incident upon the geomagnetic field from the optical solar direction, both suggest that the magnetospheric contribution possesses the appropriate pat' .. (The Walters effect suggests that the free stream solar wind makes an angle of about 4 degrees to the optical sun-earth direction and it has been argued that when the solar wind flows through the bow shock, the angle between its flow direction and the optical sun-earth line may exceed 10 degrees).

VARIABILITY IN THE S_q PATTERN

In order to quantitatively study the day to day variability in S_q, it was necessary first to develop a data base containing ground based magnetometer observations and information on the variability of solar wind parameters. Of course, both data sets must be for the same period of time. We were able to obtain IMP-8 solar wind data for 1977 and 1978 from NASA Goddard. We were also able to obtain mean hourly values of the three components of the vector magnetic field observed at about two dozen mid-latitude magnetic observatories during the same two year time period from the World Data Center in Boulder, Colorado. We had hoped with these data sets to be able to show that the day to day variability in the earth's surface magnetic field is caused primarily

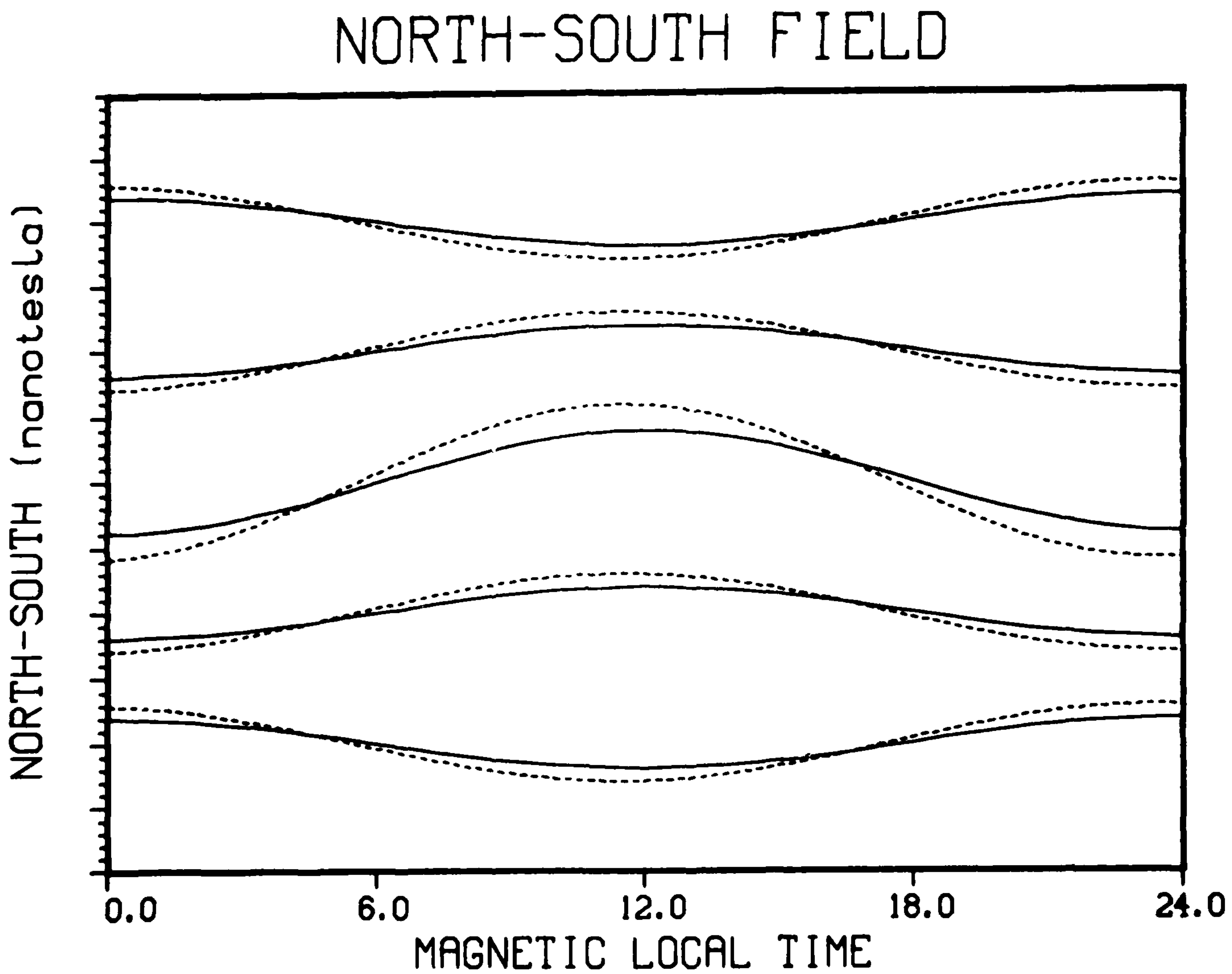

NORTH-SOUTH FIELD
NORTH-SOUTH (nanotesla)
0.0
6.0
12.0
18.0
24.0
MAGNETIC LOCAL TIME

FIGURE 10

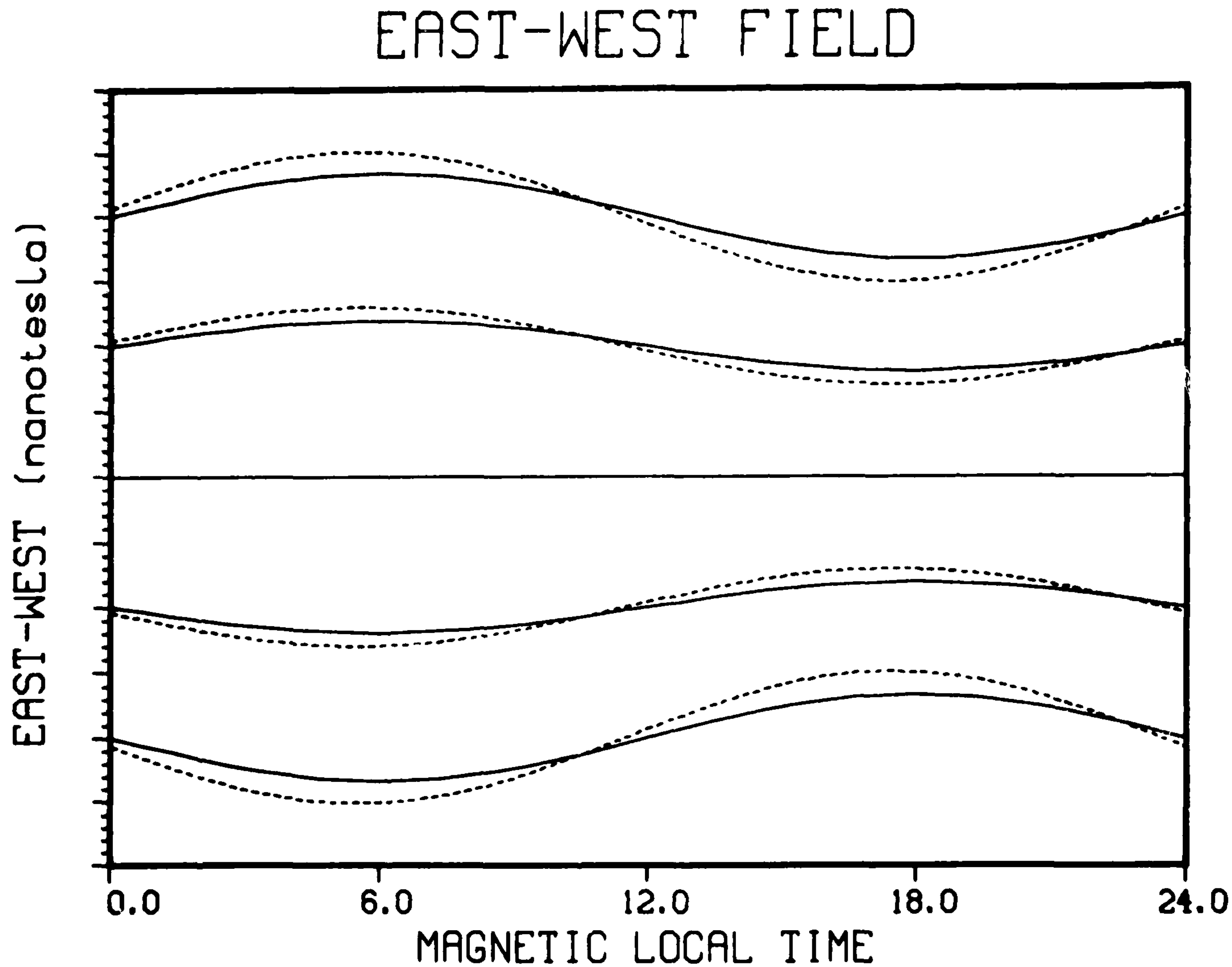

EAST-WEST FIELD
EAST-WEST (nanotesla)
0.0
6.0
12.0
18.0
24.0
MAGNETIC LOCAL TIME

FIGURE 11

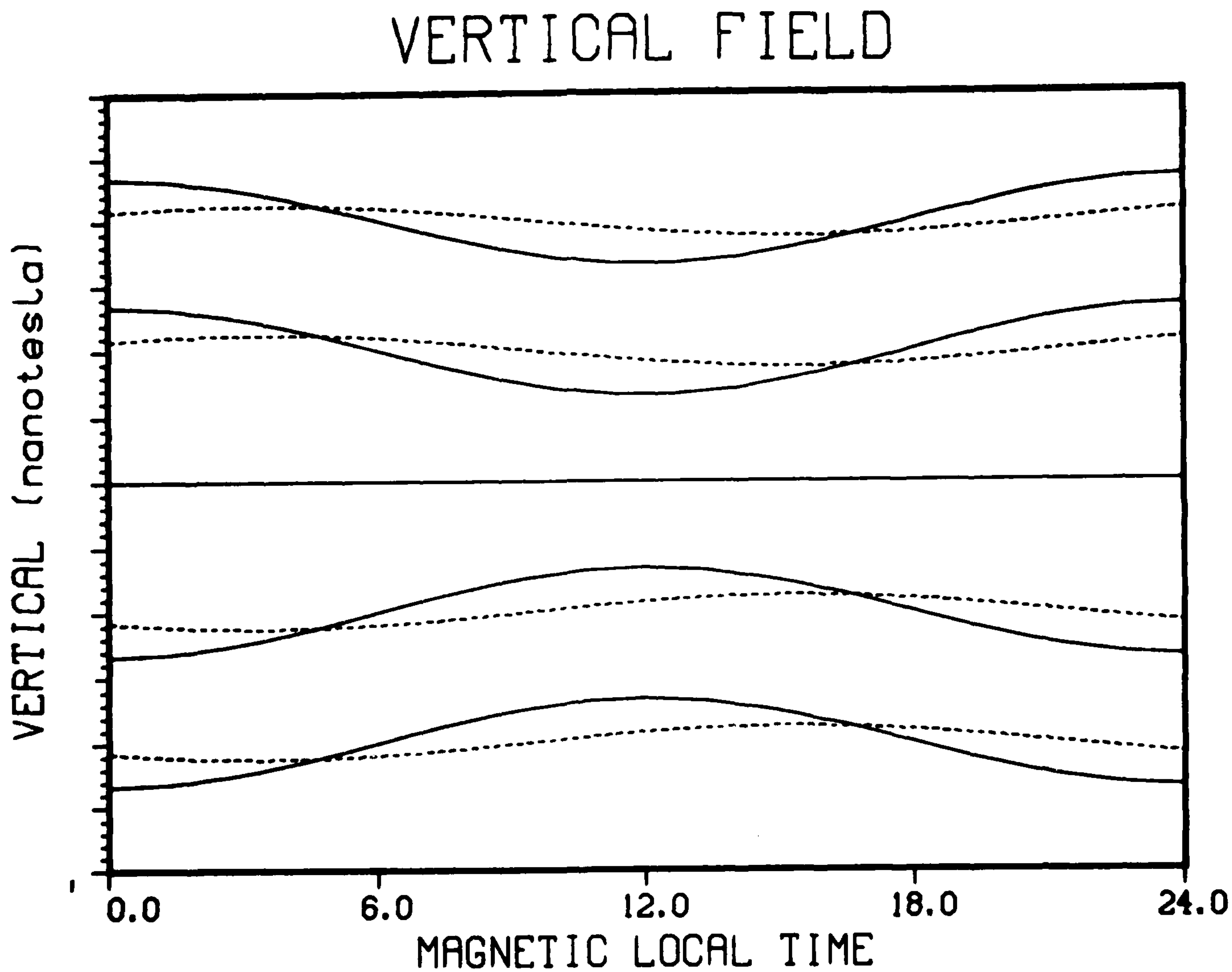
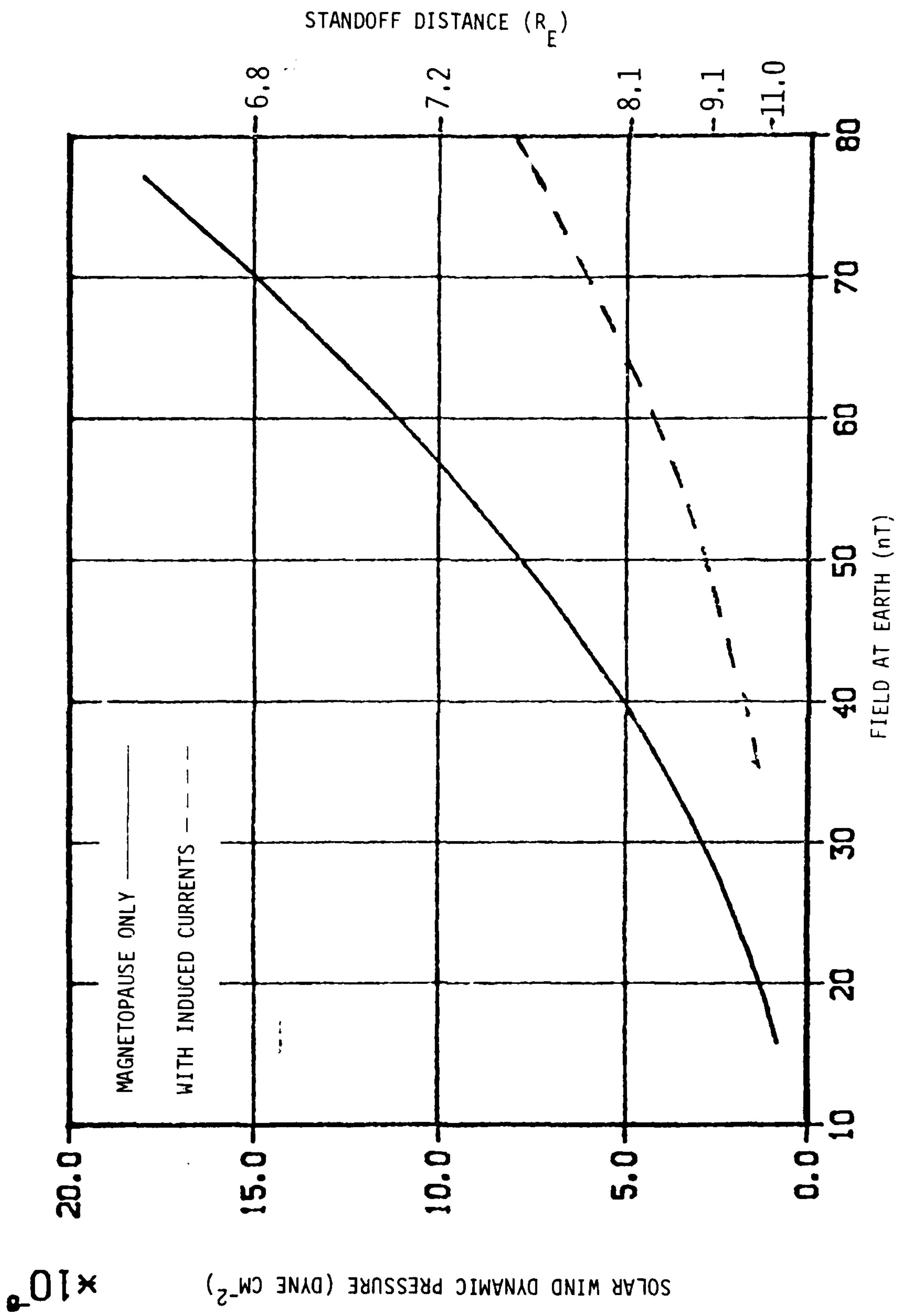

VERTICAL FIELD
VERTICAL (nanotesla)
0.0
6.0
12.0
18.0
24.0
MAGNETIC LOCAL TIME

FIGURE 12

by changes in the magnetopause magnetic field. We used solar wind data to describe changes in the magnetopause standoff distance (and also the strength of the magnetopause current system). The change in the magnetic field produced by the magnetopause currents at the earth's surface at noon in reponse to solar wind pressure is shown in Figure 13. The solid curve shows simply the magnetic field from the magnetopause currents; the dashed lines shows the total magnetic field when earth currents are included. It is seen that as the magnetopause standoff distance changes from about 10-1/2 down to 7 earth radii, the contribution from the magnetopause currents to the earth's surface magnetic field changes by approximately 60 nT. Thus we hoped to find this variation present in the day to day changes in the surface magnetic field. The data from the ground observatories was restricted to quiet magnetic conditions, as evidenced by low Kp values. Further, in order to remove the contribution from the ring current, the Dst value was removed from the data (at the earth's surface, Dst directly represents the magnetic field produced by the ring current). The horizontal component of the field at San Juan is shown at noon for quiet days with the ring current contribution removed. The expected strong dependence on solar wind dynamic pressure was not detected, as is shown in Figure 14. A similar plot is shown for Newport in Figure 15. The dependence on the magnetopause currents would tend to make the data points lie along the curve, which shows the computationally determined contribution of magnetopause currents at the earth's surface at noon as a function of dynamic solar wind pressure. Note that the curve could be moved horizontally. We also note that if such a correlation can be filtered out of the data, it could be used to uniquely determine the baseline of S_q. We are unaware of any other way to do this quantitatively, thus our study of the dependence of the surface magnetic field on solar wind variations will continue because an accurate determination of the earth's main field is important for many theoretical and commercial reasons.

OVERALL ASSESSMENT OF MAGNETOSPHERIC CONTRIBUTION TO THE SURFACE MAGNETIC FIELD

It is difficult to assess precisely what fraction of the observed S_q pattern these magnetospheric currents produce. This is because there is at present some indecision concerning the magniiude of the S_q currents. Classically, the mid latitude S_q pattern exhibits a peak to peak variation of about 25 gammas. However, recent investigations of S_q suggest that the pattern for



STANDOFF DISTANCE (R_E)
6.8
7.2
8.1
9.1
11.0
MAGNETOPAUSE ONLY
WITH INDUCED CURRENTS
10
20
30
40
50
60
70
80
FIELD AT EARTH (nT)
20.0
15.0
10.0
5.0
0.0
SOLAR WIND DYNAMIC PRESSURE (DYNE CM^{-2})
$\times 10^{-9}$

Figure 13

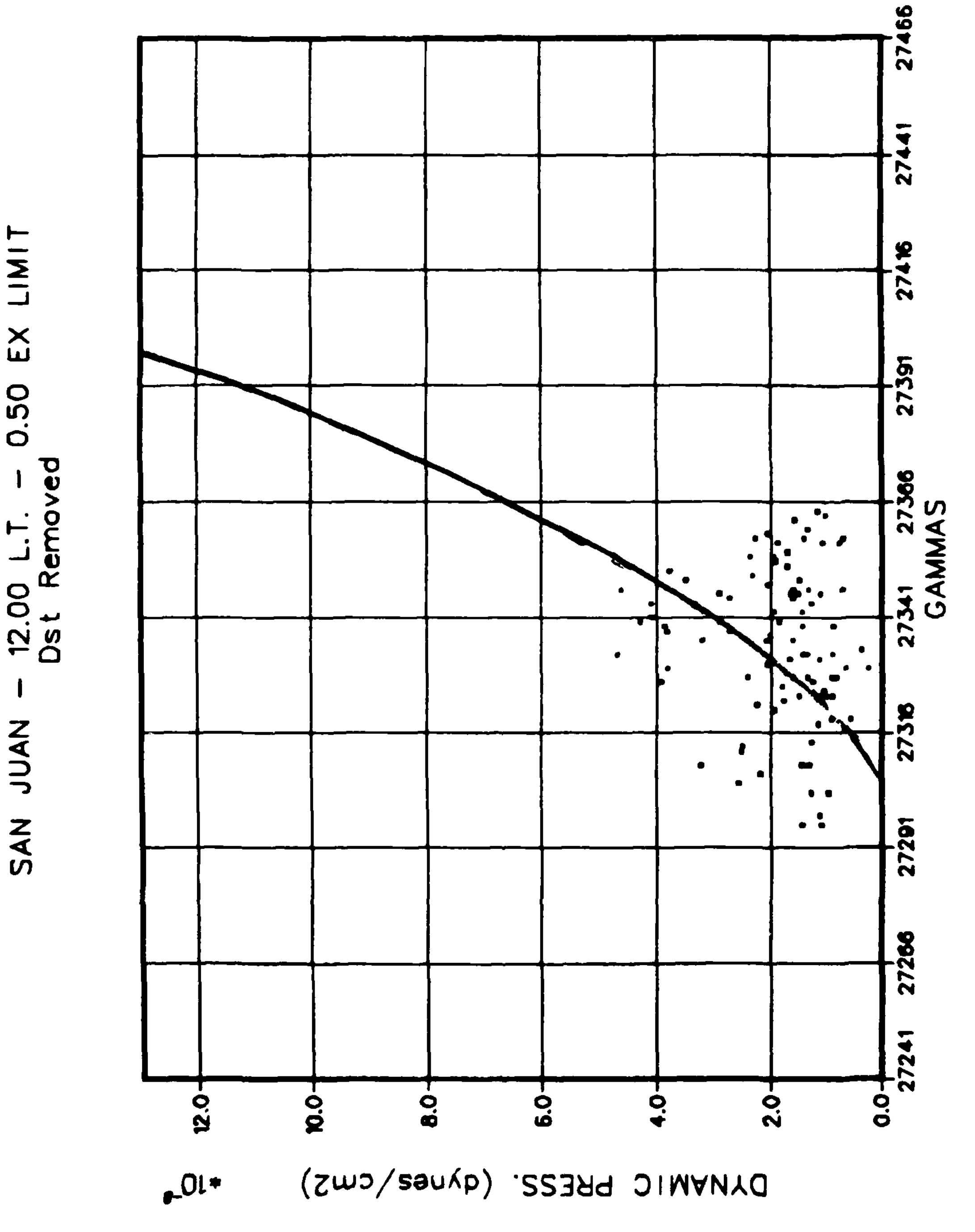

SAN JUAN - 12.00 L.T. - 0.50 EX LIMIT
Dst Removed
DYNAMIC PRESS. (dynes/cm2) *10^-8
12.0
10.0
8.0
6.0
4.0
2.0
0.0
27241
27266
27291
27316
27341
27366
27391
27416
27441
27466
GAMMAS

Figure 14

NEWPORT - 12.00 L.T. - 0.50 EX LIMIT
Positive Dst

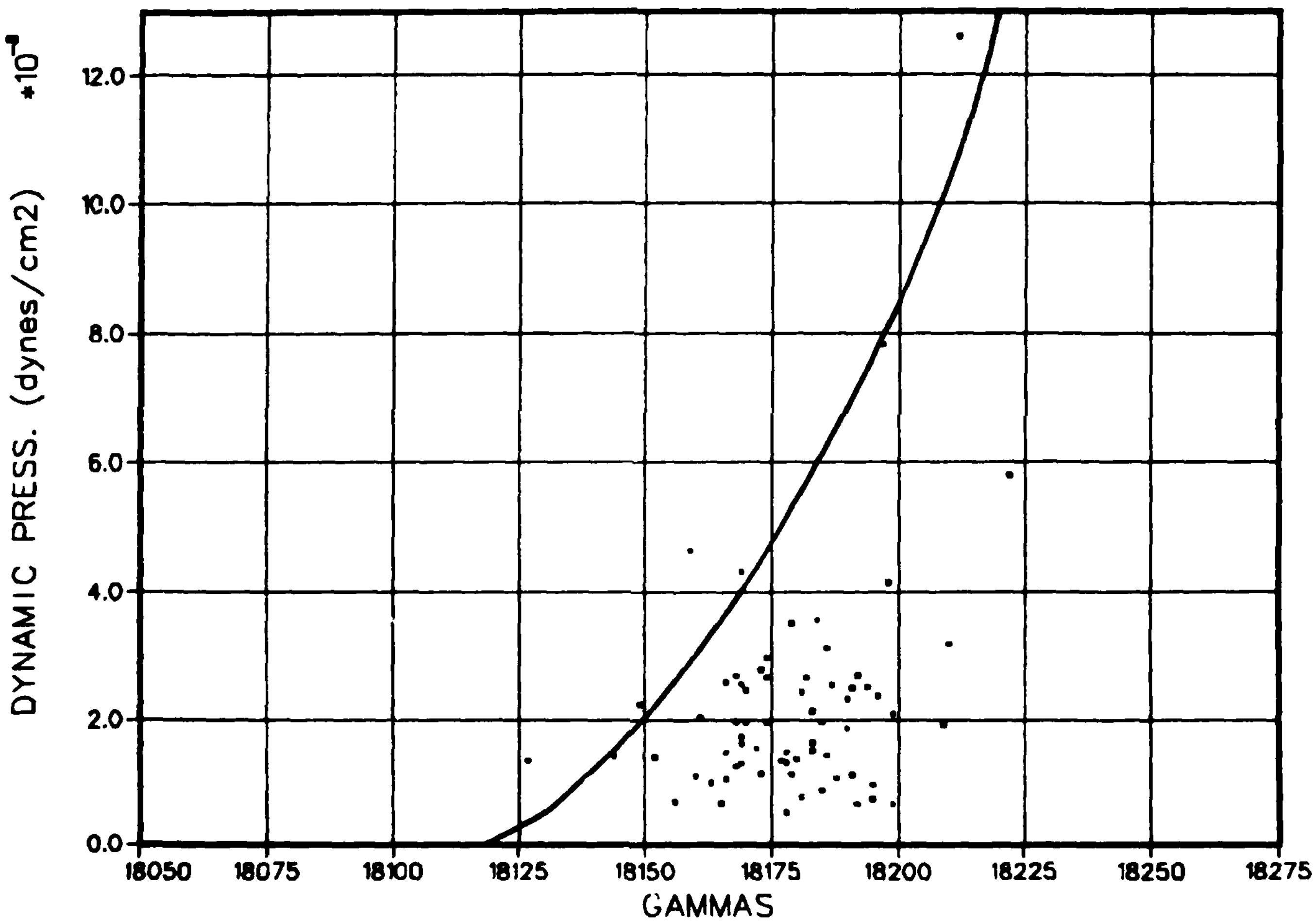


Figure 15

very quiet days during the last decade exhibits a day to night variation of approximately 45 gammas. Thus the magnetospheric contribution of approximately 12 gammas should produce somewhere between one-fourth to almost one-half the current of the S_q pattern. There is also an uncertainty in the value of the magnetopause ring and tail currents that have been used. These have been average values for each current system as defined by magnetospheric configuration for quiet times over long periods. However, it is known that the magnetosphere's current systems are quite variable as they respond to large fluctuations in solar wind and interplanetary field parameters. It would be appropriate, wherever possible, to determine the strength of the magnetospheric current systems for the same days that were used to derive the observed S_q patterns. Clearly the magnetospheric values presented here are baseline values and a detailed investigation day by day (where solar wind and interplanetary field data are available) should yield somewhat larger values for the magnetospheric contribution. We therefore remain unconvinced of the ionosphere's dominant role in producing surface magnetic field variations during quiet magnetic conditions.

Since the contribution we are now finding for these currents is approximately 2-1/2 times larger than reported earlier, it is reasonable to assume that this will also be true for the annual variation produced by the magnetospheric currents. This would suggest that the magnetospheric currents then produce approximately one-fourth the observed 8-10 nT seasonal variation.

Work done over the past two years as part of the Coordinated Data Analysis Workshops (CDAWs) has shown that although the nominal position for the magnetopause is between 10 and 11 earth radii along the sun-earth line, it is sometimes compressed to well within geosynchronous orbit. The variability in magnetospheric features during the July 29, 1977 event was extensively studied in CDAW-2. The magnetopause location ranges from about 6 R_E to over 15 R_E in just over half a day (see Figure 16). Compression of the magnetosphere is responsible for the well studied "sudden commencement phase" of the geomagnetic storm. Using pressure balance formalism it is easy to show that an inward motion of the magnetopause of 1 earth radius will increase the magnetopause contribution to S_q by approximately 35 percent. Thus increases in the surface field for magnetospheric currents as large as 50 and even 100

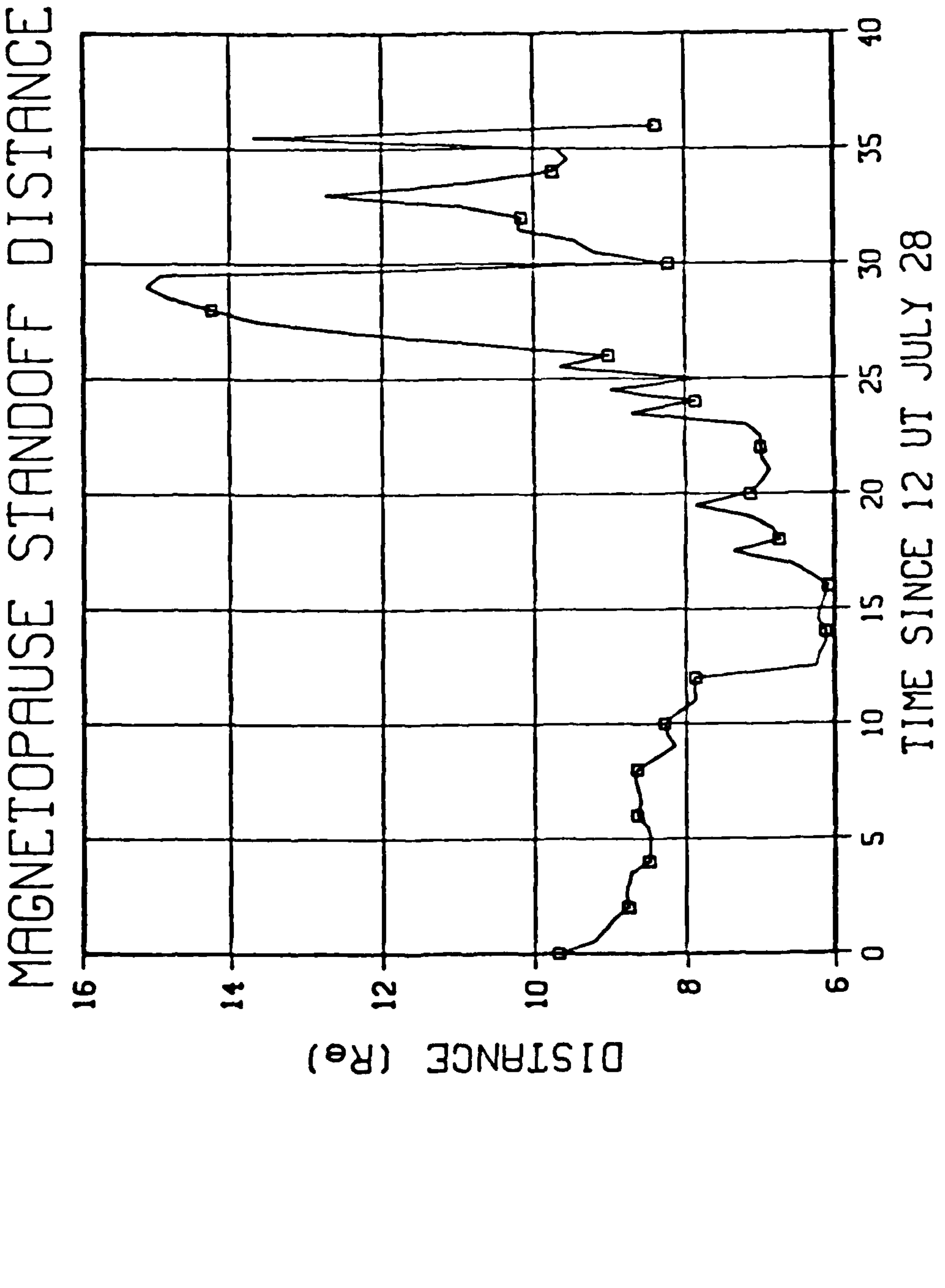

MAGNETOPAUSE STANDOFF DISTANCE
DISTANCE (Re)
16
14
12
10
8
6
0
5
10
15
20
25
30
35
40
TIME SINCE 12 UT JULY 28

Figure 16

percent are not uncommon. Studies of the solar wind suggest that there are definitely extended periods where the solar wind pressure is considerably enhanced, and yet the solar wind flow is steady. Thus it is clearly possible that the magnetospheric contribution to S_q is larger than reported here. It is also clear that the magnetospheric currents must make a significant contribution to the day to day variability in the S_q pattern. We expect that solar wind pressure variations between quiet days accounts for a large part of the range in the strength of the observed S_q pattern for those days.

In order to accurately determine the main magnetic field (as has been attempted recently with MAGSAT data), it is necessary to have a capability for representing the magnetospheric currents in real time. A problem with attempting to average magnetospheric data over some interval of time (e.g. an epoch used for the MAGSAT analysis) is that the dependence of the S_q contribution from the magnetopause location is a nonlinear function of magnetosphere size. Thus the use of magnetospheric models to accurately determine the surface magnetic field requires the availability of time varying representations of each of the magnetospheric current systems. These have been developed and are now being used as part of the CDAW process. They may also be used in conjunction with MAGSAT data sets in an attempt to accurately determine features of the earth's internal magnetic field.

SUMMARY

The purpose of this work has been to reexamine the role that the magnetosphere plays in producing magnetic variations at the earth's surface. Prior to the discovery of the magnetosphere, it had been assumed for half a century that the magnetic variations at the earth's surface during quiet times (the S_q pattern) were caused by currents flowing in the ionosphere. Some of the more astute "founding fathers" typically referred to the source of S_q as caused only by an equivalent "overhead current system" instead of specifically suggesting that all of the contribution was caused by the ionosphere. Early work on the magnetosphere suggested that it produced an inconsequential portion of this S_q pattern. However, the early magnetospheric models were developed to represent the magnetic field through the entire magnetosphere and were not constructed for accuracy at the earth's surface. Because of this and

our much better understanding of the magnetospheric currents fifteen years later, another look has been taken at the contribution of the magnetospheric currents to S_q. So far it has been found that the magnetospheric contribution to S_q, as represented in terms of a day to night range, is approximately 12 nT when the earth's finite conductivity is taken into account. (Although the earth induction problem was not a formal part of the study, significant time was spent on it because of its importance to the problem). It is also clear that the magnetospheric currents make a large contribution to the day to day variability in the observed S_q pattern. The fraction of the observed S_q pattern caused by the magnetospheric currents is still not clearly defined for two reasons. First, there is some confusion as to what the magnitude of the observed pattern really is; and second, only average values for the magnetospheric currents were used even though it is clear that for some quiet days where the observed pattern was determined, the magnetopause current system is stronger than that used in this study.

MAJOR ACCOMPLISHMENTS ON THIS CONTRACT

The major accomplishments of the work completed under this contract are listed.

- It was determined by direct integration over the magnetospheric current systems that they make a contribution of approximately 12 nT to the day to night variation in the earth's surface magnetic field. This is a minimum contribution for exceptionally quiet times when all of the magnetospheric current systems are at their minimum strength. These values are significantly different from those given by using magnetospheric magnetic field models directly, since the models were used to represent the magnetic field throughout the magnetosphere and not accurately at the earth's surface. We conclude since 12 nT is an important fraction of the total observed day to night variation in the earth's surface magnetic field at mid-latitudes that the magnetospheric currents produce at least one-third of the observed variation in S_q. This finding is important for ionospheric physicists working on the ionospheric dynamo and the global ionospheric wind systems, since the winds are not required to produce all of the observed S_q pattern.

- It is well known that currents external to the earth's surface induce currents that flow in the earth's crust which act to increase the horizontal component of the magnetic field produced by the primary current at the earth's surface. However, it is difficult to represent these induction currents accurately, thus, typically, the earth's electrical conductivity is usually represented as being a zero or infinite value. We have developed the math for representing the earth as a finite electrical conductor. It was thus possible to represent the total magnetic field produced at the earth's surfce by the ring, tail and magnetopause current systems, including the currents that they induced in the earth's crust and their contribution to the earth's surface magnetic field.

- In order to study the day to day variability in the S_q pattern, it was necessary to develop first a data base which included observations of the solar wind over a long period of time and, for the same time period, mean hourly values in the earth's surface magnetic field at many mid-latitude observatories. This data was assembled with the help of NASA Goddard and the World Data Center in Boulder.

- Preliminary computer work was done on the quantitative determination of the signature of the Birkeland current system in its "Region 2" closure in the night side of the magnetosphere. No model has been presented or published on this portion of the magnetospheric current systems since there is still some controversy over the details of its topology.

CONTINUING WORK

Work on this contract has led to several conclusions which have been listed above in the subject on Major Accomplishments. It has also led to several additional questions, which we will continue to explore. Some of these are listed below.

- Work by several investigators (especially the recent work of Matsushita and Nishida) has illustrated the importance of the field aligned (Birkeland) currents for variations in the earth's surface magnetic field in the high mid-latitude region. To do this work accurately, however, we must first better understand the entire complex Birkeland current system, including both the Region 1 and Region 2 currents and those currents that flow into the auroral ionosphere on the polar side of the day side cusps. (It is also of interest to us in our magnetospheric magnetic field modeling work to better understand the closure of the Birkeland currents in the magnetosphere. This is especially true of the Region 2 currents which carry from 1 to 2 million amperes over a relatively confined region of the magnetosphere and thus, we expect, produce a very characteristic signature in the night side of the magnetosphere).

- We expect to do a better job on finding the dependence of th surface magnetic field on solar wind dynamic pressure. Essentially, the problem to date is that there is a lot of noise in the data. This noise is typically produced by the ring and tail currents and, admittedly, also has a contribution from the ionosphere. However, we remain convinced that the signal is there, since it is so large (almost 100 gammas over the range of known magnetopause standoff locations). We have talked with John Sampson (University of Alberta) and John Olson (University of Alaska), who have developed filtering techniques which have been very helpful in digging micropulsation signals out of a large noise background. We expect their techniques may be helpful in developing unambiguously the relation between magnetic surface field variability and dynamic solar wind pressure. If such a relationship can be developed, we hope to use it to determine the S_q baseline and with it, an accurate value for the earth's main magnetic field.

- The largest uncertainty in the work performed on this contract relates to the earthward extent of the ring current. We know that it can change dramatically without changing the overall ring current signature at synchronous orbit (where most measurements of the ring current are made). However, the contribution from the ring current near the earth is expected to have an appreciable signature at mid-latitudes even during quiet times. It is important to determine this contribution to the earth's surface magnetic field in order to continue the study of the contribution of magnetospheric currents to S_q, and also for the accurate determination of the earth's main magnetic field. We have had several discussions with Sugiura and Langel of NASA Goddard on this problem.

PRESENTATIONS AND PUBLICATIONS

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The Accurate Determination of the Magnetospheric Magnetic Field in the Vicinity of the Earth (with W. P. Olson). McDonnell Douglas Astronautics Company Paper No. G8795-I. Presented to IAGA 4th Scientific Assembly in Edinburgh, Scotland, August 1981.

The Contribution of Magnetospheric Currents to the Earth's Surface Magnetic field. To be presented at the IAGA/IUGG XVIII General Assembly in Hamburg, Germany, August 1983. McDonnell Douglas Astronautics paper No. G8977-I.

The Quantitative Representation of the Magnetospheric Electromagnetic Field fo Specific Events (with S. Schneider and K. A. Pfitzer). To be presented at the 1983 Spring Meeting of AGU in Baltimore, Md.

Quantitative Modeling of the disturbed Magnetospheric Magnetic Field (with K. A. Pfitzer). To be presented at the IAGA/IUGG XVIII General Assembly in Hamburg, Germany, August 1983. McDonnell Douglas Astronautics paper No. G8980-I.

APPENDIX

THE DETERMINATION OF MAGNETIC FIELDS PRODUCED AT THE SURFACE OF A ROTATING SPHERE WITH FINITE ELECTRICAL CONDUCTIVITY

The mathematical formalism for determining the magnetic field produced by currents induced in the earth's crust is developed. These currents are formed as the earth responds to external magnetic fields as it rotates under them.

Note that this appendix was printed with the TEX typesetting program.

APPENDIX

This attachment will develop solutions for the magnetic field induced interior and exterior to a cylinder or sphere having finite conductivity. The fields are induced because the motion of the object in the presence of a static external magnetic field causes an electric field to be produced. This electric field will cause currents to flow because the object has finite conductivity, and the currents cause the induced magnetic field. We begin with Maxwell's equations in the inertial reference frame:

$$
\begin{aligned}
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{J} &= -\frac{\partial \rho}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \cdot \mathbf{D} &= \rho.
\end{aligned}
\tag{1}
$$

There are also constitutive relations that hold in the reference frame in which the material is motionless (this reference frame is denoted by an *):

$$
\begin{aligned}
\mathbf{B}^* &= \mu \mathbf{H}^* \\
\mathbf{D}^* &= \epsilon \mathbf{E}^* \\
\mathbf{J}^* &= \sigma \mathbf{E}^*.
\end{aligned}
\tag{2}
$$

The constants μ, ϵ, and σ are the material permeability, permittivity, and conductivity respectively.

The field quantities as measured in the moving frame – in which the material is motionless – are related to the fields measured in the inertial reference frame by Minkowski's equations for moving media. Letting $\mathbf{v}$ be the local material velocity, and assuming that $v/c \ll 1$ where c is the speed of light, the relations are:

$$
\begin{aligned}
\mathbf{E} &= \mathbf{E}^* - \mathbf{v} \times \mathbf{B}^* \\
\mathbf{B} &= \mathbf{B}^* \\
\mathbf{H} &= \mathbf{H}^* + \mathbf{v} \times \mathbf{D}^* \\
\mathbf{J} &= \mathbf{J}^* + \rho \mathbf{v} \\
\rho &= \rho^*.
\end{aligned}
\tag{3}
$$

From equations (2) and (3), we can derive two more useful relations:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}^* - \mu \mathbf{v} \times \mathbf{H} \\ \mathbf{H} &= \mathbf{H}^* + \epsilon \mathbf{v} \times \mathbf{E} \end{aligned} \tag{4}$$

where we also use $\epsilon\mu = 1/c^2$ and neglect terms $O(v^2/c^2)$ as was done in equations (3).

ROTATING CYLINDER OR SPHERE EQUATIONS

We assume that the material object in the static external magnetic field is a cylinder rotating about its symmetry axis, or a sphere rotating about a given axis with constant angular velocity ω. In both cases, the velocity vector is given by $\mathbf{v} = \omega \mathbf{i}_z \times \mathbf{r}$. The time dependences in Maxwell's equations will eventually damp out leaving:

$$\begin{aligned} \nabla \times \mathbf{E} &= 0 \\ \nabla \times \mathbf{H} &= \sigma \mathbf{E}^* + \rho \mathbf{v} \\ \nabla \cdot \mathbf{J} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{D} &= \rho. \end{aligned} \tag{5}$$

Using the relations in (3), (5), and $\nabla \cdot \mathbf{v} = -\omega \mathbf{i}_z \cdot \nabla \times \mathbf{r} = 0$, we can show that there is no free charge in the sphere or cylinder. We derive the equation $\rho = \frac{\omega\epsilon}{\sigma}\partial\rho/\partial\phi$ with the solution $\rho = \rho_0 e^{\phi\sigma/\epsilon\omega}$. To have continuity of ρ , we must set ρ_0 to zero, so there can be no free charge in the body.

The $\mathbf{H}$ field in the object can be shown to be approximately divergence-free as follows. From the $\nabla \cdot \mathbf{B} = 0$ equation, we can show that $\nabla \cdot \mathbf{H} - \epsilon\omega E_z = 0$. Since $\omega \approx 7.3 \times 10^{-5}$ radian/sec for one revolution per day, and $\epsilon \approx 8.9 \times 10^{-12}$ farad/meter, we have:

$$\nabla \cdot \mathbf{H} \approx (6.5 \times 10^{-16}\,\text{amp/volt-meter}) E_z$$

which approximates $\nabla \cdot \mathbf{H} = 0$ for reasonable sized $\mathbf{E}$ fields.

The $\nabla \times \mathbf{H}$ equation in (5) becomes

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \sigma\mu(\omega \mathbf{i}_z \times \mathbf{r}) \times \mathbf{H}$$

after using the $\mathbf{E}$ relation in (4) and $\rho = 0$. Taking the curl of this equation, and using the relations derived above, we get:

$$\begin{aligned} \nabla \times \nabla \times \mathbf{H} &= \sigma\mu(\omega \mathbf{i}_z \times \mathbf{H} - (\mathbf{v} \cdot \nabla)\mathbf{H}) \\ &= -\sigma\mu\omega\left(\frac{\partial H_{r\perp}}{\partial\phi}\mathbf{i}_{r\perp} + \frac{\partial H_\phi}{\partial\phi}\mathbf{i}_\phi + \frac{\partial H_z}{\partial\phi}\mathbf{i}_z\right) \quad \text{in the cylinder} \\ &= -\sigma\mu\omega\left(\frac{\partial H_r}{\partial\phi}\mathbf{i}_r + \frac{\partial H_\theta}{\partial\phi}\mathbf{i}_\theta + \frac{\partial H_\phi}{\partial\phi}\mathbf{i}_\phi\right) \quad \text{in the sphere.} \end{aligned} \tag{6}$$

It is equation (6) that will be solved to determine the induced fields in the cylinder and sphere.

SOLUTION FOR ROTATING CYLINDER

The solution for this problem is found in a manner similiar to determining the natural modes of the cylinder's oscillation in time varying fields. Letting the H field be decomposed as:

$$\mathbf{H} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \mathbf{H}_n e^{in\phi} \tag{7}$$

we find

$$\begin{aligned} \nabla \times \nabla \times \left(\mathbf{H}_n e^{in\phi}\right) &= -in\sigma\mu\omega\left(\mathbf{H}_n e^{in\phi}\right) \qquad \text{or,} \\ \nabla \times \nabla \times \mathbf{M}_n &= k_n^2 \mathbf{M}_n \qquad \text{and} \\ \nabla \times \nabla \times \mathbf{N}_n &= k_n^2 \mathbf{N}_n, \end{aligned} \tag{8}$$

where $k_n^2 = -in\sigma\mu\omega$, and $\mathbf{M}_n$ and $\mathbf{N}_n$ are two independent vector solutions for mode n. Stratton (1941) gives the solution for this problem by Hertzian vectors, and by another method (p 392) which is also applicable to the rotating sphere problem. We will follow the latter method.

Briefly, we let ξ_n be the solution of

$$\nabla^2 \xi_n + k_n^2 \xi_n = 0. \tag{9}$$

Then, two independent vector solutions to (8) are

$$\begin{aligned} \mathbf{M}_n &= \nabla \times \mathbf{i}_z \xi_n \qquad \text{and} \\ \mathbf{N}_n &= \frac{1}{k_n} \nabla \times \mathbf{M}_n. \end{aligned} \tag{10}$$

Note that both $\mathbf{M}_n$ and $\mathbf{N}_n$ are divergenceless. A solution of (9) having no z dependence is

$$\xi_n = \frac{1}{in} J_n(k_n r_\perp) e^{in\phi}, \tag{11}$$

where J_n are Bessel functions of the first kind. Therefore, the two solutions are

$$\begin{aligned} \mathbf{M}_n &= \left(\frac{1}{r_\perp} J_n(k_n r_\perp)\mathbf{i}_{r_\perp} - \frac{k_n}{in} J_n'(k_n r_\perp)\mathbf{i}_\phi\right) e^{in\phi}, \qquad \text{and} \\ \mathbf{N}_n &= \frac{k_n^2}{in} J_n(k_n r_\perp) e^{in\phi} \mathbf{i}_z. \end{aligned} \tag{12}$$

The natural modes for the rotating cylinder combine to give

$$\mathbf{H} = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} (a_n \mathbf{M}_n + b_n \mathbf{N}_n) \tag{13}$$

where a_n and b_n are determined by boundary conditions.

SOLUTION FOR ROTATING SPHERE

The solution for the rotating sphere parallels that for the rotating cylinder. We let $\mathbf{H}$ be decomposed as in (7) and must solve equation (8) in spherical geometry. Stratton has done this (p 414) by showing that if ξ_n is a solution to (9), then the two independent vector solutions of (8) are

$$\begin{aligned} \mathbf{M}_n &= \nabla \times \mathbf{i}_r r \xi_n \qquad \text{and} \\ \mathbf{N}_n &= \frac{1}{k_n} \nabla \times \mathbf{M}_n. \end{aligned} \tag{14}$$

The solution to (9) in the spherical case is

$$\begin{aligned} \xi_n &= \sum_{m=n}^{\infty} \frac{c_m}{\sqrt{r}} J_{m+\frac{1}{2}}(k_n r) P_m^n(\cos\theta) e^{in\phi} \qquad \text{or} \\ \xi_m^n &= \frac{1}{\sqrt{r}} J_{m+\frac{1}{2}}(k_n r) P_m^n(\cos\theta) e^{in\phi} \end{aligned} \tag{15}$$

where P_m^n are associated Legendre functions. The vector solutions corresponding to ξ_m^n are $\mathbf{M}_m^n$ and $\mathbf{N}_m^n$, and they are given by

$$\begin{aligned} \mathbf{M}_m^n &= \frac{1}{\sin\theta} \frac{\partial \xi_m^n}{\partial \phi} \mathbf{i}_\theta - \frac{\partial \xi_m^n}{\partial \theta} \mathbf{i}_\phi \\ \mathbf{N}_m^n &= \frac{m(m+1)}{k_n r} \xi_m^n \mathbf{i}_r + \frac{1}{k_n r} \frac{\partial^2 r \xi_m^n}{\partial r \partial \theta} \mathbf{i}_\theta + \frac{1}{k_n r \sin\theta} \frac{\partial^2 r \xi_m^n}{\partial r \partial \phi} \mathbf{i}_\phi. \end{aligned} \tag{16}$$

The natural modes combine in the spherical case to give

$$\mathbf{H} = \sum_{m=1}^{\infty} \sum_{\substack{n=-m \\ n \neq 0}}^{m} (a_m^n \mathbf{M}_m^n + b_m^n \mathbf{N}_m^n). \tag{17}$$

BOUNDARY CONDITIONS

The boundary conditions for these problems are similiar to those for scattering. We have an external forcing field, a field transmitted into the object, and a "scattered" or reflected field external to the object. The total external field is the forcing plus reflected fields. If we look to these problems as representing the S_q variation of the earth's magnetic field, it is the total external field that contains the S_q variation. The boundary conditions to be satisfied at the object surface are: 1) The tangential component of the total external **H** must match this component of the transmitted **H**, 2) The normal component of the total external **B** must match the normal component of the transmitted **B**.

We have not talked about the transmitted **B** yet but it is easy to show using (4) that to $O(v^2/c^2)$ we have $\mathbf{B} = \mu\mathbf{H}$ inside the object. In the vaccuum outside, the relation $\mathbf{B} = \mu_0\mathbf{H}$ holds exactly.

In the sections that follow, we assume that the forcing magnetic fields are caused by stationary currents that are bounded away from the cylinder or sphere. This assumption simplifies the mathematics of the problem because the forcing field can be expanded into spherical harmonics, so no coupling between the natural modes (i.e. of coefficients a_n and b_n for different n) will occur.

DETERMINING THE COEFFICIENTS FOR THE ROTATING CYLINDER

The external forcing field is assumed to have the form

$$\mathbf{H}_f = -a\nabla\Bigg(\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} d_n(r_\perp/a)^{|n|}e^{in\phi}\Bigg) \tag{18}$$

where a is the cylinder radius and the complex constants d_n are known. The reflected **H** field will have the form

$$\mathbf{H}_r = -a\nabla\Bigg(\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} g_n(a/r_\perp)^{|n|}e^{in\phi}\Bigg). \tag{19}$$

The tangential boundary condition at $r_\perp = a$ using (12) is

$$-a_n\left(\frac{k_n}{in}J'_n(k_n a)\right) = -d_n in - g_n in \tag{20}$$

and b_n is zero since we assume no forcing in the z direction. The radial boundary condition is

$$a_n\left(\frac{\mu}{a}J_n(k_n a)\right) = -d_n\mu_0|n| + g_n\mu_0|n| \tag{21}$$

The transmitted and reflected fields can be found by solving (20) and (21) simultaneously for a_n and g_n.

DETERMINING THE COEFFICIENTS FOR THE ROTATING SPHERE

In this case, the external forcing will have the form

$$\begin{aligned}\mathbf{H}_f &= -a\nabla\left(\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m} d_m^n (r/a)^{|m|} P_m^n(\cos\theta)e^{in\phi}\right)\\ &= -a\nabla\left(\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m} d_m^n \chi_m^n\right)\end{aligned} \tag{22}$$

with known complex constants d_m^n. The reflected $\mathbf{H}$ field will be

$$\begin{aligned}\mathbf{H}_r &= -a\nabla\left(\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m} g_m^n (a/r)^{|m|+1} P_m^n(\cos\theta)e^{in\phi}\right)\\ &= -a\nabla\left(\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m} g_m^n \psi_m^n\right).\end{aligned} \tag{23}$$

The tangential boundary conditions are

$$\begin{aligned}\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m}\Bigg(a_m^n\frac{1}{\sin\theta}\frac{\partial\xi_m^n}{\partial\phi} &+ b_m^n\frac{1}{k_n a}\frac{\partial^2 r\xi_m^n}{\partial r\partial\theta}\\ &+ g_m^n\frac{\partial\psi_m^n}{\partial\theta} + d_m^n\frac{\partial\chi_m^n}{\partial\theta}\Bigg) = 0\end{aligned} \tag{24}$$

for the θ direction, and

$$\begin{aligned}\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m}\Bigg(-a_m^n\frac{\partial\xi_m^n}{\partial\theta} &+ b_m^n\frac{1}{k_n a\sin\theta}\frac{\partial^2 r\xi_m^n}{\partial r\partial\phi}\\ &+ g_m^n\frac{1}{\sin\theta}\frac{\partial\psi_m^n}{\partial\phi} + d_m^n\frac{1}{\sin\theta}\frac{\partial\chi_m^n}{\partial\phi}\Bigg) = 0\end{aligned} \tag{25}$$

for the ϕ direction, where the functions are evaluated at $r = a$. Taking $\big(\partial(24)/\partial\phi\big)$—$\big(\partial\sin\theta\,(25)/\partial\theta\big)$, we get

$$\sum_{m=1}^{\infty}\sum_{\substack{n=-m\\ n\neq 0}}^{m} a_m^n\left(\frac{1}{\sin\theta}\frac{\partial^2\xi_m^n}{\partial\phi^2} + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\xi_m^n}{\partial\theta}\right)\right) = 0,$$

so $a_m^n = 0$. With $a_m^n = 0$, both (24) and (25) reduce to

$$b_m^n \frac{1}{k_n a} \frac{\partial r \xi_m^n}{\partial r} + g_m^n \psi_m^n + d_m^n \chi_m^n = 0. \tag{26}$$

The radial boundary condition completes the set needed to determine the coefficients. Matching the normal **B** components, we get

$$b_m^n \left(\frac{\mu m(m+1)}{k_n a} \xi_m^n \right) = -g_m^n \left(a\mu_0 \frac{\partial \psi_m^n}{\partial r} \right) - d_m^n a \mu_0 \frac{\partial \chi_m^n}{\partial r}. \tag{27}$$

Expressing the conditions (26) and (27) more explicitly, we have

$$b_m^n \left(\frac{1}{2k_n\sqrt{a}} J_{m+\frac{1}{2}}(k_n a) + \sqrt{a} J'_{m+\frac{1}{2}}(k_n a) \right) + g_m^n a = -a d_m^n \tag{28}$$

$$b_m^n \frac{\mu m(m+1)}{k_n a \sqrt{a}} J_{m+\frac{1}{2}}(k_n a) - g_m^n \mu_0 (m+1) = -\mu_0 m d_m^n \tag{29}$$

and the coefficients b_m^n and g_m^n are found by solving (28) and (29) simultaneously.

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J.A. Stratton, *Electromagnetic Theory,* New York, McGraw-Hill, 1941.

Note : This attachment was written using the TeX typesetting program.

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