

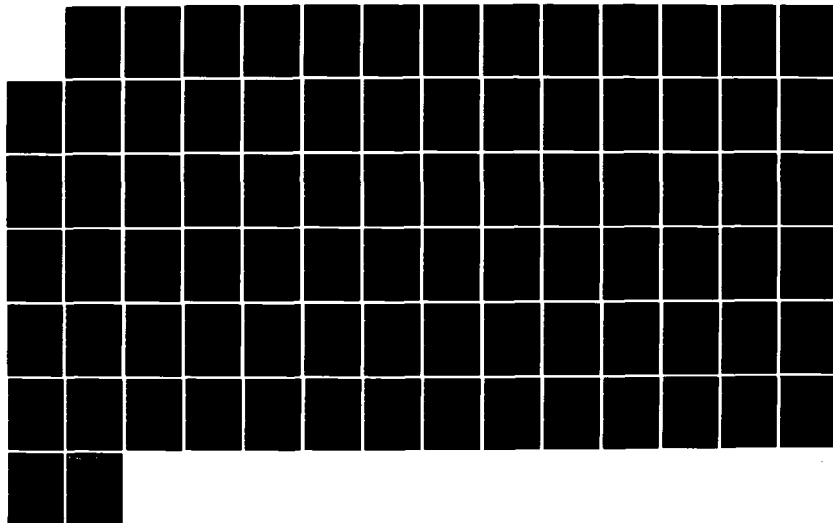
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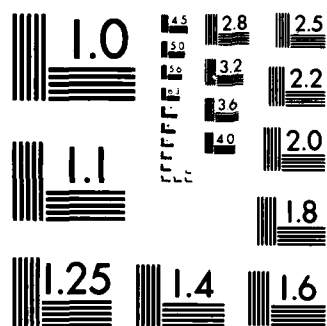
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A SPECIAL BOUNDED SEQUENTIAL PROCEDURE

CPT ROBERT G. GORRIE

22 APRIL 1983

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The number of observations, n , required for defining a course of action is random variable, often characterized by large values of n before a decision is made. Truncation rules are required to prevent unacceptable sample sizes even though convergence is guaranteed. This truncation increases the associated producer's and consumer's risk.

Herein a sequential non-probability ratio (SNPR) plan is examined which is characterized by intersecting lines and thus an implicit truncation property. A heuristic algorithm to define the parameters of the plan is introduced. The algorithm produces an SNPR plan which provides the same levels of protection as the non-truncated SPR plan.

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The Pennsylvania State University
The Graduate School
Department of Industrial and Management Systems Engineering

A Special Bounded Sequential Procedure

A Thesis in
Industrial Engineering and Operations Research

by

Robert G. Gorrie

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

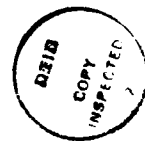
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ABSTRACT

The concept of item by item sequential sampling has been developed from the Wald sequential probability ratio (SPR) test. The SPR plan for one-sided variables inspection (a plan for testing the mean $\bar{X}'$ of a normal distribution with known variance σ'^2) is characterized by two parallel lines on a plot of the cumulative sum of observations $T(n)$ versus accumulated sample size n . The operation of the plan is described by analyzing the sequence $T(n)$. For the case where small values of $\bar{X}'$ are preferred, $T(1) = X_1$ is the first observation. If $T(1)$ falls on or above an upper bound $T_u(1)$, the lot is rejected. If $T(1)$ falls on or below a lower bound $T_l(1)$, the lot is accepted. If $T_l(1) < T(1) < T_u(1)$, another sample is taken. $T(2) = X_1 + X_2$ is then treated in a similar manner, and so on until a course of action is defined.

The number of observations, n , required for defining a course of action is a random variable, often characterized by large values of n before a decision is made. Truncation rules are required to prevent unacceptable sample sizes even though convergence is guaranteed. This truncation increases the associated producer's and consumer's risk.

Herein a sequential non-probability ratio (SNPR) plan is examined which is characterized by intersecting lines and thus an implicit truncation property. A heuristic algorithm to define the parameters of the plan is introduced. The algorithm produces an SNPR plan which provides the same levels of protection as the non-truncated SPR plan.

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CHAPTER I

INTRODUCTION

Problem Statement

Abraham Wald developed a sequential probability ratio (SPR) test in 1943 to test statistical hypotheses. Initially used in sequential acceptance sampling by attributes, it has since been adapted to variables inspection. A unique feature of Wald's SPR test is that the number of observations required by the procedure is not determined in advance of the experiment. Its ability to quickly detect good or bad quality lots makes it superior to other acceptance sampling procedures. It can materially reduce the required amount of inspection by 50% on the average when compared to the single sampling plan.

While it has been shown that the probability equals 1 that the procedure eventually terminates [9], there is no effective upper limit on the number of items to be inspected. If lots are mediocre in quality, the number of units required to be inspected can be quite large. This problem is compensated for through the use of truncation rules which prohibit sampling beyond a certain number of units by forcing a decision at some a-priori fixed integer. If the procedure is truncated prematurely, its ability to discriminate between good and bad lots is weakened. If the procedure is truncated too late, its ability to discriminate is only marginally reduced but it loses its advantage of reducing the number required for inspection.

This study provides, through a procedure proposed by Guild [5], a sequential non-probability ratio (SNPR) test which is bounded for the case of variables inspection. The specific objectives of the

thesis are:

1. Development of a heuristic procedure which specifies the arguments of an SNPR procedure for a given set of parameters.
2. Comparison of SPR and SNPR procedures.
3. Development of a computer code for the heuristic and SNPR procedure.
4. Generation of a select set of SNPR test plans.

Introductory Remarks

The purpose of an acceptance sampling plan is to define a course of action; i.e., acceptance or rejection of a particular lot of given quality. If all lots are of the same quality, the plan will indicate acceptance of some lots and rejection of others, and the accepted lots will be no better than the rejected ones. If the lots differ in quality, the plan will accept good lots more frequently than it will bad lots [1].

Usually a plan is designed so that material considered to be of good quality will have a low probability of being rejected (producer's risk) while material considered to be of bad quality will have a low probability of being accepted (consumer's risk). The grade of material considered good is called the "Acceptable Quality Level" (AQL) and the probability of rejecting material of grade AQL or better is designated by α . Similarly, the grade of material below which rejection takes place is referred to as the "Lot Tolerance Percent Defective" (LTPD) and the probability of accepting material of grade LTPD is designated as β .

The precise relationship between lot quality and the probability of acceptance (P_a) of any given lot is shown by the operating characteristic (OC) curve of the particular sampling plan. The curve graphically illustrates the plan's ability to discriminate between different quality lots with fixed σ' . The pairs $(1-\alpha, \bar{X}'_{AQL})$ and $(\beta, \bar{X}'_{LTPD})$, where $\bar{X}'_{AQL}$ is the mean locating a percent defective equal to a desired AQL and $\bar{X}'_{LTPD}$ is the mean locating a percent defective to a desired lot tolerance LTPD, specify two points on the curve as shown in Figure 1. The point on the curve where P_a equals .5 locates a mean known as the indifference point ($\bar{X}'_a$). At this point, there is no particular preference for acceptance or rejection of the lot ($\alpha=\beta$). The curve shown in Figure 1 describes a plan where low values of $\bar{X}'$ are desirable. Besides showing lot quality relationships, the OC curve is also useful in evaluating alternate sampling plans.

The Chapters to Follow

Chapter II briefly examines a single sampling plan for variables inspection and then presents the development of Wald's SPR test for variables inspection. Also discussed are the methods used to calculate the OC curve and average sample number (ASN) curve of the SPR plan. Finally various truncation rules and their effect on the plan's discriminatory power are presented. The SNPR procedure and a heuristic method of estimating the procedures arguments are presented in Chapter III followed by a comparison of SPR and SNPR procedures in Chapter IV.

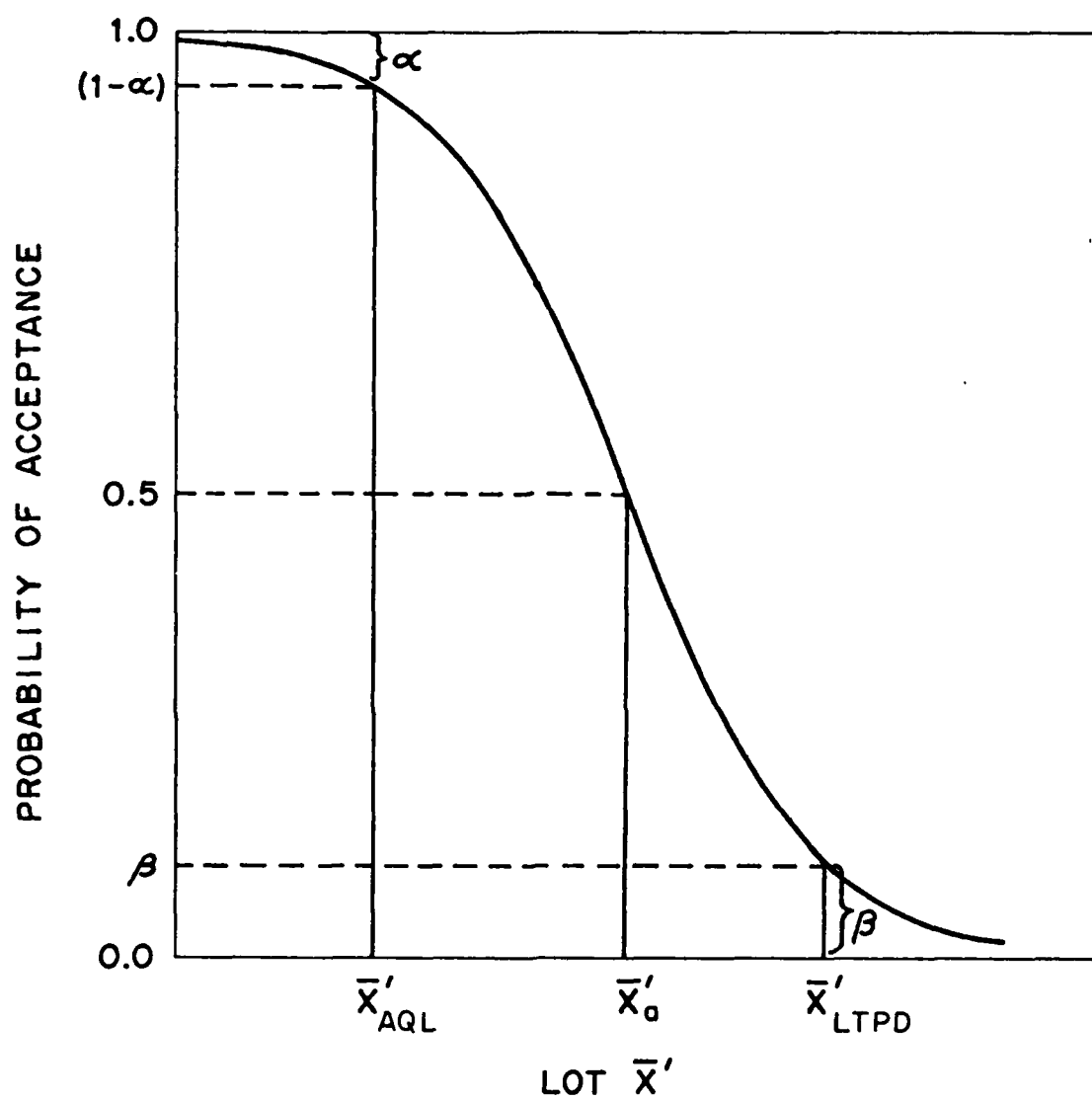


Figure 1. Operating Characteristic Curve

Throughout the thesis a sample problem is used to illustrate procedures and concepts. The problem requires the development of an acceptance sampling plan for variables inspection in which low values of $\bar{X}'$ are desirable. The following are the problem parameters:

$$\bar{X}'_{AQL} = 200 \qquad \alpha = .05 \qquad \sigma' = 30$$

$$\bar{X}'_{LTPD} = 225 \qquad \beta = .10$$

CHAPTER II

HISTORICAL PERSPECTIVE

Single Sampling Plan

A single sampling plan for variables inspection is relatively simple to design. Given $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, α , β and σ' , a sample size N and an acceptance limit $\bar{X}_a$ are determined to establish the plan. The derivation of the plan is as follows. If a lot actually has a mean grade of AQL quality, means of samples of N from the lot will have a normal distribution with mean $\bar{X}'_{AQL}$ and a standard deviation of $\sigma'/\sqrt{N}$. Therefore, if α is the probability of rejecting a lot with mean $\bar{X}'_{AQL}$, then

$$\frac{\bar{X}_a - \bar{X}'_{AQL}}{\sigma'/\sqrt{N}} = Z_{\alpha} \quad (1)$$

Similarly, a lot having a mean grade of $\bar{X}'_{LTPD}$ and a probability acceptance β requires

$$\frac{\bar{X}_a - \bar{X}'_{LTPD}}{\sigma'/\sqrt{N}} = -Z_{\beta} \quad (2)$$

When $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, α , β and σ' are defined, we have two equations in $\bar{X}_a$ and N . The simultaneous solution of which yields values of $\bar{X}_a$ and N [1].

Using the problem defined in Chapter I, $\bar{X}_a$ is computed to be 214.05 and N to be 13. Figure 2 shows the OC curve for the plan.

Sequential Probability Ratio Sampling Plan

The SPR plan for variables [1] is one which requires the sequential observation of a variable X which is normally distributed with mean $\bar{X}'$ and standard deviation σ' . The decision variable is defined as:

$$T(n) = \sum_{i=1}^n X_i \quad (3)$$

where n is the number of observations. The operation of the plan is described by analyzing the sequence $\{T(n)\}$. Consider the point $T(1) = X_1$ where X_1 is the first sample observation. If $T(1)$ falls on or above an upper bound $T_u(1)$ and low values of X are desirable, the lot is rejected. If $T(1)$ falls on or below a lower bound $T_l(1)$, the lot is accepted. If, however, $T_l(1) < T(1) < T_u(1)$, another observation is taken. Then $T(2) = X_1 + X_2$ is evaluated in a similar manner, and so on until a final course of action can be determined.

The decision variable $T(n)$ used for testing the mean of a normal distribution with σ'^2 known is derived from Wald's sequential probability ratio test [9]. The test is given as follows. Two points (θ_0, α) and (θ_1, β) are selected where α is the probability of rejecting $\bar{X}' = \theta_0$ and β is the probability of accepting $\bar{X}' = \theta_1$. The SPR of the random sample $(X_1, X_2, \dots, X_n)$ from the normal distribution is

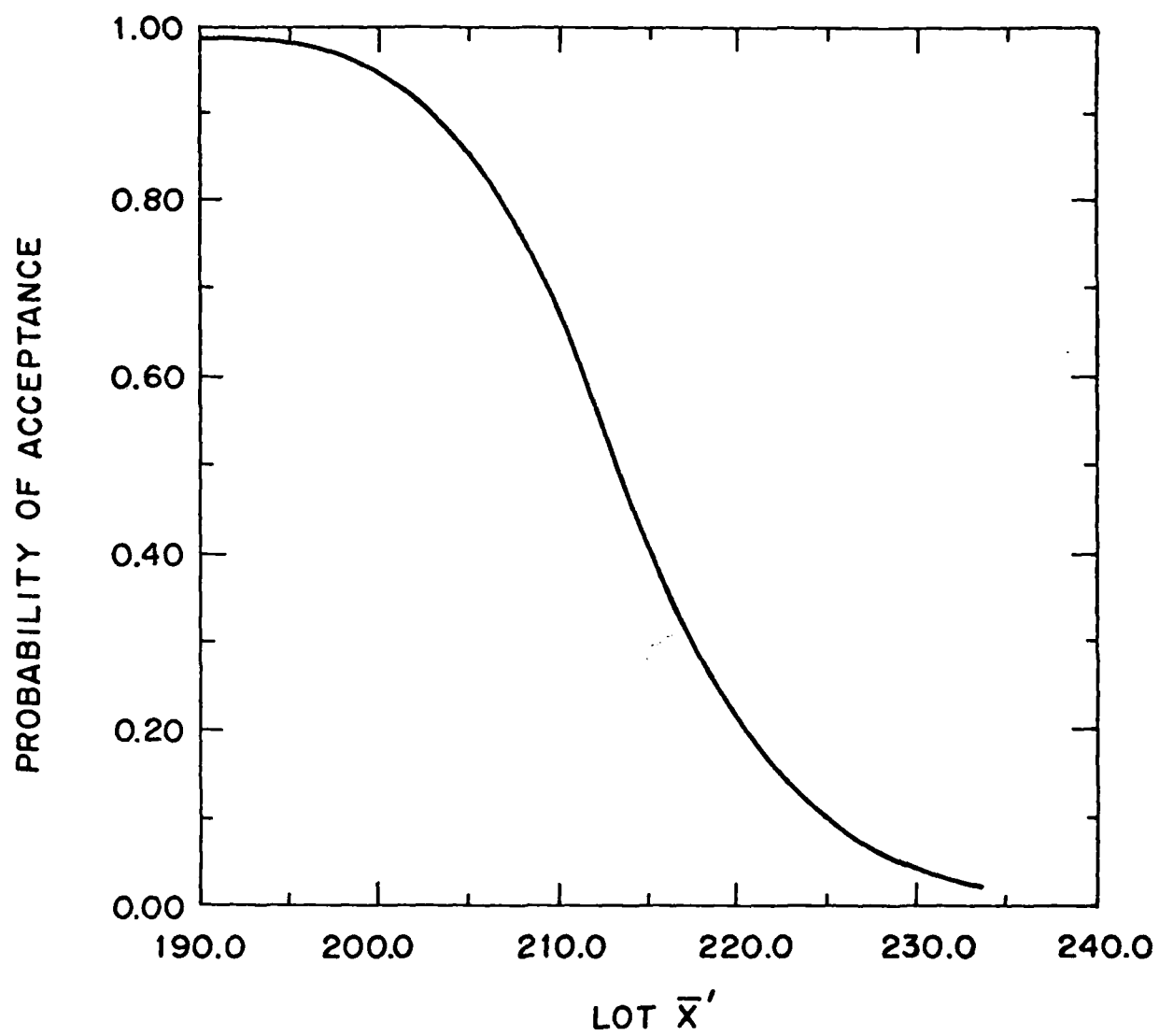


Figure 2. OC Curve: Single Sampling Plan (Sample Problem)

defined as

$$SPR = \frac{\frac{1}{(2\pi)^{n/2} \sigma^{,n}} e^{-\frac{1}{2\sigma^{,2}} \sum_{i=1}^n (X_i - \theta_1)^2}}{\frac{1}{(2\pi)^{n/2} \sigma^{,n}} e^{-\frac{1}{2\sigma^{,2}} \sum_{i=1}^n (X_i - \theta_0)^2}} \quad (4)$$

In theory, the ratio is computed at each stage of the inspection and additional observations are taken as long as

$$B < SPR < A \quad (5)$$

where B and A are appropriately defined constants. Inspection is terminated with acceptance of the lot if

$$SPR \leq B \quad (6)$$

Inspection is terminated with rejection of the lot if

$$SPR \geq A \quad (7)$$

Selecting $A = (1-\beta)/\alpha$, $B = \beta/(1-\alpha)$, taking logarithms of the inequalities and simplifying, (5), (6) and (7) become

$$\frac{\sigma^{,2}}{(\theta_1 - \theta_0)} \log \frac{\beta}{1-\alpha} + n \frac{(\theta_0 + \theta_1)}{2} < T(n) = \sum_{i=1}^n X_i < \frac{\sigma^{,2}}{(\theta_1 - \theta_0)} \log \frac{1-\beta}{\alpha} + n \frac{(\theta_0 + \theta_1)}{2} \quad (8)$$

$$T(n) = \sum_{i=1}^n x_i - \frac{\sigma'^2}{\theta_1 - \theta_0} \log \frac{\beta}{1-\alpha} + n \frac{\theta_0 + \theta_1}{2} \quad (9)$$

$$T(n) = \sum_{i=1}^n x_i \geq \frac{\sigma'^2}{\theta_1 - \theta_0} \log \frac{1-\beta}{\alpha} + n \frac{\theta_0 + \theta_1}{2} \quad (10)$$

respectively. The decision variable therefore becomes $T(n)$ rather than the SPR.

By using $\bar{X}'_{AQL} = \theta_0$ and $\bar{X}'_{LTPD} = \theta_1$, the limits for the plan from (9) and (10) become

$$T_u(n) = h_1 + S n \quad (11)$$

$$T_l(n) = h_0 + S n \quad (12)$$

where the common slope formed by the limit lines is given by

$$S = \frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD}}{2} \quad (13)$$

and the intercept of the lines when $n=0$ is given by

$$h_0 = \frac{\sigma'^2 \log B}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \quad (14)$$

and

$$h_1 = \frac{\sigma'^2 \log A}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \quad (15)$$

The SPR OC and ASN Curves

Wald [9] shows that on an OC curve for $\bar{X}' = -\infty$, $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, $(\bar{X}'_{AQL} + \bar{X}'_{LTPD})/2$, and $+\infty$, the values of $L(X')$ (the P_a of a lot whose mean equals $\bar{X}'$) are as follows:

$$L(-\infty) = 1 \quad L(\bar{X}'_{AQL}) = 1-\alpha$$

$$L\left(\frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD}}{2}\right) = \frac{\log A}{\log A - \log B} \quad (16)$$

$$L(\bar{X}'_{LTPD}) = \beta \quad L(+\infty) = 0$$

An approximate general equation is given by

$$L(\bar{X}') \sim \frac{A^h - 1}{A^h - B^h} \quad (17)$$

where

$$h = \frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD} - 2\bar{X}'}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \quad (18)$$

Wald also derived an approximate formula for the ASN curve which is given as

$$E_{\bar{X}'}(n) = \frac{h_1 + L(\bar{X}') (h_0 - h_1)}{\bar{X}' - s} \quad (19)$$

Sample Illustration

Using the values defined in Chapter I, the number of observations and acceptance and rejection limits are tabulated in Table 1 along with a sample set of observations. The same test plan is graphically illustrated in Figure 3. The corresponding OC and ASN curves are shown by Figure 4 and Figure 5 respectively. As indicated, the plan would terminate with acceptance of the lot at $n = 8$.

Truncation of the SPR Plan

As mentioned previously, the main disadvantage of the SPR plan is that there is no effective upper limit to the number of items that are required for inspection before a decision to accept or reject a lot can be made. The sample size of an SPR plan is a random variable whose value will occasionally be quite large. Truncation of the procedure is used to force a decision prior to or at $n = n'$ where n' is an a-priori fixed integer. The following is a common rule for accepting or rejecting a particular lot if a decision has not been reached for $n \leq n'$ with the regular sequential procedure. If $T(n') \geq (T_L(n') + T_U(n'))/2$ the lot is rejected, and if $T(n') < (T_L(n') + T_U(n'))/2$ the lot is accepted.

Several rules of thumb have been suggested for establishing an effective n' . Guild [5] uses n' equal to twice the maximum ASN at $\bar{X}'_{AQL}$ or $\bar{X}'_{LTPD}$. Grant and Leavenworth [4] set n' equal to three times N ; the number of samples required by a single sampling plan with the

Table 1. Acceptance and Rejection Limits of Sample Problem SPR Plan With Sample Observations

n	X(n)	T(n)	$T_L(n)$	$T_U(n)$	Decision
1	201	201	131.45	316.55	SAMPLE
2	206	407	343.95	529.05	SAMPLE
3	199	606	556.45	741.55	SAMPLE
4	204	810	768.95	954.05	SAMPLE
5	203	1013	981.45	1166.55	SAMPLE
6	197	1210	1193.95	1379.05	SAMPLE
7	199	1409	1406.45	1591.55	SAMPLE
8	203	1612	1618.95	1804.05	ACCEPT
9			1831.45	2016.55	
10			2043.95	2229.05	
11			2256.45	2441.55	
12			2468.95	2654.05	
13			2681.45	2866.55	
14			2893.95	3079.05	
15			3106.45	3291.55	
16			3318.95	3504.05	
17			3531.45	3716.55	
18			3743.95	3929.05	
19			3956.45	4141.55	
20			4168.95	4354.05	
.			.	.	
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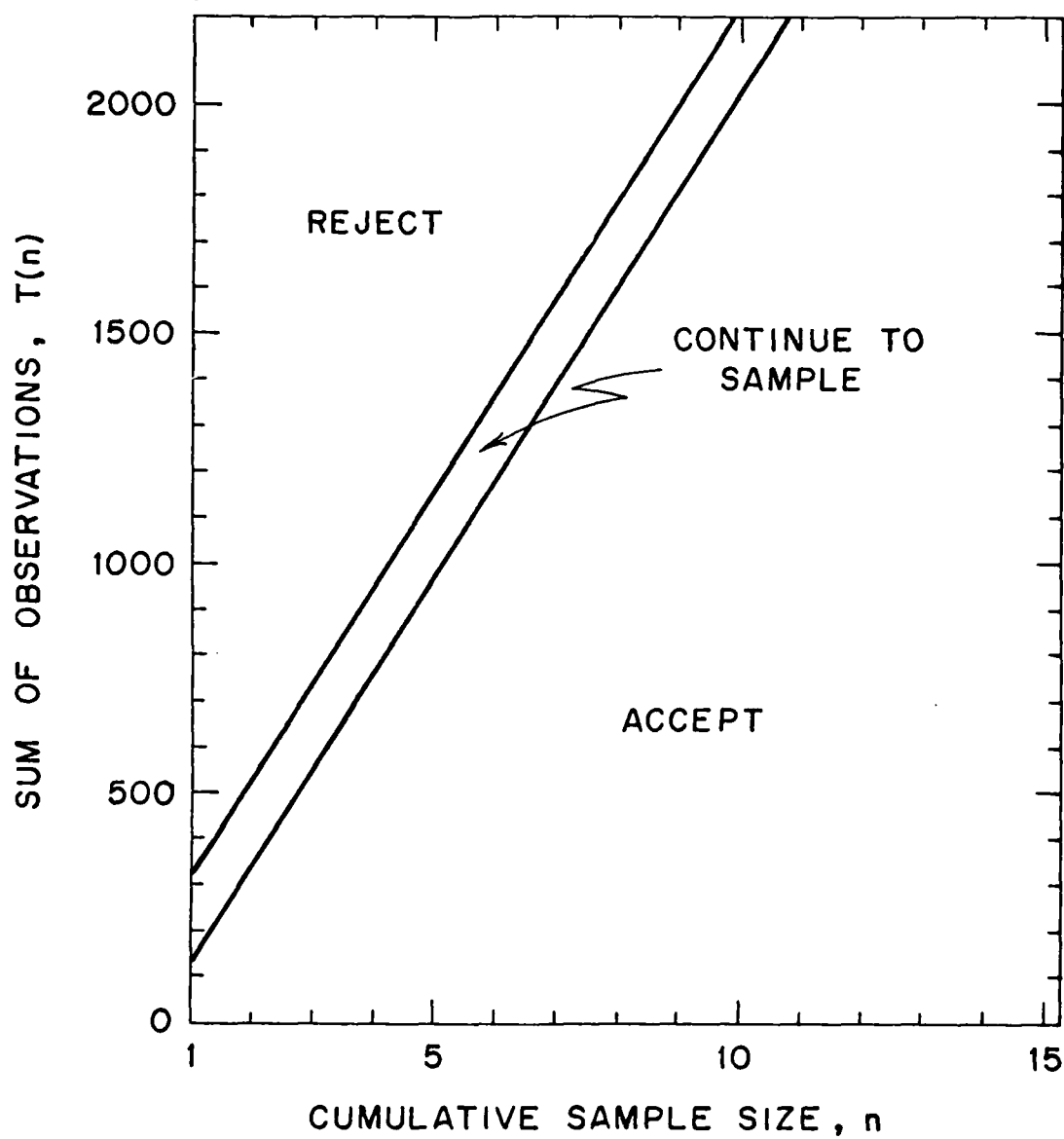


Figure 3. Plot of Sample Problem SPR Plan with Observations

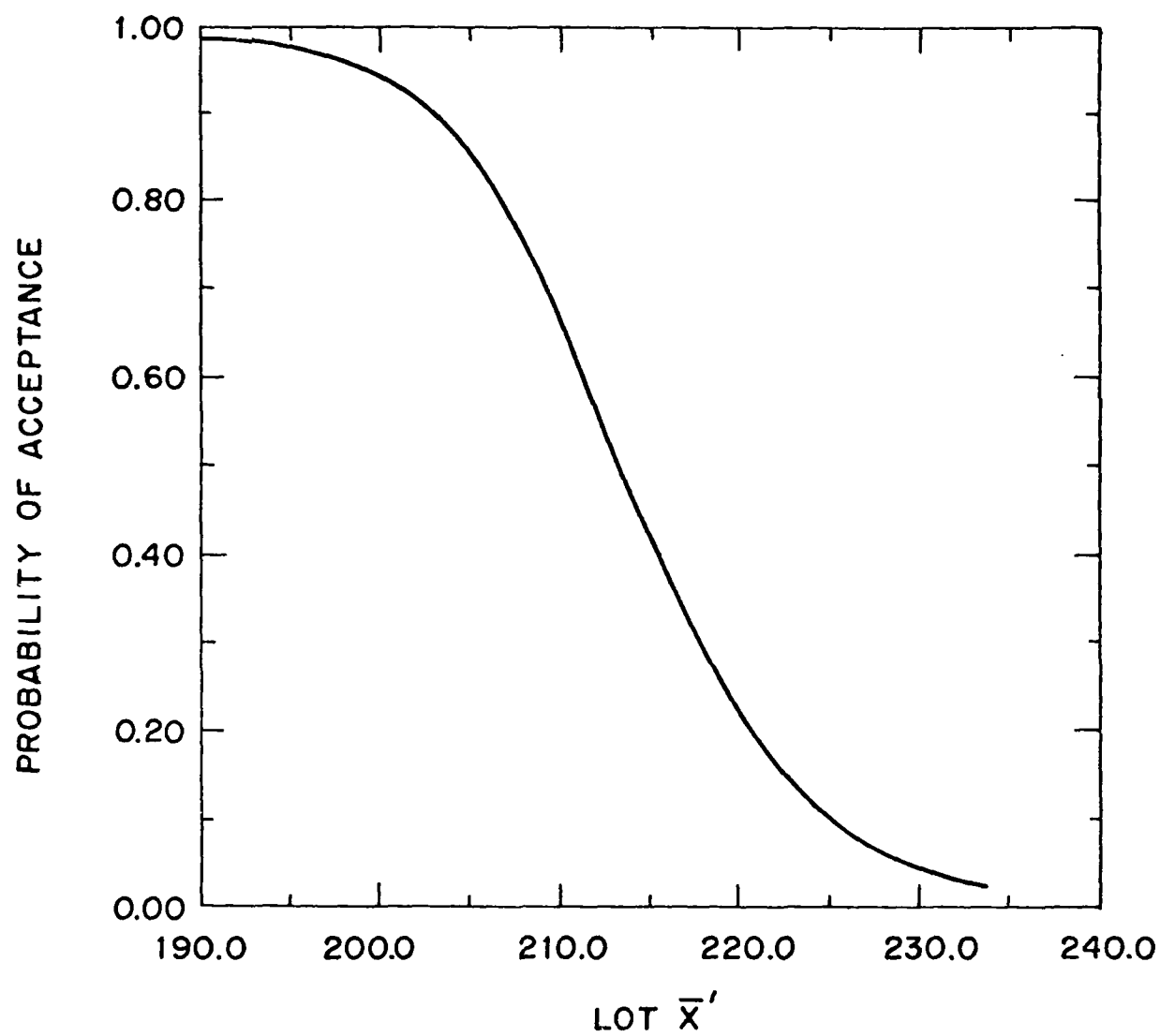


Figure 4. OC Curve: SPR Plan (Sample Problem)

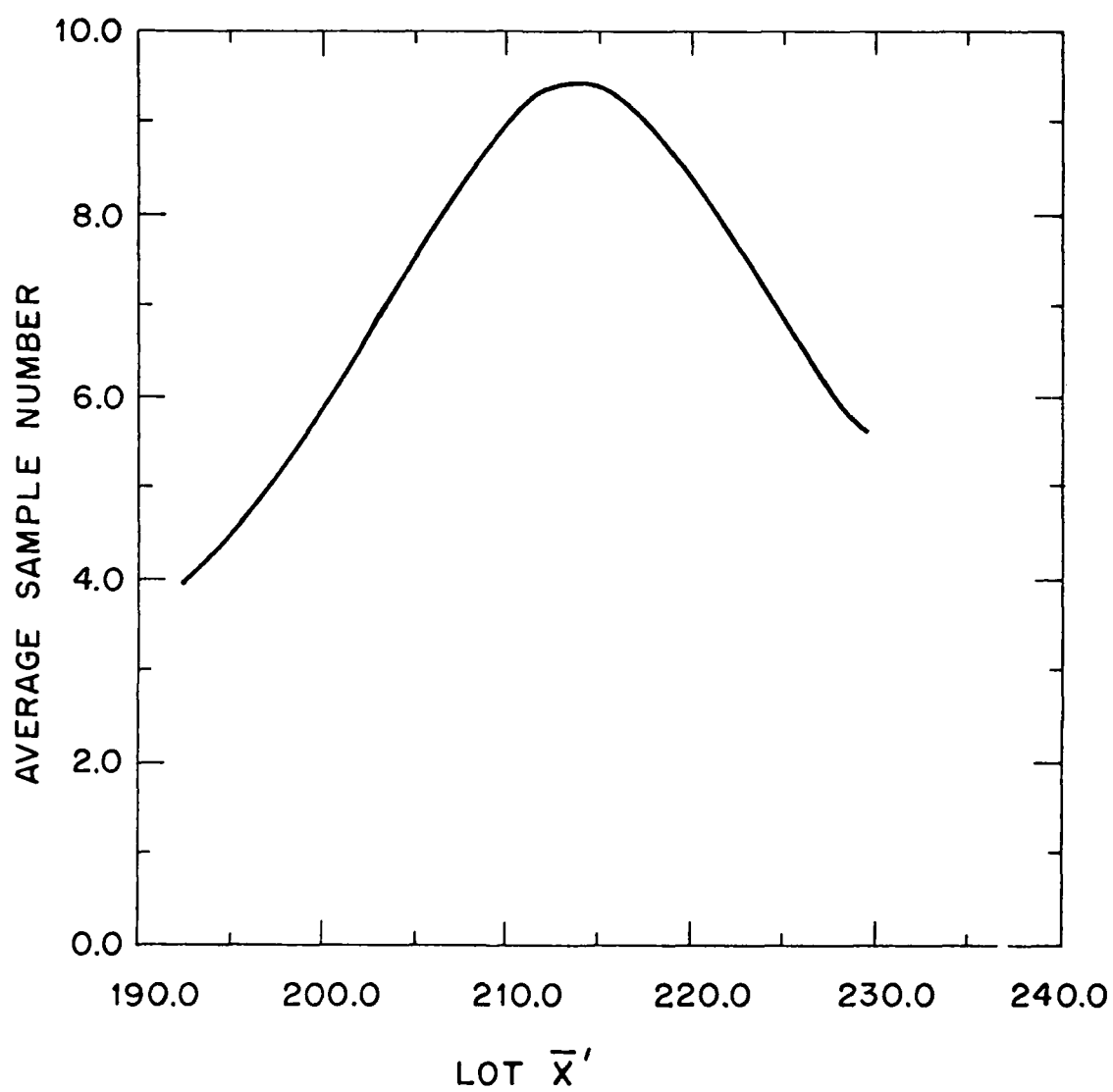


Figure 5. ASN Curve: SPR Plan (Sample Problem)

same level of protection. Wald [9] indicates that with n' put as high as three times the maximum expected value as indicated by the ASN curve, the effect of truncation on the OC curve is negligibly small since the probability is nearly one that the regular sequential procedure will terminate for $n < n'$. For the Wald method of truncation, the value of n' to be used in the sample problem is calculated to be 30. Compared to 13, the number of samples required by the single sampling plan to yield the same level of protection, 30 seems relatively large and possibly prohibitively expensive if the cost of inspection is high.

Table 2 is extracted from Wald's "Sequential Analysis" [9] and illustrates the effect of truncation on the plan for different values of α and β . If the plan is based on the values of α and β shown, but a decision is made at n' even when the regular sequential plan requires a continuation of the process, the realized values $\alpha(n')$ and $\beta(n')$ will not exceed the tabular entries. The table relates to the test of the mean of a normally distributed variate, the difference between the null and alternate hypothesis adjusted for each pair (α, β) so that the number of trials required by the single sampling plan of strength (α, β) is 1000. Notice that as α and β are relaxed, the values of $\alpha(n')$ and $\beta(n')$, even when n' is increased to twice N , vary by 15% when $\alpha = .05$ and $\beta = .05$. Wald's opinion is that the upper limits given in the table are considerably above the true values of $\alpha(n')$ and $\beta(n')$ when n' is not much higher than N . However, any difference between the pairs (α, β) and $(\alpha(n'), \beta(n'))$ may be intolerable. A more reasonable

Table 2. Effect on Risks of Error of Truncating a
Sequential Analysis at a Predetermined
Number of Trials

Number of Trials	$\alpha = .01$ and $\beta = .01$		$\alpha = .01$ and $\beta = .05$		$\alpha = .05$ and $\beta = .05$	
	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$
1000	.020	.020	.033	.070	.095	.095
1200	.015	.015	.024	.063	.082	.082
1400	.013	.013	.019	.058	.072	.072
1600	.012	.011	.016	.055	.066	.066
1800	.011	.010	.014	.053	.062	.062
2000	.010	.010	.012	.052	.058	.058
2200	.010	.010	.012	.051	.056	.056
2400	.010	.010	.011	.051	.055	.055
2600	.010	.010	.011	.051	.053	.053
2800	.010	.010	.010	.050	.053	.053
3000	.010	.010	.010	.050	.052	.052

plan would be one which guarantees the same level of protection with a commensurate savings in required sample size.

CHAPTER III

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

The SNPR plan is an acceptance sampling procedure bounded by two lines which intersect at $n = n'$, where n' is an a-priori fixed integer. Although the procedures for SPR and SNPR plans are similar, the SNPR plan requires no truncation rules. As mentioned in the previous chapter, truncation of the SPR plan makes the OC curve of the plan less discriminating. Therefore, the implicit property of the SNPR plan may be desirable if it can provide the same level of protection as the non-truncated SPR plan.

The General Approach

For the SNPR plan, $T_\ell(n)$ and $T_u(n)$ are constructed which converge at some maximum number of units, n' . The decision variable for the plan is the same as that defined in equation (1) for the SPR plan. Similarly, the operation of the plan is the same. If no decision is made prior to sampling the n' th unit, the lot is rejected if $T(n') > n'\bar{X}'$ and accepted if $T(n') \leq n'\bar{X}'$. The plan is graphically illustrated in Figure 6.

$T_u(1)$ and $T_\ell(1)$ are determined from

$$T_u(1) = \bar{X}' + Z_\alpha \sigma' \quad (20)$$

$$T_\ell(1) = \bar{X}' - Z_\alpha \sigma' \quad (21)$$

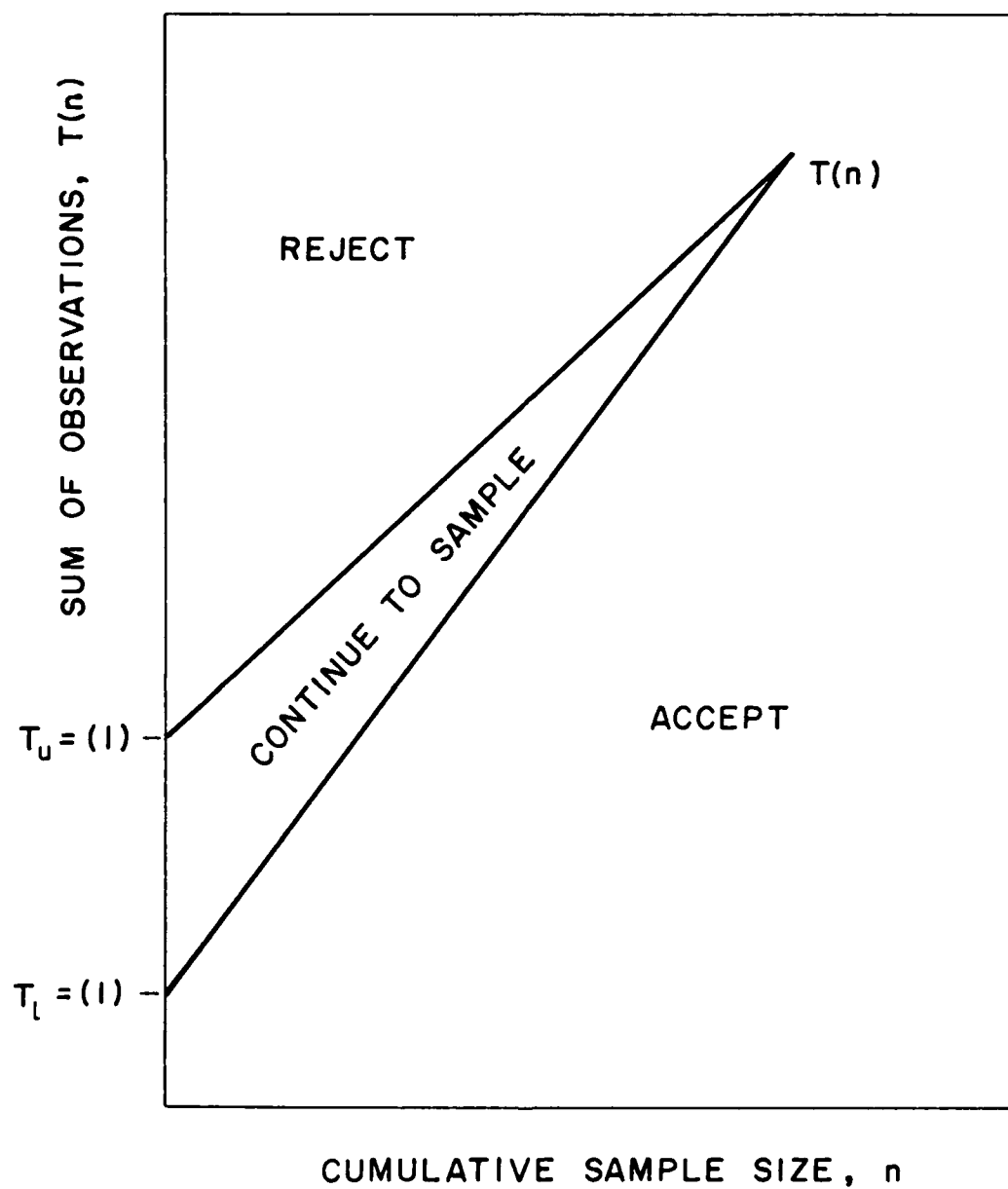


Figure 6. Plot of an SNPR Plan

so that the probability of acceptance (P_a) and the probability of rejection (P_r) are both equal to α' for X' at $n = 1$. At $n = n'$

$$T_u(n') = T_l(n') = n'\bar{X}' \quad (22)$$

The convergence by the limits on $n'\bar{X}'$ indicates a step by step reduction, d , in the size of the interval $Z_{\alpha, \sigma'}$. This is shown by

$$d = Z_{\alpha, \sigma'} / (n' - 1), \quad n' > 0 \quad (23)$$

Therefore, the calculation for the upper limit becomes

$$T_u(n) = n\bar{X}' + Z_{\alpha, \sigma'} - (n-1) Z_{\alpha, \sigma'} / (n' - 1) \quad (24)$$

or

$$T_u(n) = n\bar{X}' + (n' - n) Z_{\alpha, \sigma'} / (n' - 1), \quad 1 < n < n'$$

Similarly,

$$T_l(n) = n\bar{X}' - (n' - n) Z_{\alpha, \sigma'} / (n' - 1), \quad 1 < n < n' \quad (25)$$

$\bar{X}'$ and $Z_{\alpha, \sigma'}$ for the SNPR plan must now be established so that limits which provide the same level of protection as the SPR plan may be calculated.

Sample Illustration

Using the example, if $T_u(1)$ and $T_l(1)$ are arbitrarily set to the same values as in the SPR plan, $\bar{X}'$ and $Z_{\alpha, \sigma'}$ from equations (20) and (21) become

$$\bar{X}' = \frac{T_u(1) + T_l(1)}{2} = \frac{316.55 + 131.46}{2} = 224.01$$

$$Z_{\alpha'} = \frac{316.55 - 224.01}{2} = 3.0840, \alpha' = .0010$$

Table 3 shows the proposed plan for $n' = 10$ (10 is the maximum ASN of the SPR plan) while Figure 7 shows the plot.

α and β for the SNPR plan are calculated in the following manner.. Since $T(n) = \sum_{i=1}^n X_i$ and X is $N(\bar{X}', \sigma'^2)$, $T(n)$ is $N(n\bar{X}', n\sigma'^2)$ and $\sigma(T(n)) = \sqrt{n}\sigma'$. Therefore, the normal deviate

$$Z = (T_u(n) - n\bar{X}')/\sqrt{n}\sigma'$$

can be used to solve for α . Table 4 lists the results of calculations.

Using the equation

$$\alpha = \alpha_1 + (1-\alpha_1-\beta_1)\alpha_2 + (1-\alpha_1-\beta_1)(1-\alpha_2-\beta_2)\alpha_3 + \dots + \\ \dots (1-\alpha_9-\beta_9)\alpha_{10}$$

where $\alpha_k = P(Z > Z_{\alpha_n} | \bar{X}' = \bar{X}'_{AQL})$ and $\beta_k = P(Z < Z_{\beta_n} | \bar{X}' = \bar{X}'_{LTPD})$, α is calculated to be .005. Similarly β is calculated to be .450. Figure 8 shows the OC curves for the corresponding SPR and SNPR plans.

The arbitrary values of $\bar{X}'$ and $Z_{\alpha'}$, chosen for the SNPR plan yield an OC curve which is obviously less discriminating than that of the SPR plan. However, altering the values of $\bar{X}'$, $Z_{\alpha'}$, and n' changes the values of α and β . It will be shown that properly chosen values of $\bar{X}'$, $Z_{\alpha'}$, and n' will yield an OC curve for the SNPR plan which discriminates as well as the OC curve of the SPR plan.

Table 3. Acceptance and Rejection Limits of Sample
 Problem SNPR Plan ($\bar{X}' = 224.01$, $Z_{\alpha'} = 3.0840$
 and $n' = 10$) With Sample Observations

n	$\bar{X}(n)$	T(n)	$T_L(n)$	$T_U(n)$	Decision
1	201	201	131.49	316.53	SAMPLE
2	206	407	365.78	530.26	SAMPLE
3	199	606	600.07	743.99	SAMPLE
4	204	810	834.36	957.72	ACCEPT
5			1068.65	1171.45	
6			1302.94	1385.18	
7			1537.23	1598.91	
8			1771.52	1812.64	
9			2005.81	2026.37	
10			2240.10	2240.10	

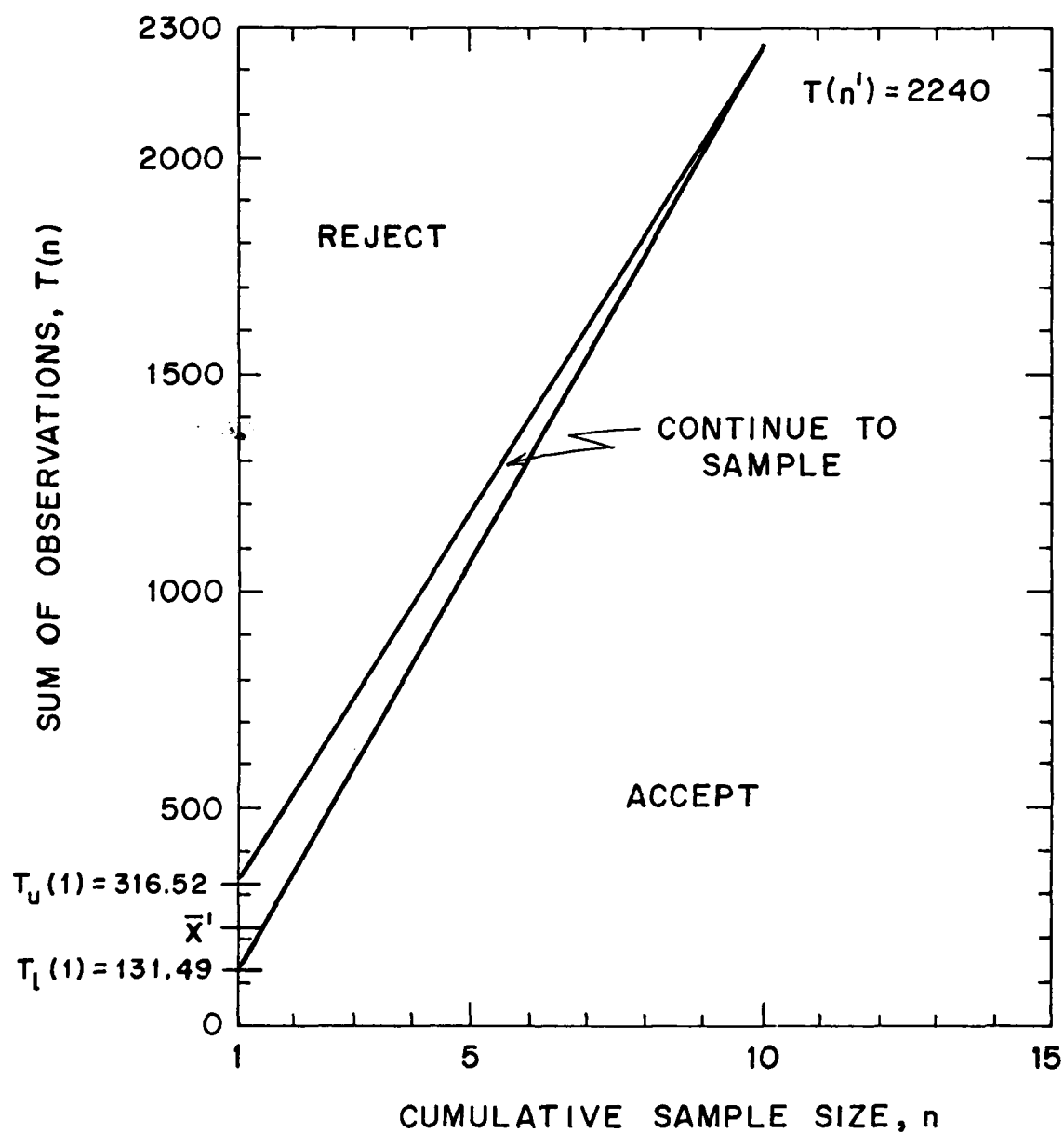


Figure 7. Plot of Sample Problem SNPR Plan ($\bar{X}' = 224.01$, $Z_{\alpha'} = 3.0840$ and $n' = 10$)

Table 4. Calculations Used to Solve for α and β in Sample Problem SNPR Plan
 $(\bar{X}' = 224.01, Z_{\alpha}' = 3.0840 \text{ and } n' = 10$

				Calculations for α						Calculations for β					
n	$\sqrt{nc'}$	$T_L(n)$	$T_U(n)$	$n\bar{x}'$	$Z_{\alpha n}$	α_n	$Z_{\beta n}$	β_n	$1-\alpha-\beta$ n	$n\bar{x}'$	$Z_{\alpha n}$	α_n	$Z_{\beta n}$	β_n	$1-\alpha-\beta$ n
1	30	131.49	316.53	200	3.88	.0000+	-2.28	.0112	.9887	225	3.05	.0011	-3.12	.0009	.9979
2	42.43	365.78	530.25	400	3.07	.0010	-.80	.2099	.7801	450	1.89	.0292	-1.98	.0236	.9979
3	51.96	600.07	743.98	600	2.77	.0027	.00+	.5005	.3874	675	1.33	.0921	-1.44	.0746	.7876
4	60.00	834.36	957.72	800	2.63	.0042	.57	.7165	.1081	900	.96	.1680	-1.09	.1370	.5474
5	67.08	1068.65	1171.45	1000	2.55	.0052	1.02	.8469	.0160	1125	.69	.2443	-.84	.2004	.3039
6	73.48	1302.94	1385.18	1200	2.52	.0058	1.40	.9194	.0012	1350	.49	.3160	-.64	.2609	.1285
7	79.37	1537.23	1598.91	1400	2.50	.0061	1.72	.9580	0.0	1575	.30	.3816	-.47	.3171	.0387
8	84.85	1771.52	1812.64	1600	2.50	.0061	2.02	.9783	0.0	1800	.15	.4408	-.33	.3686	.0074
9	90.00	2005.81	2026.37	1800	2.51	.0059	2.29	.9889	0.0	2025	.01	.4939	-.21	.4156	.0006
10	94.86	2240.10	2240.10	2000	2.53	.0056	2.53	.9943	0.0	2250	-.10	.5415	-.10	.4584	0.0

$$\begin{aligned}\alpha &= \alpha_1 + (1-\alpha_1-\beta_1)\alpha_2 + (1-\alpha_1-\beta_1)(1-\alpha_2-\beta_2)\alpha_3 + \dots + (1-\alpha_9-\beta_9)\alpha_{10} \\ &= .0000 + (.9887)(.0010) + (.9887)(.7801)(.0027) + \dots + (0) .0056 \\ &= .0056\end{aligned}$$

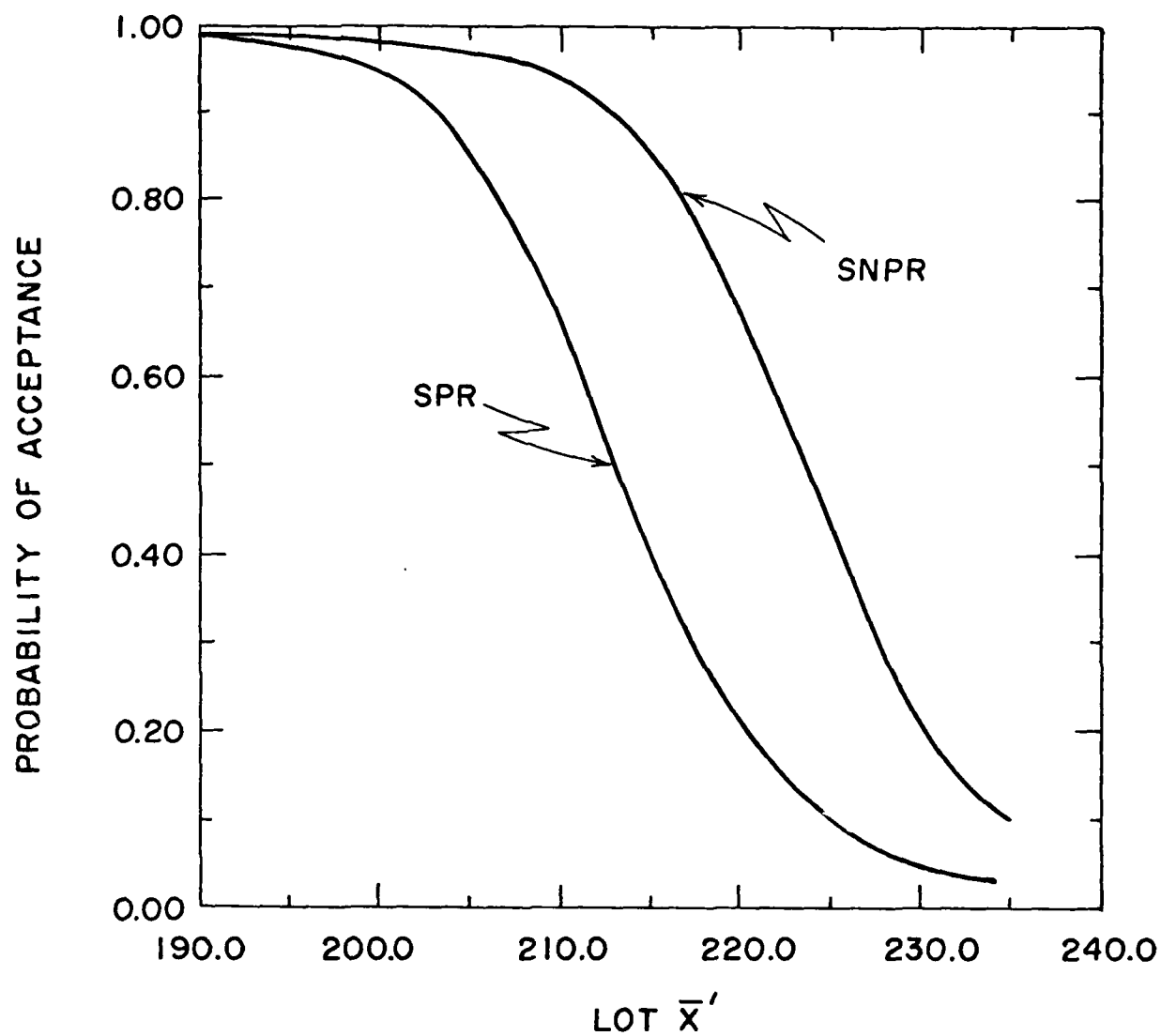


Figure 8. OC Curves: SNPR Plan (Sample Problem; $\bar{X}' = 224.01$, $Z_{\alpha'} = 3.0840$ and $n' = 10$) and SPR Plan (Sample Problem)

Preliminary Analysis

Holding n' and Z_α , constant, $\bar{X}'$ is altered to examine the effect on α and β . Table 5 lists the results. It is observed that as $\bar{X}'$ decreases, α increases and β decreases. This is intuitively reasonable because altering $\bar{X}'$ relocates the envelope formed by the acceptance and rejection limits. A downward relocation of the envelope, caused by decreasing $\bar{X}'$, increases the rejection region. If the rejection region is increased, the probability of rejecting acceptable material is increased, therefore, α is increased. Similarly β decreases as $\bar{X}'$ decreases. This is graphically illustrated in Figure 9. OC curves for the same plans are compared to the SPR plan's OC curve in Figure 10. A strong similarity exists between the OC curves of the SPR plan and the SNPR plan with $\bar{X}' = 214.05$. This particular value of $\bar{X}'$ has not been chosen arbitrarily. It is the value of $\bar{X}_a$, the acceptance limit for the single sampling plan discussed in Chapter II. $\bar{X}_a$ will be used in the development of a heuristic procedure which establishes $\bar{X}'$ for an SNPR plan equivalent to the SPR plan.

Varying Z_α , and holding $\bar{X}'$ and n' constant effects the values of α and β as shown in Table 6. As Z_α increases, α and β both decrease. Again this is intuitively reasonable because increasing Z_α widens the envelope formed by the acceptance and rejection limits. The widening decreases both the acceptance and rejection regions, as shown in Figure 11, thereby decreasing both α and β . Figure 12 shows the corresponding OC curves. Again a similarity exists between the OC

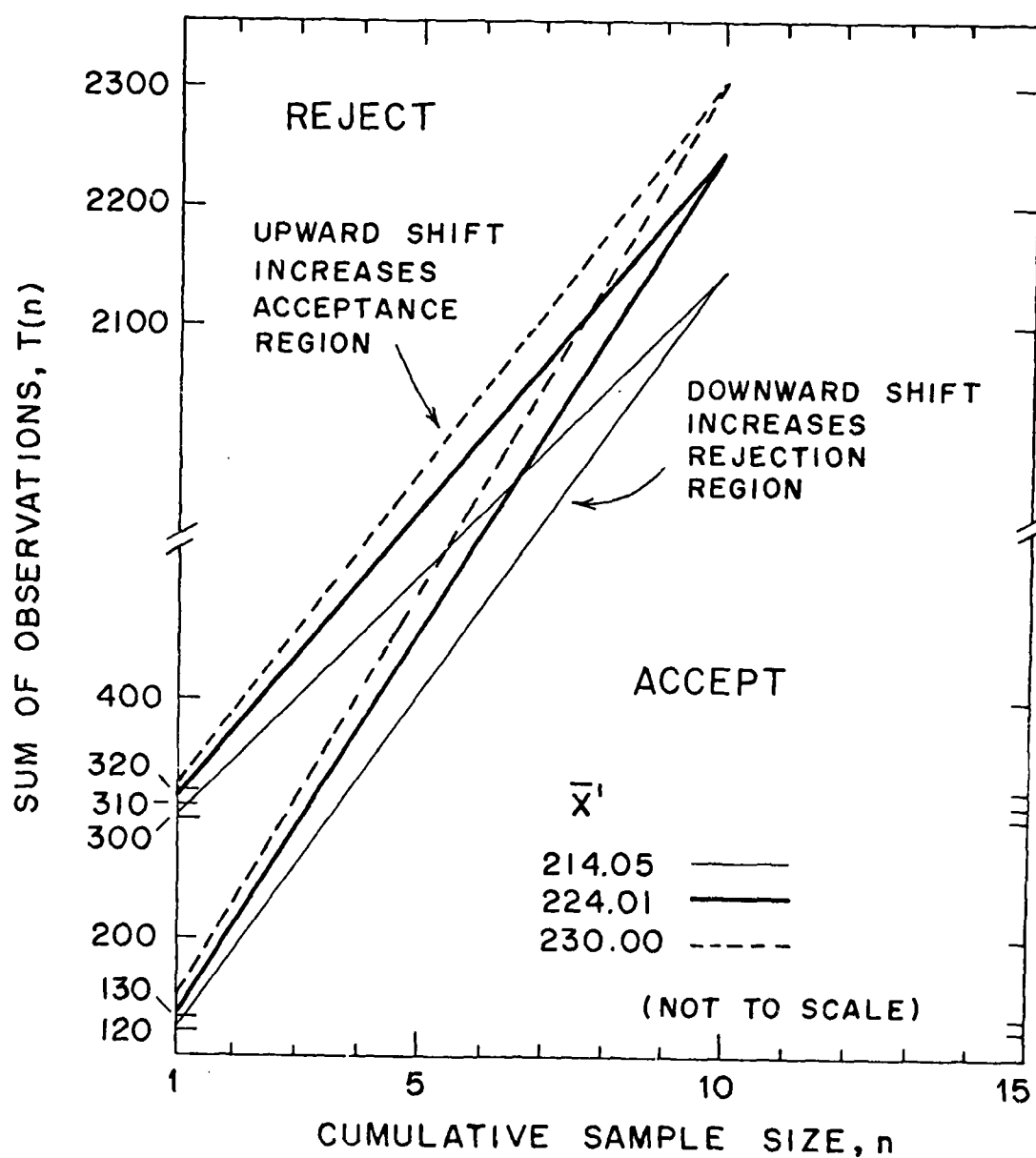


Figure 9. Plot of Sample Problem SNPR Plans ($Z_{\alpha_1} = 3.0840$, $n' = 10$ and $\bar{X}' = 214.05, 224.01$ and 230.00)

Table 5. Effect on α and β of Altering $\bar{X}'$ (Sample Problem SNPR Plan: $Z_{\alpha} = 3.0840$, $n' = 10$)

$\bar{X}'$	$T_L(1)$	$T_U(1)$	$T(n')$	α	β
214.05	121.53	306.57	2140.50	.001	.734
224.01	131.49	316.53	2240.09	.005	.450
230.00	137.48	322.52	2300.00	.050	.094

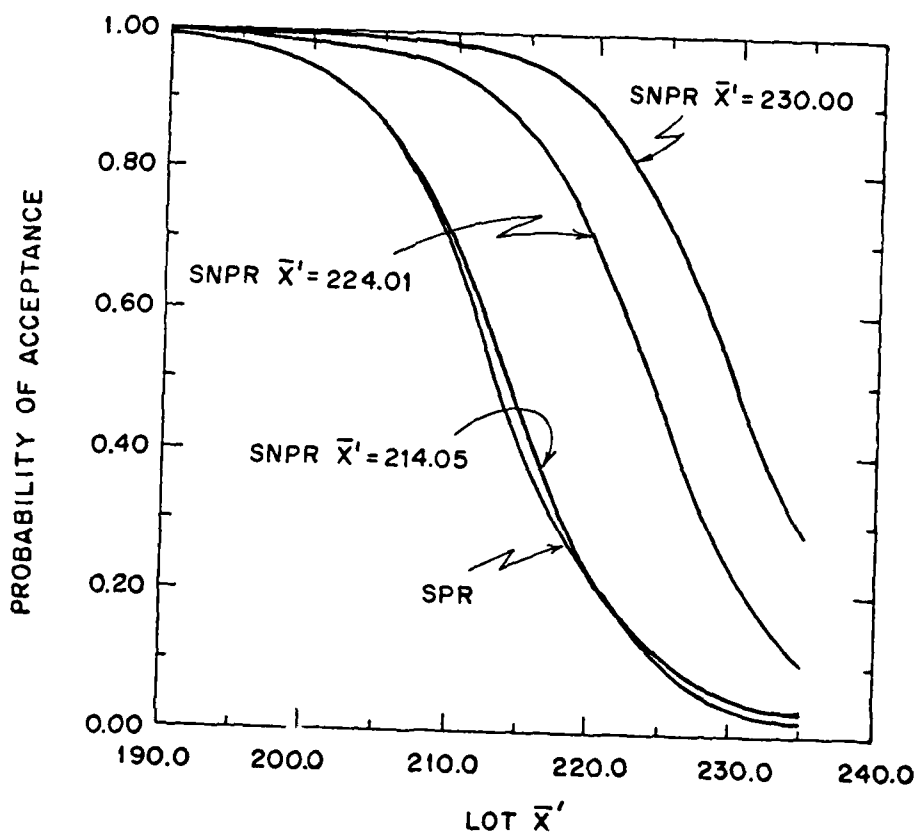


Figure 10. OC Curves: SNPR Plan (Sample Problem; $Z_{\alpha} = 3.0840$, $n' = 10$ and $\bar{X}' = 214.05$, 224.01 and 230.00) and SPR Plan (Sample Problem)

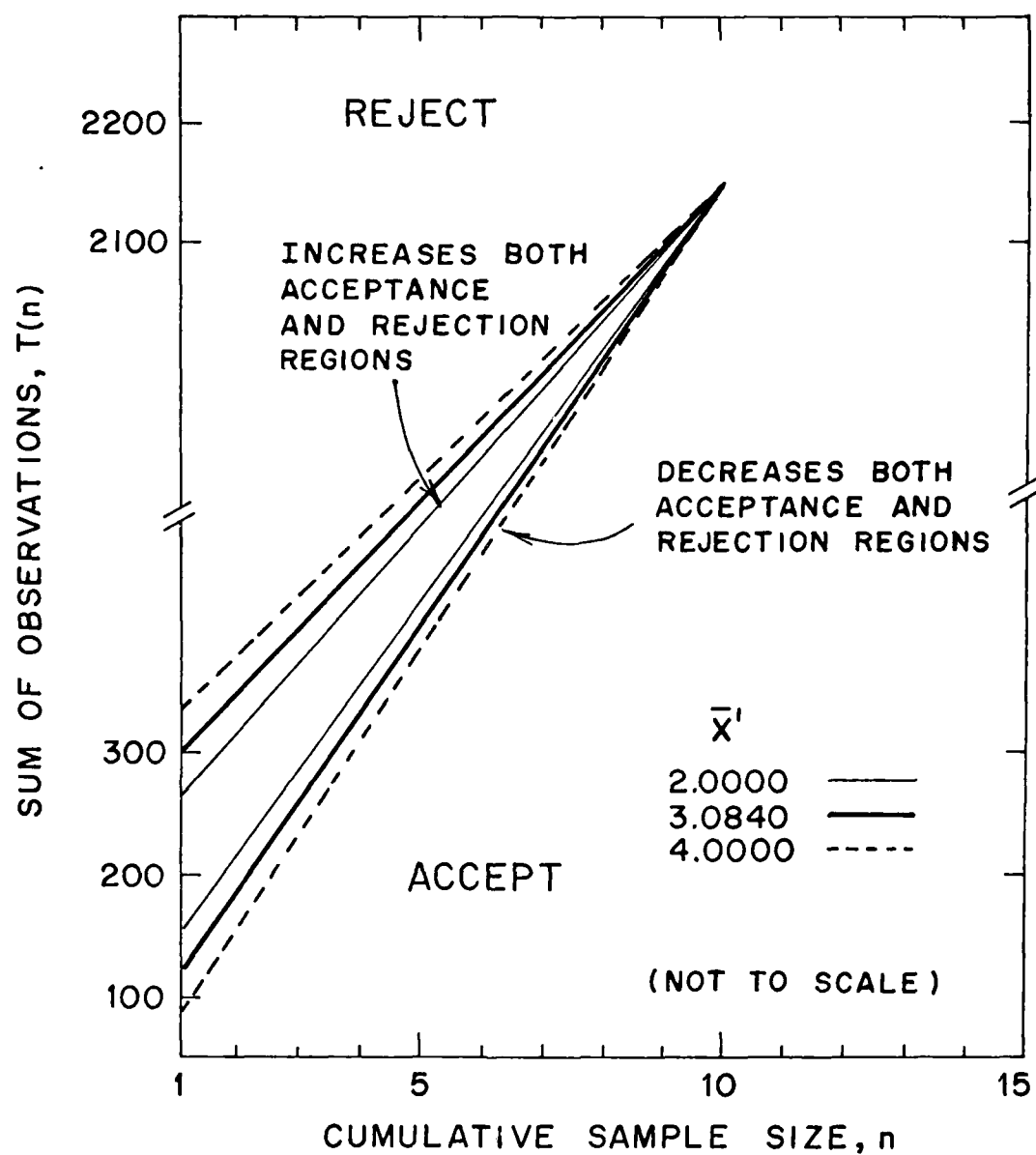


Figure 11. Plot of Sample Problem SNPR Plans ($\bar{X}' = 214.05$, $n' = 10$ and $Z_{\alpha} = 2.0000, 3.0840$ and 4.0000)

Table 6. Effect on α and β of Altering Z_{α} , (Sample Problem SNPR Plan: $\bar{X}' = 214.05$, $n' = 10$)

Z_{α}	$T_L(1)$	$T_U(1)$	$T(n')$	α	β
2.000	154.05	274.04	2140.50	.086	.137
3.084	121.53	306.57	2140.50	.050	.094
4.000	94.05	334.04	2140.50	.036	.075

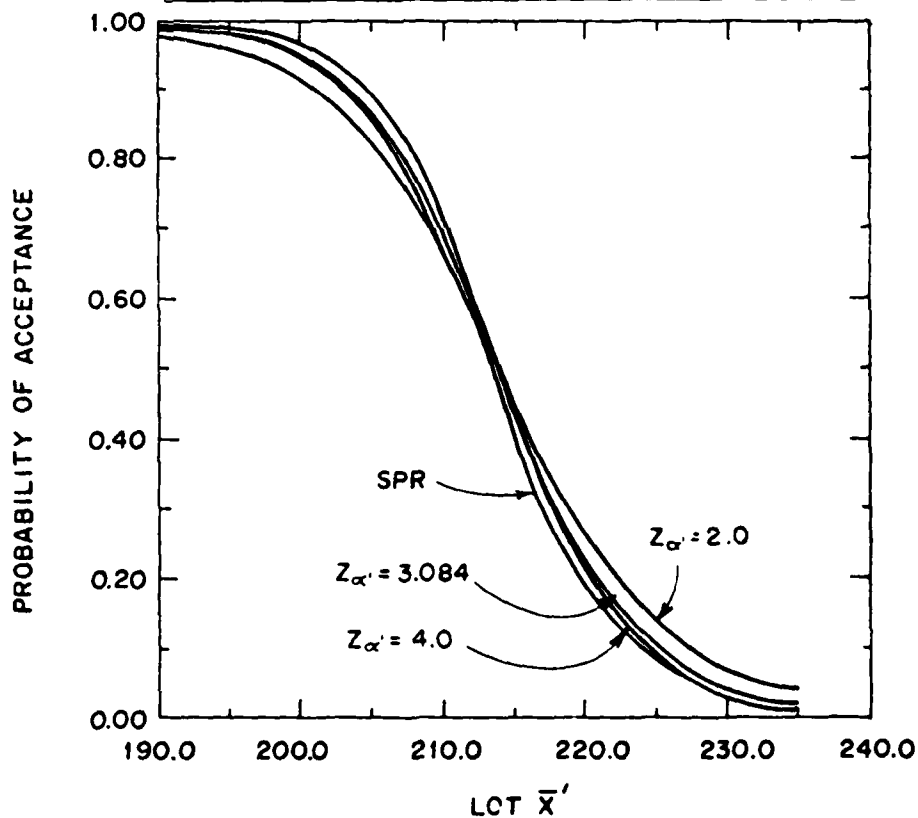


Figure 12. OC Curves: SNPR Plans (Sample Problem; $\bar{X}' = 214.05$, $n' = 10$ and $Z_{\alpha} = 2.0000$, 3.0840 and 4.0000) and SPR Plan

curves of the SPR plan and the SNPR plan with $\bar{X}' = 214.05$ and $Z_{\alpha} = 3.0840$. These values are used in the development of a heuristic which for a specified value of n' establishes $\bar{X}'$ and Z_{α} , for an SNPR plan which provides the same level of protection as the SPR plan.

Development of the Heuristic

Three important facts mentioned in the previous section are the cornerstones of the heuristic procedure:

- (1) The OC curve of an SNPR plan with $\bar{X}'$ equal to $\bar{X}_a$ of the single sampling plan, Z_{α} , equal to the Z_{α} , obtained from the limits at $n=1$ of the SPR plan and n' equal to the maximum ASN of the SPR plan closely approximates the OC curve of the SPR plan.
- (2) Increasing the value of $\bar{X}'$ in an SNPR plan decreases the value of α and increases β . Decreasing $\bar{X}'$ has the opposite effect.
- (3) Increasing the value of Z_{α} , in an SNPR plan decreases both α and β . Decreasing Z_{α} , has the opposite effect.

Using this information, Z_{α} , and $\bar{X}'$ are altered while holding n' constant at 10. By setting $\bar{X}' = 214.24$ and $Z_{\alpha} = 3.0100$, the calculated values of α and β are .05 and .10 respectively. If n is reduced to 9 (a 10% sampling reduction), the values of $\bar{X}'$ and Z_{α} , need to yield $\alpha = .05$ and $\beta = .10$ are 214.22 and 3.2400 respectively. Table 7 lists the necessary values of $\bar{X}'$ and Z_{α} , for different values of n' . Figure 13 shows the plots of two plans along with a plot of the SPR plan. Notice that as n' decreases, the plan compensates by being less discriminating in its earlier stages. Computational experience indicates that SNPR

Table 7. Values of $\bar{X}'$ and $Z_{\alpha'}$, Necessary to Obtain $\alpha = .05$ and $\beta = .10$ for Varying Values of n' in the Sample Problem

n'	$\bar{X}'$	$Z_{\alpha'}$
10	214.24	3.01
9	214.22	3.24
8	214.19	3.65
7	214.14	4.71

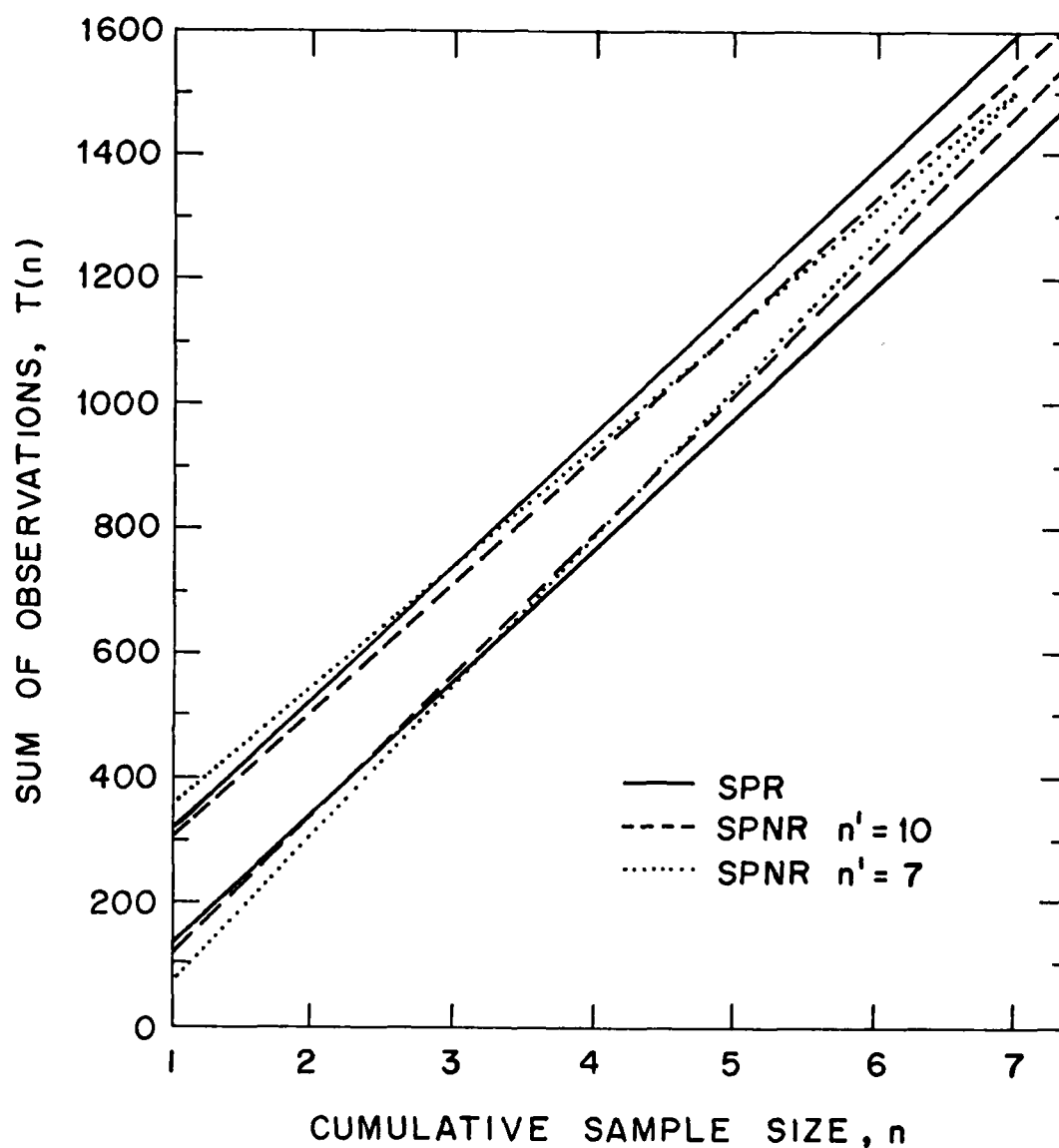


Figure 13. Plots of Sample Problem SNPR Plans ($n' = 7$ and $n' = 10$) and Sample Problem SPR Plan

plans with n' less than 70% of the SPR plan's maximum ASN cannot yield an OC curve which approximates that of the SPR plan, regardless of the values of $\bar{X}'$ and Z_{α} , chosen.

Using the estimates of $\bar{X}'$ and Z_{α} , described at the beginning of this section, a more exacting OC curve can be obtained using the following steps:

- (1) If n' is chosen other than the maximum ASN of the SPR plan ($N_{\max}$), a modified estimate of Z_{α} , is used. This value is obtained by increasing (decreasing) the initial estimate by the percentage decrease (increase) in $N_{\max}$:

$$Z_{\alpha}(n') = Z_{\alpha}(N_{\max})(1 + (1 - n'/N_{\max})) \quad (26)$$
- (2) Examine the values of α and β obtained by using the estimated parameters in the SNPR plan. Determine if they must be increased or decreased to match the objective values of α and β from the SPR plan.
- (3) If both α and β must be increased (decreased) to meet the objective values of α and β from the SPR plan, decrease (increase) Z_{α} .
- (4) If one (α or β) must be increased and the other decreased to meet the objective values of α and β from the SPR plan, alter $\bar{X}'$ accordingly.
- (5) Repeat steps 2 through 4 until the values of α and β from the SNPR test are within the desired limits.

Computational experience with the heuristic, as coded in Appendix A, reveals that usually only two or three iterations of the procedure are needed to obtain α and β within a tolerance of .001. Appendix B contains

several SNPR plans which were generated using the coded heuristic. The associated problem parameters were varied through specific ranges to demonstrate the code's flexibility. In each of the twelve plans, the exact objective values of α and β were met.

Using the sample problem, the following is an example of the heuristic procedure for $n' = 8$.

(1) Initial Estimates

$$\begin{aligned} \text{(Step 1)} \quad \bar{X}' &= 214.05 \quad Z_{\alpha'} = 3.0849 \\ Z_{\alpha'}(8) &= 3.0849 (1 + (1 - 8/10)) = 3.7019 \end{aligned}$$

$$\text{(2) With } \bar{X}' = 214.05, Z_{\alpha'} = 3.7019 \text{ and } n' = 8$$

$$\text{(Step 2)} \quad \alpha = .0507 \quad \beta = .0965$$

(3) Since α is above the objective value of .05 and β is below

(Step 4) the objective value of .10, $\bar{X}'$ is increased. (The rule for altering $\bar{X}'$ is to first alter it by $\sigma'/100$. After recalculating the values of α and β , determine the proportionate adjustment to $\bar{X}'$ which yields α and β closest to the objective values.)^{*}

$$\text{With } \bar{X}' = 214.35, Z_{\alpha'} = 3.7019 \text{ and } n' = 8$$

$$\alpha = .0474 \quad \beta = .1023$$

a proportionate adjustment yields $\bar{X}' = 214$

$$\text{With } \bar{X}' = 214, Z_{\alpha'} = 3.7019 \text{ and } n' = 8$$

$$\alpha = .0495 \quad \beta = .0993$$

(4) With α and β within .001 of the objective values of .05 and .10, respectively, the heuristic procedure is terminated.

*The rule for altering $Z_{\alpha'}$ is to increase (decrease) the value of $Z_{\alpha'}$ accordingly, recalculate the values of α and β , then determine the proportionate adjustment to $Z_{\alpha'}$ which yields α and β closest to the objective values.

CHAPTER IV

COMPARISON OF SPR AND SNPR PLANS

Because it reveals the discriminating power of a sampling plan, the OC curve is useful in evaluating alternate sampling plans. In the previous chapter a method to closely match the OC curves of the subject plans was presented and therefore voids any comparisons based on that feature. However, situations may exist where it is advantageous to quickly establish a sequential sampling plan which guarantees truncation after relatively few samples have been taken, even if the cost of truncation is a slightly altered OC curve. Two methods are available to establish a "fast truncated" sequential sampling plan. They are the truncated SPR plan described in Chapter II and the SNPR plan which uses the initial estimates of $\bar{X}'$ and Z_{α} , ($\bar{X}_a$ from the single sampling plan and Z_{α} , derived using the initial acceptance and rejection limits of the SPR plan) discussed in Chapter III.

Four sampling plans are presented in Table 8. The plans are the truncated SPR plan ($n' = 10$), the non-truncated SPR plan, the SNPR plan ($n' = 10$) generated using the coded heuristic procedure in Appendix A and the SNPR plan ($n' = 10$) generated using the initial estimates mentioned above. The first plan (truncated SPR) has $\alpha(10) = .119$ and $\beta(10) = .156$ (these values are upper limits for $\alpha(n')$ and $\beta(n')$ computed via equations developed by Wald [9]). While this is a "fast truncated" sequential plan, the resulting distortion of the OC curve leaves the practicality of this plan highly suspect. The fourth plan (SNPR generated using initial estimates) has $\alpha = .050$ and $\beta = .095$.

Table 8. Comparison of Sequential Plans Using Sample Problem

n	TRUNCATED SPR $n'=10$ $\alpha(n')=.119 \quad \beta(n')=.156$		SPR $\alpha=.050 \quad \beta=.100$		SNPR $n'=10$ $\alpha=.050 \quad \beta=.100$		SNPR (EST.) $n'=10$ $\alpha=.050 \quad \beta=.095$	
	$T_L(n)$	$T_U(n)$	$T_L(n)$	$T_U(n)$	$T_L(n)$	$T_U(n)$	$T_L(n)$	$T_U(n)$
1	131.45	3.6.55	131.45	316.55	123.97	304.54	121.53	306.57
2	343.95	529.05	343.95	529.05	348.26	508.77	345.86	510.34
3	556.45	741.55	556.45	741.55	572.55	712.99	570.19	714.11
4	768.95	954.05	768.05	954.05	796.83	917.21	794.52	917.88
5	981.45	1166.55	981.45	1166.55	1021.12	1121.44	1018.85	1121.65
6	1193.95	1379.05	1193.95	1379.05	1245.41	1325.66	1243.18	1325.42
7	1406.45	1591.55	1406.45	1591.55	1469.70	1529.89	1467.51	1529.19
8	1618.95	1804.05	1618.95	1804.05	1693.98	1734.11	1691.84	1732.96
9	1831.45	2016.55	1831.45	2016.55	1918.27	1938.34	1916.17	1936.73
10	2043.95	2229.05	2043.95	2229.05	2142.56	2142.56	2140.50	2140.50
11			2256.45	2441.55				
12			2468.95	2654.05				
13			2681.45	2866.55				
14			:	:				
15			:	:				

The OC curve of this plan, although not exactly matched to the OC curve of the non-truncated SPR plan, may reveal sufficient protection that it would not be necessary to perform the heuristic procedure to obtain a more exacting OC curve.

If exact discriminating power of a plan is an absolute requirement, Duncan [1] suggests the relative efficiency of plans with matching OC curves may be determined by examining the amount of inspection required by each plan. The ASN curve of the SPR plan shown in Figure 5 is characteristic of all SPR plans. It can be seen that decisions (generally to accept) are quickly made, on the average, when good quality lots are submitted and that decisions (generally to reject) are quickly made, on the average, when bad lots are submitted. Lots of mediocre quality require more inspection, on the average [7]. The phrase, "on the average" is stressed because the required sample size is, as mentioned previously, a random variable capable of taking on values well above the average.

Although no analytic function has been developed which describes or approximates an SNPR plan's ASN curve, it is relatively easy to envision its rudimentary shape. It is known that values on the curve cannot exceed n' but will most likely take on values well below n' . Therefore, n' represents an upper limit for the curve. In addition, Wald has proven that the efficiency of the SPR test is, "if not exactly, very nearly equal to 1 under H_0 as well as under H_1 [9]." This means that the values on the SPR test's ASN curve at $\bar{X}'_{AQL}$ and $\bar{X}'_{LTPD}$ are lower limits for all sequential tests. Next, shown in

Figure 13 are plots of three sampling plans developed from the sample problem. In the initial stages, $n = 1$ through 4, the SNPR plan ($n' = 10$) is as discriminating as the SPR plan while the SNPR plan ($n' = 7$) is less discriminating. This suggests that the ASN curve of the SNPR plan ($n' = 10$) closely resembles that of the SPR plan at the tails of the ASN curve. Conversely, the ASN curve of the SNPR plan ($n' = 7$) is above that of the SPR plan at the tails of the ASN curve. If it is assumed that incoming lots of mediocre quality force the SNPR plan's ASN curve to approach n' , the ASN curves in Figure 14 are rough approximations of what can reasonably be expected.

Examining the curves in Figure 14, an expected feature of the SNPR plan becomes evident. As n' decreases, so does the maximum average sample number. However, the curve becomes less peaked; the average sample number increases at values of $\bar{X}'$ which indicate lots of very good or very bad quality. At this point, a trade-off must be made when selecting which plan to use. If one is reasonably sure that the values of $\bar{X}'$ are clustered near one value, the selection of a plan can be made by comparing ASN curves and selecting the plan with the lowest ASN at that point. If the quality of incoming lots is unknown, the cost of inspection versus the number of inspections trade-off analysis must be made.

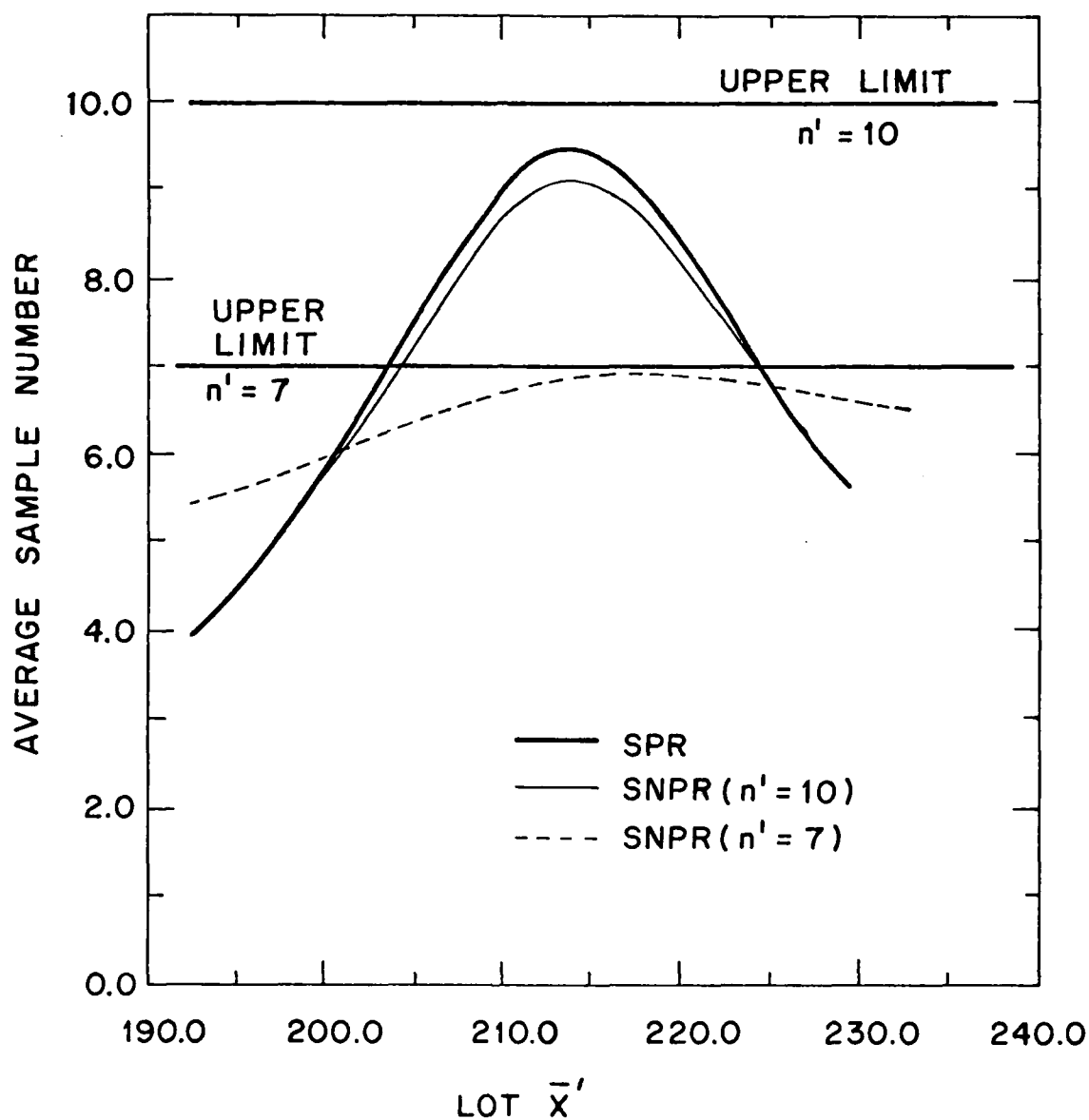


Figure 14. ASN Curves: SNPR Plans (Sample Problem $n' = 7, 10$) and SPR Plan (Sample Problem)

CHAPTER V

SUMMARY

An SPR plan can materially reduce the amount of required inspection. Studies have shown that the average decrease in sample size is often near 50% when compared with the sample size of a comparable single sampling plan [4]. An SNPR plan can achieve and better the sampling reduction obtained through the use of an SPR plan. The SNPR plan requires no truncation rules and maintains the integrity of its associated OC curve while truncation of the SPR plan makes the plan less discriminating.

Several disadvantages of sequential plans have been noted. The most severe are the cost of administering the plan and the lack of availability of information concerning prevailing levels of quality in each lot. The administrative cost of an SNPR plan can be substantially reduced through the use of the computer code of the heuristic algorithm developed in Chapter III. No bulky tables and complicated charts need be used, only a terminal display. Acceptance and rejection limits would be displayed on a screen in conjunction with input inspection variables. The savings in inspection costs and time could possibly offset any loss due to unavailability of lot quality data. The ultimate extension would be the use of the sampling plan in an automated environment where inspection is performed by machines.

Whatever its use, the SNPR plan for variables inspection is a flexible plan which in many instances outperforms other sampling plans.

Its applications are not limited to one-sided inspection and further research should be able to produce modifications which enable it to be expanded to two-sided inspection (inspection with both upper and lower quality limits). It is hoped that as a result of this research, serious consideration will be given to this plan so as to reap the potential savings that can be a by product of its use.

APPENDIX A
SEQUENTIAL NON-PROBABILITY RATIO (SNPR) AND
SEQUENTIAL PROBABILITY RATIO (SPR)
TEST PLAN GENERATOR

The following program provides a SNPR test plan, a SPR test plan or both for a given set of input parameters. It also calculates a specified number of points for the associated OC curve(s) of the requested plan(s). Additionally a specified number of points are calculated for the ASN curve of the SPR plan when that plan alone is requested.

The program has three formats:

- (1) the SPR test plan;
- (2) the SNPR test plan;
- (3) the SPR and SNPR test plans.

The following is a brief description of each format's input parameters and output. The input format is "format free," requiring only commas or spaces between input parameters for each line of data. Before input data can be read for any particular format, a numerical format code must be specified on the first data card. The format specification codes are:

CODE	FORMAT
(1)	SPR Test Plan
(2)	SNPR Test Plan
(3)	SPR and SNPR Test Plans

CARD(1): INPUT FORMAT CODE

Format 1: SPR Test Plan

The input format for the SPR test plan requires specification of:

- (1) $\bar{X}'_{AQL}$
- (2) $\bar{X}'_{LTPD}$
- (3) σ'
- (4) α (producer's risk)
- (5) β (consumer's risk)
- (6) NTRUNK
- (7) NTERM

$\bar{X}'_{AQL}$ and $\bar{X}'_{LTPD}$ are the means locating a percent defective equal to a desired AQL and LTPD respectively. The program is set up to work with $\bar{X}'_{AQL}$, however, problems where higher values of $\bar{X}'$ are desirable can be accommodated. In this instance, the output must be interpreted accordingly. σ' is the value of the population standard deviation. α is the producer's risk and β the consumer's risk. NTRUNK is the number (n) at which the plan will be terminated. NTERM is the number of points to be plotted for the ASN and OC curves. NTRUNK must be less than or equal to NTERM. A value of 0 input for NTRUNK defaults that value to twice the maximum ASN.

Figure 15 shows an example of an input list and the associated output. In the output listing, a header echoes the input. Following are columns headed by:

- N -- sample number
- TL -- lower tolerance limit
- TU -- upper tolerance limit

```
//DATA.INPUT DE *
```

```
1
```

```
200,225,30
```

```
.05,.10
```

```
20,30
```

```
*****
*
* SEQUENTIAL PROBABILITY RATIO SAMPLING PLAN *
*
*****
```

```
*****
XACL= 200.00 XLTPD= 225.00 SIGMA= 30.00
ALPHA=0.05 BETA=0.10 TRUNCATE AT X= 20
*****
```

```
*****
* N * TL TU *LCI XBAR*PRCE ACC* ASN *
*****
* 1 * 131.45 * 316.55 * 193.75 * 0.9873 * 4.20 *
* 2 * 243.95 * 529.05 * 195.21 * 0.9825 * 4.50 *
* 3 * 556.45 * 741.55 * 196.67 * 0.9757 * 4.84 *
* 4 * 768.95 * 954.05 * 198.12 * 0.9666 * 5.21 *
* 5 * 981.45 * 1166.55 * 199.58 * 0.9542 * 5.62 *
* 6 * 1193.95 * 1379.05 * 201.04 * 0.9377 * 6.07 *
* 7 * 1406.45 * 1591.55 * 202.50 * 0.9159 * 6.55 *
* 8 * 1618.95 * 1804.05 * 203.96 * 0.8877 * 7.05 *
* 9 * 1831.45 * 2016.55 * 205.42 * 0.8518 * 7.57 *
* 10 * 2043.95 * 2229.05 * 206.87 * 0.8075 * 8.07 *
* 11 * 2256.45 * 2441.55 * 208.33 * 0.7543 * 8.54 *
* 12 * 2468.95 * 2654.05 * 209.79 * 0.6928 * 8.93 *
* 13 * 2681.45 * 2866.55 * 211.25 * 0.6244 * 9.22 *
* 14 * 2893.95 * 3079.05 * 212.71 * 0.5516 * 9.38 *
* 15 * 3106.45 * 3291.55 * 214.17 * 0.4774 * 9.41 *
* 16 * 3318.95 * 3504.05 * 215.62 * 0.4051 * 9.30 *
* 17 * 3531.45 * 3716.55 * 217.08 * 0.3375 * 9.07 *
* 18 * 3743.95 * 3929.05 * 218.54 * 0.2766 * 8.75 *
* 19 * 3956.45 * 4141.55 * 220.00 * 0.2235 * 8.36 *
* 20 * 4168.95 * 4354.05 * 221.46 * 0.1786 * 7.93 *
*****
* 222.92 * 0.1414 * 7.48 *
* 224.37 * 0.1111 * 7.03 *
* 225.83 * 0.0868 * 6.60 *
* 227.29 * 0.0675 * 6.19 *
* 228.75 * 0.0524 * 5.81 *
* 230.21 * 0.0405 * 5.45 *
* 231.67 * 0.0313 * 5.13 *
* 233.12 * 0.0242 * 4.83 *
* 234.58 * 0.0186 * 4.56 *
* 236.04 * 0.0143 * 4.31 *
*****
```

Figure 15. Program Output Listing: Format 1

LOTXBAR -- values of $\bar{X}'$ on x axis of CO and ASN

PROB ACC -- probability of acceptance for $\bar{X}'$

ASN -- average sample number of $\bar{X}'$

The following are the data cards necessary for format 1 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ'

CARD (3): α , β

CARD (4): NTRUNK, NTERM

Format 2: SNPR Test Plan

The input format for the SNPR test plan requires specification of:

- (1) $\bar{X}'_{AQL}$
- (2) $\bar{X}'_{LTPD}$
- (3) σ'
- (4) n'
- (5) α
- (6) β
- (7) MAX
- (8) NTERM

All but two of the above terms have been described previously. n' is the maximum number of items to be sampled. MAX is the maximum number of iterations to be performed by the heuristic algorithm. This value should initially be set to 5 and adjusted depending on results obtained.

Figure 16 shows an example of an input list and the associated output. In the output listing, a header echoes the input and lists ZA and XABAR the values of Z_{α} and $\bar{X}'$ calculated by the heuristic

```
//DATA.INPUT DE *
2
200,225,30,10
.05,.10
5,30
```

```
*****
*
* SEQUENTIAL ACN-PROBABILITY RATIO SAMPLING PLAN *
*
*****
```

```
*****
```

```
XAOL= 200.00 XLTPD= 225.00 SIGPA= 30.00
ALPHA=0.05 BETA=0.10 NPRIME= 10
ZA= 3.01 XABAR= 214.26
```

```
*****
*****
```

```
* N *      TI      *      TU      **LOT XEAR*PROE ACC*
```

```
*****
* 1 * 123.97 * 304.54 ** 193.75 * 0.9869 *
* 2 * 348.26 * 508.77 ** 195.21 * 0.9819 *
* 3 * 572.55 * 712.99 ** 196.67 * 0.9752 *
* 4 * 796.83 * 917.21 ** 198.12 * 0.9662 *
* 5 * 1021.12 * 1121.44 ** 199.58 * 0.9541 *
* 6 * 1245.41 * 1325.66 ** 201.04 * 0.9381 *
* 7 * 1469.70 * 1529.89 ** 202.50 * 0.9173 *
* 8 * 1693.98 * 1734.11 ** 203.96 * 0.8907 *
* 9 * 1918.27 * 1938.34 ** 205.42 * 0.8572 *
* 10 * 2142.56 * 2142.56 ** 206.87 * 0.8161 *
*****
* 208.33 * 0.7669 *
* 209.79 * 0.7099 *
* 211.25 * 0.6459 *
* 212.71 * 0.5766 *
* 214.17 * 0.5045 *
* 215.62 * 0.4321 *
* 217.08 * 0.3623 *
* 218.54 * 0.2976 *
* 220.00 * 0.2396 *
* 221.46 * 0.1695 *
* 222.92 * 0.1474 *
* 224.37 * 0.1130 *
* 225.83 * 0.0856 *
* 227.29 * 0.0641 *
* 228.75 * 0.0476 *
* 230.21 * 0.0351 *
* 231.67 * 0.0258 *
* 233.12 * 0.0188 *
* 234.58 * 0.0137 *
* 236.04 * 0.0095 *
*****
```

Figure 16. Program Output Listing: Format 2

algorithm and used as arguments in the calculation of the SNPR test plan. The column headers are the same as those listed for the SPR test plan.

The following are the data cards necessary for format 2 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ' , n'

CARD (3): α , β

CARD (4): MAX, NTERM

Format 3: SPR and SNPR Test Plans

The input format for the SPR and SNPR test plans requires specification of:

- (1) $\bar{X}'_{AQL}$
- (2) $\bar{X}'_{LTPD}$
- (3) σ'
- (4) n'
- (5) α
- (6) β
- (7) MAX
- (8) NTRUNK
- (9) NTERM

Figure 17 shows an example of an input list and the associated output. In the output listing, a header which echoes common and specific input for each test plan. $\bar{X}'$ and Z_{α} , are listed for the SNPR test plan. Column headings are established for n (sample number), upper and lower tolerance limits for both plans, values of $\bar{X}'$ for the

```
//DATA.INPUT DE *
```

```
3
```

```
200,225,30,10
```

```
.05,.10
```

```
5,20,30
```

```
*****
*
*   SEQUENTIAL PROBABILITY AND NON-PROBABILITY PATIC SAMPLING PLANS
*
*****
```

```
*****
XACI= 200.00  XITFD= 225.00  SIGMA= 30.00
```

```
SNPR FLAN
```

```
ALPHA=0.05  PETA=0.10  NPRIME= 10  ZA= 3.01  XABAR= 214.26
```

```
SPR FLAN
```

```
ALPHA=0.05  PETA=0.10  TRUNCATION OCCURS AT N= 20
```

```
*****
*      SNPRT      **      SPRT      SNPRT * SPRT
*  N  **  TL  *  TU  **  TL  *  TU  **LCT XPAP*PROE ACC*PROE ACC*
*****
*  1  **  123.97 *  304.54 **  131.45 *  316.55 **  193.75 *  0.987 *  0.987 *
*  2  **  348.26 *  508.77 **  343.95 *  529.05 **  195.21 *  0.982 *  0.982 *
*  3  **  572.55 *  712.99 **  556.45 *  741.55 **  196.67 *  0.975 *  0.976 *
*  4  **  756.83 *  917.21 **  768.95 *  954.05 **  198.12 *  0.966 *  0.967 *
*  5  **  1021.12 *  1121.44 **  981.45 *  1166.55 **  199.58 *  0.954 *  0.954 *
*  6  **  1245.41 *  1325.66 **  1193.95 *  1379.05 **  201.04 *  0.938 *  0.938 *
*  7  **  1469.70 *  1529.89 **  1406.45 *  1591.55 **  202.50 *  0.917 *  0.916 *
*  8  **  1693.98 *  1734.11 **  1618.95 *  1804.05 **  203.96 *  0.891 *  0.898 *
*  9  **  1918.27 *  1938.34 **  1831.45 *  2016.55 **  205.42 *  0.857 *  0.852 *
* 10  **  2142.56 *  2142.56 **  2043.95 *  2229.05 **  206.87 *  0.816 *  0.808 *
* 11  **  2256.45 *  2441.55 **  2256.45 *  2441.55 **  208.33 *  0.767 *  0.754 *
* 12  **  2468.95 *  2654.05 **  2468.95 *  2654.05 **  209.79 *  0.710 *  0.693 *
* 13  **  2681.45 *  2866.55 **  2681.45 *  2866.55 **  211.25 *  0.646 *  0.624 *
* 14  **  2893.95 *  3079.05 **  2893.95 *  3079.05 **  212.71 *  0.577 *  0.552 *
* 15  **  3106.45 *  3291.55 **  3106.45 *  3291.55 **  214.17 *  0.504 *  0.477 *
* 16  **  3318.95 *  3504.05 **  3318.95 *  3504.05 **  215.62 *  0.432 *  0.405 *
* 17  **  3531.45 *  3716.55 **  3531.45 *  3716.55 **  217.08 *  0.362 *  0.337 *
* 18  **  3743.95 *  3929.05 **  3743.95 *  3929.05 **  218.54 *  0.298 *  0.277 *
* 19  **  3956.45 *  4141.55 **  3956.45 *  4141.55 **  220.00 *  0.240 *  0.224 *
* 20  **  4168.95 *  4354.05 **  4168.95 *  4354.05 **  221.46 *  0.189 *  0.179 *
* 21  **  222.92 *  0.147 *  0.141 *
* 22  **  224.37 *  0.113 *  0.111 *
* 23  **  225.83 *  0.086 *  0.087 *
* 24  **  227.29 *  0.064 *  0.068 *
* 25  **  228.75 *  0.048 *  0.052 *
* 26  **  230.21 *  0.035 *  0.041 *
* 27  **  231.67 *  0.026 *  0.031 *
* 28  **  233.12 *  0.019 *  0.024 *
* 29  **  234.58 *  0.014 *  0.019 *
* 30  **  236.04 *  0.010 *  0.014 *
*****
```

Figure 17. Program Output Listing: Format 3

OC curves and the associated probabilities of acceptance for each plan.

The following are the data cards necessary for format 3 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ' , n'

CARD (3): α , β

CARD (4): MAX, NTRUNK, NTERM

Program Listing

Following is a listing of the computer code which generates the test plans. It consists of six subroutines and a driver routine. Each routine is commented with a description and other documentation.

*** 20-0 (03/C4/R1--1844)

```

C               MAIN PROGRAM
C               *****
C THIS IS THE DRIVER ROUTINE FOR THE PROGRAM. IT READS DATA AND CALLS
C INDIVIDUAL SUBROUTINES BASED ON A PRESCRIBED FORMAT, IPRINT.
C THERE ARE THREE POSSIBLE FORMATS AS DESCRIBED IN THE USERS GUIDE.
C THE SPF TEST PLAN FORMAT CALLS SUBROUTINE SPRT WHERE THE TOLERANCE
C LIMITS FOR THE PLAN ARE CALCULATED. IT THEN CALLS SUBROUTINE
C OUTPUT WHERE THE CALCULATIONS ARE OUTPUT ACCORDING TO THE SPECIFIED
C FORMAT. THE SNPF TEST PLAN FORMAT CALLS SUBROUTINE SPRT TO
C OBTAIN THE TOLERANCE LIMITS FOR N=1 IN THE SPF TEST PLAN.
C NEXT, SUBROUTINE SSP (SINGLE SAMPLING PLAN) CALCULATES THE
C SEED VALUES OF ZA AND XA USED BY THE NEXT CALLED SUBROUTINE,
C SUBROUTINE SEARCH. THIS SUBROUTINE RETURNS THE UPPER AND LOWER
C TOLERANCE LIMITS FOR THE SNPF TEST PLAN. SUBROUTINE OUTPUT IS THEN
C CALLED TO PRODUCE THE REQUIRED OUTPUT LISTING. THE COMBINED SPF
C AND SNPF TEST PLAN FORMAT IS THE SAME AS THAT OF FORMAT 2 EXCEPT
C THE OUTPUT FORMAT IS MODIFIED TO HANDLE BOTH TEST PLAN FORMATS.
C
C *****
COMMON ALPHA, ASUM, BETA, BSUM, SIGMA, XA, XAQL, XASNPF, XITPD, ZASNPF,
* ZASPT, IPRINT, MAX, NMAX, NPRINT, NTERM, NTRUNK, ASKCRV (100)
* , OC (100), CCSNPF (100), SNPPTL (500), SNPRTU (500), SPPTL (500),
* SPPTU (500), XERLCT (100)
READ, IPRINT
GOTO (1,2,4), IPRINT
C PROCESSING FORMAT FOR SPRT PLAN.
1 READ, XAQL, XLTPL, SIGMA
  READ, ALPHA, BETA
  READ, NTRUNK, NTERM
  CALL SPRT
  CALL OUTPUT
  GOTO 5
C PROCESSING FORMAT FOR SNPF PLAN.
2 READ, XAQL, XITPL, SIGMA, NPRINT
  READ, ALPHA, BETA
  READ, MAX, NTERM
C ONLY THE TOLERANCE LIMITS AT N=1 ARE NEEDED FOR THIS FORMAT.
  NTRUNK=1
3 CALL SPRT
  CALL SSP
  CALL SEARCH
  CALL SNPROC
  CALL OUTPUT
  GOTO 5
C PROCESSING FORMAT FOR SPPT AND SNPPT PLANS
4 READ, XAQL, XLTPL, SIGMA, NPRINT
  READ, ALPHA, BETA
  READ, MAX, NTRUNK, NTERM
  GOTO 3
5 STOP
END

```

```

      SUBROUTINE SSP
      *****
C THIS SUBROUTINE IS A FEEDBACK ROUTINE TO SUBROUTINE SEARCH. IT CALCULATES THE SEED VALUES OF ZALPHA USING THE VALUES OF TL(1) AND TU(1) (SEE CHAPTER III OF THEESIS) GENERATED IN SUBROUTINE SPRT. IT ALSO CALCULATES THE SEED VALUE OF XBARPRIME FROM XABAR, THE INDIFFERENCE POINT OF ACCEPTANCE LIMIT, FOR A SINGLE SAMPLING PLAN.
C *****
      COMMON ALPHA, ASUM, BETA, BSUM, SIGMA, XA, XAQL, XASNPF, XLTFD, ZASNPF,
      * ZASPT, IFFINT, MAX, NMAX, NPPIME, NTERM, NTRUNK, ASNCFV(100),
      * OC(100), CCSNPF(100), SNPRTI(500), SNPRTU(500), SPRTL(500),
      * SPRTU(500), XERICI(100)
C CALCULATE THE SEED VALUE OF ZALPHA, ZASPT.
      ZASPT=(SPRTU(1)-SPRTL(1))/2/SIGMA
C CALCULATE THE REQUIRED SAMPLE NUMBER OF THE SINGLE SAMPLING PLAN
C PNSSP AND THE VALUE OF THE NEXT HIGHEST INTEGER, NSSP.
      PNSSP=((ZPRE(ALPHA)+ZPRB(BETA))*SIGMA/ABS(XAQL-XLTFD))**2
      NSSP=PNSSP
C AFTER DETERMINING WHICH VALUE IS GREATER, XAQL AND XLTFD, CALCULATE
C THE INDIFFERENCE FCINT WITH N=PNSSP AND N=NSSP. TAKE THE AVERAGE
C OF THESE TWO VALUES, XA, AND USE IT AS THE SEED, XBARPRIME.
      IF(XAQL.GE.XLTFD) GOTO 10
      XABAR=(ZPRB(ALPHA)*SIGMA/PNSSP**0.5)+XAQL
      XABI=(ZPRB(ALPHA)*SIGMA/NSSP**0.5)+XAQL
      XA=AMAX1(XABAR,XABI)-ABS(XABAR-XABI)/2
      GOTO 20
10  XABAR=(-ZPRB(ALPHA)*SIGMA/PNSSP**0.5)+XAQL
      XABI=(-ZPRB(ALPHA)*SIGMA/NSSP**0.5)+XAQL
      XA=AMAX1(XABAR,XABI)-ABS(XABAR-XABI)/2
20  RETURN
      END

```

```

      SUBROUTINE SPRJ
      *****
C THIS ROUTINE USES THE EQUATIONS DEVELOPED BY WALD (SEE CHAPTER II
C OF THESIS) FOR THE SPR TEST. IT GENERATES A TEST PLAN FOR THE
C SPECIFIED INPUT VALUES OF XAQL, XLTPD, SIGMA, ALPHA AND BETA. IT
C TRUNCATES THE PLAN AT THE INPUT VALUE OF NTRUNK OR THE DEFAULT
C VALUE WHICH IS TWICE THE VALUE OF THE MAXIMUM ASN AT XAQL OF
C XLTPD. VALUES FOR THE OPERATING CHARACTERISTIC (OC) AND AVERAGE
C SAMPLE NUMBER (ASN) CURVES ARE ALSO GENERATED.
C *****
      COMMON ALPHA, ASUM, BETA, BSUM, SIGMA, XA, XAQL, XASNPR, XLTPD, ZASNPP,
      * ZASPRJ, IPRINT, MAX, NMAX, NFEIFF, NTEFF, NTRUNK, ASNCFV(100)
      * , OC(100), OCSNPP(100), SNPRTL(500), SNPRTU(500), SPRTL(500),
      * SPRTU(500), XERLCT(100)
C DECLARATION OF FUNCTION STATEMENTS.
      ACC(N)=H0+N*S; REJ(N)=H1+A*S
      R(THETA)=(XAQL+XLTPD-2*TEFFA)/(XLTPD-XAQL)
      PACC(THETA)=(A**B(THETA)-1)/(A**H(THETA)-B**H(THETA))
      ASN(THETA)=(H1+PACC(THETA)*(H0-H1))/(TEFFA-S)
C CALCULATION OF EQUATION PARAMETERS.
      A=(1-BETA)/ALPHA
      B=BETA/(1-ALPHA)
      S=(XAQL+XLTPD)/2
      H0=SIGMA**2*ALCG(E)/(XLTPD-XAQL)
      H1=SIGMA**2*ALCG(A)/(XLTPD-XAQL)
C DETERMINATION OF TRUNCATION VALUE.
      ITERM=NTRUNK
      IF (NTRUNK.LT.0.5) ITERM=2*AMAX1(ASN(XAQL),ASN(XLTPD))
      NTRUNK=ITERM
C CALCULATION OF THE TOLERANCE LIMITS.
      DO 100 I=1,ITERM
        SPRTL(I)=ACC(I)
        SPRTU(I)=REJ(I)
100
C CALCULATION OF THE STEP FACTOR USED FOR THE XBARPFI OF THE OC
C AND ASN CURVES. THE LIMITS FOR XBARPFI EXTEND BEYOND XAQL AND
C XLTPD BY AN AMOUNT DETERMINED BY THE VALUES OF ALPHA AND BETA.
      FACT1=ALPHA*5; FACT2=BETA*5; FACTOR=(1+FACT1+FACT2)/NTEFF
      RINCRM=0; XLTPD1=XAQL-FACTOR*AES(XAQL-XLTPD); NMAX=0
C THIS SEGMENT FILLS THE ARRAY XEAFIOT WITH THE VALUES OF THE ARGUMENTS
C FOR CALCULATION OF THE OC AND ASN CURVES. ARRAYS CC AND ASN ARE
C FILLED WITH THE ASSOCIATED VALUES OF THE PROBABILITY OF ACCEPTANCE
C AND ASN.
      DO 200 I=1,NTEFF
        XERLCT(I)=XLTPD1+RINCRM
        OC(I)=PACC(XLTPD1+RINCRM)
        ASNCRV(I)=ASN(XLTPD1+RINCRM)
200
C CALCULATION OF THE MAXIMUM ASN.
      IF (ASNCRV(I).GT.NMAX) NMAX=ASNCRV(I)+1
      RINCRM=RINCRM+FACTOR*AES(XAQL-XLTPD)
      RETURN
      END

```

```

      SUBROUTINE SNPRT (ZALPHA,XBAR)
      *****
C THIS SUBROUTINE USES THE INPUT VALUES OF XACL, XLTPD, SIGMA AND NPRIME
C , ALSO THE CALLING ARGUMENTS ZALPHA AND XBAR TO GENERATE AN SNPF
C TEST PLAN WITH ASSOCIATED ALPHA (ASUM) AND BETA (ESUM).
C*****
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XACL,XASNF,XITD,ZASNF,
      * ZASPT,IEFINI,MAX,NMAX,NPRIME,NTEPM,NTRONK,ASNCV(100)
      * ,OC(100),CCSNF(100),SNFTI(500),SNPTU(500),SFTL(500),
      * SPRTU(500),XERICT(100)
C CALCULATE THE STEP INTERVAL (SEE CHAPTER III OF THESIS).
      ENTRVL=ZALPHA*SIGMA/(NPRIME-1)
C CALCULATE THE CONTROL LIMITS AND THE ALPHA AND BETA OF THE TEST AND
C STORE THE RESULTS IN THE ARRAYS SNPTU AND SNFTL.
      DO 12 J=1,NPRIME
          RJ=J
          SNPTU(J)=XBAR*J+((NPRIME-J)*ENTFVL)
          SNFTL(J)=XBAR*J-((NPRIME-J)*ENTFVL)
C CALCULATE ALPHA USING XACL.
          DEVIAT=(SNPTU(J)-J*XACL)/(SQRT(RJ)*SIGMA)
          IF (DEVIAT.GT.0.0) GOTO 1
          ALPH=1-PRBZ(-DEVIAT)
          GOTO 2
1          ALPH=PRBZ(DEVIAT)
C CALCULATE BETA USING XACL.
2          DEVIAT=(SNFTL(J)-J*XACL)/(SQRT(RJ)*SIGMA)
          IF (DEVIAT.GT.0.0) GOTO 3
          BET=1-PRBZ(-DEVIAT)
          GOTO 4
3          BET=PRBZ(DEVIAT)
C CALCULATE ALPHA SUM.
4          CONTINUE
          IF (J.GT.1.5) GOTO 5
          ASUM=ALPH
          PRCD=1-ALPH-BET
          GOTO 6
5          ASUM=ASUM+ALPH*PRCD
          PFCB=PRCD*(1-ALPH-BET)
C CALCULATE ALPHA USING XLTPD.
6          DEVIAT=(SNPTU(J)-J*XLTPD)/(SQRT(RJ)*SIGMA)
          IF (DEVIAT.GT.0.0) GOTO 7
          ALPH=1-PRBZ(-DEVIAT)
          GOTO 8
7          ALPH=PRBZ(DEVIAT)
C CALCULATE BETA USING XLTPD.
8          DEVIAT=(SNFTL(J)-J*XLTPD)/(SQRT(RJ)*SIGMA)
          IF (DEVIAT.GT.0.0) GOTO 9
          BET=PRBZ(-DEVIAT)
          GOTO 10
9          BET=PRBZ(DEVIAT)
C CALCULATE BETA SUM.
10          IF (J.GT.1.5) GOTO 11
          BSUM=BET
          PFCB=1-ALPH-BET
          GOTO 12
11          BSUM=BSUM+BET*PFCB
          PRCD=PRCD*(1-ALPH-BET)
12          CONTINUE
      RETURN
      END

```

```

      SUBROUTINE SNPROC
      *****
C THIS SUBROUTINE CALCULATES VALUES FOR THE OC CURVE OF AN SNPR TEST
C PLAK.
C
C *****
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XACL,XASNPP,XITPT,ZASNFF,
      * ZASPRF,IEFINT,MAX,NMAX,NPFIPE,KTERM,NTRUNK,ASNCRV(100)
      * ,OC(100),OCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
      * SPRTU(500),XEFLCT(100)
C CALCULATE VALUES FOR THE OC CURVE USING THE ARRAY XBRLOT(100).
      DO 8 K=1,NTERM
C CALCULATE THE PROBABILITY OF ACCEPTANCE FOR A SPECIFIC XFPLOT.
      DO 7 J=1,NPRIME
        RJ=J
C CALCULATE ALPHA USING XERLOT (K).
      1 DEVIAT=(SNERTU(J)-J*XBRLOT(K))/(SQRT(RJ)*SIGMA)
        IF (DEVIAT.GT.0.0) GOTO 2
        ALPH=1-PRBZ(-DEVIAT)
        GOTO 3
      2 ALPH=PRBZ(DEVIAT)
C CALCULATE BETA USING XEFLCT(K).
      3 DEVIET=(SNPRTL(J)-J*XBRLOT(K))/(SQRT(RJ)*SIGMA)
        IF (DEVIET.GT.0.0) GOTO 4
        BET=PRBZ(-DEVIET)
        GOTO 5
      4 BET=1-PRBZ(DEVIET)
C CALCULATE BET SUM.
      5 IF (J.GT.1.5) GOTO 6
        BETSUM=BET
        PRODB=1-ALPH-BET
        GOTO 7
      6 BETSUM=BETSUM+BET*PRODB
        PPCDR=PRODE*(1-ALPH-BET)
      7 CONTINUE
      8 CCSNPP(K)=BETSUM
      RETURN
      END

```

```

C          SUBROUTINE SEARCH
C          *****
C THIS SUBROUTINE IN CONJUNCTION WITH THE OTHER SUBROUTINES FINDS THE
C PARAMETERS ZALPHA AND XBARPRIME WHICH FOR A SPECIFIC VALUE OF
C NPRIME YIELD AN SNPR TEST PLAN WITH ASUM=ALPHA AND ESUM=BETA.
C THIS IS DONE USING THE HEURISTIC ALGORITHM DEVELOPED IN CHAPTER
C III. USING THE SEED VALUES OF ZALPHA AND XBARPRIME FROM SUBROUTINE
C SSP, A MODIFIED VALUE OF XBARPRIME (XBARPRIME(NPRIME)) IS FOUND.
C AFTER A CHECK TO SEE IF THE VALUES OF ASUM AND ESUM ARE WITHIN THE
C DESIRED TOLERANCE, THE SUBROUTINE IS TERMINATED (TOLERANCE MET) OR
C THE VALUES OF ZALPHA AND XBARPRIME ARE MODIFIED (TOLERANCE NOT
C MET). A RECALCULATION OF ASUM AND ESUM WITH THE MODIFIED CALLING
C ARGUMENTS IS FOLLOWED BY ANOTHER TOLERANCE CHECK. THE PROCESS
C CONTINUES UNTIL THE VALUES OF ASUM AND ESUM ARE WITHIN TOLERANCE OR
C THE MAXIMUM NUMBER OF ITERATIONS (MAX) HAVE BEEN PERFORMED.
C
C *****
C COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPE,ZASNPR,
C * ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERN,NTRUNK,ASNCRV(100)
C * ,OC(100),CCSNPR(100),SNERTL(500),SNERTU(500),SPRTL(500),
C * SPRTU(500),XERLCT(100)
C RNPRI=NPRIME;RNMAT=NMAX
C CALCULATE ZALPHA (NPRIME)
C ZAT=ZASPRT*(1.0+(1.0-BNPRIE/RNMAT));XBRTMP=XA;NTIME=0
C CHECK TO SEE IF ASUM AND ESUM ARE WITHIN 0.001 OF THE SPECIFIED
C ALPHA AND BETA AFTER CALLING SUBROUTINE SNPR.
C 1 CALL SNPR(ZAT,XBRTMP)
C IF (ABS(ALPHA-ASUM).LE.0.001.AND.ABS(BETA-BSUM).LE.0.001) GOTO 50
C IF (NTIME.GT.MAX) GO TO 50
C CALCULATE THE DISTANCES OF ASUM AND ESUM FROM ALPHA AND BETA.
C DIP=ABS((ASUM-ALPHA)-(ESUM-BETA))
C DETERMINE IF ASUM MUST INCREASE OR DECREASE AND ALTER ACCORDINGLY.
C CALCULATE NEW ASUM AND ESUM.
C IF ((ASUM-ALPHA).LT.(ESUM-BETA)) GOTO 10
C SIGN=1
C GOTO 20
C 10 SIGN=-1
C 20 AMARK=DIP*(1/(BETA/ALPHA+1))
C ALTER XBARPRIME BY A FIXED AMOUNT.
C SENEST=SIGMA/100.0
C XBRTM1=XBRTMP+SENEST*SIGN;ASUMPV=ASUM
C CALCULATE THE EFFECT OF THE ALTERATION.
C CALL SNPR(ZAT,XBRTM1)
C CALCULATE THE PROPORTIONATE VALUE BY WHICH XBARPRIME MUST BE ALTERED
C TO MEET THE OBJECTIVE VALUE OF ALPHA.
C SENCAL=SENEST*AMARK/ABS(ASUMPV-ASUM)
C XBRTMP=XBRTMP+SENCAL*SIGN
C CALL SNPR(ZAT,XBRTMP)
C CHECK TO SEE IF ASUM AND ESUM ARE WITHIN LIMITS.
C IF (ABS(ALPHA-ASUM).LE.0.001.AND.ABS(BETA-BSUM).LE.0.001) GOTO 50
C DETERMINE IF ASUM MUST INCREASE OR DECREASE. ALTER ZAT ACCORDINGLY
C FOR A NEW ASUM AND BSUM.
C IF ((ASUM-ALPHA).LT.0.0) GOTO 30
C SIGN=1
C GOTO 40
C 30 SIGN=-1
C 40 AMARK=ABS(ASUM-ALPHA)
C ALTER ZALPHA BY A FIXED AMOUNT.
C ZATT=ZAT+0.01*SIGN;ASUMPV=ASUM
C CALL SNPR(ZATT,XBRTMP)

```

```
C CALCULATE THE PROPORTIONATE CHANGE IN ZALPHA NECESSARY TO ACHIEVE THE
C OBJECTIVE VALUES OF ALPHA AND ETA.
  SENCAL=0.01*AMARK/ABS(ASOMPV-ASOM)
  ZAT=ZAT+SENCAL*SIGN
C INCREMENT THE ITERATION COUNTER.
  NTIME=NTIME+1
  GOTO 1
50  ZASNPP=ZAT;IASNPR=XERTMP
  RETURN
  END
```

```

      SUBROUTINE OUTPUT
      *****
C THIS SUBROUTINE FORMATS AND OUTPUTS THE RESULTS ACCORDING TO THE
C REQUESTED FORMAT.
C
C*****
COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPF,XITPD,ZASNEF,
* ZASPEI,IPRINT,MAX,NMAX,NPRIME,ITERM,NTRUNK,ASNCV(100)
* ,OC(100),CCSNEF(100),SNPEFL(500),SNPEIU(500),SPPL(500),
* SPPTU(500),XERICT(100)
C DETERMINE THE OUTPUT FORMAT.
  ITERM=NTRUNK
  IF(IPRINT.EQ.1)GOTO 1
  IF(IPRINT.EQ.2)GOTO 2
  IF(IPRINT.EQ.3)GOTO 3
C FORMAT FOR THE SPF PLAN.
1  WRITE(6,100)
   WRITE(6,101)
   WRITE(6,102)
   WRITE(6,101)
   WRITE(6,100)
   PRINT,' '
   PRINT,' '
   WRITE(6,103)
   PRINT,' '
   WRITE(6,104) XAQL,XITPD,SIGMA
   WRITE(6,105) ALPHA,EETA,ITERM
   PRINT,' '
   WRITE(6,103)
   WRITE(6,103)
   PRINT,' '
   WRITE(6,106)
   PRINT,' '
   WRITE(6,103)
   DO 10 I=1,ITERM
10  WRITE(6,107) I,SPPEI(I),SPPTU(I),XERICT(I),OC(I),ASNCV(I)
   WRITE(6,108) XERICT(ITERM+1),OC(ITERM+1),ASNCV(ITERM+1)
   ITEMP1=ITERM+2
   DO 11 I=ITEMP1,ITERM
11  WRITE(6,109) XEPLT(I),OC(I),ASNCV(I)
   WRITE(6,103)
   GOTO 1000
C FORMAT FOR THE SNF PLAN.
2  WRITE(6,200)
   WRITE(6,201)
   WRITE(6,202)
   WRITE(6,201)
   WRITE(6,200)
   PRINT,' '
   PRINT,' '
   WRITE(6,200)
   PRINT,' '
   WRITE(6,104) XAQL,XITPD,SIGMA
   WRITE(6,204) ASUM,BSUM,NPRIME
   WRITE(6,203) ZASNEF,XASNPF
   PRINT,' '
   WRITE(6,200)
   WRITE(6,200)
   PRINT,' '
   WRITE(6,205)

```

```

PRINT, ' '
WRITE(6,200)
DC 20 I=1,NPRIME
20 WRITE(6,206) I,SNFRTI(I),SNPFTU(I),XERLOT(I),CCSNPF(I)
WRITE(6,207) XERLOT(NPRIME+1),OCSNPF(NPRIME+1)
ITEMP1=NPRIME+2
DC 21 I=ITEMP1,NTERM
21 WRITE(6,208) XERICT(I),CCSNPF(I)
WRITE(6,200)
GOTO 1000
C FORMAT FOR THE SNEF AND SER PLANS.
3 WRITE(6,300)
WRITE(6,301)
WRITE(6,302)
WRITE(6,301)
WRITE(6,300)
PRINT, ' '
PRINT, ' '
WRITE(6,303)
PRINT, ' '
WRITE(6,304) XAQL,XITFD,SIGMA
WRITE(6,305)
WRITE(6,306) ASUE,BSUE,NPRIME,ZASNEF,XASNPF
WRITE(6,307)
WRITE(6,308) ALPHA,PETA,ITERM
PRINT, ' '
WRITE(6,303)
WRITE(6,303)
PRINT, ' '
WRITE(6,3091)
WRITE(6,309)
PRINT, ' '
WRITE(6,303)
DO 30 I=1,NPRIME
30 WRITE(6,310) I,SNFRTI(I),SNPFTU(I),SPFTL(I),SPFTU(I),XERLOT(I),OCS
* NPF(I),CC(I)
I=NPRIME+1
WRITE(6,311) I,SPFTL(I),SPFTU(I),XERICT(I),OCSNPF(I),OC(I)
ITEMP=NPRIME+2
DO 31 I=ITEMP,ITERM
31 WRITE(6,312) I,SPFTL(I),SPFTU(I),XERLOT(I),OCSNPF(I),CC(I)
WRITE(6,313) XERICT(ITERM+1),CCSNPF(ITERM+1),OC(ITERM+1)
ITEMP1=ITERM+2
DO 32 I=ITEMP1,NTERM
32 WRITE(6,314) XERICT(I),CCSNPF(I),CC(I)
WRITE(6,303)
GOTO 1000
100 FORMAT(6X,46(' '))
101 FORMAT(6X,'*',44X,'*')
102 FORMAT(6X,'* SEQUENTIAL PROBABILITY RATIO SAMPLING PLAN *')
103 FORMAT(1X,56(' '))
104 FORMAT(6X,'XAQL=',F7.2,2X,'XITFD=',F7.2,2X,'SIGMA=',F6.2)
105 FORMAT(6X,'ALPHA=',F4.2,2X,'PETA=',F4.2,3X,'TRUNCATE AT X= ',I4)
106 FORMAT(1X,'*',2X,'N',2X,'*',4X,'TL',8X,'TU',4X,'*ICT XBAR*PFCE AC
*C* ASN *')
107 FORMAT(1X,'*',I4,1X,'*',F9.2,'*',F9.2,'*',F7.2,'*',F6.4,'*',
*F6.2,'*')
108 FORMAT(1X,29(' '),F7.2,'*',F6.4,'*',F6.2,'*')
109 FORMAT(1X,'*',26X,'*',F7.2,'*',F6.4,'*',F6.2,'*')
200 FORMAT(1X,50(' '))

```

```

201  FORMAT(1X,'*',4EX,'*')
202  FORMAT(1X,'* SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN *')
203  FORMAT(6X,'ZA=',F6.2,2X,'XABAR=',F7.2)
204  FORMAT(6X,'ALPHA=',F4.2,2X,'BETA=',F4.2,3X,'NPRIME=',I4)
205  FORMAT(1X,'* N ',4X,'TL',4X,'*',4X,'TU',4X,'**ICT XBAR*FFCF ACC
**')
206  FORMAT(1X,'*',I4,'*',F9.2,'*',F9.2,'**',F7.2,'*',F6.4,'*')
207  FORMAT(1X,30(' '),F7.2,'*',F6.4,'*')
208  FORMAT(1X,'*',27X,'**',F7.2,'*',F6.4,'*')
300  FORMAT(8X,68(' '))
301  FORMAT(8X,'*',66X,'*')
302  FORMAT(8X,'* SEQUENTIAL PROBABILITY AND NON-PROBABILITY RATIO SAM
PING PLANS *')
303  FORMAT(1X,82(' '))
304  FORMAT(1X,'XAQI=',F7.2,2X,'XITPE=',F7.2,2X,'SIGMA=',F6.2)
305  FORMAT(10X,'SNPB PLAN')
306  FORMAT(1X,'ALPHA=',F4.2,2X,'BETA=',F4.2,2X,'NPRIME=',I4,3X,'ZA=',F
*6.2,2X,'XABAR=',F7.2)
307  FORMAT(10X,'SPF PLAN')
308  FORMAT(1X,'ALPHA=',F4.2,2X,'BETA=',F4.2,3X,'TRUNCATION OCCURS AT N
*',I4)
3091  FORMAT(17X,'SNPRT',8X,'**',9X,'SPRT',20X,'SNPRT',2X,'* SPRT')
309  FORMAT(1X,'* ',4X,'T1',4X,'*',4X,'TU',4X,'**',4X,'TL',4X,'*',
*,4X,'TU',4X,'**ICT XBAR*FFCF ACC*FFCF ACC*')
310  FORMAT(1X,'*',I4,'*',F9.2,'*',F9.2,'**',F9.2,'*',F9.2,'**',F
*7.2,'*',F5.3,'*',F5.3,'*')
311  FORMAT(1X,'*',I4,1X,25(' '),F9.2,'*',F9.2,'**',F7.2,'*',F5.3,
*','F5.3,'*')
312  FORMAT(1X,'*',I4,'*',21X,'**',F9.2,'*',F9.2,'**',F7.2,'*',F5
*.3,'*',F5.3,'*')
313  FORMAT(1X,54(' '),F7.2,'*',F5.3,'*',F5.3,'*')
314  FORMAT(1X,'*',51X,'**',F7.2,'*',F5.3,'*',F5.3,'*')
1000  RETURN

```

APPENDIX B SELECT SNPR TEST PLANS

The following pages contain SNPR Test Plans for twelve sets of sample data. In each case $\bar{X}'_{AQL} = 45$ and $\bar{X}'_{LTPD} = 60$. Three different values are used for σ' ; 15, 22.5 and 30 (these values represent 1, 1.5 and 2 times the difference $(\bar{X}'_{AQL} - \bar{X}'_{LTPD})$). For each value of σ' , two plans are generated with n' equal to the maximum ASN of the corresponding SPR test and approximately 70% of that value. Two sets of plans for $\alpha = .05, \beta = .10$ and $\alpha = .01, \beta = .01$ are generated in each of the above cases.

$\bar{X}'_{AQL}$	$\bar{X}'_{LTPD}$	α	β	σ'	n'
40	65	.05	.10	15.0	6
					7
				22.5	11
					15
				30.0	19
					27
		.01	.01	15.0	15
					22
				22.5	34
					48
				30.0	60
					85

 * SEQUENTIAL MCN-FECURABILITY RATIO SAMPLING PLAN *
 * SEQUENTIAL MCN-FECURABILITY FATIG SAMPLING PLAN *

XAQL= 45.00 XLTPD= 60.00 SIGMA= 15.00
 ALPHA=0.05 BETA=0.10 NPRINT= 6
 ZA= 2.81 XABAR= 53.52

* I * TL * TU * *LCI YEAR*PBOE ACC* * N * TL * TU * *LCI YEAR*PBOE ACC*

 * 1 * 17.23 * 89.83 * 41.25 * 0.9870 * 1 * 11.30 * 95.73 * 41.25 * 0.9881 *
 * 2 * 76.82 * 137.32 * 42.12 * 0.9819 * 2 * 73.27 * 140.81 * 42.12 * 0.9831 *
 * 3 * 136.40 * 184.80 * 43.00 * 0.9751 * 3 * 135.23 * 185.89 * 43.00 * 0.9762 *
 * 4 * 195.99 * 232.28 * 43.81 * 0.9659 * 4 * 197.19 * 230.96 * 43.87 * 0.9669 *
 * 5 * 255.57 * 279.77 * 44.75 * 0.9536 * 5 * 259.16 * 276.04 * 44.75 * 0.9542 *
 * 6 * 315.15 * 327.25 * 45.62 * 0.9373 * 6 * 321.12 * 321.12 * 45.62 * 0.9375 *
 * 7 * 374.74 * 374.74 * 46.50 * 0.9162 * 7 * 321.12 * 321.12 * 46.50 * 0.9156 *
 * 8 * 434.33 * 434.33 * 47.37 * 0.8891 * 8 * 374.74 * 374.74 * 47.37 * 0.8878 *
 * 9 * 493.92 * 493.92 * 48.25 * 0.8551 * 9 * 434.33 * 434.33 * 48.25 * 0.8530 *
 * 10 * 553.50 * 553.50 * 49.12 * 0.8136 * 10 * 493.92 * 493.92 * 49.12 * 0.8127 *
 * 11 * 613.09 * 613.09 * 50.00 * 0.7641 * 11 * 553.50 * 553.50 * 50.00 * 0.7608 *
 * 12 * 672.68 * 672.68 * 50.87 * 0.7070 * 12 * 613.09 * 613.09 * 50.87 * 0.7031 *
 * 13 * 732.27 * 732.27 * 51.75 * 0.6422 * 13 * 672.68 * 672.68 * 51.75 * 0.6402 *
 * 14 * 791.86 * 791.86 * 52.62 * 0.5744 * 14 * 732.27 * 732.27 * 52.62 * 0.5721 *
 * 15 * 851.45 * 851.45 * 53.50 * 0.5028 * 15 * 791.86 * 791.86 * 53.50 * 0.5016 *
 * 16 * 911.04 * 911.04 * 54.37 * 0.4311 * 16 * 851.45 * 851.45 * 54.37 * 0.4310 *
 * 17 * 970.63 * 970.63 * 55.25 * 0.3620 * 17 * 911.04 * 911.04 * 55.25 * 0.3620 *
 * 18 * 1030.22 * 1030.22 * 56.12 * 0.2977 * 18 * 970.63 * 970.63 * 56.12 * 0.2991 *
 * 19 * 1089.81 * 1089.81 * 57.00 * 0.2400 * 19 * 1030.22 * 1030.22 * 57.00 * 0.2416 *
 * 20 * 1149.40 * 1149.40 * 57.87 * 0.1900 * 20 * 1089.81 * 1089.81 * 57.87 * 0.1914 *
 * 21 * 1208.99 * 1208.99 * 58.75 * 0.1478 * 21 * 1149.40 * 1149.40 * 58.75 * 0.1481 *
 * 22 * 1268.58 * 1268.58 * 59.62 * 0.1133 * 22 * 1208.99 * 1208.99 * 59.62 * 0.1177 *
 * 23 * 1328.17 * 1328.17 * 60.50 * 0.0857 * 23 * 1268.58 * 1268.58 * 60.50 * 0.0855 *
 * 24 * 1387.76 * 1387.76 * 61.37 * 0.0641 * 24 * 1328.17 * 1328.17 * 61.37 * 0.0634 *
 * 25 * 1447.35 * 1447.35 * 62.25 * 0.0475 * 25 * 1387.76 * 1387.76 * 62.25 * 0.0464 *
 * 26 * 1506.94 * 1506.94 * 63.12 * 0.0349 * 26 * 1447.35 * 1447.35 * 63.12 * 0.0336 *
 * 27 * 1566.53 * 1566.53 * 64.00 * 0.0255 * 27 * 1506.94 * 1506.94 * 64.00 * 0.0241 *
 * 28 * 1626.12 * 1626.12 * 64.87 * 0.0185 * 28 * 1566.53 * 1566.53 * 64.87 * 0.0172 *
 * 29 * 1685.71 * 1685.71 * 65.75 * 0.0134 * 29 * 1626.12 * 1626.12 * 65.75 * 0.0121 *
 * 30 * 1745.30 * 1745.30 * 66.62 * 0.0096 * 30 * 1685.71 * 1685.71 * 66.62 * 0.0075 *

 * SEQUENTIAL MCN-FREQUENCY RATIO SAMPLING PLAN *
 * *****

 XACL= 45.00 XLTPD= 60.00 SIGMA= 22.50
 ALPHA=0.05 BETA=0.10 NPRINT= 11
 ZA= 5.0E XAEAR= 53.50

 * *****

* A * TL * TU * LCT XEAR*PCE ACC * I * TI * TU * LCT XEAR*PCE ACC *

1	-34.75	141.83	41.25	C.9867	1	-60.79	167.78	41.25	C.9881
2	25.10	189.06	42.12	C.9818	2	4.14	209.85	42.12	C.9831
3	84.95	236.30	43.00	C.9751	3	69.07	251.92	43.00	C.9783
4	144.80	283.53	43.87	C.9661	4	133.99	293.99	43.87	C.9669
5	204.64	330.77	44.75	C.9541	5	198.92	336.06	44.75	C.9542
6	264.49	378.00	45.62	C.9382	6	263.85	378.13	45.62	C.9375
7	324.34	425.24	46.50	C.9175	7	328.77	420.20	46.50	C.9156
8	384.19	472.47	47.37	C.8910	8	393.70	462.27	47.37	C.8877
9	444.03	519.71	48.25	C.8576	9	458.63	504.34	48.25	C.8528
10	503.88	566.94	49.12	C.8165	10	523.55	546.41	49.12	C.8104
11	563.73	614.18	50.00	C.7673	11	588.48	588.48	50.00	C.7603
12	623.58	661.41	50.87	C.7101				50.87	C.7029
13	683.42	708.65	51.75	C.6456				51.75	C.6391
14	743.27	755.88	52.62	C.5761				52.62	C.5707
15	803.12	803.12	53.50	C.5034				53.50	C.4959
			54.37	C.4307				54.37	C.4290
			55.25	C.3606				55.25	C.3606
			56.12	C.2957				56.12	C.2956
			57.00	C.2377				57.00	C.2395
			57.87	C.1878				57.87	C.1854
			58.75	C.1459				58.75	C.1471
			59.62	C.1118				59.62	C.1122
			60.50	C.0847				60.50	C.0843
			61.37	C.0635				61.37	C.0625
			62.25	C.0472				62.25	C.0457
			63.12	C.0349				63.12	C.0331
			64.00	C.0256				64.00	C.0237
			64.87	C.0187				64.87	C.0168
			65.75	C.0137				65.75	C.0119
			66.62	C.0099				66.62	C.0083

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** SEQUENTIAL ACN-EFFECTIVITY RATIO SAMPLING PLAN ** SEQUENTIAL NCN-PROBABILITY FATIG SPALLING PLAN **
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XACI= 45.70 XITPD= 60.00 SIGMA= 30.00
ALPHA=0.05 PETA=0.10 NPFIF= 27
ZA= 5.24 XAIAR= 53.55
XAOI= 45.00 XITPD= 60.00 SIGMA= 30.00
ALPHA=0.05 PETA=0.10 NPFIF= 19
ZA= 6.43 XAEAR= 53.54
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[illegible]

N	TI	TU	DEPRCE	ACC	N	TI	TU	DEPRCE	ACC	N	TI	TU	DEPRCE	ACC
1	-113.66	213.76	41.25	C.5864	1	-139.28	246.36	41.25	C.9875	1	-139.28	246.36	41.25	C.9875
2	-44.06	253.26	42.12	0.9815	2	-75.03	289.18	42.12	0.9825	2	-75.03	289.18	42.12	0.9825
3	15.53	105.77	43.00	0.9748	3	-10.78	332.01	43.00	0.9757	3	-10.78	332.01	43.00	0.9757
4	75.13	153.27	43.87	C.9655	4	53.47	374.83	43.87	C.9665	4	53.47	374.83	43.87	C.9665
5	134.71	400.77	44.75	C.9540	5	117.72	417.66	44.75	0.9541	5	117.72	417.66	44.75	0.9541
6	194.32	498.28	45.62	0.9384	6	181.97	460.48	45.62	0.9378	6	181.97	460.48	45.62	0.9378
7	253.92	495.78	46.50	C.9180	7	246.22	503.31	46.50	C.9185	7	246.22	503.31	46.50	C.9185
8	313.52	543.28	47.37	C.8918	8	310.47	546.13	47.37	0.8893	8	310.47	546.13	47.37	0.8893
9	373.11	590.75	48.25	0.8588	9	374.72	588.96	48.25	C.8553	9	374.72	588.96	48.25	C.8553
10	432.71	638.29	49.12	0.8181	10	436.57	631.78	49.12	C.8136	10	436.57	631.78	49.12	C.8136
11	492.31	685.80	50.00	0.7691	11	503.22	674.61	50.00	0.7641	11	503.22	674.61	50.00	0.7641
12	551.90	733.30	50.87	0.7120	12	567.47	717.44	50.87	0.7070	12	567.47	717.44	50.87	0.7070
13	611.53	783.80	51.75	C.6475	13	631.72	763.26	51.75	C.6433	13	631.72	763.26	51.75	C.6433
14	671.10	828.31	52.62	0.5774	14	695.97	803.06	52.62	0.5745	14	695.97	803.06	52.62	0.5745
15	730.69	875.81	53.50	0.5042	15	760.21	885.91	53.50	0.5031	15	760.21	885.91	53.50	0.5031
16	790.29	923.31	54.37	C.4305	16	824.46	888.74	54.37	C.4315	16	824.46	888.74	54.37	C.4315
17	849.89	970.82	55.25	0.3603	17	888.71	931.56	55.25	C.3624	17	888.71	931.56	55.25	C.3624
18	909.44	110.12	56.12	0.2951	18	952.96	974.39	56.12	0.2942	18	952.96	974.39	56.12	0.2942
19	969.04	1065.82	57.00	0.2370	19	1317.21	1017.21	57.00	C.2405	19	1317.21	1017.21	57.00	C.2405
20	1028.64	1113.31	57.87	0.1871	20	1317.21	1017.21	57.87	0.1903	20	1317.21	1017.21	57.87	0.1903
21	1088.27	1163.83	58.75	C.1454	21	1317.21	1017.21	58.75	C.1460	21	1317.21	1017.21	58.75	C.1460
22	1147.87	1204.33	59.62	0.1116	22	1317.21	1017.21	59.62	C.1133	22	1317.21	1017.21	59.62	C.1133
23	1207.47	1255.84	60.50	0.0847	23	1317.21	1017.21	60.50	0.0850	23	1317.21	1017.21	60.50	0.0850
24	1267.06	1303.34	61.37	C.0637	24	1317.21	1017.21	61.37	C.0638	24	1317.21	1017.21	61.37	C.0638
25	1326.66	1350.85	62.25	0.0476	25	1317.21	1017.21	62.25	0.0476	25	1317.21	1017.21	62.25	0.0476
26	1386.26	1398.35	63.12	C.0353	26	1317.21	1017.21	63.12	C.0344	26	1317.21	1017.21		

 * SEQUENTIAL MCN-RELIABILITY RATIO SAMPLING PLAN *
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YAOL= 45.00 XLTED= 60.00 SIGMA= 15.00
 ALPHA=0.01 PETA=0.01 NPRINT= 15
 ZA= 5.22 XADAR= 52.50

* I * TI * TH * *LCI XBAR*PROE ACC* * N * TL * TH * *LCI XBAR*PROE ACC*

1	-13.14	118.15	44.25	C.9533	1	-25.83	133.83	44.25	C.9533
2	42.48	167.52	44.80	C.9505	2	32.26	177.74	44.80	C.9505
3	46.11	216.89	45.35	C.9867	3	90.36	224.64	45.35	C.9867
4	153.74	266.27	45.90	C.9815	4	148.46	271.55	45.90	C.9815
5	219.36	315.64	46.45	C.9741	5	206.55	318.46	46.45	C.9735
6	264.59	365.02	47.00	C.9641	6	264.65	365.36	47.00	C.9627
7	320.61	414.35	47.55	C.9504	7	322.74	412.27	47.55	C.9482
8	376.24	463.77	48.10	C.9320	8	380.64	459.17	48.10	C.9283
9	431.87	513.14	48.65	C.9075	9	438.93	506.08	48.65	C.9024
10	487.49	562.52	49.20	C.8757	10	497.03	552.98	49.20	C.8691
11	543.12	611.89	49.75	C.8352	11	555.13	599.89	49.75	C.8274
12	598.75	661.27	50.30	C.7850	12	613.22	646.79	50.30	C.7768
13	654.37	710.64	50.85	C.7250	13	671.32	693.70	50.85	C.7172
14	710.00	760.02	51.40	C.6560	14	729.41	740.60	51.40	C.6459
15	765.63	809.39	51.95	C.5799	15	787.51	787.51	51.95	C.5766
16	821.25	858.77	52.50	C.5001				52.50	C.5001
17	876.88	908.14	53.05	C.4202				53.05	C.4236
18	932.51	957.51	53.60	C.3442				53.60	C.3503
19	988.13	1006.89	54.15	C.2752				54.15	C.2829
20	1043.76	1056.26	54.70	C.2151				54.70	C.2234
21	1099.39	1105.64	55.25	C.1650				55.25	C.1727
22	1155.01	1155.01	55.80	C.1244				55.80	C.1310
			56.35	C.0925				56.35	C.0977
			56.90	C.0681				56.90	C.0718
			57.45	C.0456				57.45	C.0521
			58.00	C.0359				58.00	C.0374
			58.55	C.0259				58.55	C.0266
			59.10	C.0166				59.10	C.0188
			59.65	C.0133				59.65	C.0131
			60.20	C.0055				60.20	C.0067

 * SEQUENTIAL NON-EQUIVALENT RATIO SAMPLING PLAN *

AAQI= 45.00 XLTED= 60.00 SIGMA= 22.50
 ALPHA=0.01 BETA=0.01 NPRINT= 34
 TA= 7.76 XAPAS= 52.50

* A * TI * TU * *LCT *NBAR*PECE ACC*

1	-122.13	227.13	44.25	C.9538	31	1611.63	1643.38	54.15	0.2770
2	-64.34	274.34	44.58	C.9923	32	1669.42	1690.55	54.48	0.2355
3	-6.55	321.55	44.91	0.9905	33	1727.21	1737.80	54.81	0.2061
4	51.24	368.76	45.24	0.9883	34	1785.00	1785.00	55.14	0.1758
5	109.04	415.97	45.57	0.9855	*****	*****	*****	55.47	0.1489
6	166.83	463.17	45.90	0.9821	*****	*****	*****	55.80	0.1234
7	224.62	510.38	46.23	0.9780	*****	*****	*****	56.13	0.1050
8	282.41	557.59	46.56	0.9730	*****	*****	*****	56.46	0.0875
9	340.20	604.80	46.89	0.9669	*****	*****	*****	56.79	0.0726
10	398.00	652.01	47.22	0.9595	*****	*****	*****	57.12	0.0599
11	455.79	699.21	47.55	0.9507	*****	*****	*****	57.45	0.0493
12	513.58	746.42	47.88	0.9401	*****	*****	*****	57.78	0.0405
13	571.37	793.63	48.21	0.9274	*****	*****	*****	58.11	0.0331
14	629.16	840.84	48.54	0.9125	*****	*****	*****	58.44	0.0270
15	686.96	888.05	48.87	0.8950	*****	*****	*****	58.77	0.0220
16	744.75	935.26	49.20	0.8746	*****	*****	*****	59.10	0.0176
17	802.54	982.46	49.53	0.8511	*****	*****	*****	59.43	0.0145
18	860.33	1029.67	49.86	0.8242	*****	*****	*****	59.76	0.0117
19	918.12	1076.88	50.19	0.7939	*****	*****	*****	60.09	0.0095
20	975.92	1124.09	50.52	0.7601	*****	*****	*****	60.42	0.0077
21	1033.71	1171.30	50.85	0.7230	*****	*****	*****	*****	*****
22	1091.51	1218.51	51.18	0.6827	*****	*****	*****	*****	*****
23	1149.29	1265.71	51.51	0.6397	*****	*****	*****	*****	*****
24	1207.08	1312.92	51.84	0.5944	*****	*****	*****	*****	*****
25	1264.83	1360.13	52.17	0.5476	*****	*****	*****	*****	*****
26	1322.67	1407.34	52.50	0.5000	*****	*****	*****	*****	*****
27	1380.46	1454.55	52.83	0.4524	*****	*****	*****	*****	*****
28	1438.25	1501.75	53.16	0.4056	*****	*****	*****	*****	*****
29	1496.01	1548.96	53.49	0.3604	*****	*****	*****	*****	*****
30	1553.84	1596.17	53.82	0.3173	*****	*****	*****	*****	*****

 * SEQUENTIAL NON-FIDELITY RATIO SAMPLING PLAN *

XACL= 45.00 XLTRE= 60.00 SIGMA= 22.50
 ALPHA=0.01 UETA=0.01 NPRIME= 48
 ZA= 6.89 XABAC= 52.53

* N * TL * TH *LCT XBARPROP ACC*

1	-102.56	207.61	44.25	0.9938	31	1572.28	1684.47	54.15	0.2747
2	-46.73	256.84	44.58	0.9924	32	1628.11	1733.70	54.46	0.2365
3	9.10	306.07	44.91	0.9906	33	1683.94	1782.93	54.81	0.2024
4	64.93	355.30	45.24	0.9885	34	1739.76	1832.15	55.14	0.1724
5	120.75	404.53	45.57	0.9859	35	1795.59	1881.38	55.47	0.1457
6	176.58	453.76	45.90	0.9827	36	1851.42	1930.61	55.80	0.1224
7	232.41	502.98	46.23	0.9789	37	1907.25	1979.84	56.13	0.1024
8	288.24	552.21	46.56	0.9742	38	1963.07	2029.07	56.46	0.0852
9	344.07	601.44	46.89	0.9685	39	2018.90	2078.30	56.79	0.0707
10	399.89	650.67	47.22	0.9616	40	2074.73	2127.53	57.12	0.0585
11	455.72	699.90	47.55	0.9533	41	2130.56	2176.75	57.45	0.0482
12	511.55	749.13	47.88	0.9434	42	2186.39	2225.98	57.78	0.0397
13	567.38	798.36	48.21	0.9315	43	2242.21	2275.21	58.11	0.0324
14	623.21	847.58	48.54	0.9174	44	2298.04	2324.44	58.44	0.0267
15	679.03	896.81	48.87	0.9008	45	2353.87	2373.67	58.77	0.0219
16	734.86	946.04	49.20	0.8812	46	2409.70	2422.90	59.10	0.0179
17	790.69	995.27	49.53	0.8585	47	2465.53	2472.13	59.43	0.0146
18	846.52	1044.50	49.86	0.8324	48	2521.35	2521.35	59.76	0.0116
19	902.34	1093.73	50.19	0.8026	*****	*****	*****	60.09	0.0057
20	958.17	1142.96	50.52	0.7652	*****	*****	*****	60.42	0.0074
21	1014.00	1192.18	50.85	0.7320	*****	*****	*****	*****	*****
22	1069.83	1241.41	51.18	0.6914	*****	*****	*****	*****	*****
23	1125.66	1290.64	51.51	0.6477	*****	*****	*****	*****	*****
24	1181.48	1339.87	51.84	0.6014	*****	*****	*****	*****	*****
25	1237.31	1389.10	52.17	0.5533	*****	*****	*****	*****	*****
26	1293.14	1438.33	52.50	0.5042	*****	*****	*****	*****	*****
27	1348.97	1487.55	52.83	0.4550	*****	*****	*****	*****	*****
28	1404.80	1536.78	53.16	0.4074	*****	*****	*****	*****	*****
29	1460.62	1586.01	53.49	0.3601	*****	*****	*****	*****	*****
30	1516.45	1635.24	53.82	0.3159	*****	*****	*****	*****	*****

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