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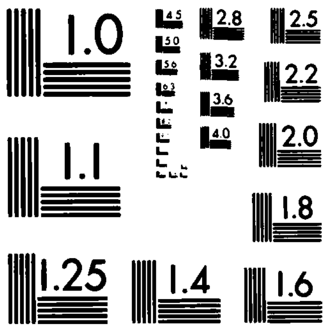

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A PROOF OF THE ODD-SYMMETRY OF THE PHASES FOR MINIMUM WEIGHT PERTURBATION, PHASE-ONLY NULL SYNTHESIS

Robert A. Shore

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441

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A proof is given of the odd-symmetry property of the phase perturbations required to impose nulls in a real linear array antenna pattern using phase-only weight control subject to minimization of the absolute weight perturbations. The perturbed weights are, therefore, conjugate symmetric with respect to a phase reference at the center of the array, and the perturbed pattern is real.

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Preface

The author wishes to express his thanks and appreciation to Dr. Ronald L. Fante for a very helpful communication, to Dr. Hans Steyskal for many stimulating conversations on the subject of this report, and to Dr. Robert J. Papa for a critical reading of the draft of this report.

Contents

A Proof of the Odd-Symmetry of the Phases for Minimum Weight Perturbation, Phase-Only Null Synthesis

1. INTRODUCTION

The current interest in phase-only control of the element weights of linear arrays for adaptive nulling[1-9] and null synthesis[10-16] is a result of the increasing importance of phased arrays and adaptive processing. Null synthesis has considerable value for adaptive nulling in that it can help determine limits to what can be achieved adaptively.

In null synthesis, as in adaptive nulling, it is generally undesirable to impose nulls in a given array antenna pattern at the expense of large distortion in regions of the pattern aside from the imposed null locations. One way to reduce such pattern distortion is to minimize the perturbations of the element weights. The imposing of nulls in the pattern of a linear array by phase-only weight control subject to minimization of the weight perturbations is, unlike for combined phase and amplitude control, a nonlinear problem and, in general, cannot be solved analytically. Numerical technques[14, 17] must be used to calculate the phases required for minimized weight perturbation, phase-only null synthesis.

If the pattern in which nulls are to be imposed is real, it is reasonable to assume that the pattern satisfying the null constraints and corresponding to minimum element weight perturbations should likewise be real. The phase perturbations of

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Because of the large number of references cited above, they will not be listed here. See References, page 19.

the element weights are then odd-symmetric with respect to a phase reference located at the center of the array. If nonlinear programming or other numerical techniques are used to calculate the phases for null synthesis, the odd-symmetry property of the phases can be used to reduce the number of unknown phases by a factor of a half, thereby significantly increasing computational efficiency. Although intuitively reasonable, nevertheless a proof of the odd-symmetry property is desirable. In an earlier report,[18] the author proved the odd-symmetry property for the case of a single imposed null. The method of proof, however, unfortunately did not generalize to multiple imposed nulls. It is the purpose of this report to give a general proof of the odd-symmetry property.

2. PROOF OF THE ODD-SYMMETRY OF THE PHASES FOR MINIMUM WEIGHT PERTURBATION, PHASE-ONLY NULL SYNTHESIS

We consider a linear array of N equispaced isotropic elements whose field pattern is given by

$$p_o(u) = \sum_{n=1}^{N} a_n e^{jd_n u} \quad . \tag{1}$$

In Eq. (1) the $\{a_n\}$ are the complex element weights,

$$d_n = (N - 1)/2 - (n - 1) = -d_{N-n+1} \quad , \quad n = 1, 2, \ldots, N \tag{2}$$

and

$$u = 2\pi/\lambda \, d \sin \theta \quad .$$

where

λ = wavelength,

d = interelement spacing, and

θ = angle measured from broadside to the array.

The phase reference is taken to be the center of the array. The pattern is assumed real so that the complex element weights satisfy the relation[19]

18. Shore, R.A. (1983) On the Odd-Symmetry of Minimum Phase-Only Perturbations, RADC-TR-83-26.

19. Oppenheim, A.V., and Schafer, R.W. (1975) Digital Signal Processing, Prentice-Hall, N.J., pp. 24-26.

$$a_{N-n+1} = a_n^* \quad , \quad n = 1, 2, \ldots, N \quad . \tag{3}$$

Let ϕ_n, $n = 1, 2, \ldots, N$ be the set of phase perturbations that (a) imposes nulls in the pattern at the locations $u = u_k$, $k = 1, 2, \ldots, K$:

$$\sum_{n=1}^{n} a_n e^{j\phi_n} e^{jd_n u_k} = 0 \quad , \quad k = 1, 2, \ldots, K \tag{4}$$

and (b) minimizes the weighted sum of the squares of the absolute values of the element weight perturbations

$$F = \sum_{n=1}^{N} c_n \left| a_n \left(e^{j\phi_n} - 1 \right) \right|^2 \quad . \tag{5}$$

The weighting coefficients, $\{c_n\}$, in Eq. (5) are assumed real, positive, and symmetric:*

$$c_{N-n+1} = c_n \quad , \quad n = 1, 2, \ldots, N \quad . \tag{6}$$

We wish to show that the phase perturbations are odd-symmetric:

$$\phi_{N-n+1} = -\phi_n \quad , \quad n = 1, 2, \ldots, N \quad , \tag{7}$$

so that the perturbed pattern,

$$p(u) = \sum_{n=1}^{N} a_n e^{j\phi_n} e^{jd_n u}$$

is real.

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*The choice of $c_n = 1$ for all n makes F the sum of the squares of the absolute values of the weight perturbations. For half wavelength spacing of the array elements, this is equivalent to minimizing the mean square pattern perturbation for θ from $-\pi/2$ to $+\pi/2$. Other choices of the c_n with practical application to null synthesis are also possible;[13] for example, $c_n = 1/|a_n|^2$.

The proof of Eq. (7) uses the method of Lagrangian multipliers.[20] We first rewrite the null constraints of Eq. (4) in the form

$$\sum_{n=1}^{N} a_n \left(e^{j\phi_n} - 1\right) e^{jd_n u_k} = -p_o(u_k) \quad , \quad k = 1, 2, \ldots, K \tag{8}$$

and then make the constraints purely real by multiplying the left-hand side of Eq. (8) by its complex conjugate and squaring the (real) right-hand side, thus obtaining

$$\sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m^* \left(e^{j\phi_n} - 1\right) \left(e^{-j\phi_m} - 1\right) e^{j(d_n - d_m)u_k} - p_o^2(u_k) = 0 \quad , \quad k = 1, 2, \ldots, K \quad . \tag{9}$$

Since

$$F = \sum_{n=1}^{N} c_n |a_n|^2 \left(e^{j\phi_n} - 1\right) \left(e^{-j\phi_n} - 1\right)$$

$$= \sum_{n=1}^{N} c_n |a_n|^2 (2 - 2 \cos \phi_n)$$

$$= 2 \sum_{n=1}^{N} c_n |a_n|^2 - 2 \sum_{n=1}^{N} c_n |a_n|^2 \cos \phi_n \quad , \tag{10}$$

minimizing F is equivalent to maximizing

$$F' = \sum_{n=1}^{N} c_n |a_n|^2 \cos \phi_n \quad . \tag{11}$$

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20. Fletcher, R. (1981) Practical Methods of Optimization; Vol. 2, Constrained Optimization, John Wiley & Sons, New York, Ch. 9.

We now form the Lagrangian

$$L = F' + \sum_{k=1}^{K} \lambda_k C_k$$

where the constraint functions, C_k, from Eq. (9) are given by

$$C_k = \sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m^* \left(e^{j\phi_n} - 1\right) \left(e^{-j\phi_m} - 1\right) e^{j(d_n - d_m)u_k} - p_o^2(u_k) \quad , \qquad k = 1, 2, \ldots, K \tag{12}$$

and the $\{\lambda_k\}$ are the (real) Lagrangian multipliers. For the $\{\phi_n\}$ to locally maximize (or minimize) F' subject to satisfying the constraints $C_k = 0$ it is necessary that the gradient of L with respect to the $\{\phi_n\}$ be zero. Accordingly we differentiate L with respect to an arbitrary one of the $\{\phi_n\}$, say ϕ_p, and equate the derivative to zero:

$$\frac{\partial F'}{\partial \phi_p} = - \sum_{k=1}^{K} \lambda_k \frac{\partial C_k}{\partial \phi_p} \quad . \tag{13}$$

From Eq. (11)

$$\frac{\partial F'}{\partial \phi_p} = - c_p |a_p|^2 \sin \phi_p \quad . \tag{14}$$

To differentiate C_k with respect to ϕ_p we group the terms of the double summation in Eq. (12) containing ϕ_p according as to whether (1) $n = p$, $m \neq p$; (2) $n \neq p$, $m = p$, or (3) $n = m = p$. For the first group of terms

$$\frac{\partial}{\partial \phi_p} \left[a_p \left(e^{j\phi_p} - 1\right) e^{jd_p u_k} \sum_{\substack{m=1 \\ m \neq p}}^{N} a_m^* \left(e^{-j\phi_m} - 1\right) e^{-jd_m u_k} \right]$$

$$= j a_p e^{j\phi_p} e^{jd_p u_k} \left[\sum_{m=1}^{N} a_m^* \left(e^{-j\phi_m} - 1\right) e^{-jd_m u_k} - a_p^* \left(e^{-j\phi_p} - 1\right) e^{-jd_p u_k} \right]$$

$$= -j\,p_o(u_k)\,a_p\,e^{j(\phi_p+d_pu_k)} - j|a_p|^2\left(1 - e^{j\phi_p}\right) \quad , \qquad (15a)$$

where we have used Eq. (8) and the fact that $p_o(u)$ is real. The contribution of the second group of terms to $\frac{\partial C_k}{\partial \phi_p}$ is simply the complex conjugate of that of the first group

$$j\,p_o(u_k)\,a_p^*\,e^{-j(\phi_p+d_pu_k)} + j|a_p|^2\left(1 - e^{-j\phi_p}\right) \quad , \qquad (15b)$$

while the contribution of the one term in group (3) is

$$\frac{\partial}{\partial\phi_p}\left[\,|a_p|^2\left(e^{j\phi_p} - 1\right)\left(e^{-j\phi_p} - 1\right)\right] = |a_p|^2\,\frac{\partial}{\partial\phi_p}\,(2 - 2\cos\phi_p)$$

$$2|a_p|^2\sin\phi_p \quad . \qquad (15c)$$

Summing the three contributions [Eqs. (15a) through 15(c)] we obtain

$$\frac{\partial C_k}{\partial\phi_p} = 2p_o(u_k)\,\mathrm{Im}\left\{a_p\,e^{j(\phi_p+d_pu_k)}\right\} \quad . \qquad (16)$$

Substituting Eq. (14) and Eq. (16) in Eq. (13) then yields

$$c_p|a_p|^2\sin\phi_p = 2\sum_{k=1}^{K}\lambda_k\,p_o(u_k)\;\mathrm{Im}\left\{a_p\,e^{j(\phi_p+d_pu_k)}\right\} \quad . \qquad (17)$$

Now write the complex amplitude a_p in the magnitude-and-phase form

$$a_p\;\;|a_p|\,e^{j\alpha_p} \quad . \qquad (18)$$

Equation (3) implies that

$$|a_{N-p+1}| = |a_p| \tag{19a}$$

$$\alpha_{N-p+1} = -\alpha_p \quad . \tag{19b}$$

Substituting Eq. (18) in Eq. (17),

$$\begin{aligned} c_p|a_p|\sin\phi_p &= 2\sum_{k=1}^{K} \lambda_k p_o(u_k)\sin(\phi_p + d_p u_k + \alpha_p) \\ &= \left[2\sum_{k=1}^{K} \lambda_k p_o(u_k)\cos(d_p u_k + \alpha_p)\right]\sin\phi_p \\ &\quad + \left[2\sum_{k=1}^{K} \lambda_k p_o(u_k)\sin(d_p u_k + \alpha_p)\right]\cos\phi_p \end{aligned}$$

from which, rearranging, we obtain

$$\tan\phi_p = \frac{2\sum_{k=1}^{K} \lambda_k p_o(u_k)\sin(d_p u_k + \alpha_p)}{c_p|a_p| - 2\sum_{k=1}^{K} \lambda_k p_o(u_k)\cos(d_p u_k + \alpha_p)} \quad . \tag{20}$$

But then, letting $p \to N - p + 1$,

$$\tan\phi_{N-p+1} = -\tan\phi_p$$

since, in view of Eqs. (2), (6), and (19), the numerator of Eq. (20) is changed in sign only, while the denominator remains unchanged.

Equation (21) implies that either

$$\phi_{N-p+1} = -\phi_p \tag{22}$$

or

$$\phi_{N-p+1} = -\phi_p + \pi \quad . \tag{23}$$

Equation (22) is the odd-symmetry property we are attempting to demonstrate. To complete the proof we show that if there is a solution to the constrained optimization problem containing one or more pairs of phases that satisfy Eq. (23), then by subtracting $\pi/2$ from these phases we obtain a new solution that satisfies Eq. (22) and gives the same value of the maximized function F' of Eq. (11). It then follows that the phase perturbations for minimized weight perturbation null synthesis can always be assumed to satisfy Eq. (22) without the risk of discarding a solution with a larger value of F'.

Suppose that there is a solution to the constrained optimization problem containing one or more pairs of phase perturbations that satisfy Eq. (23). (We do not claim that one actually exists.) We first note that the contribution to the total array pattern of those pairs of elements whose phase perturbations satisfy Eq. (23) is purely imaginary, while the contribution of the element pairs whose phase perturbations obey Eq. (22) is purely real. The two patterns, imaginary and real, are completely uncoupled since they derive from distinct element sets, and hence both patterns must have nulls at the imposed null locations $\{u_k\}$ for the constraints, Eq. (9), to be satisfied. It follows that if the weights of all the elements whose phase perturbations satisfy Eq. (23) are multiplied by the same constant, the resulting total pattern will still have nulls at all the imposed null locations. We now form a new set of phase perturbations by subtracting $\pi/2$ — equivalently multiplying the respective element weights by $e^{-j\pi/2} = -j$ — from all the phases satisfying Eq. (23). Denoting the new phase perturbations by ϕ'_p,

$$\phi'_p = \phi_p - \frac{\pi}{2}$$

and

$$\phi'_{N-p+1} = \phi_{N-p+1} - \frac{\pi}{2} = (-\phi_p + \pi) - \frac{\pi}{2} = -\left(\phi_p - \frac{\pi}{2}\right) = -\phi'_p \quad , \quad p \epsilon P' \quad , \tag{24}$$

where P' denotes the set of indices of all the phases satisfying Eq. (23). Thus the altered phase perturbations, $\{\phi'_p\}$, obey Eq. (22). The new total pattern is now purely real and, from the remark made just above about multiplying the element weights by a constant, has nulls at all the imposed null locations $\{u_k\}$ so that the null constraints, Eq. (9), continue to be satisfied. It remains to consider the value of F'. The contribution of the phases satisfying Eq. (23) to the value of F' is zero since, using Eqs. (6) and (19a),

$$c_p|a_p|^2 \cos \phi_p + c_{N-p+1}|a_{N-p+1}|^2 \cos \phi_{N-p+1}$$

$$= c_p|a_p|^2 \left[\cos \phi_p + \cos(-\phi_p + \pi)\right] \quad 0 \quad . \tag{25}$$

The contribution of the altered phase pairs to the value of F' is, using Eqs. (6), (19a), and (24)

$$\frac{1}{2} \sum_{p \epsilon P'} \left(c_p|a_p|^2 \cos \phi'_p + c_{N-p+1}|a_{N-p+1}|^2 \cos \phi'_{N-p+1} \right)$$

$$= \frac{1}{2} \sum_{p \epsilon P'} 2c_p|a_p| \cos\left(\phi_p - \frac{\pi}{2}\right) = \sum_{p \epsilon P'} c_p|a_p|^2 \sin \phi_p \quad ; \tag{26}$$

the factor of 1/2 is included because the phase pairs are counted twice, once for each member of the pair. Now if Eq. (17), the equation obtained by setting the derivative of the Lagrangian equal to zero, is summed with respect to p from 1 to N we obtain

$$\sum_{p=1}^{N} c_p|a_p|^2 \sin \phi_p = 2 \sum_{k=1}^{K} \lambda_k p_o(u_k) \left[\mathrm{Im} \left\{ \sum_{p=1}^{N} a_p e^{j(\phi_p + d_p u_k)} \right\} \right] \quad .$$

from which it follows by substituting Eq. (4) that

$$\sum_{p=1}^{N} c_p|a_p|^2 \sin \phi_p = 0 \tag{27}$$

Since the contribution to the sum on the left-hand side of Eq. (27) of all phase pairs satisfying Eq. (22) is zero, it follows that the contribution of the remaining pairs to the sum must also be zero. But this sum is precisely the sum on the right-hand side of Eq. (26). Hence, subtracting $\pi/2$ from all phases satisfying Eq. (23) does not alter the fact that their contribution to the value of F' is zero. We have thus shown, as claimed above, that if there is a solution to the constrained optimization problem with phases satisfying Eq. (23), then a new solution can be obtained by subtracting $\pi/2$ from all these phases that gives the same value of F' and that satisfies Eq. (22). Therefore, if we assume that the phase perturbations for minimized weight perturbation null synthesis are odd-symmetric, there is no possibility of inadvertently discarding a solution without odd-symmetry that gives a smaller sum of weighted squared absolute weight perturbations.

3. CONCLUDING REMARKS

In this section we comment on some aspects and implications of the proof given in Section 2.

(1) It is worth noting that it is not possible to reverse the procedure we have used in the last part of the proof to show that any solution with phases satisfying Eq. (23) can be replaced by a purely odd-symmetric solution with the same value of F'. If $\pi/2$ is added to all phases satisfying Eq. (22), we do indeed obtain a solution that satisfies Eq. (23) and the null constraints. However, letting P denote the set of indices of all phases that satisfy Eq. (22), the contribution of these phases to the value of F' is

$$\sum_{p \epsilon P} c_p |a_p|^2 \cos \phi_p \quad ,$$

while the contribution of the altered phases to F' is zero by Eq. (25), and there is now no way of arguing that the contribution to F' remains unchanged or increases as a result of having altered the phases.

(2) In the proof we have not attempted to eliminate the possibility of there actually being a solution to the minimized weight perturbation, null synthesis problem without odd-symmetry of the phase perturbations. What we have shown is that one cannot possibly minimize the weight perturbations more without odd-symmetry than with it and hence, since odd-symmetry and a real perturbed pattern is a simpler and more desirable situation than non-odd-symmetry and a complex pattern, one should feel free to assume odd-symmetry from the outset in numerical calculations. The proof in fact suggests that a stronger result is true: namely, that one can always do better with a purely odd-symmetric solution than with a solution containing some phase perturbations that satisfy Eq. (23), and hence that Eq. (23) can be dismissed as an "extraneous" solution of Eq. (21), or quite possibly a solution that maximizes rather than minimizes the weight perturbations.

As we have noted in the proof, if there were a solution to the minimized weight perturbation nulling problem with some, but not all, phase pairs odd-symmetric, then the array would be decomposable into two subarrays, each consisting of a set of pairs of symmetrically-placed elements. One subarray would have a purely real and the other a purely imaginary pattern, and each pattern would have nulls at all the imposed null locations. Hence, the original problem of imposing nulls in a pattern with minimized weight perturbations would be replaced by two separate nulling problems. An immediate consequence of this is that the upper limit on the possible number of imposed nulls would be reduced from the usual $N/2$ with phase-only nulling to $N_{min}/2$, with N_{min} the number of elements in the smaller of the two subarrays. No decomposition into two subarrays could allow more than $N/4$ imposed nulls. Furthermore, since the same nulls would be imposed in the two independent subarray patterns, it is to be expected that the resultant perturbations of the overall pattern and array weights would have to be larger than they would be if the most efficient use were made of all array elements as one group to impose pattern nulls. It is also important to note that in a very real sense, minimization of the weight perturbations with phases satisfying Eq. (23) is impossible. The contribution of each pair of elements with phases satisfying Eq. (23) to the sum of squared weight perturbations given by Eq. (10) is, using Eq. (25) and the symmetry of the $\{c_n\}$ and the $\{a_n\}$,

$$2\left(c_p|a_p|^2 + c_{N-p+1}|a_{N-p+1}|^2\right)$$

$$- 2\left(c_p|a_p|^2 \cos\phi_p + c_{N-p+1}|a_{N-p+1}|^2 \cos\phi_{N-p+1}\right) = 4c_p|a_p|^2 \quad .$$

This contribution is not only a fixed quantity that cannot be decreased by varying ϕ_p, but also represents a large perturbation of the pair of array weights being exactly one half of the maximum possible perturbation. For these reasons, although not proved rigorously, it is plausible that the odd-symmetric solution, Eq. (22), to Eq. (21) is not only at least as good as any solution containing phases that satisfy Eq. (23), but is in fact the only valid form of the phase perturbations for minimized weight perturbation, null synthesis.

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