

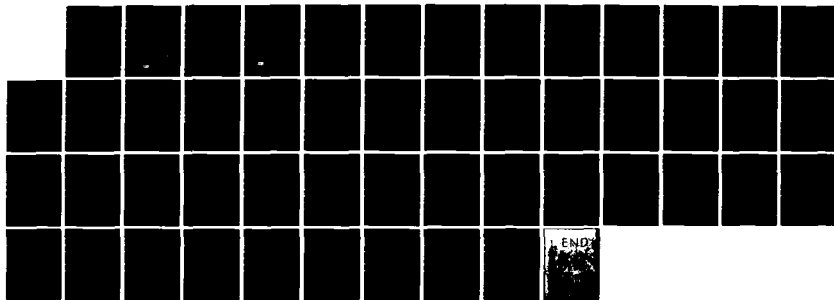
AD-A130 559

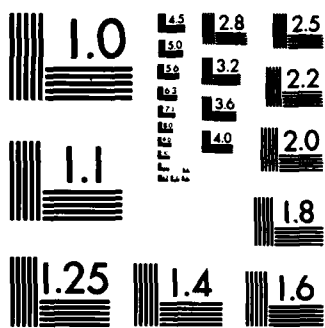
PROPERTIES OF QUICK LOOK PASSIVE LOCALIZATION(U) CENTER 1/1
FOR NAVAL ANALYSES ALEXANDRIA VA OPERATIONS EVALUATION
GROUP J A THOMAS ET AL. MAY 83 CNA-PP-383
N00014-76-C-0001

UNCLASSIFIED

F/G 19/5

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

PROPERTIES OF QUICK LOOK PASSIVE LOCALIZATION

James A. Thomas, Jr.
Marc Mangel

ADA 130559

N00014-76-C-0001

JUL 22 1983
A

DTIC FILE COPY



CENTER FOR NAVAL ANALYSES

This document is available
for public release
distribution is unlimited

**APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED**

**The ideas expressed in this paper are those of the authors.
The paper does not necessarily represent the views of either
the Center for Naval Analyses or the Department of Defense.**

PROFESSIONAL PAPER 383 / MAY 1983

PROPERTIES OF QUICK LOOK PASSIVE LOCALIZATION

James A. Thomas, Jr.
Marc Mangel



Operations Evaluation Group

CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

ABSTRACT

This paper examines the problem of target localization based solely upon bearing information obtained by a single platform over short observation times. A number of Monte Carlo and analytical techniques for the construction of statistical distributions of target ranges are presented and compared.

NTIS GRA&I		☒
DTIC TAB		☐
Unannounced		☐
Justification		☐
<i>Let's on point</i>		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or	Special
A		



1. INTRODUCTION

If an observer travels in a straight path at a known constant velocity and observes a stationary target at two different exact bearings separated by time t , the range to the target is given by

$$R_{\text{predicted}} = V_A \cdot t \frac{\sin \phi_0}{\sin (\phi_1 - \phi_0)} \quad (1.1)$$

Here V_A is the velocity of the observer and ϕ_1 and ϕ_0 are the final and initial observed bearings, respectively. If, on the other hand, the target moves with velocity V_S and each observed bearing can differ from the actual bearing to the target by as much as $\pm\delta$, the range to the target could be as great as

$$R_{\text{max}} = \frac{[V_A \sin(\phi_0 + \delta) + V_S] t}{\sin(\phi_1 - \phi_0 - 2\delta)} \quad , \quad (1.2)$$

or as little as

$$R_{\text{min}} = \frac{[V_A \sin(\phi_0 - \delta) - V_S] t}{\sin(\phi_1 - \phi_0 + 2\delta)} \quad . \quad (1.3)$$

(see figure 1).

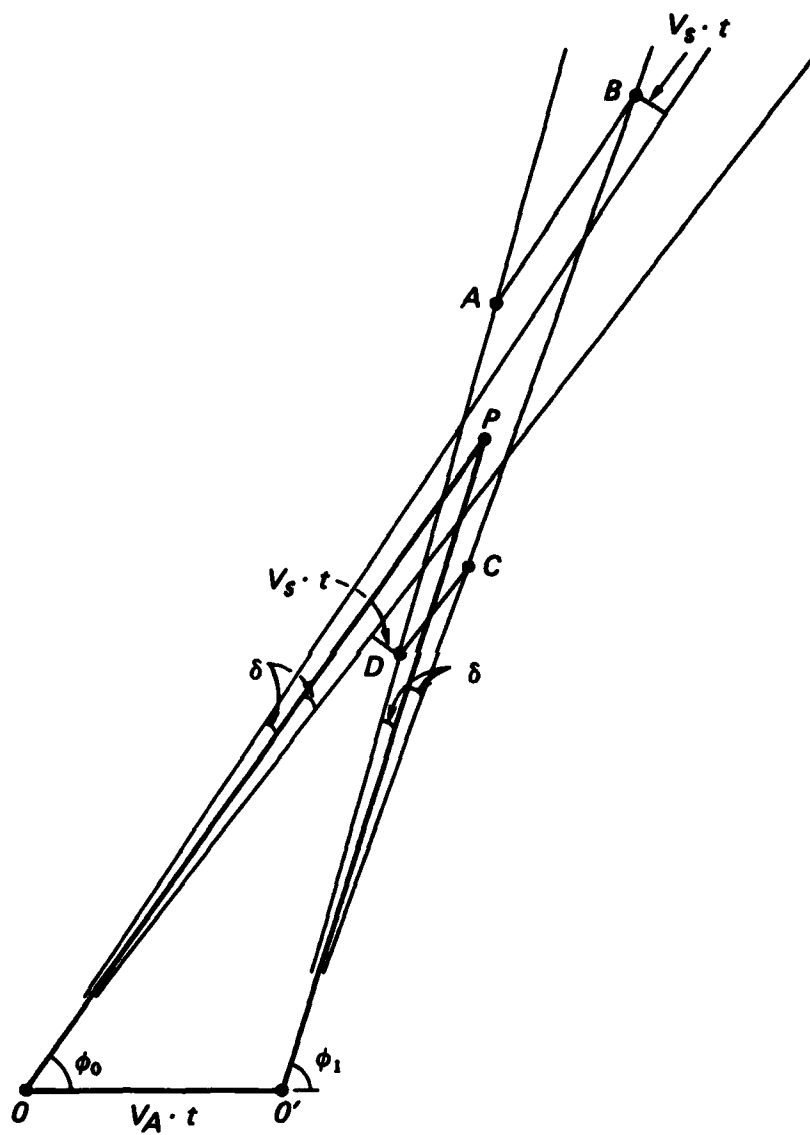


FIG. 1: GEOMETRY OF TWO-BEARING TRIANGULATION

The actual range to the target depends upon the target motion and the actual bearing errors; if the bearings are noisy, an appropriate question is: what is the distribution of range, given the observed bearings?

In this paper, a number of Monte Carlo and analytical techniques for the construction of the range distribution are given. Sections 2 and 3 contain descriptions and results of the Monte Carlo simulations. Sections 4 and 5 contain analytical results. In section 2 a "random range" simulation is described. In section 3, a "random bearing" simulation is described. In section 4, a method for approximately calculating the mean and variance of the range, given the observations, is presented. In section 5, a method for approximately calculating the distribution on the range is presented. Section 5 contains a discussion of the tactical use of the results presented in sections 2 to 4.

2. THE RANDOM RANGE SIMULATION

Description of the Simulation

This simulation runs for specified values of the observed initial and final bearings. The actual final bearings are assumed to be uniformly distributed between $+\delta$ and $-\delta$ of the observed final bearings. The actual initial bearings are assumed to be within $\pm\delta$ of the observed initial bearings with no assumption as to the distribution. Final target ranges are

assumed to be uniformly distributed between $R_{\min}$ and the lesser of $R_{\max}$ or a specified range bound (hence, the name random range simulation.) Target motion is assumed to be in a uniformly random direction at a constant velocity, V_s .

The simulation begins with the final observed bearing. It selects a bearing error to determine the actual final bearing, then selects a final target range, R . It selects the direction of target motion, then traces motion of target and observer backwards in time to their positions when the initial bearing was observed. If the actual bearing to the target at this point is within $+\delta$ or $-\delta$ of the observed initial bearing, R is retained as a valid sample. Otherwise, it is discarded. The procedure is repeated until the desired sample size has been obtained.

Simulation Results

Assume that the observer is an aircraft and the target is a ship. The aircraft ground velocity was 420 kts, the target velocity 30 kts, and the target range was not allowed to exceed 75 nautical miles. The time between bearing observations ranged from 30 to 120 seconds in 15 second increments. The initial observed bearing was 30 degrees in all instances, with bearing changes ranging from 2 to 10 degrees. Final bearing errors were assumed to be uniformly distributed up to 1 degree either side of the observed bearing. One thousand replications were collected

for each combination of bearing change and time between bearings. Results of these simulations are shown in and figures 2 through 8.

3. RANDOM BEARING SIMULATION

Description of the Simulation

The simulation described in this section works directly with the distribution on R , instead of assuming a distribution and then rejecting trials in which the assumptions are not met. Assume that the bearing changes are ϕ_0 and ϕ_1 , that the aircraft and ship speeds are V_A and V_S , that the observation time is t , and that the target moves in the direction α . In the next section it is shown that the range R to the target after the second bearing is

$$R = \frac{V_A t \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - \frac{V_S t \sin(\phi_0 - \alpha)}{\sin(\phi_1 - \phi_0)} \quad (3.1)$$

The simulation proceeds as follows:

1. Initialize, $N=0$. Input observed bearings $\hat{\phi}_0, \hat{\phi}_1$.
2. Select α , where α is uniformly distributed on $[0, 360^\circ]$.

TABLE 1

FRACTION OF TRIALS WITH RANGE GREATER THAN R^*
 $V_A = 420$ kts $V_S = 30$ kts $t = 30$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)								
	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
5									
10						1.0	1.0	1.0	1.0
15		1.0	1.0	1.0	1.0	.986	.969	.896	.801
20	1.0	.998	.996	.965	.907	.820	.669	.450	.232
25	.989	.979	.938	.828	.682	.487	.260	.127	.036
30	.946	.903	.825	.636	.434	.215	.058	.012	0
35	.874	.792	.677	.439	.235	.065	.008	0	
40	.779	.672	.519	.293	.100	.016	0		
45	.669	.528	.416	.184	.049	.001			
50	.546	.429	.309	.128	.010	0			
55	.413	.329	.213	.070	0				
60	.301	.227	.131	.036					
65	.203	.147	.076	.011					
70	.103	.075	.027	.003					

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 2

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

$V_A = 420$ kts $V_S = 30$ kts $t = 45$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)								
	3.0	3.5	4.0	4.5	5.0	5.5	6.0	7.0	8.0
5									
10								1.0	1.0
15				1.0	1.0	1.0	1.0	.996	.951
20		1.0	1.0	.999	.998	.979	.936	.752	.442
25	1.0	.993	.987	.948	.890	.762	.623	.287	.055
30	.987	.958	.894	.799	.651	.422	.279	.037	0
35	.934	.883	.734	.555	.384	.203	.078	0	
40	.846	.756	.558	.360	.172	.055	.013		
45	.727	.592	.395	.205	.066	0	0		
50	.592	.444	.266	.097	.016				
55	.431	.296	.171	.047	.003				
60	.314	.191	.089	.016	0				
65	.174	.115	.044	.001					
70	.088	.056	.019	0					

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 3

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

 $V_A = 420$ kts $V_S = 30$ kts $t = 60$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)								
	4.0	4.5	5.0	5.5	6.0	7.5	8.0	9.5	10.0
5									
10									1.0
15						1.0	1.0	1.0	.993
20			1.0	1.0	1.0	.992	.951	.813	.582
25	1.0	1.0	.999	.992	.676	.853	.577	.284	.097
30	.997	.991	.953	.902	.798	.521	.186	.027	0
35	.960	.933	.823	.662	.492	.199	.026	0	
40	.895	.814	.630	.413	.257	.041	0		
45	.762	.649	.418	.207	.105	.002			
50	.610	.475	.260	.103	.035	0			
55	.455	.315	.140	.045	.006				
60	.309	.186	.071	.009	0				
65	.187	.094	.028	0					
70	.077	.036	.008						

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 4

FRACTION OF TRIALS WITH RANGE GREATER THAN R^* $V_A = 420$ kts $V_S = 30$ kts $t = 75$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)								
	4.0	4.5	5.0	5.5	6.0	7.0	8.0	9.0	10.0
5									
10									
15								1.0	1.0
20						1.0	1.0	.999	.949
25				1.0	1.0	.997	.961	.828	.578
30		1.0	1.0	.998	.982	.903	.689	.424	.125
35	1.0	.996	.985	.942	.904	.642	.336	.102	.002
40	.990	.969	.923	.825	.713	.335	.091	.004	0
45	.947	.885	.799	.652	.480	.124	.014	0	
50	.845	.735	.621	.447	.262	.039	0		
55	.699	.568	.427	.270	.132	.002			
60	.531	.381	.274	.149	.059	0			
65	.364	.225	.154	.066	.022				
70	.181	.090	.056	.029	.002				

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 5

FRACTION OF TRIALS WITH RANGE GREATER THAN R^* $V_A = 420$ kts $V_S = 30$ kts $t = 90$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)									
	5.0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5										
10										
15										
20							1.0	1.0	1.0	1.0
25				1.0	1.0	1.0	.999	.990	.960	.935
30		1.0	1.0	.996	.973	.946	.894	.796	.692	.572
35	1.0	.988	.968	.930	.842	.728	.598	.440	.307	.203
40	.996	.919	.870	.746	.608	.423	.273	.153	.075	.028
45	.953	.778	.659	.490	.333	.168	.076	.023	.005	0
50	.858	.591	.443	.259	.146	.052	.007	0	0	
55	.701	.386	.241	.109	.007	0				
60	.516	.227	.110	.046	.010	0				
65	.304	.114	.043	.007	0					
70	.152	.051	.014	0						

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 6

FRACTION OF TRIALS WITH RANGE GREATER THAN R^* $V_A = 420$ kts $V_S = 30$ kts $t = 105$ seconds $R < 75$ n.mi.

R (n.mi.)	Bearing change (degrees)									
	5.0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5										
10										
15										
20										
25							1.0	1.0	1.0	1.0
30			1.0	1.0	1.0	1.0	.995	.980	.937	.892
35		1.0	.999	.998	.983	.957	.875	.822	.686	.630
40	1.0	.993	.975	.945	.892	.796	.633	.515	.350	.283
45	.997	.949	.897	.801	.692	.538	.382	.220	.110	.070
50	.956	.826	.728	.617	.461	.294	.150	.055	.012	.000
55	.859	.651	.532	.382	.251	.109	.042	.007	0	0
60	.710	.444	.340	.214	.128	.033	.004	0		
65	.488	.258	.180	.095	.037	.004	0			
70	.234	.106	.066	.030	.004	0				

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

TABLE 7

FRACTION OF TRIALS WITH RANGE GREATER THAN R^* $V_A = 420$ kts $V_S = 30$ kts $t = 120$ seconds $R < 75$ n.mi.

<u>R</u> (n.mi.)	<u>Bearing change (degrees)</u>									
	<u>5.0</u>	<u>6.0</u>	<u>6.5</u>	<u>7.0</u>	<u>7.5</u>	<u>8.0</u>	<u>8.5</u>	<u>9.0</u>	<u>9.5</u>	<u>10.0</u>
5										
10										
15										
20										
25									1.0	1.0
30							1.0	1.0	.998	.991
35				1.0	1.0	1.0	.988	.965	.935	.945
40		1.0	1.0	.979	.977	.953	.896	.825	.717	.565
45	1.0	.997	.979	.954	.879	.794	.696	.560	.422	.252
50	.994	.858	.912	.836	.694	.582	.435	.300	.184	.066
55	.955	.881	.778	.659	.469	.355	.216	.100	.051	.009
60	.857	.707	.580	.442	.276	.187	.072	.020	.003	0
65	.684	.486	.357	.260	.137	.082	.018	.002	0	
70	.372	.236	.163	.097	.042	.024	0	0		

*1,000 replications per bearing change with $\phi_0 = 30^\circ$.

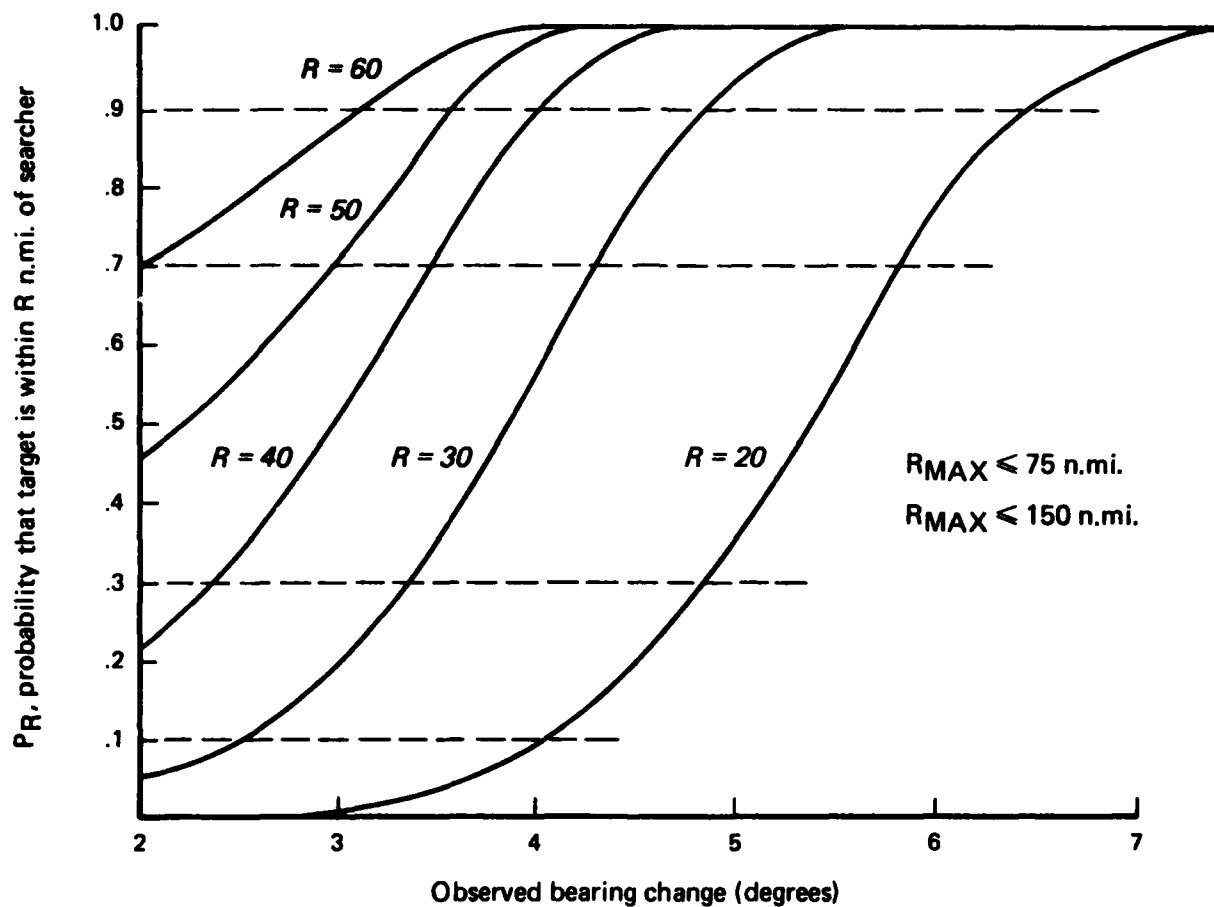


FIG. 2: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 30$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

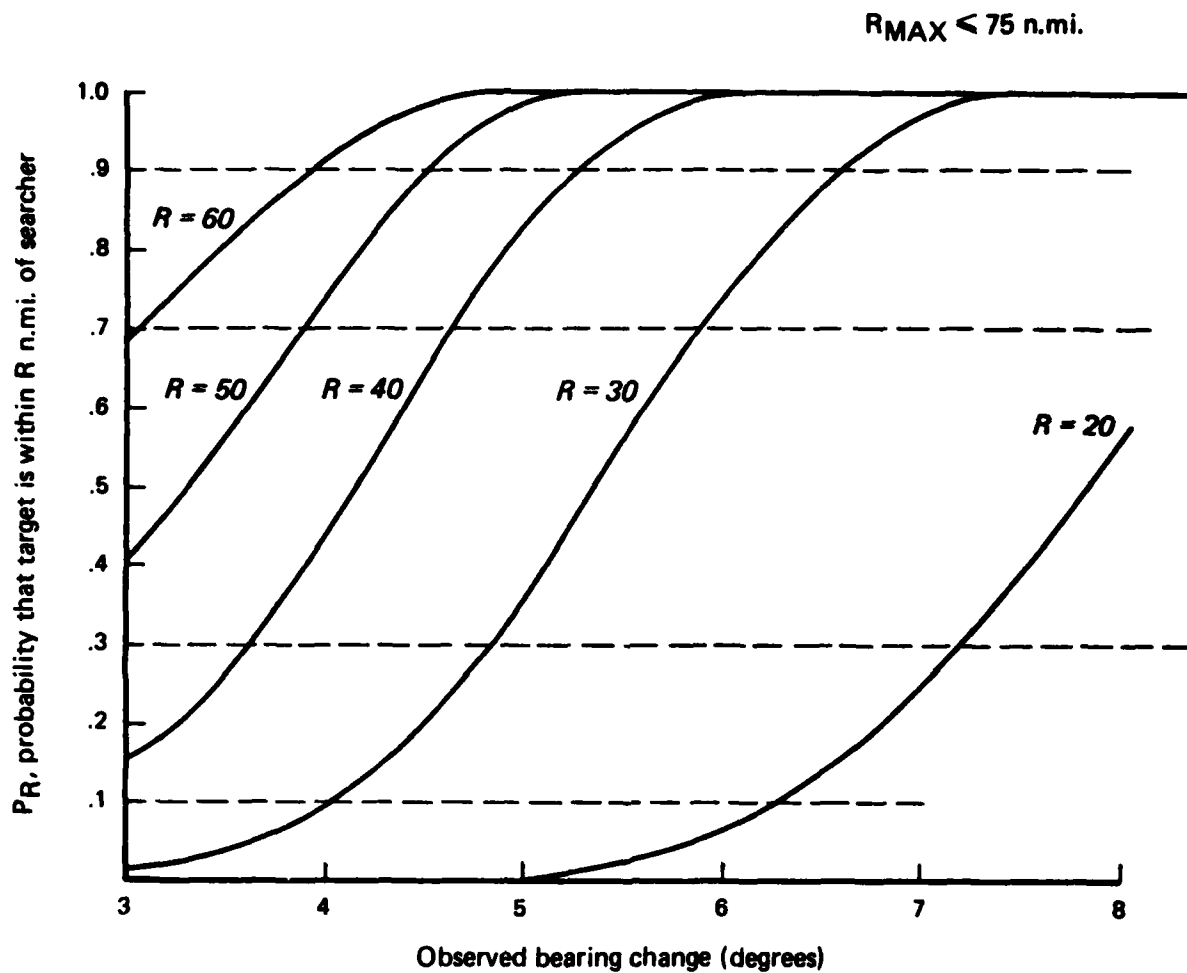


FIG. 3: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 45$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

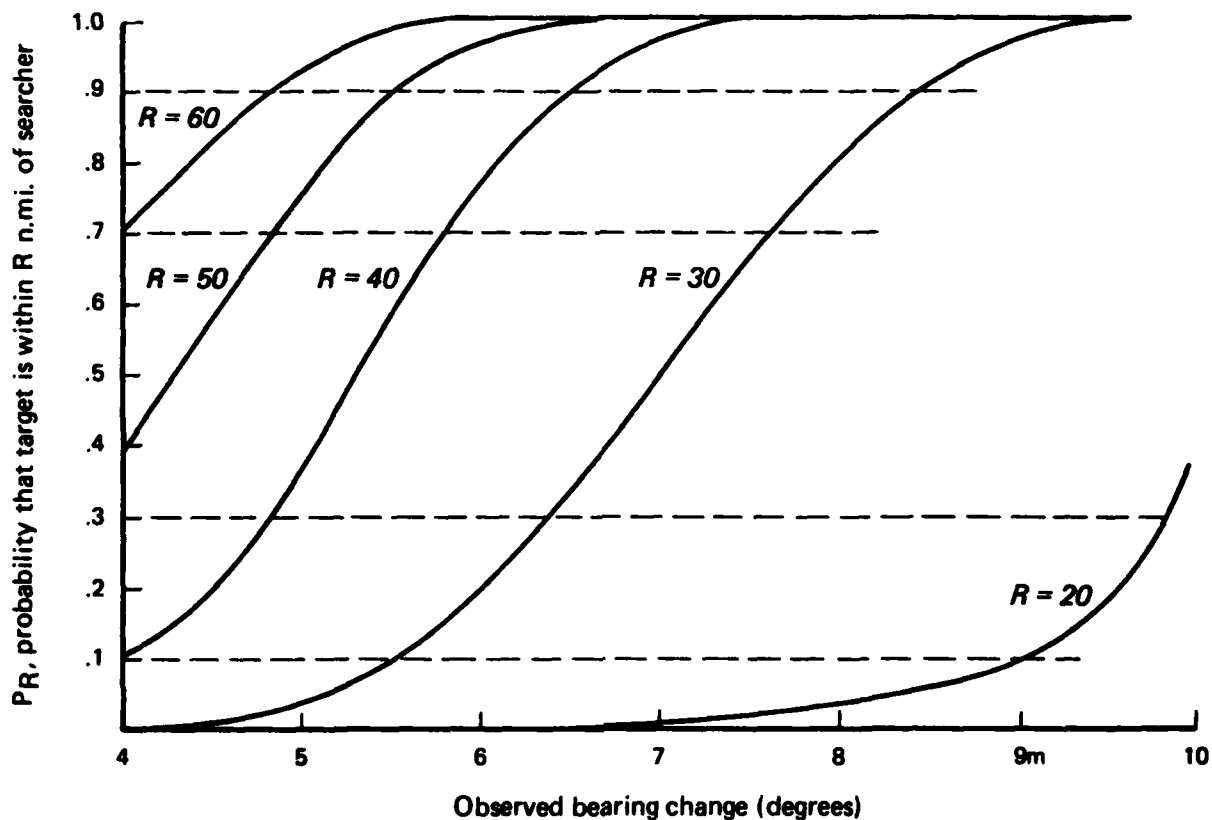


FIG. 4: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 60$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

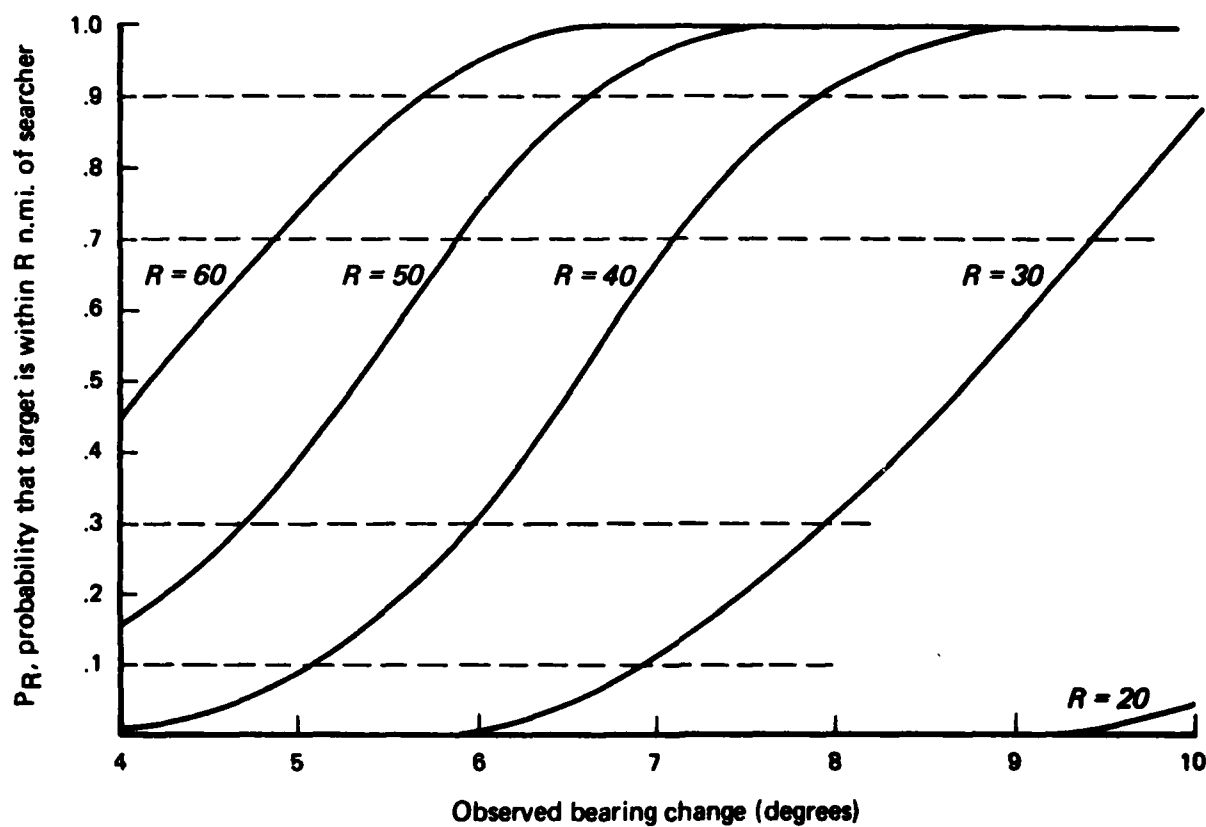


FIG. 5: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 75$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

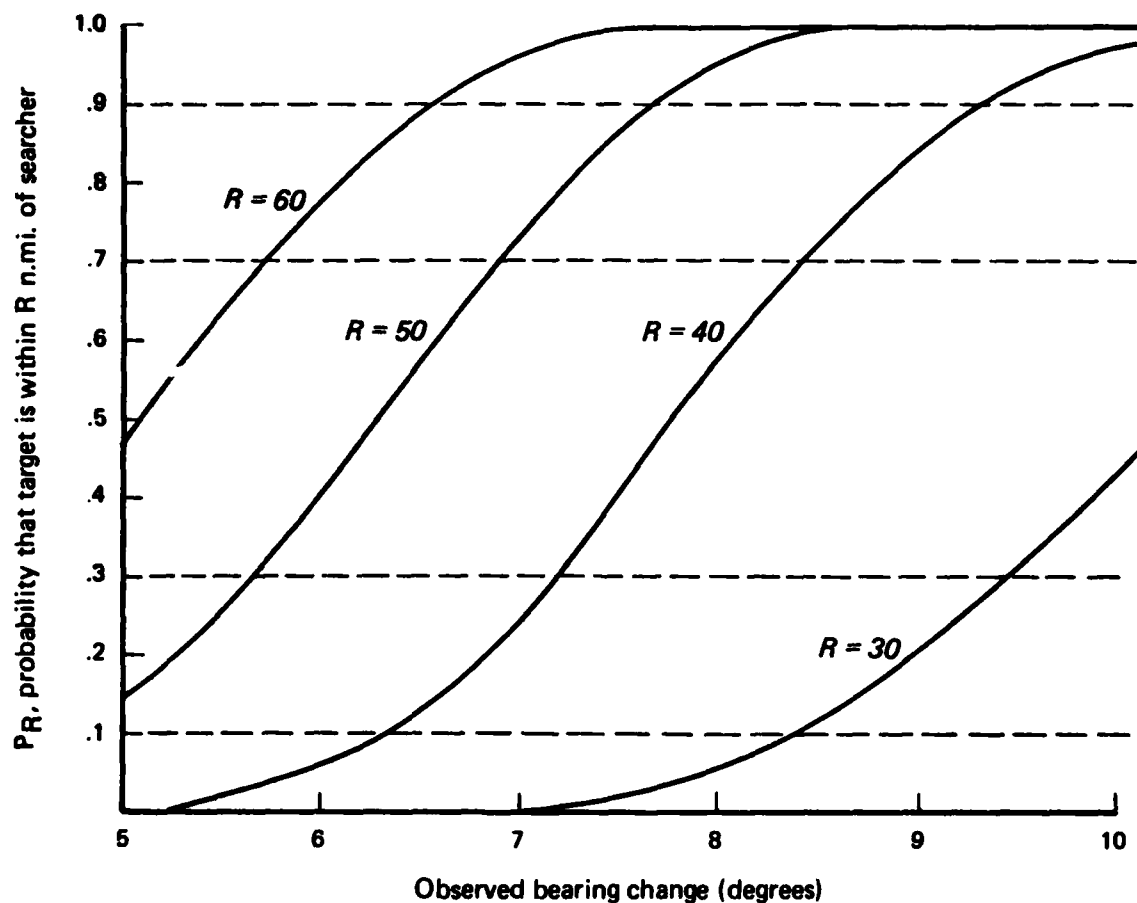


FIG. 6: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_t = 30$ KTS, $t = 90$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

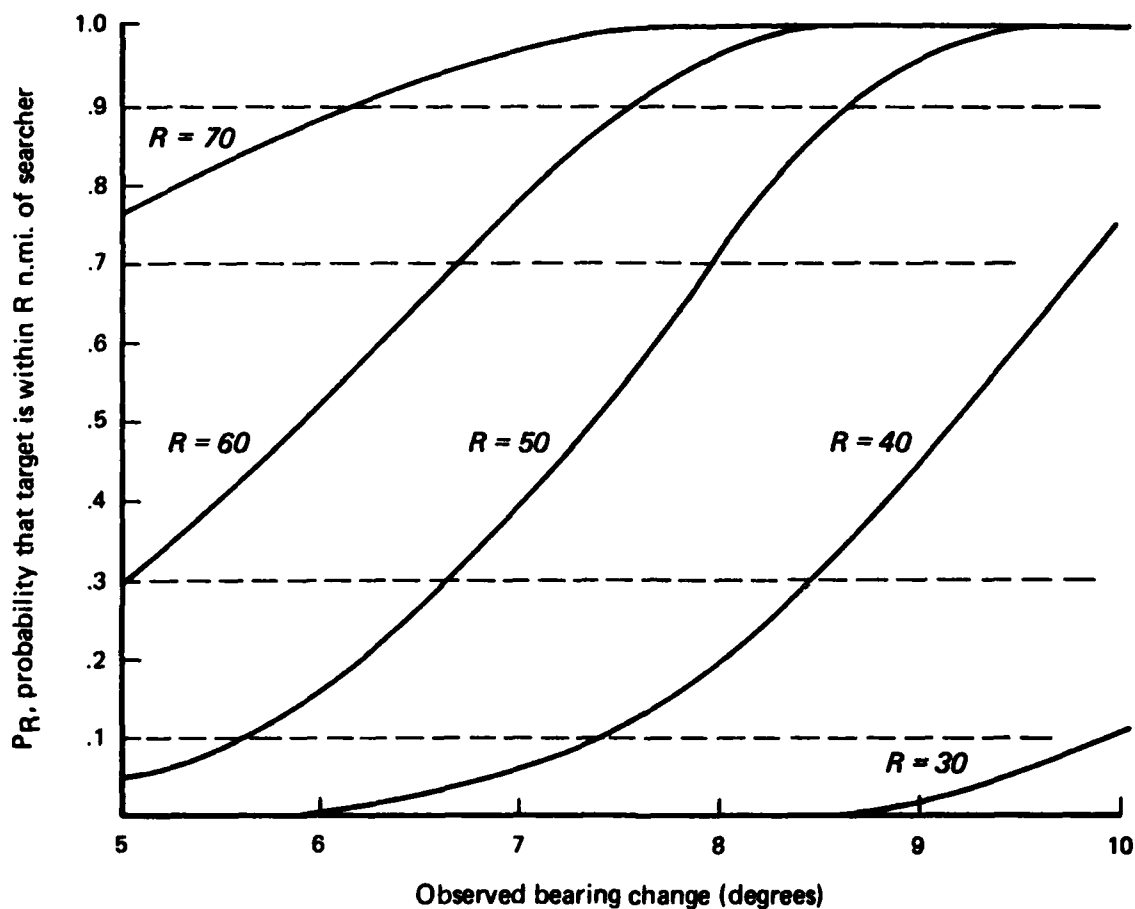


FIG. 7: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 105$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

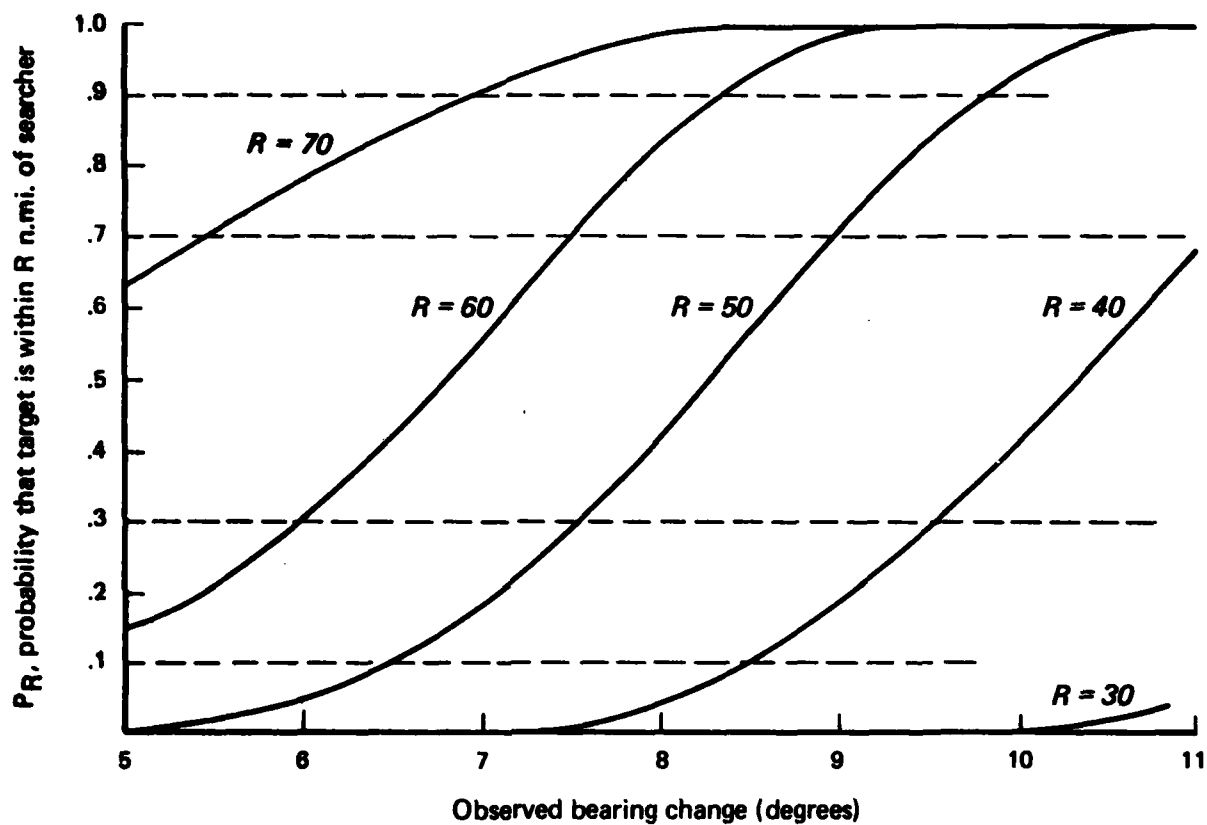


FIG. 8: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH $V_A = 420$ KTS, $V_s = 30$ KTS, $t = 120$ SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN $\pm 1^\circ$ AND $R_{MAX} < 75$ n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

3. Compute real bearings according to

$$\begin{aligned}\phi_0 &= \hat{\phi}_0 + \delta_1 \\ \phi_1 &= \hat{\phi}_1 + \eta_1\end{aligned}$$

here δ_1 and η_1 are independently identically distributed random variables, with a uniform distribution on $[-\delta^\circ, \delta^\circ]$. Here δ is a specified angle.

4. Compute R from equation (3.1).

5. Recompute statistics for the distribution on R .

6. N is replaced by $N+1$; if the new value is less than the cutoff, return to step 2, otherwise exit.

Simulation Results

It is worthwhile to consider how the random range and random bearing simulations differ. In the random bearing simulation, the conditional distribution function for R , given the observations

$$H(r) = \Pr\{R \leq r | \hat{\phi}_0, \hat{\phi}_1\} \quad (3.3)$$

is computed directly. In the random range simulation, the distribution function on R is initially assumed to be uniform on $[R_{\min}, \hat{R}_{\max}]$ (where $\hat{R}_{\max}$ is either $R_{\max}$ from equation (1.2) or an artificially imposed cut-off). Then trials that do not conform to the assumption that $-1 < \phi_0 - \hat{\phi}_0 < 1$ are rejected. The differences in the two procedures should be reflected most in differences in the tails of the range distributions.

Figure 9 shows the results of the two kinds of simulations. It appears that for most ranges, the R simulation underestimates the closeness of the target.

4. ANALYTICAL RESULTS: MEAN AND VARIANCE OF THE RANGE

First consider the derivation of equation (3.1). The geometry is shown in figure 10. Set $S = V_A t$ and $d = V_{St}$. Applying the law of sines to the small triangle gives

$$\frac{d}{\sin(\phi_1 - \phi_0)} = \frac{Q}{\sin(\phi_0 - \alpha)} \quad (4.1)$$

For the large triangle, the law of sines gives

$$\frac{S}{\sin(\phi_1 - \phi_0)} = \frac{Q+R}{\sin(\phi_0)} \quad (4.2)$$

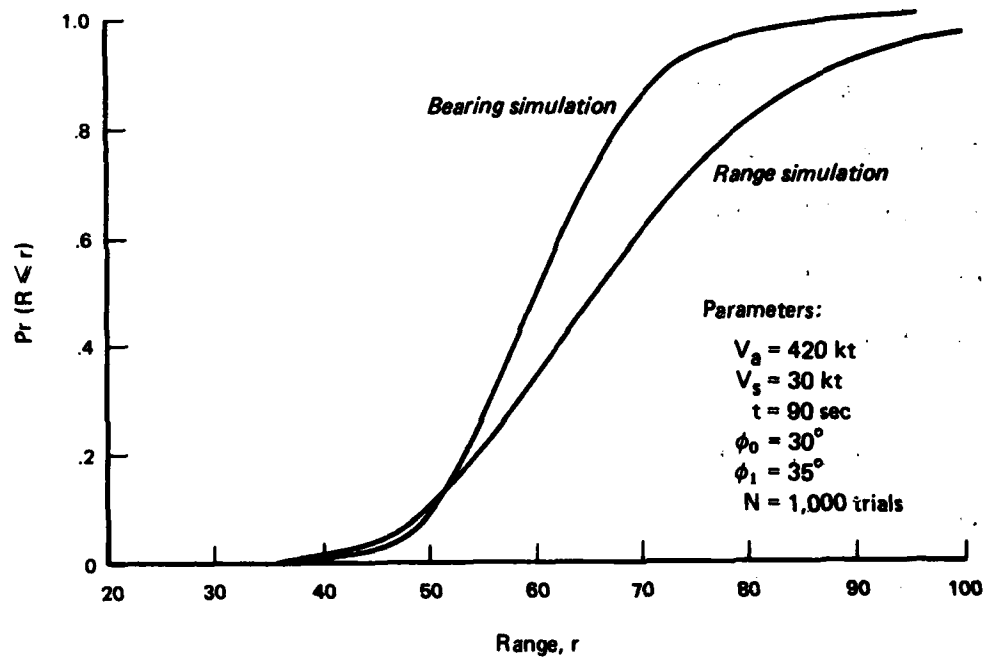


FIG. 9: COMPARISON OF RANGE AND BEARING SIMULATIONS

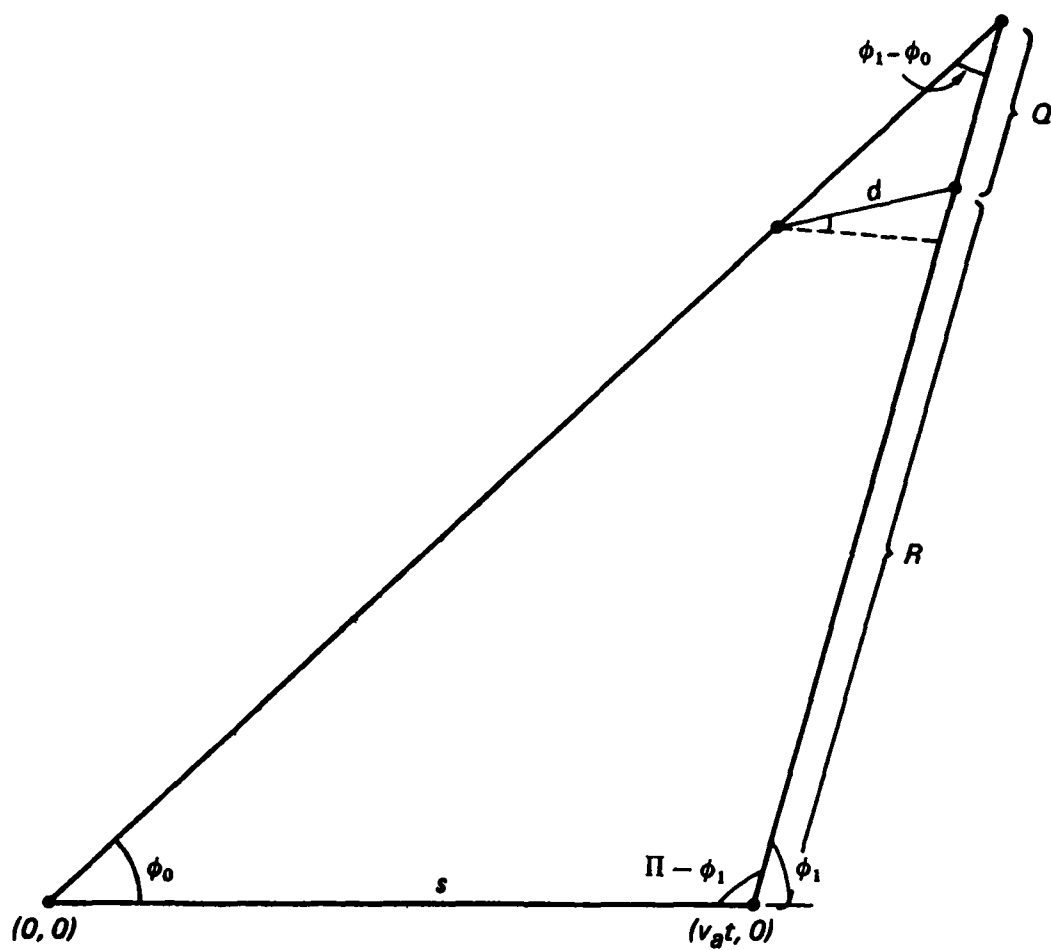


FIG. 10: GEOMETRY USED IN DERIVATION OF EQUATION 3.1.

Solving equations (4.2) and (4.3) for R gives

$$R = \frac{S \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - Q$$

$$= \frac{S \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - \frac{d \sin(\phi_0 - \alpha)}{\sin(\phi_1 - \phi_0)} \quad (4.3)$$

Now replace ϕ_0 and ϕ_1 with random variables, X and Y. R is a random variable too. Furthermore, R is a nonlinear function of the random variables X and Y. In order to calculate the mean and variance of R, the following approximation is used. If $f(X,Y)$ is any twice differentiable function, and $\bar{X} = E\{X\}$, $\bar{Y} = E\{Y\}$, then a Taylor expansion gives the approximation

$$E\{f(X,Y)\} \approx E\{f(\bar{X},\bar{Y}) + f_x(\bar{X},\bar{Y}) [X-\bar{X}] + f_y(\bar{X},\bar{Y}) [Y-\bar{Y}]$$

$$+ \frac{1}{2} f_{xx}(\bar{X},\bar{Y}) [X-\bar{X}]^2 + f_{xy}(\bar{X},\bar{Y}) [X-\bar{X}][Y-\bar{Y}] \quad (4.4)$$

$$+ \frac{1}{2} f_{yy}(\bar{X},\bar{Y}) [Y-\bar{Y}]^2\}$$

so that

$$E\{f(X,Y)\} \approx f(\bar{X},\bar{Y}) + \frac{1}{2}[f_{xx}(\bar{X},\bar{Y}) \text{Var } X \quad (4.5)$$

$$+ 2f_{xy}(\bar{X},\bar{Y}) \text{Cov}(X,Y) + f_{yy}(\bar{X},\bar{Y}) \text{Var } Y]$$

In the same way one finds that

$$E\{f^2(X,Y)\} \approx f^2(\bar{X},\bar{Y}) + [f_x^2(\bar{X},\bar{Y}) + f(\bar{X},\bar{Y}) f_{xx}(\bar{X},\bar{Y})] \text{Var } X \quad (4.6)$$

$$+ [f_y^2(\bar{X},\bar{Y}) + f(\bar{X},\bar{Y}) f_{yy}(\bar{X},\bar{Y})] \text{Var } Y$$

$$+ 2 [f_x(\bar{X},\bar{Y}) f_y(\bar{X},\bar{Y})$$

$$+ f(\bar{X},\bar{Y}) f_{xy}(\bar{X},\bar{Y})] \text{Cov } (\bar{X},\bar{Y})$$

In the case of interest here, $f(X,Y)=R=[S \sin(X) - d \sin(X-\alpha)][\sin(Y-X)]^{-1}$, and assuming $\text{Cov}(X,Y)=0$, the derivatives of interest are

$$R_y = -[S \sin(x) - d \sin(x-\alpha)] \sin(y-x)^{-2} \cos(y-x)$$

$$R_{yy} = -[S \sin(x) - d \sin(x-\alpha)] \{-2[\sin(y-x)^{-3} \cos^2(y-x) \\ - [\sin(y-x)^{-1}]\}$$

(4.7)

$$R_x = [S \cos(x) - d \cos(x-\alpha)] \sin(y-x)^{-1} \\ + [S \sin(x) - d \sin(x-\alpha)] \sin(y-x)^{-2} \cos(y-x)$$

$$R_{xx} = 2[S \cos(x) - d \cos(x-\alpha)] \sin(y-x)^{-2} \cos(y-x) \\ + 2[S \sin(x) - d \sin(x-\alpha)] \sin(y-x)^{-3} \cos^2(y-x)$$

The derivatives (4.7) involve $\cos(x-\alpha)$ and $\sin(x-\alpha)$. Assuming the $\alpha \sim U[w_0, w_1]$, table 1 shows the averages of the functions of interest.

TABLE 8

<u>Function</u>	<u>w₀</u>	<u>w₁</u>	<u>Average</u>
$\cos(x-\alpha)$	0	360°	0
$\sin(x-\alpha)$	0	360°	0
$\cos(x-\alpha)$	0	360°	$2 \sin x / \pi$
$\sin(x-\alpha)$	0	360°	$-2 \cos x / \pi$

Note that if α is uniformly distributed on $[0, 360^\circ]$, then the target motion makes essentially no contribution to the results of this level of approximation. Using equations (4.5) - (4.7) and table 8, assuming X and Y are independent, and $X, Y \sim U[-1^\circ, 1^\circ]$ leads to figures 11, 12, and 13. In these figures, the mean of R is plotted against $\Delta\phi \equiv \phi_1 - \phi_0$ for $t = 30, 40$, and 50 seconds, with $\alpha \sim U[0, 2\pi]$, $V_A = 420$ kts, and $V_S = 30$ kts. The error bars indicate one standard deviation.

The considerable size of the error bars suggests that distributional results are very important, since knowledge of the mean and variance is not sufficient in this case to give accurate predictions.

5. ANALYTICAL RESULTS: APPROXIMATE CALCULATION OF THE DISTRIBUTION ON R

Since R is a nonlinear function of X and Y , it is difficult to calculate its distribution exactly. In this section, an approximate method for the calculation of the distribution is given. The starting point is equation (4.3) for R with X and Y replacing ϕ_0 and ϕ_1 :

$$R = \frac{S \sin(X)}{\sin(Y-X)} - \frac{d \sin(X-\alpha)}{\sin(Y-X)} \quad (5.1)$$

The approximation used here is the small noise approximation, in which X and Y are replaced by

$$\begin{aligned} X &= \bar{X} + \delta \\ Y &= \bar{Y} + \eta \end{aligned} \quad (5.2)$$

where $\delta \sim U[-\delta_1, \delta_1]$ and $\eta \sim U[-\eta_1, \eta_1]$

are the deviations from the average, and a Taylor expansion of (5.1) is used to write R as a linear function of δ and η . Such a procedure is valid if δ and η are sufficiently small relative to other terms in the expansion. Although used mainly for mathematical convenience, if such an approximation is not valid the entire concept of getting useful information from two noisy bearings may not be valid, since then the noise overrides the deterministic terms.

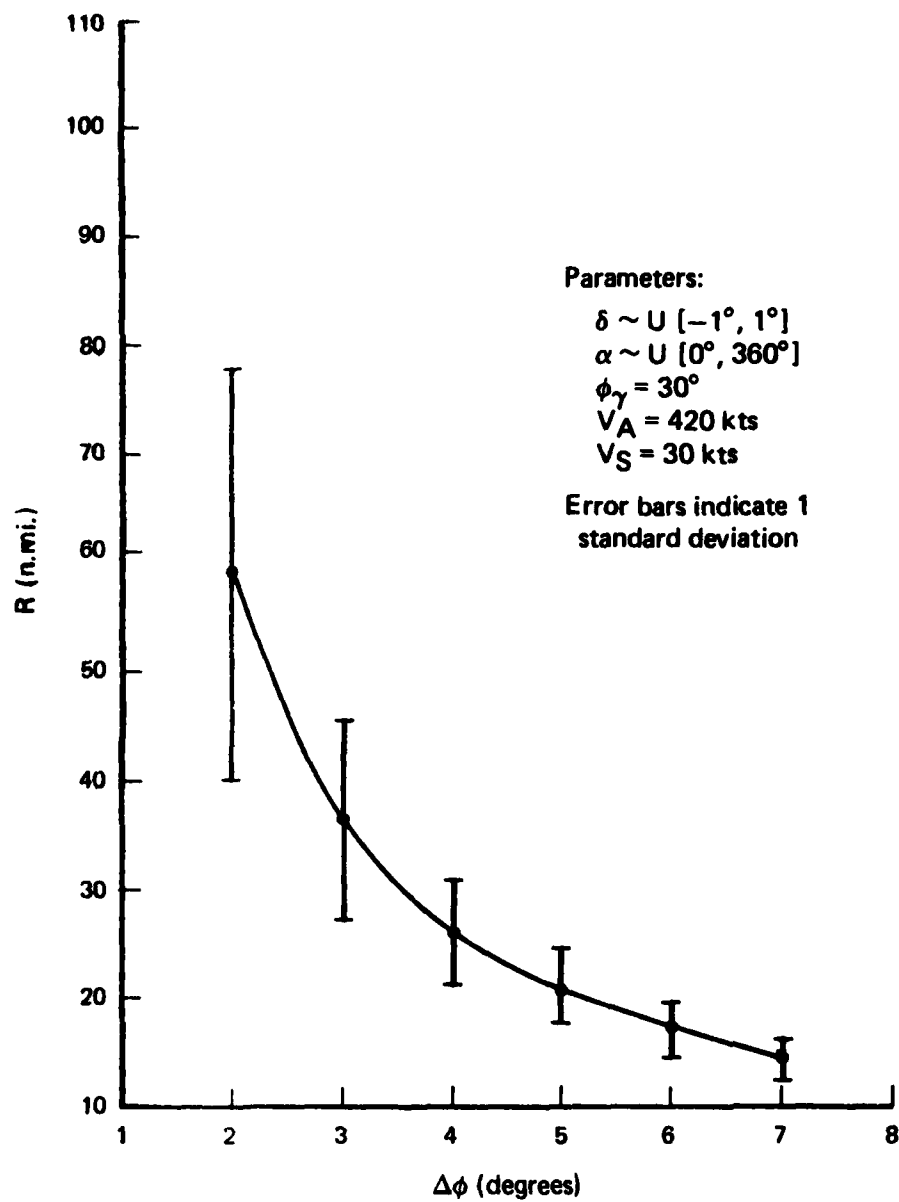


FIG. 11: R vs., $\Delta\phi$ WITH $t = 30$ SECONDS

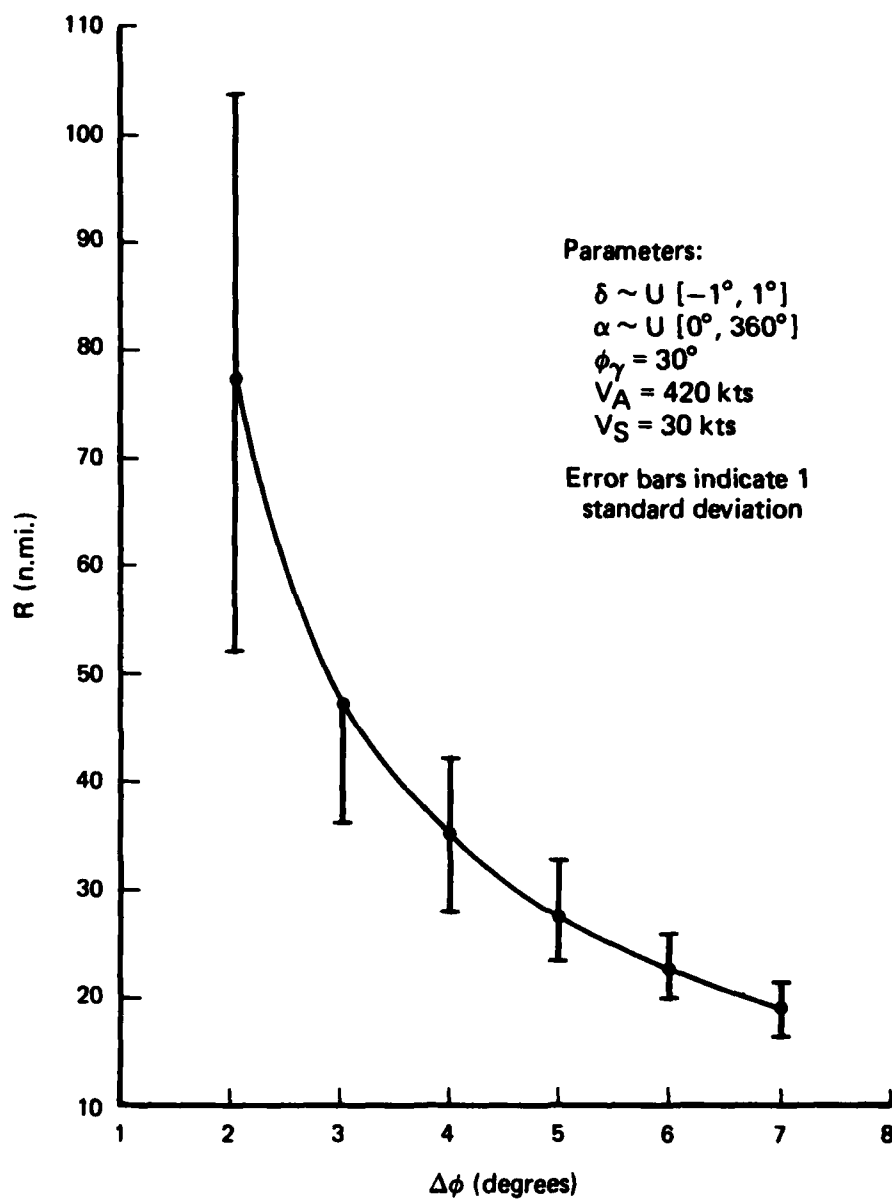


FIG. 12: R vs. $\Delta\phi$ WITH $t = 40$ SECONDS

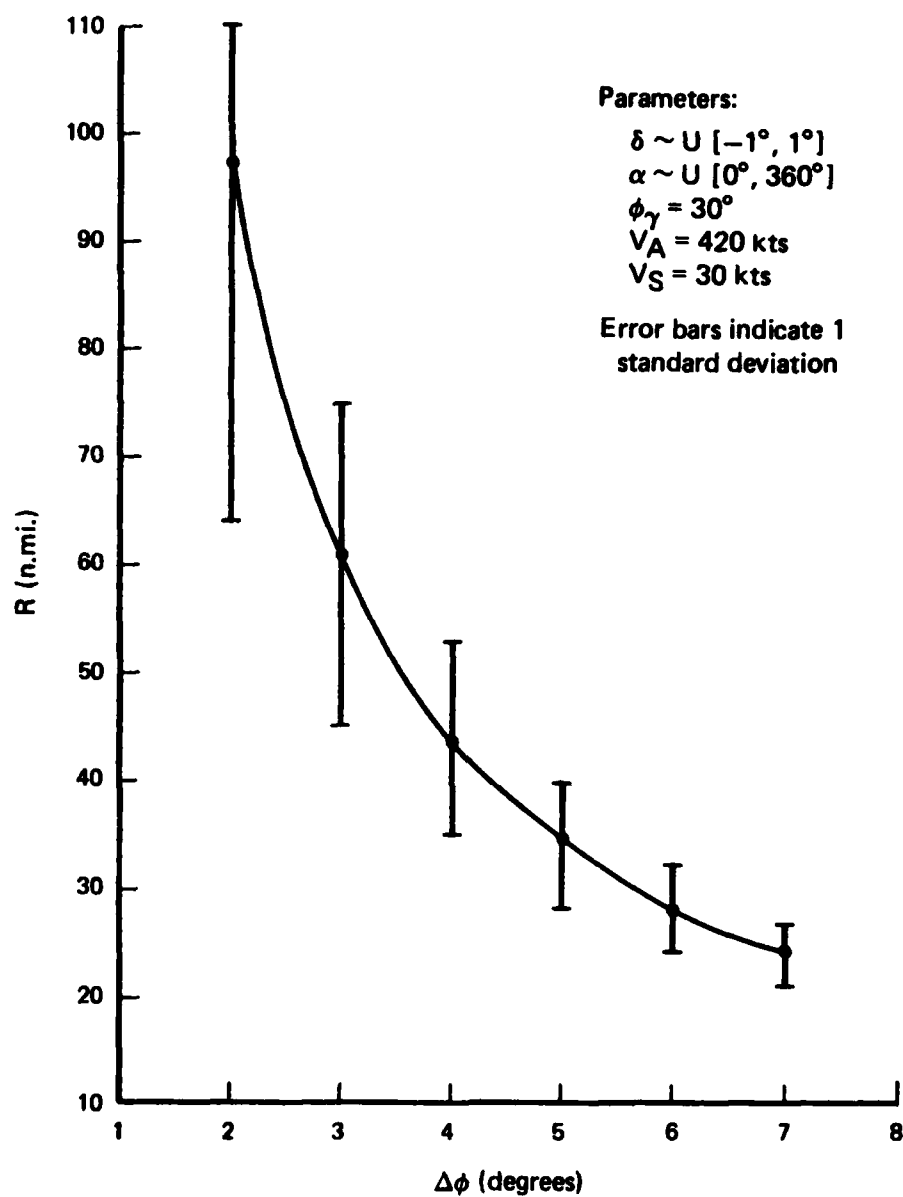


FIG. 13: R vs. $\Delta\phi$ WITH $t = 50$ SECONDS

In this approximation

$$\sin(X) \approx \sin(\bar{X}) + \delta \cos(\bar{X}) \equiv A_1 + A_2 \delta$$

$$\sin(X-\alpha) \approx \sin(\bar{X}-\alpha) + \delta \cos(\bar{X}-\alpha) \equiv C_1 + C_2 \delta$$

(5.3)

$$[\sin(Y-X)]^{-1} \approx [\sin(\bar{Y}-\bar{X})]^{-1}$$

$$- \frac{\cos(\bar{Y}-\bar{X})}{[\sin(\bar{Y}-\bar{X})]^2} (\eta-\delta) \equiv B_1 - B_2 (\eta-\delta)$$

Using (5.3) in (5.1) gives

$$R = \bar{R} + E\delta + F\eta \quad (5.4)$$

where

$$\bar{R} = B_1(SA_1 - dC_1)$$

$$E = B_1(SA_2 - dC_2) + B_2(SA_1 - dC_1) \quad (5.5)$$

$$F = B_2(dC_1 - SA_1)$$

The distributional quantity of interest $H(r)$ is then

$$H(r) = \Pr\{R < r\} = \Pr\{\bar{R} + E\delta + F\eta < r\} \quad (5.6)$$

It is easy to show that, for the operational values of interest, $E > 0$ and $F < 0$. In light of the form of R in (5.4), there are maximum and minimum possible values of R given by

$$R_{\max} = \bar{R} + E\delta_1 + |F|\eta_1 \quad (5.7)$$

$$R_{\min} = \bar{R} - E\delta_1 + F\eta_1$$

Note that $R_{\max}$ and $R_{\min}$ still depend upon α through the coefficients in (5.3) and (5.5).

Define new random variables, U and V by

$$U = E\delta \quad V = |F|\eta \quad (5.8)$$

so that

$$U \sim U[-E\delta_1, E\delta_1] \quad (5.9)$$

$$V \sim U[+F\eta_1, -F\eta_1]$$

In the sequel it is helpful to set $u_1 = E\delta_1$, $v_1 = |F|\eta_1$.

Defining $Z \equiv r - \bar{R}$, equation (5.6) becomes

$$\Pr\{R < r\} = \Pr\{U - V < Z\} \quad (5.10)$$

In order to compute (5.10), refer to figure 14. For a given Z , $\Pr\{U - V < Z\}$ is the fraction of the area of the rectangle above the line $u - v = Z$. There are three cases. Set $Z_1 = u_1 - v_1$ and $Z_2 = v_1 - u_1$; then the three cases are

Case I : $Z > Z_1$

Case II: $Z < Z_2$

Case III: $Z_2 < Z < Z_1$

In Case I

$$\Pr\{U - V < Z\} = 1 - \frac{1}{8u_1v_1} (u_1 + v_1 - Z)^2 \quad (5.11)$$

In Case II,

$$\Pr\{U - V < Z\} = \frac{1}{8u_1v_1} (Z + u_1 + v_1)^2 \quad (5.12)$$

In Case III,

$$\Pr\{U - V < Z\} = 1 - \frac{1}{4u_1v_1} \{2v_1^2 + 2v_1(u_1 - Z - v_1)\} \quad (5.13)$$

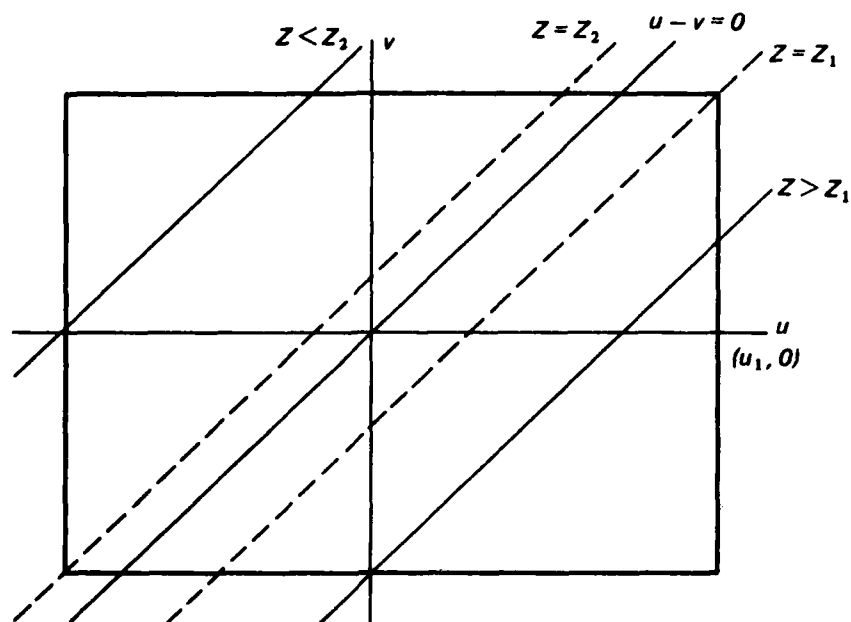


FIG. 14: RECTANGLE USED TO COMPUTE $\Pr U - V < Z$

Figure 15 shows a comparison of the theory developed here with the range and bearing simulations for the same parameters as in figure 9. Further comparisons of the theory and bearing simulation, with the noise uniformly distributed on $\pm .5^\circ$ or $\pm .25^\circ$ showed that the agreement between theory and simulations was very good.

Further comparisons of the theory and the random range simulation showed that the predicted minimum values for the ranges agreed closely, but that the predicted maximum values disagreed. As an example, for the parameter values in figure 9, the minimum range by the theory is 34 n.mi. and the minimum range by the range simulation is 35 n.mi. The maximum ranges, however, are 86 n.mi. and 117 n.mi. respectively. The cause of the long tail on the range simulation is not known.

TACTICAL IMPLICATIONS AND USES

One tactical use of the techniques described here is the determination of confidence levels on the range to a target. Referring to figure 16, if the observer wished to know that he was within 60 miles of the target with .9 probability, he would need to observe a bearing after t seconds that was above the $P_{60} = .9$ curve. If the observer wished to know that he was outside 30 miles of the target with .9 probability, he would need to observe a bearing after t seconds that was below the $P_{30} = .1$ curve. Finally, if the observer wished to know that he

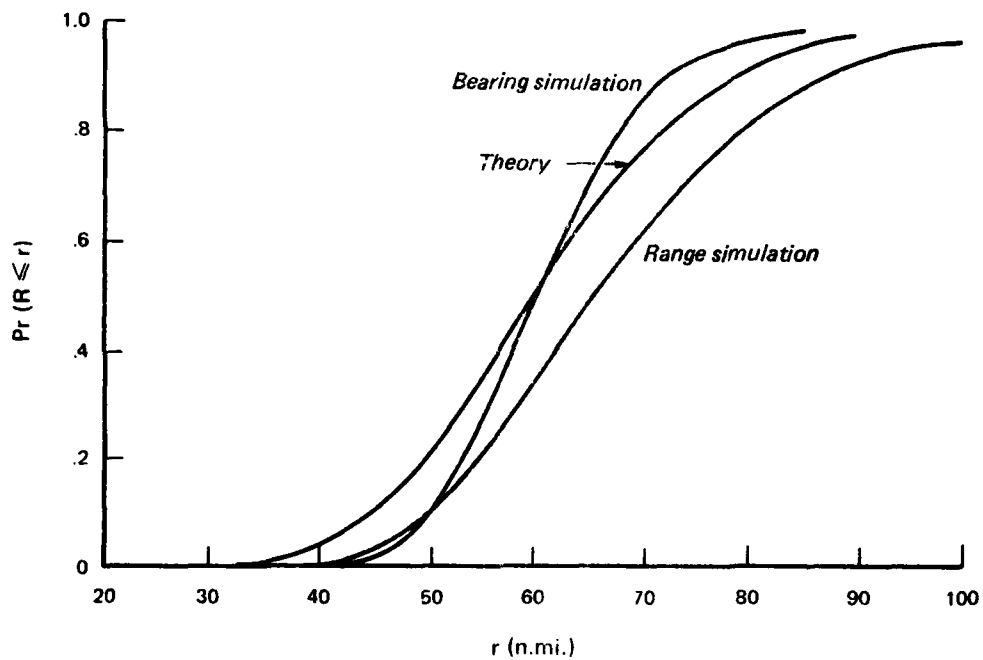


FIG. 15: COMPARISON OF THEORY WITH RANGE AND BEARING SIMULATIONS

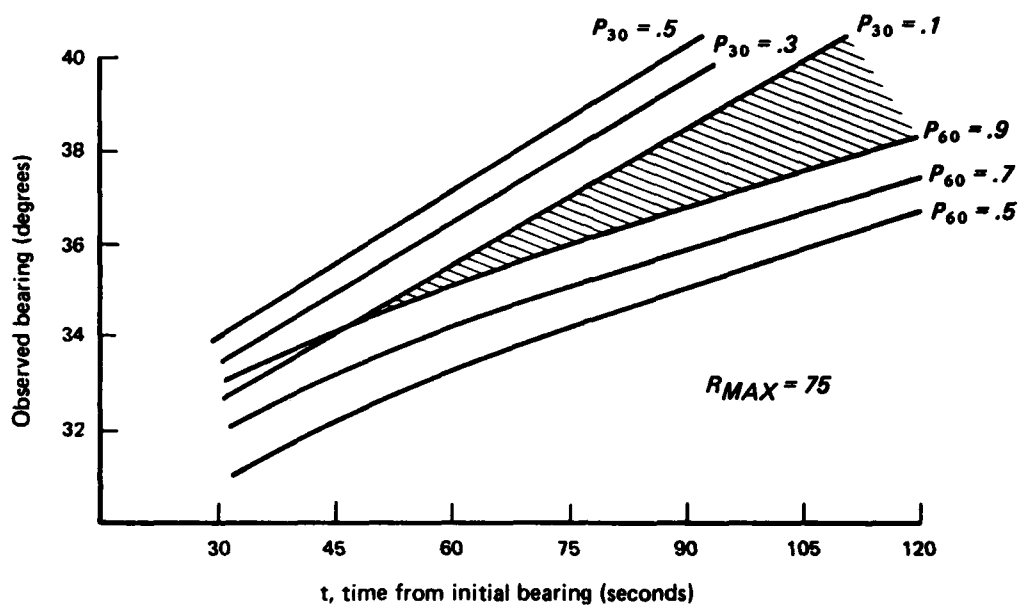


FIG. 16: P_{60} AND P_{30} PROFILES FOR BEARING OBSERVED t SECONDS AFTER THE INITIAL BEARING OF 20 DEGREES

was somewhere between 30 and 60 miles from the target with .9 probability, his bearing observation after t seconds would need to be below the $P_{30} = .1$ curve but above the $P_{60} = .9$ curve. This is the shaded region shown in the figure.

There may be other applications for this sort of information. For any application to be operationally useful it will be necessary to determine through operational testing the actual distribution of bearing errors encountered in the bearing observations.

CNA PROFESSIONAL PAPERS - 1978 TO PRESENT*

- PP 211
Mizrahi, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978, AD A054 429
- PP 212
Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solution," 20 pp., Jun 1978, AD A058 535
- PP 213
Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lanchester Equations," 12 pp., Feb 78 (Presented at the First Annual Workshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978), AD A071 472
- PP 214
Weinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1976 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1976), AD A056 228
- PP 215
Collie, Russell C., "Comments on: Principles of Information Retrieval by Manfred Kochen," 10 pp., Mar 78 (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301), December 1975), AD A054 426
- PP 216
Collie, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978 (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 366-370, November 1977), AD A054 425
- PP 217
Collie, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78 (Presented at the Annual meeting of the American Society for Information Science held in San Francisco, California, October 1976), AD A054 442
- PP 218 - Classified
- PP 219
Huntzinger, R. LeVar, "Market Analysis with Rational Expectations: Theory and Estimation," 60 pp., Apr 78, AD A054 422
- PP 220
Maurer, Donald E., "Diagonalization by Group Matrices," 26 pp., Apr 78, AD A054 443
- PP 221
Weinland, Robert G., "Superpower Naval Diplomacy in the October 1973 Arab-Israeli War," 76 pp., Jun 1978 (Published in Superpower in the Mediterranean: Political Utility and Military Constraints, The Washington Papers No. 61, Beverly Hills and London: Sage Publications, 1979) AD A055 564
- PP 222
Mizrahi, Maurice M., "Correspondence Rules and Path Integrals," 30 pp., Jun 1978 (Invited paper presented at the CNRS meeting on "Mathematical Problems in Feynman's Path Integrals," Marseille, France, May 22-26, 1978) (Published in Springer Verlag Lecture Notes in Physics, 106, (1979), 234-253) AD A055 536
- PP 223
Mangel, Marc, "Stochastic Mechanics of Molecule Molecule Reactions," 21 pp., Jun 1978, AD A056 227
- PP 224
Manger, Marc, "Aggregation, Bifurcation, and Extinction in Exploited Animal Populations," 48 pp., Mar 1978, AD A058 536
*Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada
- PP 225
Mangel, Marc, "Oscillations, Fluctuations, and the Hopf Bifurcation," 43 pp., Jun 1978, AD A058 537
*Portions of this work were completed at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada.
- PP 226
Ralston, J. M. and J. W. Mann, "Temperature and Current Dependence of Degradation in Red-Emitting GaP LEDs," 34 pp., Jun 1978 (Published in Journal of Applied Physics, 50, 3630, May 1979) AD A058 538
*Bell Telephone Laboratories, Inc.
- PP 227
Mangel, Marc, "Uniform Treatment of Fluctuations at Critical Points," 50 pp., May 1978, AD A058 539
- PP 228
Mangel, Marc, "Relaxation at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978, AD A058 540
- PP 229
Mangel, Marc, "Diffusion Theory of Reaction Rates, I: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978, AD A058 541
- PP 230
Mangel, Marc, "Diffusion Theory of Reaction Rates, II Ornstein-Uhlenbeck Approximation," 34 pp., Feb 1978, AD A058 542
- PP 231
Wilson, Desmond P., Jr., "Naval Projection Forces: The Case for a Responsive MAF," Aug 1978, AD A054 543
- PP 232
Jacobson, Louis, "Can Policy Changes Be Made Acceptable to Labor?" Aug 1978 (Submitted for publication in Industrial and Labor Relations Review), AD A061 528

*CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311. An Index of Selected Publications is also available on request. The index includes a Listing of Professional Papers; with abstracts; issued from 1969 to June 1981.

- PP 233
Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quits," 23 pp., Sep 1978
- PP 234 - Revised
Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditures: The Case of Construction Grants," 23 pp., Oct 1979, AD A061 529
- PP 235
Mizrahi, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (Published in Journal of Mathematical Physics 20 (1979) pp. 844-855), AD A061 538
- PP 237
Maurer, Donald, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979 (Published in the Illinois Journal of Mathematics, Vol. 22 (1978), pp. 672-681)
- PP 238
Utgoff, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978 (Presented at the Conference on Economic Indicators and Performance: The Current Dilemma Facing Government and Business Leaders, presented by Indiana University Graduate School of Business), AD A061 527
- PP 239
Troost, R. P. and Warner, J. T., "The Effects of Military Occupational Training on Civilian Earnings: An Income Selectivity Approach," 38 pp., Nov 1979, AD A077 831
- PP 240
Powers, Bruce, "Goals of the Center for Naval Analyses," 13 pp., Dec 1978, AD A063 759
- PP 241
Mangel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in Journal of Chemical Physics, Vol. 69, No. 6, Oct 15, 1978), AD A063 787
- PP 242
Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978, AD A063 760
- PP 243
Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems," 18 pp., Dec 1978, AD A063 761
- PP 244
Sherman, Allen and Horowitz, Stanley A., "Maintenance Costs of Complex Equipment," 20 pp., Dec 1978 (Published By The American Society of Naval Engineers, Naval Engineers Journal, Vol. 91, No. 6, Dec 1979) AD A071 473
- PP 245
Simpson, William R., "The Accelerometer Methods of Obtaining Aircraft Performance from Flight Test Data (Dynamic Performance Testing)," 403 pp., Jun 1979, AD A075 226
- PP 246
Brechling, Frank, "Layoffs and Unemployment Insurance," 35 pp., Feb 1979 (Presented at the Nber Conference on "Low Income Labor Markets," Chicago, Jun 1978), AD A096 629
- PP 248
Thomas, James A., Jr., "The Transport Properties of Dilute Gases in Applied Fields," 183 pp., Mar 1979, AD A096 464
- PP 249
Glasser, Kenneth S., "A Secretary Problem with a Random Number of Choices," 23 pp., Mar 1979
- PP 250
Mangel, Marc, "Modeling Fluctuations in Macroscopic Systems," 26 pp., Jun 1979
- PP 251
Troost, Robert P., "The Estimation and Interpretation of Several Selectivity Models," 37 pp., Jun 1979, AD A075 941
- PP 252
Munn, Walter R., "Position Finding with Prior Knowledge of Covariance Parameters," 5 pp., Jun 1979 (Published in IEEE Transactions on Aerospace & Electronic Systems, Vol. AES-15, No. 3, Mar 1979)
- PP 253
Glasser, Kenneth S., "The d-Choice Secretary Problem," 32 pp., Jun 1979, AD A075 225
- PP 254
Mangel, Marc and Quenbeck, David B., "Integration of a Bivariate Normal Over an Offset Circle," 14 pp., Jun 1979, AD A096 471
- PP 255 - Classified, AD B051 441L
- PP 256
Maurer, Donald E., "Using Personnel Distribution Models," 27 pp., Feb 1980, AD A082 218
- PP 257
Thaler, R., "Discounting and Fiscal Constraints: Why Discounting is Always Right," 10 pp., Aug 1979, AD A075 224
- PP 258
Mangel, Marc S. and Thomas, James A., Jr., "Analytical Methods in Search Theory," 86 pp., Nov 1979, AD A077 832
- PP 259
Glass, David V.; Hou, Ih-Ching; Munn, Walter R., and Parin, David A., "A Class of Commutative Markov Matrices," 17 pp., Nov 1979, AD A077 833
- PP 260
Mangel, Marc S. and Cope, Davis K., "Detection Rate and Sweep Width in Visual Search," 14 pp., Nov 1979, AD A077 834
- PP 261
Villa, Carlos L.; Zvijec, David J. and Ross, John, "Frenck-Condon Theory of Chemical Dynamics. VI. Angular Distributions of Reaction Products," 14 pp., Nov 1979 (Reprinted from Journal Chemical Phys. 70(12), 15 Jun 1979), AD A076 287
- PP 262
Petersen, Charles C., "Third World Military Elites in Soviet Perspective," 50 pp., Nov 1979, AD A077 835
- PP 263
Robinson, Kathy L., "Using Commercial Tankers and Container-ships for Navy Underway Replenishment," 25 pp., Nov 1979, AD A077 836

- PP 264
Weinland, Robert G., "The U.S. Navy in the Pacific: Past, Present, and Glances of the Future," 31 pp., Nov 1979 (Delivered at the International Symposium on the Sea, sponsored by the International Institute for Strategic Studies, The Brookings Institution and the Yomiuri Shimbun, Tokyo, 16-20 Oct 1978) AD A066 837
- PP 265
Weinland, Robert G., "War and Peace in the North: Some Political Implications of the Changing Military Situation in Northern Europe," 18 pp., Nov 1979 (Prepared for presentation to the Conference of the Nordic Balance in Perspective: The Changing Military and Political Situation, Center for Strategic and International Studies, Georgetown University, Jun 15-16, 1978) AD A077 838
- PP 266
Utgoff, Kathy Classen, and Brechling, Frank, "Taxes and Inflation," 25 pp., Nov 1979, AD A081 194
- PP 267
Trost, Robert P., and Vogel, Robert C., "The Response of State Government Receipts to Economic Fluctuations and the Allocation of Counter-Cyclical Revenue Sharing Grants," 12 pp., Dec 1979 (Reprinted from the Review of Economics and Statistics, Vol. LXI, No. 3, August 1979)
- PP 268
Thomson, James S., "Seaport Dependence and Inter-State Cooperation: The Case of Sub-Saharan Africa," 141 pp., Jan 1980, AD A081 193
- PP 269
Weiss, Kenneth G., "The Soviet Involvement in the Ogaden War," 42 pp., Jan 1980 (Presented at the Southern Conference on Slavic Studies in October, 1979), AD A082 219
- PP 270
Rasmek, Richard, "Soviet Policy in the Horn of Africa: The Decision to Intervene," 52 pp., Jan 1980 (To be published in "The Soviet Union in the Third World: Success or Failure," ed. by Robert M. Donaldson, Westview Press, Boulder, Co., Summer 1980), AD A081 195
- PP 271
McConnell, James, "Soviet and American Strategic Doctrines: One More Time," 43 pp., Jan 1980, AD A081 192
- PP 272
Weiss, Kenneth G., "The Azores in Diplomacy and Strategy, 1940-1945," 46 pp., Mar 1980, AD A085 094
- PP 273
Makade, Michael K., "Labor Supply of Wives with Husbands Employed Either Full Time or Part Time," 39 pp., Mar 1980, AD A082 220
- PP 274
Nunn, Walter R., "A Result in the Theory of Spiral Search," 9 pp., Mar 1980
- PP 275
Goldberg, Lawrence, "Recruiters Advertising and Navy Enlistments," 34 pp., Mar 1980, AD A082 221
- PP 276
Goldberg, Lawrence, "Delaying an Overhaul and Ship's Equipment," 40 pp., May 1980, AD A085 095
- PP 277
Mangel, Marc, "Small Fluctuations in Systems with Multiple Limit Cycles," 19 pp., Mar 1980 (Published in SIAM J. Appl. Math., Vol. 38, No. 1, Feb 1980) AD A086 229
- PP 278
Mizrahi, Maurice, "A Targeting Problem: Exact vs. Expected-Value Approaches," 23 pp., Apr 1980, AD A085 096
- PP 279
Walt, Stephen M., "Causal Inferences and the Use of Force: A Critique of Force Without War," 50 pp., May 1980, AD A085 097
- PP 280
Goldberg, Lawrence, "Estimation of the Effects of A Ship's Steaming on the Failure Rate of its Equipment: An Application of Econometric Analysis," 25 pp., Apr 1980, AD A085 098
- PP 281
Mizrahi, Maurice M., "Comment on 'Discretization Problems of Functional Integrals in Phase Space'," 2 pp., May 1980, published in "Physical Review D", Vol. 22 (1980), AD A094 994
- PP 283
Dismales, Bradford, "Expected Demand for the U.S. Navy to Serve as An Instrument of U.S. Foreign Policy: Thinking About Political and Military Environmental Factors," 30 pp., Apr 1980, AD A085 099
- PP 284
J. Kallison, W. Nunn, and U. Sumita, "The Laguerre Transform," 119 pp., May 1980, AD A085 100
*The Graduate School of Management, University of Rochester and the Center for Naval Analyses
**The Graduate School of Management, University of Rochester
- PP 285
Rasmek, Richard B., "Superpower Security Interests in the Indian Ocean Area," 26 pp., Jun 1980, AD A087 113
- PP 286
Mizrahi, Maurice M., "On the MKB Approximation to the Propagator for Arbitrary Hamiltonians," 23 pp., Aug 1980 (Published in Journal of Math. Phys., 22(1) Jan 1981), AD A091 307
- PP 287
Cope, Davis, "Limit Cycle Solutions of Reaction-Diffusion Equations," 35 pp., Jun 1980, AD A087 114
- PP 288
Golman, Walter, "Don't Let Your Slides Flip You: A Painless Guide to Visuals That Really Aid," 28 pp., (revised Aug 1982), AD A092 732
- PP 289
Robinson, Jack, "Adequate Classification Guidance - A Solution and a Problem," 7 pp., Aug 1980, AD A091 212
- PP 290
Watson, Gregory M., "Evaluation of Computer Software in an Operational Environment," 17 pp., Aug 1980, AD A091 213
- PP 291
Maddala, G. S., and Trost, R. P., "Some Extensions of the Marlowe Press Model," 17 pp., Oct 1980, AD A091 946
*University of Florida

- PP 292
Thomas, James A., Jr., "The Transport Properties of Binary Gas Mixtures in Applied Magnetic Fields," 10 pp., Sept 1980 (Published in Journal of Chemical Physics 72(10), 15 May 1980)
- PP 293
Thomas, James A., Jr., "Evaluation of Kinetic Theory Collision Integrals Using the Generalized Phase Shift Approach," 12 pp., Sept 1980 (Printed in Journal of Chemical Physics 72(10), 15 May 1980)
- PP 294
Roberts, Stephen S., "French Naval Policy Outside of Europe," 30 pp., Sept 1980 (Presented at the Conference of the Section on Military Studies, International Studies Association Kiewah Island, S.C.), AD A091 306
- PP 295
Roberts, Stephen S., "An Indicator of Informal Empire: Patterns of U.S. Navy Cruising on Overseas Stations, 1869-1897," 40 pp., Sept 1980 (Presented at Fourth Naval History Symposium, US Naval Academy, 26 October 1979, AD A091 316)
- PP 296
Dismukes, Bradford and Petersen, Charles C., "Maritime Factors Affecting Iberian Security," (Factores Maritimos que Afectan la Seguridad Iberica) 14 pp., Oct 1980, AD A092 733
- PP 297 - Classified
- PP 298
Mizrahi, Maurice M., "A Markov Approach to Large Missile Attacks," 31 pp., Jan 1981, AD A096,159
- PP 299
Jondrow, James M. and Levy, Robert A., "Wage Leadership in Construction," 19 pp., Jan 1981, AD A094 797
- PP 300
Jondrow, James and Schmidt, Peter, "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model," 11 pp., Jan 1981, AD A096 160
*Michigan State University
- PP 301
Jondrow, James M.; Levy, Robert A. and Hughes, Claire, "Technical Change and Employment in Steel, Autos, Aluminum, and Iron Ore," 17 pp., Mar 1981, AD A099 394
- PP 302
Jondrow, James M. and Levy, Robert A., "The Effect of Imports on Employment Under Rational Expectations," 19 pp., Apr 1981, AD A099 392
- PP 303
Thomson, James, "The Rarest Commodity in the Coming Resource Wars," 3 pp., Aug 1981 (Published in the Washington Star, April 13, 1981), AD A104 221
- PP 304
Duffy, Michael K.; Greenwood, Michael J. and McDowell, John M., "A Cross-Sectional Model of Annual Interregional Migration and Employment Growth: Intertemporal Evidence of Structural Change, 1958-1975," 31 pp., Apr 1981, AD A099 393
*University of Colorado
*Arizona State University
- PP 305
Nunn, Laura M., "An Introduction to the Literature of Search Theory," 32 pp., Jun 1981, AD A100 420
- PP 306
Anger, Thomas E., "What Good Are Warfare Models?" 7 pp., May 1981, AD A100 421
- PP 307
Thomson, James, "Dependence, Risk, and Vulnerability," 43 pp., Jun 1981, AD A102 698
- PP 308
Mizrahi, M.M., "Correspondence Rules and Path Integrals," Jul 1981, Published in "Nuovo Cimento B", Vol. 61 (1981), AD A102 699
- PP 309
Weinland, Robert G., "An (The?) Explanation of the Soviet Invasion of Afghanistan," 44 pp., May 1981, AD A100 422
- PP 310
Stanford, Janette M. and Tai Te Wu, "A Predictive Method for Determining Possible Three-dimensional Foldings of Immunoglobulin Backbones Around Antibody Combining Sites," 19 pp., Jun 1981 (Published in J. theor. Biol. (1981) 88, 421-439, AD A100 423
*Northwestern University, Evanston, IL
- PP 311
Bowes, Marianne, Brechling, Frank P. R., and Utgoff, Kathleen P. Classen, "An Evaluation of UI Funds," 13 pp., May 1981 (Published in National Commission on Unemployment Compensation's Unemployment Compensation: Studies and Research, Volume 2, July 1980), AD A100 424
- PP 312
Jondrow, James; Bowes, Marianne and Levy, Robert, "The Optimum Speed Limit," 23 pp., May 1981, AD A 100 425
- PP 313
Roberts, Stephen S., "The U.S. Navy in the 1980s," 36 pp., Jul 1981, AD A 102 696
- PP 314
Jehn, Christopher; Morowitz, Stanley A. and Lockman, Robert F., "Examining the Draft Debate," 20 pp., Jul 1981, AD A106 192
- PP 315
Buck, Ralph V., Capt., "Le Catastrophe by any other name...", 4 pp., Jul 1981, AD A102 697
- PP 316
Roberts, Stephen S., "Western European and NATO Navies, 1980," 20 pp., Aug 1981, AD A104 223
- PP 317
Roberts, Stephen S., "Superpower Naval Crisis Management in the Mediterranean," 35 pp., Aug 1981, AD A104 222
- PP 318
Vego, Milan N., "Yugoslavia and the Soviet Policy of Force in the Mediterranean Since 1961," 187 pp., Aug 1981

- PP 319
Smith, Michael W., "Antiair Warfare Defense of Ships at Sea," 46 pp., Sep 1981 (This talk was delivered at the Naval Warfare System and Technology Conference of the American Institute of Aeronautics and Astronautics in Washington on December 12, 1980; in Boston on January 20, 1981; and in Los Angeles on June 12, 1981.), AD A106 191
- PP 320
Trost, R. P.; Lurie, Philip and Berger, Edward, "A Note on Estimating Continuous Time Decision Models," 15 pp., Sep 1981, AD A106 193
- PP 321
Duffy, Michael K. and Ladman, Jerry R., "The Simultaneous Determination of Income and Employment in United States--Mexico Border Region Economies," 34 pp., Sep 1981
"Associate Professor of Economics, Arizona State University, Tempe, AZ., AD A106 540
- PP 322
Warner, John T., "Issues in Navy Manpower Research and Policy: An Economist's Perspective," 66 pp., Dec 1981, AD A110 221
- PP 323
Bosma, Frederick M., "Generation of Correlated Log-Normal Sequences for the Simulation of Clutter Echoes," 33 pp., Dec 1981.
- PP 324
Horowitz, Stanley A., "Quantifying Seapower Readiness," 6 pp., Dec 1981 (Published in Defense Management Journal, Vol. 18, No. 2), AD A 110 220
- PP 326
Roberts, Stephen S., "Western European and NATO Navies, 1981," 27 pp., Jul 1982, AD A118 703
- PP 327
Hammon, Collin, Capt., USN and Graham, David R., Dr., "Estimation and Analysis of Navy Shipbuilding Program Disruption Costs," 12 pp., Mar 1980, AD A112 514
- PP 328
Weinland, Robert G., "Northern Waters: Their Strategic Significance," 27 pp., Dec 1980, AD A112 509
- PP 329
Mangel, Marc, "Applied Mathematicians And Naval Operators," 40 pp., Mar 1982 (Revised), AD A116 598
- PP 330
Lockman, Robert F., "Alternative Approaches to Attrition Management," 30 pp., Jan 1982, AD A112 510
- PP 331
Roberts, Stephen S., "The Turkish Straits and the Soviet Navy in the Mediterranean," 15 pp., Mar 1982 (Published in Navy International)
- PP 332
Jahn, Christopher, "The RDF and Amphibious Warfare," 36 pp., Mar 1982, AD A 113 592
- PP 333
Lee, Lung-Fel and Trost, Robert P., "Estimation of Some Limited Dependent Variable Models with Application to Housing Demand," 26 pp., Jan 1982. (Published in Journal of Econometrics 8 (1978) 357-382), AD A 112 536
- PP 334
Kenny, Lawrence W., Lee, Lung-Fel, Maddala, G. S., and Trost R. P., "Returns to College Education: An Investigation of Self-Selection Bias Based on the Project Talent Data," 15 pp., Jan 1982. (Published in International Economic Review, Vol. 20, No. 3, October 1979), AD A112 480
- PP 335
Lee, Lung-Fel, G.S. Maddala, and R. P. Trost, "Asymptotic Covariance Matrices of Two-Stage Probit and Two-Stage Tobit Methods for Simultaneous Equations Models with Selectivity," 13 pp., Jan 1982. (Published in Econometrica, Vol. 48, No. 2 March, 1980), AD A112 483
- PP 336
O'Neill, Thomas, "Mobility Fuels for the Navy," 13 pp., Jan 1982. (Accepted for publication in Naval Institute Proceedings), AD A112 511
- PP 337
Warner, John T. and Goldberg, Matthew S., "The Influence of Non-Pecuniary Factors on Labor Supply," 23 pp., Dec 1981, AD A113 094
- PP 339
Wilson, Desmond P., "The Persian Gulf and the National Interest," 11 pp., Feb 1982, AD A112 505
- PP 340
Lurie, Philip, Trost, R. P., and Berger, Edward, "A Method for Analyzing Multiple Spell Duration Data," 34 pp., Feb 1982, AD A112 504
- PP 341
Trost, Robert P. and Vogel, Robert C., "Prediction with Pooled Cross-Section and Time-Series Data: Two Case Studies," 6 pp., Feb 1982, AD A112 503
- PP 342
Lee, Lung-Fel, Maddala, G. S., and Trost, R. P., "Testing for Structural Change by D-Methods in Switching Simultaneous Equations Models," 5 pp., Feb 1982, AD A112 482
- PP 343
Goldberg, Matthew S., "Projecting the Navy Enlisted Force Level," 9 pp., Feb 1982, AD A112 484
- PP 344
Fletcher, Jean, W., "Navy Quality of Life and Reenlistment," 13 pp., Nov 1981, AD A113 095
- PP 345
Utgoft, Kathy and Thaler, Dick, "The Economics of Multi Year Contracting," 47 pp., Mar 1982. (Presented at the 1982 Annual Meeting of the Public Choice Society, San Antonio, Texas, March 5-7, 1982), AD A114 732
- PP 346
Rostker, Bernard, "Selective Service and the All-Volunteer Force," 23 pp., Mar 1982, AD A113 096
- PP 347
McConnell, James, M., "A Possible Counterforce Role for the Typhoon," 24 pp., Mar 1982, AD A116 601
- PP 348
Jondrow, James, Trost, Robert, "An Empirical Study of Production Inefficiency in the Presence of Errors-in-The-Variables," 14 pp., Feb 1982, AD A113 591

- PP 349
W. H. Breckenridge, O. Kim Mainin, "Collisional Intra-multiplet Relaxation of $Cd(5s5p^3P_{0,1,2})$ by Alkane Hydrocarbons," 7 pp., Jul 1981. (Published in Journal of Chemical Physics, 76(4), 15 Feb 1982), AD A115 093
- PP 350
Levin, Marc, "A Method for Increasing the Firepower of Virginia Class Cruisers," 10 pp., Apr 1982. (To be published in U.S. Naval Institute Proceedings), AD A116 602
- PP 351
Coutre, S. E.; Stanford, J. M.; Movis, J. G.; Stevens, P. W.; Wu, T. T., "Possible Three-Dimensional Backbone Folding Around Antibody Combining Site of Immunoglobulin MDPC 167," 18 pp., Apr 1982. (Published in Journal of Theoretical Biology), AD A116 603
- PP 352
Berfoot, C. Bernard, "Aggregation of Conditional Absorbing Markov Chains," 7 pp., June 1982 (Presented to the Sixth European Meeting on Cybernetics and Systems Research, held at the University of Vienna, Apr 1982.), AD A116 603
- PP 353
Berfoot, C. Bernard, "Some Mathematical Methods for Modeling the Performance of a Distributed Data Base System," 18 pp., June 1982. (Presented to the International Working Conference on Model Realism, held at Bad Honnef, West Germany, Apr 1982.), AD A116 604
- PP 354
Hall, John V., "Why the Short-War Scenario is Wrong for Naval Planning," 6 pp., Jun 1982., AD A118 702
- PP 356
Cylke, Steven; Goldberg, Matthew S.; Hogan, Paul; Mairs, Lee; "Estimation of the Personal Discount Rate: Evidence from Military Reenlistment Decisions," 19 pp., Apr 1982., AD A122 419
- PP 357
Goldberg, Matthew S., "Discrimination, Nepotism, and Long-Run Wage Differentials," 13 pp., Sep 1982. (Published in Quarterly Journal of Economics, May 1982.).
- PP 358
Akst, George, "Evaluating Tactical Command And Control Systems--A Three-Tiered Approach," 12 pp., Sep 1982., AD A122 478
- PP 359
Quester, Aline; Fletcher, Jean; Marcus, Alan; "Veteran Status As A Screening Device: Comment," 26 pp., Aug 1982, AD A123 656
- PP 361
Quenbeck, David B., "Methods for Generating Aircraft Trajectories," 31 pp., Sep 1982., AD A122 366
- PP 362
Morowitz, Stanley A., "Is the Military Budget Out of Balance?," 10 pp., Sep 1982., AD A122 368
- PP 363
Marcus, A. J., "Personnel Substitution and Navy Aviation Readiness," 35 pp., Oct 1982., AD A122 420
- PP 364
Quester, Aline; Nakada, Michael; "The Military's Monopsony Power," 29 pp., Oct 1982., AD A123 657
- PP 365
Greer, William L.; Bartholomew, James C., Cdr.; "Psychological Aspects of Mine Warfare," 15 pp., Oct 1982
- PP 366
Sprulli, Nancy L.; Gastwirth, Joseph L.; "On the Estimation of the Correlation Coefficient From Grouped Data," 9 pp., Oct 1982. (Published in the Journal of the American Statistical Association, Sep 1982, Vol. 77, No. 379, Theory and Methods Section.), AD A122 362
- PP 368
Weinland, Robert G., "The Evolution of Soviet Requirements for Naval Forces--Solving the Problems of the Early 1960s," 41 pp., Dec 1982, AD A123 655
- PP 369
Quester, Aline; Lockman, Robert; "The All-Volunteer Force: A Positive Perspective," 29 pp., Nov 1982
- PP 370
Rostker, Bernard D., "Human Resource Models: An Overview," 17 pp., Nov 1982., AD A123 656
- PP 372
Hurley, William J., "An Overview of Acoustic Analysis," 46 pp., Jan 1983
- PP 373
Jacobson, Louis, "Research to Quantify the Effect of Permanent Change of Station Moves on Wives' Wages and Labor Supply," 35 pp., Jan 1983
- PP 375
Feldman, Paul, "Privatizing Airports in Washington, D.C.," 17 pp., Feb. 1983
- PP 376
Weiss, Kenneth G., "Power Grows Out of the Barrel of a Gunboat: The U.S. in Sino-Soviet Crises," 136 pp., Dec 1982
- PP 379
Jondrow, James M.; Chase, David E.; Gamble, Christopher L.; "The Price Differential Between Domestic and Imported Steel," 17 pp., May 1983.
- PP 380
Bells, Ellen, "Balancing Accession and Retention: Cost and Productivity Tradeoffs," 38 pp., March 1983.
- PP 381
Reeves, John M. L., "ONA's Conceptual Design and Cost Models for High-Speed Surface Craft," 23 pp., Apr 1983
- PP 382
Levy, Robert A.; Jondrow James M.; "The Adjustment of Employment to Technical Change in the Steel and Auto Industries," 40 pp., May 1983
- PP 383
Thomas, James A., Jr; Mangal, Marc.; "Properties of Quick Look Passive Localization," 39 pp., May 1983

