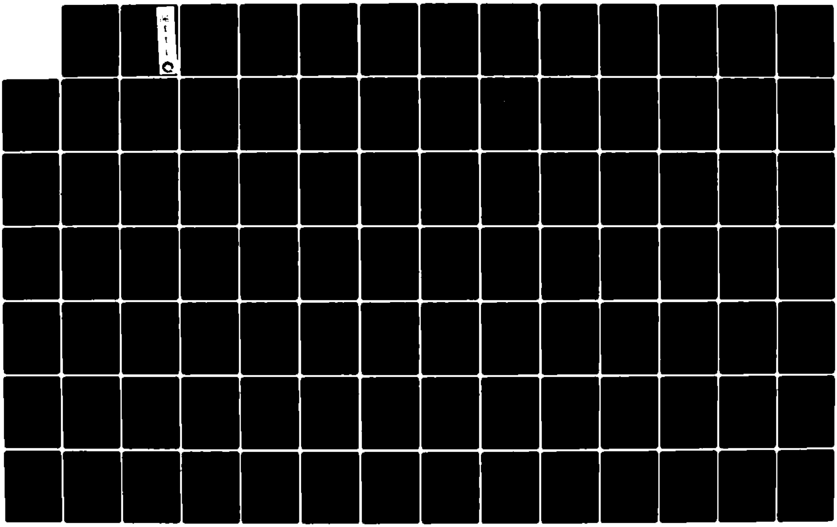
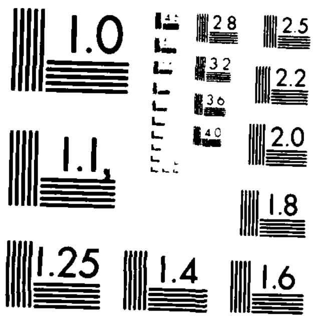
AD-A131 286 ASSESSMENT OF MEANS FOR DETERMINING DEFLECTION OF THE VERTICAL(U) ANALYTIC SCIENCES CORP READING MA W G HELLER ET AL. AUG 82 TASC-TR-4051-1 ETL-0303 1/3

UNCLASSIFIED DAAK70-82-C-0011 F/G 8/5 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

ETL-0303

Assessment of means for determining deflection of the vertical

W. G. Heller

A. R. LeSchack

S.F. Cushman

The Analytic Sciences Corporation
One Jacob Way
Reading, Massachusetts 01867

AUGUST 1982

Approved for Public Release. Distribution Unlimited

Prepared For

U.S. ARMY CORPS OF ENGINEERS
ENGINEER TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VIRGINIA 22060

E T L

U.S. ARMY ENGINEER TOPOGRAPHIC LABORATORIES

ETL

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER ETL-0303	2. GOVT ACCESSION NO. AD-A131286	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Assessment of Means For Determining Deflection of The Vertical		5. TYPE OF REPORT & PERIOD COVERED Interim Technical Report (1 Dec 1981 to 31 May 1982)
		6. PERFORMING ORG. REPORT NUMBER TR-4051-1
7. AUTHOR(s) W.G. Heller A.R. LeSchack S.F. Cushman		8. CONTRACT OR GRANT NUMBER(s) DAAK70-82-C-0011
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Analytic Sciences Corporation One Jacob Way Reading, Massachusetts 01867		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62707A; 4A762707AB55; A; 00013
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, VA 22060		12. REPORT DATE August 1982
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Deflection of the vertical	Photographic zenith tubes
Astrogeodetic surveys	Horrebow-Talcott method
Gravity anomalies	Sterneck's method
Gravimetric techniques	Astrolabes
	Theodolites

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This Interim Technical Report documents the first phase of a study of methods for determining deflections of the vertical. In this phase, the astrogeodetic and gravimetric methods have been examined. The objectives of these investigations have been: assessment of the accuracy of alternative astrogeodetic procedures; identification and analysis of potential improvements in astrogeodetic instrumentation; documentation of the

20. (Continued)

gravimetric method and identification of achievable accuracy; development of error models for astrogeodetic and gravimetric techniques.

The second phase of this program will address techniques for determining deflections of the vertical from combinations of astrogeodetic, gravimetric, inertial, and gradiometric techniques.

TR-4051-1

ASSESSMENT OF MEANS FOR DETERMINING DEFLECTION OF THE VERTICAL
(INTERIM TECHNICAL REPORT)

August 1982

Prepared under

Contract No. DAAK70-82-C-0011

for

U.S. ARMY ENGINEER TOPOGRAPHIC LABORATORIES
Fort Belvoir, Virginia

Prepared by:

W.G. Heller
A.R. LeSchack
S.F. Cushman

Approved by:

J.H. Fagan
R.F. Brammer

THE ANALYTIC SCIENCES CORPORATION
One Jacob Way
Reading, Massachusetts 01867

PREFACE

This document was prepared as an Interim Technical Report under contract DAAK70-82-C-0011 for the U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia. The contracting officer's technical representative is Mr. William A. Allen.

RE: Classified Reference, Distribution Unlimited
No change per Mr. Wm. A. Allen, AETL/ETL-TD-EA

A

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1.1 BACKGROUND

The U.S. Army Engineer Topographic Laboratories (USAETL) are currently supporting research designed to provide the Defense Mapping Agency (DMA) with advanced approaches for mapping the earth's deflection field. Four methods are being considered. They are: astrogeodetic, gravimetric, inertial, and gradiometric. These different approaches are depicted in Fig. 1.1-1.

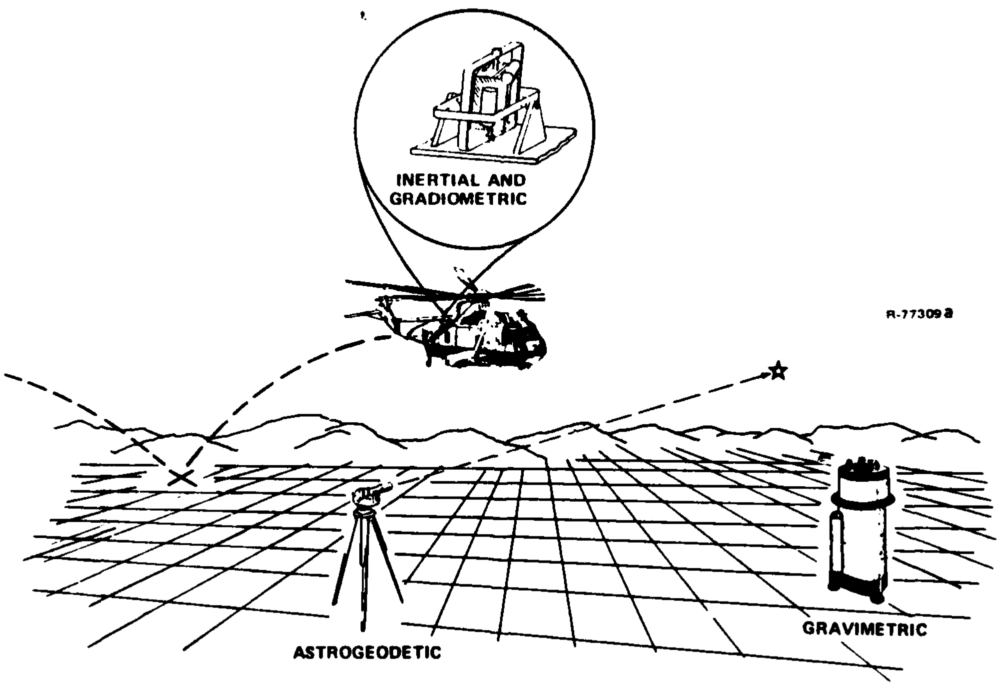


Figure 1.1-1 Techniques for Determining Deflections of the Vertical

This interim report details TASC studies of the astrogeodetic and gravimetric methods for determining deflection of the vertical. A companion work (Ref. 50) treats inertial and gradiometric methods. The second phase of this program will address synergistic combinations of all four techniques in the context of accuracy, cost of survey products, and overall logistical simplicity.

1.2 PURPOSE AND SCOPE

The objectives of the investigations reported herein are summarized below:

- To assess the accuracy and accuracy potential of alternative astrogeodetic techniques and procedures
- To identify and quantify improvements in accuracy, speed, cost, and reliability associated with new or modified astrosurvey instrumentation
- To investigate modern applications of the gravimetric method and identify achievable accuracies
- To develop astrogeodetic and gravimetric error models suitable for quantifying measurement errors.

Insofar as astrogeodetic procedures enjoy widespread use for both routine and high-accuracy determinations of deflections of the vertical, astrogeodesy receives main emphasis in this report. In addition, there are many new developments which offer potential advances in the practice of astrogeodesy. Gravimetric geodesy, while benefiting from recent improvements in global and distant zone gravity modeling, does not currently involve the wide diversity of measurement techniques and approaches seen within the set of astrogeodetic options. A possible exception to this lack of diversity, not discussed herein, is the use of gravimetric data in combination with other types of information. These and related investigations will be conducted during the Phase Two portion of this study.

This report is intended to draw together a wide range of results reported in the literature, technical "lore," mathematical analysis, and simulation to provide estimates of the state of the art in astrogeodesy and gravimetry. Much of this material is provided as a basis for the error model development and quantitative discussions which follow. A secondary objective is to document a significant amount of material which is either unavailable or scattered throughout the published literature.

1.3 TECHNICAL APPROACH

The principal analysis techniques employed in arriving at the results presented in this interim report are

- Critical evaluation and analysis of existing literature
- Quantification of errors in alternative measurement and data reduction techniques for deflections of the vertical
- Simulation of deflection survey and adjustment processes to provide estimates of the standard errors associated with particular optimal measurement approaches.

The second and third of these techniques will also be used as the basis for evaluating multisensor combinations of astrogeodetic, gravimetric, inertial, and gradiometric means for determining deflections of the vertical during the second phase of this effort.

1.4 SUMMARY OF RESULTS

Results are summarized at the end of each chapter and in Chapter Five. Key findings are previewed below.

Conventional astrogeodetic observing techniques involving the Wild T-4 theodolite represent the most often used current means for determining the deflection of the vertical at a point. Incremental improvements in accuracy and productivity appear likely with the near-term application of charged coupled device (CCD) imaging technology to the T-4 instrument as well as to astrolabe equipment. In addition, the application of geodetic network adjustment procedures to fields of closely spaced astronomic observations promises benefits in both accuracy and confidence of measured deflection data.

A potentially promising approach to deflection determination is to apply small, portable photographic zenith tube (PPZT) technology to make field measurements. While such devices appear to offer accuracies comparable with present T-4 techniques, improvements in productivity may be possible through shortened site occupation times and simplified operating procedures (although few data are yet available to support such contentions).

Gravimetric determinations of the deflections of the vertical, which utilize recently acquired worldwide gravity data, indicate that significant improvements have been made over the past several years. However, available data densities and accuracies in distant zones, as well as requirements for inner zone coverage, place gravimetrically achievable accuracies somewhat behind those of current astrogeodetic capabilities. The logistical difficulties of acquiring extremely dense inner zone anomaly data, as well as the need to increase gravity library holdings in remote land areas, militate further against the gravimetric method. Under special circumstances, as discussed in Chapter 3, the gravimetric method may be a useful adjunct in particular geographic areas. The approach also enjoys current use for determining deflections of the vertical at sea.

1.5 OVERVIEW OF THIS REPORT

This report is organized as follows. Chapter Two details astrogeodesy procedures, equipment, practices, and techniques. It also assesses technological approaches and develops error model descriptions for sources of uncertainty which significantly affect deflection survey accuracy. Chapter Three describes and analyzes gravimetric procedures for deflection determination. Principal numerical studies and survey simulation findings are presented in Chapter Four. Chapter Five summarizes the findings and conclusions of this report.

2. THE ASTROGEODETIC METHOD

2.1 INTRODUCTORY MATERIAL

The concept of deflection of the vertical has been defined in a number of ways, of which two -- the topocentric deflection and the gravimetric (or geodetic) deflection -- are relevant in the context of this report. These definition are illustrated in Fig. 2.1-1. The topocentric deflection, chiefly the subject of the present chapter (on the astrogeodetic method), is defined as the difference in direction between the true vertical (plumb line) at a point on the earth's surface and the corresponding normal to the ellipsoid. The gravimetric deflection, of principal concern in Chapter 3 (the gravimetric method) involves the plumb line direction at the geoid (reduced vertical) and the normal to the ellipsoid. The difference between the topocentric and gravimetric deflections depends on the curvature of the plumb line, and can be significant in rugged mountainous terrain.

The topocentric vertical is not a quantity that can be measured directly. The classical approach to its evaluation has been to determine the astrogeodetic latitude and longitude at a point, and to compare these coordinates with the geodetic latitude and longitude obtained by precise ground surveying techniques. However, geodetic surveys are based on an origin which can be, and occasionally is, changed. A shift in origin changes all of the geodetic coordinates, and hence all of the deflections as well.

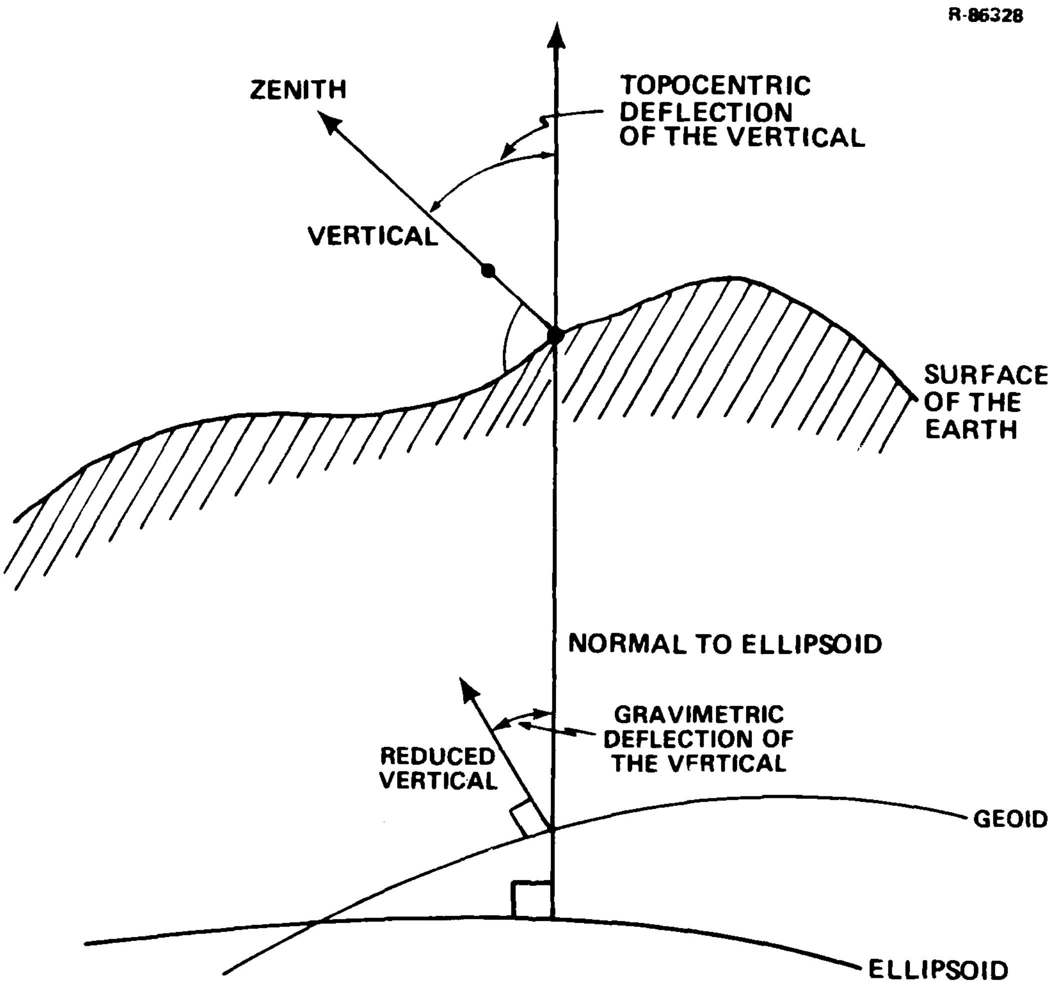


Figure 2.1-1 Deflection of the Vertical

Presently, geodetic coordinates are based on data which are global in extent, and the approximation to some universal figure and orientation of figure of the earth is realized much more closely than in the past. The fact remains that the geodetic coordinates are based upon a mathematical statement which is an approximation of reality. As surveys become better, geodetic coordinates can and will change. As these geodetic coordinates change, so will the deflections.

One quantity, generally considered invariant for the short term, is the position of a place determined from observations of the stars and the direction of gravity. The quality of these determinations will no doubt improve as instrumentation and technique improve. If closely scrutinized, even these astrogeodetic coordinates are time dependent. But the concept of determining the position of a place on the basis of astrogeodetic observations is useful at the accuracy levels needed today and those likely in the foreseeable future. This chapter is devoted to a discussion of surface position determinations from observations of gravity and the stars.

2.1.1 Background

In preparing this chapter of the report the following special considerations are taken into account.

- The primary emphasis is on portable equipment suitable for field use, as opposed to equipment used in fixed observatories. Of course, less portable equipment may be useful in establishing master or reference stations (Ref. 26)
- The "baseline" includes only equipment and techniques usually employed by U.S. agencies which routinely determine astronomic position. Both equipment and technique are subject to occasional modification, hence the use of "usually" in the description
- Astronomic positions are instantaneous and topocentric - that is, referred to the epoch of observation and the location of the intersection of the principal axes of the instrument used. Corrections are customarily applied to remove time dependencies and to refer the position to the geoid or ellipsoid. In the context of this report, topocentric deflections are required (Fig. 2.1-1)

- The primary geographic area considered is the contiguous continental United States. Thus the region of interest is the land area lying between latitudes 23°N and 50°N, and longitudes 66°W and 126°W.

2.1.2 Baselines for Astrogeodetic Procedures

For many years, first-order astronomic latitudes have been determined by the Horrebow-Talcott method, and astronomic longitudes have been determined by the Mayer method of observing stellar transits. Since control surveys were first performed by U.S. agencies, star catalogues have become more accurate, optical instruments have been improved, electronic equipment has been invented, and time has been redefined. All these changes have affected astrogeodesy in major ways. The fact remains that the two methods have remained standard for over a century. As a result there has developed a moderately-sized astrogeodetic data base involving many stations, many observers, and many instruments.

Recently, in anticipation of a new and more refined use of these data in the readjustment of the North American Datum, the National Geodetic Survey undertook a lengthy examination of all astrogeodetic data in their files. The published results (Ref. 3) form the basis for the analysis in this report. It is one of the most comprehensive and detailed evaluations of practices in the United States.

Historically the military branches of the U.S. Government have maintained a capability of their own to perform precise surveys of all types. The capability is ongoing. The services have also performed extensive research into astrogeodesy. On the basis of this research, Sterneck's method for determining latitude was adopted in 1964 as the standard for military surveys. Many determinations of latitude have been made since, and today the National Geodetic Survey is considering the Sterneck method as a companion or possible alternative to the Horrebow-Talcott method.

In this report, the Horrebow-Talcott method for determining latitude has been selected as the baseline against which other methods are compared. This is because experience in this country with the Horrebow-Talcott method extends for over 130 years, during which many observers of varying backgrounds have used several different types of instruments made by assorted manufacturers. By comparison, U.S. experience with Sterneck's method has been mostly in the last two decades using the Wild T-4 Theodolite. Indications are that Sterneck's method now provides results comparable in precision to those using the older method. However, the confidence with which judgments can be made about the Horrebow-Talcott method make it ideally suited for the baseline. (See also Section 2.6.)

2.2 BASIC ASTRONOMY

Spherical trigonometry terminology and notation vary in the literature. To provide appropriate background, a review of the topic is given in this section, and notation is introduced.

2.2.1 Some Definitions

For the purposes of this report certain definitions are included here (see Fig. 2.2-1). Note that some of these definitions may involve conditions sometimes not included in textbooks on geodetic astronomy.

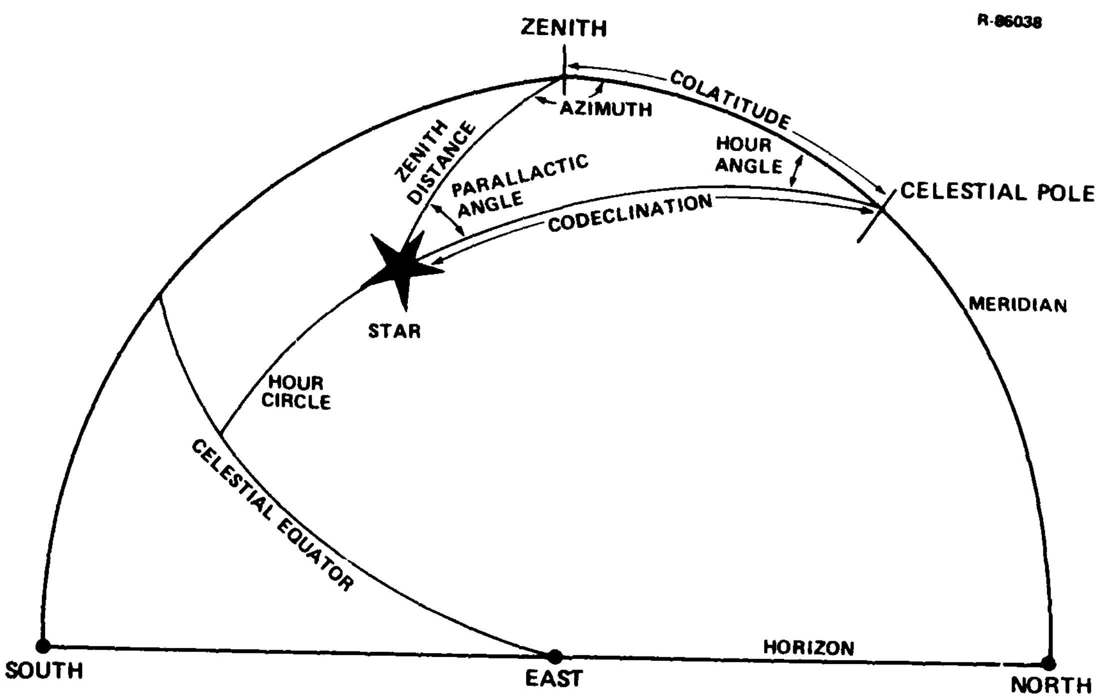


Figure 2.2-1 The Celestial Sphere

- Celestial Sphere - A sphere of infinite radius the center of which is located at the intersection of the principal axes of the instrument used to determine astronomic position

- Zenith - Intersection of the celestial sphere and the upwardly projected tangent to the plumb line. The point of tangency is the center of the celestial sphere

- Vertical Circle - A great circle passing through the zenith

- Zenith Distance - The (smallest) angle from the zenith to the star of interest. It is measured along a vertical circle (in the plane of the star, the observer, and the zenith). Symbol: z. $0 \leq z \leq 180$ deg (although $z < 90$ for visible objects)

- Celestial Horizon - Intersection of the celestial sphere and the central plane perpendicular to the gravity vector. The horizon is a great circle. Its zenith distance is always 90 deg

- Celestial Pole - That point on the celestial sphere about which the stars appear to rotate

- Celestial Equator - That great circle related to the celestial pole as the horizon is to the zenith

- Hour Circle - The great circle passing through the celestial pole and the star of interest

- Declination - The (smallest) angle between the celestial equator and the star of interest. Declination is measured along an hour circle, positive towards the North Celestial Pole. The declination of the North Pole is always 90 deg. Symbol: δ. $-90 \leq \delta \leq 90$

- Meridian - That great circle containing the zenith, the nadir (z=180 deg), and both celestial poles

- Astronomic Latitude - The instantaneous magnitude of the elevation of the upper celestial pole. The latitude is also equal to the zenith distance of the upper culmination of the celestial equator, and to the declination of the zenith. Symbol: ϕ. $-90 \leq \phi \leq 90$

- Azimuth - The angle from the meridian to the vertical circle passing through the star of interest, measured at the zenith, positive from north towards east. Symbol: $\bar{A}$.* $0 \leq \bar{A} \leq 360$

*The symbol A is used in this report to designate the azimuth factor (see Section 2.3-6).

- Hour Angle - The angle, with vertex at the celestial pole, between the observer's meridian and the great circle passing between pole and star (hour circle), measured from the observer's meridian westward from 0 to 360 deg. (Frequently expressed in equivalent time units from 0 to 24 hr)

- Astronomic Longitude - The angle between the observer's meridian and the standard zero meridian (Greenwich). In geodetic practice, longitude is measured positive to the east from 0 to 360 deg (frequently 0 to 24 hr). Other conventions are sometimes used in astronomy and geography

- Transit - The coincidence of a star with a (specified) meridian

- Prime Vertical - The vertical plane perpendicular to the meridian. Only those stars for which the declination is less than the latitude (for north latitudes) will cross the prime vertical

- Elongation - The condition that the azimuth of a star be an extremum. Only those stars for which the declination is greater than the latitude (for north latitude) will elongate

- Almucantar - A (small) circle of constant zenith distance. Any almucantar is parallel to the celestial horizon.

2.2.2 The Astronomical Triangle

The astronomical triangle is fundamental to astrogeodesy. Its three apices are the celestial pole, the observer's zenith, and whatever celestial object is observed. For observations near the meridian, the astronomical triangle is nearly collinear - that is, the hour angle is nearly zero. This permits the use of linear corrections and makes it unnecessary to resolve the entire triangle.

The three coordinate systems widely used in geodetic astronomy are:

- The horizon system
- The hour angle system
- The right ascension system.

They are defined in detail, along with the transformations between them, in Appendix A.

2.3 AN ASTROGEODETIC SURVEY BASELINE

The overall purpose of this part of the study is to point the way towards methods and equipment for field determination of astronomic positions of high precision. Described first are the methods and equipment used at present by organizations performing geodetic surveys for this purpose. Following are analyses of sources of errors for these procedures and a discussion of each of them, which will form the basis for efforts at improvement of techniques for astrogeodetic positioning.

2.3.1 Introduction

Current American practice separates latitude and longitude into distinct procedures. Each is described and analyzed individually although they are interrelated. Since positions relative to the pole, as opposed to relative rotation angles, are simpler to describe and analyze, latitude is discussed first.

The specifications on which the following descriptions are based are those of Ref. 23. Military organizations base their procedures on Ref. 22, which differ slightly. Aside from the use of Sterneck's method for determining latitude instead of the Horrebow-Talcott method, these differences do not alter the conclusions of this report.

2.3.2 Latitude by the Horrebow-Talcott Method

The Horrebow-Talcott method for determining latitude was first described two hundred and fifty years ago (Ref. 1). Except for some improvement in equipment no basic changes have occurred since. The method depends on a measurement of the difference in zenith distance of two stars as they pass the meridian, or transit, one south and the other north of the observer. The two stars are preselected so that:

- They appear in the list prepared by the National Geodetic Survey, which is a selection (Ref. 2) of stars from the Smithsonian Astrophysical Observatory Catalogue using the FK4 coordinate system
- They have nearly the same zenith distance (north and south)
- Their expected difference in zenith distance is less than the useful field of view of the telescope used to observe them.

This technique allows the observer to set the telescope to the expected mean zenith distance, and thereafter to determine by means of an eyepiece micrometer the deviation of each star from this value. (The instrument is reversed about its vertical axis between the stars.) The actual value of the mean zenith distance is not determined, since it appears with opposite sign in the observation equation for each star. Then, for each pair observed, the latitude is half the sum of the apparent declinations corrected by half the difference of observed zenith distances. Sixteen pairs of stars are usually observed on a single night, their selection being further restricted by a requirement assuring that the algebraic sum of the differences of zenith distance does not exceed a specific value.

Equipment for making these observations is usually specified to be the Universal Theodolite T-4 made by Wild Heerbrugg Instruments of Switzerland. This theodolite is basically an altazimuth telescope to which are added several devices for simplifying astronomic observations. Of these, two are of particular interest.

- Horrebow levels (customarily two, for greater precision) are mounted perpendicular to the horizontal axis of the telescope. They permit detection of small deviations from the expected mean zenith distance set prior to observation
- A micrometer eyepiece makes possible precise measurement of position in the field of view. Two such measurements determine the difference in zenith distance between two stars of a pair. This is the principal measurement for determination of latitude.

Additionally, a horizontal circle allows retention of the meridian, and a small vertical circle allows setting the expected mean zenith distance. This value need only be set accurately enough to assure appearance of both stars in the field of view, its exact retention being the function of the Horrebow level.

The basic Horrebow-Talcott solution involves the declinations of the stars and their difference in zenith distance. Up to three corrections are applied:

- A change in the zenith distance of the telescope between the two observations for a single pair
- The difference in the astronomic refraction between stars of a pair
- If necessary, a correction to the difference in zenith distance if the star has been observed off the meridian.

Other corrections are customarily applied to refer the observed positions to a uniform pole of rotation and to the geoid, but the geoid correction need not be applied in the present context since interest is in the topocentric deflection.

The ideal procedure for the determination of latitude would be to measure either the zenith distance of the celestial equator or the elevation of the pole. Either of these observations involves determination of vertical angles, a difficult survey measurement. The Horrebow-Talcott method takes advantage of:

- Minimization of instrumental errors, since only differential measurements are made
- Only minimal dependence on a model of refraction, since the difference in zenith distance is small
- Straightforward solution for latitude, since the corrections seldom require more than three significant figures.

A major weakness of the method is its requirement for many more stars than are contained in the FK4 catalogue; position data for these stars must be extracted from other catalogues of lesser accuracy than the FK4.

In Ref. 3, the method described above gives rise to standard deviations of the determined latitudes which can be represented by:

$$\sigma_\phi = [(\sigma_r)^2/n + (\sigma_s)^2]^{\frac{1}{2}} \qquad (2.3\text{-}1)$$

where

σ_r = 0.72 $\widehat{\sec}$, defined as the standard deviation of the latitude from the observation of a single star pair

σ_s = 0.26 $\widehat{\sec}$, defined as the between-determinations standard deviation of the latitude

n = 16, the customarily accepted minimum number of pairs of stars observed in the determination.

Sixteen pairs of stars usually take about three hours to observe. Further increases in n are usually considered to offer diminishing returns.

In an unpublished expansion of Ref. 3, σ_r is examined further by dividing it into two parts:

- Errors associated with star positions
- Errors associated with all other factors.

From an analysis of variance of both present day data and historical observations (which are complicated by changes in catalogues and reductions for polar motion), the conclusion is reached that these two error sources are approximately equal to each other at 0.5 $\widehat{\sec}$, for a single star pair.

2.3.3 Errors with the Horrebow-Talcott Method

Latitude is computed from the following equation, based on observations with the T-4.

$$\phi = \frac{1}{2}(\delta_N+\delta_S) + \frac{1}{2}R(M_W-M_E) + \frac{1}{16}(d+d')\,(\Sigma b_W-\Sigma b_E) + \frac{1}{2}(r_N-r_S) + \frac{1}{2}(m_N+m_S) \tag{2.3-2}$$

where

N,S refer to the north/south star of a pair

E,W refer to the east/west location of eyepiece at observation*

ϕ is the observed astronomic latitude

δ is the apparent declination of star at epoch of observation

R is the run of the micrometer in $\widehat{\text{sec}}$ per full turn of control knob

M is the micrometer reading at observation

d,d' are the sensitivities of first and second (primed) Horrebow levels in $\widehat{\text{sec}}$ per division (nominally 1 $\widehat{\text{sec}}$ per 2 mm div.)

Σb is the sum of the four readings of ends of bubbles of Horrebow levels at observation (typically estimated to 0.1 division)

r is the refraction

m is the ex-meridian correction†

---

*The T-4 is a "broken telescope" instrument, so that the eyepiece is always horizontal; the preset zenith distance is retained for both stars by rotation about the vertical axis.

†Ex-meridian correction is required when the star is not observed at transit, and adjusts the observed zenith distance to its (smaller) value at actual transit.

These terms are now examined in detail for their contribution to the total error.

The standard error of the star positions used in the actual observations is about 0.5 $\widehat{sec}$, or slightly less (see Section 2.4.1). This uncertainty represents the accumulation of errors from the following sources:

- Original observations for declination (few of epoch later than 1930)
- Original determinations of proper motion
- Constants for updating positions to 1950.0 (the catalogue epoch)
- Corrections applied to the original catalogue to change from the original coordinate system to approximately that of the FK4
- Constants for updating positions to epoch of observation
- Accumulated rounding error in computations performed before the advent of modern computing equipment.

These errors are "ultimate" in the sense that no improvement appears to be practical until the formulation and publication of a new catalogue*.

*It is possible, usually, to use the FK4 as a source of stars for the Horrebow-Talcott method, but the limited number of stars even in northern latitudes necessitates observations on two nights (or more) to obtain data from the minimum of 16 pairs required by specifications. The cost effectiveness of this procedure would appear low. Use of off-meridian stars offers some relief from the catalogue density required to support astrogeodetic surveying.

The primary measurement, difference of zenith distance (the second term of Eq. 2.3-2), is subject to a single error*:

> $\sigma_{\Delta z}$, which is the uncertainty in making the principal observation.

A typical value of $\sigma_{\Delta z}$ is 0.5 $\widehat{\text{sec}}$.

The level correction (the third term in Eq. 2.3-2) is subject to two errors:

> σ_d, which is the uncertainty in determination of the sensitivity of the level vial
>
> $\sigma_{\Sigma b}$, which is the uncertainty in level bubble observation.

The uncertainty in the level correction may be calculated by taking the root of the sum of squares of the partial derivatives of the appropriate part of Eq. 2.3-2:

$$\sigma_B = \{[\tfrac{1}{16}(\Sigma b_W - \Sigma b_E)\ \sigma_d]^2 + [\tfrac{1}{16}(d+d')\ \sigma_{\Sigma b}]^2\}^{\frac{1}{2}}. \qquad (2.3\text{-}3)$$

In this equation, realistic values are:

$$\Sigma b_W - \Sigma b_E < 8 \text{ divisions}$$
$$d+d' < 2\ \widehat{\text{sec}} \text{ per division}$$
$$\sigma_d < 0.03\ \widehat{\text{sec}} \text{ per division}$$
$$\sigma_{\Sigma b} < 1 \text{ division}$$

then

$$\sigma_B < [(0.015)^2 + (0.125)^2]^{\frac{1}{2}} \qquad (2.3\text{-}4)$$

---

*R is determined in the overall solution for latitude, and is assumed to contribute nothing here.

Thus, the uncertainty in the latitude due to uncertainty in determining the correction for mislevelment is 0.126 $\widehat{\sec}$. Note that reductions in the overall value of σ_B can be made by reducing each term. However, it is instructive to consider each individually. The quantity $(\Sigma b_W - \Sigma b_E)$ is routinely reduced in field practice by manipulation of the zenith distance to bring it more closely into agreement between stars of a pair. The quantity $\sigma_{\Sigma b}$ is particularly subject to uncertainty. As level vials age, the bubbles tend to stick in place, a condition easily discovered in a laboratory but not in the field. Some evidence points to an aggravation of this sticking by changes in temperature to which the equipment is routinely subjected in the field. It is doubtful that this error source, $\sigma_{\Sigma b}$, can be reduced below 0.1 $\widehat{\sec}$ using level vials.

The correction for difference in astronomic refraction (the fourth term in Eq. 2.3-2) is calculated from:

$$R = \frac{1}{2}\,(r_N - r_S) = \frac{1}{2}\,C \sin \Delta z \sec^2 z \qquad (2.3\text{-}5)$$

where

- C is the constant of astronomic refraction, with a nominal value of 57.9 $\widehat{\sec}$
- Δz is the difference in zenith distance between the two stars of a pair, taken from the observations themselves (twice the second term of Eq. 2.3-2)
- z is the (preset) mean zenith distance of the pair of stars observed.

The uncertainty, σ_R, is determined by root-sum-of-squares of the partial derivatives of Eq. 2.3-5 as:

$$\sigma_R = \left[\left(\frac{1}{2}\,\sigma_c \sin \Delta z \sec^2 z\right)^2 + \left(\frac{1}{2}\,C\,\sigma_{\Delta z} \cos \Delta z \sec^2 z\right)^2 + \left(C\,\sigma_z \sin \Delta z \sin z \sec^3 z\right)^2\right]^{\frac{1}{2}} \qquad (2.3\text{-}6)$$

For numerical evaluation, the following values are used

$\Delta z = 20\ \widehat{\text{min}}$ by limitations of the telescope,

$z = 30$ deg which is the recommended z_{max}

$\sigma_c < 0.2\ \widehat{\text{sec}}$ (uncertainty in the value of C)

$\sigma_{\Delta z} < 0.5\ \widehat{\text{sec}}$ from the discussion above

$\sigma_z < 120\ \widehat{\text{sec}}$ from expected mean zenith distance and setting circles

(Note that $\sigma_{\Delta z}$ and σ_z must be converted to radians in the computation.) Then

$$\sigma_R < [(0.00078)^2 + (0.00009)^2 + (0.00015)^2]^{\frac{1}{2}} \qquad (2.3\text{-}7)$$

Thus the uncertainty in the latitude due to uncertainty in determining the correction for refraction is less than 0.001 $\widehat{\text{sec}}$.

The small value cited for σ_R assumes normal refraction effects in a symmetric atmosphere that are well modeled by the tan z variation (see Ref. 6) upon which Eq. 2.3-5 is based. Anomalous effects sometimes encountered in the field may introduce unmodeled errors as large as 0.1 $\widehat{\text{sec}}$.

The final correction, for ex-meridian observation, is calculated for each star from:

$$m = \sin^2 \tfrac{1}{2}t \sin 2\delta \qquad (2.3\text{-}8)$$

where

t is the actual hour angle at epoch of observation

δ is the declination.

Each correction, m, is subject to error from two sources:

σ_δ the uncertainty in the declination

σ_t the uncertainty in the hour angle.

The contribution of each component is determined from the root-sum-of-squares of partial derivatives of Eq. 2.3-8. However, t is always small; hence cos $\frac{1}{2}t$ is taken as unity, and for numerical convenience in determining an upper bound, sin 2δ and cos 2δ are each replaced by unity. Then:

$$\sigma_m \leq [(\sigma_t \sin \tfrac{1}{2}t)^2 + (2\sigma_\delta \sin^2 \tfrac{1}{2}t)^2]^{\frac{1}{2}} \qquad (2.3\text{-}9)$$

For numerical evaluation, take

$t \doteq 2$ minutes of time

$\sigma_t < 0.5$ seconds of time (7.5 $\widehat{\text{sec}}$)

$\sigma_\delta < 0.5$ $\widehat{\text{sec}}$, as before.

Then

$$\sigma_m = [(0.03272)^2 + (0.00002)^2]^{\frac{1}{2}} \qquad (2.3\text{-}10)$$

Thus the uncertainty in the latitude due to uncertainty in determining the ex-meridian correction is 0.033 $\widehat{\text{sec}}$.* Usually,

*Note that σ_m has been calculated for a single star, rather than for the pair. Examination of the final term in Eq. 2.3-2 reveals that σ_m applies to the latitude as calculated.

t is large only on nights when rapidly moving small clouds interfere occasionally with observations at transit.

The contributors of various error sources to the latitude are summarized in Table 2.3-1. The root sum square of these numbers gives 0.72 $\widehat{sec}$ as the internal standard deviation of a determination of latitude from a single pair of stars, in agreement with the value determined in Ref. 3 for σ_r (the corresponding value).

TABLE 2.3-1
SOURCES OF ERROR IN LATITUDE

SOURCE	STANDARD ERROR	
	SYMBOL	MAGNITUDE ($\widehat{sec}$)
Declination	σ_δ	0.5
Difference of zenith distance	$\sigma_{\Delta z}$	0.5
Correction for levels	σ_B	0.126
Correction for differential astronomic refraction	σ_R	0.001
Correction for ex-meridian observation	σ_m	0.033
Root sum square	-	0.72

In summary, three sources are the principal contributors to uncertainty in a latitude determined by the Horrebow-Talcott method:

- Declination (σ_δ = 0.5 $\widehat{sec}$)
- Pointing accuracy by the observer ($\sigma_{\Delta z}$ = 0.5 $\widehat{sec}$)

- Use of a bubble level to maintain the mean zenith distance ($\sigma_B = 0.13\ \widehat{sec}$).

See Section 2.4 for further discussion of these sources of errors.

2.3.4 Latitude by Sterneck's Method

Latitude can be determined directly by measuring the zenith distance of a known star at its transit, from which the latitude is either the sum or difference of the declination and the measured zenith distance. The principal reasons for not using this simple method have been

- Imperfectly graduated vertical circles
- Uncertain correction for refraction.

As the result of modern technology, graduations on circles can be placed within 0.3 $\widehat{sec}$ with electronic control of the manufacturing process. For the T-4, the precision of measurement in a vertical plane is governed by the standard level vial attached to the vertical circle rather than by the graduations.

The refraction correction, while improved, is still uncertain. To reduce residual errors of refraction, observations are taken of a pair of stars, one north of the zenith and one south, as with the Horrebow-Talcott method. However, since the field of view of the telescope is not a limiting factor, the difference in zenith distances is arbitrary. The refraction is computed using any model, but only the <u>difference</u> in refraction between the two observations is of significance.

In Sterneck's method, the orientation of the zero of the vertical circle is unimportant. Two stars are observed at the transit of each. By reading the vertical circle, which is not moved between the two transits, the sum of the zenith distances is determined. Assuming that refraction is known, and the pointings are perfect, then the latitude is the mean declination plus half the difference of the readings. Also, if desired, the zenith of the vertical circle can be determined as half the mean reading plus half the difference in declinations. Thus a star pair permits determination of both latitude and the orientation of the vertical circle. For statistical purposes, a set is usually defined as several pairs of stars, all observed without changing the orientation of the vertical axis.

If there is a restriction on selection of stars (for example, mean declination of all stars not to differ from latitude by more than some tolerance), the refraction correction is limited to that for the residual difference in zenith distance. See, for example, Fig. 2.3-1.

These criteria are so general that a rather small catalogue of stars suffices. This implies that the FK4, currently regarded as the best general-purpose list of stellar positions, can be used. Practically, the density of stars is great enough to ensure that a set of Sterneck observations will take about as long as a corresponding set of Horrebow-Talcott observations using the NGS star list.

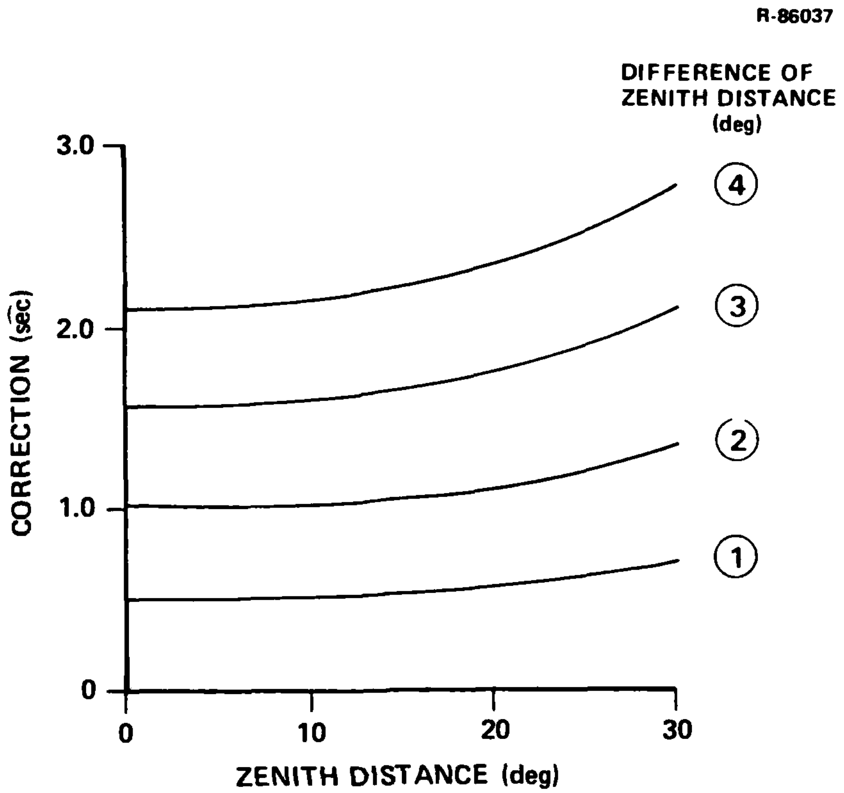


Figure 2.3-1 The Correction for Differential Refraction for a Pair of Stars

The key sources of error in Sterneck's method are comparable to those of the Horrebow-Talcott method, namely:

- The sensitivity of the uncertainty in latitude to errors in declination is one-to-one
- The sensitivity of the uncertainty in latitude to errors of pointing is one-to-one
- The maintenance of orientation of the vertical circle (or mean zenith distance) is wholly dependent on a level vial.

Regarding the last item, the standard T-4 can be equipped with a level vial on the vertical circle with a sensitivity of 1 to 2 sec per division. It is normally used in a coincidence method - that is, both ends of the bubble are viewed simultaneously and normally made to coincide. In the procedure described in Ref. 7 this vial is replaced with a Horrebow level having about twice the sensitivity. The level is read as usual - that is, the positions of both sides are compared to a scale. As a further refinement, the sensitivity of this level is determined at each field site just before or just after completing the observation for latitude. This procedure guarantees that the value of the sensitivity used in computing corrections to observations at this site is free of changes caused by differences in temperature and barometric pressure between this site and the laboratory where sensitivity has been determined. Alternatively, calibration curves can be generated and updated as appropriate.

Another procedural variation is the requirement for two observations on each star, with the telescope reversed about its vertical axis between them, thereby removing any collimation error. For these observations, the fixed central wire is used, the vertical circle is oriented relative to the replaced level, and azimuth of the instrumental meridian is established by the same method as for observations for longitude (see, for example, Ref. 6, p. 315), and maintained by using the horizontal circle.

The errors of this method can be assigned to the following source categories:

- Declination
- Zenith distance
- Correction for levels
- Correction for differential refraction

- Correction for ex-meridian observations (as defined in Section 2.3.3, page 2-17, footnote at the bottom of the page).

The analysis is the same as that for the Horrebow-Talcott method.

Errors of declination are unlikely to improve more than 30 percent by replacing the quasi-FK4 system found in the NGS/SAO catalogue with the actual system. Thus, the uncertainty from the Horrebow-Talcott method is to be reduced to 70 percent, and

$$\sigma_\delta = 0.5 \times 0.70 = 0.35 \ \widehat{sec} \qquad (2.3\text{-}11)$$

Pointing errors are unlikely to be reduced at all since the two pointings are opposite ends of the same measurement*, and the individual error of each pointing will combine statistically with its companion. In addition, the measurement is now dependent on circle graduations rather than an eyepiece micrometer. Thus the uncertainty of the Horrebow-Talcott method is to be increased by the root of two.

$$\sigma_z = 0.5 \times \sqrt{2} = 0.71 \ \widehat{sec} \qquad (2.3\text{-}12)$$

Correction for inclination cannot improve in quality over that for the Horrebow level, since only one vial is used instead of two. Thus the uncertainty of the Horrebow-Talcott method, similarly, is increased by the root of two:

$$\sigma_B = 0.126 \times \sqrt{2} = 0.18 \ \widehat{sec} \qquad (2.3\text{-}13)$$

*The measurement is made in two stages: 1) Point at the star and read the vertical circle. 2) Rotate the instrument 180 deg in azimuth, point again at the star, and make a second reading of the vertical circle. Small corrections for the apparent motion of the star are applied. Then half the difference between readings is the observed zenith distance.

The specification for keeping the mean declination for all stars within one degree of the observed latitude (Ref. 7) indicates that a correction for differential refraction may have as much as three times the error of the corresponding correction for latitudes determined by the Horrebow-Talcott method. Star pairs may also be separated by greater intervals of time, allowing an uncompensated change in meteorological conditions, which is arbitrarily assigned a factor of the square root of two. Thus the uncertainty of the Horrebow-Talcott method is to be increased by factors of both 3 and $\sqrt{2}$:

$$\sigma_R = 0.001 \times 3 \times \sqrt{2} = 0.0042 \ \widehat{sec} \qquad (2.3\text{-}14)$$

Just as the pointing error increases, so does the contribution of errors in the ex-meridian correction. However, timing is improved by use of an electric chronograph. Taking the improvement as 50 percent, and considering the additional observation, the uncertainty in the Horrebow-Talcott method is increased by the square root of two and decreased by 50 percent, so

$$\sigma_m = 0.033 \times \sqrt{2} \times 0.5 = 0.023 \ \widehat{sec} \qquad (2.3\text{-}15)$$

These figures can be compiled to give an overview of Sterneck's method, as shown in Table 2.3-2. The root sum square of these values is 0.81 $\widehat{sec}$. This figure, based on traditional field practice, may be somewhat conservative; recent improvements (cf. Ref. 26) may justify a value as low as 0.70 $\widehat{sec}$. A total of 32 stars are expected but 26 on a single night will be accepted (Ref. 7). Using the latter figure, the final expected internal error of a Sterneck latitude is 0.16 $\widehat{sec}$, which is to be compared to the value of $0.72/\sqrt{16}$ or 0.18 $\widehat{sec}$ for latitudes determined by the Horrebow-Talcott method. The improvement is modest. Procedures in the field are similar and the time spent observing is comparable. Compilation of an observing list for Sterneck's method can be done by field personnel, while computers are generally employed for compilation of observing lists for the Horrebow-Talcott method. However, this is not a limiting criterion since the locations at which astrogeodetic observations are to be made are known well in advance of the actual observations. Sterneck's method employs the same catalogue of stellar positions as is used for determining longitudes. In conclusion, this by itself is desirable even without any reduction in resultant uncertainties.

TABLE 2.3-2
ERROR SOURCES FOR STERNECK'S METHOD

ERROR SOURCE	SYMBOL	MAGNITUDE ($\widehat{sec}$)
Declination	σ_δ	0.35
Zenith Distance	σ_z	0.71
Levels	σ_B	0.18
Residual Differential Refraction	σ_R	0.0042
Ex-meridian Correction	σ_m	0.023
Root Sum Square	-	0.81

2.3.5 Longitude by Observations of Transits

The longitude of a place can be defined in terms of a difference in local time between the place and the Greenwich meridian. Thus, longitude can be measured by determining the local sidereal time at some epoch and comparing it with the sidereal time at Greenwich for that same epoch.

Historically, the method by which local time has been determined has hardly changed in a century. By definition, the local apparent sidereal time is equal to the apparent right ascension of a star at the epoch of its local transit, or meridian crossing. Improvement in quality of longitudes determined at field sites has come about almost solely by improvements in the method for determining (at the field site) what the local apparent sidereal time is at any given epoch at the standard (Greenwich) meridian. This has been the navigator's problem since he first took to sea, and the cartographer's problem since he first sketched a map in the sand. Mechanical chronometers represented an enormous advance. Now electronic clocks are so accurate that the determination of local epoch is the principal source of error. Clock errors are typically two orders of magnitude less. Thus the following discussion includes no analysis of determining time at the reference meridian, under the assumption that all field parties are equipped with a clock which records epochs that can be related to that time to any needed precision. (The necessary precision is one millisecond, equivalent to 0.015 $\widehat{sec}$.)

A simple technique for determining local time is to place the vertical cross-wire of a telescope on the observer's meridian and to observe the coincidence of the image of an identifiable star with the wire. An improvement on this is to add several pairs of wires each located symmetrically with respect to the center wire, so the mean of times at which the image crosses each wire reduces the uncertainty of determination. The next improvement is the use of a single moving wire which the observer superimposes on the moving image of the star for a suitable interval of time. Movement of the wire causes periodic recording of the epoch. The position of the image in the field can be exactly correlated to each epoch thus recorded. Again, the mean of all of the measurements gives the final observed time of transit.

The final improvement is reversal of the instrument during observations of each star by its rotation about the vertical axis. The implications of this procedure are:

- The star is tracked <u>in</u> and <u>out</u> over the same locations in the field
- Thus the observations are made symmetrically with respect to the meridian of the instrument. The instrumental meridian is defined as that plane perpendicular to the horizontal axis in which the telescope moves when rotated about the axis
- The star is not observed as it makes its actual transit.

The results of these improvements need analysis and some do not increase the precision with which longitude is determined as much as originally expected. The improvement mechanisms are supposed to be:

- Instrumental collimation error is removed by the reversal of the instrument
- Multiple pairs (usually ten but up to thirty) of symmetric timings reduce the uncertainty of the final determination
- Timings occur without anticipation by the observer so that the measurements are almost impersonal
- Radial distortions in the optics are totally eliminated
- Nonlinearities in the lead screw are totally eliminated. (In fact only a nominal value of the run of the micrometer is needed in the computation.)

Coincidentally, asymmetry of procedure (turn knob one way for in, reverse for out) is eliminated by the practice of observing the first, third, and fifth stars at one end of the field, and the second, fourth, and sixth stars at the opposite end.

Stars used are restricted to the 1535 that form the FK4 system, except that, under certain circumstances, stars from the FK4 Supplement may be used. Two unknowns, the longitude and the azimuth of the instrumental meridian, are to be determined. Customarily at least six stars are observed in a set. Within a set, at least two stars must be observed on each side of the prime vertical plane. All are selected to have azimuth factors (see Section 2.3.6) less than a specified value, and the algebraic sum of these factors should approach zero. (The sign changes at the zenith and at the pole.) Further, at least six sets have traditionally been specified,* no fewer than two of which are observed on a given night. Each set usually requires about an hour but, under optimal conditions, can take as little as half that. More stringent modern requirements may call for 40 to 60 stars.

At present the instrument specified for determination of longitude is the T-4, described in section 2.3.2. Of several ancillary devices mounted on the instrument three are of primary concern during longitude observations: an eyepiece micrometer, a hanging level mounted parallel to and sensitive to tilts of the horizontal axis, and a horizontal circle.

The eyepiece consists of three parts. Mounted on the telescope in the focal plane of the objective is a fixed reticle which defines the field in which all observations are to be made. This is divided in two by a meridian line: all observations on a given star are made on one side only of the field.

*NGS specifications.

The second part consists of a magnification (eyepiece) lens mounted above a single wire. This wire is also in the focal plane of the telescope but the assembly can be moved perpendicular to the optical axis and parallel to the apparent motion of the image of a star. The third part is a drive mechanism to effect movement of the second part relative to the first and to produce the signals which identify the location of the movable wire. Two opposing finger wheels are attached to a shaft through the housing of the eyepiece so that both hands can be used to maintain continuous motion.

The hanging level is suspended from the same parts of the horizontal shaft as are used for supporting the telescope. This system permits detection, as directly as possible, of the location of the horizontal axis relative to the direction of local gravity. The sensitivity of this level is nominally one second of arc per division of two millimeters, and it is routinely estimated to one tenth of a division.

The horizontal circle is used to maintain the orientation of the instrument about its vertical axis. The basic graduations are at intervals of two minutes of arc about a circle of 240 mm diameter. The optical micrometer associated with the circle is marked directly to 0.1 $\widehat{sec}$. In addition there is a small vertical circle which permits setting the zenith distance easily and sufficiently accurately to find the star in the field.

Analysis of the data collected follows the procedure of Mayer. Imperfections in the observations can be introduced from three sources associated with the instrument:

- The horizontal axis of the theodolite can be rotated about a line connecting zenith and nadir, so that the instrumental meridian (see above) and the true meridian intersect only at these points, producing an azimuth error, a

- The horizontal axis of the theodolite can be rotated about a line connecting the north point and the south point, so that the instrumental meridian and the true meridian intersect only at these points, producing a level error, b

- The optical axis can be displaced from the instrumental meridian so that it sweeps out a small circle parallel to the instrumental meridian, producing a collimation error, c.

Of these, the last is removed by reversing the instrument, and is therefore ignored in reduction of observations.

Thus, in addition to the longitude, the two small angles, a and b, must be determined. The instrumental azimuth is poorly determined, however, since stars are selected near the zenith to reduce the influence of this error on the determination of longitude. The hanging level is used to determine b directly. Since the instrument is reversed as a part of the observational procedure, there is no need to reverse the level vial, and it remains in place on the theodolite after initialization.

Other small corrections are required. The measurement is made parallel to the rotation of the earth, hence a correction for aberration must be applied. The traditional form of electrical contact in the eyepiece is a commutator, which has physical width, hence a correction for <u>width of contact</u> is applied. Further, it is customary to visualize some displacement of the contact wheel relative to the moving wire caused by the dynamics of gears and lead screws, hence a small correction for <u>lost motion</u> is applied.

It is difficult to design a more direct measurement of longitude, short of replacing the observer with an automated device. The quantity desired is the quantity observed. The instrument is well balanced. The observer is comfortable. Procedures are symmetrical. Refraction has no effect (except for long term asymmetries of the atmosphere). But the determination leaves much to be desired. Even large meridian observatories (which determine local apparent sidereal time rather than longitude) improve the situation only slightly with their more extensive facilities and vastly greater time to make observations.

In Ref. 3, it is observed that longitudes determined by the method just described have standard deviations which can be represented by:

$$\sigma_\lambda = [(\sigma_r)^2/n + (\sigma_s)^2/k]^{\frac{1}{2}} \qquad (2.3\text{-}16)$$

where

σ_r = 0.24 $\widehat{\text{sec}}$, which is the standard deviation of a single set of stars

σ_s = 0.37 $\widehat{\text{sec}}$, which is the between-determinations standard deviation

n = 6, the customary number of star sets observed in the determination of longitude

k = 1, the customary number of instrument/observer combinations used in the determination of longitude.

The inclusion of k takes note of bias effects associated both with the individual instrument and with the individual observer using it. So called <u>personal equations</u> have been discussed since transit instruments were first analysed and various techniques have been devised to measure them. Generally, the results have been uncertain, varying in magnitude and sign in irregular fashion. They remain a problem. See Section 2.7.4 for further discussion.

2.3.6 Errors with Transit Observations

Longitude is computed from the following equation

$$\lambda = \alpha - [\theta + I + K + B + A] \qquad (2.3\text{-}17)$$

where

λ is the observed astronomic longitude

α is the apparent right ascension of star at epoch of observation

θ is the local apparent sidereal time on the reference meridian at epoch of observation

I is the instrumental correction

K is the correction for aberration

B is the correction for tilt of horizontal axis

A is the correction for azimuth of instrumental meridian

The individual terms are examined in detail for their contribution to the overall error in determination of longitudes. For numerical evaluation, latitude is limited as before (Section 2.1.1) and declination is limited to the range from 13 deg S to 65 deg N, which is dictated by limitations imposed by the azimuth factor (discussed below). Values which maximize the derived uncertainty are used in each computation.

One might expect that, just as errors in the declinations of stars observed for latitude contribute to the error budget of that determination, errors in the right ascensions of stars observed for longitude would contribute to the error budget in determination of longitude. However, this is not the case, since catalogue errors in right ascension affect the determination of sidereal time at the observer's meridian and at the Greenwich meridian by the same amount. The longitude, determined from time differences, is not affected.

In discussing the error from timing, two separate functions must be distinguished:

- Determining the precise instant at which the event to be timed has occurred
- Assigning a number which is called the "time" of the event.

For determining longitude, the event of interest is the transit of a known star. The number assigned is, ideally, a linear function of the time at the reference meridian.

Today, with modern technology, the second problem is essentially solved, since Universal Time (from which the Greenwich apparent sidereal time can be computed directly) is available in the field to well within the required accuracy for longitude determination.

Determining the instant of transit at the local meridian is subject to a number of random effects, and systematic effects and quasi-random changes of the systematic effects, all of which have been exhaustively studied by many investigators, mostly in the last century. (See, for example, the section on "personality" in Ref. 32.) The device mounted on the T-4 for making the observation is the impersonal micrometer.

If the assignment were perfect -- that is, if the observer kept the movable wire exactly centered on the image for the entire interval -- then the measurement could be diagrammed as in the top frame of Fig. 2.3-2. The slope is dependent on the declination of the star. The systematic effects referred to in the previous paragraph result in a translation of this line of points up or down, the magnitude being a measure of the individual observer's bias error, known as personal equation. The line of points can lead to the incorrect conclusion that the tracking of the image is the straight line connecting these points. Different observers have different techniques. Some try to keep the motion of the wire continuous (middle frame) and others merely approximate smooth motion by a series of tiny, but discontinuous, rotations of the handwheels (bottom frame). In both cases it is possible to move the wire back, thereby violating one necessary condition for obtaining satisfactory data.

The data sampling rate is sufficiently small (about once a second for half a minute) so that very little information is really available about the observer's performance. However, some estimate can be gained by examining residuals of individual pairs against the grand mean (see Ref. 23). From such investigations it may be concluded that the uncertainty in the determination of local apparent sidereal time is of the order $\sigma_\theta = 0.35\ \widehat{\text{sec}}$.

There have been investigations of the feasibility of replacing the standard micrometer with either a linear position encoder or a shaft angular encoder, in order to increase the data sampling rate. What appears more promising is the automation of the observer's function, as discussed in Sections 2.5.1 and 2.8.2.

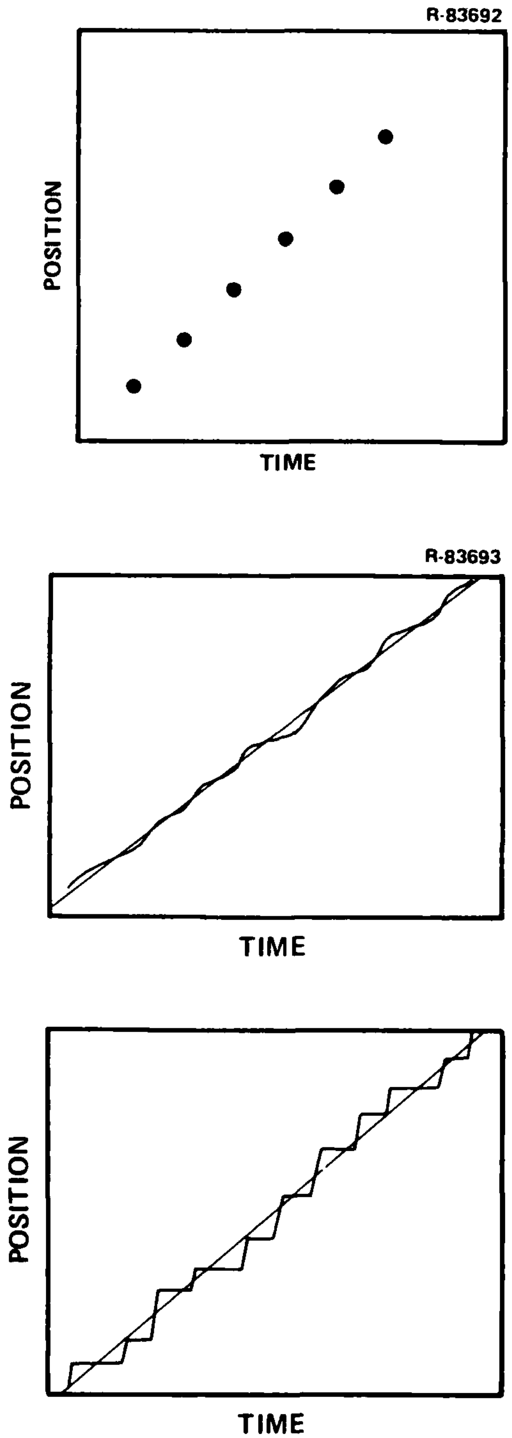


Figure 2.3-2 Use of the Impersonal Micrometer

A small correction for two instrumental constants can be derived from consideration of the mechanical design of the impersonal micrometer. The impersonal micrometer of the T-4 consists of a lead screw which drives the carriage supporting the movable wire (and the eyepiece lens). Since the lead screw is what provides the motion, and the wire is displaced perpendicularly to the lead screw, there is the possibility of differential movement between the two parts. Attached directly to the shaft of the lead screw is the commutator rotor, and the dial from which measurements of the position of objects in the field are determined. The traditional way of measuring lost motion is a quasi-static process, "... since a careful pointing or bisection requires a very slow motion of the screw." (Quoted from Ref. 23.) The measurement involves reading the position of the lead screw as the wire is brought into coincidence with a fixed wire in the field, the coincidence being effected in both directions of movement. The difference between values is the lost motion. The two constants, lost motion, ℓ, and width of contact strip, s, are combined in one formula:

$$I = \frac{1}{2} \cdot \frac{R}{100} \cdot (\ell + s) \cdot \widehat{\sec}\ \delta \qquad (2.3\text{-}18)$$

in which

$\frac{1}{2}$ adjusts the edge of the contact to its center

100 is the number of divisions per turn of the micrometer

R is the run of the micrometer in $\widehat{\sec}$ per turn.

The other variables have been defined previously.

Four individual sources of errors must now be combined:

σ_R, which is the uncertainty in determining the run of the micrometer

σ_ℓ, which is the uncertainty in determining the lost motion

σ_s, which is the uncertainty in determining the width of the contact strip

σ_δ, which is the uncertainty in the declination.

Their contribution, by taking the root sum of squares of partial derivatives of Eq. 2.3-18, is:

$$\sigma_I = \{[\frac{1}{200}(\ell+s)\,\sigma_R \sec\delta]^2 + [\frac{R}{200}\,\sigma_\ell \sec\delta]^2 + [\frac{R}{200}\,\sigma_s \sec\delta]^2 + [\frac{R}{200}(\ell+s)\,\sigma_\delta \sec\delta\,\tan\delta]^2\}^{\frac{1}{2}} \qquad (2.3\text{-}19)$$

To evaluate this expression, note that the nominal run of the T-4 micrometer is 150 $\widehat{sec}$ per turn. The combined term $(\ell+s)$ is taken as one division. In order to maximize the effect of this term, the declination is also maximized (65 deg). Further:

$\sigma_R < 0.063\ \widehat{sec}$ per turn,

$\sigma_\ell < 0.05$ div,

$\sigma_s < 0.03$ div,

$\sigma_\delta < 0.50\ \widehat{sec} = 2.4 \times 10^{-6}$ rad,

so that

$$\sigma_I < \{[0.00075]^2 + [0.08873]^2 + [0.05324]^2 + [9.2\times10^{-6}]^2\}^{\frac{1}{2}} \qquad (2.3\text{-}20)$$

Thus the uncertainty in longitude due to the uncertainty in determining the instrumental correction is 0.104 $\widehat{sec}$. Of the individual contributors, the second is the largest. This is a conservative estimate, and σ_I customarily will be smaller than the figure given.

The correction for diurnal aberration is determined from

$$K = 0.021 \cos\phi \sec\delta \qquad (2.3\text{-}21)$$

where the units are seconds of time (equivalent to 0.315 $\widehat{sec}$). It has three sources of error: in the determination of the constant (σ_K = 0.0005 $\widehat{sec}$), in latitude (σ_ϕ = 60 $\widehat{sec}$), and in declination (σ_δ = 0.5 $\widehat{sec}$)*. Then using the root-sum-square of the partial derivatives of Eq. 2.3-21:

$$\begin{aligned}\sigma_K = \{&[\sigma_K \cos\phi \sec\delta]^2 \\ &+ [0.315\, \sigma_\phi \sin\phi \sec\delta]^2 \\ &+ [0.315\, \sigma_\delta \cos\phi \sec\delta \tan\delta]^2\}^{\frac{1}{2}}\end{aligned} \qquad (2.3\text{-}22)$$

For quantitative evaluation, the same reasoning is used as before. Inserting the numerical values above, Eq. 2.3-22 becomes

$$\sigma_K < \{[0.001]^2 + [2\times10^{-4}]^2 + [4\times10^{-6}]^2\}^{\frac{1}{2}} \qquad (2.3\text{-}23)$$

Thus the uncertainty in longitude resulting from uncertainty in determining the correction for diurnal aberration is no more than 0.001 $\widehat{sec}$.

---

*These latter two values are used as conservative estimates, to establish that this source of error is minor.

The nonverticality of the ideal instrumental meridian is detected by a hanging level. This sensitive level is suspended from the horizontal axis at points directly above the bearings holding the axis. Thus a common surface is used both to guide rotations about the horizontal axis and to detect variations in level of this axis. Ideally the circumstances of observation are at least:

- The base of the instrument is solidly emplaced so that there are neither rotations nor translations (stability)
- The vertical axis contained therein is truly parallel to the local gravity vector at the intersection of the two axes (levelness)
- The supports of the horizontal axis, called wyes, are so fixed as to hold the horizontal axis truly perpendicular to the vertical axis and hence tangent to the geop passing through the intersection of axes (orthogonality)
- The bearing surfaces on the horizontal axis are cylindrical, coaxial, and of the same radius (trunnion wobble)
- The optical axis of the telescope is truly perpendicular to the horizontal axis (collimation).

Stability is encouraged by field procedures which are designed to prevent poorly set-up instruments. Collimation is cancelled by reversing the instrument in midobservation. Orthogonality is tested each time the instrument is set up and is, in fact, quite stable. However, it and trunnion wobble are both involved in the determination of levelness, since the hanging level is the only (standard) device used to detect changes in any of these conditions. In fact, stability can be involved, since a rotation of the entire instrument can not be differentiated immediately from, say, an elongation of one wye.

A level vial is a circular section of glass tube. A bubble is introduced into the enclosed fluid and its position is determined by averaging the positions of the ends of the bubble. Divisions are normally two millimeters long; hence, a one-second vial will have a radius of over 400 m. By whatever method a curve of this radius is produced within a glass tube, and fluid sealed therein, the assumption is always made that the curve is circular, so that tilts are linearly correlated with positions of the bubble. Such is not the case, as is shown in Ref. 4. However, good practice in the field insures that tilts, from all causes, are kept small. Consequently, the range of readings from the vial stays small. A linear model is therefore appropriate.

The formula for this correction is

$$B = \frac{1}{4}(|\Delta b_W| - |\Delta b_E|)\ d\ (\cos\phi + \tan\delta\ \sin\phi) \qquad (2.3\text{-}24)$$

where

4 is the number of readings involved,

$|\Delta b_W|$ is the absolute difference of two readings of the west end of the bubble,

$|\Delta b_E|$ is the absolute difference of two readings of the east end of the bubble,

d is the sensitivity of the level vial in $\widehat{\text{sec}}$ per division,

and the remaining variables are as before. The sign convention implied in W-E is derived from Mayer's equation. The last parentheses delimit the level correction, which is taken as 2.3 in numerical evaluations. This is the maximum value (for ϕ = 50 deg and δ = 66 deg, which are regarded as operational maximums).

There are four contributors to the uncertainty in the level correction, σ_B, which are:

σ_d, the uncertainty in determining the sensitivity of the level vial

σ_b, the uncertainty in reading the level vial

σ_ϕ, the uncertainty in the assumed latitude

σ_δ, the uncertainty in the declination.

They contribute in the following fashion (by root-sum-of-squares of the partial derivatives of Eq. 2.3-24):

$$\begin{aligned}\sigma_B = \{ & [\tfrac{1}{4}(|\Delta b_W| - |\Delta b_E|)(\cos\phi + \tan\delta\sin\phi)\sigma_d]^2 \\ & + [\tfrac{1}{4}d(\cos\phi + \tan\delta\sin\phi)\sigma_b]^2 \\ & + [\tfrac{1}{4}(|\Delta b_W| - |\Delta b_E|)d\,(\tan\delta\cos\phi - \sin\phi)\sigma_\phi]^2 \\ & + [\tfrac{1}{4}(|\Delta b_W| - |\Delta b_E|)d\,(\sec^2\delta\sin\phi)\sigma_\delta]^2\}^{\frac{1}{2}}\end{aligned} \tag{2.3-25}$$

For quantitative evaluation, trigonometric functions are maximized as above. In addition, the following values are assumed:

$$|\Delta b_W| - |\Delta b_E| = 10 \text{ div}$$
$$d = 1.0\ \widehat{\text{sec}}/\text{div}$$

and the individual uncertainties are

$$\sigma_d < 0.02\ \widehat{\text{sec}}/\text{div}$$
$$\sigma_b < 0.4 \text{ div}$$
$$\sigma_\phi < 3\ \widehat{\text{sec}} = 1.45 \times 10^{-5} \text{ rad}$$
$$\sigma_\delta < 0.05\ \widehat{\text{sec}} = 2.4 \times 10^{-7} \text{ rad.}$$

Then by inserting the above values, Eq. 2.3-25 becomes

$$\sigma_B < \{[0.1182]^2 + [0.3012]^2 + [2.5\times10^{-5}]^2 + [2.8\times10^{-6}]^2\}^{\frac{1}{2}} \tag{2.3-26}$$

Thus the uncertainty in longitude caused by uncertainty in determining the correction for mislevelment is 0.32 $\widehat{\text{sec}}$. Obviously, this is a major source of error. Because the level factor exceeds unity, the inclination ought to be determined with high precision. It is not. Problems include at least the following:

- Nonlinearities are discussed in Ref. 4
- Bubbles stick in vials (see discussion in Section 2.4.3)
- Thermal gradients affect level vials more rapidly and sensitively than the instrument proper
- Bubbles respond slowly (~20 seconds) to small changes, hence produce smoothed data.

One feature of observations for longitude with a T-4 is beneficial: the vial is mounted eccentrically to the vertical axis with the result that, at every reversal, the fluid is well stirred and sticking is minimized. In contrast, the observer is always located close to the same end of the vial so effects of body heat accumulate. During observations, the program does not leave much time for reading the vial, particularly in the first half of the observation on each star. The conclusion, based on field experience, is that the level correction is a significant source of error and the magnitudes given above are reasonable.

The final correction is for instrumental azimuth. (See section 2.7.1 for further discussion.) The correction is:

$$A = a(\sin \phi - \cos \phi \tan \delta) \qquad (2.3\text{-}27)$$

in which a is the instrumental azimuth.

The quantity, $\sin \phi - \cos \phi \tan \delta$, is referred to as the <u>azimuth factor</u>. Stars are selected so that the absolute magnitude of this factor is less than 0.6 and the algebraic sum of the factors within each set should be less than one. A plot of the azimuth factor over the entire meridian is given in Fig. 2.3-3 for latitude 40 North.

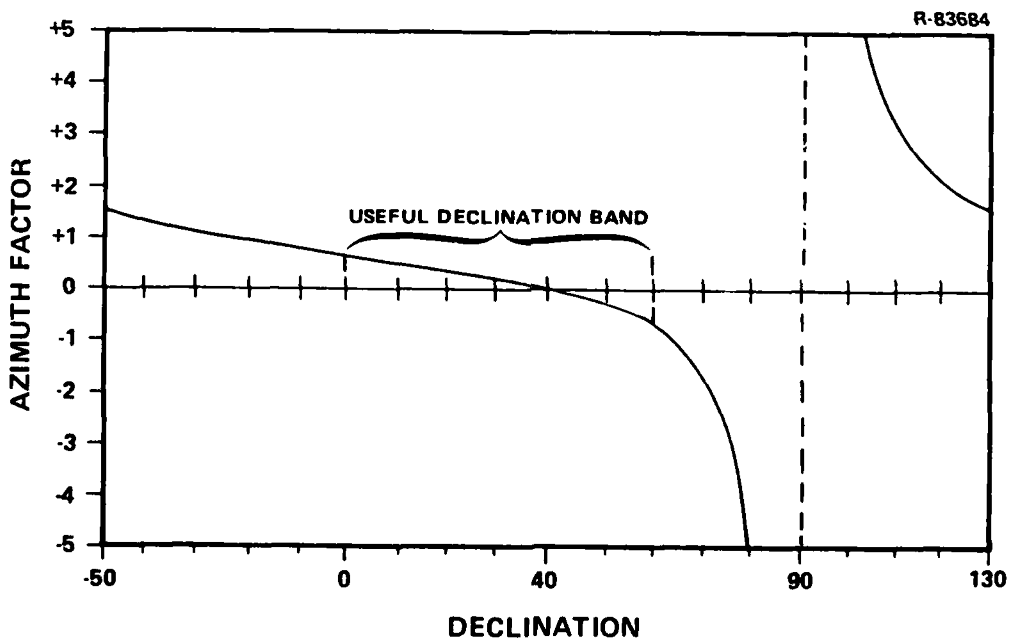


Figure 2.3-3 Azimuth Factor for Latitude 40 North

Note that the useful band of declination closer to the pole is narrower than the equatorial band, an unfortunate circumstance since the density of stars decreases as the pole is approached. The problem is aggravated as $|\phi|$ approaches 90 deg.

Since a is determined as an unknown in the solution for longitude, it contributes nothing to the error in longitude. There remain as sources of uncertainty:

σ_ϕ, the uncertainty in the assumed latitude

σ_δ, the uncertainty in the declination.

They contribute in the following manner (using the partial derivatives of Eq. 2.3-27):

$$\sigma_A = \{[a(\cos\phi + \sin\phi\tan\delta)\sigma_\phi]^2 + [a(\cos\phi\sec^2\delta)\sigma_\delta]^2\}^{\frac{1}{2}} \tag{2.3-28}$$

Inserting numerical values used before, Eq. 2.3-28 becomes

$$\sigma_A = \{[0.030]^2 + [0.003]^2\}^{\frac{1}{2}} \tag{2.3-29}$$

Thus the uncertainty in longitude due to uncertainty in determining the instrumental azimuth is 0.03 $\widehat{\text{sec}}$.

Individual contributions to the total error of a longitude measurement are summarized in Table 2.3-3. The root sum square is σ_λ = 0.49 $\widehat{\text{sec}}$ <u>per star</u>. Then $0.49/\sqrt{6}$ = 0.20 $\widehat{\text{sec}}$, which is consistent with the value stated for σ_r in Ref. 3.

Use of the level vial is the major contributor to the total uncertainty, with the skill of the observer the second most important. Level vials are discussed in more detail in

Table 2.3-3
SIGNIFICANT SOURCES OF ERROR
IN LONGITUDE

SOURCE	STANDARD ERROR	
	SYMBOL	MAGNITUDE ($\widehat{sec}$)
Determination of Epoch	σ_θ	0.35
Instrumental Constants	σ_I	0.10
Level	σ_B	0.32
Azimuth	σ_A	0.03
Root Sum Square	-	0.49

Section 2.4.3, and the possibilities for replacement of the observer with an automatic eyepiece are described in Section 2.5.1. In actuality, most determinations of longitude are as good as they are because the instruments are set up well on firm supports, and because of the effectiveness of the various compensation procedures.

2.3.7 The Baseline in Brief

Standard procedures in the United States divide the process of determining astrogeodetic positions into two parts, one for each coordinate. Transits of stars are used to determine longitude. Traditionally the Horrebow-Talcott method has been used to determine latitude, but Sterneck's method may totally replace it in the next decade. An extensive summary comparison is given in Section 2.9.2. As a rule of thumb, baseline methods result in astrogeodetic positions having uncertainties of the order of 0.3 $\widehat{sec}$.

2.4 PRINCIPAL SOURCES OF ERRORS

The analyses of the several methods considered in the last section pointed up the sources of error which contribute most to total error budgets. Since there are several sources which are common to all, they are brought together in this section for further discussion.

2.4.1 Catalogues of Stellar Positions

The sensitivity of astrogeodetic positions to uncertainties in the positions of stars observed is high. In determining latitude, the dependency is very nearly one to one no matter what technique is used for observation. In determining longitude, there is less dependency on the absolute positions. What is less apparent is the dependence on the uniformity of the system of right ascension. At various times, various observers and various organizations have attempted to select (or reject) stars for their own observations. Sometimes this is absolutely necessary as, for example, when one is using a narrow-field PZT at a single site. The resulting catalogue is developed over many years and many observations. For those making observations for astrogeodetic positions, catalogue preparation is seldom part of the job at hand, and it is necessary to rely on catalogues prepared by others.

Two principal difficulties arise in using catalogues prepared by astrometrists:

- The catalogue lists too few stars to be completely useful for the astrogeodetic observations to be made
- The positions of the stars in the catalogue are derived, in part, from outdated data.

It is easy enough to say that either or both of these criticisms can be met in a simple way. Calls for better catalogues of positions of stellar objects have been made many times in the past and no doubt will continue to be made in the future. Even when catalogues are updated, the quality of the analysis from which improved positions are derived is questioned. (See Ref. 15.)

The two principal catalogues in common use by the geodetic community are:

- The Fourth Fundamental Catalogue (FK4), (Ref. 16)
- The General Catalogue (GC), (Ref. 17)*.

Of these two, the first (FK4) is recognized as the catalogue that establishes the most rigorously defined coordinate system. It contains 1535 stars selected to cover the sky with a fairly even distribution of locations. Most of the stars visible to the naked eye are included, but other fainter stars are added partly to fulfill the requirements for even spatial distribution. The GC is also a fundamental catalogue in the astrometric sense. (See Ref. 18 for a complete discussion of the different types of catalogues, as well as a critical evaluation of existing catalogues.) It contains 33342 stars, and includes nearly every star down to a limiting magnitude of about 7.5. Unfortunately, the positions in this catalogue are increasingly questionable, because of the antiquity of the data used to form them. It appears now in the guise of a third catalogue, the Smithsonian Astrophysical Observatory

*Unfortunately, this catalogue is frequently referred to as "The Boss Catalogue" after its principal author, who with his son produced several other catalogues of stellar positions which could be referred to by the same title.

Catalogue (SAOC), Ref. 19, which contains positions and proper motions for about 259,000 stars. This catalogue is not a fundamental catalogue in that no attempt is made to define a coordinate system; rather the intent is to assemble several catalogues into one by applying corrections to the positions of each catalogue to transform to the system of the FK4. Thus the SAO catalogue is referred to as having FK4 positions even though the vast majority of stars in the catalogue have nothing whatever to do with the FK4 system proper. A selection was made from the SAOC to form what has been referred to as the NGS star list (Ref. 2), but the positions and proper motions in this (NGS) list are identical to those in the SAOC.

Mention should be made of another catalogue which has been used by various organizations making astrogeodetic observations, the FK4 Supplement (FK4 Sup), Ref. 20. This list is not a fundamental catalogue and adds 1987 stars to the basic FK4. It is in the nature of a compilation catalogue since the positions were obtained by transformation of data from two other catalogues to place them in the FK4 system. It has little to recommend it over the SAOC except having been produced by the same organization that developed the FK4. The stars in the FK4 Sup were not used in the formation of the FK4 system; hence, in a sense, its title is misleading.

Considering the importance of stellar coordinate systems (all earthbound coordinates are, in the limit, dependent upon stellar coordinates), and considering that astrometry is a continuing occupation of some (few) people, it is a wonder that no new catalogue has appeared that satisfies the requirements of geodetic users. The task is not at all unusual nor does it require a tremendous allocation of resources. The Fifth Fundamental Catalogue (FK5) is in preparation, and will be adopted as a replacement for the FK4 immediately on appearance. Whether it will replace the GC as well remains to be seen. In any event, if the observational technique for latitude is changed from that of Horrebow-Talcott to that of Sterneck, then the number of stars required for astrogeodesy is reduced, and in this regard the FK5 will be satisfactory. However, until the catalogue has actually been in use for a while, it will be impossible to estimate the magnitude of errors involved. Obviously the intent is to reduce the errors known to exist in the FK4, and the authors of the FK5 will, no doubt, publish their own estimates of what they have accomplished.

Ideally, the coordinate system of a catalogue would be inertial -- that is, fixed in space. The existing fundamental catalogues all suffer in this regard by including only the brighter stars which are associated with the local galaxy and therefore partake of its rotation. Only fainter objects are far enough away to be essentially motionless, which is to say that their motions are so small as to be undetectable. The so-called <u>proper motion</u> of local stars can be determined well only by comparison of observations widely separated in time. For these comparisons to be made, the coordinate systems as established at the two epochs must be comparable. In theory it is possible to use existing (though largely unreduced) data for the early determinations of positions, to repeat the observations in much the same manner for current positions, and to determine therefrom, in one huge simultaneous solution, the positions and proper motions of all stars down to, say, fourteenth magnitude. The desirable feature of the process is the inclusion of many of the faint objects mentioned above, which would form the basis of an inertial coordinate system, or at least a better approximation than now exists. To date, such a project has not been undertaken.

Just as astrogeodetic observations are statistical in nature, so are astrometric. A perfect coordinate system in which to locate the stars can not be obtained. It is unlikely, using present methods, that the uncertainty in stellar positions can be halved. (The revision of the General Catalogue now in progress promises standard deviations of 0.25 $\widehat{\text{sec}}$ for epoch 1985.) It is conceivable that the uncertainties could be reduced by an order of magnitude, if the necessary resources were to be devoted to the task.

2.4.2 Observers

The successful use of almost every instrument now applied to astrogeodetic observations is dependent upon the skill of the observer. Not every person who is interested in making observations can be successfully trained to more than limited competence, and even those who are capable of being good observers are not always motivated to become such. In today's world, the prospect of doing nothing but observing stars in various remote locations for many years is not enough to lure new people to the work. In civilian mapping agencies, the recent reduction in astrogeodetic observing has meant that there is no reason to provide training for students in this skill.

Comparison of the results of observations taken with field instruments and those taken with observatory instruments shows that it is unlikely that improvement can be effected in the precision with which individual observers make their measurements. The difference between results with the two types of instruments has much to do with the more uniform working conditions in observatories, with the larger optics, and with longer series of observations. It is remarkable that field instruments perform so well.

Elsewhere a comment has been made that possibly the old zenith telescopes used a century ago were better instruments than the modern T-4. However, another possibility is that the observers make better use of their instruments. Certainly in determining latitude, a single observation was not considered enough with the zenith telescope, while today, in the Horrebow-Talcott method, usually a single bisection of the star with the movable wire is considered a measurement. The observer, of course, has ample time to perfect his bisection. The preliminary regulations of the National Geodetic Survey for Sterneck latitudes (Ref. 7) take notice of this fact and require that multiple observations be taken, the actual transit being ignored as with longitude observations in favor of several bisections before and after transit. Since no allowance is made for errors of the vertical circle*, the position is being taken that the observer is contributing more to the overall uncertainty in this measurement than is the instrument - at least this portion of it. It would be interesting to compare Horrebow-Talcott latitudes taken with multiple bisections to the many existing latitude measurements in which single bisections are allowed. Unfortunately, very little relevant information is available.

Much has been written over the past century about personal equation. That it exists for longitude observations seems well established. The observer contributes a more or less constant bias to every longitude he observes. This bias

*The vertical circle of the T-4 is adjustable, just as is the horizontal circle. Specifications for Sterneck latitudes usually require observations in sets (see, for example, Ref. 22), with the vertical circle rotated specific amounts between sets. This rotation compensates approximately for errors of graduation in the circle itself.

has been shown to depend also upon health and personal circumstances. So far no one has developed a method for the determination of this quantity at the site. Training and motivation can make it stable for long periods of time. Only a machine seems likely to stabilize it entirely.

The observer contributes most to this quantity, even though of late there is evidence that the instrument also contributes some. Biases in instruments are difficult to model. It is easier to visualize some feature of an instrument that makes individual observers <u>use</u> it in a systematically different manner from other instruments: the handwheel of the eyepiece could be a little tighter, or the field of view a little less uneven, or the vertical axis a little smoother allowing easier reversal. If this is true, then only a machine is likely to overcome it.

Many attempts have been made to aid the observer. Multiple cross wires, the Hunter shutter, and the impersonal micrometer now found on the T-4 are all devices which are supposed to make the observer more machine-like. Attempts have also been made to replace the observer entirely. Usually the photomultiplier tube has been used since the available light levels are low. These tubes are applied to field conditions with difficulty, the skill of observing being replaced by the skill of manipulating the controls of the tube. These attempts have offered only modest improvements.

With the application of the charge-coupled device to astrogeodesy, there is at last the real possibility that the observer can be replaced. The device is, by its nature, capable of synchronizing location and time for solving the very problem which limits the human observer. It is not yet clear that the level of skill required in the field can be much reduced, although the kind of skill (technological vs observing) may differ. At present, the electronic equipment that must accompany the eyepiece proper is complex and difficult to operate efficiently. However, recent history of design of electronic equipment shows high promise for early production of simple and reliable field equipment to accompany the automatic eyepiece. (See Section 2.5.1, which discusses this device in more detail.)

2.4.3 Level Vials

All methods for determing position astrogeodetically require a reference to the direction of the plumb line or vertical. For centuries, the method used has been to enclose a bubble in a tube bent to an arc of a circle. The greater the radius of the arc, the greater the sensitivity assigned to the vial. The manufacture of very sensitive vials requires great skill in grinding surfaces on glass, and in heat-treating the glass to seal the vial. In theory the arc is polished by a curved mandrel mounted so as to swing about the center of curvature. It is charged with rouge and introduced into a tube of glass and then withdrawn. This process must be done slowly, with no force, and must be repeated until the interior of the tube has worn to the governing shape of the mandrel. The entire inside of the tube must be polished to shape. This polishing must be done sufficiently gently that no frictional heat is generated, otherwise residual nonconformities appear when the polishing is stopped. Since the interior surface of the tube cannot be tested optically or mechanically, dependence must be placed on the grinding process to produce the desired surface.

When the polishing has been completed, a chamber is usually introduced to allow adjustment of the size of the bubble. The glass is then cleaned, sometimes treated, and filled with ether, or a combination of ether and other similar volatile chemicals. The filling mixture is designed not only to remain fluid but also to keep the interior surface of the glass clean. The tube is sealed by sudden heating, drawing, and annealing to minimize residual heat strains. The vial is then spring-mounted in an encasing metal tube so that no mechanical strains can be imparted to the glass.

Art is very much a part of the process of manufacture. Even relatively coarse vials with the same nominal parameters (sensitivities of a third of a minute of arc per division) react differently to an imposed tilt. It is not unusual for one to reach equilibrium in a small fraction of the time required by another. These effects increase with increased sensitivity: a nominal one $\widehat{\text{sec}}$ vial takes about two minutes (time) to come to equilibrium after being tilted only a few seconds of arc. Another effect well known to users of the T-4 is movement of the bubble caused by uneven heating at one end by the focused beam of a flashlight. The movement can be as much as several divisions, and is an effect of the level vial, not of the T-4.

Because art is so much a part of the manufacturing process, secrecy surrounds much of it. Wild is known not to make their own vials, but does not reveal their source. Zeiss makes (buys?) somewhat better vials than Wild, but without quite the sensitivity of either the hanging level or the Horrebow levels of the T-4. Because the radius is so great on the hanging level (nearly half a kilometer) and the resultant suspension for the mandrel so complicated, it is suspected that instead two intersecting straight cylinders are ground, the angle between their axes being about one second. Since large deviations from center are never used on very sensitive vials, the nonlinearity of such a system is of no concern, and in fact is probably lost in the imprecision of manufacture.

Thermal response is significant. The bubble responds to a beam of light. Also well known is the lengthening of the bubble that accompanies the common drop in temperature of the air as observations proceed into the night. The effect of this change of length is well documented (Ref. 4; see also Ref. 26). And there is no reason to believe that, if similarly tested again today, the results of the tests described in Ref. 4 would be the same, since change in sensitivity with age is another characteristics of vials. This is due, no doubt, to slowly occurring changes in the glass itself. The fluid also changes, becoming more prone to sticking to the glass, thus restricting the movement of the bubble.

It is interesting that so much is known about the deficiencies of level vials, and so little is done to replace them with something better. Automatic (pendulum) leveling instruments have no need for such high precision and have not, in any case, succeeded in attaining it, perhaps because of size limitations. Pendulums offer not only the sensitivity but the possibility of both optical and mechanical magnification to make the determination of dislevelment more sensitive. They are nonlinear devices but this cannot be considered a drawback since the need here is to <u>retain</u> an orientation as closely as possible. The use of a pool of mercury for sensing direction of gravity is discussed in Section 2.5.2. Another possible approach to leveling which may be of significance in the future involves the use of accelerometer "tilt meter" technology.

2.4.4 The Atmosphere

Basic to almost all theories of refraction in the atmosphere of the earth is the assumption that the refractive index is a function only of the distance from the center of the earth. Thus the angle of refraction, the angle by which the light from a star is bent in passing through all of the earth's atmosphere, is represented by:

$$\Delta z = r_0 n_0 \sin z \int_1^{n_0} \frac{dn}{n(r^2 n^2 - r_0^2 n_0^2 \sin z)^{\frac{1}{2}}} \qquad (2.4\text{-}1)$$

in which r is the distance from the center of the earth, n is the refractive index, z is the observed zenith distance, and the zero subscript refers to the position of the observer. Since the nature of the relationship between the refractive index and radial distance is never known exactly, the major difference between various published theories lies in approximations made to evaluate first the function and then the integral. Generally, the refraction has been expressed in series of the form

$$\Delta z = \sum_0^{\infty} (-1)^i R_{i+1} \tan^{(2i+1)} z \qquad (2.4\text{-}2)$$

with the R_i being functions of various meteorological parameters, the color of the light involved, and occasionally of the zenith distance. Various theories use different standard numerical values of the R_i and different expressions or tables to adjust the standard values to local weather conditions. The end results, no matter what theory is used, vary from one to the other by small fractions of seconds of arc over the range $0 < z < 60$ deg, which is a greater range than that customarily used for astrogeodetic observations.

Why then is refraction such a problem? Everyone knows that when a line of sight passes close to heated pavement, shimmer and mirages result. This is a violation of radial symmetry. The same effect can be observed in a different direction by looking along a heated vertical wall. Desert mirages, as another example, are large-scale effects in which light is bent by a number of degrees. It takes no effort to realize that smaller effects exist, that they are not generally noticed because of their small magnitude, and that measurement of these effects is difficult. The solution adopted in positional astronomy has been to attempt to design observing programs that avoid measurements involving refraction, at least the assumed radially symmetric refraction. Where this is impossible, observations are balanced about the zenith as much as possible, under the theory that time variations in the atmospheric irregularities will average to zero.

But consider the observations taken in the meridian to determine longitude. Radially symmetric refraction makes the stars appear to be higher than they really are, but it does not affect the time at which they cross the meridian; hence the observations are, in theory, entirely free of refractive effects. But in a long series of observations for longitude taken both before and after passage of a hurricane over the observing site, systematic displacements in the calculated longitudes can easily be attributed to the loss of radial symmetry in the atmosphere surrounding the storm. In fact, they appear to bear a close relationship to the symmetry of the storm itself.

From analysis of leveling observations, it is known that refraction in the atmosphere close to the earth's surface is correlated with topography. Generally, leveling involves only the first three meters of the atmosphere above the topography. But in mountainous areas refraction becomes a serious problem in the determination of precise elevations. The data can not be extended to deduce quantitative information about astronomic refraction, but since results of the highest precision are being sought, these deviations from the assumption of radial symmetry must be considered.

The problem is not easily solved. What is needed is a method for precise determination of the amount by which the light from a star has been deviated by the atmosphere at any time and any place. An approach to the solution is to take advantage of the dispersion of light -- that is, the dependence of the refraction upon the wavelength of the light involved. Cauchy originally expressed the refractive index, from a purely physical analysis, in the form

$$n = \sum_{i=0}^{\infty} a_i \lambda^{-2i} \qquad (2.4\text{-}3)$$

where λ refers to the wavelength of light. The a_i are coefficients which may not be constant. (Cauchy's equation is now usually written in terms of another variable which is related to the wavelength but which gives a closer representation of effects noted in spectroscopy. This change of variable does not affect the present argument.)

Cauchy's equation can be used to express the dispersion, which is the derivative with respect to wavelength:

$$D = dn/d\lambda = -2 \sum_{0}^{\infty} i \, a_i \, \lambda^{-(2i+1)} \qquad (2.4\text{-}4)$$

It is evident that the image of a star, in passing through the atmosphere, is spread out exactly as if it had been passed through a prism. Technologically, the difficulty with using this differential refraction to measure the total refraction has been the large ratio between the two quantities, on the order of 1 to 40, over the range of visible light. This difficulty, while perhaps not totally removed, has been greatly reduced by using computer processing of the output of image sensing devices, and by employing nulling techniques to measure the differential refraction. (See Section 2.5.4 on the two-color refractometer.)

2.4.5 Instruments

The astrogeodetic procedures described above are based on observations made with the telescope pointed at the observer's meridian as closely as possible. Observations may also be made with the star off the meridian, but corrections must then be applied in one form or another to simulate an observation made at the instant the star crosses the meridian (transits). The requirements placed on the instrument with which these observations are to be made are the following:

- The optical axis must always point exactly at the meridian
- The instrument must be capable of reversal through 180 deg in azimuth

- Timing of the apparent transit must be possible
- Differential measurements within the field of the telescope must be possible
- Absolute determinations of zenith distance must be possible.

The first requirement creates the need for the precision horizontal axis, which allows the optical axis to swing so as to sweep out a great circle. To insure that the great circle is a vertical circle, a hanging (or striding) level must be mounted on the axis itself. And to prevent the azimuth of the great circle from changing, some provision must be made to lock the horizontal axis in a fixed vertical plane.

The second requirement creates the need for some sort of vertical axis. Note that this axis need not be of precision construction, and in fact many of the early types of zenith telescopes and transit instruments did not have this axis constructed with more than minimum care. There is also the need for an indication of 180 deg difference in the horizontal direction, but the intervening arc need not be divided at all. Early zenith telescopes employed stops that were adjustable, and transit instruments for years used only a pair of fixed wyes that established this condition automatically. The telescope always rested in the wyes; it was reversed by lifting it, swinging it through 180 deg, and lowering it into the same supports.

The third requirement has come to mean the impersonal micrometer. This device involves motion of a single wire in the field of view, which remains fixed with respect to the telescope. Timing comes from a commutator attached to the mechanism that moves the wire.

The impersonal micrometer can also be operated as an eyepiece micrometer for measuring positions of objects within the field. For latitude observations, the direction of motion of interest is perpendicular to that for longitude, so a provision must be made in the mount for rotation of the eyepiece by 90 deg.

Finally, the absolute measurement of zenith distance requires a graduated circle suspended in a vertical plane and some means of reading it. The usefulness of this vertical circle arises only with the use of Sterneck's method for determining latitude. It serves no other purpose. Note that with Sterneck's method there is no need to rotate the eyepiece since the micrometer is <u>not</u> used.

The T-4 includes several other features that are not required in the list above. The vertical axis is constructed with great care because the T-4 is a <u>theodolite</u>, not just a transit instrument. A very good horizontal circle is also mounted, with provisions for reading it and changing its orientation within its mount. There is also a provision for adjusting the orthogonality of the vertical and horizontal axes. This is a great convenience, and simple to build on, but is not truly a necessity. It is to be noted, however, that the T-4 contains both the precise vertical axis and a horizontal circle, neither of which is used in the course of the observations described.

The quality of the instrument is high and its ruggedness is seldom questioned. So many have been sold (in comparison to any other astrogeodetic instrument for precise observations) that it is nearly a production line item. But it is still made with highly skilled hand labor on the parts that demand the utmost precision of construction.

A detailed study of the quality of the instrument is described in Ref. 25. Minor complaints are reported in this reference, but it is clear that the T-4 is neither better nor worse than the other two instruments (each of different manufacture) to which it was compared. It should be noted that with so much hand labor going into the construction, each T-4 is unique. Shortfalls in the performance of the T-4 tested for Ref. 25 can not be considered representative of T-4s in general. Strictly, each instrument bears extensive testing, both in the laboratory and in the field.

Note that Ref. 25 implies that the tests described report on the performance of the instrument in general. Field conditions are dirtier, more subject to large fluctuations of temperature, and are often more humid. The performance of the T-4 in the field can only be worse than that determined from laboratory tests. Comparison is also made in Ref. 25 with comparable astrogeodetic instruments. While the main thrust of the investigation is towards the ability of the various instruments to determine horizontal angles, some thought is also given to the use of the instruments for astrogeodesy. While different, the three instruments are shown to be comparable in capabilities. Thus, one concludes that the T-4 is as good as any theodolite available, even if there are exchanges to be made among specific details of specific instruments of different manufacture.

The fact remains that the T-4 does not do as good a job as can be done by other, admittedly larger, instruments. Partly this is due to dependence on the skill of the manufacturer using essentially traditional methods, and then on the skill (and motivation) of a human operator who probably can find a more interesting job that will pay him more and allow him to be home part of every day.

2.5 REMEDIAL EQUIPMENT

This section presents several approaches to the reduction of astrogeodetic instrument error sources through the use of equipment modifications or the introduction of new equipment. These approaches may be summarized as follows:

- Automatic eyepiece - to reduce observer error and permit more accurate timing
- Mercury leveling - to provide better performance than level vials
- Axis mirror - to improve instrument azimuth and inclination alignments
- Two-color refractometer - to reduce the effects of anomalous refraction.

2.5.1 The Automatic Eyepiece System

When observations for positions are made, the skill of the observer at coordinating movements of his fingers to changes in what he sees enters as a direct source of error in the final results. This skill can be learned -- can be developed with practice -- but to different levels in different individuals. Traditionally, women have been considered to possess better eye/hand coordination than men, but very few women make astrogeodetic observations. In Sections 2.3.3 and 2.3.6 it has been established that the observer contributes significantly to the error budgets. The magnitude of these errors has hardly changed with time. Occasionally an unusually well-coordinated person makes observations for extended periods, but there appears no way to train (and consistently motivate) a person to perform significantly better than what has been documented over the past century. (Compare Refs. 3 and 5, for example.)

Many attempts have been made to replace the observer with an automatic device. The principal limiting factor has been the small amount of light to be discriminated. The human eye has remarkable ability to detect and to point at objects despite large variations in brightness. The two functions -- finding and pointing -- have been produced in man-made devices historically by artifice. The charge coupled device, or CCD, has changed this.

Basically the CCD is a rectangular array of photosensitive elements, each of which builds a charge the magnitude of which depends on the amount of light impinging on it. If the amount of charge is measured, the amount of light can, in principal, be determined, and if the measurement is digitized, a computer can perform analyses. When an image is projected on a CCD, the electrical charges produced can be extracted sequentially, along each row and row by row, so that the time elapsed since extraction of charges began gives positional information. The frequency at which the extraction process can be performed depends on the number of elements, or pixels, and the degree to which the charge on each pixel is digitized. The system discussed here operates at 40 Hz.

CCDs are currently being manufactured with pixels that are typically 10 to 20 micrometers (μm) on a side and arranged in rectangular arrays of a few hundred on each side. To compare pixel size with the diameter of a stellar image, it is convenient to use the size of the central Airy disk in the star's diffraction pattern as a lower bound. Assuming perfect (diffraction-limited) optics for the T-4, this diameter would be about 15 μm. It may be concluded that the stellar image is about the same size as, or somewhat larger than, a single pixel; hence it is expected that the star is seen by several pixels simultaneously. Computer analysis of the illumination reaching adjacent pixels allows positional resolution at subpixel levels.

Charge coupled devices are being applied to the T-4 by Dr. Douglas Currie, physicist at the University of Maryland. The following discussion refers to this research and numerical values derived in this report relate only to this application. Currie locates the CCD at the focal plane of the objective, to form an exact replacement of the micrometer eyepiece. At a minimum, the array covers the rectangular area of the eyepiece customarily referred to as the usable field. This limits the time to adjust the telescope, if necessary, prior to actual observations, but is not a serious shortcoming.

Note that, as presently configured, the T-4 equipped with the CCD must be manipulated by an attendant operator. This specifically includes initial orientation, which must be accomplished using the normal eyepiece. The operator must also manipulate electrical controls for the CCD, since the CCD completely obstructs the light path when it is in place, and the operator must observe a television screen to monitor the field of view. The <u>only</u> function performed by the CCD is tracking -- that is, that part of the observer's duties requiring greatest skill. The major success in applying the CCD to astrogeodetic observations has been in determining transits, where the coordinate being measured changes rapidly with time. At present, a human observer can, given sufficient time, measure a slowly changing coordinate (such as zenith distance near transit) almost as well as the CCD. For this reason, the greatest potential for CCD application is in the determination of longitude.

Several aspects of the automatic eyepiece deserve special mention. Since both the manual and the CCD eyepieces are required presently, orientation of the CCD must be assured each time it is employed. Currently, placing the plane of the array in the focal plane of the telescope is done by viewing the image on the monitor. Two alternatives suggest themselves: using an adjustable stop to determine the proper location, or using an external collimator to examine the array through the objective. Eventually the CCD will probably be used for the initial orientation of the T-4 also, in which case there should be stops just like those now employed for the visual eyepiece. The rotation of the array about the optical axis is also adjustable, and eventually stops should be employed for this as well, similar to those for the visual eyepiece.

Although the spectral sensitivity of typical CCD devices peaks in the far red and infrared, the intrinsic quantum efficiency is so high that there should be adequate response over the whole visual range. Thus spectral sensitivity issues would not appear to preclude the use of CCD sensors in an automatic eyepiece for the T-4.

Almost all photoelectrical surfaces have residual noise -- that is, even in total darkness, some photoelectrons are emitted. This background pattern is unimportant if there is a bright image to be detected, but, for faint stars, the signal-to-noise ratio can be improved by subtracting the background. The concept is simple, but execution requires a large buffer, a microcomputer to coordinate flow of data, and appropriate power supplies. This assembly is too heavy to be supported by the T-4 and is, therefore, external to it. Information is transmitted through one of three fiber optic cables between the CCD and its supporting components.

Photoelectric surfaces are sensitive to changes in temperature, so some stabilization is required. In the CCD eyepiece, minimal temperature control is provided by a thermoelectric device which keeps the photoelectric surface close to ambient temperature and allows only slow changes if air temperatures vary greatly during the course of observations. Cooling below the ambient would be expected to increase the sensitivity of the device to faint stars.

As mentioned above, the sampling frequency at which data are extracted from the CCD is governed by several considerations. First, there is the total time required just to remove all the information from the array. This is done sequentially, pixel by pixel. The greater the number of pixels, the longer it takes to read the data. Second, the information from each pixel is digital (an integral number of electrons) and is treated as an analog signal which is more coarsely redigitized. Sixteen bits are used in this application, as an adequate level of digitization considerating the additional time required for increased resolution. Third, required preprocessing of the data takes a significant amount of time.

The rationale for frequent extraction and preprocessing of the information from the CCD merits further discussion. An array of 100 by 100 pixels, each digitized to 16 bits, produces at least 4×10^4 bits during each imaging cycle. Of these, perhaps 100 bits contain significant information -- interest is limited to the position of one star. Following transfer of the data from CCD to appropriate buffers, a computer program is used to:

- Detect the approximate location of the image of interest
- Delimit a small region enclosing this image

- Reject data external to this region
- Predict the motion of the image across the array so as to simplify the preceding functions in subsequent cycles.

These four functions set an upper limit on the frequency of data extraction.

A guide to the lowest sampling frequency feasible for imaging is set by atmospheric activity. Images of stars change in size, brightness, and location as a result of turbulence. If a stellar image is imposed on a CCD for a long interval, the number of pixels that become involved in the image -- that is, the apparent size of the image -- increases because the star has moved (earth rotation), the image has moved (atmospheric turbulence), and the space charges of the most brightly illuminated pixels have leaked into surrounding pixels. (CCDs do not detect high contrast well.) Position as a function of time can be determined more accurately if the image is small. Thus the data collected by the CCD should be sampled frequently enough to avoid the movement of the image from any source. According to Ref. 14, "Essentially all [image] motion is confined to frequencies below 20 Hz...." So the sampling rate must be at least that high. Sidereal motion in 50 msec is equivalent to about one fifth of a pixel at the focus of the T-4. Scintillation (changes in brightness) will not affect the CCD, which is an integrating device. Actually, the automatic eyepiece integrates the image for 20 msec, and transmits for the next 5 msec, giving an active sampling rate of 40 Hz.

In this application, the CCD and some immediately necessary electronic circuits are mounted in place of the visual eyepiece of the T-4. This assembly weighs about five pounds, only slightly more than the visual eyepiece. The added weight requires an adjustment to the weight-bearing spring at the appropriate wye (see Section 2.3.6). Three fiber optic cables connect this assembly to the other equipment. One, as mentioned, provides the background signal for subtraction. A second carries the actual signal, less background, to the data processor. The third is for timing signals. A multiconductor power cable is also attached to the eyepiece. No matter how light and flexible, these cables tend to interfere with manipulation of the T-4, and provide the possibility of severe disruption if a cable snags or is tripped over. The standard T-4 is quite free of such encumbrance since all wires customarily run through commutators within the theodolite, and can be enclosed within the tripod and under duckboards, until safely out of the way of the observer.

The four cables from the eyepiece end at a local control unit which must be near the CCD. The data are transmitted to a small computer for preprocessing and storage. The stored data are reduced by a central computer facility after the mission.

Reliable data on which to base a quantitative evaluation of the automatic eyepiece are limited. What data exist involve the use of assorted components in all tests to date. As a result, the contributions of the individual components to the error budget cannot be easily estimated. It appears, however, that a reasonable guess for the improvement effected by the CCD in tracking is a factor of at least two, possibly more. It is a simple matter to show that tracking is not the only source of error. From Ref. 3 the internal standard error of the longitude as determined from one set of six stars is 0.24 $\widehat{\text{sec}}$, and multiplying by $\sqrt{6}$, one can conclude that a single star is generally tracked well enough to determine the transit to within about 0.6 $\widehat{\text{sec}}$. In fact, from actual samples of manually tracked stars, it appears that a star is seldom tracked with an overall precision worse than 0.3 $\widehat{\mathrm{sec}}$. Between the individual star and the six-star set, other sources of errors are involved, and these other errors <u>are not reduced</u> by the presence of an automatic eyepiece. The contribution of the CCD is compared to a human observer in Table 2.5-1, which has been compiled from Currie's data. In this table, note that the entry "Single Measurement" refers to one specific timing determination for a particular star, while "Single Star" refers to the computed meridian crossing time for that star, based on a number of timing determinations.

TABLE 2.5-1

RMS ERROR COMPARISON
HUMAN OBSERVER VS. CCD EYEPIECE

STANDARD DEVIATION	HUMAN OBSERVER ($\widehat{\mathrm{sec}}$)	CCD-EQUIPPED T-4 ($\widehat{\mathrm{sec}}$)	PERCENT IMPROVED
Single Measurement	0.83	0.60	28
Single Star	0.36	0.33	8
6-Star Set	0.13	0.13	0

While it is certainly interesting and useful to experiment with an automatic eyepiece mounted on a T-4, the future of the project seems limited if no attempt is made to remove the other sources of errors. These sources include current procedures for determining the instrumental azimuth (see Sections 2.7.1 and 2.7.2), the limitations of level vials in indicating inclinations (see Sections 2.4.3, 2.5.2, and 2.5.3), and limitations of technology in detecting imperfect performance of instrumental bearings (see Section 2.5.3). The human observer is still very much a part of the observations with a CCD eyepiece since it is he who manipulates the T-4 for every operation but tracking. Some attempts to design observing routines or observing lists for the CCD eyepiece application have been carried out at ETL and other government agencies. Meridian observations may not be the best source of longitude data with this device.

The key superiority of an automatic eyepiece using a CCD or other technology lies in its ability to determine the time at which an image of a star is at a specific position in the optical field of view with great precision. To take full advantage of this precision, the supporting instrument should be able to maintain high correlation between the position in the field of view and some desired location on the celestial sphere. The T-4 has limited capability to do this, as has been amply demonstrated in this report and elsewhere. In this regard there is little hope that much improvement can be effected with the T-4 as currently designed. Nonetheless, much potential is associated with CCD technology. Other applications of CCD technology with greater promise are discussed in Sections 2.8.1 and 2.8.3.

2.5.2 Leveling with Mercury

One alternative to use of level vials on theodolites is to autocollimate against the surface of a pool of mercury. With the proper optical elements, autocollimation can attain alignment to about 0.1 sec (Ref. 25). Reference 12 reports the manufacture of one model of mercury level for the Kern DKM3A Theodolite. All that is required is a telescope with a Gaussian eyepiece. This is attached to whatever part of the theodolite is to be leveled, in the same manner as the corresponding level vial is now. The telescope points directly down at a small pool of mercury. The surface of the mercury need only just clear the objective lens of the telescope.

With the Gaussian eyepiece, both sides of the same crosshairs are viewed, and when the images are superimposed, the optical axis is truly vertical.

There are three applications of interest. In Ref. 12, the Horrebow level (which is used on any theodolite to retain a fixed zenith distance) has been replaced. This is a straightforward procedure since the intent is to retain a preset zenith distance, and there is usually time to bring the telescope into exact adjustment before the observation must take place. A second, equally simple, procedure is to replace the vertical circle level. Again, the intent is to place the index of the vertical circle in the same orientation every time the vertical circle is to be read. With the standard Wild T-4 it is possible to read data for a correction to be applied to the location of the index when it is out of position. This is necessary because the bubble responds so slowly to adjustments that exact alignment is not only tedious, but also disturbs the orientation of the instrument. Therefore, the application of a correction is a simplification. Such would not be the case with a mercury pool level.

The final replacement is of the hanging level. This is more problematical. The device should hang below the horizontal axis where access would be difficult. Secondly, accommodation must be made for determining the existing inclination. In this application, the hanging level is used in observations for longitude; there is not enough time to adjust the orientation of the T-4 as well. Instead, the out-of-level tilt is measured, and a calculated correction is applied to the observed transit time. In this case, a Gaussian eyepiece is inadequate and must be replaced with a double reticle eyepiece (one reticle being a scale). This type of eyepiece adds a small degree of uncertainty since the two reticles can have a relative misadjustment. However, tests can be devised for calibration. Access to a suspended level can be provided with suitable optical trains. These usually restrict the field of view, and in this application a restriction would have to be considered carefully. The field of view should include at least 30 $\widehat{\text{sec}}$ range. Replacement of the hanging level is so complicated that an axis mirror, discussed in the following section, offers greater promise.

One of the standard problems with mercury surfaces is corrosion of the mercury. The application in Ref. 12 encloses the entire mercury pool in a capsule one side of which is the objective of the autocollimator. The mercury is thoroughly cleaned, as is the capsule, and then the remaining space is charged with an inert gas prior to sealing the assembly permanently. Since mercury reacts with almost everything (including iron, contrary to tradition) materials for the capsule should be selected carefully.

Note that leveling with mercury provides nonlinear indications of tilt. The relative error is the tilt less its tangent divided by the tilt, which at a tilt of 30 $\widehat{\text{sec}}$ equals one part in 7×10^{-9}. Also of concern is the size and shape of the container holding the mercury. It should be large enough to avoid edge effects from the meniscus which, at the required level of sensitivity, extends well away from the wall. And it should not be shallow, depending on submerged partitions for damping rather than the usual rise in the bottom.

Important advantages are expected with use of a mercury level:

- A higher quality image is viewed, because
  - A pointer replaces the meniscus

- A photoetched scale replaces the scale etched on the surface of the glass vial
- Coincidence between the pointer and scale is created optically to replace the displacement between meniscus and scale

- Reliability is increased, because
  - Viewing is at a convenient location for the observer through use of appropriate optical elements
  - The scale can be made to move rather than the indicator
  - Defocussing of the image occurs if the system is disturbed
  - Response is quicker
  - Sensor is not dependent on the skill of the manufacturer for its accuracy
  - Minimum response is greater - twice the tangent of the tilt as compared to just the tilt.
- Increased ruggedness is assured, because
  - Assembly of components is noncritical
  - Adjustment of components is not required after manufacture
- Decreased cost or manufacture is assured, because
  - Components are readily available
  - Assembly is according to common technological processes.

Development and testing should be encouraged, with the replacement of all level vials of sensitivity greater than, say, 5 $\widehat{\text{sec}}$ per division as the ultimate goal.

2.5.3 The Axis Mirror

In a meridian transit of the type used in an observatory, trunnion wobble refers to rotation of the telescope (and its attached horizontal axis) about any axis except the ideal horizontal axis. In Mayer's analysis, the two rotations of interest change the value of the instrumental azimuth, a, and/or the inclination, b. The term trunnion is a carry-over from artillery terminology, referring there to the knob, one on either side, which supports the cannon and permits changes in elevation. If the trunnion was not perfectly round and properly located relative to its companion, a change in elevation would produce an undesirable change in azimuth as well. The comparison to the meridian transit is exact.

Before it was possible to manufacture precision ball bearings, the supports for the horizontal axis of a meridian transit were shaped like a capital Y. Moreover, the two wyes (as they are still called) were more or less permanently located in the plane of the prime vertical. Only small adjustment in azimuth was provided for, and this motion was heavily clamped when not in use. Reversal of the telescope, to remove collimation, was effected by lifting the horizontal axis out of the wyes, rotating it, and then gently lowering it back in place. Generally the wyes were deliberately made of soft brass so that foreign matter would embed itself rather than change the orientation of the telescope. The trunnion was always of iron or, as it became available, steel. Corrosion was an ever present source of foreign matter. Wear of the wye was expected, but assumed to be even on both faces.

In the T-4, the wye has been replaced by three precision ball bearings. One is spring-mounted on a vertical axis and bears almost all the weight of its end of the axis, half that of the telescope, and all that of either the eyepiece or vertical circle, depending on which end of the horizontal axis is considered. By design, the two remaining bearings support a total of no more than one kilogram and are mounted so as to support the trunnion exactly as a wye would. They are in line in order to bear on the same narrow portion of the trunnion; the hanging level bears on exactly the same band, but on the upper side. (These two bearing points on the hanging level are rounded pads of brass just as the wyes used to be.) Modifications to the horizontal axis of the T-4 must maintain this loading of the wyes if the instrument is to continue to perform properly.

Trunnion cross sectional shapes differing from a perfect circle will generate both azimuth and inclination errors. An irregularity on one trunnion of one micrometer (or a foreign particle of this dimension) will cause an undesired rotation of nearly a second of arc. The bearings forming the wye are located so that an irregularity affecting only one point of support will induce equal changes in both azimuth and inclination.

Only one end of the horizontal axis has been considered. If the degree of out-of-roundness is equal (and in phase) at both trunnions, compensatory motions occur at both supports and rather than a rotation, a translation of the entire telescope has occurred. But the magnitude is several orders too small to be detected in the resultant position. Only residual rotations about axes orthogonal to the horizontal axis are of consequence.

These small rotations can be detected by autocollimation to an optical flat mounted normal to the horizontal axis. This procedure is proposed in Ref. 9 for use in observations for azimuth. The concept is applicable as well to meridian observations for either latitude or longitude, where it would provide a direct measure of errors of instrumental azimuth and inclination.

The instrumental setup is exactly the same as without the axis mirror. Two autocollimating telescopes are mounted east and west of the T-4. All three instruments are arranged to be collinear. The two external telescopes establish and maintain a level east-west line throughout the interval during which observations for position are in progress. The T-4 must have an optical flat with reflective coating mounted on the end of the horizontal axis, presumably in place of the vertical circle. As an observation on a star is being made, a supplemental observation is made through the appropriate autocollimator to measure deviations of the horizontal axis from the line established by the autocollimators. Its actual azimuth, obviously, remains an unknown and must be determined either as part of the solution for position or by a separate procedure.

The disadvantages of using an axis mirror include at least the following:

- Use of the Sterneck method for determining latitude is precluded
- Extra equipment is involved
- There is no guarantee of stability of the autocollimators
- Extra personnel may be involved.

The Sterneck method involves measurement of absolute zenith distances, for which the vertical circle is needed. To mount an axis mirror, the physical end of the horizontal axis must be exposed. A standard T-4 has this end of the axis covered by the viewing apparatus for the vertical circle, though not by the circle proper. Redesign of this end of the axis to permit both mounting an axis mirror and viewing the vertical circle is possible, at least in concept. Thus it might not be necessary to accept the disadvantage of losing use of the vertical circle.

At current prices, capital investment in all equipment for a field party is in excess of one hundred thousand dollars. Two autocollimators add less than ten percent to this cost.

Since the purpose of erecting the autocollimators is to detect small variations in the orientation of the T-4, there is every reason to expect motion of the autocollimators as well. There are, however, three separate elements, each surrounded by duckboards or some other platform for the observer. It is unlikely that these three telescopes will rotate as a whole or by equal amounts in the same direction. (A differential rotation of one element relative to the other two is always detectable, although it is not always possible to identify which element actually rotates.)

In spite of well-built pillars and carefully isolated platforms for the observer, the motion of the observer around the T-4 is the most likely cause of motion of the instrument either by disturbance of the ground or by thermal irregularities. The observer cannot avoid moving about (short of complete automation as has been done for meridian transits at observatories). But during use of the T-4, there is no need to move about either autocollimator. This is the reason for adding personnel to the observing party. An observer would be seated at each collimator, at least through each set of observations, but preferably continuously during use of the T-4. The resulting avoidance of motion near the secondary pillars would help insure their stability.

The autocollimation procedure is capable of reliably detecting rotations of the order of 0.1 $\widehat{sec}$ (see Ref. 25), when components are properly designed and applied. In this application, protocols need to be provided for establishing the line of sight initially, for monitoring the T-4 during its use, and for confirming its maintenance at the end of the night. The problem remains of what to do when there is evidence of rotation of the reference line. Since the autocollimators must be located at the same height as the axis of the T-4, it is possible to collimate the T-4 with each secondary telescope. This allows the horizontal circle of the T-4 to be employed (and the vertical circle if it has been modified rather than removed) in the overall process of surveillance.

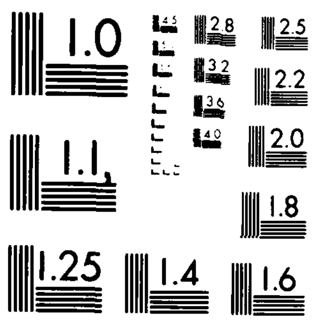
According to Ref. 9, mounts have already been designed and built for various theodolites, including the T-4, which hold the normal of an optical flat approximately parallel to the horizontal axis. Stability of the mount, counterbalancing the horizontal axis, and use for observations in meridian are not described. A single autocollimator of excellent manufacture is used in the preliminary test described. Curiously, it appears that the ability of the collimator to detect any inclination is overlooked. A mercury leveler, as described, is wholly unnecessary if the autocollimation process is employed.

The advantages associated with the use of an axis mirror include the following:

- A direct measure of discrepancies in the instrumental azimuth <u>at the time of observation</u> is available.
- A direct measure of the azimuth of the telescope <u>at the zenith distance of observation</u> is available.

AD-A131 286 ASSESSMENT OF MEANS FOR DETERMINING DEFLECTION OF THE VERTICAL(U) ANALYTIC SCIENCES CORP READING MA W G HELLER ET AL. AUG 82 TASC-TR-4051-1 ETL-0303 2/3

UNCLASSIFIED DAAK70-82-C-0011 F/G 8/5 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

- A direct measure of the inclination is available independent of a level vial.

- Variations in inclination (and/or azimuth) during the observation can be detected if present. (A level vial has a time constant which is a direct function of sensitivity. If read immediately after an observation for longitude, the hanging level will indicate a transient which is settling toward the mean inclination for that observation. Note that a one-sec level has a time constant of several seconds of time.)

- These advantages are gained with little increase in the time required for observations for position.

Development of techniques for using an axis mirror on the T-4 should be expedited.

2.5.4 The Two-Color Refractometer

The significance of the two-color refractometer development effort is summarized by the following principles:

- Unmodeled refraction effects considerably larger than the normal effects discussed in Sections 2.3.3 and 2.4.4 -- arising, for example, from strong local inhomogeneities in the atmosphere -- may represent significant sources of error

- The actual refraction may be computed with a high degree of accuracy from a direct measurement of the dispersion.

These concepts are discussed in the remainder of this section.

The degree to which an optical medium refracts light depends on the wavelength of the light. Cauchy gave an explicit description of this dependence, known as the dispersion, more than a century ago. In the earth's atmosphere, the dispersion is slight, amounting to less than one sec for small zenith distances, but it offers the possibility of determining the total refraction. Light from a common source is separated into widely different pass bands, and the angle between the two nearly monochromatic images is measured. This angle permits the computation of the total refraction if only visible light is of interest.

The exact form of the relationship between measured dispersion and computed total refraction depends on how the effective index of refraction in the atmosphere varies with wavelength. Using the simplest model that is physically realistic,

$$n = 1 + A\left(1 + \frac{B}{\lambda^2}\right) \tag{2.5-1}$$

where

n is index of refraction

A and B are empirically determined constants

λ is the wavelength of the light

The relation between dispersion and refraction takes the form:

$$R = K\,(\delta z)\,\frac{\lambda_1^2 \lambda_2^2}{\lambda_1^2 - \lambda_2^2} \tag{2.5-2}$$

where

R is the actual refraction

(δz) is the measured dispersion

λ_1 and λ_2 are the two wavelengths detected by the refractometer

K is a constant computed from local atmospheric properties

The short response time of photoelectric devices is the single feature most responsible for assuring some success at determining the total refraction. By employing a nulling technique, it is possible to separate the light from a single star into components at far ends of the visible spectrum, to determine the degree to which they are separated by measuring the deviation needed to reunite the images, and to deduce from this the total angle by which the light from the star has been bent in passing through the atmosphere.

An astrometric interferometer, based on the principle of a two-color refractometer, has been built and made to operate quite well. There is no new technology involved. What has not yet been done is to make a direct application of the technology to astrogeodetic positioning. The refractometer could be applied easily to larger optics than are customary in geodetic instruments. Prof. Douglas Currie of the University of Maryland, developer of the two-color refractometer, has calculated that the light gathering power of the T-4 is near the lower limit of usefulness of the currently existing refractometer, which is almost as heavy as the T-4 telecope itself.

On the other hand, the two-color refractometer offers sufficient potential that, if developed around an instrument capable of bearing the weight comfortably and with suitable light gathering power, virtual elimination of refraction as an error source should be possible. One promising solution is the application of the refractometer to the astrolabe, which is a stable instrument capable of bearing rather large loads with no distortion, since the gravity vector always affects the instrument in the same manner no matter what star is observed. Large optics can be included easily, since the light path within an astrolabe is usually folded. The resulting instrument could be automated entirely, timing stars with a CCD, determining the total refraction with a refractometer, and acquiring stars for tracking by means of azimuth information provided by a dedicated microcomputer. While this instrument does not yet exist, development efforts are under way at the University of Maryland.

2.5.5 Remedial Equipment in Brief

Four devices are discussed in this section which offer promise of reduction of uncertainty in astrogeodetic positions. The devices address three of the five principal sources of error discussed in Section 2.4. All of the devices are attractive. In the case of observers (Section 2.4.2), the human skills which are required are in essence removed from the field to the laboratory where they can be applied more reliably. In the case of level vials (Section 2.4.3) a natural sensor replaces a man-made sensor. The two-color refractometer makes possible a measurement of a previously imponderable effect, the refraction (Section 2.4.4). All of the improvements offer reduction of major existing causes of the overall uncertainty of astrogeodetic positions. A summary comparison is given in Section 2.9.3.

2.6 ALTERNATIVE PROCESSES

The published literature contains many papers dealing with new ways to deduce position from assorted varieties of observations. Most are based upon some observations and considerable investigation of the algebra involved in the solution. The dearth of observational data means that a worthwhile statement of the possible quality is difficult to produce. Generally, these methods are noteworthy more for their curiosity than for their significant contribution to astrogeodesy. To remove potential new techniques from curiosity status, it is necessary to produce a large quantity of data, taken under a variety of circumstances, by an assortment of observers, using several different instruments (even if only of one manufacturer). If these data consistently show a high precision of final results, they they should be considered further.

As noted elsewhere in this report, it is a general principle of mensuration theory that the quantity to be measured should be measured as directly as possible unless there are overwhelming reasons for the introduction of indirect measuring procedures. Longitudes are measured almost purely when they are determined from observations of transits, and latitude is measured almost purely by determining zenith distances near transit. No more direct measurements seem possible. The alternatives discussed in this section usually have the merit of ancient invention so that there is at least some data to investigate. Unfortunately, the data are mostly as old as the method. For a proper analysis, data taken with modern instruments are needed in large volumes. The one promising process, the astrolabe, seems promising not because the existent data are good; rather application of modern technology promises efficient and economical increase in the quality of the data.

2.6.1 Observations at the Prime Vertical

If the interval in time, 2t, between the two crossings (first east, then west) of the prime vertical (which is perpendicular to the meridian) by a celestial body is measured, then the latitude of the observer can be determined. For such observations, an analysis similar to that applied to transit observations can be made for an imperfect instrument. This has been done in Ref. 5. With appropriate changes in notation, equation 170, Vol. II, of Ref. 5, becomes

$$\phi = \phi' + b + c \sin \phi'/\sin \delta \qquad (2.6\text{-}1)$$

in which

$$\tan \phi' = \tan \delta \cos \psi/\cos t = \tan \phi \cos \psi$$

b is the level correction,

c is the collimation,

ψ is the hour angle of the instrumental zenith.

Equation 2.6-1 governs every observation. The approximations involved, which are small, are shown in Section 177 of Vol. II, Ref. 5. The right ascension and declination of the body are assumed constant throughout the interval, although a correction, if required, could be applied for a change in position. As with with meridian observations, the inclination of the instrument can be determined from a level vial. Also, the collimation error can be eliminated by appropriate observing procedure.

Since the T-4 is a common and readily available astro-geodetic instrument, the purpose of this section is to examine its applicability to observations on stars as they cross the prime vertical. The T-4 is well designed for such a use, except for a particular detail of the eyepiece which is discussed below.

To put the T-4 in line with the prime vertical, the instrument is first put <u>in meridian</u> by the usual procedure, described, for example, in Ref. 6, p. 315. Then it can be turned 90 deg about its vertical axis and be nearly as closely parallel to the prime vertical as it was to the meridian. The "broken telescope" makes observations at any zenith distance as easy as for meridian observations. The hanging level can be used to determine b just as it is in observations in the meridian. Since the timing is being made for coincidences with a vertical circle, there is no need for precision in setting zenith distances. Thus the small vertical circle can be used as in regular observations for latitude and longitude.

The variation comes in the direction of motion of the star in the eyepiece. The pattern of the reticle in the standard eyepiece is wholly rectangular and some effort is expended by the observer in making one part of the pattern parallel to the meridian. Stars cross the meridian perpendicular to it. They cross the prime vertical at angles which are functions of their zenith distances, hence the motion through the field is diagonal to the normally placed field. If the field is rotated parallel to the motion of the star, however, there is nothing to mark the prime vertical except the central intersection of the crosswires. Under this condition it is imperative that the instrument be adjusted so that the apparent motion of the star is along the central wire.

The interest is not in determining the time at which the star reaches a certain almucantar, but rather the important event is when it reaches a certain vertical circle. Thus the eyepiece must be oriented so that the measuring wires are parallel to the prime vertical. There is no theoretical difficulty with this procedure, but there are practical problems:

- Tracking is difficult near the zenith

- The motion being tracked is slow and therefore difficult to follow (or time) well
- There is a major component of motion parallel to the measuring wires which constitutes a distraction to the measurement procedure
- The measurement takes place over an extended range of the wires.

The first consideration is well known to those with experience in determining longitude: equatorial (as opposed to circumpolar) stars require less concentration to track smoothly and well. The last consideration, however, implies that an error analysis must include not only a determination of the separation of the wires but also an analysis of their failure to be strictly straight and parallel lines. An error model which incorporates these effects should be used in computing latitude. Notice, also, that latitude, a quantity which is essentially independent of time, is being measured by determining the times of related events, passages of stars across the prime vertical. As shown in Section 2.3.6, timing astronomical events is difficult.

The principal information taken from observation of the passage of a star across the prime vertical is time. By differentiating Eq. 2.6-1 with regard to time, two terms are developed. The second of these terms arises from the misalignment of the instrument; its magnitude is on the order of $c\psi$ (both of which are small) and is ignored. The first term of the derivative, on development, gives:

$$\sigma_\phi = \frac{\sin\phi}{\sin\delta}\left(\sin^2\phi - \sin^2\delta\right)^{\frac{1}{2}}\sigma_t \qquad (2.6\text{-}2)$$

This expression can be minimized only by making δ approach ϕ so that the observations are taken close to the zenith. An added attraction is that the time between observations on the same star (if both passages, east and west, are required) is minimized. Figure 2.6-1 plots latitude against declination for representative values of the ratio of σ_ϕ to σ_t.

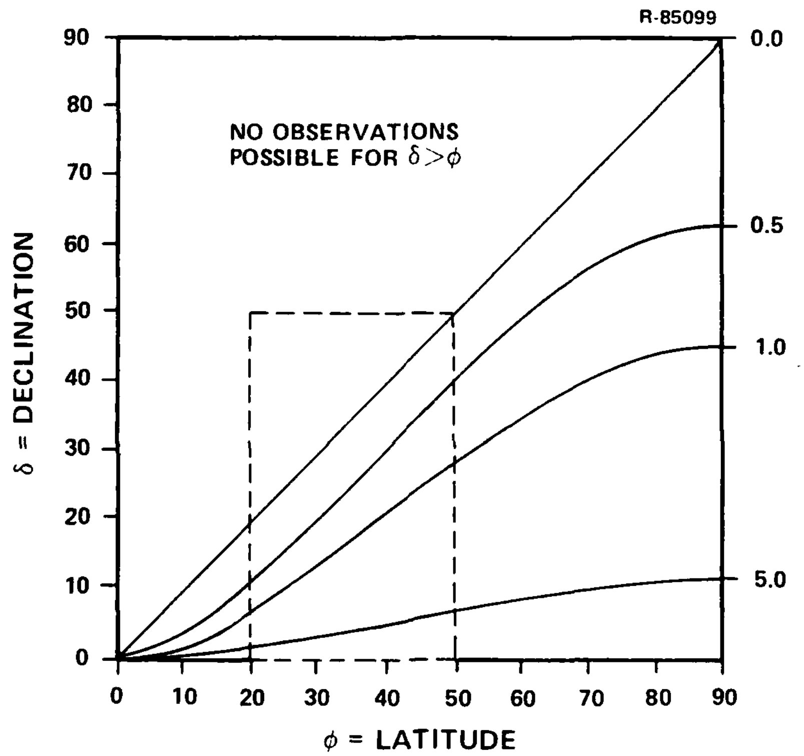


Figure 2.6-1 Error in Prime Vertical Latitude

The next source of error considered is the declination. By differentiating Eq. 2.6-1 with respect to δ and making some simplifications, one obtains

$$\sigma_\phi = \frac{\sin 2\phi}{\sin 2\delta}\,\sigma_\delta \qquad (2.6\text{-}3)$$

Equation 2.6-3 implies that only north of the boundary between Wyoming and Montana can $\sigma_\phi/\sigma_\delta$ be made less than unity. That is, for most of the United States, the contribution of errors in the declinations of the stars observed to the uncertainty of the latitude deduced from observations of passages of the prime vertical will be greater than the errors in the declinations themselves. This is diagrammed in Fig. 2.6-2. For observations in the prime vertical to be possible, the declination must be between zero and the latitude. Thus only stars below line "A" in Fig. 2.6-2 are suitable. The condition that $\sigma_\phi/\sigma_\delta$ be less than unity can be restated as: the latitude must exceed the polar distance*. This condition occurs only above line "B". Thus the shaded area is the area of interest.

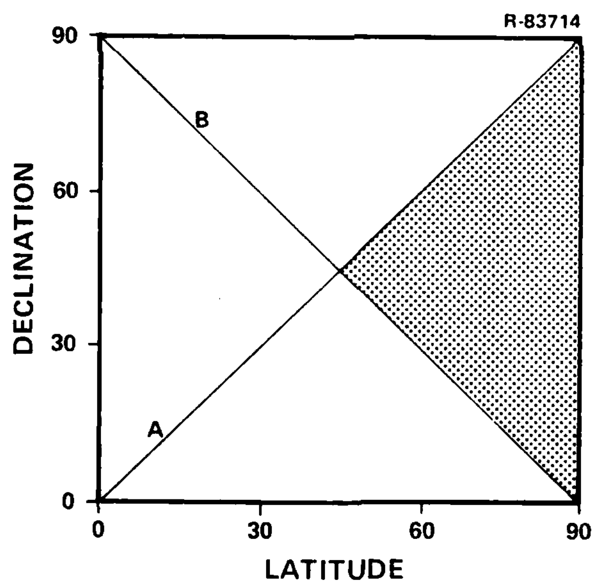


Figure 2.6-2 Latitude - Declination Relationship

*The polar distance is)0 deg .ss the declination.

By inspection it is apparent that errors in the level reading contribute directly to the errors of the final latitude. Thus the method of observing in the prime vertical offers no advantage in this regard over any other method for determining latitude.

Data for quantitative error analysis of this type of observation are nearly nonexistent. The method was standard in the United States for the few astronomic latitudes determined in the early history of the Coast Survey (as it was then called). It was replaced in 1851 by the Horrebow-Talcott method. The change in methods was made for reasons that have not changed in the 130 years since. Equipment has not improved that much.

There is no advantage to using the method for determination of longitude. The track of the star is diagonal to the component of motion desired, and therefore more difficult to observe and analyze. The reduction is similar to that for the astrolabe, which is treated in Section 2.6.3 of this report.

Attention is invited to two early comments which are germane. The process of determining latitude by observations in the prime vertical is briefly described in Ref. 21, the concluding statement of which is: "If the only instrument available is a theodolite having a good striding level, but not equipped for observations by the zenith telescope method, observations in the prime vertical will give the best possible determination of latitude." The T-4, a universal theodolite, is equipped to be used like a zenith telescope. Finally, "... the great superiority of this method of finding the latitude [Horrebow-Talcott]..." is discussed in the last three paragraphs of Section 232, Ref. 5. Those comments appear to be as valid today as when originally written.

2.6.2 Latitude by Observations at Elongation

Elongation is the condition of being at the extremum of azimuth. Elongation is observed only for stars having declinations greater than the latitude of the observer. At the instant of elongation, the astronomical triangle becomes a right triangle, the parallactic angle (at the star) being 90 deg. There are three equations of interest to be derived from this condition:

$$\phi = \text{arc cos } (\cos \delta \csc \bar{A}) \qquad (2.6\text{-}4)$$

$$\phi = \text{arc sin } (\cos z \sin \delta) \qquad (2.6\text{-}5)$$

$$\phi = \text{arc tan } (\cos t \tan \delta) \qquad (2.6\text{-}6)$$

The declination, δ, is to be determined from the catalogue. The hour angle, t, is not easily observed directly, although it can be inferred from other measurements. While the zenith distance, z, could be measured in theory, its observation is complicated by uncertainty in time and by refraction. Thus, Eq. 2.6-4 is the most useful.

To determine the actual magnitude of $\bar{A}$ would require a target on earth of known azimuth. However, as in observations at the prime vertical, observations at both elongations (east and west) provide a measure of $2\bar{A}$, thus giving a solution. Since the time interval involved is many hours, stability of the orientation of the horizontal circle becomes a concern. A simple solution is to measure the horizontal angle between the same ground target (azimuth unknown) and the star at each elongation. This could be done as if an astronomic azimuth observation was being performed following standard specifications. The observing period should extend both before and after elongation, so least squares techniques could be used to determine the extreme horizontal angle. Then half the sum (if the target is inside the elongations) or half the difference (for an external target) of observed horizontal angles is the $\bar{A}$ of Eq. 2.6-4, thereby also giving the azimuth of the target. This method thus joins the group of simultaneous solutions, most of which can easily be shown to give less than optimum results.

Analysis of elongation observations concentrates on the two principal error sources, azimuth and declination. Reference 3 quotes a value of 1.46 $\widehat{sec}$ as the accepted standard error of an azimuth. This figure assumes Polaris as the target; other stars at elongation do not provide as much accuracy. There are two azimuths to be observed; hence, for independent errors, the uncertainty of the azimuth is given by

$$\sigma_{\bar{A}} = 1.46 \times \sqrt{2} = 2\ \widehat{sec} \tag{2.6-7}$$

By differentiating Eq. 2.6-4 with respect to azimuth, the rms latitude error resulting from the uncertainty in measuring the star's azimuth is

$$\sigma_{\phi} = \text{ctn}\ \phi\ \sec\ \delta\ [\cos^2 \phi - \cos^2 \delta]^{\frac{1}{2}}\ \sigma_{\bar{A}} \tag{2.6-8}$$

Inspection of Eq. 2.6-8 reveals that, as the declination increases above the latitude, the factor by which $\sigma_{\bar{A}}$ is multiplied rises from zero, and a convenient maximum is reached at that value for which $\sigma_{\bar{A}} = \sigma_{\phi}$. This value is presented in Table 2.6-1. Observations should be made only on stars for which the declination lies between the latitude and δ_{max}, in order to keep this component tractable.

TABLE 2.6-1
MAXIMUM DECLINATION OF STARS
FOR LATITUDES BY ELONGATION

LATITUDE, ϕ (deg)	DECLINATION, δ_{max} (deg)	INCLINATION CORRECTION (ctn z)
23	32	1.1
25	35	1.1
30	41	1.2
35	48	1.2
40	54	1.3
45	60	1.4
50	66	1.6

Note that the observing method for determining latitude by elongations is essentially that of azimuth, which requires a correction for inclination of the horizontal axis, approximated by i ctn z, where the inclination, i, is determined by a level vial. Since ctn z increases as z decreases and z decreases as δ approaches ϕ, the value of ctn z for δ_{max} is actually the minimum value. This is shown in Table 2.6-1 ($\cos z = \sin \phi \csc \delta_{max}$).

The conclusion is that, to maintain $\sigma_{\bar{A}}$ at the value estimated in Eq. 2.6-7, the inclination correction, which contributes directly to $\sigma_{\bar{A}}$, must be very well determined. Note that the sensitivity of the computed azimuth to errors in determining the inclination correction is greater than one. The importance of this problem is recognized by the surveying community. For example:

- Canadian specifications for azimuth include a requirement that only the T-4 be used, since, in part, it is the only instrument for which the inclination is well enough determined to permit use of Polaris at Canadian latitudes.

- U.S. specifications recommend use of the so called micrometer method at high latitudes (that is, in Alaska) for the same reason.

The zenith distance of Polaris in Alaska is comparable to that required for the observations discussed in this section when applied in the "lower 48" states; hence great care must be taken with the inclination correction.

The second and last error source is the catalogued declination. To investigate the effect of errors in declination, Eq. 2.6-4 is differentiated with respect to declination, to get

$$\frac{d\phi}{d\delta} = \frac{\tan \delta}{\tan \phi} \quad (2.6\text{-}9)$$

But $\delta > \phi$; therefore, $\tan \delta > \tan \phi$ and

$$\sigma_{\phi} > \sigma_{\delta} \quad (2.6\text{-}10)$$

Equation 2.6-10 applies for all values of latitude and declination, and for any variation of observational procedures that can be devised.

In short, latitudes determined from observations on stars at elongation can never be as accurate as current first order techniques for the same expenditure of effort, even though a standard T-4 would be the best instrument to use, and the observations with it would be quite routine.

2.6.3 Circumzenithal Observations

In the discussion so far, a "moderately good" knowledge of the location of the station at which precise astrogeodetic observations are to be taken is a prerequisite to the observations themselves. More specifically, it is necessary to know the longitude to make observations for latitude, and the latitude to make observations for longitude. This leads to the concept of the existence of some correlation between the two components of positions, at least in the measurement procedure. One method for determining astrogeodetic positions explicitly admits this correlation and solves simultaneously for both components. This is the method of equal altitudes, or, from the name of the principal instrument used in the observations, the astrolabe method. (See also Section 2.8.3.)

Basically, an astrolabe is an optical device for preserving a fixed zenith distance with variable azimuth. The time at which a known star reaches the almucantar to which the astrolabe is set is the only observed quantity. The data are solved for the latitude and longitude of the observing site, and usually allowance is made for a variation of the actual zenith distance involved in the observations. The components of position are correlated by functions of the azimuth of the star as it is observed. A good estimate of the position speeds the solution since the model must be linearized for an adjustment.

The process of solution can be visualized as a navigational problem. Each observation produces a line of position (LOP). If stars at varying azimuths are observed, the LOPs will form a circle approximately. The center of the circle is the geographic position of the observer, and the radius of the circle is the zenith distance observed. If an approximate position and an assumed zenith distance are used at the start, corrections to the position are deduced from the displacement of the center of the circle and its radius is a correction to the zenith distance. A least squares solution is relatively straightforward. (See, for example, Ref. 6, page 523.)

The actual zenith distance at which observations are to be taken is arbitrary. Generally a zenith distance of 30 degrees is the most frequently used value, although zenith distances of forty-five degrees are also used. Note that the greater the zenith distance, the larger are corrections for refraction. The advantage of a large zenith distance is, of course, the greater range of declination of stars which is made available. Using forty-five degrees, there is no doubt that the FK4 can be used to supply all the stars necessary. It is less certain that this catalogue can be used with a thirty-degree zenith distance.

A theodolite clamped on its horizontal axis, but free to rotate about its vertical axis, suffices, in principle, for an astrolabe. Images of the stars move through the field at angles which depend on the azimuth, making tracking of the motion of the image difficult. The T-4 can be employed as an astrolabe without change, by using the Horrebow levels to maintain the zenith distance. The horizontal exit of the optical axis is advantageous. This use of the Horrebow levels for maintaining the zenith distance also means that this angle can be varied at will, but the advantages of a variable zenith distance are not apparent.

Various procedures are used to determine the time at which the star coincides with the desired almucantar. As with observations in the meridian for longitude, a single wire, several wires, or dynamic tracking all provide a solution each with its own problems. The single wire is seldom used in astrolabes, as with transit instruments. Observations can be made by human observers or by various photoelectronic devices.

Today, almost all astrolabes used for precise observations split the light coming from a star into two bundles, one of which is reflected directly into the objective of a horizontal telescope, and the other of which is reflected first from a pool of mercury (or some other horizontal reflecting surface) and then into the objective. This procedure provides two images, one of which moves downward, and the other upward. The rate at which the images merge is, of course, dependent on the azimuth of the star as it approaches or leaves the desired almucantar. The use of a theodolite clamped in a fixed zenith distance does not provide two images. Devices, usually called astrolabe attachments, are manufactured for attachment to a surveying level or to a theodolite clamped so that its optical axis is horizontal. These devices comprise a prism and a pool of mercury, thereby providing two images.

Further refinements are also possible. Variable filters can be interposed in the light path to keep the apparent magnitude of all stars the same. The astrolabe can be driven in azimuth at a rate calculated to keep the motion of the two images of the star nearly vertical in the field. An impersonal micrometer can be interposed to produce variable and opposite vertical displacements of the images. The micrometer allows the observer to maintain the images in a fixed position in the field for long periods of time during which multiple determinations of time are made. Prior to averaging, the observations must be corrected for the deviation of the image at the moment time is recorded.

Circumzenithal observations require two conditions of the instrument used:

- A fixed zenith distance
- Rotation of the optical axis into any azimuth.

These requirements are less restrictive than those imposed on a theodolite, and can be satisfied with a simpler, less expensive, device. Successful astrolabes have been made by affixing an astrolabe attachment to a geodetic level, such as the Wild N-3, or Keuffel and Esser 91 30 10. (An automatic level is preferred, however, because the line of sight is kept level with minimal attention from the observer.) Logistic factors and accuracy levels for this method are summarized in Table 2.9-4.

2.6.4 Observations of Stellar Rate

With modern technology, dividing intervals of time into very small fractions has become an easy and highly precise operation. Conventional astrogeodetic techniques generally deal with epochs rather than with intervals of time, but with the advent of this modern timing equipment, the suggestion can be made that stellar rate observations need to be investigated for their ability to reveal astronomical positions.

Unfortunately, rates involve not only measurement of an interval of time but measurement of extent in space as well. A rate is the ratio of one interval to another interval. In this case, stellar rate must be interpreted as the ratio of an angle to the interval of time the star requires to move from one end of the angle to the other. Two coincidences must be made for a single measurement, the start and the stop. It has already been shown that one of the problems with determining longitudes is the limited ability of an observer to establish one coincidence accurately.

This difficulty suggests automation of the observation. The Automatic Astronomic Positioning System (see Section 2.8.2) has already used this technique. Its reticle effectively projects several vertical circles onto the celestial sphere. The epoch at which a star crosses any particular vertical circle forms the basic datum. By subtracting two epochs the interval is determined, and this rate information is used by the program to deduce positional information. The AAPS uses an optical system with a short focal length. This implies that the reticle should be constructed with extreme precision so that the rate information is not degraded by angular errors. The impression given by the available literature, however, is that the reticle was indeed unsatisfactory because it is a simple gelatin reticle. It is interesting to note that the cost of a well made, metal-deposition reticle, even today, represents a small percentage of the total cost of the AAPS project.

In essence, observations at the prime vertical use the stellar rate technique. As a star crosses the prime vertical, the epoch is determined, but the time interval between the two crossings, east and west, is a datum also used in the solution.

For visual observations, the problem is the same: determination of the epoch of coincidence of a star with some location in the sky. Irregularities in the construction and operation of the instrument affect automated observations in the same manner regardless of the observing scheme.

In summary, observations of stellar rate provide no special advantage when using currently available instruments. However, the design of a special instrument which specifically takes advantage of the precision possible when creating and measuring time intervals could provide fruitful. In such a design, little is to be gained if the other interval, be it angle or whatever, is not reproduced and maintained with equivalent precision. No near-term technological developments are available which offer high confidence for achieving the incremental precision or logistical improvement in observations of stellar rate which warrant the replacement of conventional astrogeodetic procedures.

2.6.5 Alternative Processes in Brief

Four processes, alternate to the baseline techniques discussed in Section 2.3, and viewed with the T-4 as the instrument employed, have been investigated in Section 2.6. They are briefly compared in Section 2.9.4. However interesting any process is, this report recommends none of them. This lack of enthusiasm is based on both theoretical analyses which indicate no clear cut advantage in accuracy, simplicity, or speed as well as on a lack of data of sufficient historical extent to support a contention of relative superiority in comparison to the baseline techniques.

2.7 REMEDIAL PROCESSES

This section could easily be extended to a full-scale investigation in its own right. However, since the results of the investigations reported here indicate that these remedial processes are less promising than some of the technology development alternatives described in the foregoing, the next sections simply state what was examined and how. Emphasis has been placed on the determination of longitude because this area seems to offer the greatest possibility for improvement. The analysis has been directed toward existing techniques for the reason stated earlier - few data exist for evaluating more modern equipment.

2.7.1 Analysis of Longitude Reductions

The usual method for reducing data taken for determination of longitude involves the application of Mayer's equation (Eq. 2.7-1). This equation considers three sources of instrument error, discussed earlier in Section 2.3.5, and diagrammed in detail in Fig. 2.7-1.

- Rotation about the vertical, a,
- Rotation about the north-south line, b,
- Nonperpendicularity of the axes, c.

The first error source is referred to as an error of azimuth. The angle between the true meridian and that of the instrument is designated a, positive when the instrument has been rotated against the sun -- that is, counterclockwise when looking down from the zenith. The second source of error is called an error of level. The angle between the true meridian and that of the instrument is designated b, positive when the instrument has been tipped toward the east. The third error is the collimation error -- that is, the displacement of the optical axis from the meridian of the instrument. The angle, c, is measured from the instrumental meridian perpendicularly to the optical

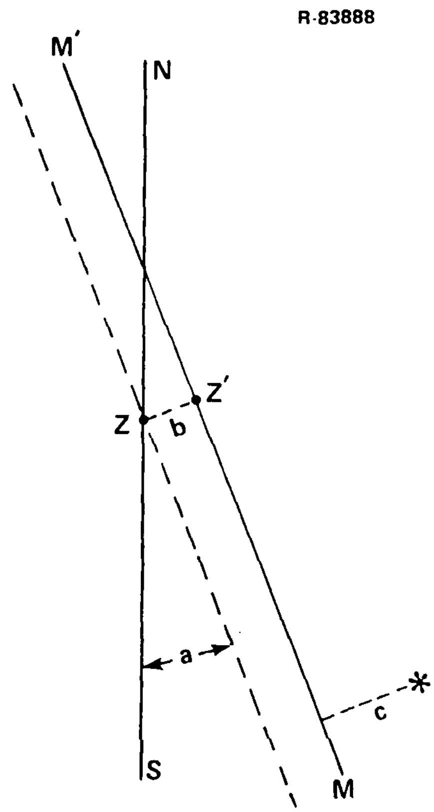


Figure 2.7-1 Geometrical Configuration of the Near-Zenith Region

axis. Given these quantities, Mayer's equation relates the apparent transit of a star to these quantities in the following fashion:

$$aA + bB + cC = \alpha - \theta \qquad (2.7\text{-}1)$$

in which

θ is the local apparent sidereal time of the observation

$A = \sin z \sec \delta$

$B = \cos z \sec \delta$

$C = \sec \delta$

α and δ are the right ascension and declination of the star, and z its zenith distance. Equation 2.7-1 can be compared to Eq. 2.3-17. Note that in Eq. 2.7-1, corrections for aberration, K, and instrument construction, I, are being ignored and that the use of the symbols A and B is more restrictive. Obviously a, b, and c must be measured in units of time rather than arc. Mayer's equation is an admitted approximation. An exact relation is now developed.

Figure 2.7-1 represents the region near the celestial zenith, viewed from outside the celestial sphere. In this figure,

The line, NZS, is the celestial meridian

The point, Z, is the zenith

The line, M'Z'M, is the meridian of the instrument which has been rotated by a, and inclined by b.

The meridian of the instrument refers to the plane (or its intersection with the celestial sphere) which is perpendicular to the horizontal axis of the instrument and which passes through the intersection of that axis and the optical axis. This definition permits application of Mayer's equation to the old Bamberg transits which had no vertical axis in the usual sense, and to the Kern DKM-3A in which the optical axis and vertical axis are intentionally skewed by several millimeters. The zenith of the instrument, Z', is not a clearly defined point, but can be described as that point on MM' closest to Z. It becomes thereby the zero point of the vertical circle of the instrument.

The classical astronomical triangle (described in Section 2.2.2 and Appendix A) relates hour angle, t, and declination, δ, via latitude, ϕ, to azimuth, $\bar{A}$ (angle NZ* in

Fig. 2.7-1), and zenith distance, z (arc Z*). (See Eq. A-15.) It is useful to describe the instrumental zenith distance, z' (arc Z'* in Fig. 2.7-1), and the observed azimuth, $\bar{A}'$ (angle M'Z'*), but further analogy to the astronomical triangle is not needed.

Two Cartesian coordinate systems are required. The first is System 1 of Appendix A. The second, designated System 4, is oriented to the instrument. The transformation from a vector in System 4, $\underline{V_4}$, to a vector in System 1, $\underline{v_1}$, is

$$T_1^4 = \begin{pmatrix} \cos a & \sin a \cos b & \sin a \sin b \\ -\sin a & \cos a \cos b & \cos a \sin b \\ 0 & -\sin b & \cos b \end{pmatrix} \qquad (2.7\text{-}2)$$

Note that a and b have been defined in Mayer's equation so that the corrections to the observed local apparent sidereal time, aA and bB, are positive if both a and A, and both b and B, are positive. In this connection, more useful definitions of A and B are

$$A = \sin \phi - \tan \delta \cos \phi \qquad (2.7\text{-}3)$$

$$B = \cos \phi + \tan \delta \sin \phi \qquad (2.7\text{-}4)$$

since the proper sign is automatically produced if correct signs for latitude and declination are used.

The interest is in determining the hour angle of the star at the epoch of observation. Coordinates in the celestial system are commonly described in terms of azimuth and zenith distance, but they may be described as well in terms of latitude, declination, and hour angle through the astronomical triangle. (In the ideal -- on the meridian -- the hour angle is zero.) If the replacements from the astronomical triangle are made, and the rotations applied to the instrumental coordinates, then the x_{12} part of the transformation defined in Eq. 2.7.2 is

$$
\begin{aligned}
-\sin t \cos \delta = & - \sin a \sin z' \cos \bar{A}' \\
& + \cos a \cos b \sin z' \sin \bar{A}' \qquad (2.7\text{-}5) \\
& + \cos a \sin b \cos z'
\end{aligned}
$$

in which the minus sign on the left accounts for the fact that the azimuth and the hour angle are measured from opposite sides of the meridian. The effect of collimation can be introduced by noting that a right spherical triangle on the instrumental meridian containing both z' and the star gives

$$\sin c = \cos(90-z') \cos[90-(180-\bar{A}')] \qquad (2.7\text{-}6)$$

or

$$\sin c = \sin z' \sin \bar{A}' \qquad (2.7\text{-}7)$$

Solving for the hour angle then gives

$$
\begin{aligned}
\sin t = & \sin a \sin z' \cos \bar{A}' \sec \delta \\
& - \cos a \cos b \sin c \sec \delta \qquad (2.7\text{-}8) \\
& - \cos a \sin b \cos z' \sec \delta
\end{aligned}
$$

Equation 2.7-8 is exact.

A comparison can now be made with Mayer's equation. If the observation is perfect, the right ascension equals the local apparent sidereal time. Thus the hour angle must be zero. The three terms are used to compute corrections, and hence provide the opposite sign to the actual hour angle, the calculated quantity. For comparisons, approximations similar to those in Mayer's equation can be introduced into the exact equation. The quantities a, b, and c are all sufficiently small so that, to first order, their cosines are unity and their sines can be replaced by the angles themselves. Further, the two azimuths and zenith distances are approximately equal; hence, the functions at z' are replaced by the corresponding equations from the astronomical triangle. When these substitutions are made,

$$t = - aA - cC - bB \qquad (2.7\text{-}9)$$

in which the sign of the azimuth term has been changed to accomodate the quadrant of $\bar{A}'$, which is nearly identical to $\bar{A}$. Thus Mayer's equation is valid, provided that the approximations are valid. The error in the correction due to the assumption that Mayer's equation applies is

$$\begin{aligned} E_t &= (a - \sin a)A \\ &+ (b - \sin b \cos a)B \\ &+ (c - \sin c \cos a \cos b)C \end{aligned} \qquad (2.7\text{-}10)$$

For a bounding numerical evaluation, take

$|a| = 45\ \widehat{sec}$ (good practice has it less),

$|b| = 5\ \widehat{sec}$ (a large value),

$|c| = 75\ \widehat{sec}$ (the limit of the field of view),

and numerical values of the factors A, B, and C, derived in Section 2.3.6 as maxima. Thus the error must be less than $8.9 \times 10^{-6}\ \widehat{sec}$, which is quite tolerable.

One other consideration should be mentioned. Solutions in spherical trigonometry can break down in the vicinity of poles. Both the zenith and the celestial pole are poles of a coordinate system. However, Mayer's equation is written in terms of the hour angle/declination system, hence only stars near the celestial pole cause concern. Indeed, the correction for collimation ($c \sec \delta$) seems to allow the full range of declination. Note, however, that the exact equation,

$$\sin t = \sin c \sec \delta \tag{2.7-11}$$

does not, since t does not exist for $\delta > 90-c$.

The zenith <u>can</u> be observed, and in fact is an area of interest for observations, since the azimuth factor is zero. However, as stated above, Mayer's equation is written in terms of the variables of System 2 (Appendix A), hence no problems arise at this pole.

The computational model for the longitude solution merits inspection. The standard deviation of the longitude is equal to

$$\sigma_\lambda = \sigma_o \left[\frac{\Sigma A^2}{n\Sigma A^2 - (\Sigma A)^2} \right]^{\frac{1}{2}} \tag{2.7-12}$$

where σ_o = standard error of unit weight

n = at least 6 stars observed in one longitude set

ΣA^2 = sum of squares of azimuth factors

$(\Sigma A)^2$ = square of algebraic sum of azimuth factors

The sum of the azimuth factors of the six stars observed is, by specification, expected to lie between plus and minus one.

Suppose that ΣA is zero -- north and south stars have been exactly matched -- then $\sigma_\lambda = \sigma_o/\sqrt{6} = 0.41\ \sigma_o$. This is the minimum value of σ_λ for a given quality of observations. If $\Sigma A \neq 0$ then the denominator in Eq. 2.7-12 must be reduced and the ratio σ_λ/σ_o becomes greater than 0.41.

There are two conclusions to be drawn from this analysis:

- The algebraic sum of azimuth factors should <u>always</u> be as small as possible, not just between +1 and -1
- Individual azimuth factors should be as large as possible so that ΣA^2 is maximized -- which implies that stars near the zenith should be avoided, especially if $\Sigma A \neq 0$.

Another prevailing view holds that an appropriate procedure is to use stars near the zenith, but to maintain a balance between north and south stars. However, no matter how stars are selected, there is an augmented sensitivity to level corrections for far north stars. (The range for the continental limits of the U.S. is $0.83 < B < 2.27$.)

Since, in general, the correlation coefficient between the two unknowns, a and λ, is not zero (that is, $\Sigma A \neq 0$), the value of a direct measurement of the instrumental azimuth merits consideration, subject to the constraint that manipulation of the instrument should be minimized. This is an application of a Kalman filter, from which it can be deduced that the improved standard error of the longitude, $\sigma_\lambda(+)$, (improved by a measurement of a) is related to its value, $\sigma_\lambda(-)$, before the measurement, in the following way

$$\sigma_\lambda(+) = \sigma_\lambda(-)\left[\frac{\sigma_a^{2}(1-\rho^2) + r_a}{\sigma_a^2 + r_a}\right]^{\frac{1}{2}} \quad (2.7\text{-}13)$$

in which ρ is the correlation factor and r_a is the noise in the measurement of a (Ref. 13, p. 114).

Consider that the measurement of the instrumental azimuth, a, is perfect, so $r_a = 0$. Then the improvement is $\{1-\rho^2\}^{\frac{1}{2}}$, from which it is seen that for small ρ, the improvement provided by this measurement is slight. Reversing the argument, if ρ is small, keeping the instrumental azimuth, a, as an unknown in the solution does not affect precision. On the other hand, if ρ is large (because ΣA has not been minimized), even a noisy measurement of the instrumental azimuth would be advantageous.

The instrumental azimuth, a, is not a straightforward concept. Since the T-4 has a vertical axis, exact reversal of the instrument depends on the skill of the operator. Secondly, neither wyes nor horizontal circles prevent trunnion wobble. (Observatory transits frequently employ azimuth marks, but these can be used only at a fixed zenith distance which must be quite different from that of the stars being observed. Trunnion wobble remains a consideration in determining, or more importantly, maintaining a fixed azimuth.) Even if the observer is careful in setting the horizontal circle, trunnion wobble can introduce a change in the actual azimuth for each setting of the telescope. In a complete model, each star is observed in two different azimuths. Since, during a set of observations, the last portion of the procedure for tracking one star and the first portion of the procedure of tracking the next star are generally done without changing the vertical axis, these two azimuths differ as the zenith distance is changed only by an amount induced by trunnion wobble. The conclusion is that the instrumental azimuth is a noisy quantity at best -- that is, σ_a is large -- and if correlated (by poor selection of stars) with the longitude, will increase the formal standard error of the longitude. No weighting scheme can change this, although it might, for purposes of analysis, allow formal recognition of the correlation between stars.

2.7.2 Weights in the Solution for Longitude

A frequent complaint about the use of Mayer's equation in solving for longitude is that each observation is assigned the same weight. To rectify this, studies (Refs. 23 and 26) have been made of many observations of transits of stars made with two meridian instruments. The weighting function as derived in Ref. 23 is:

$$p = 1/(1 + 0.32653 \tan^2 \delta) \qquad (2.7\text{-}14)$$

where the numerical coefficient is the result of the analysis of the observations. The weight, p, represents a recognition that stars with large declinations determine longitude poorly. However, these weights are described as being used only for observations taken in Alaska - that is, in high latitudes.

Another procedure is given in Ref. 24, where the cosine of the declination is used rather than the more complicated function above. The algorithm in Ref. 24 eases the computational load somewhat, but it solves for the component of deflection of the vertical rather than the longitude directly. (This component is derived only if the geodetic longitude is assumed as the starting point, a procedure which is also possible using Mayer's equation.) The Geodetic Survey of Canada and U.S. military surveying units use this procedure routinely, even though the stars are selected according to the same criteria that were described in Section 2.3.5. This is worth noting here, because mention has been made in the previous section that the correlation between the unknowns, when solving Mayer's equation, is proportional to the sum of the azimuth factors. If the procedure in Ref. 24 (Gregerson's solution) is used, the correlation is proportional to the sum of the sines of twice the zenith distances, instead. (The zenith distance must be a signed quantity, as before. Thus, two stars selected so that their zenith distances are of equal magnitude but opposite signs will give zero correlation by Gregerson's solution, but not using Mayer's equation.

Examination of algebraic expressions for the uncertainties (σ_λ from solution of Mayer's equation and σ_η from Gregerson's procedure) does not reveal an obvious systematic difference which could be used to rgue for one system as opposed to the other. Professional opinion is that Gregerson's procedure will give a slightly better solution for the longitude if the stars have been selected poorly -- that is, if the set is badly mismatched (as could happen, for example, if an otherwise satisfactory set is found on reduction to include one poorly observed star which should be rejected). However, when observations are taken to maximize the information about the deflection of the vertical, poorly matched observations can <u>not</u> be tolerated at all. Furthermore, in this study, the latitudes have been limited in range to those of the continental United States. The conclusion is that no significant benefit is to be obtained from Gregerson's solution, aside from a slight easing of the computational load, which is not significant for implementation on digital computers.

Proposals have also been made concerning nonzero off-diagonal terms in the weight matrix. The observing procedure requires reversal of the instrument in the middle of observations on a single star. Only adjustment in zenith distance is made between stars. Thus, the instrumental azimuth is a constant for the end of observations on one star and the beginning of observations on the next, providing correlation between the two stars. This correlation should be explicitly recognized, but is not in either of the two algorithms discussed above: in both cases a unique value of the instrumental azimuth is modeled. Without a wholly new model which includes multiple values of the instrumental azimuth, explicit inclusion of the correlation between stars provides no advantages.

2.7.3 Data: Quantity vs. Quality

Occasionally proposals are made to improve astronomic observations by taking more data of the type already collected. See, for example, Ref. 26, p. 27. This is the standard statistical problem, which can be summed up as "More = Better." If the data are not correctly treated, the results may appear to have smaller statistical uncertainty, but what is accomplished in fact is the precise computation of a biased quantity.

The difficulty with the concept of taking more of the same data can be diagrammed as shown in Fig. 2.7-2*. Both axes are logarithmic. The abscissa represents time in seconds required for observations while the ordinate is the standard error of the process in seconds of arc. The individual points on the left line represent the standard error of the longitude from timing one, two, three, etc., stars. The slope of this line is determined by the average rate at which stars can be observed with the T-4, and is not otherwise significant. The values, however, come from the usual statistical formula which, for uncorrelated errors of measurement, shows that the standard

*Credit is given to Prof. Douglas Currie for the basic idea of this diagram.

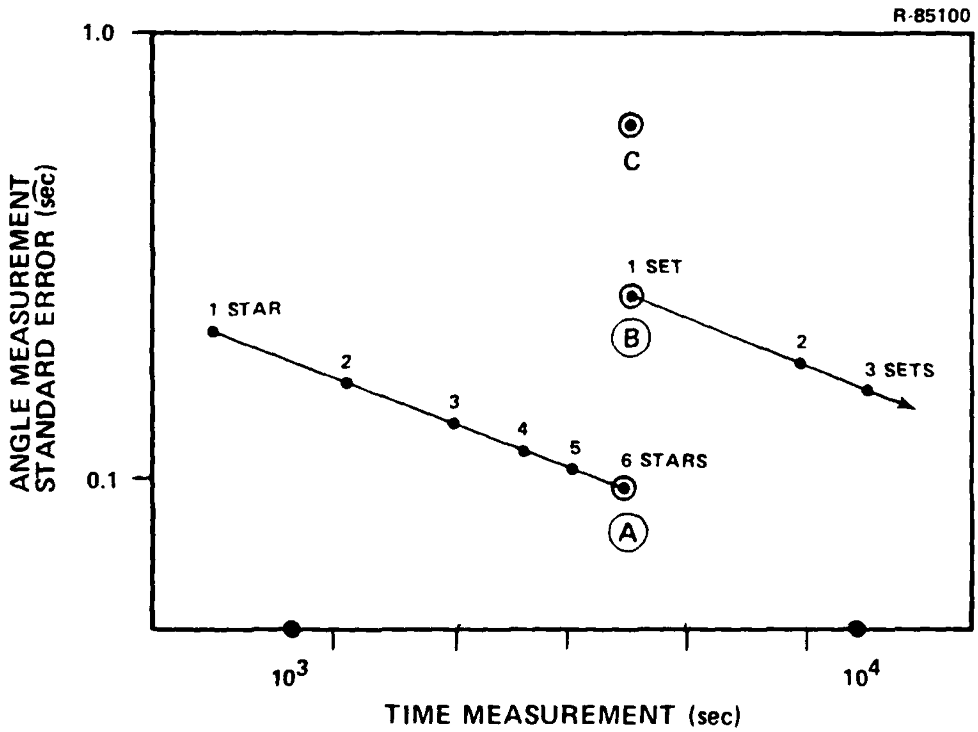


Figure 2.7-2 Error Diagram for Astronomic Observations

deviation decreases according to the ratio $[N/(N+1)]^{\frac{1}{2}}$ as the number of stars observed, N, is increased. But according to Ref. 3, after six stars (which constitute a set) have been observed, the standard deviation of the longitude is the value at point 'B' in the figure, not that at point 'A'. According to the same formula, this value, in turn, should decrease as the number of sets increases (indicated by the right line). In fact, working back from Eq. 2.3-16, point 'C' actually represents the standard deviation of the longitude determined from one set.

The difference is caused by the use of an incorrect model at each tip of the sawtooth. If all sources of error are considered at each step of the observation, then indeed a single line* can be drawn which represents the improvement as more data are taken. The line should pass through point 'C'. The conclusion is that little precision is gained by taking large quantities of data if those data are not reduced with a statistical model which incorporates all sources of error. By tracking a star for many turns of the micrometer, one increases the knowledge of the epoch at which that particular star reached some unknown point in the sky. This point is unknown because the instantaneous value of the midpoint of the tracking process has not been determined. Further, the process by which an estimate is reached for the location of this point in the sky actually estimates a mean of several midpoints, which are correlated with each other in some manner which will not be explicitly stated in the computation.

Since the data are not being computed with an appropriate statistical model, an automatic eyepiece, discussed in Section 2.5.1, can contribute little to precision in determining longitude. To apply this device properly to the T-4, the reduction of the data taken should be performed according to some yet-to-be-determined algorithm which properly models, in particular, the several instrumental azimuths involved in each longitude set.

It may be that collimation, for example, could be treated by analytical methods with greater success, rather than by procedural techniques. Similarly, if an axis mirror could be used to monitor the position of the horizontal axis during the observations of stars, the solution would involve a single instrumental azimuth, with small corrections provided by the axis mirror. This is a far more realistic model than the one represented by Eq. 2.3-17.

---

*A single line applies only if the data are taken continuously. The requirement that observations be taken on each of two nights is ignored.

In short, more data are valuable only if they can be treated properly. Only then will multiple observations result in higher precision.

2.7.4 Multiple Observers and Instruments

In Ref. 3, the quantity k is included in the expression for the expected standard deviation of an observation for longitude, where k is defined as the number of observers. Reference 26 discusses evidence for biases associated with individual instruments and recommends a requirement for a minimum of two observers and two instruments for what is defined there as Superior Astronomic Positions. These statements bring out the need for redundant observations, especially during determination of longitude, and suggest the need for use of several instruments in these observations. There is little advantage in the context of this report, however, in separating the individual contributions of observers and instruments. The recommendation is made that an observer and his instrument be kept together as a team -- that if any given observer is assigned to make an observation, he must use the same instrument used at all previous sites, specifically including base stations at which observers are certified and personal equations are determined. This does not preclude more than one observer being assigned to one instrument -- observers get sick more often than do T-4s.

The variables n (number of sets observed) and k in the equation for standard error can be varied relatively easily. If two observers (and two instruments) are used on several nights, each could observe the same number of sets of stars. Table 2.7-1 demonstrates the improvement gained by this method. It is derived by changing only n and k in Eq. 2.3-16. The strategy which gains the most for the least effort is that

TABLE 2.7-1

STANDARD DEVIATIONS OF LONGITUDE ($\widehat{sec}$) FOR MULTIPLE SETS, n, OBSERVED BY MULTIPLE OBSERVERS, k

n=	6	8	9	12	18
k= 1	0.38	0.38	0.38	0.37	0.37
2	-	0.28	-	0.27	-
3	-	-	0.23	-	0.22

of using two observers, each occupying every station one night each, and each observing two sets each night (n = 8, k = 2).

In turn, this strategy leads to the following proposal, which can be referred to as translocation. Each pair of stations would be occupied simultaneously and, insofar as possible, the same observing list would be used at each station. Several weeks later, the same pair of stations would be reoccupied by the same observers, but with their locations reversed. Arguments supporting this procedure are the following. Clear weather at both sites would suggest a high correlation between the conditions of refraction at both locations as influenced by the presence of a single high pressure system covering the area. The difference in deflection at the two sites would be free of the effects of anomalous refraction. The common observing list would make it possible to determine the difference in longitude between the stations totally free of errors in the catalogued positions of the stars employed. These errors would affect latitudes in minimal fashion. Reversal of occupation would provide a difference in position which would be totally free of personal (and instrumental) equation.

The single argument that negates the proposal of the preceding paragraph is the following one. In this application the requirement is the absolute deflection of the vertical at a specific point on the surface of the earth. The arrangements suggested above would help provide a detailed and cohesive knowledge of the geoid over whatever area of the earth the set of observations encompassed. That is, the shape of the geoid in some restricted region would become well known. But the process does not increase knowledge of the absolute deflection at any single point in the region. For geophysical studies a contribution has been made, but for the present purposes, there is little improvement. Translocation is discussed further in Section 4.3.2, where it is shown that while the accuracy improvements expected from multiple measurements are realized, the availability of differential measurements offers insignificant advantages.

While translocation offers little benefit in the context of this study, observations made by more than one observer using more than one instrument can contribute significantly. Enough is known about bias produced in determining astronomic positions to emphasize the need for optimal procedures both for taking measurements and for reducing them. If standard T-4s are employed, regardless of observational scheme, the advantages of at least two instruments sent to every site are significant, as shown in Table 2.7-1. This study indicates no advantage in separating the components of bias contributed by the observer and his instrument. The recommendations are made that each observer be assigned a particular T-4 permanently, that at least two observer/instrument teams be sent to each station, and that these occupations should be separated in time by long intervals to average out temporary meteorological effects. Benefits to be gained from multiple observations are discussed further in Section 4.3.1.

2.8 ALTERNATIVE EQUIPMENT

There are other instruments (manufactured, for example, by Kern and Zeiss) that can be considered functionally equivalent to the T-4. They are not addressed explicitly in this report for two reasons:

- The T-4 is the most common field instrument used in the United States
- The other instruments have been shown to be equivalent in performance in other studies (Ref. 25, for example).

Consequently, this section deals mainly with devices that take a wholly different approach to determining position from stellar observations. The first device discussed, the portable photographic zenith tube (PZT), has not yet been used as the basis for automated determination of position; it could, however, be adopted for such application and seems already to have produced data of useful quality. The second device, the Automatic Astronomic Positioning System (AAPS), depends heavily on the existence of sophisticated electronic equipment for the observation and reduction of large volumes of data.

2.8.1 The Photographic Zenith Tube

Positions on the earth are generally described in terms of latitude and longitude. However, given knowledge of the apparent time at some reference meridian, an observer who determines the right ascension and declination of the zenith has completely described his location on earth -- the latitude is equal to the declination, and the longitude is equal to the difference between the apparent sidereal time at the reference meridian and the right ascension of the zenith. This is the basic computation used in employing the photographic zenith tube (PZT) as an astrogeodetic instrument.

In concept, it is necessary merely to photograph an area of the sky which includes some identifiable stars in order to locate the zenith relative to these stars, and to determine its position from them. The epoch of the exposure gives the necessary connection to the reference meridian.

The assumption made is that the relationship between the actual positions of the stars and their images on the plate is defined exactly by a gnomonic projection, in which every image on the photograph can be connected to its corresponding star on the celestial sphere by a single straight line. All such lines pass precisely through the nodal point of the camera lens. Considerable effort is required to insure that this is the case, but these efforts are mostly during construction of the camera. The lens must be excellent, its support rigid, and the film must be located relative to the lens with high precision. In fact, in spite of these efforts, the projection is never exactly gnomonic and this implies computational procedures to accomodate the discrepancies. The precision with which the zenith can be located is also a cause for concern. Level bubbles do not permit satisfactory accuracy. Mercury surfaces provide a more reliable reference to the direction of gravity (see Section 2.5.2), usually by incorporation as an optical element in the light path from star to film. Every PZT requires field procedures (usually reversal) to accomodate collimation. But the interval between the two exposures required allows sufficient earth rotation, so that each exposure records a different field of stars.

The term PZT usually suggests the large observatory instruments employed for years by government agencies charged with determining variations in earth's rotation for precise timekeeping purposes. Focal lengths of twelve meters are currently in use. Clearly, such instruments can not be considered for determining astrogeodetic positions at widely separated locations in the field.

Several attempts have been made to produce small instruments with shorter focal lengths and smaller mass than those in observatories, but with comparable precision. Most of these attempts have failed, and in many cases the failure can be attributed to the use of level vials as the device to sense the direction of gravity. The two promising techniques to be described in this section avoid use of level vials. The first instrument was actually completed and tested. It is reported to be packed in shipping cartons and stored somewhere in the Washington area, although concerted efforts to find it have failed*. The second device was never completed, largely because the inventor came to the conclusion during construction that determining positions astrogeodetically is no longer necessary†. These two portable PZTs are described now.

In the early 1960s, Raytheon, then owners of Autometrics, funded development of a portable PZT. The project was directed by Everett Merritt, who had conceived the basic idea of a special camera as an improvement on the clearly unsuccessful PZT developed previously by the David W. Mann Company. In Merritt's view, the failure of the earlier camera was due to improper mecnanical construction and dependency on level vials. Merritt additionally developed a new algorithm

*These have involved contacts with the following companies -- Raytheon; Autometrics, Inc.; and Information Development and Applications, Inc. of Beltsville, MD.

†It is likely that in this case, also, appropriate steps could be taken to complete the device. By the inventor's estimate, it is two-thirds complete.

for reduction of the plates taken with his camera to reduce the influence of the radial distortions which were known to be present in the lens used.

The final version of this PZT used an existing lens of 30 cm focal length with a focal ratio of f/2.0. Standard 4 by 5 glass plates were used without reseau. Exposures were a few seconds long. The zenith was assumed in first approximation to be the principal point of the camera, and this was guaranteed by an autocollimating device rigidly attached to the camera body. The autocollimator used folded optics to attain a focal length of one meter without extending the size of the camera. The mirror was the surface of a pool of mercury enclosed in a vial, one face of which was the objective lens of the autocollimator. This enclosure meant that the surface of the mercury seldom needed cleaning in the field. Thus this device predates that described in Section 2.5.2, for the same purpose.

At first, two exposures on the same plate were taken with a reversal of the camera by 180 deg between them. Eventually three exposures were taken with a rotation of 120 deg between them. This made it possible to separate asymmetries of the atmosphere as required. Since this was a developmental project, measurement was performed by hand on a good Mann comparator. The positions of about 100 stellar images on the plate relative to the principal point were punched on cards, which were input for a FORTRAN program in an IBM 1620 computer. Solution for a position was an operation of several hours, and produced the latitude and longitude of the observational site, corrected focal length of the camera, location of the principal point on the plate, three components of radial distortion, and two of tangential distortion.

Various tests were made with the camera before the project was abandoned. Final tests were made on the grounds of the US Naval Observatory, and it has been stated that the device produced astrogeodetic positions with uncertainties near 0.1 sec, based on comparisons with observatory instruments. This figure is mentioned as both an absolute accuracy and as relative repeatability. Since nothing was published in the literature, the memories of the people involved must be relied upon for these figures. The project was abandoned because the parent company was unable to predict future profits from the venture.

A more recently developed PZT is in Australia, partially complete, but also in storage. Anthony Sprent, now Lecturer in Surveying at the University of Tasmania, built a camera intended for astrogeodetic use. His approach is quite different from that of Merritt. The plate is moved across the field at the sidereal rate, but only a narrow band containing the image of the meridian of the site is exposed. The exposure thus becomes a photograph of the star field at midexposure, extended in right ascension on both sides of the meridian by as much as 15 minutes of time. The declination range is one deg. The 48 inch focal length is folded so that the camera and its support are only about three feet tall.

The unique contribution of the Australian PZT is the development of a device for sensing the vertical to within 0.01 $\widehat{\text{sec}}$ for deviations as large as a half degree*. It consists of a collimator with a small strobe light at its focus, flashing once every thirty seconds in synchronism with a radio time signal. These times relate the exposure to the reference meridian. The optical path from the strobe to the film includes the gravity sensing compensator located between the objective

*These are Sprent's figures.

of the collimator and that of the PZT. This compensator depends upon several compartments of water and air separated by optical flats. The combination of glass, water, and air, with the water seeking the local geop, produces an emergent beam which is parallel to the direction of gravity within the specified tolerance. The flashes appear as a line of dots across the exposure. The instrument is reversed after each flash to accomodate collimation.

The assembly is estimated to weigh 30 kg, compared to 50 kg for a Wild T-4. Ancillary equipment and shipping crates would increase the total weight to be transported. Since the camera was not completed, tests have not been made with it, so that quantitative evaluation is difficult. The project was abandoned several years ago.

Both of these PZTs depend upon an orientation of the film which can not be guaranteed. The importance of the location of the emulsion in photogrammetric applications has been analyzed most recently in Ref. 27, where emphasis is on loss of precision when glass plates are used. The errors that arise from incorrect assumptions concerning the location of the emulsion increase in importance as the radial distance from the principal point increases; the need studied in this report requires both wide fields of view and the utmost precision. Special cameras have been designed using film locked in place with a vacuum platen during exposure, and depending upon a reseau for accomodating dimensional instabilities (Ref. 28). Both PZTs could benefit from the incorporation of the technology.

Mr. Sprent writes in a personal communication that he would no longer use film at all in a PZT, but would instead prefer to apply CCD technology. If one compares the apparent precision with which Prof. Currie has determined position on the array (Section 2.5.1) with the requirements of the PZT, Mr. Sprent's concept seems sound. If the same precision can be obtained from a CCD as is described in Section 2.5.1, the array makes it possible to locate the sensitive elements permanently in the focal plane and to calibrate their actual location. The number of images to be measured depends on factors yet to be evaluated.

Quantitative evaluation of the PZT as an instrument for use in the present application can not be made for lack of data. It appears that Merritt's camera approached positional uncertainties of about 0.1 $\widehat{\text{sec}}$, over a decade ago. The advantages of PZTs are quite simply:

- Ease of operation
- Minimization of time at the field site.

In contrast, the major disadvantage has been the magnitude of the effort involved to reduce the data. Formerly, the measurement of plates was considered a task of substantial magnitude. Now, automatic plate measuring devices can be ordered from several manufacturers. The precision of these devices is in every case as great as that attained by skilled human operators using manual measuring engines, and the time required for measurement is less. Computers routinely handle similar quantities of data. The process of data transfer, from measuring engine to computer, has been automated also. Thus the PZT does not now pose as significant a problem in this regard as it did. In addition, the application of a sensor inherently amenable to digitization is attractive.

The field of view of the two PZTs described is estimated to be of the order of 6 to 8 deg. This is not really great enough to allow use of the FK4 without waiting for suitable stars to become available. If a more extensive catalogue becomes available, as suggested in Section 2.4.1, the narrow field will be much less a drawback. In summary, transportable PZTs are recommended for reconsideration as devices for determining astrogeodetic positions.

2.8.2 The Automated Astronomic Positioning System

Development of the Automated Astronomic Positioning System (AAPS) was begun about a decade ago. In concept it was meant to be a self-contained device for determining the astronomic latitude and longitude. As such it contains, in two interconnected units, the equivalent of telescope, clock, observer, recorder, star catalogue, and computer. Once in place, the device is entirely automatic, from the start of observations to the announcement of latitude and longitude.

The observations depend on photoelectric detection of passages of stellar images across several lines which are nominally fixed vertical circles. These passages are brief events in terms of the photocell's response. By processing this signal, the epoch of the passage is reliably and precisely determined. The time interval between passages over adjacent vertical circles -- that is, the stellar rate -- is a function of both latitude and declination, and the mean epoch of related passages is a function of longitude and right ascension. Since the coordinates of the star are available, the location of the AAPS is the major unknown. The device could also identify stars in the course of observations, but utilization of the approximate location of the AAPS and local time minimized the computational load in the reduction of data.

The orientation of the AAPS relative to the vertical is detected and compensated by a Hughes Level Control System. This device electrically senses the movement of a bubble over a flat plate and produces signals that are used in a feedback loop to move the AAPS back to level. Initial orientation in azimuth need only be approximate, and once observations are begun, reversals are automatically controlled.

The system achieved precision comparable to current first-order techniques using the T-4 (Ref. 10). This is a remarkable achievement, in light of the complexity of the concept and the level of technical competence when the design began. However, development was suspended several years ago because of cost overruns.

In an evaluation of the results then available (Ref. 10), the admission is made that considerable massaging of the small amount of data then available was necessary to attain the claimed precision. This is not difficult to understand since operating procedures may not have been optimal. For example, the photograph in Ref. 10 illustrates the AAPS sitting on a piece of plywood on the ground. The location of the AAPS directly on the surface of the earth maximizes problems with refraction. The first meter of the atmosphere introduces more irregularities in any line of sight, even vertical, than the next ten. However, much of the development remaining to be done involves the sensor rather than the reduction procedures (Ref. 11).

However competent the AAPS is, or could become, the system is costly and complex. Unattended, clandestine operations offer a potential justification for resuming the project, but such an application obviates the need for reduction of the data at the site, since storage for later retrieval, or transmission to a secure computer, would suffice. In short, the key factors which would justify deployment of the AAPS are:

- Need for totally self-contained and continuous operation
- Lack of economic restraints.

2.8.3 The Astrolabe

The most common astrolabe in terms of the number of devices manufactured is the Danjon astrolabe, a French instrument described in Ref. 8. This type of astrolabe has been operated by observatories in many countries ever since it was first designed. Considerable experience has been gained in its use. In an observatory setting, it is capable of determining position to approximately 0.1 $\widehat{\text{sec}}$. This figure is derived from observations on many nights at the same location, and from an observing procedure using 28 stars per set (each set requiring about two hours to observe), with at least two sets observed each night. The Danjon astrolabe has not been used very much for astrogeodetic applications, the interest at observatories being restricted almost entirely to the determination of time and polar motion. However, the device separates easily into two parts, either of which can be comfortably moved by two people. It has the advantage over the T-4 that the two parts are <u>not</u> separated by a precision surface which must be treated with great care. As a result, maintenance and assembly in the field is eased.

A Czech astrolabe is described in Ref. 44. This device uses two mirrors to provide the beam splitting. The mercury surface is relocated to a position on the vertical axis, which means that centrifugal forces are symmetrical when the instrument is swung rapidly in azimuth from one star to the next, thereby minimizing disturbances to the reflecting surface. The micrometer is of a different design, also, but the effect of keeping the images in the same place in the field for long periods of time is the same. Both the Danjon and the Czech astrolabes are driven in azimuth during the observations, to compensate for that component of the motion of the images which is not of interest. The Czech astrolabe was designed to be used as a portable (transportable) instrument for the determination of astrogeodetic positions, and has been tested as such by both Czech and German geodesists. Reported precisions in the determination of longitudes are nearly 0.1 $\widehat{sec}$ (Ref. 45).

The Chinese have also built a large astrolabe which is supposed to be an improvement on the Danjon version (Ref. 46). This device, while appearing to be quite successful, is specifically designed for use in an established observatory, and requires extensive ancillary equipment. It is not considered adaptable for field use and is not discussed further.

As with a transit instrument, both the Danjon and Czech astrolabes are labor intensive, requiring continuous attendance during observations, usually by two people: the observer and a recorder. Also, not every person who sets out to become an observer succeeds in becoming sufficiently skilled (or sufficiently motivated to keep at the work even if skilled) to assure a continuing program of observations for either time or position. The training required is similar to that for a transit instrument and it is likely that a good observer could adapt easily to either instrument. Some experience indicates that astrolabe operation is, in fact, somewhat easier to teach.

The advantages of the astrolabe include the following:

- The direction of gravity is determined simply and with high precision by the pool of mercury

- The line of sight is determined very precisely by the rigid construction of the prism or mirror assembly
- Time is the only observed quantity for both components of position
- The astrolabe is at all times balanced and subject to the same uniform strains, no matter what star is being observed
- The timing apparatus is the same as that for the transit instrument and its technology is up to date
- Very little ancillary equipment is required in the field, provided that observing lists can be supplied from, and reduction of data can be acceptably accomplished at, a central facility.

In addition to these advantages is the particular adaptability of astrolabes to automation. The CCD eyepiece (see Section 2.5.1) can be integrated with an astrolabe with a high probability of success. As applied to the T-4, CCDs have already been shown to be competent at relating time and position. Software has not yet been written to make measurements of the relative displacement of two simultaneous images in a common field, but the development of such programs is a logical near-term possibility. The design of the astrolabe makes the attachment of electrical and optical cables easier than does the design of the T-4, where rotations about two axes must always be considered. It is probably possible to mount the cables so that no imbalance of the instrument is ever produced. Since the automatic eyepiece requires the support of a small computer in the field, the same computer could be used to drive the astrolabe in azimuth, both during observations, and from star to star according to a given observing list. A key attraction of this approach is that the operation can become wholly independent of the skill of a human, and, in fact, the instrument could be left to operate on its own.

Further improvement is also possible. An astrolabe is rugged enough to carry the additional complexity of a refractometer (see Section 2.5.4). Such a combination of automatic observations corrected by absolute measurements of refractive effects would make a powerful tool for determining astrogeodetic positions. Given approximate coordinates of an observing site, sufficient computing power would be present to produce a complete observing list, and probably to make preliminary reductions of the data as well. With upgrades to computer capability, software could be designed to produce improved positions, sequentially, as the observational data from each star become available. The ultimate advantage offered is the possibility of writing specifications such that field personnel need only monitor the necessary statistical quantities to be assured that adequate observations have been made and that the occupation has been completed. Such a device is probably less than five years in the future if pursued vigorously.

In summary, the astrolabe is currently capable of providing astrogeodetic positions of precision comparable to that attained by baseline techniques, with position errors of 0.3 $\widehat{\text{sec}}$. More importantly, it holds significant promise of improvement in precision, for less expenditure of funds and of time, than the standard T-4. By its nature, the astrolabe lends itself to complete automation better than the T-4.

2.8.4 Alternative Equipment in Brief

Three devices, the photographic zenith tube, the automatic astronomic positioning system, and the astrolabe, have been reviewed. The AAPS is looked upon as a predecessor of what the other two might become -- wholly self-contained, fully automatic devices. None of the three has succeeded in replacing baseline techniques, although all three have the capability. The astrolabe is recommended as having the greatest potential for timely, economical development into the best possible system for determining astrogeodetic positions.

2.9 SUMMARIES AND RECOMMENDATIONS

In this section, an attempt is made to tabulate the major findings of this chapter. Since the baseline is the most important topic, it is the foundation on which the table was designed. Hence, the table has restricted application to the other topics. Many of the findings may be considered conservative, in that they are based on established procedures and the documented results of field practice. Recent contributions* indicate that modern military practice does somewhat better.

Since the conclusions of Section 2.7 do not lend themselves to tabular representation, they are discussed separately in Section 2.9.6. The tables which follow are preceded by a separate description of the column headings.

*From R. Salvermoser (DMAHTC) and others.

2.9.1 Description of Tables of Comparison

0. Descriptor	Keyword to describe the type of observation, the method or instrument used in the determination of an astrogeodetic position
1. Catalogue	The name of the organized catalogue of positions of stars used in the reduction of observations
1a. Right ascension	The sensitivity of the method to errors in the right ascension of stars used
1b. Declination	The sensitivity of the method to error in the declination of stars used
2. Instrument	The instrument normally used, or recommended for use, in the method discussed
2a. Collimation	The sensitivity of the method to collimation of the instrument
2b. Circles	The sensitivity of the method to errors in the graduation of the circles of the instrument
2c. Optics	The sensitivity of the method to errors in the optical parts of the instrument
2d. Mechanics	The sensitivity of the method to errors in the mechanical parts of the instrument
3. Operation	The principal duties of the operator
3a. Azimuth	The sensitivity of the method to errors of azimuth in the instrumental alignment
3b. Level	The sensitivity of the method to errors of level in the instrumental alignment
3c. Timing	The sensitivity of the method to errors in timing the observations
4. Weather	The requirements of the method as to local weather conditions (cloud cover is customarily indicated in tenths of the full sky)
5. Field time	The usual amount of time at the site to perform the normally required observations
6. Solution	Brief description of the type of solution used to obtain the astrogeodetic position
6a. Unknowns	Unknowns solved for in usual solution
6b. Latitude	The sensitivity of the method to errors in the assumed latitude
6c. Longitude	The sensitivity of the method to errors in the assumed longitude
7. Quality	The expected standard deviation of the final value of the coordinate(s) as derived from the solution, including only the single improvement under discussion, if applicable

2.9.2 Summary of the Baseline

0. Descriptor	Latitude - Horrebow-Talcott	Latitude - Sterneck	Longitude - Meridian transits
1. Catalogue	NGS/SAO	FK4	FK4
1a. Right ascension	Zero	Zero	Zero - except for periodic errors
1b. Declination	One to one	One to one	Zero
2. Instrument	T-4	T-4	T-4
2a. Collimation	Zero by procedure	Zero by procedure	Zero by procedure
2b. Circles	Zero	One to one	Zero
2c. Optics	One to one	Zero	Zero by procedure
2d. Mechanics	Nearly zero	One to one	Troublesome
3. Operation	Alignment of T-4, pointing at star perpendicular to to motion	Alignment of T-4, pointing at star perpendicular to motion	Alignment of T-4, tracking of motion of star
3a. Azimuth	Zero	Zero	a (sin z sec δ)
3b. Level	One to one	One to one	b (cos z sec δ)
3c. Timing	Zero	Zero	One to one

2.9.2 Summary of the Baseline (Continued)

0. Descriptor	Latitude - Horrebow-Talcott	Latitude - Sterneck	Longitude - Meridian transits
4. Weather	Clouds < 5/10	Clouds < 5/10	Clouds < 3/10 for one hour
5. Field time	2 hours, 2 nights	2 hours, one night	Two hours minimum on 3 nights, or three hours on 2 nights
6. Solution	Differences of zenith distances yield latitude	Zenith distances yield the latitude directly	Mayer's equation with unit weights, or Gregerson's solution with near unit weights for well selected stars
6a. Unknowns	Latitude, micrometer rate	Latitude (circle zenith)	Latitude instrumental azimuth
6b. Latitude	Insensitive except for observing list	Insensitive except for observing list	Insensitive, ±0.5 deg
6c. Longitude	Insensitive except for observing list	Insensitive except for observing list	Insensitive except for observing list
7. Quality	$\sigma_\phi^2 = 0.518/N + 0.068$, for N = 16 star pairs: $\sigma_\phi = 0.32$ sec	$\sigma_\phi^2 = 0.656/N + 0.068$, N = 32 star pairs: $\sigma_\phi = 0.30$ sec	$\sigma_\lambda^2 = 0.040/N + 0.137/K$, for N = 6 star sets and K = 1 $\sigma_\lambda = 0.38$ sec

2.9.3 Summary of Remedial Equipment

0. Descriptor	Automatic Eyepiece	Mercury level	Axis mirror	Refractometer
1. Catalogue	N/A	N/A	N/A	N/A
2. Instrument	T-4	T-4	T-4	Refractometer
2a. Collimation	Expected to be zero	Zero, by procedure	Zero, by procedure	N/A
2b. Circles	N/A	N/A	N/A	N/A
2c. Optics	Zero, by procedure	Zero	Zero	Possibly serious
2d. Mechanics	N/A	No change from level vial with bubble	Expected to nullify common defects	N/A
3. Operation	Significant detailed operation of electronic equipment	Static positioning of cross hairs	Reading position of cross hairs	Operation of electronic equipment
3a. Azimuth	N/A	N/A	Expected to remove azimuth misalignment	N/A
3b. Level	N/A	Expected to remove level misalignment	Expected to remove azimuth misalignment	N/A
3c. Timing	Expected to remove timing errors	N/A	N/A	N/A

2.9.3 Summary of Remedial Equipment (Continued)

0. Descriptor	Automatic Eyepiece	Mercury level	Axis mirror	Refractometer
4. Weather	No change from human observer	N/A	N/A	N/A
5. Field time	Potential slight decrease	No change from common procedures	No change from common procedures	N/A
6. Solution	N/A	N/A	N/A	N/A
7. Quality	σ_λ = 0.37 sec σ_ϕ = 0.27 sec using Sterneck method	σ_λ = 0.37 sec σ_ϕ = 0.30 sec using Sterneck method	σ_λ = 0.37 sec σ_ϕ = 0.30 sec using Sterneck method	Imponderable since total refraction cannot be measured now

2.9.4 Summary of Alternative Processes

0. Descriptor	Prime Vertical	Elongation	Circumzenithal	Stellar Rate
1. Catalogue	FK4	FK4	FK4	FK4
1a. Right ascension	Zero, if t minimized	Zero	Formulas are complex; correlation properties unfavorable	Depends on specific program of observation
1b. Declination	$\sigma_\phi = \sigma_\delta \sin 2\phi / \sin 2\delta$	$\sigma_\phi = \sigma_\delta \tan \delta / \tan \phi$		
2. Instrument	T-4	T-4	Astrolabe	Unknown
2a. Collimation	Zero, by procedure	Zero, by procedure	Zero, by procedure	Zero
2b. Circles	Zero	Minimized, by procedure	Zero	Unknown
2c. Optics	Zero	Zero	Minimized, by construction	Unknown
2d. Mechanics	Minimal	Orthogonality of axes a necessity	Minimal	Unknown
3. Operation	Orient instrument, time passage(s)	Orient instrument, determined directions	Time passage(s)	Probably, time passages
3a. Azimuth	Minimal, if z small	Approximately one to one	Zero	Unknown
3b. Level	One to one	One to one	Zero, by procedure	Probably zero
3c. Timing	$\sigma_\phi = \sigma_T \tan t$	Zero	Complex formula depends on azimuth	Critical

2.9.4 Summary of Alternative Processes (Continued)

0. Descriptor	Prime Vertical	Elongation	Circumzenithal	Stellar Rate
4. Weather	Each star observed at two passages, up to several hours apart	Each star observed at both elongations, 12 hours apart	Cloud cover < 2/10	Unknown
5. Field time	Six hours	Many hours	Two hours	Unknown
6. Solution	Latitude from interval measurements	Latitude from direction measurements	Position from timing passages of almucantar	Interval measurements
6a. Unknowns	Latitude, orientation of instrument	Latitude, azimuth	Latitude, longitude zenith distance	Latitude, and probably instrument constants
6b. Latitude	Minimal	Minimal	Minimal	Unknown
6c. Longitude	Minimal	Minimal	Minimal	Unknown
7. Quality	$\sigma_{\phi} > 0.3 \widehat{sec}$	$\sigma_{\phi} > 0.6 \widehat{sec}$ per star	$\sigma_{\phi,\lambda} < 0.2 \widehat{sec}$ if automated	Unknown

2.9.5 Summary of Alternative Equipment

0. Descriptor	PZT	AAPS
1. Catalogue	NGS	FK4
1a. Right ascension	Zero	Zero
1b. Declination	One to one, in latitude	One to one, in latitude
2. Instrument	Photographic zenith tube	Automatic astronomic positoning system
2a. Collimation	Zero, by procedure	Unknown
2b. Circles	N/A	N/A
2c. Optics	One to one	One to One
2d. Mechanics	Inappreciable, by design	Significant
3. Operation	Level, start, and stop	Set up and start
3a. Azimuth	Zero	Important
3b. Level	One to one	Zero
3c. Timing	Automatic	Automatic
4. Weather	Zenith clear for 5 sec	Clouds < 9/10 for several hours
5. Field time	Ten minutes	All night
6. Solution	Determine right ascension and declination of zenith	Essentially stellar rate
6a. Unknowns	Latitude, longitude, camera lens parameters, plate parameters	Latitude, longitude, and possibly others
6b. Latitude 6c. Longitude	Good assumed position speeds solution by minimizing iterations of nonlinear model	Good assumed position speeds solution by minimizing iterations of nonlinear model
7. Quality	$\sigma_{\phi,\lambda} \approx 0.1\ \widehat{sec}$	$\sigma_{\phi,\lambda} \approx 0.3$ to $0.4\ \widehat{sec}$

2.9.6 Recommendations

General - The development of an astrolabe incorporating a mercury pool, a charge-coupled device, and a two-color refractometer should be pursued vigorously. This product should reach fruition in less than five years and offers the greatest opportunity for accurately and economically (i.e. low variable costs) determining astrogeodetic positions.

The portable photographic zenith tube fitted with a charge-coupled device should be given serious consideration as an astrogeodetic instrument. This is a potentially viable technological alternative to the astrolabe.

For the present, until one of these devices proves dependable, the baseline techniques described in Section 2.3 should be continued. Recommendations for certain changes are given below in order of decreasing emphasis.

> Latitude - Multiple observations should be made on every star observed. This recommendation applies both to the Horrebow-Talcott method and the Sterneck method.
>
> Development of the technology for leveling with mercury should be encouraged.
>
> Longitude - Each observer should always use the same instrument.
>
> Longitude should be determined from the mean of:
>
> - Two observers, each with a unique T-4
> - Two nights for each observer
> - Two sets minimum per observer per night.
>
> The occupations should be separated by several weeks.
>
> Development of the technology for use of an axis mirror should be encouraged, both to replace the hanging level and as a superior monitor of instrumental azimuth.

3. THE GRAVITATIONAL METHOD

The subject of this chapter is the computation of the deflection of the vertical at an arbitrary point by the use of gravity data alone. In theory, a knowledge of gravity everywhere on the earth is required; in practice, the available data include point gravity anomalies near the desired deflection station and mean gravity anomalies, at various grid spacings, covering the rest of the surface. Mixed-data methods, involving gravity measurements combined with other kinds of data, will be treated in a later phase of this study effort and are not included in this chapter. In particular, the method of astro-gravimetric leveling -- a mixed-data technique combining gravity anomaly data with direct astrogeodetic deflections -- will receive detailed consideration because preliminary evaluation suggests that this method, also known as gravimetric interpolation, may offer important practical and logistical advantages under certain field conditions.

A theoretical overview of the basis for using gravity data to compute deflections of the vertical is found in Section 3.1. The treatment is brief, since detailed developments are readily available in standard reference sources. Section 3.2 describes the practice of gravimetric determination of deflection as currently implemented under optimal conditions that define the present state of the art; these considerations include the type, density, and quality of the gravity data employed, as well as the selection and implementation of computational procedures. In Section 3.3 the attainable level of accuracy for the gravimetric method is discussed. The conclusions in Section 3.3 are based on relevant citations from the current scientific literature, as verified by simulation studies carried out as part of this study effort. The results and findings of this examination of the gravimetric method are summarized in Section 3.4, along with a brief review of key logistical factors that will influence decisions about the part (if any) to be played by the gravimetric method in future deflection surveys.

In anticipation of the detailed findings presented in the sections enumerated above, the following conclusions are stated:

- As a general approach, the gravimetric method can not attain the accuracy levels which can be obtained by using in-place astrogeodetic surveying.
- Under ideal conditions, in certain geographic areas, the possibility exists of computing deflections of the vertical from gravity data at accuracy levels comparable to those obtained from astrogeodetic surveying; logistical considerations may then favor the use of the gravimetric method as a local replacement for, or adjunct to, dense astrogeodetic surveys.

In the discussions of this chapter, it is assumed that existing gravity data bases will suffice to support gravimetric computations in the continental United States, subject to a possible requirement for densification in the immediate vicinity of a computed deflection station. Gravimetric surveying on land is a mature technology capable of accuracy levels generally considered to be more than adequate for the near-field requirements of the gravimetric method (Refs. 33 and 41). As a consequence, neither the subject of gravimeter hardware and survey methodology (described in Refs. 51 and 52), nor the analysis and modeling of gravimeter instrument error sources, receives significant attention in the remainder of this chapter.

The computational techniques reviewed in this chapter are deterministic in nature and are to be distinguished from modern statistically oriented approaches such as minimum variance estimation (Ref. 53) and least squares collocation (Ref. 46), which make use of empirical or theoretical covariance properties of gravity field quantities. In the context of the pure gravimetric method as considered in this chapter, the theoretical and computational advantages of the statistical approaches are not realized in practice. For example, a review in Ref. 33 of collocation methods as an alternative to the computational procedure described in Section 3.2 suggests a potential accuracy limitation that is worse, by nearly an order of magnitude, than the accuracy levels discussed in Section 3.3. However, minimum variance estimation methods do offer an optimal basis for combining data provided by multisensor gravity field survey methods such as those to be considered in the second phase of this study program. More attention will be focused on multisensor techniques in the coming study phase.

3.1 THEORETICAL BASIS FOR GRAVIMETRIC DETERMINATION OF THE DEFLECTION OF THE VERTICAL

The theoretical basis for determining the deflection of the vertical at a given point from global gravity data is closely related to the classical boundary value problem of physical geodesy -- to compute the geometrical form of the physical surface of the earth, using as data the values of the gravity vector and the geopotential. In its pristine form, the boundary value problem of physical geodesy is intractable for a number of reasons:

- The physical surface of the earth cannot be represented mathematically
- The only gravity field quantity that can conveniently be measured to a high degree of precision on or near the physical surface is scalar gravity
- Gravity is measured only at a limited number of discrete points, the distribution of which, for practical reasons, is highly irregular.

The modifications required to make the mathematical problem solvable in a practical sense, using available data, take the form of linearizations around assumed standard solutions. Two approaches are outlined in the following; one is the classical solution of Stokes and Vening-Meinesz (Section 3.1.1); the second is the modern solution of Molodensky, as modified by Pellinen.

3.1.1 The Classical Solution

The assumptions underlying the classical solution, in the sense of Stokes and Vening-Meinesz, to the problem of computing the geoid (and, hence, its slope - the deflection of the vertical) from worldwide gravity data, as well as the mathematical derivations of the formulas involved, are treated thoroughly in the standard reference sources (Ref. 42, for example). It is important to note that four surfaces are involved in the classical solution. These are:

- The physical surface of the earth
- The geoid
- The reference ellipsoid
- A unit sphere associated with the reference ellipsoid.

The input quantities (measured gravity) are available on, or close to, the physical surface of the earth. Since the variables involved in the Stokes solution are, however, linearized small quantities corresponding to the difference in shape and gravity field between the ellipsoid and the geoid, it is necessary to reduce measured gravity to the geoid (free-air reduction) for comparison with normal gravity on the ellipsoid. Then the integral formula of Stokes

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \; S(\psi) \; d\sigma \qquad (3.1\text{-}1)$$

where

- R is an assumed mean radius for the earth (usually taken as the radius of a sphere having the same volume as the ellipsoid)
- γ is, likewise, an assumed mean value for gravity
- σ represents the surface of a sphere approximating the ellipsoid
- Δg is the free-air anomaly
- ψ is the angular separation, on the surface of the sphere, between the point for which the solution is being computed and the element of integration, $d\sigma$
- $S(\psi)$ is the Stokes function

relates the separation between geoid and ellipsoid at a point (the undulation N) to the gravity anomaly on the entire geoid. Similarly, the Vening-Meinesz formulas

$$\xi = \frac{1}{4\pi\gamma} \iint_{\sigma} \Delta g \left(\frac{dS}{d\psi}\right) \cos\alpha \; d\sigma \qquad (3.1\text{-}2)$$

$$\eta = \frac{1}{4\pi\gamma} \iint_{\sigma} \Delta g \left(\frac{dS}{d\psi}\right) \sin\alpha \; d\sigma \qquad (3.1\text{-}3)$$

where

$\frac{dS}{d\psi}$, the derivative of the Stokes function, is the Vening-Meinesz function

α is the azimuth of the great-circle arc (of length ψ) joining the solution point and the element of integration, $d\sigma$

relate the north-south (ξ) and east-west (η) components of the deflection of the vertical at a point on the geoid to worldwide gravity anomalies.

The output quantities that are actually desired, in the gravimetric method for computing deflections, are ξ and η at the physical surface of the earth, rather than the geoid. Thus a final reduction for curvature of the plumb line is required, in principle, to relate the results of the Vening-Meinesz computation to the surface of the earth. This reduction is particularly significant in mountainous terrain (Ref. 35), but is usually, as a practical matter, overshadowed by the various sources of error in Eqs. 3.1-2 and 3.1-3 and the approximations that are inevitable in their numerical evaluation.

Major error sources in the Vening-Meinesz solution are:

- The reduction of observed gravity to the geoid; in particular, the proper evaluation of topographic effects
- The use of a sphere as the integration surface.

Terrain and topographic corrections on the one hand, and ellipsoidal corrrections on the other, have been worked out to improve the accuracy of the Vening-Meinesz solution

(Refs. 42, 43, 44, and 45), but the effort appears to be better spent in the development and application of the modern (or contemporary) solution summarized in Section 3.1.2. The limitations imposed by the numerical evaluation of the integral formulas using incomplete worldwide gravity data remain essentially the same in the modern solution.

3.1.2 The Modern Solution

The modern solution to the problem of determining the shape of the earth (and hence the deflection of the vertical) from gravity measured on the surface departs from the classical approach in that the use of the geoid is dispensed with; the desired output quantities (such as height anomaly or deflection at the surface) as well as the input quantities (measured gravity) are referred to the physical surface of the earth. The modern solution was introduced by Molodensky in 1945 and has undergone considerable development and improvement since that time. An up-to-date reference source for the modern solution is Part D of Ref. 44, while earlier presentations are found in Ref. 43 and Chapter 8 of Ref. 42.

The Original Molodesky Solution

Molodensky's original formula for the determination of the deflection of the vertical from worldwide gravity data takes the form (Ref. 42):

$$\xi = \frac{1}{4\pi\gamma} \iint_{\sigma} (\Delta g + G_1) \left(\frac{dS}{d\psi}\right) \cos\alpha \, d\sigma - \frac{\Delta g}{\gamma} \tan\beta_1 \quad (3.1\text{-}4)$$

$$\eta = \frac{1}{4\pi\gamma} \iint_{\sigma} (\Delta g + G_1) \left(\frac{dS}{d\psi}\right) \sin\alpha \, d\sigma - \frac{\Delta g}{\gamma} \tan\beta_2 \quad (3.1\text{-}5)$$

where the symbols have the same meaning as in the corresponding Eqs. 3.1-2 and 3.1-3, except as indicated in the discussion to follow.

The free-air anomaly Δg appearing in Eqs. 3.1-4 and 3.1-5 differs from the classical free-air anomaly (as used in Eqs. 3.1-2 and 3.1-3) in that it is an anomaly referred to ground level rather than to the geoid:

$$\Delta g = g_p - \gamma_Q \tag{3.1-6}$$

where

- g_p is actual gravity measured at a point P on the surface of the earth
- γ_Q is normal gravity at a point Q, on the reference surface known as the telluroid, corresponding to P

Furthermore, the anomaly referred to ground level is modified by G_1, which is a terrain (and local mass anomaly) correction term to account for the departure of the earth's physical surface from a surface of equal potential. This term is calculated by approximating an integral of the form:

$$G_1 = \frac{R^2}{2\pi} \iint_\sigma \frac{h-h_p}{\ell_o^3} \Delta g \, d\sigma \tag{3.1-7}$$

which involves the terrain relief $(h-h_p)$ between the point for which the correction is being determined, P, and other points in its vicinity; as well as the gravity anomaly Δg. In Eq. 3.1-7 the chordal distance between the point P and the element of integration is ℓ_o; the other symbols have been defined above. In theory the integral in Eq. 3.1-7 is extended over the entire sphere. Practical calculations, however, generally do not extend beyond 50 km from the computation point.

The second term in Eqs. 3.1-4 and 3.1-5 represents the effect of the slope, or tilt, of the terrain in the vicinity of the computation point, with β_1 representing the inclination of a north-south terrain profile with respect to horizontal, and β_2 representing the corresponding east-west inclination.

The Pellinen Version

A subsequent modification of the Molodensky solution by Pellinen (Ref. 37) offers computational advantages as well as a simpler formulation. This is the formulation which is the basis for the present state of the art, as reviewed in Section 3.2, with respect to gravimetric determination of deflections.

Pellinen's formulas

$$\xi = \frac{1}{4\pi\gamma} \iint_{\sigma} (\Delta g)' \left(\frac{dS}{d\psi}\right) \cos\alpha \, d\sigma \quad (3.1\text{-}8)$$

$$\eta = \frac{1}{4\pi\gamma} \iint_{\sigma} (\Delta g)' \left(\frac{dS}{d\psi}\right) \sin\alpha \, d\sigma \quad (3.1\text{-}9)$$

have the same form as the formulas of Vening-Meinesz (Eqs. 3.1-2 and 3.1-3), except for the use of the modified gravity anomaly, $(\Delta g)'$, in place of the classical gravity anomaly. This modified anomaly incorporates all effects of the topography and is computed as follows:

$$(\Delta g)' = \Delta g + C \quad (3.1\text{-}10)$$

where

Δg is the free-air anomaly referred to ground level (Eq. 3.1-6)

C is the topographic correction term.

The correction term C is computed in theory by evaluating the integral

$$C = \frac{R^2}{2\pi} \iint_{\sigma} \frac{(h-h_p)(\Delta g-\Delta g_p)}{\ell^3} d\sigma \qquad (3.1\text{-}11)$$

This resembles the Molodensky terrain correction term, G_1, as defined in Eq. 3.1-7, except for the use of a <u>relative gravity anomaly</u>, $\Delta g-\Delta g_P$, referred to the anomaly at the computation point, P. In practice (as was pointed out in the discussion of Eq. 3.1-7), the integral over the entire sphere appearing in Eq. 3.1-11 is replaced by a summation involving points in the immediate vicinity of P.

Accuracy of the Theoretical Solutions

The Stokes formula (Eq. 3.1-1) involves approximations that limit its accuracy to a level given traditionally as 0.3 percent, assuming perfect knowledge of the worldwide free-air anomaly. Thus errors in the computed undulation could be expected to reach a level of 2 to 3 meters. The relative accuracy of the Vening-Meinesz formulas (Eqs. 3.1-2 and 3.1-3) is lower, by almost one order of magnitude, but should still be sufficient, in principle, to predict deflections of the vertical from perfect worldwide gravity data, at an accuracy level of 0.2 to 0.3 sec (Ref. 44).

How much difference, in theory, does the use of the Molodensky or Pellinen solution, rather than the classical Vening-Meinesz formula, make in the accuracy of the computed deflections? The answer depends strongly on the nature of the topography in the vicinity of the computation point, but can easily amount to 0.3 $\widehat{sec}$ in mountainous terrain (Ref. 44). Thus the modern solutions are emphatically preferable. Moreover, since the modern solutions involve linear approximations, they can be improved by the computation of correction terms of second and higher order. Currently, this is unnecessary because accuracy is governed by the limitations imposed by imperfect knowledge of Δg and of the topography.

Alternative theoretical approaches are reviewed and evaluated in Ref. 45, but it is noteworthy that the solution incorporated in Eqs. 3.1-8 through 3.1-11 has been adopted uniformly for recent state-of-the-art gravimetric determinations of deflection of the vertical (Refs. 33, 39, and 41).

3.2 COMPUTATIONAL IMPLEMENTATION OF THE GRAVIMETRIC METHOD FOR DETERMINING THE DEFLECTION OF THE VERTICAL

The implementation described in this section as representative of the present state of the art in the computation of deflections of the vertical by the gravimetric method is based on the Pellinen modification (Eqs. 3.1-8 through 3.1-11) of the modern (Molodensky) solution. It is being used in Australia for deflection determination and is documented in Refs. 33 and 41.

Essentially there are two sets of integrals that must be evaluated numerically. At each gravity station a topographic correction term, C, is computed by approximating the integral of Eq. 3.1-11, using gravity and terrain relief data in the vicinity of the gravity station. This is applied as an adjustment to the ground level free-air anomaly. With the gravity data thus adjusted, the integrals in Eqs. 3.1-8 and 3.1-9 are then evaluated by a second numerical procedure, in which the integral over the entire sphere is partitioned into five zones at increasing radial distances from the computation point. The representation and treatment of the gravity data, as well as the details of approximating the integral by a finite sum, vary from zone to zone.

3.2.1 The Concept of Rice Rings

The evaluation of both types of integrals enumerated above involves the use of a circular template system of the type introduced by Rice (Ref. 36). In its original form, this system consisted of 57 concentric rings surrounding the computation point, the innermost ring beginning at a distance of 100 m from the center, and the outermost ring ending at a distance of 1094.3 km from the center (see Table I of Ref. 36). Note that because of the singularity of the Vening-Meinesz function when ψ, the angular separation, approaches zero, a special treatment was used to incorporate the effect of gravity stations within 100 m (see the discussion of the innermost zone below); and, in his original treatment, Rice did not account for the effects of anomalies beyond 1094.3 km, accepting the contribution of the distant zone as an error source. The system of rings is further divided into zones by a set of radial lines projecting from the center at an angular separation of 10 deg. There are, then, a total of 2052 zones (excluding the innermost zone) in the Rice template system.

The key point in this system is the choice of the radii of the circles bounding the various rings. These radii are chosen (based on the variation of the Vening-Meinesz function with distance from the center) so that each zone contributes the same amount of deflection at the center for a given mean value of gravity anomaly over the zone. In particular, each Rice zone contributes 0.001 $\widehat{\sec}$ of radial deflection if the mean gravity anomaly for the zone is 1.0 mgal.

With the use of this template system, the numerical evaluation of an integral (Eq. 3.1-2, etc.) reduces to two simple steps:

- The determination of a mean (or other representative) value of the gravity anomaly within each zone
- The summation of the contributions from all of the zones.

3.2.2 Computation of the Topographic Correction

For the evaluation of the topographic correction term (Eq. 3.1-11), a template system is devised in Ref. 33 that consists of 90 zones, having the characteristics summarized in Table 3.2-1 (Ref. 33).

Evaluation of the topographic correction term, using this template system, is a two-stage process:

- Determine suitable mean values, within each zone, of
  - gravity anomaly difference
  - terrain height difference
- Compute a weighted sum of the individual zone contributions.

Reference 33 recommends a uniform application of the topographic correction computation to <u>each gravity station</u> in the point gravity anomaly data base. In Ref. 41, on the other hand, the correction is applied much more selectively (with due regard to computational resources) at points where, based on the nature of the surrounding terrain and the amount of short-wavelength

TABLE 3.2-1

TEMPLATE SYSTEM FOR TOPOGRAPHIC CORRECTION

RING NUMBER (Note 1)	CONTRIBUTION TO TOPOGRAPHIC CORRECTION (mgal) (Note 2)	INNER RADIUS (km)	OUTER RADIUS (km)	NUMBER OF ZONES
2	7.5×10^{-5}	0.100	0.137	6
3	7.5×10^{-5}	0.137	0.217	6
4	7.5×10^{-5}	0.217	0.526	6
5	1.5×10^{-5}	0.526	0.734	9
6	1.5×10^{-5}	0.734	1.218	9
7	1.5×10^{-5}	1.218	3.556	9
8	4.0×10^{-6}	3.556	7.287	9
9	1.0×10^{-6}	7.287	9.878	12
10	1.0×10^{-6}	9.878	15.330	12
11	1.0×10^{-6}	15.330	34.210	12

Note 1: The effects of Ring 1 (points within 100 m), if significant, require special treatment because of the divergence of the integral

Note 2: This is the contribution to the correction term from a zone for which the mean height difference is 100 m and the mean gravity anomaly difference is 1 mgal.

energy in the gravity anomaly field, the topographic correction is expected, a priori, to be significant.

It may be noted that other approaches to the implementation of topographic corrections are currently subjects of research (Refs. 34 and 44).

3.2.3 Evaluation of the Pellinen Integral Formula

The first step in the evaluation of the integral formulas of Eqs. 3.1-8 and 3.1-9 is the partitioning of the domain of integration into five zones:

- <u>Innermost</u> - within 100 m of the computation point
- <u>Inner</u> - a 3-deg square centered on the one-deg square containing the computation point
- <u>Near</u> - the 15-deg square centered on the 5-deg square containing the computation point
- <u>Middle</u> - the 45-deg square centered on (but excluding) the near zone
- <u>Outer</u> - the rest of the earth's surface, beyond the middle zone.

Within each of the zones, the contribution to the integral is evaluated as outlined below.

<u>Innermost Zone</u> - A special treatment is required for the innermost zone, which must actually be excluded from the domain of integration in Eqs. 3.1-8 and 3.1-9 to avoid divergence of the integral, since the Vening-Meinesz function behaves asymptotically as

$$\frac{dS}{d\psi} \sim - \frac{2}{\psi^2} \tag{3.2-1}$$

as ψ approaches zero. Within this innermost zone there must be sufficient gravity data to permit an estimate of the north-south and east-west components of the horizontal gradient of the gravity anomaly. The traditional approach (Ref. 36) is to use a square mesh of 8 points arranged symmetrically within the innermost zone for the gradient estimation. Quite clearly, the use of gradiometer data, if available, would be particularly appropriate in this application. The innermost zone contributions are then:

$$\xi_{IM} = - \frac{1}{2\gamma} (\text{grad } \Delta g)_{NS} \; r_{IM} \tag{3.2-2}$$

$$\eta_{IM} = - \frac{1}{2\gamma} (\text{grad } \Delta g)_{EW} \, r_{IM} \qquad (3.2\text{-}3)$$

where

γ	is normal gravity at the computation point
r_{IM}	is the radius of the innermost zone (in this application, r_{IM} is 100 m)
$(\text{grad } \Delta g)_{NS}$	is the north-south component of the horizontal gradient of the gravity anomaly
$(\text{grad } \Delta g)_{EW}$	is the east-west component of the horizontal gradient of the gravity anomaly

Inner Zone - Within the inner zone, a circular template system based on the concept of Rice rings (Section 3.2.1) is employed to organize the computations. The smallest of the 21 rings begins at a distance of 100 m from the computation point, while the outermost ring ends at a distance of 128.6 km. Since the rings are subdivided uniformly by radial lines separated by 10 deg, the total number of compartments is 756. The radii defining the rings are chosen so that any compartment will contribute 0.002 $\widehat{\text{sec}}$ of radial deflection if the modified gravity anomaly (Eq. 3.1-10) within that compartment has a uniform value of 1 mgal.

The computational procedure for the inner zone consists of these stages:

- Sorting the point gravity anomalies into the appropriate compartments
- Using interpolation procedures to compute a best estimate of the gravity anomaly at the center of each compartment. Note that this will not, in general, be equal to the mean gravity anomaly within the compartment
- Summing the contributions from all of the compartments.

Transition Area - Beyond the inner zone, the gravity data are used in the form of mean gravity anomalies defined for square compartments. The contribution from the area of overlap between the inner-zone ring template and the block pattern of the near zone is evaluated by an elaborate special procedure (Refs. 33 and 41) to insure that no area is omitted and no contribution is duplicated.

Near Zone - Within the near zone, the gravity field is represented by 0.5-deg mean gravity anomalies; the integrals (Eqs. 3.1-8 and 3.1-9) reduce to simple sums over the individual blocks, with the kernel function

$$K = \left(\frac{dS}{d\psi}\right) \cos \alpha \qquad (3.2\text{-}4)$$

or

$$K = \left(\frac{dS}{d\psi}\right) \sin \alpha \qquad (3.2\text{-}5)$$

evaluated at the center of the block.

Middle and Outer Zones - Mean gravity anomalies defined for one-deg squares are used for the middle zone; five-deg mean gravity anomalies are used in the outer zone. The evaluation of the contribution of each individual block is essentially as described for the near zone.

Summary - The salient features of the evaluation procedure described above, which is presented in detail in Refs. 33 and 41, are summarized in Table 3.2-2.

TABLE 3.2-2

OVERVIEW OF THE EVALUATION OF THE PELLINEN INTEGRAL FORMULA

ZONE	DATA USED	COMPUTATIONAL METHOD
Innermost	Point gravity anomalies (note possibility of using gradiometer data)	Eqs. 3.2-2 and 3.2-3
Inner	Point gravity anomalies, terrain height data	Circular template system (Rice rings)
Near	0.5-deg mean gravity anomalies	Summation over individual blocks
Middle	1-deg mean gravity anomalies	Summation over individual blocks
Outer	5-deg mean gravity anomalies	Summation over individual blocks

3.3 ACCURACY OF THE GRAVIMETRIC METHOD

It is helpful to note that there have been a number of recent investigations, both theoretical and experimental, of the accuracy of the gravimetric method, the results of which are directly applicable to the conditions of the present study. These include Refs. 33, 38, 39, and 41.

References 33 and 41 report extensive experimental results carried out in various areas of the Australian continent, in which direct comparisons were made between deflections of the vertical as measured by baseline astrogeodetic techniques (Chapter 2 of this report) and as computed by the gravimetric techniques discussed in Section 3.2. Existing gravity and terrain height data bases were densified, in some cases, to meet the data requirements for innermost and inner zone computations. Several different sets of mean gravity anomalies were used for middle and outer zone computations, to determine whether there were systematic differences in the computed deflections of the vertical resulting from these different representations of the worldwide field.

The most significant systematic errors in the gravimetrically determined deflections of the vertical, as reported in Refs. 33 and 41, are attributed to inadequate knowledge of gravity in ocean areas adjacent to the Australian continent. These effects have been studied further in Ref. 38, where a detailed analysis is made of the influence on computed deflections of the poorly surveyed (or unsurveyed) ocean to the east, south, and west of Australia. Modeling of these systematic effects leads to a set of adjustments, called pseudocorrections in Ref. 38, that are applied to the oceanic mean gravity anomalies. In essence this process uses the observed astrogeodetic deflections to calibrate the gravity anomaly field by a numerical inversion of the Vening-Meinesz equations.

With systematic errors removed, the agreement between astrogeodetic and gravimetric deflections -- in the best cases -- achieved the accuracy level of 0.1 to 0.2 $\widehat{\text{sec}}$ that is predicted by the theoretical analyses of Ref. 33.

Another recent investigation of the accuracy of the gravimetric method is reported in Ref. 39. This analysis is based on the use of the Tscherning-Rapp global covariance function for free-air anomalies, following the classical methodology described, for example, in Ref. 42. The results that are most important in the present context are these:

- The major source of error in the computed deflections of the vertical is the uncertainty in the mean free-air gravity anomalies worldwide
- The magnitude of this effect is about 0.2 $\widehat{\text{sec}}$.

The relevant conclusions of these investigations can be summarized succinctly. The gravimetric method, in principle, is capable of attaining accuracy levels comparable to astro-geodetic techniques. The limiting factors are the availability of sufficiently dense gravity anomaly and terrain height data in near zones (to permit the evaluation of the topographic correction terms as well as the near-zone contribution to the Pellinen integrals), and the availability of worldwide mean gravity anomaly data of sufficient density and accuracy for distant-zone computations. The near-zone problem can, in principle, be addressed directly; whether this is worth doing becomes a question of logistics and allocation of resources. How serious a limitation is imposed by the problem of distant-zone mean gravity anomaly data depends to a great extent on the location of the point for which the deflection of the vertical is to be computed. In the Australian studies, for example, relatively poorly surveyed ocean areas are not that far away from points at which the deflection is to be computed, and can thus have a considerable effect (of the order of 1 $\widehat{\text{sec}}$ or more), while, in the interior of the United States or in central Europe, this is a less significant problem.

Where all factors are favorable, both theoretical analyses and the experimental results show that gravimetrically computed deflections of the vertical can reach accuracies of 0.2 $\widehat{\text{sec}}$. When data distribution and density limitations are taken into account, an appropriate accuracy goal is 0.2 to 0.3 $\widehat{\text{sec}}$.

Many of the earlier studies of the attainable accuracy of the gravimetric method had, as a major concern, the issue of truncating the far-zone computation. Two reasons apply: computational burden and the scarcity and poor quality of worldwide mean gravity anomaly data as input to far-zone computations.

Such studies (Ref. 40, for example) have become entirely academic in the present context, since for accuracy levels comparable to those expected from state-of-the art astrogeodetic technology (Chapter 2), no distant-zone truncation is acceptable. The entire worldwide field must be represented, either directly in the form of mean gravity anomalies (as in the implementation described in Section 3.2) or indirectly in some other form, such as the use of a spherical harmonic gravity model to provide information about the distant zone.

3.4 CONCLUSIONS

In general, considering the availability and accuracy of gravity anomaly and terrain height data, the gravimetric method cannot, at present, be considered as a substitute for direct measurement of deflections of the vertical by the astrogeodetic method. The limitation is not the theory or its implementation, but the availability of gravity and height data at sufficient density and accuracy levels.

Under favorable circumstances, where appropriately dense data coverage is available -- for example, in nonmountainous terrain in continental interiors -- the gravimetric method may provide a useful alternative to astrogeodetic procedures. It is possible, for example, by making dense gravity measurements in the vicinity of -- but not directly at -- some inaccessible point, to compute the deflection of the vertical at that point without a direct occupation.

In deciding whether to invest time and resources in direct astronomic measurements of the deflection of the vertical, or in the densification of existing gravity (and terrain height) data bases in order to use the gravimetric method, one must be aware of the logistical factors that may become operative. These include the comparative cost factor -- one astrogeodetic station is estimated to cost somewhere between 30 and 40 times as much as a single gravity station (Ref. 33); and the relative time involved -- an order of magnitude more required for an astrogeodetic station than for a gravimetric station. In addition, the skill level, training requirements, and availability of qualified personnel are all far less critical for gravimetric surveying in the field than for astrogeodetic measurements.

Finally, it is noted that these conclusions apply only to the pure gravimetric method. Mixed data approaches involving the combination of astrogeodetic and gravimetric measurements will be treated in the next phase of this study.

4. NUMERICAL STUDIES OF ALTERNATIVE DEFLECTION OF THE VERTICAL SURVEY APPROACHES

4.1 INTRODUCTION

Alternative approaches to deflection of the vertical surveys were studied and simulated. Key performance considerations included accuracy improvement, productivity, cost, and time required for completion. These alternative approaches included:

- Use of wide-area astrogeodetic deflection data to improve accuracy of individual point values
- Use of a modest number of nearby stations to improve accuracy.

The numerical simulation studies explore variants of the techniques reported in Chapter 2. They also provide insight into the performance of different astrogeodetic survey approaches and quantify accuracy sensitivity to various relevant parameters such as:

- The character (frequency content) of the gravity field
- Measurement grid spacing.

In addition to the techniques listed above, a quantitative study was performed which treated measurement of the deflection of the vertical by translocation -- that is, using techniques involving the differencing of simultaneous deflection observations at different sites.

4.2 SIMULATION STUDIES

4.2.1 Astrogeodetic Survey Simulation

The work reported in the following sections describes several mathematical techniques which have been used to model alternative astrogeodetic survey strategies and simulate the propagation of measurement errors. All of these techniques are based on the use of minimum-variance processing of measured quantities and involve algorithms which optimally account for the statistical characteristics of both the gravity disturbances (i.e., the deflection of the vertical "signal") and the measurement error "noise."

Although, in practice, an astrogeodetic measurement is combined with geodetic position data to obtain the deflection of the vertical, for modeling purposes it is convenient to treat the deflections as measured quantities. Accordingly, the measurement errors are lumped together into appropriate measurement noise models. Two models used to characterize instrument errors include a white noise model and a first-order Markov model. A description of these models is provided.

4.2.2 Error Models

Note that white noise corresponds to random, uncorrelated measurement errors. Such errors are completely unpredictable from one observation to the next, regardless of how closely spaced the observation points may be. The correlation function, ϕ, of a white noise process is given by

$$\phi = A\,\delta(x) \tag{4.2-1}$$

where x is the shift distance, A is the strength of the process and $\delta(\ldots)$ is the Dirac impulse function. Typical astrogeodetic error sources modeled well by white noise are atmospherics, long term temperature effects, and certain elements of observer error.

A first-order Markov noise model can account for correlation properties of measurement noise processes and is particularly appropriate when the measurements are close together both in distance and time. This is often the case for errors driven by environmental variables, such as temperature, pressure, humidity, and atmospheric refraction. Selection of a Markov error model's variance, σ^2, and correlation distance*, d, provides a basis for characterizing the correlated error in astrogeodetic measurements.

The expression for the autocorrelation, ϕ, of a first-order Markov process is:

$$\phi = \sigma^2 e^{-x/d} \qquad (4.2\text{-}2)$$

where x is the shift distance.

4.2.3 Signal Models

Four representations for the deflection field were used in the studies. They are:

- "Baseline" gravity disturbance model -- representative of a typical local gravity field
- "Active" gravity disturbance model -- representative of an extreme gravity disturbance field such as might be encountered in mountainous areas
- Attenuated white noise model -- based on worldwide average data and representative of geophysically "mild" areas

*i.e., the 1/e point of the autocorrelation function.

- White Sands gravity disturbance model -- based on gravity data in the area of Holloman Air Force Base.

The equations for the power spectral densities (PSDs) of the gravity disturbance potential for the different models, as well as the model parameters, are summarized in Table 4.2-1. In the equations given in Table 4.2-1, β and σ are model parameters and ν is frequency .

TABLE 4.2-1
GRAVITY MODEL EQUATIONS AND PARAMETERS

MODEL	EQUATION FOR SPECTRAL DENSITY OF ANOMALOUS SURFACE POTENTIAL	MODEL PARAMETERS [β^{-1}(km); D(km); σ(m^2/sec^2)]
Baseline	$\phi(\nu) = \sum_{k=1}^{2} \frac{10\pi\sigma_k^2\beta_k^5}{[\beta_k^2+(2\pi\nu)^2]^{7/2}}$	$\beta_1^{-1} = 27.78$; $\sigma_1 = 16$ $\beta_2^{-1} = 370.4$; $\sigma_2 = 91.43$
Active	Same as above	$\beta_1^{-1} = 22.22$; $\sigma_1 = 29.98$ $\beta_2^{-1} = 350.03$; $\sigma_2 = 103.68$
White Sands	$\phi(\nu) = \frac{10\pi\sigma^2\beta^5}{[\beta^2+(2\pi\nu)^2]^{7/2}}$	$\beta^{-1} = 17.9$ $\sigma = 3.47$
Attenuated White Noise	$\phi(\nu) = \sum_{k=1}^{5} 8\pi D_k^2\sigma_k^2 e^{-4\pi D_k \nu}$	$D_1 = 10$ $\sigma_1 = 0.721$ $D_2 = 76$ $\sigma_2 = 23.03$ $D_3 = 376$ $\sigma_3 = 53.50$ $D_4 = 1055$ $\sigma_4 = 55.36$ $D_5 = 2189$ $\sigma_5 = 278.3$

Graphs of these PSDs are presented in Fig. 4.2-1.

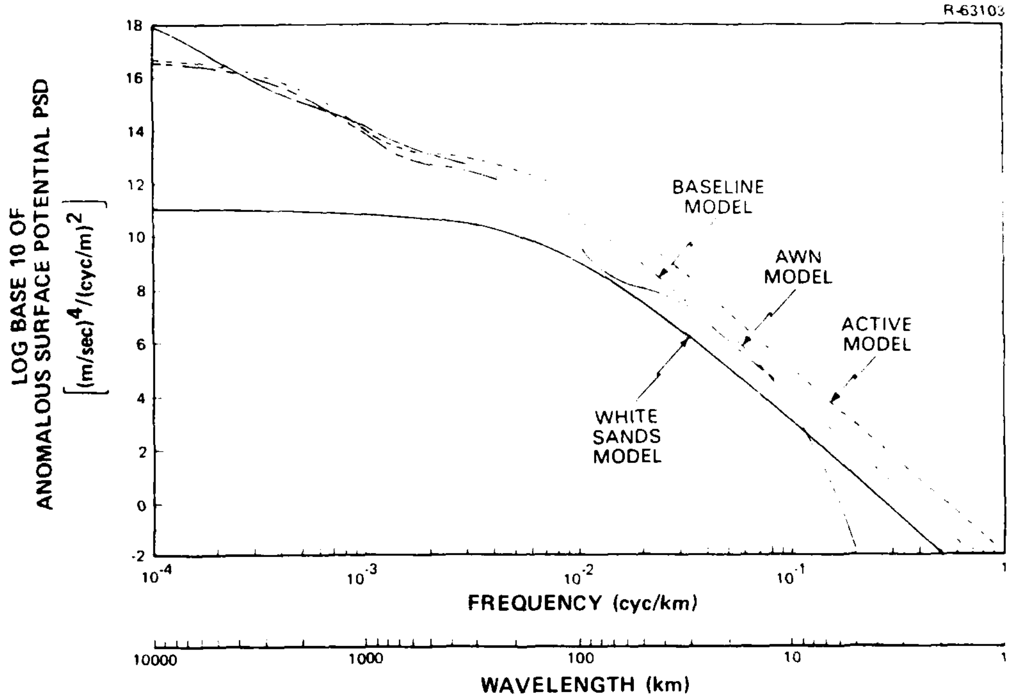


Figure 4.2-1 Spectral Densities of the Anomalous Surface Potential for the AWN, Baseline, White Sands, and Active Models

4.3 SIMULATION RESULTS

Two simulation methodologies were employed, both based on reducing deflection data using optimal minimum variance processing. Both simulation methodologies involve the solution of error covariance equations to estimate rms post-survey reduction errors resulting from a particular survey approach. The survey error covariance equations are of the form

$$\Phi_{\varepsilon,\varepsilon} = \Phi_{x,x} - \Phi_{x,z}(\Phi_{zz}+R)^{-1}\,\Phi_{z,x} \tag{4.3-1}$$

where quantities ϕ_{--} represent covariance or power spectra of quantities defined below

$\phi_{\varepsilon,\varepsilon}$ - mean squared, post-measurement error of the quantity estimated (e.g., deflection of the vertical)

$\phi_{x,x}$ - mean squared a priori uncertainty of the quantity estimated (e.g., the unsurveyed deflection field)

$\phi_{x,z}$ and $\phi_{z,x}$ - correlation between the quantity being estimated (e.g., deflections of the vertical) and the quantity measured (e.g., nearby vertical)

$\phi_{z,z}$ - mean squared value of the measurement quantity (e.g., nearby deflections)

R - mean squared value of the measurement error, (e.g., noise).

The two methodologies involved include

1) Solving Eq. 4.3-1 directly in the space domain. In this instance the arguments of the ϕ functions are shift distance and the rms estimation errors are the diagonal elements of $\phi_{\varepsilon\varepsilon}$. This procedure is known as space domain collocation (Refs. 46, 47) and is usually best suited to the regional reduction of modest numbers of astrogeodetic station measurements.

2) Solving Eq. 4.3-1 in the two-dimensional Fourier frequency domain where the spectrum of the covariance quantity, $\Phi_{x,y}$, is defined by

$$\Phi_{x,y}(\nu_1,\nu_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_{x,y}\, e^{-2\pi j(u\nu_1+v\nu_2)}\, du dv \tag{4.3-2}$$

In Eq. 4.3-2 the quantities u and v are orthogonai shift directions and the corresponding frequency axes are ν_1 and ν_2*. After the computations implied by Eq. 4.3-1 are carried out in the frequency domain (where, for appropriate data distributions, they take on computationally convenient forms -- Refs. 47, 48), the estimation error variance is found by the inverse transform,

$$\sigma_\varepsilon^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{\varepsilon\varepsilon}(\nu_1, \nu_2) \, d\nu_1 \, d\nu_2 \qquad (4.3\text{-}3)$$

In practice Eq. 4.3-3 is represented as a discrete sum (Ref. 49). The frequency domain approach is well suited to use of the gravimetric method where large amounts of data over the whole earth are involved.

4.3.1 Deflection of the Vertical Interpolation

The purpose of this set of simulations was to investigate the post-adjustment accuracy of deflections associated with different measurement densification and estimation strategies. The set of study cases was based on a data field consisting of gridded† deflection stations. Simulation of the optimal adjustment procedure described above was performed to correspond to the following survey approaches.

Utilize existing deflection data to maximum extent possible - The concept is to interpolate existing deflection data to estimate deflection at a site of interest. Limited survey resources could then be applied first to those sites for which existing data indicates the highest post-interpolation deflection uncertainty. To ascertain the efficacy of this

*Note that the gravity disturbance models given in Table 4.2-1 are isotropic, i.e., $\nu^2 = \nu_1^2 + \nu_2^2$.

†Gridding was chosen as a convenient way to express station density. In practice, astrogeodetic data are not gridded. This poses no difficulty for the space domain data reduction algorithm or the error analysis (e.g., Eq. 4.3-1).

approach, simulations were performed which determined the rms estimation error when deflections are optimally interpolated at a point. In this study case, the estimation point can, with equal probability, be situated anywhere within the perimeter defined by points for which the deflections are known.

Figure 4.3-1 presents deflection interpolation accuracy as a function of the density of existing deflection data which have rms uncorrelated errors (per individual point) of 0.3 sec. Two cases are illustrated, one in which both north and east components of the deflection are available, the other where one deflection component is not available or has suffered severe degradation. The data extent (at the same average density) is taken to be sufficient to preclude limited data field effects from significantly affecting deflection estimation errors. Ordinarily this corresponds to several deflection correlation distances (e.g., about 100 km). The baseline gravity model of Table 4.2-1 was used as a statistical representation for the earth's deflection field.

Note that, as expected, increased density (i.e., a closer average spacing between the estimated point and its neighboring measurements) provides increased accuracy. The effect of averaging the survey errors over many measurements is also apparent as a result of the close spacing of the observation stations. For station spacings less than six km, the regional adjustment provides estimation point accuracy which is better than that of the individual measurements.

It is also interesting to note in Fig. 4.3-1 that a relatively small penalty is paid for the omission of one deflection component from the adjustment; the increment of error which results is given by the vertical span between the lines on the graph. In the case where only a single deflection component was used to estimate deflections at an interpolation

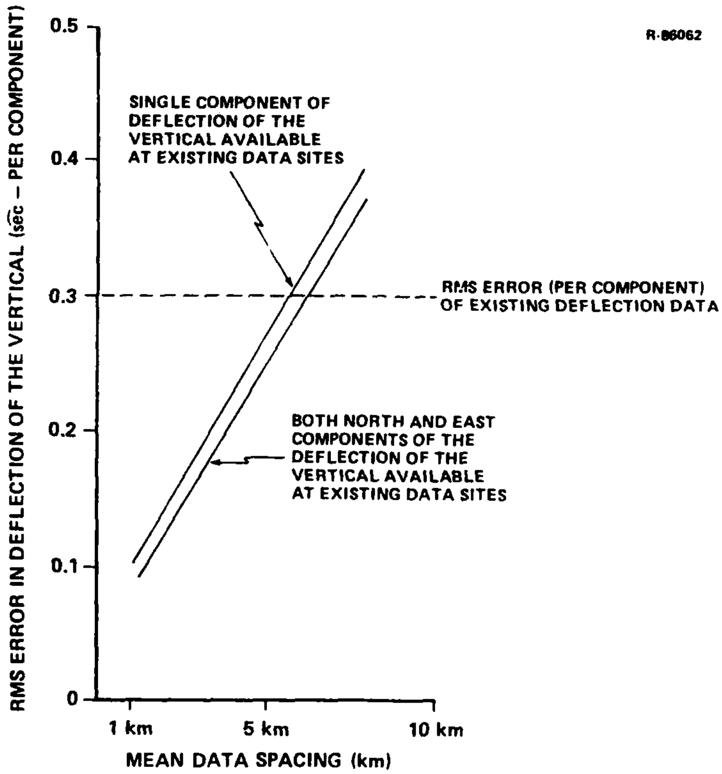


Figure 4.3-1 Post-regional Adjustment Error at Unsurveyed Points After Optimal Interpolation

site, the same component being estimated was presumed to be available at the nearby measurement sites. For example, north deflection estimates presume the availability of north deflection data at nearby sites.

Figure 4.3-1 extends only to spacings as close as 1 km because the error models developed in Chapter 2 indicate the onset of systematic effects* (which limit further error reduction) as regionally-adjusted deflection accuracies near 0.1 sec rms are reached. The resulting conclusion is that densification of high-quality astrogeodetic surveys to station spacings less than one km provides diminishing returns in the accuracy of interpolated points.

*These are addressed in subsequent pages.

Augment Existing Data with New Measurements of Selected Accuracy - This simulation series treated the case where available data accuracy and density would be insufficient to support vertical deflection accuracy requirements at a given site. The concept studied is to occupy the site at which the deflection is to be determined, then optimally combine the new measurement with the existing surrounding data.

For the cases presented in Fig. 4.3-2, the rms measurement errors for both the new measurement and surrounding data were taken as 0.3 $\widehat{\sec}$ per station. Figure 4.3-2, like Fig. 4.3-1, illustrates the benefit of a data base containing highly densified vertical deflection measurements. It also indicates minimal sensitivity to missing deflection components in the data field if their direction is opposite to the component being sought.

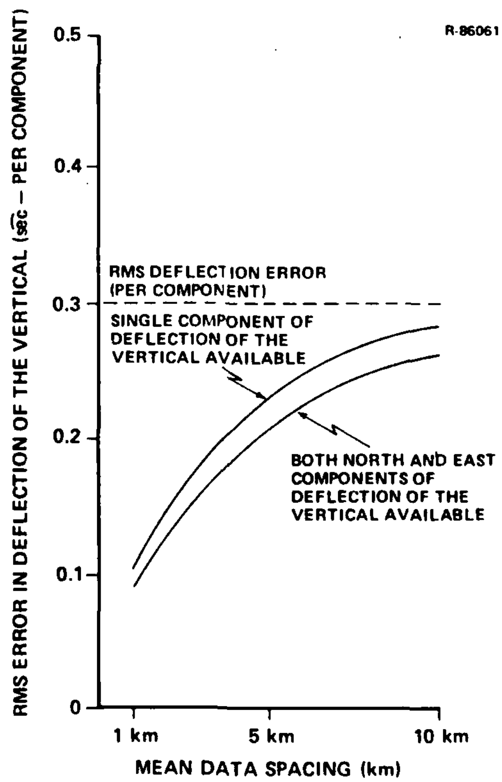


Figure 4.3-2 Post-regional Adjustment Error at Points Where New Measurements Have Been Made

One key difference between Figs. 4.3-2 and 4.3-1 merits mention: for widely spaced data, Fig. 4.3-2 indicates that the accuracy at the regional adjustment about the desired point relaxes to the single-point measurement accuracy. This is not surprising since a single point is the limiting case. In contrast, for the situation described in Fig. 4.3-1, where no measurements are made at the estimation point, the errors due to wide data spacing can reach the uncertainty of the unsurveyed deflection field.

An overlay of Figs. 4.3-2 and 4.3-1 illustrates that for data spacings approaching 1 km, the distinction between the presence or absence of a measurement at any given site bears minimally on post-adjustment accuracy. This is expected since, for both of these survey situations, in the limit of very densely spaced uncorrelated measurements, the error at any point approaches zero. The limit, in which the interpolation adjustment (Fig. 4.3-1) and the adjustment involving a new station (Fig. 4.3-2) are equivalent, is, for practical purposes, reached for data spacings of 1 km. This further strengthens the conclusion that densification of deflection stations beyond one measurement per km^2 will provide diminishing improvements to post-adjusted regional deflection data.

Sensitivity to Measurement Noise Correlation - The survey simulation results presented in Fig. 4.3-3 indicate the limits to accuracy improvement offered by repetitive measurements when the measurement errors are correlated. As described in Chapter 2, residual errors in high-accuracy determinations of deflection of the vertical have both random errors and unknown systematic uncertainties. The systematic effects are generally accepted as the key accuracy-limiting factors in present day deflection measurement techniques. The upper fork of the error curve in Fig. 4.3-3 corresponds to a first-order

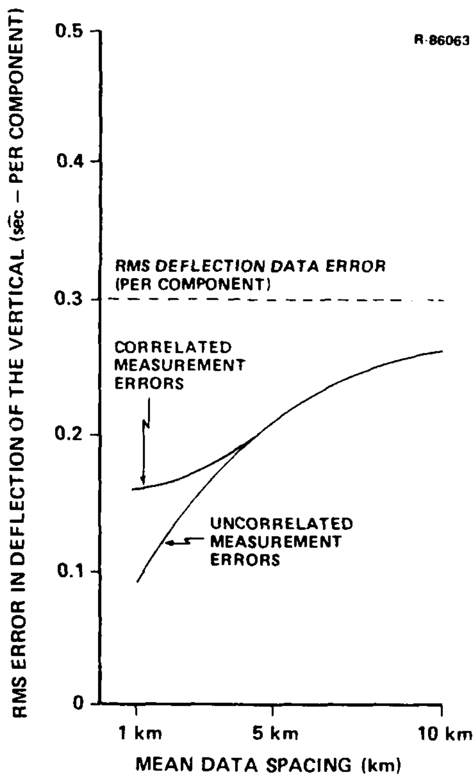


Figure 4.3-3 Effects of Correlated Measurement Noise

Markov measurement error model (Eq. 4.2-2). The rms measurement error is 0.3 sec (as in the previous study cases) and the correlation distance (1/e point of the correlation function) is one km. Figure 4.3-3 clearly illustrates the difficulty associated with overcoming systematic errors vs those which are uncorrelated. These results motivate conclusions supporting the development of technology designed to mitigate systematic errors -- including effects such as residual refraction, star catalogue error, and certain portions of instrument error.

Sensitivity to Different Gravity Field Models - The studies described in the preceding pages used a "typical" gravity representation, that of the Baseline Gravity Model given in Table 4.2-1. Tables 4.3-1a and 4.3-1b present the sensitivity of the results reported in Fig. 4.3-2 to the frequency distribution of the gravity field in the area surveyed. In

TABLE 4.3-1a

SENSITIVITY OF POST-REGIONAL ADJUSTMENT DEFLECTION ACCURACY TO GRAVITY FIELD STATISTICS WHEN BOTH NORTH AND EAST DEFLECTIONS ARE AVAILABLE

STATISTICAL MODEL USED TO CHARACTERIZE DEFLECTIONS OF THE VERTICAL	POST-ADJUSTMENT RESIDUAL ERROR (sec)
Attenuated White Noise	0.18
White Sands	0.18
Baseline	0.20
Active	0.27

TABLE 4.3-1b

SENSITIVITY OF POST-REGIONAL ADJUSTMENT DEFLECTION ACCURACY TO GRAVITY FIELD STATISTICS WHEN ONLY ONE COMPONENT OF VERTICAL DEFLECTION IS AVAILABLE

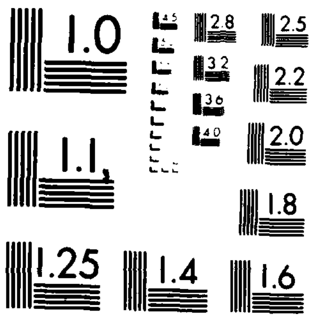
STATISTICAL MODEL USED TO CHARACTERIZE DEFLECTIONS OF THE VERTICAL	POST-ADJUSTMENT RESIDUAL ERROR (sec)
Attenuated White Noise	0.21
White Sands	0.20
Baseline	0.23
Active	0.28

particular, the variation of post-adjustment rms deflection errors for a measurement point spacing of 5 km is indicated. The table entries are ordered according to increasing "roughness" (i.e., more high-frequency content) of the gravity disturbance field. The "Baseline Model" case corresponds to the values given in Fig. 4.3-2 at the 5 km spacing. Other survey

AD-A131 286 ASSESSMENT OF MEANS FOR DETERMINING DEFLECTION OF THE VERTICAL(U) ANALYTIC SCIENCES CORP READING MA W G HELLER ET AL. AUG 82 TASC-TR-4051-1 ETL-0303 3/3

UNCLASSIFIED DAAK70-82-C-0011 F/G 8/5 NL

END
DATE
FILMED
9 83
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

quantities are the same as in the simulation results presented in Fig. 4.3-2.

The tables indicate less than 0.03 $\widehat{\text{sec}}$ of variability in post-adjustment deflection accuracy for areas which are described by the smoothest three gravity models. In areas that are very active gravimetrically, such as high mountains, an increase in residual (over the baseline model) deflection error of 0.07 $\widehat{\text{sec}}$ could be expected. The relative insensitivity of these results to such a wide range of different models provides confidence that robust regional adjustment algorithms can be implemented to achieve the deflection accuracies indicated for Figs. 4.3-1 through 4.3-3.

4.3.2 Translocation of Astrogeodetic Observations

Numerical investigations were carried out to evaluate the prospects for controlling systematic errors in astrogeodetic measurements caused by refraction, temperature, and personal equation. Several variants of the concept of performing simultaneous stellar observations at two sites were studied in the context of reducing the data differentially to remove common-mode error sources.

The findings of these studies indicate only modest improvements in post-adjustment accuracy when a translocation approach is used. In effect, for close station spacings (e.g., two km), there is almost no differential mode signal content. Even though the common-mode measurement errors do not appear, the differential-mode errors remain. The effect is a larger signal-to-noise ratio in the translocation measurement than in the single point measurement. For moderate spacings (of the order of ten km), the differential-mode signal content improves; however, less of the deflection error content at these distances is common mode. Nonetheless, some overall improvement is noted at the more extended spacings.

Figure 4.3-4 illustrates one sample translocation survey geometry studied. The data reduction approaches simulated were

- Establish a baseline case by computing the accuracy of the deflection of the vertical at the central point based on optimal regional adjustment of all five stations as well as a repeated measurement at the central site (north or east component of the deflection is measured at each site. In addition, a repeated center measurement of a single deflection component is taken). This is the conventional processing case.

- Measure the same quantities as for the baseline and, in addition, the set of differential astronomic positions indicated by the connecting arrows in Fig. 4.3-4. The differential measurements define the deflection component in the direction of each differential station pair. Both the conventional and differential measurements are processed as an overall regional adjustment.

Table 4.3-2 summarizes the results of the translocation simulation studies for three cases:

1) The conventional processing case applied to the baseline gravity disturbance model of Table 4.2-1. The uncorrelated measurement error ascribed to each deflection value is 0.3 $\widehat{\text{sec}}$

2) Same as 1) but with the addition of the differential astrogeodetic measurements indicated in Fig. 4.3-4. The uncorrelated measurement noise associated with the

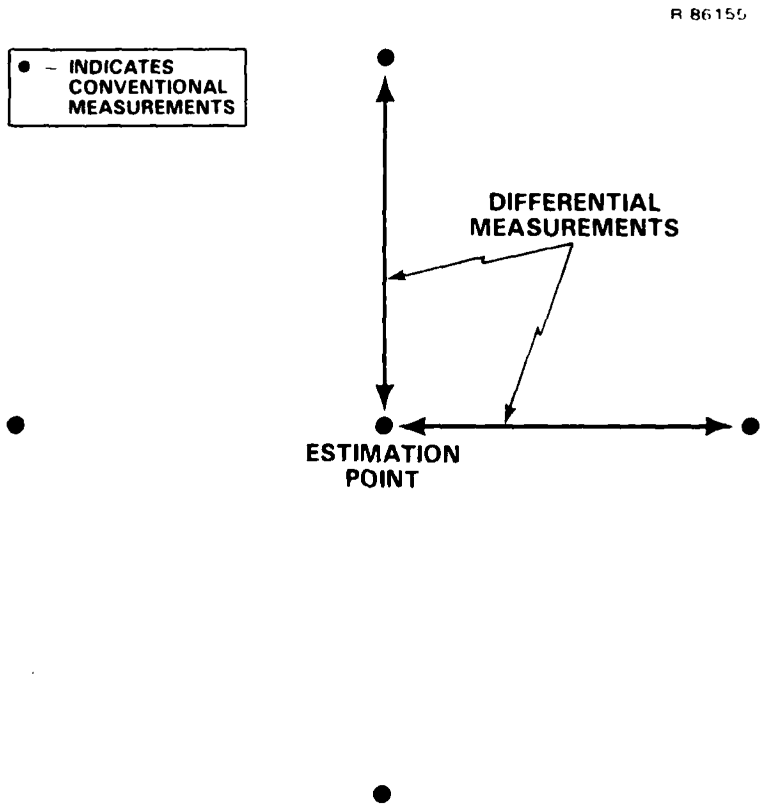


Figure 4.3-4 Translocation Method Geometry

differential astrogeodetic measurements was 0.3 $\widehat{sec}$

3) Same as 2) but with error-free differential astrogeodetic measurements.

Study case 3 is particularly interesting because it offers an upper bound on the potential of the translocation technique. The improvement is disappointing, although understandable in light of the earlier discussion. Considering the logistical difficulty of implementing sychronized observations by two teams of observers as well as increased data processing complexity, the alternative of multiple occupations at each site using conventional techniques appears to be more productive than translocation.

TABLE 4.3-2

RESULTS OF ASTROGEODETIC TRANSLOCATION STUDY

STUDY CASE	DISTANCE, d, BETWEEN OUTER SURVEY STATIONS (km)	DIFFERENTIAL MEASUREMENT ERROR ($\widehat{sec}$)	POST-ADJUSTMENT rms DEFLECTION ERROR (per component - $\widehat{sec}$ rms)
Conventional	2	∞	0.145
Noisy Translocation	2	0.3	0.141
Noiseless Translocation	2	0.0	0.136
Conventional	10	∞	0.208
Noisy Translocation	10	0.3	0.172
Noiseless Translocation	10	0.0	0.149

4.4 NUMERICAL STUDY PERSPECTIVE

The results of this chapter serve two purposes. One is to provide insights into alternative astrogeodetic survey approaches and procedures. In particular, the results establish a basis for tradeoffs among parameters such as station spacing, measurement error, and repeated site occupation strategies in order to meet a specified post-adjustment "per-site" deflection accuracy requirement most efficiently.

The second purpose, which is accomplished by the assembly of astrogeodetic observation and error models required for the simulation studies, is preparation for the second study phase. The second phase will utilize the models and simulation procedures described herein to address optimal multisensor survey techniques for determining deflections accurately, at minimum cost and in a logistically efficient, timely manner.

5. SUMMARY AND CONCLUSIONS

This interim report has examined astrogeodetic approaches, procedures, and equipment to determine the best means to perform future surveys of deflections of the vertical for military applications. Quantitative error models have been developed for the various astrogeodetic data acquisition alternatives and applied to numerical studies of particular sensors and geometries. Particularly noteworthy findings*, along with their key impact or promise, are:

- Significant improvements in accuracy or productivity cannot be accomplished by viewing procedure or data reduction algorithm changes alone (except for repetitive site occupations). The limitations of the standard astrogeodetic techniques (Horrebow-Talcott, Sterneck, measurement of meridian transits) as well as various special approaches (prime vertical, elongation, and circumzenithal observations), are imposed by effects such as atmospheric refraction, star catalogue error, and limitations inherent to the human viewer.

- Accuracies approaching 0.1 $\widehat{sec}$ can be obtained using T-4 theodolite technology by multiple reoccupations† of astrogeodetic observing sites, spaced over a several month span. Such observations must operate at the limits of the state of the art, and be performed by highly trained personnel. The productivity of this approach is unattractive. Also, a timely response

---

*A more detailed summary of astrogeodetic findings is presented in Section 2.9.

†Beyond the multiple measurements currently called out in standard astrogeodetic operating procedures.

to potential new requirements for additional deflection of the vertical surveys is difficult with current techniques. Area adjustment of densified fields of astrogeodetic stations, using minimum-variance techniques and statistical measurement error models, offers accuracies which can approach 0.1 sec (e.g., Figs. 4.3-1, 4.3-2, and 4.3-3 in Chapter 4). If accomplished through current T-4-related observational techniques, the productivity and timeliness comments offered above also relate to the area adjustment approach. However, if the individual observations can be obtained with greater ease (e.g., by one or more of the technologies mentioned below), the incremental improvement offered by network adjustment techniques is attractive.

- Combination of repeated site reoccupations and area adjustment procedures could provide accuracies marginally better than 0.1 sec. Of course, the comments in the previous two items about productivity and timeliness apply to the combination approach as well.

- Several technologies offer the potential to reach or exceed accuracies of 0.1 sec. Although yet to be assessed developmental and testing costs to perfect each of the technologies listed below probably exceed one million dollars over the long term, the incremental cost per "production-mode" astrogeodetic station would decline by an order of magnitude. Using any of the advances listed below, significantly higher production rates probably can be realized than are achievable with current T-4-related procedures. The technologies are:

  Astrolabe with CCD eyepiece

  Two-color refractometer

  Portable photographic zenith tube

  T-4 with CCD eyepiece

Note that accuracy improvements offered through two-color refractometry and use of a CCD eyepiece are complementary; a system which employs both technologies would realize greater accuracy improvement than either the two-color refractometer or the CCD eyepiece alone.

- Star catalogue error is a fundamental error source which continues to increase. Currently used catalogues can contain systematic errors of 0.1 $\widehat{\text{sec}}$, or more under special conditions. Localized errors in catalogue positions can be higher. The key problem is that present catalogues are based on stellar observations which were taken in the 1920s and 1930s. As the catalogues age, the extrapolations of star positions based on the original observations vary increasingly from their actual positions. The following two steps should be taken

    1) utilize routine observatory astrometric measurements to update (or recreate) star catalogues. It is recognized that this could be a demanding effort.

    2) implement detailed error analyses of existing catalogues to ascertain the time evolution of both systematic and localized errors, particularly as they would be impacted by current Defense Mapping Agency astrogeodetic operating procedures.

- Deflection of the vertical determination by techniques using gravity measurements alone do not offer the accuracy achievable by the astrogeodetic approaches previously described. However, with modern measurement and data base approaches to storing worldwide and local data, gravimetric techniques can support 0.2 to 0.3 $\widehat{\text{sec}}$ deflection of the vertical accuracies in most reasonably smooth topographical areas. Because of the ease of making gravity measurements, cost and logistic factors

> can favor a gravimetric approach. Nonetheless, considerable time would be required to complete a gravimetric survey configured to support the computation of suitably accurate deflections of the vertical at missile launch sites.

It is appropriate to reiterate that, except as indicated, these comments relate to individual technologies and approaches which have been examined singly. The next phase of this study will treat selected combinations of methods for determining deflections of the vertical.

APPENDIX A

The coordinate systems used in geodetic astronomy are defined in this appendix, along with the transformations between them. More detailed discussions can be found in Ref. 6.

System 1, the horizon system, is left handed and has the fundamental plane in the horizon, positive x_{11} through the north point, positive x_{12} goes east, and positive x_{13} passes through the zenith. Its principal angles are the azimuth, A, and the zenith distance, z. Thus a unit vector in System 1, $\underline{v_1}$, comprises the coordinates:

$$x_{11} = \sin z \cos \bar{A} \qquad (A\text{-}1)$$

$$x_{12} = \sin z \sin \bar{A} \qquad (A\text{-}2)$$

$$x_{13} = \cos z \qquad (A\text{-}3)$$

System 2, the hour angle system, is also left handed and has the fundamental plane in the celestial equator, positive x_{21} towards the upper intersection of equator and meridian, positive x_{22} westerly, and positive x_{23} through the north celestial pole. The principal angles are the hour angle, t, and the declination, δ. A unit vector in System 2, $\underline{v_2}$, comprises the coordinates:

$$x_{21} = \cos \delta \cos t \qquad (A\text{-}4)$$

$$x_{22} = \cos \delta \sin t \qquad (A\text{-}5)$$

$$x_{23} = \sin \delta \qquad (A\text{-}6)$$

System 3, <u>the right ascension system</u>, is right handed, and also has its fundamental plane in the celestial equator, positive x_{31} through the vernal equinox, positive x_{32} to the east (increasing right ascension), and positive x_{33} also through the north celestial pole. The principal angles are the right ascension, α, and the declination, δ. A unit vector in System 3, $\underline{v_3}$, comprises the coordinates:

$$x_{31} = \cos\delta \cos\alpha \tag{A-7}$$

$$x_{32} = \cos\delta \sin\alpha \tag{A-8}$$

$$x_{33} = \sin\delta \tag{A-9}$$

The relationship between the first two systems is such that the angle between the axes x_{13} and x_{23} is the colatitude, $\bar{\phi}$, and a rotation of 180 deg completes the transformation. Thus the transformations between the two are defined by

$$\underline{v_1} = T_1^2\, \underline{v_2} \tag{A-10}$$

$$\underline{v_2} = T_2^1\, \underline{v_1} \tag{A-11}$$

where

$$T_2^1 = T_1^2 = \begin{pmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & -1 & 0 \\ \cos\phi & 0 & \sin\phi \end{pmatrix} \tag{A-12}$$

The different systems can be compared to reveal the basic equations of astrogeodesy.

$$\begin{pmatrix} \sin z \cos A \\ \sin z \sin A \\ \cos z \end{pmatrix} = \begin{pmatrix} -\sin \phi & 0 & \cos \phi \\ 0 & -1 & 0 \\ \cos \phi & 0 & \sin \phi \end{pmatrix} \begin{pmatrix} \cos \delta \cos t \\ \cos \delta \sin t \\ \sin \delta \end{pmatrix} \tag{A-13}$$

$$= \begin{pmatrix} -\sin \phi \cos \delta \cos t + \cos \phi \sin \delta \\ -\cos \delta \sin t \\ \cos \phi \cos \delta \cos t + \sin \phi \sin \delta \end{pmatrix} \tag{A-14}$$

Similarly

$$\begin{pmatrix} \cos \delta \cos t \\ \cos \delta \sin t \\ \sin \delta \end{pmatrix} = \begin{pmatrix} -\sin \phi \sin z \cos \bar{A} + \cos \phi \cos z \\ -\sin z \sin \bar{A} \\ \cos \phi \sin z \cos \bar{A} + \sin \phi \cos z \end{pmatrix} \tag{A-15}$$

Equations A-14 and A-15, which were derived from matrix rotations, can be compared to the equations found in most older texts, derived using classical trigonometry. The top lines are the five-part formulas, while the last two lines are the law of sines and the law of cosines, respectively. Notice that the algebraic signs relate the quadrant of t to that of $\bar{A}$, and vice versa. Note also that there are numerous other applications of the basic spherical trigonometric formulas, but they all involve the third, or parallactic, angle in the astronomical triangle. Since, in general, this angle cannot be measured, it is seldom referred to in astrogeodesy.

The relationship between systems two and three is such that the angle between the two x_1 axes is the local apparent sidereal time, θ. A change of handedness completes the transformation. Thus

$$\underline{v_2} = T_2^3\,\underline{v_3} \tag{A-16}$$

$$\underline{v_3} = T_3^2\,\underline{v_2} \tag{A-17}$$

The transformation matrix, T_j^i, in each case is

$$T_3^2 = T_2^3 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A-18}$$

In meridian observations, all measurements are in the x_{11} x_{13} plane, so the sine law does not apply. However, the five-part formula turns the zenith distance into a signed quantity. This causes no difficulty as the formulas are seldom used; it is easier just to add and subtract the angles as needed. Note that, if the quadrants of both $\bar{A}$ and t are strictly observed, z is always positive.

Three special circumstances are of interest, in addition to observations in the meridian. Observations in the prime vertical are made in the x_{12} x_{13} plane - that is, $\bar{A} = \pm 90$ deg. In this case, the law of cosines points to the basic condition that the declination must always be between zero and the latitude. Observations at elongation require consideration of the parallactic angle. The sine law can be used to relate the latitude, declination, azimuth, and parallactic angle. This is the only expression containing the two fixed quantities and only two variables. When the derivative is set to zero, the condition for elongation, that the parallactic angle be right, is deduced. The same derivative sets the magnitude of the declination greater than that of the latitude for observations to be possible.

REFERENCES

1. Horrebow, Peter, Atrium Astronomiae, 1732.

2. Bossler, J.D., "The SAO Catalogue, its qualitative and quantitative value to the C&GS Satellite Triangulation Program," ESSA, Institute for Earth Sciences, Coast and Geodetic Survey, U.S.D.C., 1966.

3. Pettey, J.E., and Carter W.E., "Uncertainties of Astronomic Positions and Azimuths," Proc. 2nd Symposium on Problems Related to Redefinition of N.A.D., 1978.

4. Barnes, G.L., and Mueller, I.I., "The Dependence of Level-Sensitivity on the Position and Length of the Bubble on the Wild T-4 Theodolite," Bulletin Géodésique 81, pp. 277-286, 1966.

5. Chauvenet, William, A Manual of Spherical and Practical Astronomy, 2 vols., republished 1960, New York: Dover, 1891.

6. Mueller, I.I., Spherical and Practical Astronomy as Applied to Geodesy, New York: Frederick Ungar, 1969.

7. "Latitude Instructions for Modified Sterneck Method," Unpublished Memorandum, National Geodetic Survey, N.O.S. U.S.D.C.

8. Danjon, André, "The Impersonal Astrolabe," in Stars and Stellar Systems, Vol. I, Telescopes, G.P. Kuiper, Ed., and B.M. Middlehurst, Assoc. Ed., Chicago: University of Chicago, 1960.

9. Kivioja, L.A., "First Order Astro-Azimuth Observations Using an Axis Mirror," Proc. ACSM, St. Louis, MO, March 1974.

10. Hunt, Mahlon S., "An Evaluation of AAPS Acceptance Performance," Project 7600, Terrestrial Sciences Laboratory, AFCRL, Hanscomb AFB, 1975.

11. Carroll, Joseph E., "Engineering Improvements: Automated Astronomic Positioning System," Control Data Corporation, Report No. AFCRL-TR-75-0156, 1975.

REFERENCES (Continued)

12. Kivioja, L.A., "New Mercury Leveler for the Horrebow-Talcott Method," Surveying and Mapping, 35, pp. 145-146, 1975.

13. Gelb, Arthur, Ed., Applied Optimal Estimation, Cambridge, Mass.: MIT Press, 1974.

14. Bufton, Jack and Genatt, Sol H., "Simultaneous Observations of Atmospheric Turbulence Effects on Stellar Irradiance and Phase," Astronomical Journal, 76, pp. 378ff, 1971.

15. Eichhorn, Heinrich, "On the Construction of a Comprehensive General Catalogue of Star Positions," Celestial Mechanics, 22, pp. 127-135, 1980.

16. Fricke, W., and Kopff, A., Fourth Fundamental Catalogue (FK4), Veröff. des Astron. Rechen-Instituts Heidelberg, No. 10, 1963.

17. Boss, Benjamin, General Catalogue of 33342 Stars for the Epoch 1950, Pub. No. 486, Washington: Carnegie Institution of Washington, 1937.

18. Eichhorn, Heinrich, Astronomy of Star Positions, New York: Frederick Ungar, 1974.

19. Smithsonian Astrophysical Observatory (Staff of), Star Catalog, Publications of the Smithsonian Institution of Washington No. 4562, Washington: Smithsonian Institution, 1966.

20. Astronomisches Rechen-Institut, Preliminary Supplement to the FK4 (FK4 Sup), Veröff. des Astron. Rechen-Instituts Heidelberg, No. 11, 1963.

21. Bowie, William, Determination of Time, Longitude, Latitude, and Azimuth, Coast and Geodetic Survey Special Publication No. 14, Washington: Government Printing Office, 1914.

22. Precise Astronomic Surveys, Department of the Army, Technical Manual 5-442, 1970.

23. Hoskinson, Albert J., and Duerksen, J.A., Manual of Geodetic Astronomy, Coast and Geodetic Survey Publication No. 237, Washington: Government Printing Office, 1947.

REFERENCES (Continued)

24. Gregerson, L.F., "Weighted Solution of Longitude Equations," Canadian Surveyor, Vol. 21, No. 5, pp. 370-375, 1967.

25. Schwebel, R., "On the Investigation of Instrumental Errors of Universal and Transit Instruments by Means of Autocollimation," AFCRL-70-0097, Translation No. 49, Air Force Cambridge Research Laboratory, 1970.

26. Gilbert, Paul F., Astronomic Position Accuracy Capability Study, Technical Report No. DMAHTC 79-002, Defense Mapping Agency, 1979.

27. Brown, Duane C., "Application of Close-Range Photogrammetry to Measurements of Structures in Orbit," GSI Technical Report No. 80-012, Melbourne, Fla: Geodetic Services, Inc., 1980.

28. Brown, Duane, "STARS, a Turnkey System for Close-Range Photogrammetry," GSI Report No. 82-007, Melbourne, Fla.: Geodetic Services, Inc., 1982.

29. Vondrák, J., "The New Circumzenithal of the Research Institute for Geodesy, Topography and Cartography in Prague," Bul. Astronomical Institutes of Czechoslovakia, 21, 4, 1970.

30. Soltau, Gerhard, "On the Accuracy of Longitude Determinations Using the VUGTK-CSSR Circumzenithal Astrolabe," Proc. Gen. Assembly International Assoc. of Geodesy, 1975.

31. The Photoelectric Astrolabe Research and Production Group, "The Photoelectric Astrolabe, Type II," Acta Astronom. Sinica, 16, 115-122, 1975, republished in Chinese Astronomy, 1, 79-87, Great Britain: Pergamon, 1977.

32. Gething, P.J.D., "The Collimation Error of the Airy Transit Circle," Mon. Not. Roy. Astron. Soc., 114, pp. 415ff, 1954.

33. Kearsley, A.H.W., The Computation of Deflection of the Vertical from Gravity Anomalies, Report UNISERV S-15, School of Surveying, University of New South Wales, 1976.

REFERENCES (Continued)

34. Forsberg, R., and Tscherning, C.C., "The Use of Height Data in Gravity Field Approximation by Collocation," Journal of Geophysical Research, 86, B9, pp. 7843-7854, 1981.

35. Groten, E., Determination of Plumb Line Curvatures by Astronomical and Gravimetric Methods, NOAA Technical Memorandum NOS NGS 30, National Oceanic and Atmospheric Administration, 1981.

36. Rice, D.A., "Deflections of the Vertical from Gravity Anomalies," Bulletin Géodesique, pp. 285-308, 1952.

37. Pellinen, L.P., "Expedient Formulae for Computation of Earth's Gravitational Field Characteristics from Gravity Anomalies," Bulletin Géodesique, pp. 327-333, 1964.

38. Clarke, F.L., "Modelling Position Dependent Errors in Gravimetric Deflections of the Vertical," Bulletin Géodesique, pp. 1-16, 1981.

39. Ihde, J., "Investigations on the Accuracy Concerning the Derivation of Absolute Gravimetric Height Anomalies and Deflections of the Vertical Based on the Theory of Molodensky," Bulletin Géodesique, pp. 99-110, 1981.

40. Kaula, W.M., Accuracy of Gravimetrically Computed Deflections of the Vertical, Transactions of the American Geophysical Union, 38, pp. 297-305, 1957.

41. Van Gysen, H., Precise Deflections of the Vertical in the Canberra Area, University of New South Wales, 1980.

42. Heiskanen, W.A., and Moritz, H., Physical Geodesy, W.H. Freeman and Company, 1967.

43. Molodenskii, M.S., Eremeev, V.F., and Yurkina, M.I., Methods for Study of the External Gravitational Field and Figure of the Earth, translated by the Israel Program for Scientific Translations, 1962 (University Microfilms International).

44. Moritz, H., Advanced Physical Geodesy, Herbert Wichmann Verlag, Karlsruhe, 1980.

REFERENCES (Continued)

45. Chen, J.Y., "Methods for Computing Deflections of the Vertical by Modifying Vening-Meinesz' Function," Bulletin Géodesique, pp. 9-26, 1982.

46. Moritz, H., "Least-Squares Collocation as a Gravitational Inverse Problem," Report No. 249, Department of Geodetic Science, Ohio State University, Columbus, OH, November 1976.

47. Thomas, S.W. and Heller, W.G., "Efficient Estimation Techniques for Integrated Gravity Data Processing," The Analytic Sciences Corporation Technical Report, TR-680-1, September 1976.

48. Heller, W.G., Tait, K.S., and Thomas, S.W., "GEOFAST - A Fast Gravimetric Estimation Algorithm," The Analytic Sciences Corporation Technical Report, AFGL-TR-77-0195, August 1977.

49. Goldstein, J.D., "Analysis and Simulation of Multisensor Gravity Surveys and Associated Ballistic Missile Impact Errors (U)," The Analytic Sciences Corporation, Technical Report TR-868-1-3, 1980 (SECRET).

50. Baumgartner, S.L., Capp, D.W., and Heller, W.G., "Assessment of Inertial Technology for Gravity Survey Applications," The Analytic Sciences Corporation, Technical Report TR-3937 (ETL-0291-1), 1982.

51. Dobrin, M.B., Introduction To Geophysical Prospecting, 3rd edition, McGraw-Hill, 1976. (Chapters 11 and 12).

52. Nettleton, L.L., Gravity and Magnetics in Oil Prospecting, McGraw-Hill, 1976. (Chapters 3 and 4; Chapter 5 deals with airborne and at-sea gravimetry).

53. Gentry, D.E. and Nash, R.A., Jr., "A Statistical Algorithm for Computing Vertical Deflections Gravimetrically," Journal of Geophysical Research, Vol. 77, No. 26, September 1972.

EN

DAT
FILME

9-8

DTI