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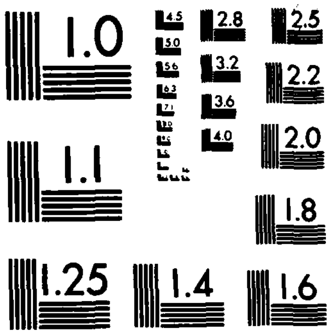


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# ON THE OPTIMAL DESIGN OF BIDIRECTIONAL COMPOSITES

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June 1983

Report for the Period June 1982 - May 1983

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The intention of this report is to develop a feeling for the directional properties of laminated composites. Easy to understand and to use rules for the versatile application of anisotropic, though orthotropic bidirectional laminates are presented. Simple examples and charts demonstrate the possible weight advantages over quasi-isotropic configurations. Explicit use is made of the Mohr's circle representation. Multiple load g is considered and a graphic solution is indicated.

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## FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio.

The work reported herein was performed during the period June 1982 to May 1983 and is part of the work done by the author while being a visiting scientist to the MLBM. He is a scientist from the Institute for Structural Research and Design Development of the DFVLR, Stuttgart, FRG.

Many thanks to Stephen W. Tsai for his encouragement and guidance in the course of this work.

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## SECTION I
## INTRODUCTION

This work mainly concentrates on the strength of symmetric bidirectional composites, namely cross-ply and angle-ply laminates, which are orthotropic and hence easy to handle. When used in the right way they may well offer weight-advantages over quasi-isotropic lay-ups. Studying bidirectional composites can also promote the understanding of the basic characteristics of laminated composites and develop a feeling for their directional properties. The sizing of bidirectional configurations is comparatively easy and can be done with a programmable pocket calculator. The analytical procedure is explicitly described in Reference 1. All sample calculations and charts, based on T300/5208 graphite epoxy, were done with a Tl-59 calculator using the Composite Materials Module and the Air Force Materials Laboratory Combo cards.[2] Extensive use was made of the Mohr's circle representation of a a state of stress. This representation was considered a valuable help to visualize some of the basic relations. All examples are for in-plane loading of flat and unstiffened panels. Multiple loading is also considered. Optimization was mainly based on the first ply failure strength, employing the quadratic interaction failure criterion. Use of other failure criteria might lead to different numerical results, but the basic relations would remain unchanged though.

## SECTION II
## BASIC CONSIDERATIONS

### 1. MOHR'S CIRCLE AND PRINCIPAL ORIENTATION

A given state of stress can graphically be represented by a Mohr's circle. The three stress components needed to describe this state of stress in a given coordinate system may be represented by the following sets of variables:[1]

Set 1: $\sigma_1, \sigma_2, \sigma_6$

Set 2: p, q, r

$$p = \frac{1}{2}(\sigma_1 + \sigma_2),\ q = \frac{1}{2}(\sigma_1 - \sigma_2),\ r = \sigma_6 \qquad (1)$$

Set 3: I, R, $\theta_o$

$$I = \frac{1}{2}(\sigma_1 + \sigma_2),\ R = \sqrt{p^2 + q^2} \qquad \theta_o = \frac{1}{2}\cos^{-1}\frac{q}{R} \qquad (2)$$

The geometric relations are given in Figure 1.

For the same state of stress any change of the coordinate system will change the variables in the three sets. The stress transformation can be performed in terms of each of the above sets. The new stress components are depicted by the same Mohr's circle.

As is obvious from Fig. 1, the phase angle $\theta_o$ is the only variable in set 3; I and R are invariants. Set 3 is therefore well suited for use with a Mohr's circle. It is also evident that there is one orientation where there is no shear. This is the principal orientation or direction and the normal stress components $\sigma_1$ and $\sigma_2$ reach their maximum and minimum values and become the principal stress components $\sigma_I$ and $\sigma_{II}$. In the case of a bidirectional laminate, provided the state of stress is known, the laminate orthotropy axes can be orientated in the principal directions, eliminating shear stress. In the case of a cross-ply laminate the principal stress components will thus act in the fiber directions. A Mohr's circle can also be drawn in strain space. The principal axes of stress and strain are coincident.

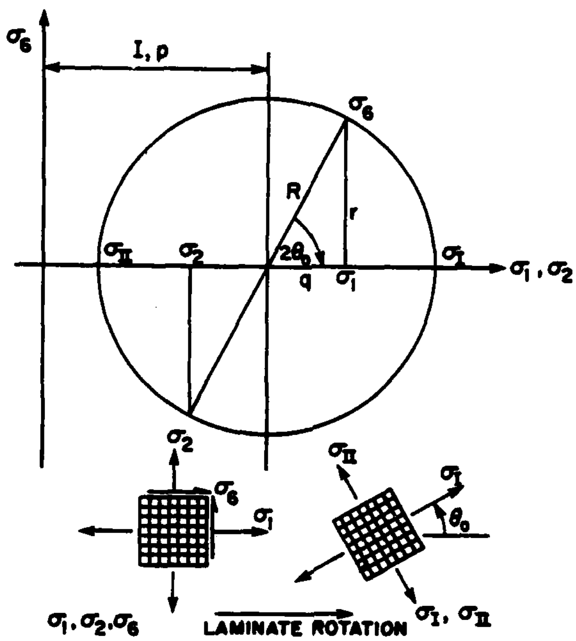


Figure 1. Mohr's Circle in Stress Space.

## 2. IN-PLANE STRENGTH OF LAMINATES

The stress distribution across the thickness of multidirectional laminates is not constant because of the different ply moduli. However, a stress resultant can be defined

$$N_i = h\,\bar{\sigma}_i \qquad (3)$$

which is the average stress per unit width of a laminate with thickness h. To determine the in-plane strength of a laminate for given stress resultants, the in-plane strain is calculated first.

$$\varepsilon_i^{o} = a_{ij} N_j \qquad (4)$$

Assuming strain is constant through the thickness, simple strain transformation will yield the ply strains. Then the strength ratios of the individual plies can be calculated. Here the quadratic interaction failure criterion in strain space is used. Failure occurs when

$$G_{ij}\varepsilon_i\,\varepsilon_j + G_i\,\varepsilon_i = 1 \qquad (5)$$

Expanded for two-dimensional stress on a unidirectional composite in its orthotropic axes

$$G_{xx}\varepsilon_x^2 + 2G_{xy}\varepsilon_x\varepsilon_y + G_{yy}\varepsilon_y^2 + G_{ss}\varepsilon_s^2 + G_x\varepsilon_x + G_y\varepsilon_y = 1 \qquad (6)$$

Introducing the strength ratio

$$\dot{R} = \frac{\sigma_{allowable}}{\sigma_{applied}} = \frac{\varepsilon_{allowable}}{\varepsilon_{induced}} \qquad (7)$$

the failure criterion can be solved for $\dot{R}$

$$[G_{ij}\;\varepsilon_i\;\varepsilon_j]\;\dot{R}^2 + [G_i\;\varepsilon_i]\;\dot{R} - 1 = 0 \qquad (8)$$

A direct link between the stress resultants and the strength ratios is gained when equation (4) is substituted in the strength ratio equation. For a ply with $\theta$ orientation we get

$$[H_{kf}^{(\theta)}\;N_k N_f]\;\dot{R}_{(\theta)}^2 + [H_j^{(\theta)}\;N_j]\;\dot{R}_{(\theta)} - 1 = 0 \qquad (9)$$

The coefficients of the H - function are calculated from the in-plane compliance of the laminate and the transformed strength parameters of the ply.[3] Because of the quadratic form there will be two solutions $\dot{R}$ and $\dot{R}'$ corresponding to strain vectors piercing the failure envelope in opposite directions. The ply with the lowest strength ratio fails first. There is an envelope for each ply orientation; the first ply failure (FPF) envelope is the innermost boundary of the superposed ply failure surfaces. Once the lowest strength ratio of a given laminate configuration (thickness h) for given stress resultants is known the minimum ply number can be calculated

$$n_{req} = \frac{h}{h_o R} \quad (h_o = \text{single ply thickness}) \tag{10}$$

Designing a laminate for FPF strength is a conservative approach. A laminate can still support loads after FPF, on the other hand it is rather likely there is no damage or damage initiation if loads are kept below the FPF level.

## SECTION III
## STRENGTH OF BIDIRECTIONAL LAMINATES

### 1. FAILURE SURFACES OF CROSS-AND ANGLE-PLY LAMINATES

The failure envelopes in normal stress resultant space - or the principal stress plane - for various cross-ply configurations are shown in Fig. 2. Basically quadrants I (tension-tension) and III (compression - compression) are similar, and there is symmetry between II (compr. - tens.) and IV (tens. - compr.). For clarification the $[0_3/90]$ - laminate is taken as example, its envelopes are shown in bold lines, the upper ellipse presenting the 90° plies, the lower one the 0° plies. The shaded area is the FPF envelope. In quadrants I and III the two failure envelopes intersect. For a stress ratio vector intersecting the curves in either point all plies will fail simultaneously. Of all possible stress ratio vectors in both quadrants, those to these points are the longest, which means an optimum condition for laminate design. We see the $[0_3/90]$ - laminate offers the highest strength for a stress ratio of about 3:2. As it turns out a fairly good approximation to the optimum is achieved by matching the ply ratio to the load ratio, a procedure also called netting analysis.(1,3) This also will produce simultaneous or close to simultaneous failure. When a 1:1 ply ratio is used for a 1:1 stress resultant ratio all plies will fail simultaneously. For higher stress resultant ratios netting analysis will become increasingly less accurate. When a 10:1 stress resultant ratio is met with a 10:1 cross-ply ratio, the strength ratios of the two orientations will differ about 25%. There is no coincidence of failure envelopes in quadrants II and IV and consequently no simultaneous failure. But as before the optimum design is characterized by the longest stress ratio vector. For a given stress ratio the cross-ply arrangement whose inner failure envelope is most distant from the origin of the coordinate is the optimum. This is shown in Fig. 3. A chart according to Fig. 3 may be used for optimizing a cross-ply in the IV and, observing symmetry rules, II quadrant.

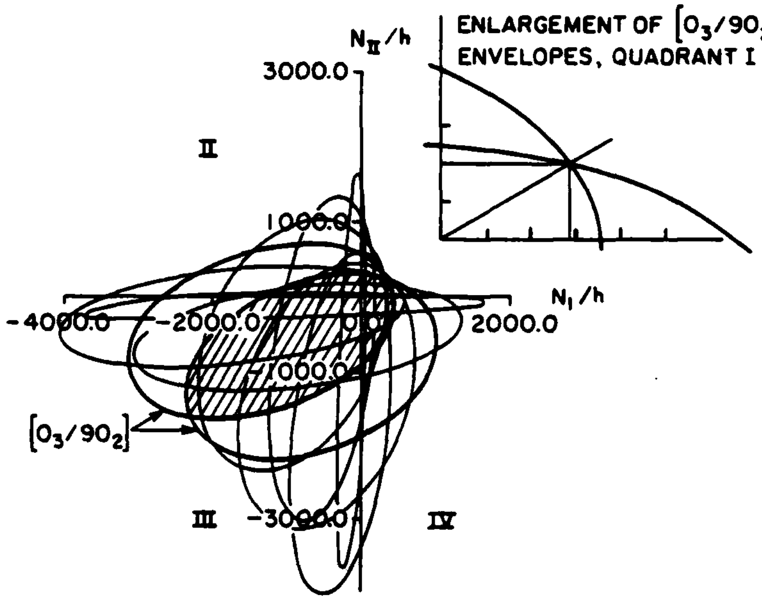


Figure 2. $[0_m/90_n]$ Cross-Ply Failure Envelopes in Stress Space, m/n Ratio in 20%-Steps.

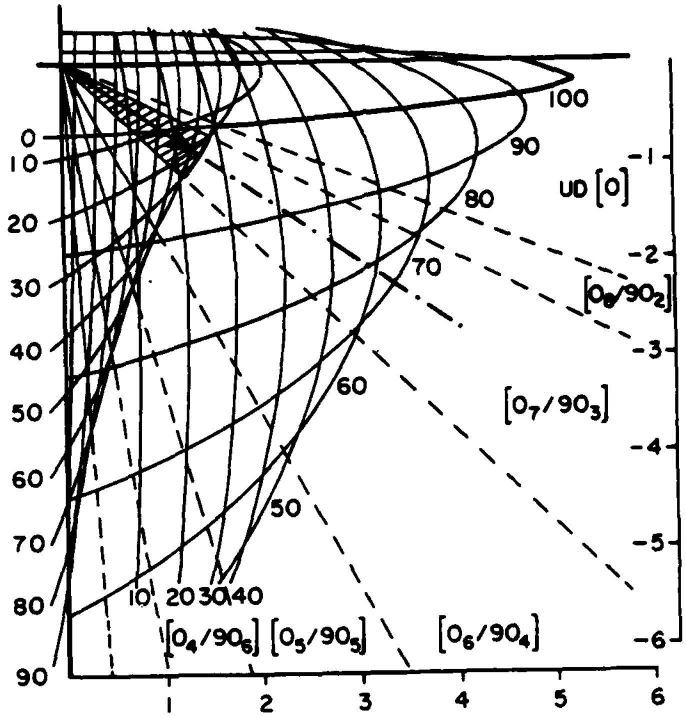


Figure 3. $[0_m/90_n]$ Cross-Ply Failure Envelopes in Quadr. IV; m/n Ratio in 10%-Steps (Example: Optimum for 3/-2-Stress Ratio is $[0_7/90_3]$.

The failure envelopes were drawn for 10% changes of the constituent layers. For example a $[0_1/90_3]$ laminate is best for a stress ratio of 3:-2. The failure envelopes for angle-ply laminates in normal stress resultant space are given in Fig. 4. There is no FPF as all plies must fail simultaneously with the absence of shear. The failure envelopes are narrow, making angle-ply configurations rather susceptible to failure for small changes in stress ratio or ply angle. An optimum ply angle is clearly found in quadrants I and III. However, angle-plies behave very poor in quadrants II and IV as compared to cross-ply laminates. The generation of the failure envelopes is described in Ref. 4; ways to optimize angle plies are shown in Ref. 5 and 6.

## 2. MINIMUM PLY CONSIDERATIONS

The impact of the individual failure envelopes on minimum ply requirements for angle-and cross-ply laminates in the principal stress plane is depicted in Fig. 5 and 6 for quadrants I and IV. For reasons of comparison $N_I$ was kept constant at 1 MN/m and $N_{II}$ was changed in discrete steps. Fig. 5 shows the required ply number vs. the ply angle. A minimum ply number corresponding to an optimum ply angle exists for tension-tension (I quadr), none, however, for tension-compression (IV quadr). At small $N_I:N_{II}$ ratios (e.g. higher $N_{II}$ values) little deviations from the optimum lamination angle may strongly increase the ply number required. To obtain the curves for reversed stress resultant ratios the angle coordinates have to run from right to left; in Fig. 6 the cross-ply ratio must be reversed. Fig. 6 gives ply number vs. cross-ply ratio. Minimum ply numbers corresponding to optimum cross-ply ratios exist in both the first and fourth quadrant. In tension-compression generally less plies are required. Influence on ply number requirements from deviation from optimum cross-ply ratio is less pronounced. Again it is shown that netting analysis results in the lowest ply number in tension-tension. A comparison of Fig. 5 and 6 indicates that optimized angle-ply configurations require less plies at small

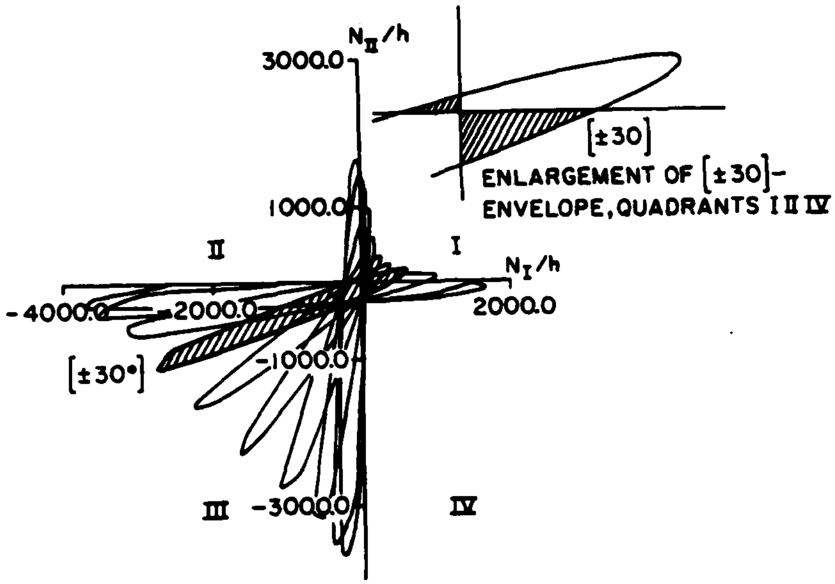


Figure 4. Angle-Ply Failure Envelopes in Stress Space, θ in 10°-Steps.

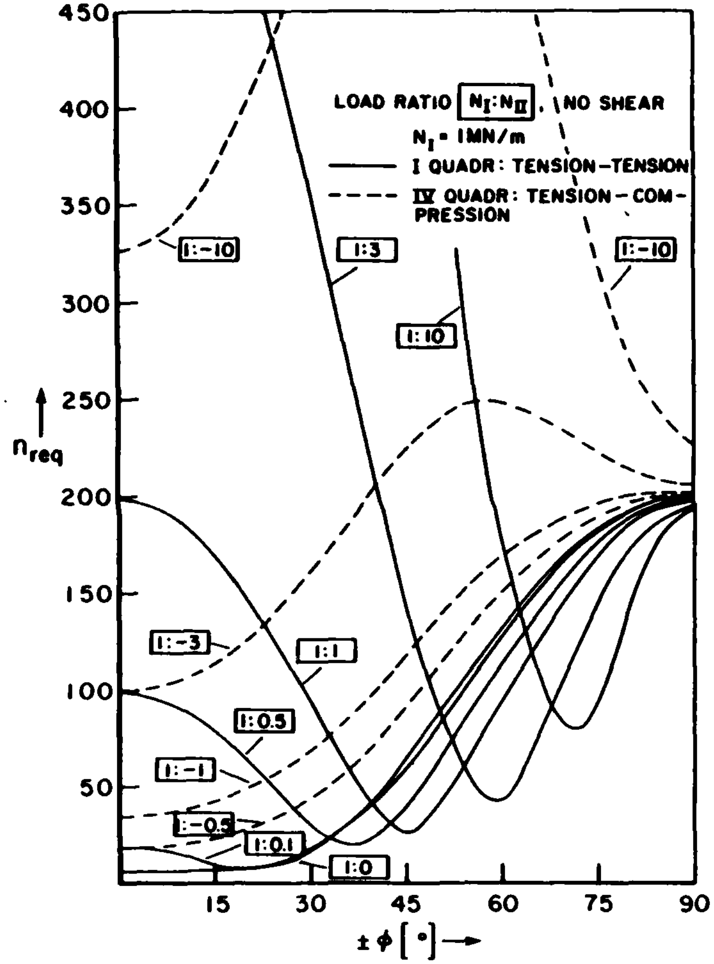


Figure 5. Angle-Ply: Required Ply Number vs. Ply-Angle for Various Normal Stress Resultant Ratios.

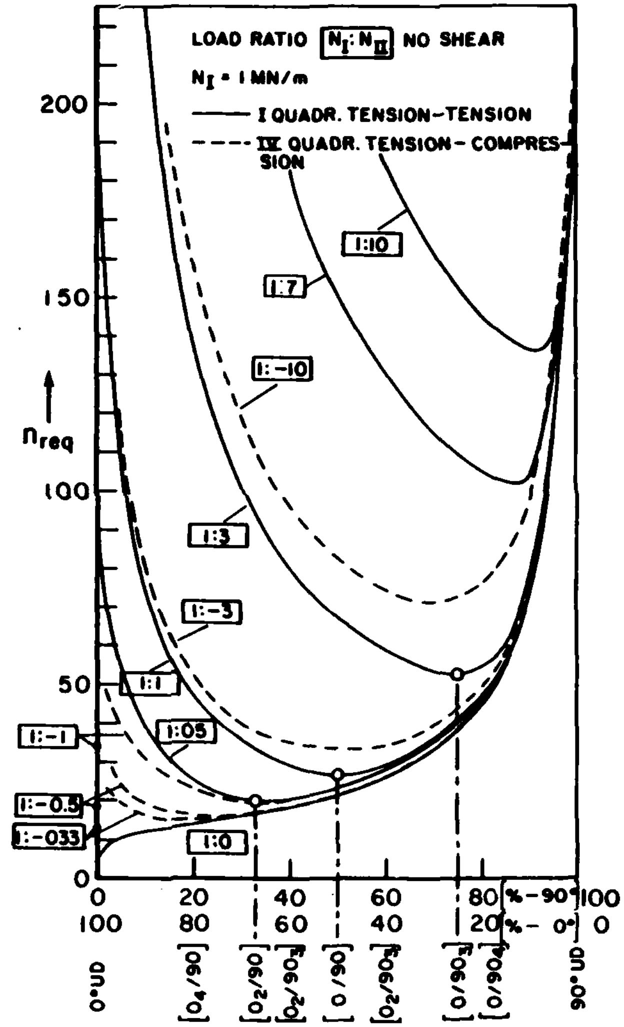


Figure 6. Cross-Ply: Required Ply Number vs. Cross-Ply Ratio for Various Normal Stress Resultant Ratios.

stress ratios. Because of the limitation to the first quadrant and the aforementioned sensitivity to changes in both stress ratio and orientation mainly cross-plies will be considered in the following.

## 3. CROSS-PLY VS. QUASI-ISOTROPIC LAMINATE

For a given state of stress the required ply number is nearly independent of all possible orientations when a quasi-isotropic laminate is used. This is not true for cross-ply configurations. Provided the existing state of stress is well known and does not change, the ply number needed can be significantly reduced, however, when the orientation with no shear is found and a cross-ply laminate is orientated in the principal directions. Netting analysis will drastically improve the results. (Only in the first quadrant and especially at high stress ratios a further reduction in ply number may be achieved with an optimized angle ply configuration). See Fig. 7.

The Mohr's circle representation again is helpful in illustrating the influence of different states of stress on the required ply number, also the terms of equations 1 and 2 are employed. In Fig. 8 the required ply number is plotted over p, with r = 0 and q = const and arbitrarily set at 0.5. This corresponds to Mohr's circles of constant radii shifted towards higher principal stresses; there is no shear, the stress components act in the principal directions. In tension-tension $n_{req}$ increases about linearly with p. The ply number for the quasi-isotropic laminate is an upper boundary. The least ply number is needed when q = 0, that is Mohr's circles are reduced to dots. The difference in ply number for quasi-isotropic and cross-ply configurations is accounted for in Fig. 9. Hence, the ply number dependence on q for a fixed p (arbitrarily set 1.5) is shown. These stress states are illustrated by Mohr's circles of identical origin but different radii. Stress states A are the shear free stress states after coordinate transformation. In

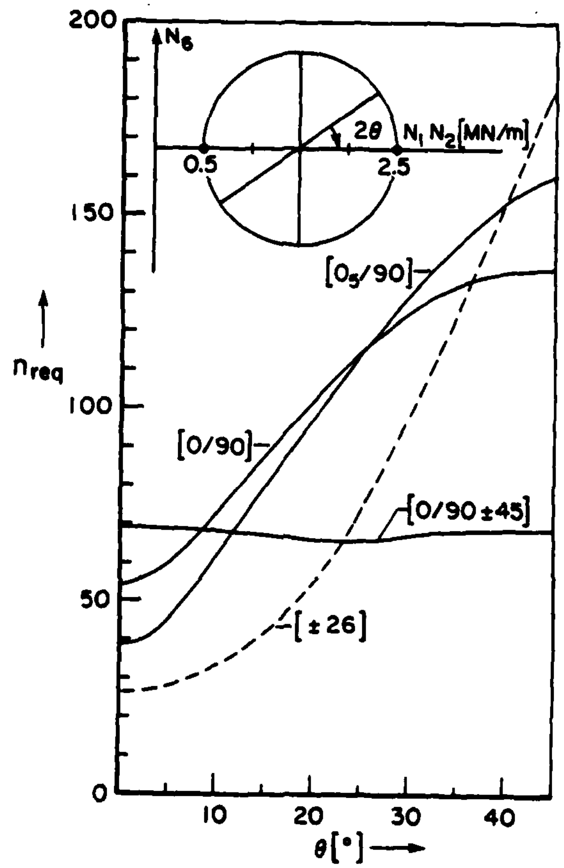


Figure 7. Ply Requirements for Quasi-Isotropic and Cross-Ply Configurations for a Given State of Stress.

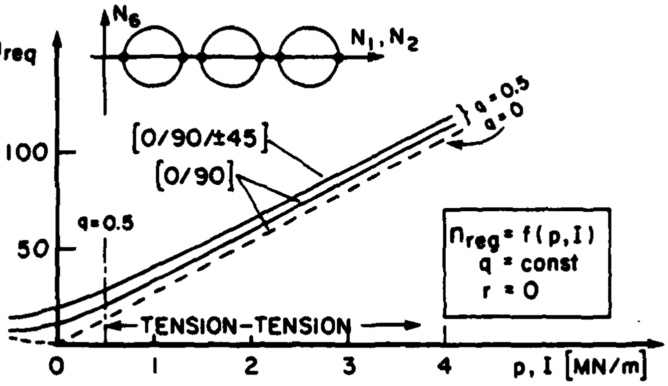


Figure 8A. Ply Number vs. I, p in Quadr. I, IV for Quasi-Isotropic and Cross-Ply Laminates.

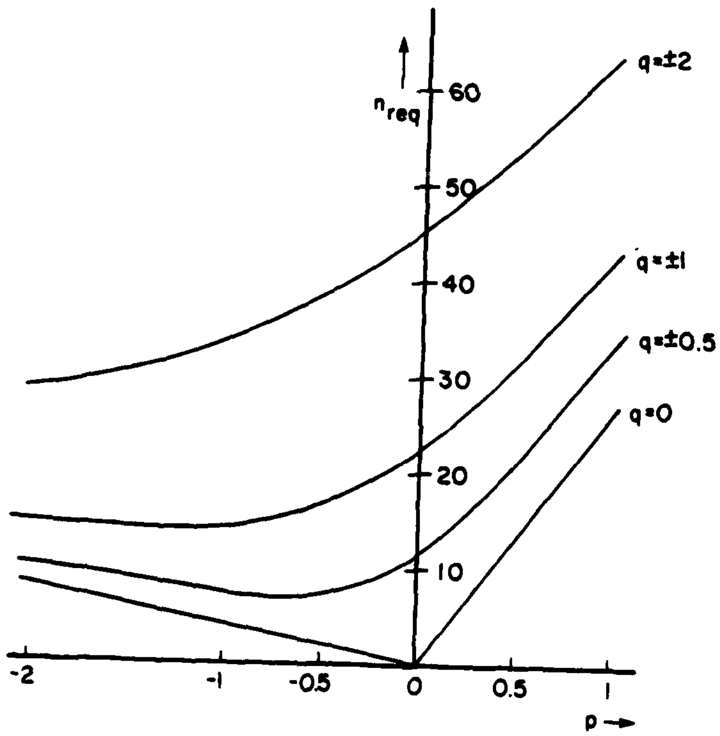


Figure 8B. (Enlarged Section of Lower Left of Fig. 8A): Ply Number vs. p for Different Values of q.

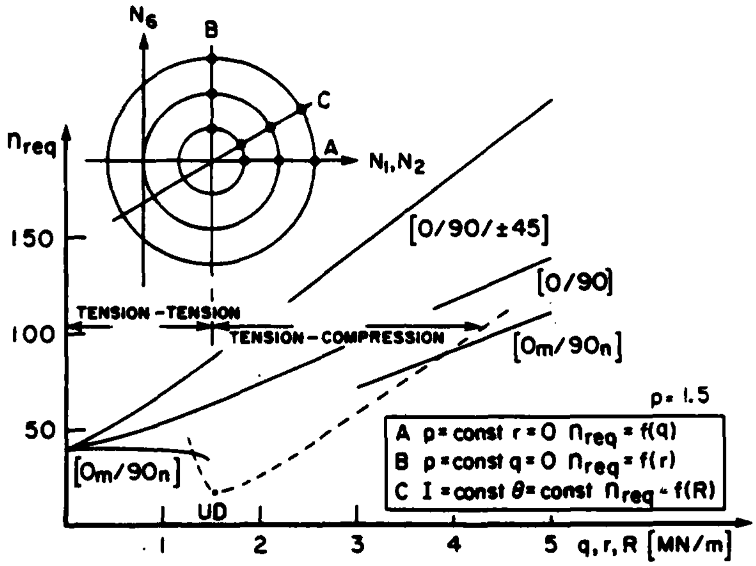


Figure 9. Ply Number vs. q, r, R in Quadr. I, IV for Quasi-Isotropic and Cross-Ply Configurations. Stress Transformation will Change B,C to A.

the tension-tension (compr. - compr.) domain and excluding unidirectional loading, the minimum required ply number for optimized cross ply configurations in the principal directions is practically independent of q and r or the radii of the equivalent Mohr's circles. In tension-compression unidirectional laminate construction will prevail for small q/r/R's before cross-ply ratios as gained from Fig. 3 deliver the optimum eg. smallest ply number. Example: For a stress resultant state of (2.5: .5: 0), in MN/m, corresponding to p = 1.5, q = 1, the required ply number is 69 for a [0/90/±45] quasi-isotropic laminate vs. 55 for a [0/90] cross-ply (20% weight savings) or vs. 40 for an optimized [$0_5$/90] cross-ply configuration (42% weight savings). In reality the actual ply number of a symmetric cross-ply must be an even multiple of the cross-ply ratio. In the given example, this would lead to required ply numbers of 72, 56 and 48 and weight savings of 22 and 33%, respectively.

## SECTION IV

## MOHR'S CIRCLES FOR PLY STRESSES AND STRAINS

In the preceding section use of the Mohr's circle representation was made for stress resultants as defined in equ. 3. Mohr's circles can also be used with the stresses in each ply. For cross-ply configurations this means we have to deal with two different circles for a given state of stress, one for each ply orientation. Under these circumstances, working with Mohr's circles may be time-consuming and less transparent. In the case of angle-ply configurations under bidirectional loading in the principal directions (no external shear), the circles are concentric and of equal diameter; this will change with the introduction of an average shear stress, Fig. 10 (the dots indicate $\varepsilon_6$ on the circle, $\varepsilon_1$ and $\varepsilon_2$ are found corresponding to Fig. 1).

Mohr's circle representation is also possible in strain space. As strains resulting from in-plane loading are assumed to be constant across the thickness of a laminate, only one Mohr's circle is needed to characterize each state of strain. In Fig. 11 Mohr's circles in strain space are drawn for various cross-ply and angle-ply configurations for a sample unidirectional and bidirectional loading including shear. As a reference the Mohr's circle for a quasi-isotropic laminate (dashed line) is also shown in each case. The quasi-isotropic T300/5208 laminate incidentally has about the same stiffness and hence the same strains under equal loading as aluminum. It can be noticed that the angle plies experience generally higher strains than cross-plies, both under unidirectional and bidirectional principal loads. However, when shear is added this will vastly increase the corresponding Mohr's circles for cross-plies; the cross-ply ratio will lose its influence.

Instead of drawing a separate Mohr's circle for each state of strain, a more efficient method of showing location and radius, though not the phase angle, may be used. This is done in Fig. 12A where I and R are plotted vs. the cross-ply ratio for two unidirectional loads and their combination.

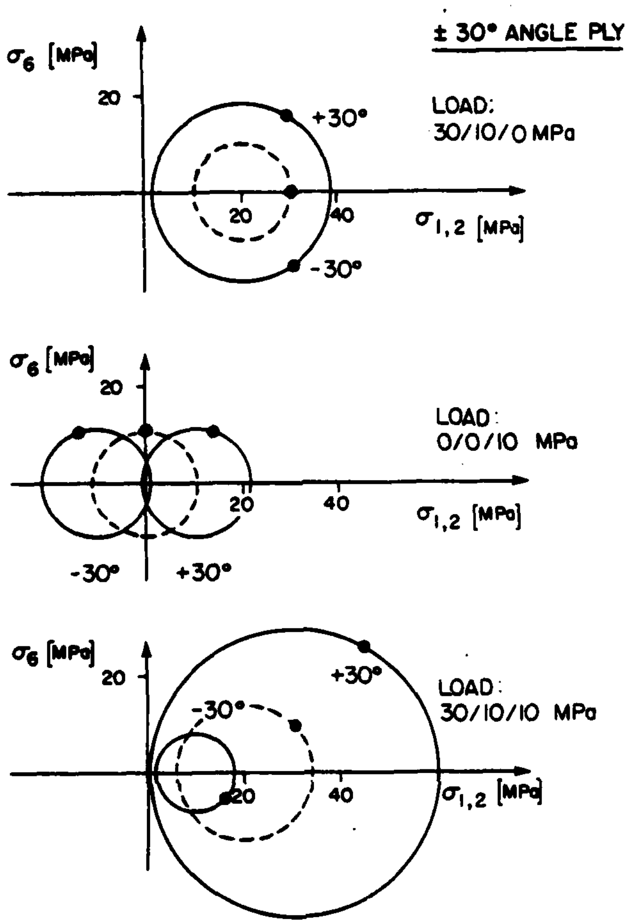


Figure 10. Ply Stresses in an Angle Ply Configuration Under Bidirectional Loading Without and With Shear. Dashed curves indicate laminate stresses.

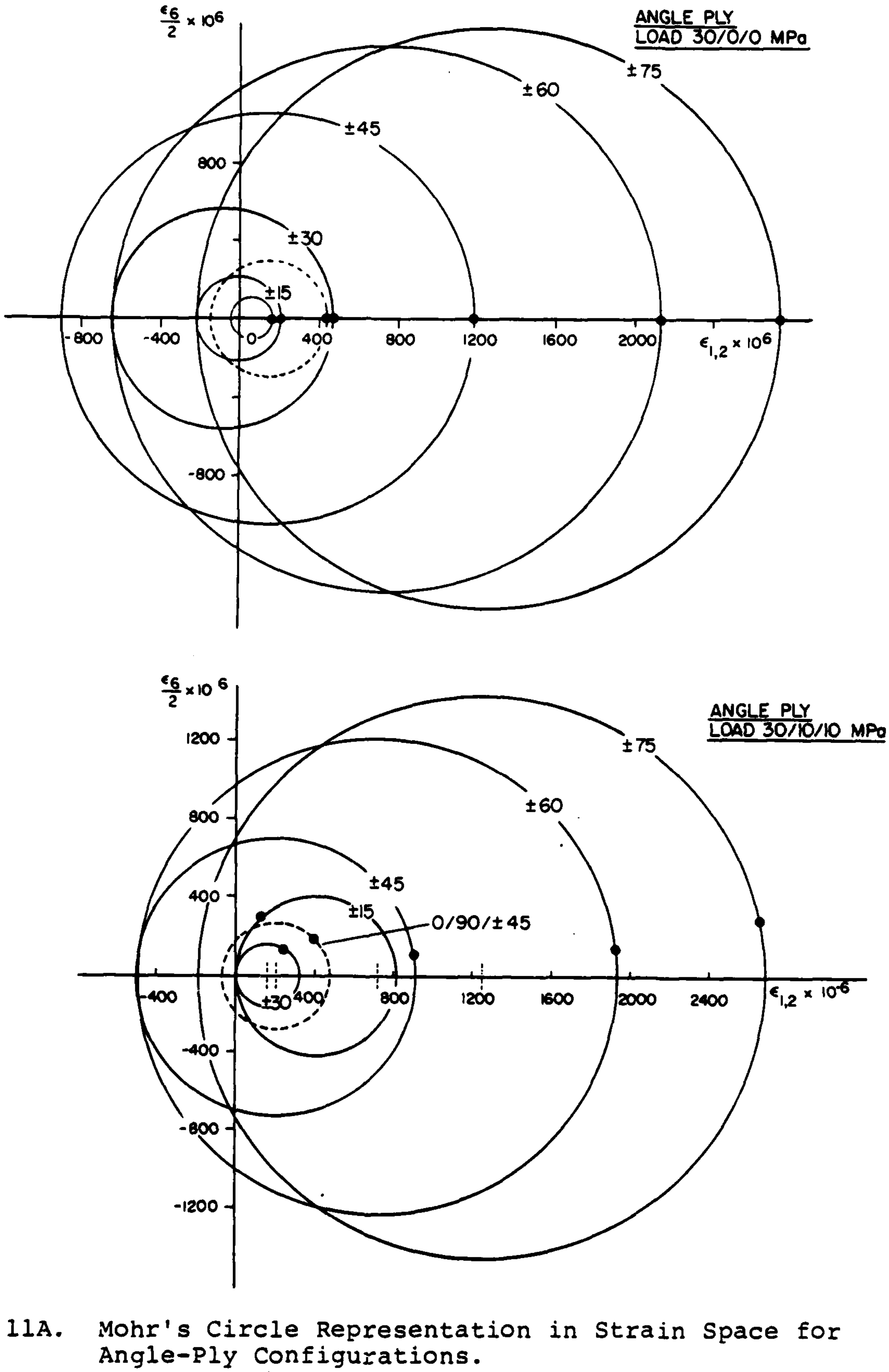


Figure 11A. Mohr's Circle Representation in Strain Space for Angle-Ply Configurations.

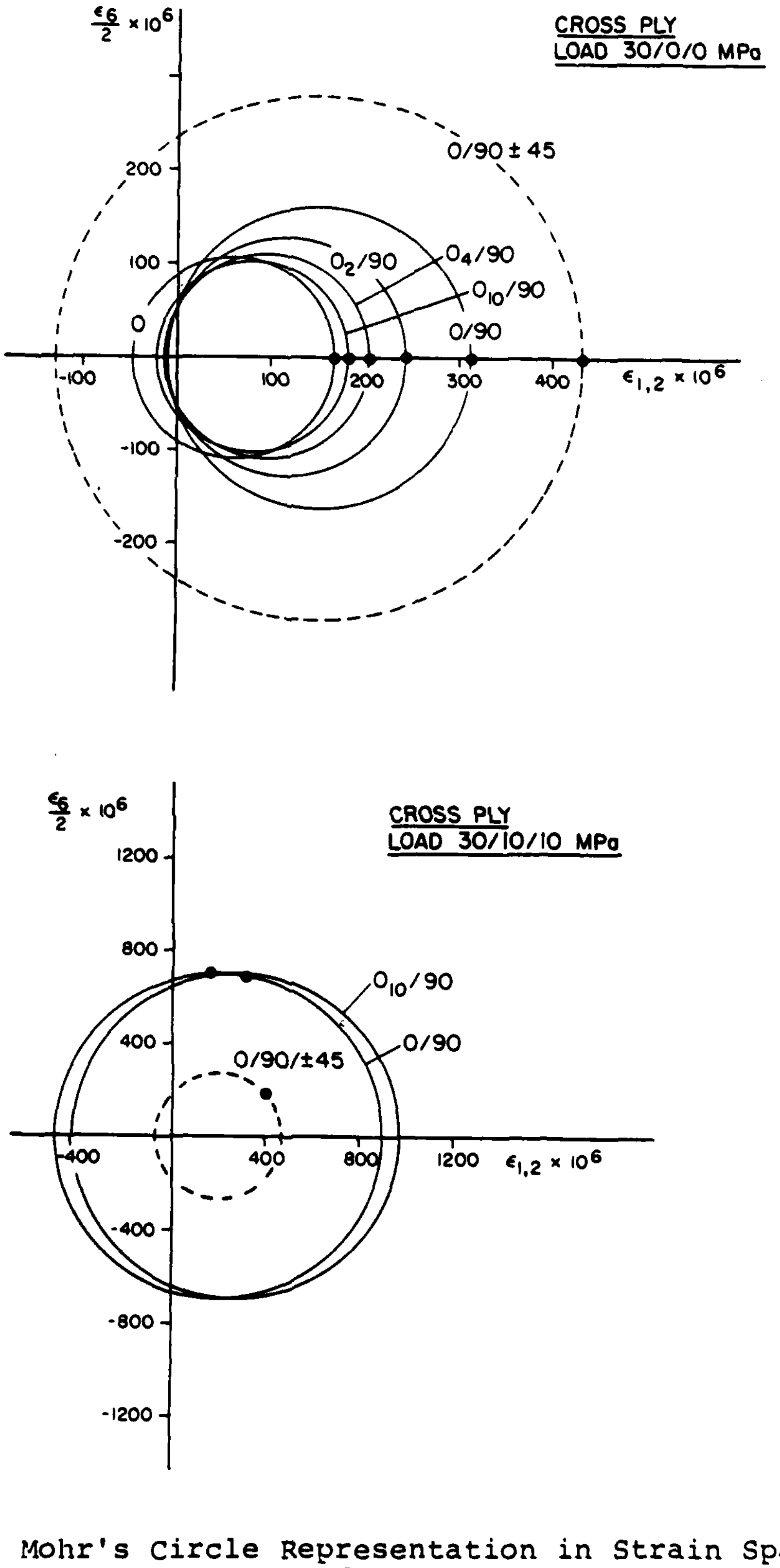


Figure 11B. Mohr's Circle Representation in Strain Space for Cross-Ply Configurations.

It can be shown that for a load combination which results from adding other loads or load combinations the p, q, and r values of the new combination are found by adding p, q, and r, respectively, of the individual loads. This means that the new I-plot, too, can be derived from the I-plots of the previous loads. Usually this is not true for R. With no shear strain and principal strains only, however, the radii may be simply added or subtracted, too. In Fig. 12A, adding the 30/0/0 MPa plots to the 0/10/0 MPa plots rendered the 30/10/0 MPa plots for I and R. Dashed lines indicate negative values for R. Caution must be used with the sign, which can mathematically be positive or negative. In Fig. 12A the positive amount is plotted, which makes sense because of its geometrical meaning. When the (positive) amount of R is added to I, we get the curves shown in Fig. 12B for the 3 sample loads. A look at these curves shows we find a minimum for 30/0/0 MPa loading with a UD laminate in the 1 direction, for 0/10/0 it is a UD laminate in the 2 direction and for 30/10/0 the minimum is found close to a cross-ply configuration considered optimum when netting analysis is used. Corresponding results were found for other loadings and also for angle plies, Fig. 13. The minimum of I + R in strain space under principal loads is linked to R reaching a zero-value, the corresponding Mohr's circle is reduced to a dot and the principal strains assume equal values.

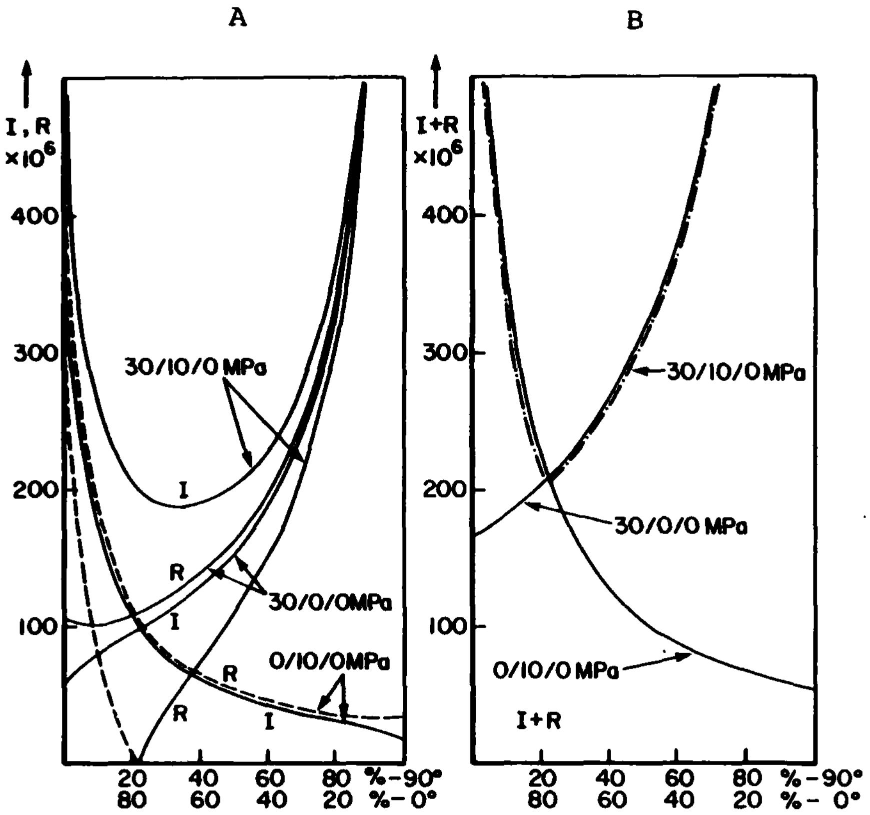


Figure 12. Mohr's Circle Representation in Terms of I, R (left) and I + R (right) for Cross-Ply Configurations.

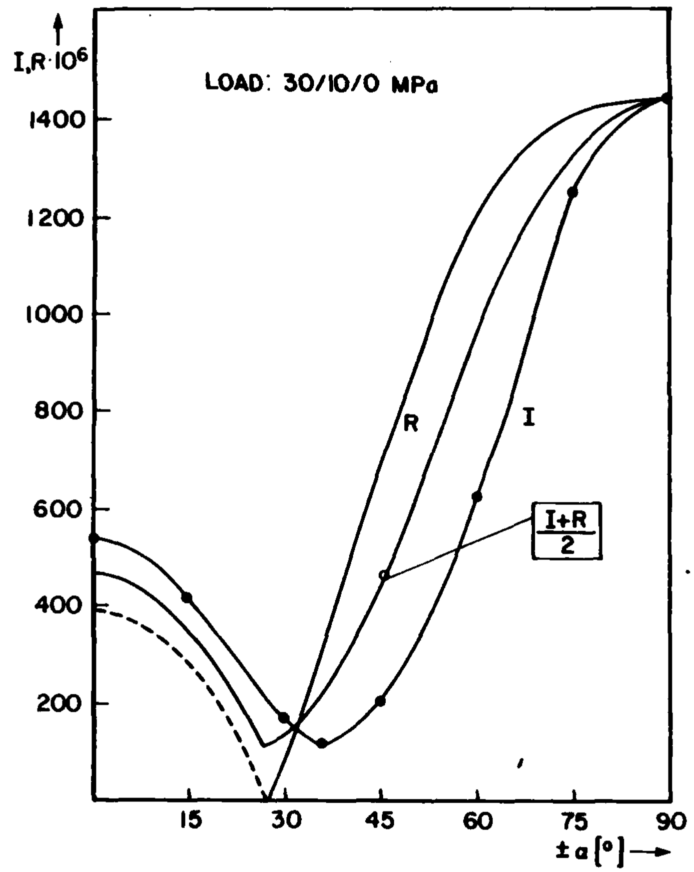


Figure 13. Mohr's Circle Representation in Terms of I, R for Angle-Ply Configurations.

## SECTION V
## MULTIPLE LOADING

An optimized cross-ply can be subjected to multiple loading, provided the different states of stress are well known.

If multiple loading means adding shear to given principal stress resultants, the optimized cross-ply will very soon be outperformed by a quasi-isotropic configuration, because the shear carrying capability of a cross-ply-configuration is rather limited as shown before. If multiple loading means adding shear combined with a decrease in the originally given principal loads, then the optimized cross-ply may be a good solution, Fig. 14. If multiple loading comprises changing load ratios with no change in directions in normal stress resultant space, optimized cross plies may also do better than a quasi-isotropic laminate. Take a load ratio of $N_{II}: N_{I} = 3:1$, the optimized cross-ply is $[0/90_3]$; the load ratio may now change from 3:1 past 0:1 to even -5:1 without failure, a wider range than the $[0/90/\pm 45]$ laminate offers, and with less plies. An optimized angle ply having the minimum required ply number for this 3:1 [MPa] load ratio would not sustain a change in load ratio; if, for example, the load ratio were changed to 0:1 [MPa], which actually means a significant load reduction, this angle ply laminate would fail because it is way off the optimum, Fig. 15.

Finally, a method shall be shown which enables a quick graphic overview on optimum cross-ply ratio and minimum ply number with adequate accuracy in simple multiple loading problems. If a detailed chart similar to Fig. 6 is available or generated for the material concerned, the curves for the individual loading combinations can be taken and superposed according to Figs. 16 and 17. The point of coincidence will indicate the cross-ply ratio that sustains either loading and also the minimum ply number needed. Let the multiple loads be 3:1:0 MPa and 1:3:0 MPa. The solution to this multiple loading problem is a $[0/90]_3$ cross-ply of not less than 68 plies, Fig. 16. When the multiple

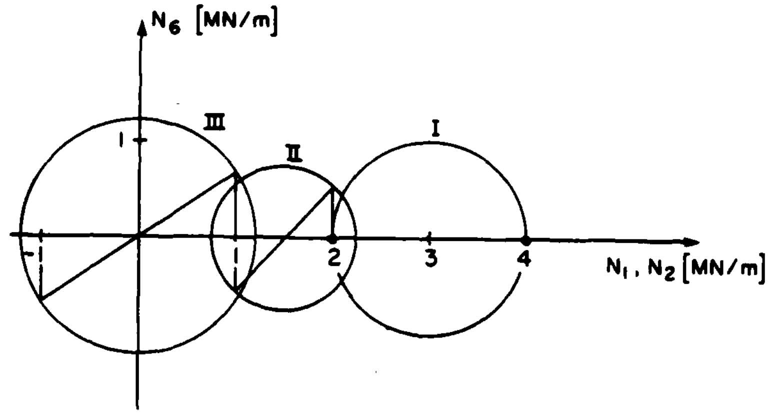


Figure 14. Multiple Loading; Shear Carrying Capability of $[0_2/90]$ Laminate, Optimized for $N_I/N_{II}=4/2$, Stress State I, at Different Stress States (II,III).

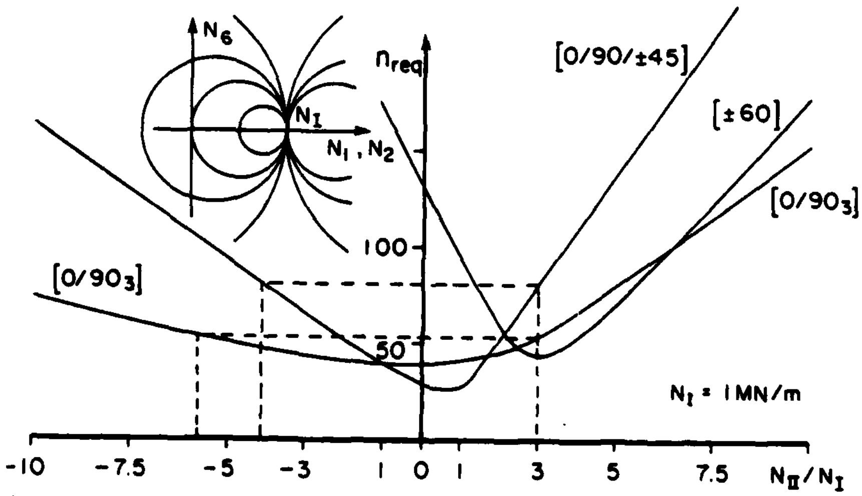


Figure 15. Multiple Loading; Ply Requirements for Different Load Ratios, Principal Directions do not Change.

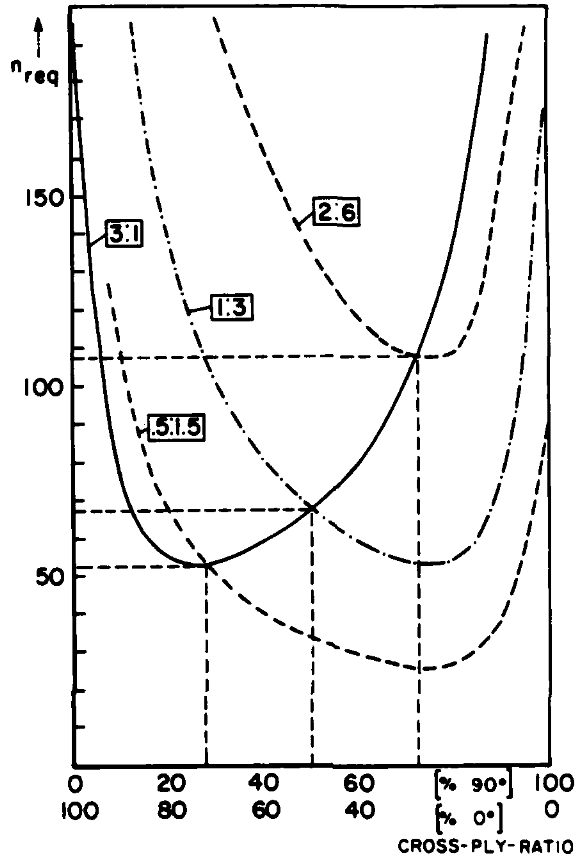


Figure 16. Graphical Determination of Optimum Cross-Ply Ratio and Ply Number (I).

loads are 3:1:0 MPa and 1:7:0 MPa the solution would be a cross-ply with 27% of all plies in 0° orientation, 73 in 90° orientation, minimum ply number about 112, Fig. 17. The curves for load combinations which are multiples of known combinations can fairly easily be generated by multiplying the existing values accordingly. This is also shown in Fig. 16; multiples of the 1:3:0 MPa loading combination, namely 2:6:0 MPa and 0.5:1.5:0 MPa, were gained by doubling and halving the original values. The new curves again are superposed to the 3:1:0 MPa curve. As before the point of coincidence yields the solutions.

The same results concerning the optimum cross-ply ratio were obtained for the sample loadings used in Figs. 16 and 17 when the (I + R) - curves first introduced in Section IV were superposed instead. This is demonstrated in Fig. 18 for the 3:1:0 MPa and 1:7:0 MPa multiple loads. However, there is no information on the minimum required ply number. As mentioned earlier, the minima of the corresponding individual curves also differ slightly according to which method is used.

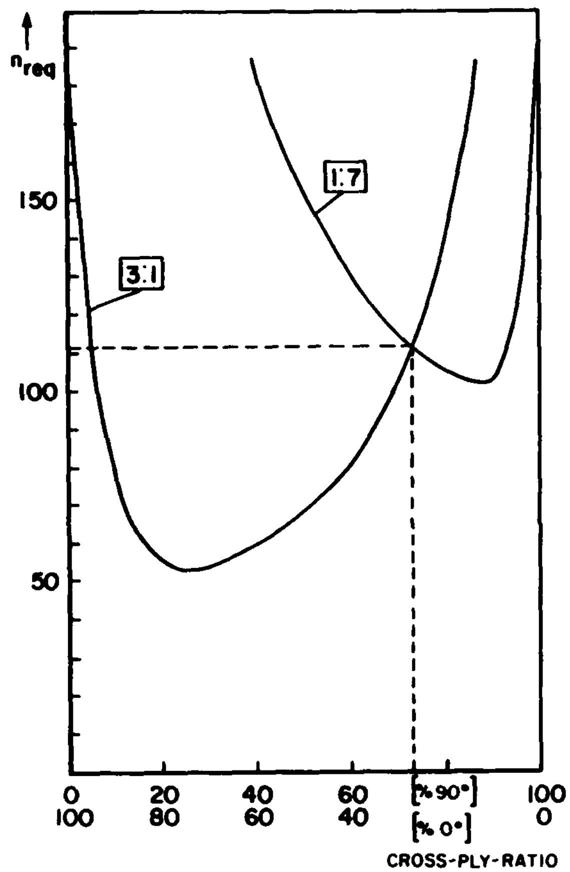


Figure 17. Graphical Determination of Optimum Cross-Ply Ratio and Ply Number (II).

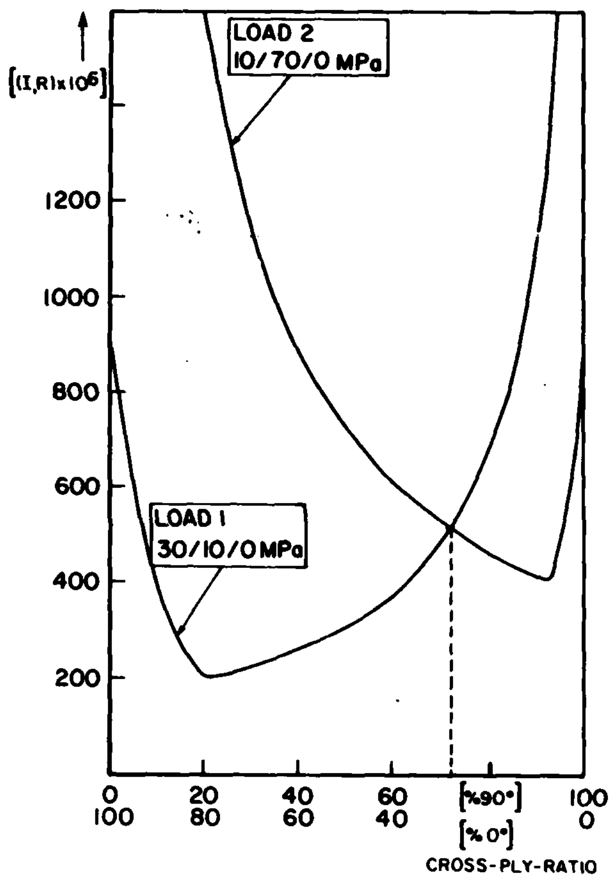


Figure 18. Graphical Determination of Optimum Cross-Ply Ratio Based on (I+R) Curves.

## SECTION VI
## CONCLUSION

When designing for weight, the introduction of quasi-isotropic composite configurations will but render the upper bound of the weight savings possible. It is shown that considerable higher weight savings may be obtained with symmetric, bidirectional laminate configurations, especially with cross-plies oriented in the shear-free directions. Though still orthotropic and thus easy to handle, they offer some of the advantages available when the anisotropy of composite materials is fully exploited. Simple charts can be generated that help the designer find the optimum cross-ply configuration when single or multiple loading is involved. Working with bidirectional laminates will facillitate a better understanding of the directional characteristics of fiber reinforced materials.

## REFERENCES


1. S. W. Tsai and H. T. Hahn, Introduction to Composite Materials, Technomic Publishing Co., Westport, CT, 1980.

2. S. L. Donaldson, Revised Instruction for Tl-59 Combined Card/Module Calculations for In-Plane and Flexural Properties of Symmetric Laminates, AFWAL-TR-82-4081, Air Force Materials Laboratory, 1982.

3. N. Balasubramanian, Failure Envelopes and Laminate Optimization for Composite Materials, 13th National SAMPE Technical Conference, Mt. Pocono, PA, Oct. 13-15, 1981.

4. Som Soni, Composite Laminate Analysis Program CLAP, AFWAL-Technical Report, Air Force Materials Laboratory (in preparation).

5. Won J. Park, An Optimal Design of Simple Symmetric Laminates Under the First Ply Failure Criteria, AFWAL-TR-81-4175, Air Force Materials Laboratory, 1982.

6. K. Ikegami et. al, Failure Criteria of Angle Ply Laminates of Fibre Reinforced Plastics and Applications to Optimize the Strength, Fibre Science and Technology, 1982, pp. 175-190.


* USGPO: 1983-659-062-1041

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