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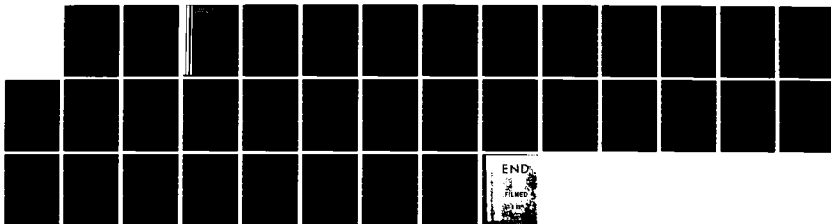
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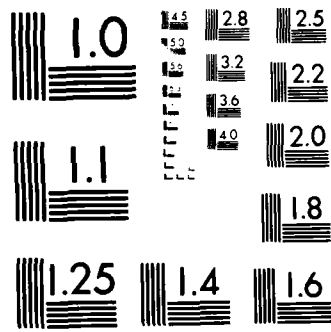
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RAYS AND MODES ON CONCAVE SURFACES

by

L. B. FELSEN

FINAL REPORT

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A

Summary

This Final Report summarizes the accomplishments under a research program concerned with the properties of electromagnetic fields, especially at high frequencies, in the presence of concave boundaries or interfaces. The program was initiated under Grant No. DAHC-04-75-G-0152 and continued under the present Contract No. DAAG-29-79-C-0005.

During the predecessor contract, a basic and new formulation for source-excited fields near a concave boundary was established. It had previously been recognized that ray methods break down when source and observer are near the boundary, but procedures to cope with this failure have been of limited utility. A fundamental achievement in our research effort was the recognition that spectral regions containing the "illegitimate" rays could alternatively be filled with guided modes, in this instance the whispering gallery modes, subject to a remainder field that can be made small but is nevertheless readily computable. This recognition then led to a hybrid ray-mode formulation that offers not only computational convenience but also grants new physical insight into the propagation process. These conclusions were reached by detailed study of the canonical problem comprising a line source in the presence of a perfectly conducting cylindrical surface.

The basic principles having been established, the hybrid ray-mode technique was then explored and extended under the present contract for a variety of different configurations. The two-dimensional generalizations have included circular boundaries with impedance conditions (possibility of having a surface wave), circularly stratified regions (with application to tropospheric propagation), and excitation of a cylindrically curved strip by edge diffraction of an incident field (application to curved reflector antennas). The method has also been extended to three dimensions by use of electric or magnetic dipole excitation instead of the line source. Each of these studies has contained not only the analysis but also extensive numerical comparisons that attest to the effectiveness of the hybrid scheme.

When both sides of a curved boundary are accessible (for example, on an interface between two different media or on a curved strip configuration), fields guided along the (penetrable) concave side or along the convex

side exhibit leakage due to the surface curvature. The commonly employed high-frequency description of these phenomena in terms of surface rays has limitations which had been incompletely explored and understood. Accordingly, we have undertaken to study this phenomenology in detail by considering again the circular boundary, with subsequent extension to non-circular smooth shapes. We have found that a proper ray description of the guiding process requires complex rays, which characterize the leaky guided (also the creeping wave) modes. Only at appropriately large distances from the boundary can the leakage field be described by the surface rays of GTD. There has therefore emerged a basic understanding of the high-frequency behavior of peripherally guided fields, which has direct relevance to the construction of a hybrid ray-mode field incorporating leaky and creeping wave modes.

A very recent application of the hybrid method to transient scattering has led to a deeper understanding of the relation between wavefronts (rays) and SEM (singularity expansion method) body resonances (modes), besides holding out a promise of more efficient calculation of the transient response. This new technique remains to be further explored.

The hybrid approach to guided propagation and to scattering, first formulated under this program, has had an impact on other disciplines and problem areas, including underwater acoustics and seismology. Extensions to still other applications are under consideration.

The accomplishments summarized above have been documented in articles published, or accepted for publication, in recognized journals. Those pertaining to the current contract period are listed below. For articles accepted but not yet published, the Abstracts and Introductions are included in an Appendix. Also included is a list of papers presented at technical meetings. Finally, to grant appreciation of the scope of the hybrid ray-mode method, a list of relevant publications pertaining to other problem areas is appended as well.

1. Publications During this Contract Period

E. Heyman and L. B. Felsen, "Creeping Waves and Resonances in Transient Scattering by Smooth Convex Objects," accepted for publication in IEEE Trans. on Antennas and Propagation.

E. Heyman and L. B. Felsen, "Evanescent Waves and Complex Rays for Modal Propagation in Curved Open Waveguides." Abstract and Introduction only. To appear in SIAM J. on Appl. Math.

L. B. Felsen and E. Heyman, "Traveling Wave and Oscillatory Formulations of Scattering Problems," To appear in Handbook on Acoustic, Electromagnetic and Elastic Wave Scattering--Theory and Experiment.

C. G. Migliora, L. B. Felsen and S.H. Cho, "High-Frequency Propagation in an Elevated Tropospheric Duct," IEEE Transactions on Antennas and Propagation, AP-30 (1982).

E. Topuz, E. Niver, and L. B. Felsen, "Electromagnetic Fields Near a Concave Perfectly Conducting Cylindrical Surface," IEEE Transactions on Antennas and Propagation, Vol. AP.30, No. 2, March 1982, pp. 280-292.

M. Idemen and L. B. Felsen, "Diffraction of a Whispering Gallery Mode by the Edge of a Thin Concave Cylindrical Surface," IEEE Trans. on Antennas and Propagation, AP-29, 1981, pp. 571-579.

E. Topuz and L. B. Felsen, "High Frequency Electromagnetic Fields on Perfectly Conducting Concave Cylindrical Surfaces," IEEE Transactions on Antennas and Propagation, Vol. AP-28, November 1980, No. 6.

2. Papers Presented at Technical Meetings

L. B. Felsen, "Progressive and Oscillatory Formulation of Propagation and Scattering," (invited; planary session) IEEE/AP-S and URSI Symposium, U. of Houston, Texas, May 1983.

E. Heyman and L. B. Felsen, "Evanescent Waves and Complex Rays for Curved Dielectric Waveguides," presented at the National Radio Science Meeting, Boulder, Colorado, January 13-15, 1982.

E. Heyman and L. B. Felsen, "A Hybrid Creeping Wave and SEM Approach to Transient Field Scattering by a Circular Cylinder," presented at the National Radio Science Meeting, Boulder, Colorado, January 13-15, 1982.

L. B. Felsen, "Ray and Modal Approaches to Guided Propagation," 20th URSI General Assembly, Washington, D. C., August 1981.

L. B. Felsen, "Hybrid Ray-Mode Method," (invited) 20th URSI General Assembly, Washington, D.C. 1981.

L. B. Felsen, "High-Frequency Signal Propagation and Scattering in Guiding Channels," AGARD Symposium on Terrain Profiles and Contours in Electromagnetic Wave Propagation, Spatind, Norway, September 10-14, p. 7, 1-3, 7, 1979.

S. H. Cho, C.G. Migliora and L. B. Felsen, "Hybrid Ray-Mode Formulation of Tropospheric Propagation," AGARD Symposium on Special Topics in HF Propagation, Lisbon, Portugal, May 28-June 1, 1979, p. 11, 1-8, 15.

L. Felsen, "Propagation Along Concave Surfaces," presented at URSI meeting, Seattle, Washington, June 18-22, 1979.

3. Publications in Related Areas, Not Sponsored Under this Program

L. B. Felsen and T. Ishihara, "Hybrid Ray-Mode Formulation of Ducted Propagation," J. Acoust. Soc. Am., March 1979.

L. B. Felsen, "Hybrid Ray-Mode Fields in Inhomogeneous Waveguides and Ducts," J. Acoust. Soc. Am., 69(2), February 1981, 352-361.

L. B. Felsen and A. Kamel, "Hybrid Ray-Mode Formulation of Parallel Plane Waveguide Green's Functions," IEEE Trans. on Antennas and Propagation, AP-29 (1981) 637-649.

A. Kamel and L. B. Felsen, "Hybrid Ray-Mode Formulation of SH Motion in a Two-Layer Half Space," Bull. Seismol. Soc. Am., 71, 6: 1763-1781 (1981).

E. Niver, S. H. Cho and L. B. Felsen, "Rays and Modes in an Acoustic Channel with Exponential Velocity Profile," Radio Science, Vol. 16, No. 6, 963-970 (November-December 1981).

A. Kamel and L. B. Felsen, "On the Ray Equivalent of a Group of Modes," J. Acoust. Soc. Am. 71 (1982) 1445-1452.

A. Kamel and L. B. Felsen, "Hybrid Green's Function for SH Motion in a Low Velocity Layer," Wave Motion 5 (1983) pp. 83-97.

APPENDIX

1. E. Heyman and L. B. Felsen, "Evanescent Waves and Complex Rays for Modal Propagation in Curved Open Waveguides." Abstract and Introduction only. To appear in SIAM Journal on Applied Math.
2. L. B. Felsen and E. Heyman, "Traveling Wave and Oscillatory Formulations of Scattering Problems." Introduction only. To appear in Handbook on Acoustic, Electromagnetic and Elastic Wave Propagation, V.K. and V.V. Varadan (editors), North Holland Publishing Co.
3. E. Heyman and L. B. Felsen, "Creeping Waves and Resonances in Transient Scattering by Smooth Convex Objects," to appear in IEEE Transactions on Antennas and Propagation.

Traveling Wave and Oscillatory Formulations of
Scattering Problems

by

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I. Introduction

Transient radiation from, and scattering by, objects of finite size is important for applications pertaining to antennas and to target identification. The behavior of the radiated or scattered field depends markedly on the spectral range of the exciting illumination. At very high frequencies or short wavelengths, geometric-optical methods^{1, 2} based on local properties of the radiator or scatterer are accurate and efficient whereas at low frequencies or long wavelengths, the quasi-static volume effect of the object plays a crucial role.³ In the intervening range, the so-called "resonance region" where object dimensions are comparable to the wavelength, neither of these approximate analytical techniques applies but numerical methods may be employed to advantage. These features become operative collectively when the exciting transient field resembles an impulse so that its spectrum spans the entire frequency range. Here, the singularity expansion method (SEM),⁴ which represents the object response as a series of damped oscillations, is most effective at the intermediate and lower frequencies relevant to intermediate and long observation times although efforts have been made to show that it can in principle (but with some difficulty) accommodate even the high frequency regime descriptive of the wavefront and early time response. More effective, and physically more transparent during early times, is the ray-like and causal tracking of the wavefronts per se, and the field not too far behind the wavefronts. This procedure, however, becomes cumbersome at moderately late times when very many such ray-type fields must be considered, and high frequency approximations are no longer applicable. Most promising in this connection is a hybrid formulation which combines SEM resonances and wavefront representations within a single framework that seeks to exploit the advantages of each.⁵

This chapter deals with the detailed description of the scattering processes outlined above, concentrating in the time-harmonic regime on high frequencies where the local plane wave behavior of the fields makes ray formulations possible. Relevant concepts are illustrated in their most fundamental form by consideration of a two-dimensional cylindrical smooth convex scattering object. Before presenting the analytical details, we give a physically motivated overview of the wave phenomena to be encountered. The scatterer is assumed to be

excited by a prescribed incident field which is specularly reflected according to the rules of the geometrical theory of diffraction (GTD) and, through glancing incidence, diffracted in the form of creeping waves (see Fig. 1).^{1, 6} While the creeping waves, which incorporate the above-mentioned wavefronts in the transient regime, can be expressed as surface ray fields progressing circumferentially around the object and shedding energy tangentially, their properties are inferred from those of waves that are guided around the object due to the surface curvature. This conclusion emerges from the rigorous analysis of circular cylindrical canonical prototype problems. Thus, an understanding of the creeping wave surface ray fields of GTD requires a treatment of these fields as guided modes⁷ synthesized by self-consistent modal-ray congruences.⁸ These modal considerations, to be elaborated on below, clarify the excitation and shedding mechanisms of the resulting surface rays of GTD.

When an object is penetrable, the reflected fields of GTD are augmented by refracted and internally reflected rays which eventually find their way back to the outside. We shall not deal further with these conventional ray types. Concerning diffracted ray fields, a glancing incident ray also excites a lateral ray which sheds energy into the interior at the angle of critical refraction, in addition to leaking into the exterior because of surface curvature (Fig. 2).^{9, 10} This "leaky lateral" ray field is closely related to the creeping rays on a penetrable boundary. Moreover, sources near the scatterer may excite internal whispering gallery (WG) modes which are guided along the concave boundary by local total reflection and are evanescent (decay exponentially) away from the boundary on the outside (Fig. 2). Similar evanescent waves may be generated on an impedance boundary which has an appropriate reactive component. Because of the surface curvature, these exterior waves become less evanescent with distance from the surface and eventually become propagating when the local circumferential phase velocity equals the velocity of propagation in the ambient medium (Fig. 2).^{11, 12}

Because of the external leakage, the circumferential propagation coefficients of the above-noted surface guided fields are complex. Therefore, the description of these high-frequency guided waves involves local plane waves with complex phase. Such wave fields can be expressed either in terms of local evanescent plane waves in the physical coordinate space^{13, 14} or, more generally and effectively, as local complex plane waves in a complex coordinate space.^{14, 15} The theory of complex rays may be treated as an analytic extension of the conventional theory for ordinary real rays; the

physically meaningful evanescent wave fields are recovered by appropriate intersections of the complex rays with real space. The discussion of these general ray fields must proceed along two separate routes depending on whether such fields arise from exciting sources or are source-free. In the first instance, the prescribed excitation by a localized source or by a surface distribution gives rise to an initial value problem. The ray fields traveling away from the source region, obtained by local ray tracing as schematized in Figs. 1 and 2, are determined from these initial values.¹⁵⁻¹⁹ In the second instance, the existence of a source-free solution as in a guided mode requires globally self-consistent ray congruences which convert coherently one into the other so as to synthesize a standing wave field that satisfies the required boundary conditions.^{8, 20} This process is schematized in Fig. 3(a) for the interior problem of a lossless impenetrable concave boundary. Here, an ordinary real ray incident on the boundary gives rise to a specularly reflected real ray which travels away from the boundary to a maximum distance d and thereafter approaches the boundary again. The same considerations apply to any other incident and reflected ray combination. The system of real rays generated in this manner describes incident and reflected ray congruences, respectively, if both ray systems are tangent to a single real envelope, the caustic, $d(\ell)$. The caustic may therefore be regarded as the generating surface for both the incident and reflected ray systems whereon rays reflected geometrically at the boundary are converted into incident rays. If the global systems of incident and reflected rays are phase coherent so that a surface perpendicular to each ray congruence is an equiphase surface, then the resulting closed ray system synthesizes a source-free field. Phase coherence may be phrased to require that the total phase accumulation along a full cycle of incidence and reflection, including phase change at the boundary and at the caustic, is a multiple p of 2π (see Fig. 3(a)); each integer multiple defines its own caustic $d_p(\ell)$.

The field u_p resulting by superposition of the incident and reflected contributions is oscillatory and represents a guided mode confined essentially to the region d . On the "dark" side of the caustic, which is not penetrated by real rays, the fields are evanescent; their description requires the introduction of complex rays (see Fig. 3(a)).^{21, 22}

The same considerations can be applied to guided modes with leakage. Here, the congruences involve complex rays generated by a complex caustic and reflected from a boundary surface extended into a complex coordinate space (see Fig. 3(b)). The various types of guided modes are characterized by the relative location of the caustic with respect to the boundary.

The preceding discussion has dealt with the surface diffracted ray fields (i. e., the guided modal fields) as traveling waves which circumnavigate the scatterer repeatedly without constraint of periodicity (see Fig. 4). Yet the actual fields in the presence of a scattering object with finite cross section must be periodic in the peripheral coordinate. This implies that the totality of individually non-periodic surface ray fields is periodic. The collective behavior of the surface ray fields is, in fact, found to form the basis for synthesis of body resonances, the SEM resonances mentioned earlier, that occur at real or complex frequencies for non-leaky internal or leaky external wave processes, respectively.²³ For a perfect electrically conducting or an acoustically soft circular cylinder and an assumed harmonic time dependence $\exp(-i\omega t)$, these complex resonances for the exterior problem are arranged throughout the complex frequency plane as shown in Fig. 5.

Under transient conditions of excitation, the wave field constituents described above for the time-harmonic regime may be resolved individually because different phase accumulations along ray paths are then translated into different arrival times of causal fields at the observation point. The first appearance of a particular field along a ray trajectory, i. e., the arrival of its wavefront, is synthesized by waves with high frequency content. This validates use of the various high-frequency asymptotic ray and guided mode descriptions for observations times near a first arrival.^{24, 25} For later observation times, when lower frequencies play a dominant role, high-frequency ray fields are no longer applicable and are also inconvenient because with reference to the diffracted fields, contributions from many circumnavigations of the scatterer must be included. Here, a more effective description is based on the collective treatment of the circumferentially traveling guided wave fields since the resulting wave types then account for the object response as a whole rather than by successive circumferential samples. This collective description is in terms of the above-mentioned body resonances at real or complex characteristic frequencies, which depend on the object shape and composition but

not on the form of the excitation.⁴ When the transient field is described as a Laplace inverse of the time-harmonic response, the characteristic frequencies appear as pole singularities in the complex frequency plane, and their temporal contribution is in terms of undamped or damped sinusoidal oscillations due to real and complex poles, respectively. This method of representing the transient field, the singularity expansion method (SEM), can be validated in principle over the entire range of observation times if the resonant frequencies are determined with sufficient accuracy²⁶ but its implementation is difficult at early times when the incident wavefront has not yet fully traversed the object surface (see Fig. 6).²³ Many body resonances are required to synthesize a zero field on the non-illuminated portion of the object, and to describe the abrupt transition across the impinging wavefront. A hybrid formulation,⁵ which retains some of the traveling wave constituents effective at early times, near the wavefront, and those few of the SEM resonances effective at later times (see Fig. 7), forms an alternative that makes the transient field calculation more efficient and also describes the relevant physical wave processes in their most fundamental form.

The phenomenology of scattering by a smooth convex object thus having been described in physical terms, we now proceed to the analysis that justifies the description. It is assumed that the reader is familiar with asymptotic ray theory as presented, for example, in references 2,27. Therefore, the treatment of ray theory per se is restricted here to summarizing the fundamental concepts and defining the quantities needed in the subsequent ray formulation of guided wave fields.

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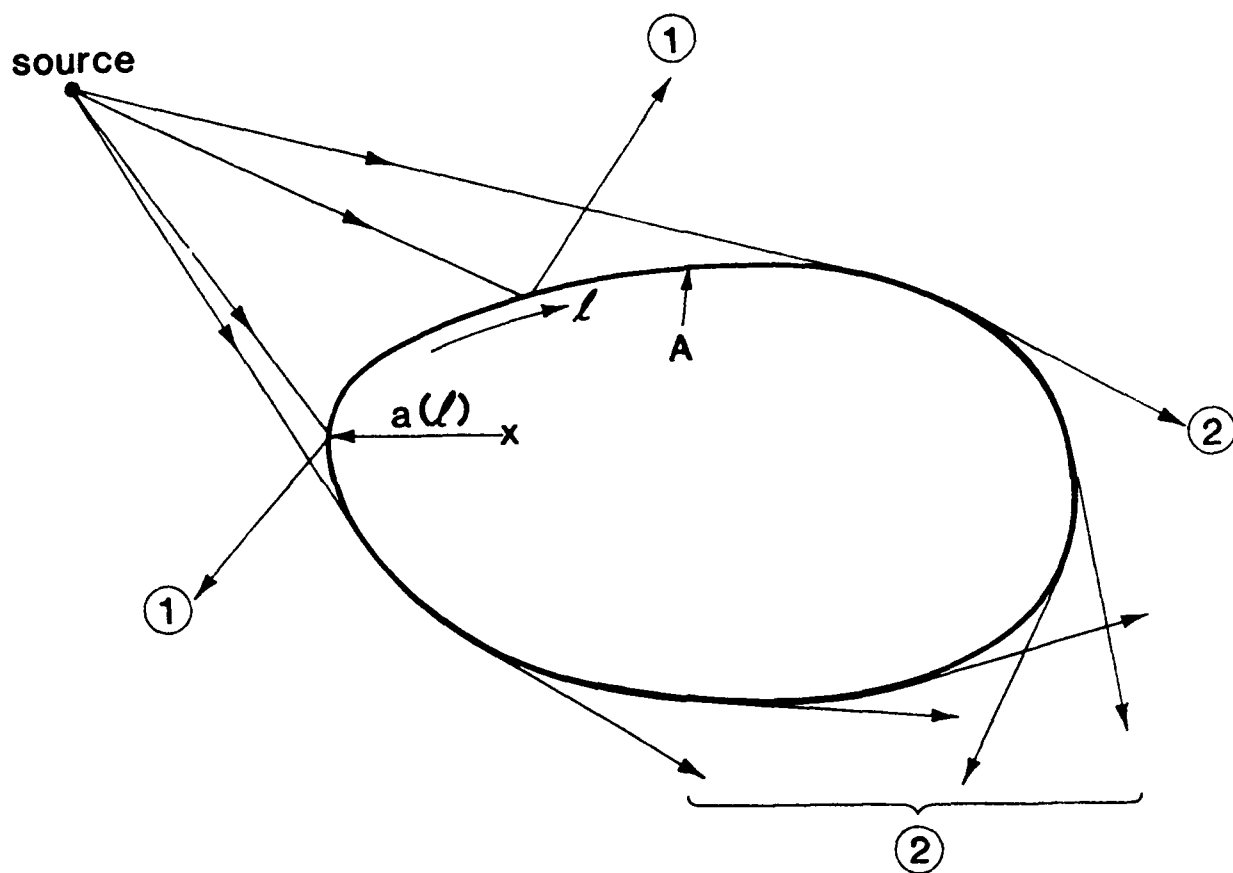


Fig. 1 Ray field contributions to high-frequency scattering by an impenetrable convex body. Distance on boundary A is measured by l and the local radius of curvature is $a(l)$. (1) Reflected rays. (2) Diffracted rays (creeping waves). For the detailed propagation mechanism for the creeping waves, see (Fig. 3(b)).

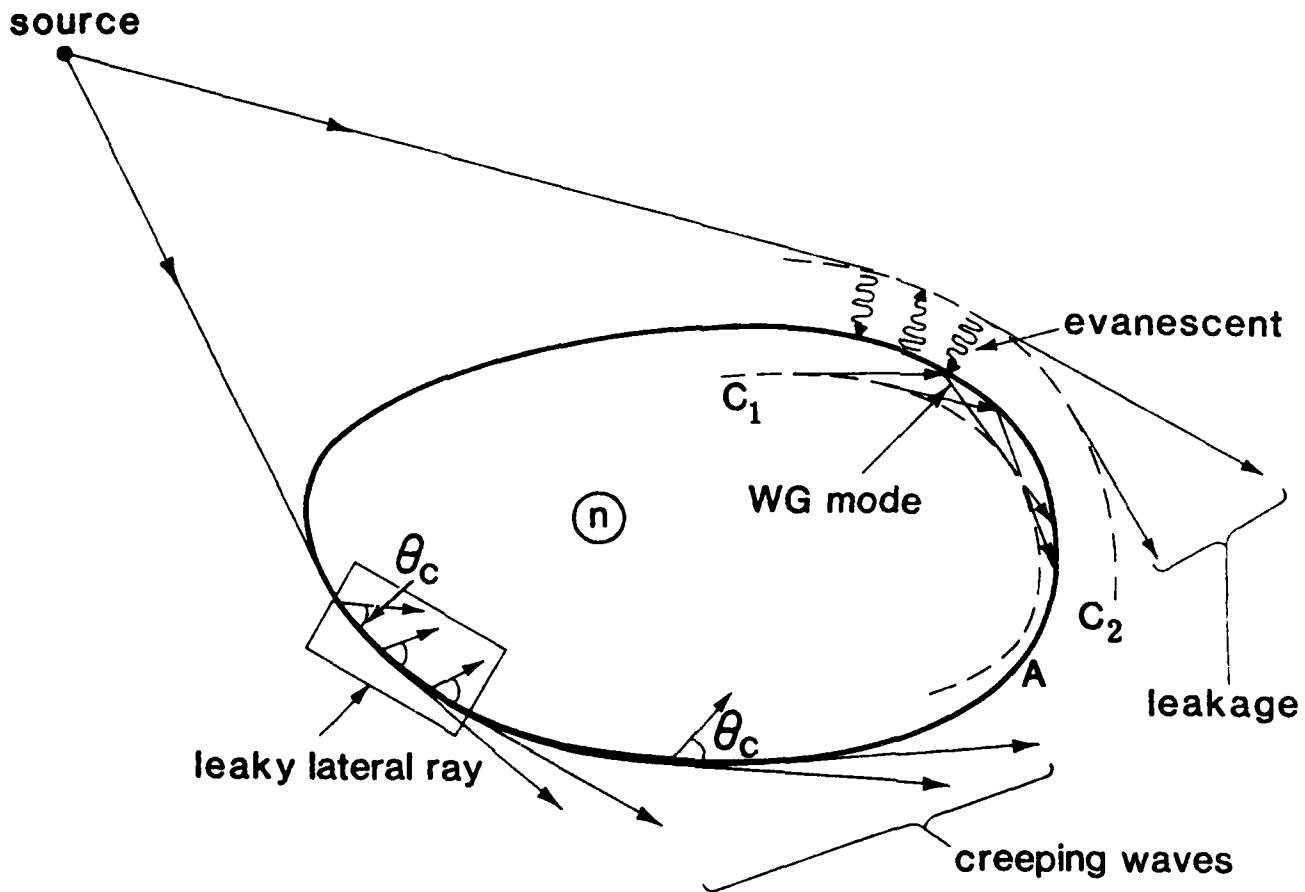
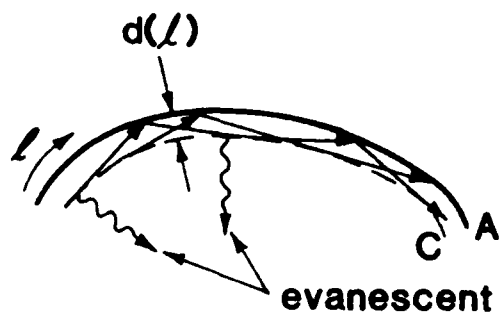
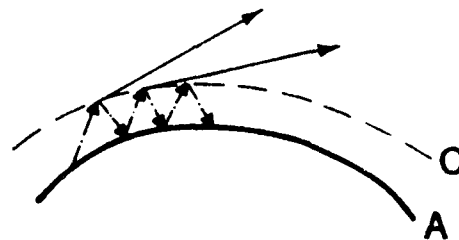


Fig. 2

Diffracted ray field contributions to high-frequency scattering by a penetrable convex body with refractive index n . A glancing incident ray excites a leaky lateral ray which is subsequently converted into creeping waves; here, θ_c denotes the critical angle. An almost glancing ray from a nearby source also excites, by evanescent tunneling, internally trapped WG modes which, in turn, leak energy to the exterior by the same tunneling mechanism. The internally guided field for a typical mode is confined between the boundary B and the internal caustic C_1 , while radiation in the exterior occurs from the caustic C_2 . The rays inside schematize the two congruences which synthesize the WG modal field (see also Fig. 3).



(a) Whispering gallery mode



(b) Creeping wave mode

Fig. 3 Real (\rightarrow) ray congruences, complex (\dashrightarrow) ray congruences and caustics ($- - -$) for modal propagation along (a) a concave and (b) a convex boundary. Distance along the boundary B is measured by l , and the distance between B and a caustic C is $d(l)$. For the creeping wave, the caustic and the outgoing exterior ray congruence are slightly complex.

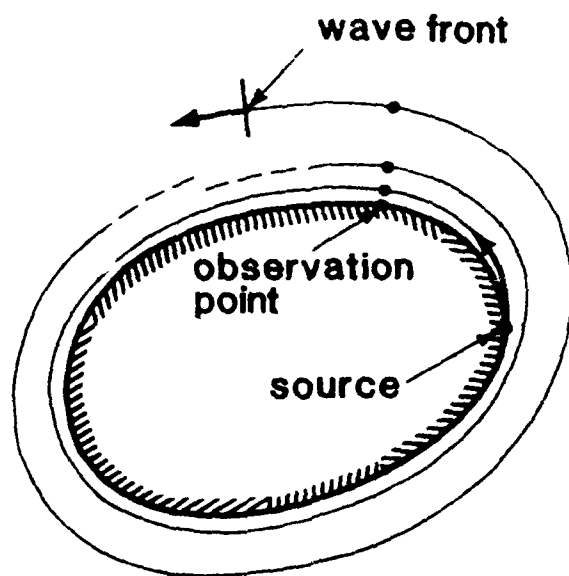


Fig. 4 Time evolution a surface guided mode. The field at the observation point is the sum of contributions due to each passage of the wave front.

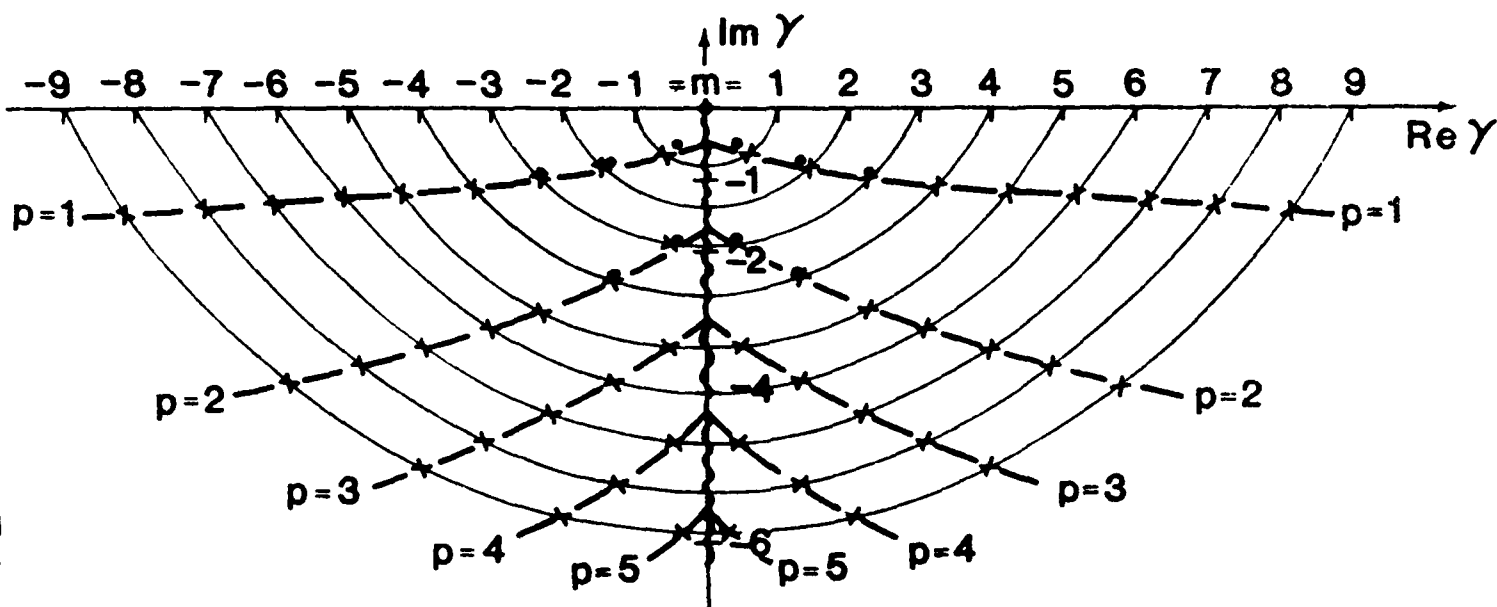


Fig. 5

Complex resonances of perfect electrically conducting, or acoustically soft, circular cylinder in the $\gamma = ka$ plane. Here, $k = \omega/v$, with ω denoting the complex frequency, v the wave propagation speed in the exterior medium, and "a" the radius of the cylinder. A resonance γ_{mp} (identified by x) is defined by two indexes m and p . The resonances can be ordered along "arcs" and "layers" characterized by $m = \text{constant}$ and $p = \text{constant}$, which describe standing wave and traveling (creeping) wave processes, respectively. For detailed discussion, see Sec. III. 2. 2, equations (184) and (194).

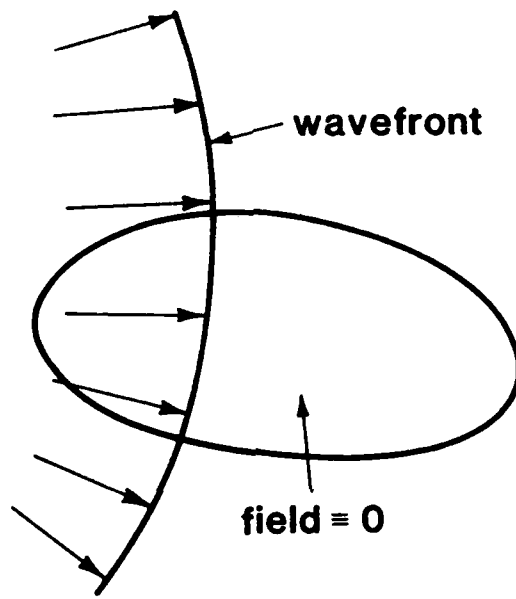


Fig. 6 Early time illumination.

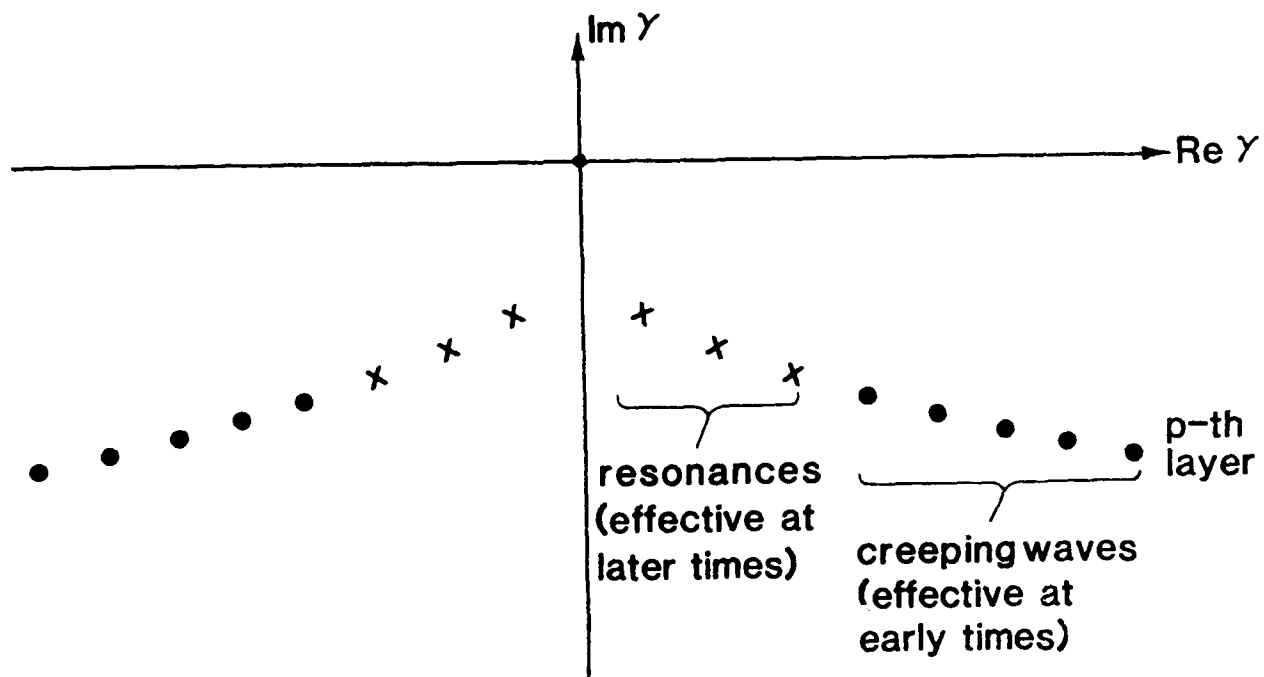


Fig. 7 Hybrid decomposition of resonances in the layer for the p-th creeping wave. \times - resonances retained explicitly. \bullet - resonances summed collectively and expressed in terms of contributions from the latest arrivals of the creeping wave.

EVANESCENT WAVES AND COMPLEX RAYS FOR MODAL
PROPAGATION IN CURVED OPEN WAVEGUIDES*

by

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ABSTRACT

Evanescent wave tracking (EWT) has provided a new and effective method for analyzing high-frequency trapped mode propagation in longitudinally homogeneous open dielectric waveguides. It has therefore been suggestive to extend EWT so as to account for mode field leakage when the waveguide is curved. A circularly bent two-dimensional guiding structure has served as a prototype in the present study. Even in this canonical configuration, direction generalization of the EWT procedure, attempted first, has not been achieved so as to bridge the transition from the predominantly evanescent to the predominantly radiating regime as the observer moves away from the guide axis on the convex side. Therefore, the problem has been analyzed alternatively by complex ray tracing (CRT) that involves continuation of initial conditions, boundaries, etc., into a complex coordinate space. This approach, structured around the complex ray congruences generated by a complex caustic, has been successful. By self-consistent closure of the complex ray system, it has been possible to describe all of the various types of guided modes that may arise on a circular guiding structure. When the waveguide axis deviates weakly from circularity, local closure by CRT defines the properties of local modes that adapt continuously to the changing guiding environment. The CRT procedure therefore provides a unified and systematic treatment of a broad class of high-frequency guiding problems, which can then be physically interpreted in terms of EWT.

I. Introduction

Natural and man-made open guiding configurations often exhibit deviations of the waveguide axis from a straight path. A guided mode, which is trapped in the straight structure so that its transverse field decays exponentially toward infinity, becomes leaky on the convex side when the guiding axis possesses curvature. Even a perfectly conducting boundary, when convex, may support guided creeping waves that shed energy continually as they progress. Various mechanisms have been proposed to explain the leakage phenomena in physical terms and to construct a mathematical description accordingly. The recently developed evanescent wave tracking (EWT) theory [1,2] seemed to hold promise for providing an alternative interpretation that grants new physical insights as well as a mathematical framework for quantitative calculation of leaky propagation at high frequencies. By EWT, a field that is locally evanescent near a curved guiding structure becomes less so as the observer moves away on the convex side, and the local evanescent plane waves are converted eventually into the local non-evanescent plane waves of conventional ray theory. The rules of EWT permit the tracking of local evanescent plane waves in real space along phase paths or attenuation paths that define the propagation or attenuation properties, respectively, of the complex phase as well as the algebraic amplitude of the local field. However, EWT as developed in [1,2], does not accommodate the evanescent to non-evanescent transition that is characterized by an almost real "quasi-caustic" of the leaky ray structure.

It was suggestive to seek a generalization of EWT to describe the transition behavior. The canonical problem of a leaky "slow wave" complex phase distribution on an initial convex circular boundary was chosen as a rigorously solvable prototype, with which the generalized EWT was to be compared. The EWT generalization, attempted by solving for the phase paths and attenuation paths (phase fronts) by perturbing the known trajectories corresponding to real initial phase, is described in Section II of this paper. Even in the small leakage limit, the perturbation fails near the quasi-caustic of the evanescent to radiating transition. Although a uniform Airy-type transition function can be extracted from the rigorous solution, we have not found a simple way to patch such a function onto the EWT evanescent and non-evanescent local plane wave fields on either side when the boundary curvature is non-circular.

To overcome these difficulties, we decided to abandon the conceptually simple but mathematically involved tracking of evanescent waves in real space in favor of ray tracing along straight trajectories in complex space [3]. This is done in Section III by extending the initial complex phase condition into a complex coordinate space and studying the corresponding complex ray configurations. The complex caustic of the complex ray system and the ray congruences in its interior [4, 5] and exterior provide the building blocks from which one may construct the phase paths and phase fronts of EWT in real space. Since the caustic is now complex, no patching is required in real space; instead, the local plane wave transition from evanescent to radiating, as described by strongly and weakly complex rays, respectively, occurs continuously. The complex caustic forms the envelope of outgoing and incoming ray congruences and may be regarded as the contact surface whereon an incoming ray is converted into an outgoing ray. When a circular boundary (also extended into complex space) is placed concentric with the caustic, conversion from incoming to outgoing rays with respect to this boundary takes place by reflection. This double conversion, at the caustic and the boundary, may form the basis for invoking a closure condition that makes the ray fields self-consistent [6]. These self-consistent fields then represent an eigenmode of the guiding structure. As shown in Section IV, the relative location of the caustic with respect to the boundary determines whether an externally guided mode is essentially trapped (weak leakage) or essentially radiating (strong leakage) [7]. Various surface conditions, ranging from perfectly conducting through surface impedances capable (or not) of supporting a surface wave to transparent access to a second medium, illustrate the variety of possible mode solutions of the exterior (strongly and weakly leaky) and interior (whispering gallery) type. These results for circular azimuthally symmetric configurations can be generalized to boundaries deviating weakly from circularity [8]. It is found that ray closure can be invoked locally provided that the complex caustic is located near the bounding surface, i. e., when a boundary layer exists near the surface. In the absence of a boundary layer, when the caustic is far from the boundary, we have not been able to achieve closure and thereby show

existence of a local guided mode.

The complex ray construction of the modal fields is presented in Section V. A uniform complex ray representation of the Kravtsov-Ludwig type [9, 10] in Section VI, required near an "almost real" caustic or when the caustic is near the boundary, refines our presentation to yield a formulation in agreement with previous results [11, 12]. The paper concludes with a summary in Section VII.

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CREEPING WAVES AND RESONANCES IN TRANSIENT SCATTERING BY SMOOTH CONVEX OBJECTS*

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ABSTRACT

Scattering by smooth convex objects excited by a transient field with broad spectral content has been analyzed either by ray formulations, which are useful at observation times descriptive of the early arrivals, or by the complex resonances of the singularity expansion method (SEM), which are most appropriate at intermediate and late observation times. Within the framework of SEM, efforts have recently been made to show that in a grouping of the resonances along "layers" rather than along the conventional "arcs" based on an angular harmonic field representation, the higher order resonances behave collectively like a wave traveling circumferentially around the object. This observation has provided the stimulus for the present investigation to place the relation between the wavefront arrivals (creeping waves) and the SEM resonances on a rigorous basis. Using a perfectly conducting circular cylinder as a canonical model, this is done by direct application of the theory of characteristic Green's functions to construct alternative field solutions, and by collective summation of groups of wavefront arrivals or groups of resonances. The connection between creeping waves and resonances thus having been established, hybrid formulations are developed that combine the creeping waves and the SEM resonances within a single rigorous framework so as to maximize the utility of each over the entire range of observation times. These results are then generalized to smooth cylindrical objects with non-circular convex shape.

I. Introduction

Transient radiation from, and scattering by, objects of finite size is becoming increasingly important in applications pertaining to antennas and to target identification. The behavior of the radiated or scattered field depends markedly on the spectral range of the exciting illumination. At very high frequencies or short wavelengths, geometric-optical methods based on local properties of the radiator or scatterer are accurate and efficient whereas at low frequencies or long wavelengths, the quasi-static volume effect of the object plays a crucial role. In the intervening range, the so-called "resonance region" where object dimensions are comparable to the wavelength, neither of these approximate analytical techniques applies but numerical methods may be employed to advantage. When the exciting transient field resembles an impulse so that its spectrum spans the entire frequency range, no single method has been found fully acceptable. The singularity expansion method (SEM) [1], which represents the object response as a series of damped oscillations, is most effective at the intermediate and lower frequencies relevant to intermediate and long observation times although efforts have been made to show that it can in principle (but with some difficulty) accommodate even the high frequency regime descriptive of the wavefront and early time response. More effective, and physically more transparent during early times, is the ray-like and causal tracking of the wavefronts per se, and the field not too far behind the wavefronts [2]. This procedure, however, becomes cumbersome at moderately late times when very many such ray-type fields must be considered, and high frequency approximations are no longer applicable.

The preceding observations indicate why transient scattering problems have usually been attacked by two disjoint procedures: a) ray expansions for very early times, and b) SEM expansions for later times [3]. Attention has recently been given to how the two procedures can be related. Studies have been performed on canonical configurations such as a perfectly conducting sphere or cylinder for which rigorous solutions can be constructed in alternative forms. With respect to SEM, the conventional grouping of the pole singularities in the complex frequency (ω) plane, which furnish the resonant damped oscillations, has been carried out along "arcs" that are identified by the index m of the angular harmonics in the periodic angular

coordinate space (see Fig. 1). Within every arc (i.e., for every m), there exists a finite number of poles tagged by different values of the index p . Thus, every complex frequency pole $\omega_{m,p}$ may be identified by its indexes (m,p) . It has recently been suggested that an alternative grouping in "layers" identified by the index p provides a better understanding and physical interpretation of the relevant wave processes; in every layer, p is constant and there exists an infinite number of poles identified by different values of the index m . In fact, numerical experiment has revealed that the higher order (large m) poles on a layer interfere collectively in such a manner as to simulate the peaked behavior of the field associated with a wavefront traveling around the object [4,5]. Attempts have also been made to relate this phenomenon to a "creeping wave" representation of the transient field, whereby the synthesis over the (m,p) constituents is first carried out by summing over p (which identifies the creeping waves) and then over m , instead of following the conventional reverse procedure by summing first over the angular harmonics identified by m .

Motivated by these earlier investigations, we seek in the present paper to place the creeping wave representation of transient scattering and its relation to the angular harmonic SEM representation on a systematic and rigorous basis, using directly the various alternative eigenfunction expansions available for the transient field in the presence of a perfectly conducting circular cylinder. The SEM resonances are obtained by imposing a self-consistency condition on the creeping waves which revolve around the cylinder; i.e., after one revolution these fields should exhibit phase coherence with the fields at the starting point. The asymptotic properties of this creeping wave expansion yield naturally the physical "turn-on times" of the transient SEM series, which describe the arrivals of the wavefronts at the observation point [6]. Accordingly, there are two SEM series: one for the clockwise and the other for the counterclockwise propagating waves. The analysis also shows that the transient SEM series can be applied at a certain time before these turn-on times, yielding zero field during that interval.

Having established this connection, which also identifies the role of causality in the SEM treatment, we then give attention to the formulation of a hybrid procedure wherein both the creeping wave and SEM formulations

are combined within a single framework that draws upon the relative advantages of each [7]. The hybrid formulations are based on the collective treatment of either a selected group of creeping waves or a selected group of SEM resonances. At long observation times, when the creeping waves have undergone many circumnavigations of the cylinder, one may construct a hybrid field wherein a few of the latest creeping wave arrivals are kept explicitly but all of the remaining earlier arrivals, which contain low frequency constituents, are expressed collectively and efficiently by lower order (small m) SEM resonances. A "collective" creeping wave accounts in this formulation for the truncation of the creeping wave series. By an alternative route, starting from the complete SEM series, one may sum the higher order (large m) contributions on the p -th layer collectively to obtain a "collective" SEM field whose peak travels around the cylinder with the speed approximating the group velocity of the last included SEM pole. This representation, which involves again the explicit lower order SEM resonances effective for the low frequency spectral components and the collective SEM field efficient for the high frequency components, puts the results of the previously noted numerical experiment [5] on a rigorous foundation. These aspects being clarified for the canonical circular cylinder, they are then extended to a smoothly curved cylinder of arbitrary convex shape.

The above-described objectives are addressed here as follows. The selected prototype problem, formulated in Section II, seeks the determination of the surface currents induced by an impulsive line source of magnetic currents located on the cylinder surface. The transient field is Fourier transformed to reduce it to the time-harmonic domain. Alternative field representations, based on the method of characteristic Green's functions, are constructed in Section III directly and without intervention of the Watson transformation as in reference 4 to yield expansions in terms of angular eigenfunctions (conventional angular harmonics), radial eigenfunctions (creeping waves), and resonant eigenfunctions (SEM). This presentation emphasizes particularly the various ways of dealing with the creeping waves in the time-harmonic and the transient regimes, and the connection between the creeping waves and the SEM resonances. These basic results are then employed in Section IV to develop alternative hybrid formulations

by collective treatment of a group of creeping waves or of a group of SEM resonant solutions. The generalization to scattering by cylindrical objects with arbitrary convex curvature is given in Section V, and the paper concludes with a summary in Section VI.

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