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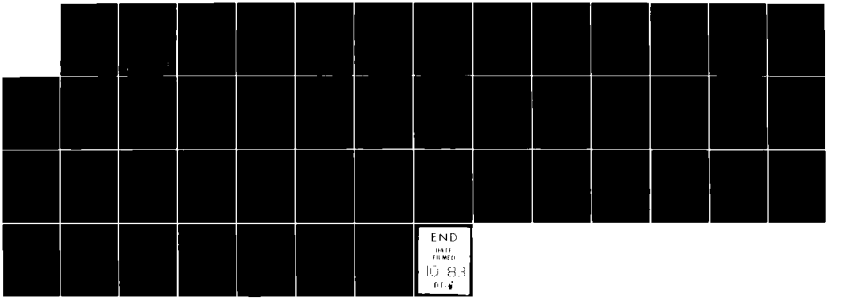

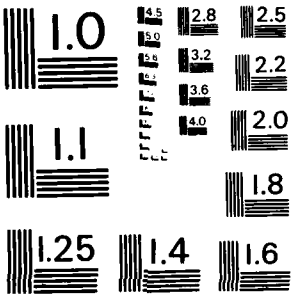

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ADAPTIVE ARRAY BEHAVIOR WITH PERIODIC PHASE MODULATED INTERFERENCE

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16. Abstract (Limit: 200 words)

This report discusses the behavior of an LMS adaptive array [1] when receiving a phase modulated interference signal. A mathematical technique is presented for computing the time behavior of the array weights with such interference. This technique is then used to study the behavior of a 2-element array receiving an interference signal with sinusoidal phase modulation. The effects of the interference phase modulation on desired signal modulation, output SINR (signal-to-interference-plus-noise ratio) and desired signal bit error probability are discussed.

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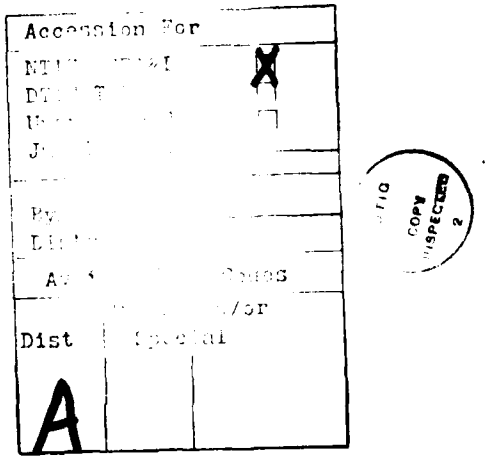

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I. INTRODUCTION

The performance of an LMS (least mean square) adaptive array [1] can be influenced by the modulation on an interference signal. For example, pulsed interference can make the weights in the array alternate back and forth between two sets of values, one when the interference is on and the other when it is off [2]. Interference modulation has two deleterious effects on the array. First, it can make the array modulate the desired signal. Second, it can cause the array output SINR (signal-to-interference-plus-noise ratio) to vary with time. In a digital communications system, such SINR variation usually results in an increased bit error probability. For these effects to occur, the modulation parameters must be properly chosen. For example, the modulation rate must be commensurate with the array speed of response.

In previous studies, the authors have examined the effects on an adaptive array of interference with pulsed modulation [2], with sinusoidal, suppressed carrier envelope modulation [3] and with arbitrary periodic envelope modulation [4]. Each of these studies involved interference with envelope modulation but not phase modulation.

The purpose of the present report is to extend the earlier work to handle interference with phase modulation. We assume an adaptive array receives an interference signal with a periodic but otherwise arbitrary phase modulation. We present a method of solving for the resulting time-varying array weights. Using this method, we then study as an

example the behavior of a 2-element array that receives an interference signal with sinusoidal phase modulation. We show how each interference signal parameter affects the desired signal modulation, the SINR variation and the bit error probability.

In Section II, we formulate the problem and present the mathematical technique for solving for the array weights. In Section III, we describe the 2-element array and the phase modulated interference signal and present our numerical results. Section IV contains the conclusions.

II. FORMULATION OF THE PROBLEM

Assume an adaptive array consists of J isotropic elements with half wavelength spacing, as shown in Figure 1. Let $\tilde{x}_j(t)$ be the analytic signal received on element j. $\tilde{x}_j(t)$ is multiplied by a complex weight w_j and then summed to produce the array output $\tilde{s}(t)$. The array weights are controlled by LMS (least mean square) feedback loops [1], which obtain each weight w_j by integrating the product of $\tilde{x}_j(t)$ with the error signal $\tilde{\varepsilon}(t)$. The error signal is the difference between the reference signal $\tilde{r}(t)$ and the array output $\tilde{s}(t)$. The array weights satisfy the differential equation

$$\frac{dW}{dt} + k\Phi W = kS \ , \tag{1}$$

where $W = [w_1, w_2, \ldots, w_J]^T$ is the weight vector, t is time, Φ is the the covariance matrix,

$$\Phi = E(X^* X^T) \quad , \tag{2}$$

S is the reference correlation vector,

$$S = E[X^* \tilde{r}(t)] \quad , \tag{3}$$

and k is the LMS loop gain. In these equations, X is the signal vector,

$$X = [\tilde{x}_1(t), \tilde{x}_2(t), \ldots, \tilde{x}_J(t)]^T \quad , \tag{4}$$

T denotes transpose, * complex conjugate and $E(\cdot)$ expectation.

We assume that a desired signal and an interference signal are incident on the array and that thermal noise is present in each element signal. The signal vector then contains three terms,

$$X = X_d + X_i + X_n \quad , \tag{5}$$

where X_d, X_i, and X_n are the desired, interference and noise vectors, respectively.

Let the desired signal be a CW (single frequency) signal incident from angle θ_d relative to broadside. (θ is defined in Figure 1.) The desired signal vector is then

$$X_d = A_d \, e^{j(\omega_0 t + \psi_d)} U_d \quad , \tag{6}$$

where A_d is the amplitude, ω_0 is the carrier frequency, ψ_d is the carrier phase angle, and U_d is a vector containing the interelement phase shifts,

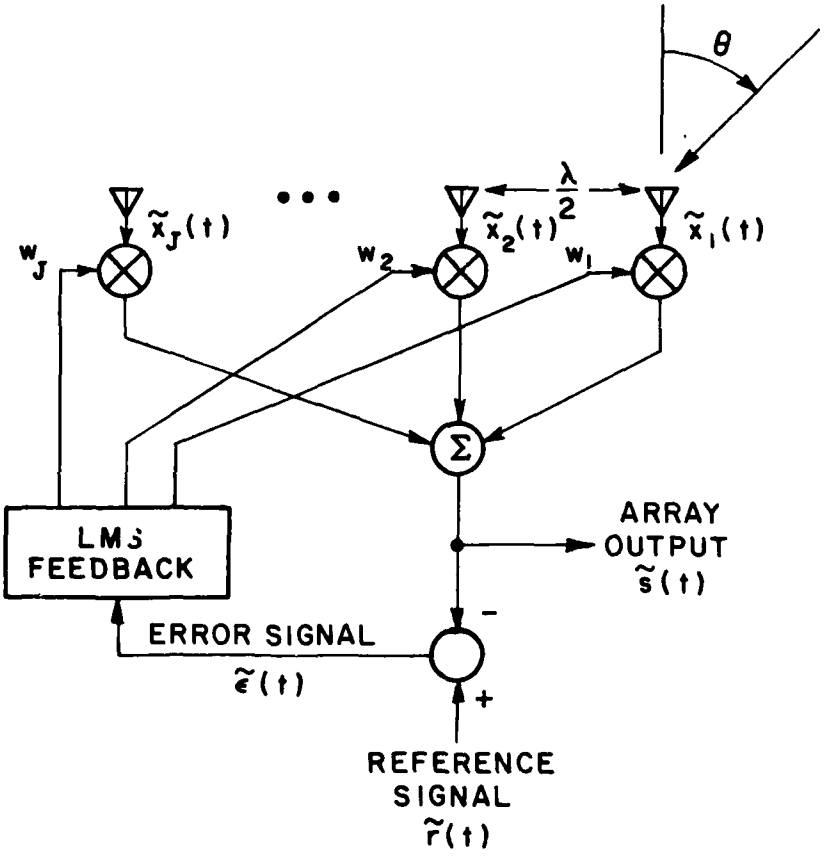

Figure 1. The LMS Adaptive Array.

$$U_d = [1,\ e^{-j\phi_d},\ \ldots,\ e^{-jK\phi_d}]^T \quad , \tag{7}$$

where

$$\phi_d = \pi \sin \theta_d \quad , \tag{8}$$

and where, to simply notation, we let

$$K = J - 1 \quad . \tag{9}$$

We assume ψ_d is a random variable uniformly distributed on $(0,2\pi)$.

Next, assume an angle modulated interference signal arrives from angle θ_i. The interference signal vector is

$$X_i = A_i e^{j(\omega_0 t + \psi_i)} U_i(t) \quad , \tag{10}$$

with

$$U_i(t) = \begin{pmatrix} e^{j\gamma_i(t)} \\ e^{j[\gamma_i(t-T_i)-\phi_i]} \\ e^{j[\gamma_i(t-2T_i)-2\phi_i]} \\ \cdot \\ \cdot \\ \cdot \\ e^{j[\gamma_i(t-KT_i)-K\phi_i]} \end{pmatrix} \quad , \tag{11}$$

where $\gamma_i(t)$ is the angle modulation as received on element 1, A_i is the interference amplitude and ψ_i is the carrier phase angle. T_i and ϕ_i are the interelement time delay and carrier phase shift,

$$T_i = \frac{\pi}{\omega_o} \sin \theta_i \quad , \tag{12}$$

and

$$\phi_i = \omega_o T_i = \pi \sin\theta_i \quad . \tag{13}$$

We assume ψ_i is a random variable, uniformly distributed on $(0,2\pi)$ and statistically independent of ψ_d.

Finally, we assume each element signal contains a zero-mean, independent Gaussian thermal voltage $\tilde{n}_j(t)$ of power σ^2. The noise vector is

$$X_n = [\tilde{n}_1(t), \tilde{n}_2(t), \ldots, \tilde{n}_J(t)]^T \quad , \tag{14}$$

where

$$E[\tilde{n}_j^*(t)\tilde{n}_k(t)] = \sigma^2\delta_{jk} \quad , \quad 1 \leq j, k \leq J \quad , \tag{15}$$

with δ_{jk} the Kronecker delta. The $\tilde{n}_j(t)$ are assumed statistically independent of both ψ_d and ψ_i.

Under these assumptions, the covariance matrix in (2) is the sum of a desired, an interference and a noise term,

$$\Phi = \Phi_d + \Phi_i + \Phi_n \quad . \tag{16}$$

The desired signal term is

$$\Phi_d = E(X_d^* X^T) = A_d^2 U_d^* U_d^T \quad . \tag{17}$$

The interference term is

$$\Phi_i = E(X_i^* X_i^T) = A_i^2 \begin{bmatrix} 1 & e^{j[x_{01}(t)-\phi_i]} & \cdots & e^{j[x_{0K}(t)-K\phi_i]} \\ e^{j[x_{10}(t)+\phi_i]} & 1 & \cdots & e^{j[x_{1K}(t)-(K-1)\phi_i]} \\ \vdots & \vdots & & \vdots \\ e^{j[x_{K0}(t)+K\phi_i]} & e^{j[x_{K1}(t)+(K-1)\phi_i]} & \cdots & 1 \end{bmatrix} \tag{18}$$

where

$$x_{pq}(t) = \gamma_i(t-qT_i) - \gamma_i(t-pT_i) \quad , \tag{19}$$

and where p and q are integers between 0 and K. The noise term is

$$\Phi_n = \sigma^2 I \quad , \tag{20}$$

with I the identity matrix.

To compute the reference correlation vector S in (3), we must specify the reference signal $\tilde{r}(t)$. In practical applications, the reference signal is derived from the array output [5,6,7]. To make the array perform properly, it must be a signal correlated with the desired signal and uncorrelated with the interference. Here we assume the reference signal to be a replica of the desired signal,

$$\tilde{r}(t) = A_r \, e^{j(\omega_o t + \psi_d)} \quad , \tag{21}$$

where A_r is its amplitude. Equation (3) then yields

$$S = A_r \, A_d \, U_d^* \quad . \tag{22}$$

When (16) and (22) are substituted into (1), we find that the weights satisfy a system of differential equations with constants on the right but with time-varying coefficients (due to Φ_i). In the rest of this section, we develop a method for solving this system of equations.

To solve (1), we do several things. First, we write the interference covariance matrix in a more compact form. We define J vectors,

$$V_o = (1, \, 0, \, 0 \ldots \, 0,0)^T \quad ,$$

$$V_1 = (0, \, e^{-j\phi_i}, \, 0, \, \ldots \, 0,0)^T \quad ,$$

$$V_2 = (0, \, 0, \, e^{-j2\phi_i}, \, \ldots, \, 0,0)^T \quad ,$$

$$\vdots$$

and

$$V_K = (0, 0, 0 \ldots 0, e^{-jK\phi_i})^T \quad . \tag{23}$$

Note that these J vectors form an orthonormal set,

$$V_j^T V_k^* = \delta_{jk} \ , \quad 0 \leq j,k \leq K \quad . \tag{24}$$

By using these vectors, Φ_i in (18) may be written

$$\Phi_i = A_i^2 \sum_{p=0}^{K} \sum_{q=0}^{K} e^{jx_{pq}(t)} V_p^* V_q^T \ , \tag{25}$$

so (1) becomes

$$\frac{dW}{dt} + k \left[A_d^2 U_d^* U_d^T + A_i^2 \sum_{p=0}^{K} \sum_{q=0}^{K} e^{jx_{pq}(t)} V_p^* V_q^T + \sigma^2 I\right] W = kA_r A_d U_d^* \quad . \tag{26}$$

Next, we normalize (26). Dividing by $k\sigma^2$ gives

$$\frac{dW(t')}{dt'} + \left[\Phi_2 + \xi_i \sum_{p=0}^{K} \sum_{q=0}^{K} e^{jx_{pq}(t')} V_p^* V_q^T\right] W(t') = \frac{A_r}{\sigma} \sqrt{\xi_d}\, U_d^* \ , \tag{27}$$

where

$$\Phi_2 = I + \xi_d U_d^* U_d^T \ , \tag{28}$$

$$\xi_d = \frac{A_d^2}{\sigma^2} = \text{input signal-to-noise ratio (SNR) per element,}$$

$$\xi_i = \frac{A_i^2}{\sigma^2} = \text{input interference-to-noise ratio (INR) per element,}$$

and where

$$t' = k\sigma^2 t = \text{normalized time} \ . \tag{29}$$

Also, we note that the constant $\frac{A_r}{\sigma}$ on the right will just appear as a scale factor in the solution for $W(t')$. It has no effect on the array output SINR to be discussed below. Hence we arbitrarily set $\frac{A_r}{\sigma} = 1$ to eliminate it. Equation (27) is then

$$\frac{dW(t')}{dt'} + [\Phi_2 + \xi_i \sum_{p=0}^{K} \sum_{q=0}^{K} e^{j\chi_{pq}(t')} V_p^* V_q^T]W(t') = \sqrt{\xi_d}\, U_d^* \ . \tag{30}$$

Next, in order to be able to solve (30), we make the important assumption that the angle modulation $\gamma_i(t)$ is periodic. If $\gamma_i(t)$ is periodic, the functions $e^{j\chi_{pq}(t')}$ are also periodic. Therefore we may expand each of them in a Fourier series. In order to simplify notation later, we shall also include the constant ξ_i in this expansion. Thus, we write

$$\xi_i\, e^{j\chi_{pq}(t')} = \sum_{\ell=-\infty}^{\infty} f_{\ell pq}\, e^{j\ell\omega_m' t'} \ , \tag{31}$$

where $f_{\ell pq}$ is the ℓ^{th} Fourier coefficient of $\xi_i\, e^{jx_{pq}(t')}$ and ω_m' is the normalized fundamental frequency of $\gamma_i(t')$, i.e.,

$$\omega_m' = {}^{\omega_m}/_{k\sigma^2} \quad , \tag{32}$$

where ω_m is the fundamental frequency of $\gamma_i(t)$. We note for later use that if p=q, then $x_{pq}(t') = 0$, so the series in (31) contains only a zero frequency term, i.e.,

$$f_{\ell pp} = \xi_i\, \delta_{\ell o} \quad . \tag{33}$$

Also, it is easily shown that

$$f_{\ell qp} = f^*_{-\ell pq} \quad , \tag{34}$$

and

$$f_{\ell(p+1)(q+1)} = f_{\ell pq}\; e^{-j\ell\omega_m' T_i'} \quad . \tag{35}$$

In addition to assuming $\gamma_i(t)$ periodic, we shall also assume that the bandwidth of the interference is finite, i.e., we assume that only a finite number of terms are nonzero in (31). Suppose the coefficients in (31) are zero for $|\ell| > L$, where L is some integer.

Then

$$\xi_i\, e^{jx_{pq}(t')} = \sum_{\ell=-L}^{L} f_{\ell pq}\, e^{jn\omega_m' t'} \quad . \tag{36}$$

Since (30) is a linear differential equation with periodic coefficients, the solution for W(t') will be a periodic function of time after any initial transients have died out [8]. In this paper, we shall not attempt to solve for initial transients, but shall consider only the periodic steady-state solution. Once any initial transients are over, W(t') can be written as a Fourier series,

$$W(t') = \sum_{n=-\infty}^{\infty} C_n \, e^{jn\omega_m' t'} \quad , \tag{37}$$

where C_n is a vector Fourier coefficient. Substituting this series into (30) and enforcing the resulting equation for each frequency component separately gives

$$(\Phi_2 + jn\omega_m' I)C_n + \sum_{\ell=-L}^{L} \sum_{p=0}^{K} \sum_{q=0}^{K} f_{\ell pq} V_p^* V_q^T C_{n-\ell} = \sqrt{\xi_d} U_d^* \delta_{no}, \quad -\infty < n < \infty \; . \tag{38}$$

Equation (38) is an infinite system of vector equations, one for each n. To solve for the C_n, we first assume that there is some integer N such that the Fourier coefficients C_n are negligible for $|n| > N$. In other words, we assume W(t') can be adequately approximated by a finite sum

$$W(t') = \sum_{n=-N}^{N} C_n \, e^{jn\omega_m' t'} \quad . \tag{39}$$

Such an approximation is reasonable because the feedback loops controlling the array weights in W(t') are lowpass filter loops that cannot respond above a certain speed.

If we set $C_n=0$ for $|n| > N$, then (38) yields a finite system of equations for the remaining C_n. Each vecton C_n has J scalar components, so the result is a system of $(2N+1)J$ linear equations in the unknown scalar components. One could solve for $W(t')$ by solving this system of equations numerically.

However, rather than work with (38) directly, we shall instead transform variables first. We do this because the resulting equations have a more systematic form. Specifically, we shall express each C_n in terms of its components along the vectors V_k in (23). (Since each C_n has J components, the J orthonormal vectors V_k can serve as basis vectors.) We write

$$C_n = \sum_{k=0}^{K} \alpha_{n,k} V_k^* \quad , \tag{40}$$

where the $\alpha_{n,k}$ are scalar coefficients. $\alpha_{n,k}$ is the component of C_n along the unit vector V_k^*. Substituting (40) into (38), multiplying the result on the left by V_a^T (for a=0,1,2, ..., K) and using (24) yields the equation

$$jn\omega_m' \alpha_{n,a} + \sum_{k=0}^{K} Q_{ak} \alpha_{n,k} + \sum_{\ell=-L}^{L} \sum_{q=0}^{K} f_{\ell aq} \alpha_{n-\ell,q} = \sqrt{\xi_d} (V_a^T U_d^*) \delta_{no} \quad , \tag{41}$$

where

$$Q_{ak} = V_a^T \Phi_2 V_k^* \quad . \tag{42}$$

In (41), we have $0 \leq a \leq K$ and it is understood that $\alpha_{m,q} = 0$ whenever $|m| > N$. Q_{ak} is readily found from (23) and (28):

$$Q_{ak} = \delta_{ak} + \xi_d \, e^{j(a-k)(\phi_d - \phi_i)} \quad . \tag{43}$$

Also, we find from (7) and (23) that

$$V_a^T \, U_d^* = e^{ja(\phi_d - \phi_i)} \quad , \quad 0 \leq a \leq K \quad . \tag{44}$$

Equation (41), when written out, yields a finite system of equations of the form

$$MA = B \quad , \tag{45}$$

where A is a vector containing the unknown coefficients $\alpha_{n,k}$,

$$A = (\alpha_{N,0}, \alpha_{N,1}, \cdots, \alpha_{N,K}, \alpha_{N-1,1}, \cdots, \alpha_{N-1,K}, \cdots, \alpha_{-N,1}, \cdots, \alpha_{-N,K})^T \, , \tag{46}$$

B is a vector obtained from the right hand side of (41), and M is the matrix of coefficients obtained from the left side of (41). We give an example of M and B below in Section III. The numerical results presented below have been obtained by solving (41) numerically.

For this method to yield accurate results, N must be chosen large enough that at least 2LJ of the $\alpha_{n,k}$ are essentially zero on each end of the vector A in (46). If this vector has 2LJ zeros on each end, the solution obtained from (41) will yield the same result as the solution

of the infinite system in (38). In practice, a suitable value of N may be determined by increasing N until the A vector has 2LJ negligible terms on each end and until the values of the $\alpha_{n,k}$ in the middle of the vector A are unaffected by further increases in N. Experience shows how large N must be in specific cases*. Once the $\alpha_{n,k}$ have been found, the C_n may be found from (40) and W(t') from (39).

The time-varying array weights have two effects on array performance. They cause the array to modulate the desired signal, and they cause the array output SINR to vary with time. In the remainder of this section, we discuss the calculation of these effects.

Given a time-varying weight vector W(t'), the desired signal component of the array output is

$$\tilde{s}_d(t') = A_d\, W^T(t')\, U_d\, e^{j(\omega_o' t' + \psi_d)} \quad , \tag{47}$$

where $\omega_o' = \omega_o/k\sigma^2$. To study the modulation on $\tilde{s}_d(t)$, we define

$$a_d(t')\, e^{j\eta_d(t')} = A_d W^T(t') U_d \quad . \tag{48}$$

Then $a_d(t') = A_d|W^T(t')U_d|$ is the envelope modulation and $\eta_d(t') = \langle W^T(t')U_d$ is the phase modulation. Furthermore, we define $a_{dn}(t')$ to be

*The numerical solution of (41) was done using a Gauss elimination routine with full pivoting [9]. We have not examined the eigenvalue behavior of M in detail, but we did not experience any convergence problems or other unusual behavior due to ill-conditioning with this approach.

the envelope normalized to its value in the absence of interference, i.e.,

$$a_{dn}(t') = \frac{a_d(t')}{A_d |W_o^T U_d|} \quad , \tag{49}$$

where W_o is the steady-state weight vector that would occur without interference,

$$W_o = (\Phi_d + \Phi_n)^{-1} S \quad . \tag{50}$$

(Φ_d, Φ_n and S are given in (17), (20) and (22).) Our results below are presented in terms of $a_{dn}(t')$, because the effect of the interference can be seen directly by comparing $a_{dn}(t')$ with unity.

The output desired signal power is

$$P_d(t') = \frac{1}{2} E[|s_d(t')|^2] = \frac{1}{2} A_d^2 |W^T(t') U_d|^2 \quad . \tag{51}$$

The output interference power is

$$P_i(t') = \frac{1}{2} A_i^2 |W^T(t') U_i(t')|^2 \quad , \tag{52}$$

where $U_i(t')$ is the vector of phasors given in (11) but written in terms of normalized time $t' = k\sigma^2 t$. The output thermal noise power is

$$P_n(t') = \frac{\sigma^2}{2} W^T(t') W^*(t') \quad . \tag{53}$$

From these quantities, the output interference-to-noise ratio (INR),

$$\text{INR} = \frac{P_i(t')}{P_n(t')} \quad , \tag{54}$$

and the output signal-to-interference-plus-noise ratio (SINR),

$$\text{SINR} = \frac{P_d(t')}{P_i(t') + P_n(t')} \quad , \tag{55}$$

may be computed as functions of t'.

In the next section, we present an example using this technique and discuss the effects of a phase modulated interference signal on the adaptive array.

III. AN EXAMPLE

Consider an array with two elements, so J=2 and K=1. Let the interference have sinusoidal phase modulation,

$$\gamma_i(t') = \beta \sin \omega_m' t' \quad , \tag{56}$$

where β is the maximum phase deviation (or the modulation index) and ω_m' is the normalized modulation frequency. Then one finds from (19) and standard trigonometric identities that

$$e^{j\chi_{01}(t')} = e^{j\beta' \sin[\omega_m' (t' - \frac{T_i'}{2}) - \frac{\pi}{2}]} \quad , \tag{57}$$

where

$$\beta' = 2\beta \sin \frac{\omega_m' T_i'}{2} \quad , \tag{58}$$

(and $T_i' = k\sigma^2 T$). Using the Fourier Series expansion [10],

$$e^{jz\sin\rho} = \sum_{\ell=-\infty}^{\infty} J_\ell(z) e^{j\ell\rho} \quad , \tag{59}$$

we find

$$\xi_i \, e^{j\chi_{01}(t)} = \sum_{\ell=-\infty}^{\infty} f_{\ell 01} \, e^{j\ell\omega_m' t'} \quad , \tag{60}$$

with

$$f_{\ell 01} = \xi_i J_\ell(\beta') \, e^{-j\ell \left(\frac{\omega_m' T_i'}{2} + \frac{\pi}{2}\right)} \quad . \tag{61}$$

From (34), we also have

$$f_{\ell 10} = f^*_{-\ell 01} = (-1)^\ell f_{\ell 01} \quad , \tag{62}$$

since

$$J_{-\ell}(\beta') = (-1)^\ell J_\ell(\beta') \quad . \tag{63}$$

To truncate the series in (60), we use the fact that

$$J_\ell(z) \cong 0 \text{ for } |\ell| > z + 1 \quad . \tag{64}$$

Thus, we approximate

$$\xi_i e^{jx_{01}(t')} = \sum_{\ell=-L}^{L} f_{\ell 01} e^{j\ell\omega_m' t'} \quad , \tag{65}$$

with

$$L = \{\beta' + 1\} = \{2\beta \sin \frac{\omega_m' T_i'}{2} + 1\} \quad , \tag{66}$$

where $\{r\}$ denotes the smallest integer greater than r.

Writing out (41) with L=2 as an example and with K=1 gives the following matrix of coefficients M in (45):

$$\begin{bmatrix}
 & & & & & & & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & & & & & & & & \\
.. & 0 & 0 & 0 & F_{-2} & 0 & F_{-1} & d_2 & Z & 0 & F_1 & 0 & F_2 & & & & & & & & \\
.... & 0 & F_2^* & 0 & F_1^* & 0 & Z^* & d_2 & F_{-1}^* & 0 & F_{-2}^* & 0 & 0 & 0 & ..... & & & & & & \\
 & .... & 0 & 0 & 0 & F_{-2} & 0 & F_{-1} & d_1 & Z & 0 & F_1 & 0 & F_2 & 0 & .... & & & & & \\
 & & .... & 0 & F_2^* & 0 & F_1^* & 0 & Z^* & d_1 & F_{-1}^* & 0 & F_{-2}^* & 0 & 0 & 0 & .... & & & & \\
 & & & .... & 0 & 0 & 0 & F_{-2} & 0 & F_{-1} & d_0 & Z & 0 & F_1 & 0 & F_2 & 0 & .... & & & \\
 & & & & .... & 0 & F_2^* & 0 & F_1^* & 0 & Z^* & d_0 & F_{-1}^* & 0 & F_{-2}^* & 0 & 0 & 0 & .... & & \\
 & & & & & .... & 0 & 0 & 0 & F_{-2} & 0 & F_{-1} & d_{-1} & Z & 0 & F_1 & 0 & F_2 & 0 & .... & \\
 & & & & & & .... & 0 & F_2^* & 0 & F_1^* & 0 & Z^* & d_{-1} & F_{-1}^* & 0 & F_{-2}^* & 0 & 0 & 0 & .... \\
 & & & & & & & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & & & & & & & &
\end{bmatrix}$$

The vectors A and S in (45) are

$$A = \begin{bmatrix} \alpha_{N,0} \\ \vdots \\ \alpha_{2,0} \\ \alpha_{2,1} \\ \alpha_{1,0} \\ \alpha_{1,1} \\ \alpha_{0,0} \\ \alpha_{0,1} \\ \alpha_{-1,0} \\ \alpha_{-1,1} \\ \vdots \\ \alpha_{-N,1} \end{bmatrix} , \qquad S = \sqrt{\xi_d} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 1 \\ e^{j(\phi_d - \phi_i)} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} , \tag{67}$$

where

$$F_\ell = \quad \ell 01 \quad , \tag{68}$$

which, from (34), also implies

$$F^*_{-\ell} = f^*_{-\ell 01} = f_{\ell 10} \quad , \tag{69}$$

and where

$$d_n = f_{000} + Q_{00} + jn\omega'_m = 1 + \xi_d + \xi_i + jn\omega'_m \quad , \tag{70}$$

and

$$Z = Q_{01} + f_{001} = \xi_d e^{j(\phi_i - \phi_d)} + \xi_i J_o(\beta') \quad . \tag{71}$$

To obtain the results presented below, we have solved this system (with the appropriate values of L) numerically for the $\alpha_{n,k}$, and then evaluated C_n and $W(t')$ from (40) and (39).

Let us now discuss the results obtained from these calculations. In part A below, we show typical curves of desired signal modulation, output INR and SINR as functions of time. In parts B-E, we describe the effect of each signal parameter on the desired signal modulation. In part F, we assume the array is used in a DPSK (differential phase shift keyed) digital communication system [11] and show how the bit error probability is affected by the interference signal parameters ξ_i, β and f_m'.

A. Typical Waveforms

Figure 2 shows typical curves of the normalized envelope modulation $a_{dn}(t')$ for θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB, $f_m' = \frac{\omega_m'}{2\pi} = 10^2$, $f_o' = \frac{\omega_o'}{2\pi} = 10^8$, and β=10^2, 10^3, 10^4 and 10^5. These curves illustrate that the envelope modulation on the desired signal can be substantial for larger values of β.

It turns out that for this 2-element array, angle modulated interference does not cause phase modulation on the desired signal. This result was discovered by calculating $\eta_d(t')$, for numerous values

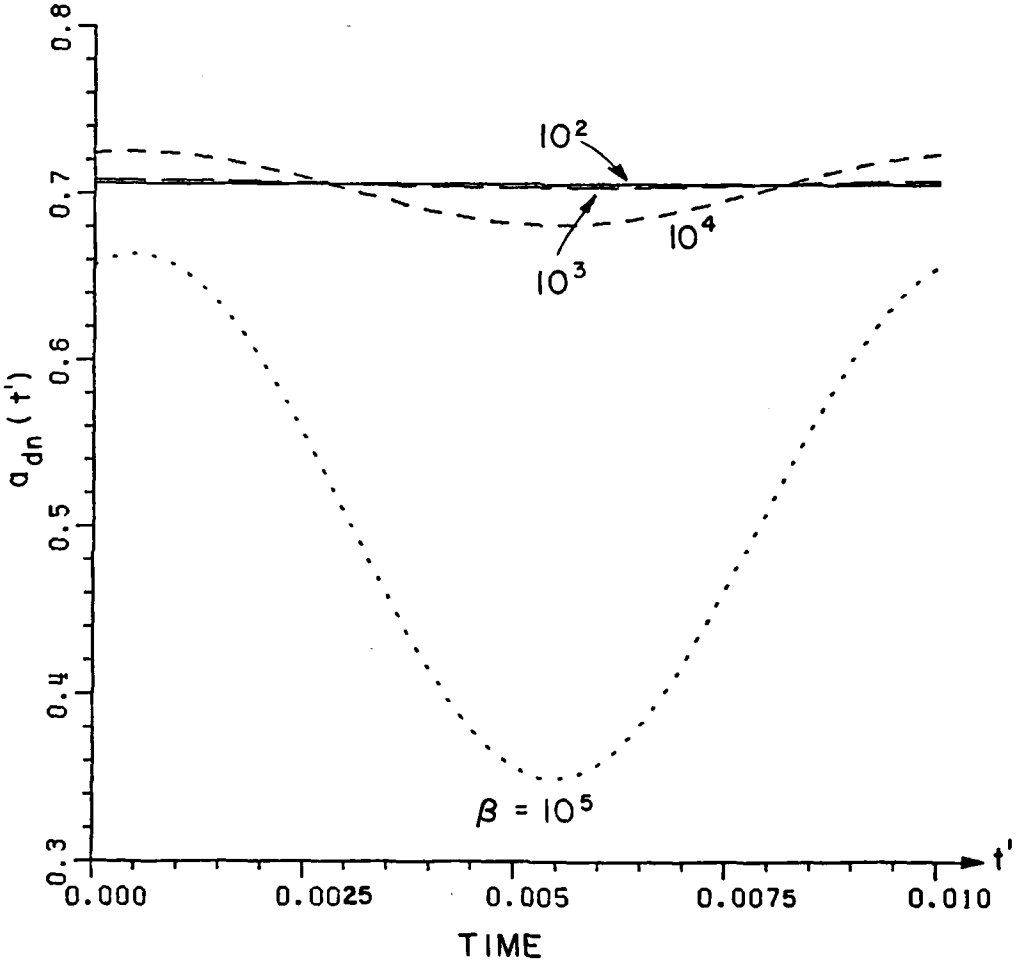

Figure 2. $a_{dn}(t')$ versus time.

θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB,

$f'_m=10^2$, $f'_o=10^8$.

of θ_i, β, f_m' and ξ_i. These calculations show that $\eta_d(t')$ does not change with time regardless of the signal parameters. The reason for this behavior is as follows. Both the amplitude and phase of each weight in the array vary sinusoidally with time. The amplitudes of the two weights are equal at each instant of time. The phase angle of each weight contains a term constant with time, which depends on interference arrival angle, and a term that varies sinusoidally with time. The sinusoidally varying term is 180° out of phase on the two weights. When a desired signal is passed through these weights the phase modulation produced on the desired signal by one weight is 180° out of phase with that produced by the other weight, and the sum signal at the array output contains no phase modulation.

Figures 3 and 4 show typical curves of the output INR and SINR as functions of time, over one period and for the same signal parameters as in Figure 2. Figure 3 shows that the average INR increases and the array SINR decreases as β increases. Also, as β increases, the INR and SINR variations with time are more pronounced. At low β, the INR and SINR are constant, because the array feedback tracks the incoming phase modulation.

In general one finds that the array behavior changes substantially as the signal parameters θ_d, ξ_d, θ_i, ξ_i, f_m' and β are varied. In parts B-E of this section, we examine the effect of each signal parameter on the desired signal modulation. In part F, we show how bit error probability is affected by the angle modulated interference when the array is used in a DPSK communication system.

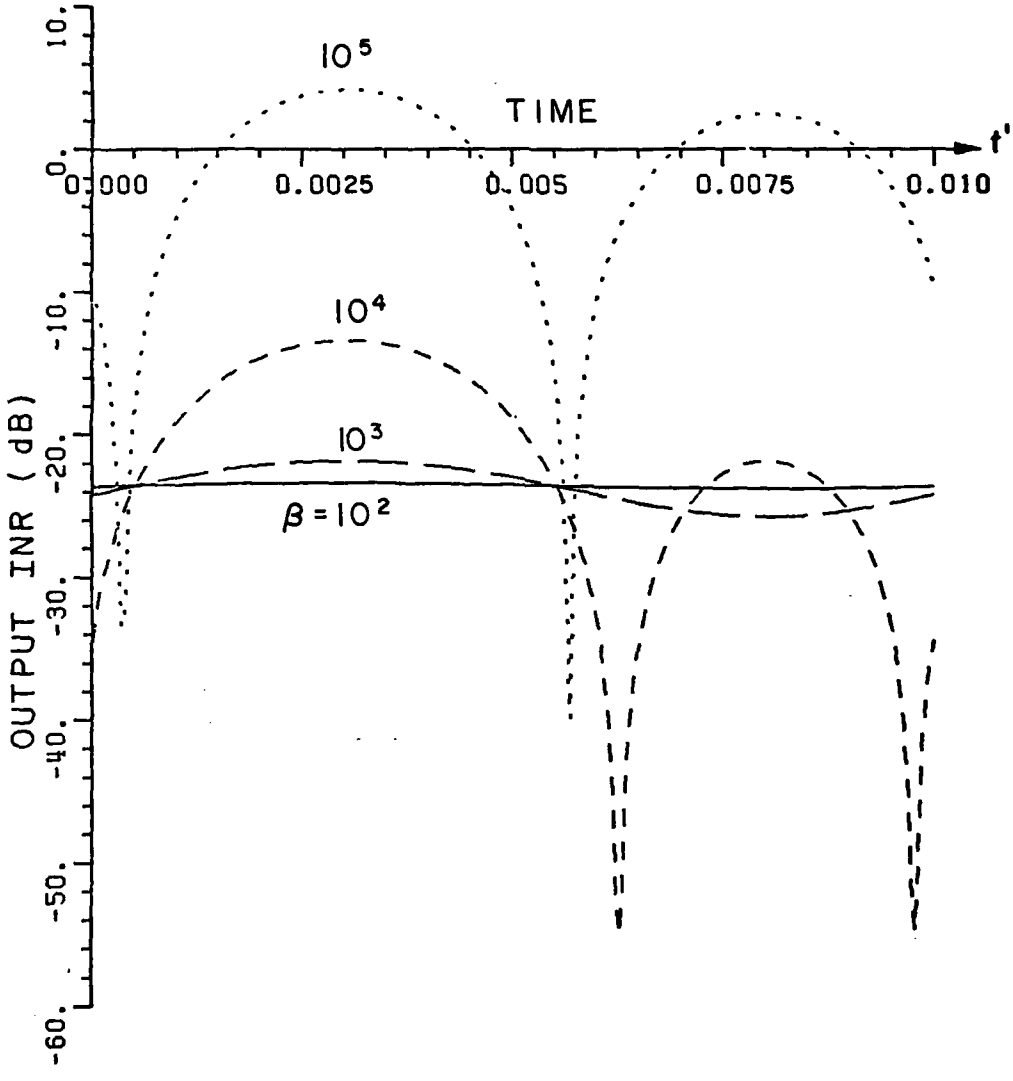

Figure 3. Output INR versus time.

θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB,

$f'_m = 10^2$, $f'_o = 10^8$.

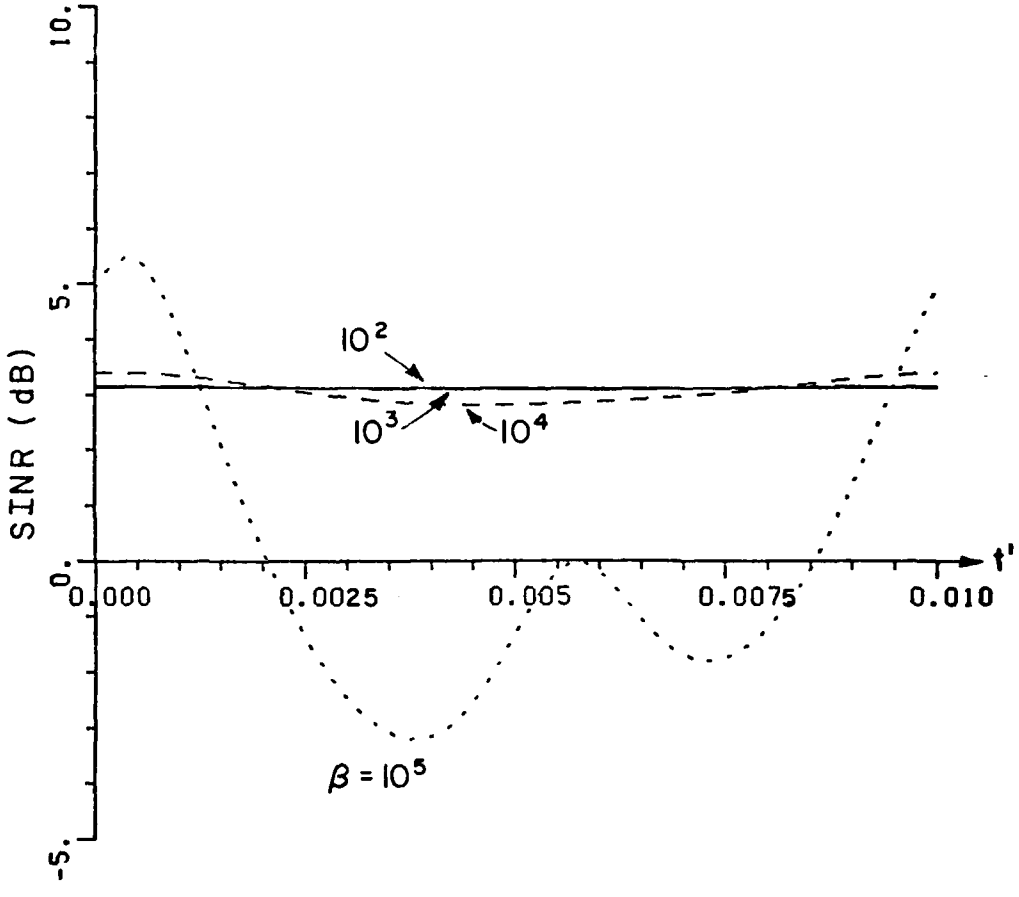

Figure 4. SINR versus time.

θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB,

$f'_m=10^2$, $f'_o=10^8$.

To describe the desired signal modulation, we shall define three quantities. First, we let a_{min} and a_{max} be the minimum and maximum values of $a_{dn}(t')$ during the modulation period. Then, we define

$$m = \frac{a_{max} - a_{min}}{a_{max}} \quad . \tag{72}$$

m is the envelope variation normalized to its peak. It may be thought of as "fractional modulation", analogous to percentage modulation in AM. In Parts B-E below, we describe the effect of each signal parameter on a_{max} and m.

B. The Effect of Angle of Arrival

Desired signal modulation effects are small when θ_i is far from θ_d. When θ_i is close to θ_d, the envelope variation m is large and the peak a_{max} is small. However, when θ_i is equal to θ_d, the desired signal is nulled, so m drops to a small value.

Figures 5 and 6 show typical curves of m and a_{max} as functions of θ_i for θ_d=0°, ξ_d=10 dB, ξ_i=40 dB, f_m'=10^3 and f_o'=10^8. Four different curves are shown, for β = 2×10^3, 4×10^3, 8×10^3 and 10^4. It is seen that m is large and a_{max} is small when θ_i is near θ_d. We note also that when θ_i is extremely close to θ_d, m drops to zero. This behavior occurs because as θ_i becomes very close to θ_d, the peak-to-peak variation (a_{max}-a_{min}) approaches zero more quickly than a_{max} does, so m goes to zero. However, this behavior of m is no importance, since the desired signal is nulled anyway.

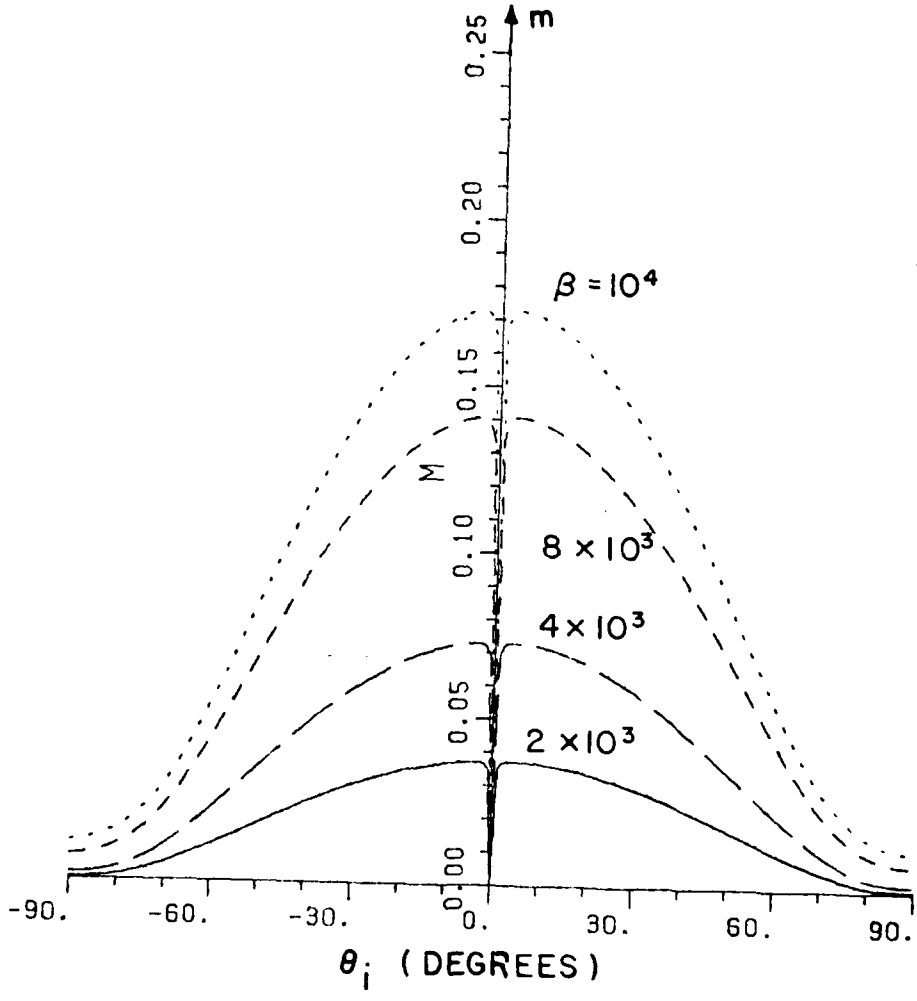

Figure 5. m versus θ_i.

θ_d=0°, ξ_d=10 dB, ξ_i=40 dB,

$f'_m=10^3$, $f'_o=10^8$.

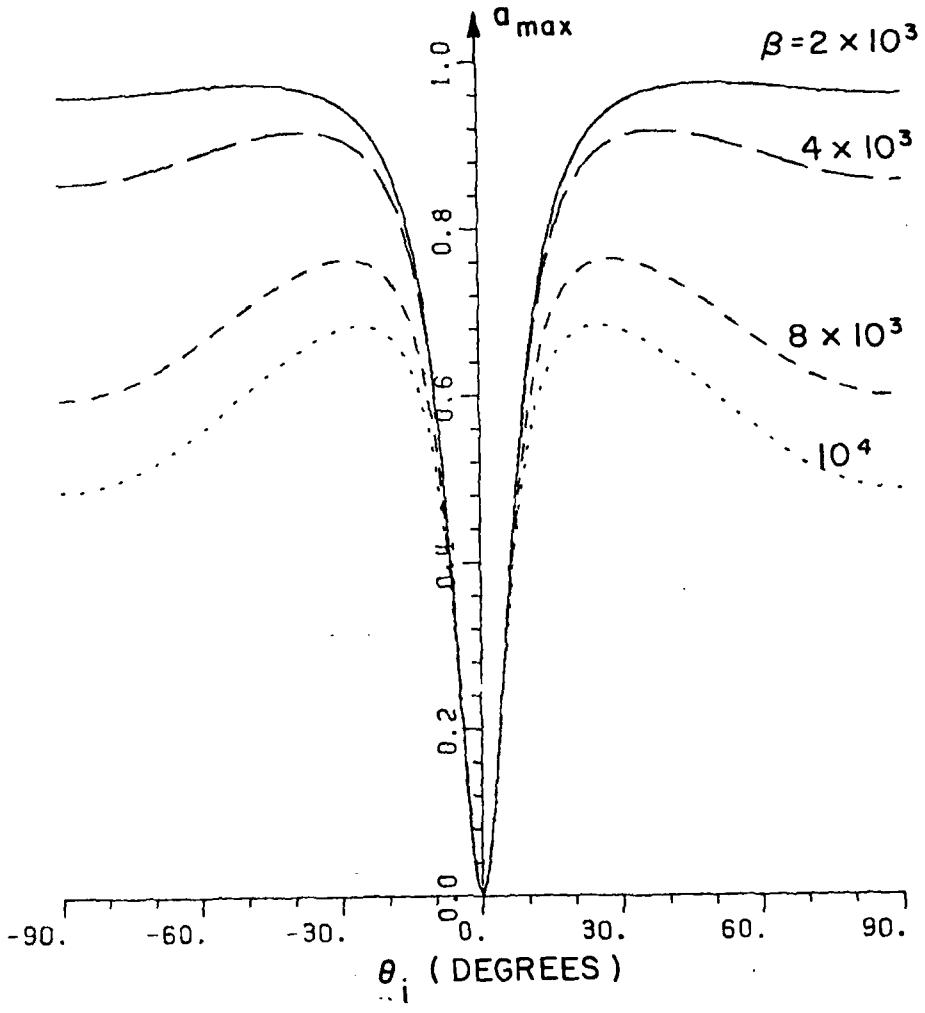

Figure 6. a_{max} versus θ_i.

$\theta_d = 0°$, $\xi_d = 10$ dB, $\xi_i = 40$ dB,

$f_m' = 10^3$, $f_o' = 10^8$.

C. The Effect of Modulation Index and Frequency

The peak a_{max} is large at low f_m' and drops as f_m' increases. The higher β, the farther a_{max} drops at high f_m'. The variation m is small at low f_m', peaks at intermediate values of f_m' and drops as f_m' increases. For low and intermediate f_m', m is largest for high β. At high f_m', m drops more rapidly with larger values of β.

Figures 7 and 8 illustrate these results for the case θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB, $f_o'=10^8$ and for values of β between 2×10^3 and 10^4. It is seen in Figure 7 that a_{max} is large for low f_m' and drops as f_m' increases. This behavior is due to the array speed of response, which limits its ability to track the modulation at high f_m'. m, seen in Figure 8, peaks at intermediate f_m'.

D. The Effect of Interference-To-Noise Ratio

For all values of f_m', the variation m is largest for high INR. a_{max} is high at low f_m' and drops at high f_m'.

Figures 9 and 10 show the behavior of a_{max} and m versus f_m' for θ_d=30°, θ_i=45°, ξ_d=10 dB, $\beta=10^4$, $f_o'=10^8$ and for four values of ξ_i between 20 and 45 dB. It may be seen in Figure 9 that a_{max} peaks slightly at high INR, for values of f_m' around 400. This behavior is due to the change in array speed of response with INR.

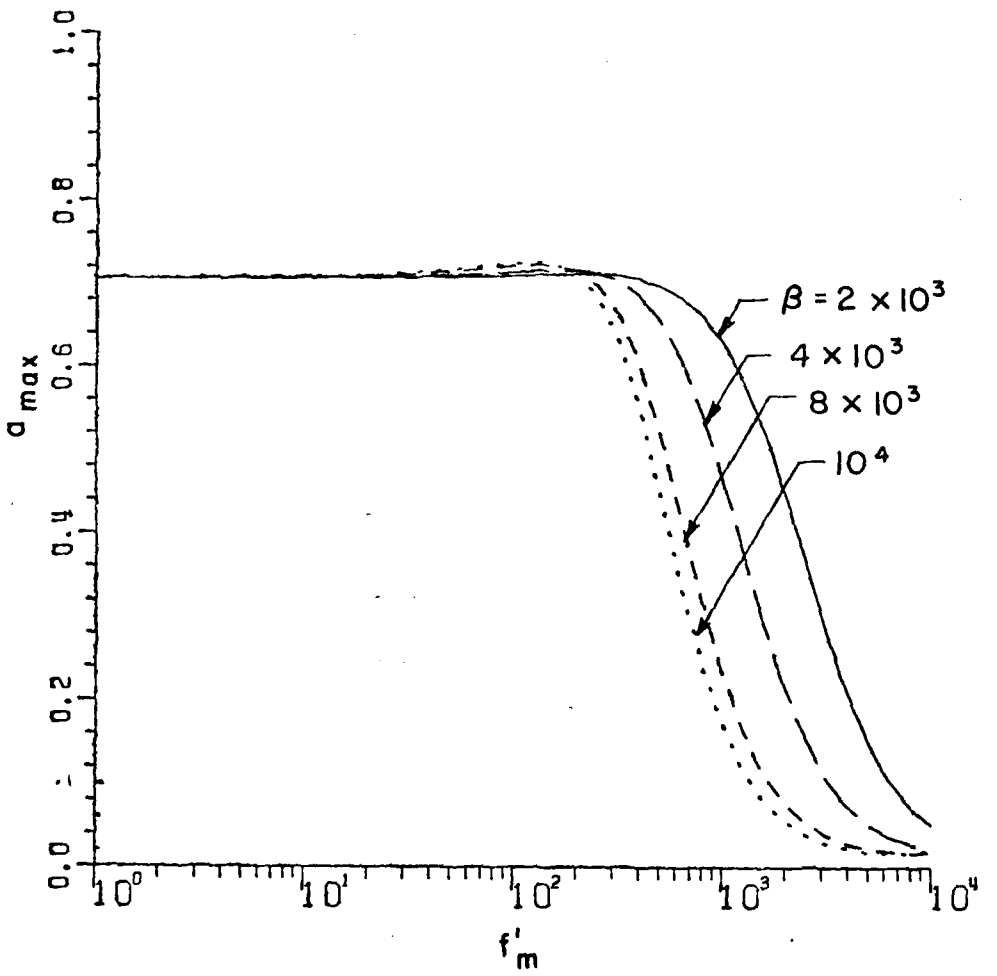

Figure 7. a_{max} versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB,

$f'_o=10^8$.

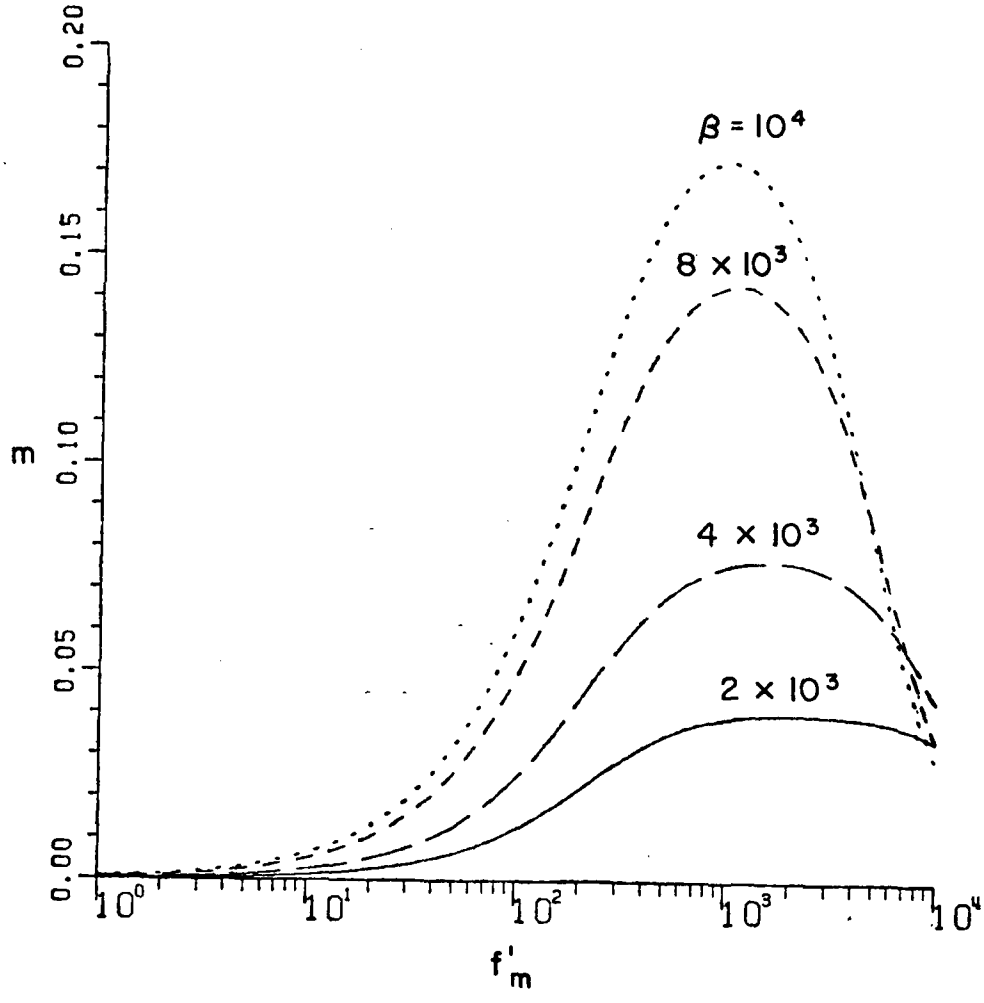

Figure 8. m versus f'_m.

θ_d=30°, θ_f=45°, ξ_d=10 dB, ξ_f=30 dB,

$f'_o=10^8$.

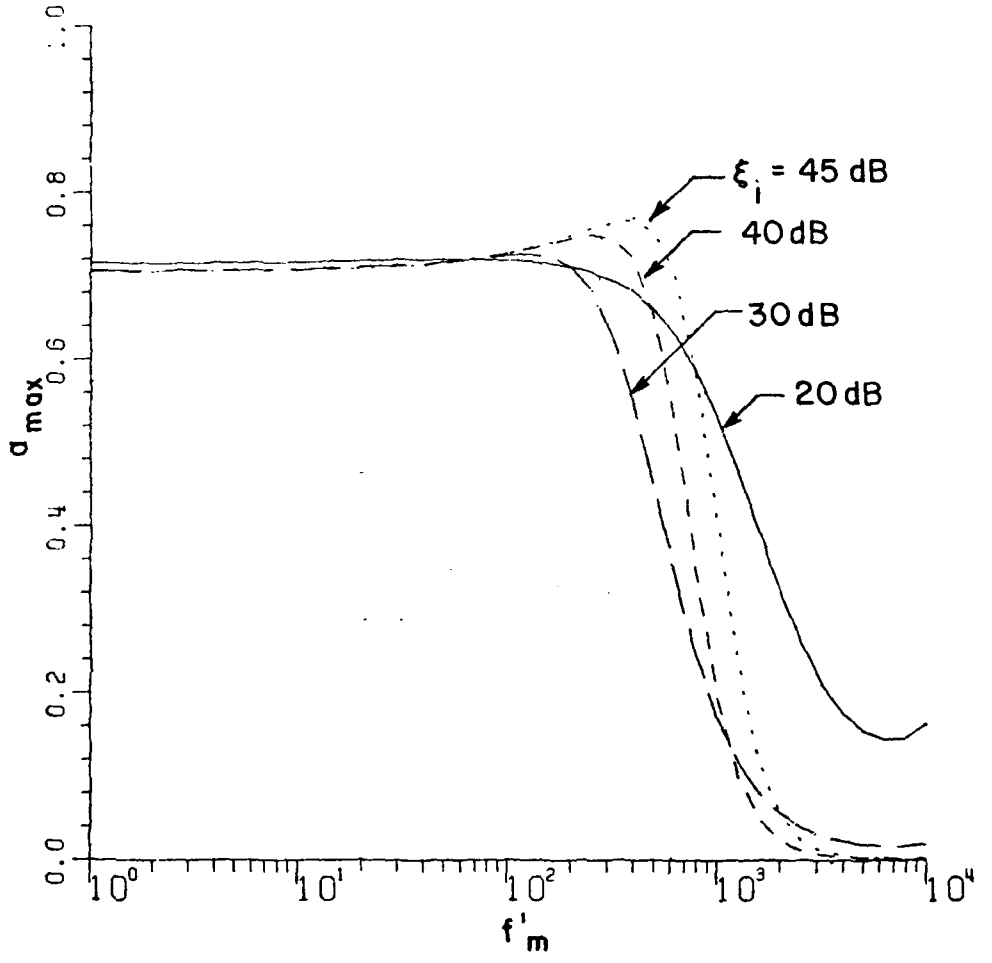

Figure 9. a_{max} versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=10 dB,

$\beta=10^4$, $f'_o=10^8$.

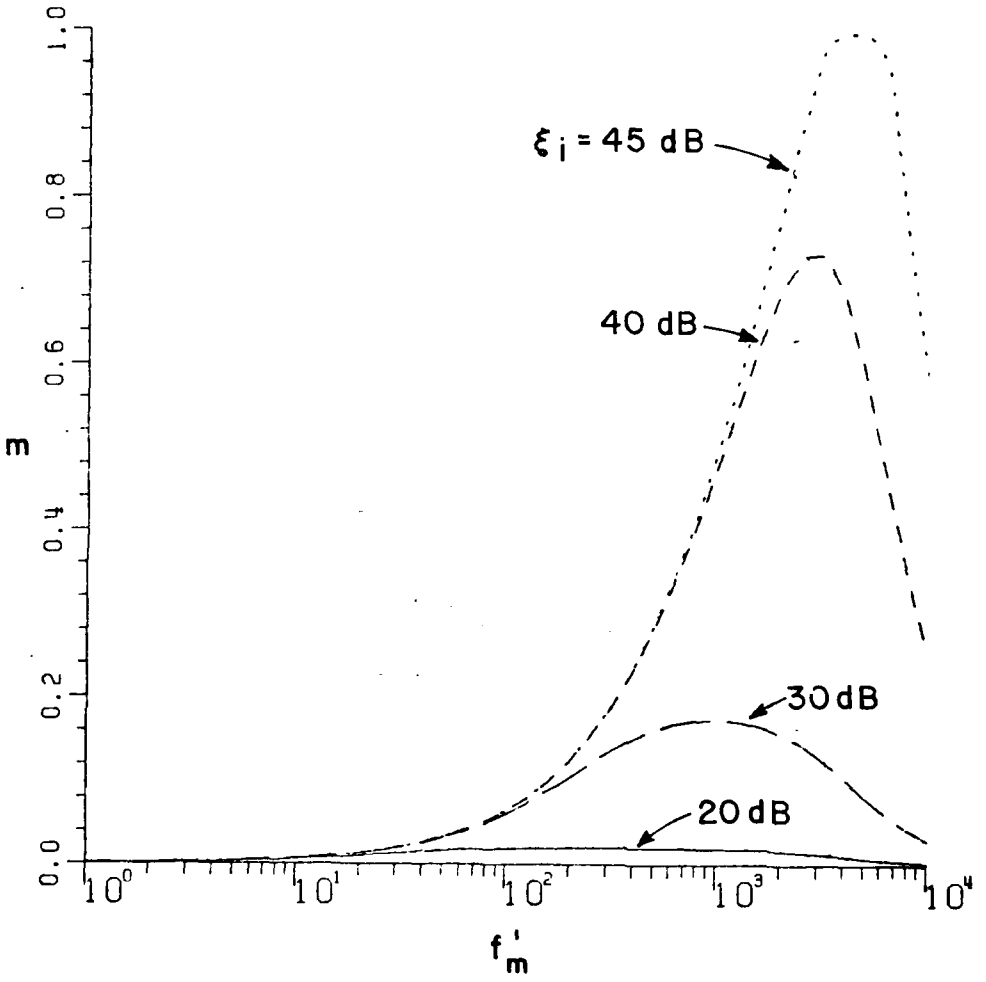

Figure 10. m versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=10 dB,

β=10^4, f'_o=10^8 .

E. The Effect of Desired Signal-To-Noise Ratio

m is largest and a_{max} is smallest for low ξ_d. As ξ_d is increased, m decreases and a_{max} increases.

Figures 11 and 12 illustrate this behavior. Figure 11 shows m and Figure 12 shows a_{max}, both versus f_m' for θ_d=30°, θ_i=45°, ξ_i=30 dB, $f_o'=10^8$, $\beta=10^4$, and for four values of ξ_d between 10 and 40 dB. It is seen that m peaks at intermediate f_m' with the value of f_m' at the peak dependent on ξ_d.

F. Bit Error Probability

To evaluate the effect of the time-varying SINR, we have computed the bit error probability when the desired signal is DPSK biphase modulated signal. We assume the desired signal bit duration is large compared with the propagation time across the array but small compared to the interference modulation period $2\pi/\omega_m$. Also, we assume the reference signal is replica of the desired signal. Under these assumptions the array weights with the DPSK biphase modulated desired signal are the same as those produced with a CW signal. The effective bit error probability $\bar{P}_e$ has by obtained by averaging the instantaneous bit error probability over one period o* the interference modulation. The instantaneous bit error probability $P_e(t')$ is related to the SINR by [12],

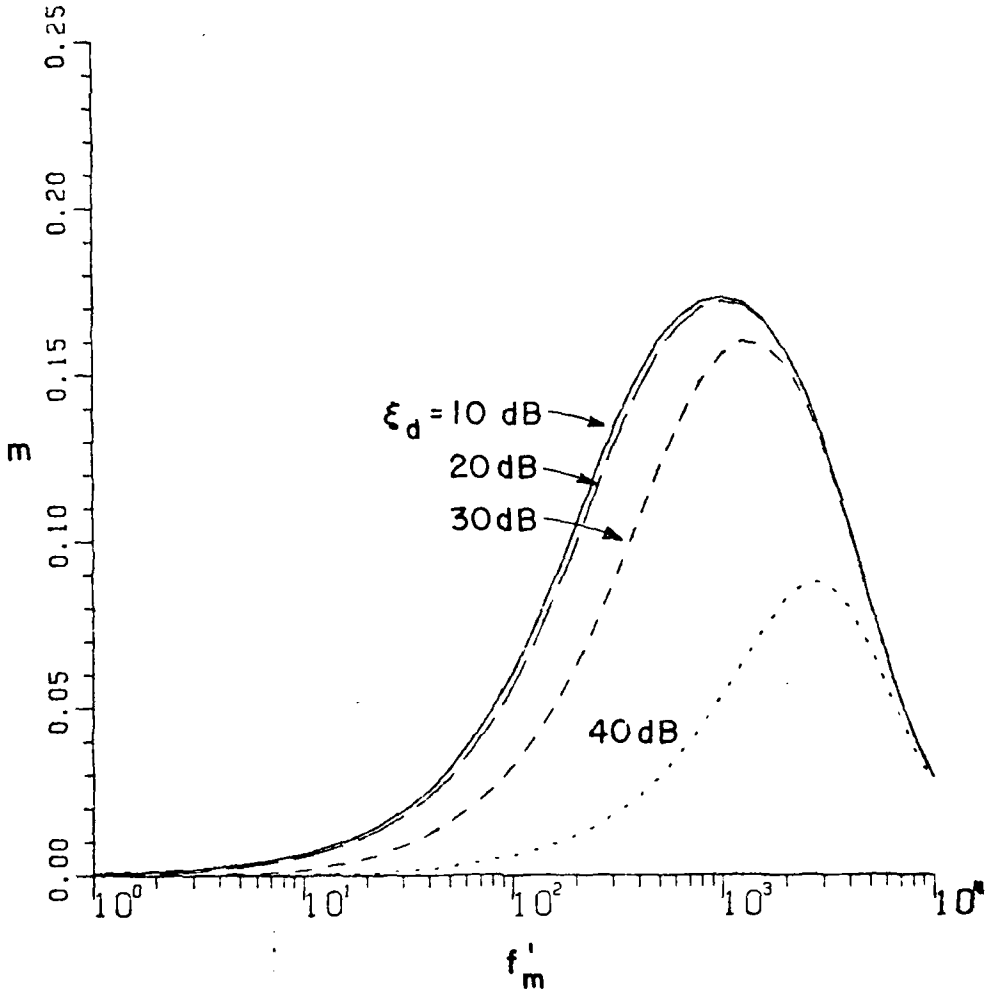

Figure 11. m versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=30 dB,

$\beta=10^4$, $f'_o=10^8$.

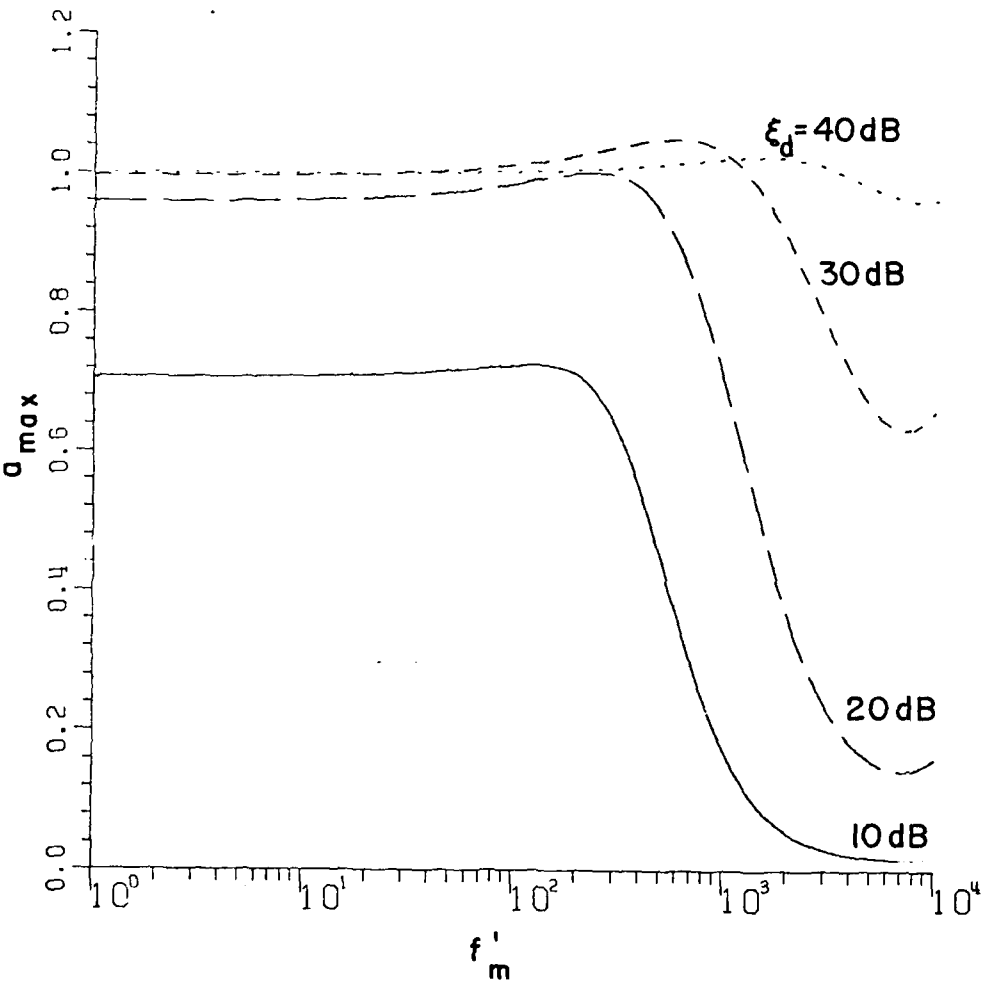

Figure 12. a_{max} versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=30 dB,

$\beta=10^4$, $f'_o=10^8$.

$$P_e(t') = \frac{1}{2} e^{-SINR(t')} \quad . \tag{73}$$

Figure 13 shows typical curves of $\bar{P}_e$ as a function of f_m' for θ_d=30°, θ_i=45°, ξ_d=10 dB, ξ_i=30 dB, $f_o'=10^8$ and for four values of β between 2×10^3 and 10^4. At low f_m', where the interference bandwidth is too small to cause any degradation in the array performance, $\bar{P}_e$ stays constant at a value that does not depend on β. This value of $\bar{P}_e$ is determined simply by the input INR. As f_m' increases the interference bandwidth has more effect. At high f_m', P_e approaches 0.5.

Figure 14 shows typical curves of $\bar{P}_e$ versus f_m' for different values of ξ_i and for θ_d=30°, θ_i=45°, ξ_d=10 dB, $f_o'=10^8$ and $\beta=10^4$. These curves illustrate the dependence of $\bar{P}_e$ on the interference power. At low f_m', the interference has small bandwidth and $\bar{P}_e$ depends slightly on ξ_i. This behavior is the same as with CW interference.

IV. CONCLUSIONS

We have developed a method for finding the periodic steady-state weights of an adaptive array receiving an interference signal with periodic angle modulation. We then used this method to study the behavior of a two-element array subjected to an interference signal with sinusoidal phase modulation. This interference causes the array weights to cycle sinusoidally with time, both in magnitude and phase. The modulated weights cause the desired signal to have envelope (but not phase) modulation at the array output.

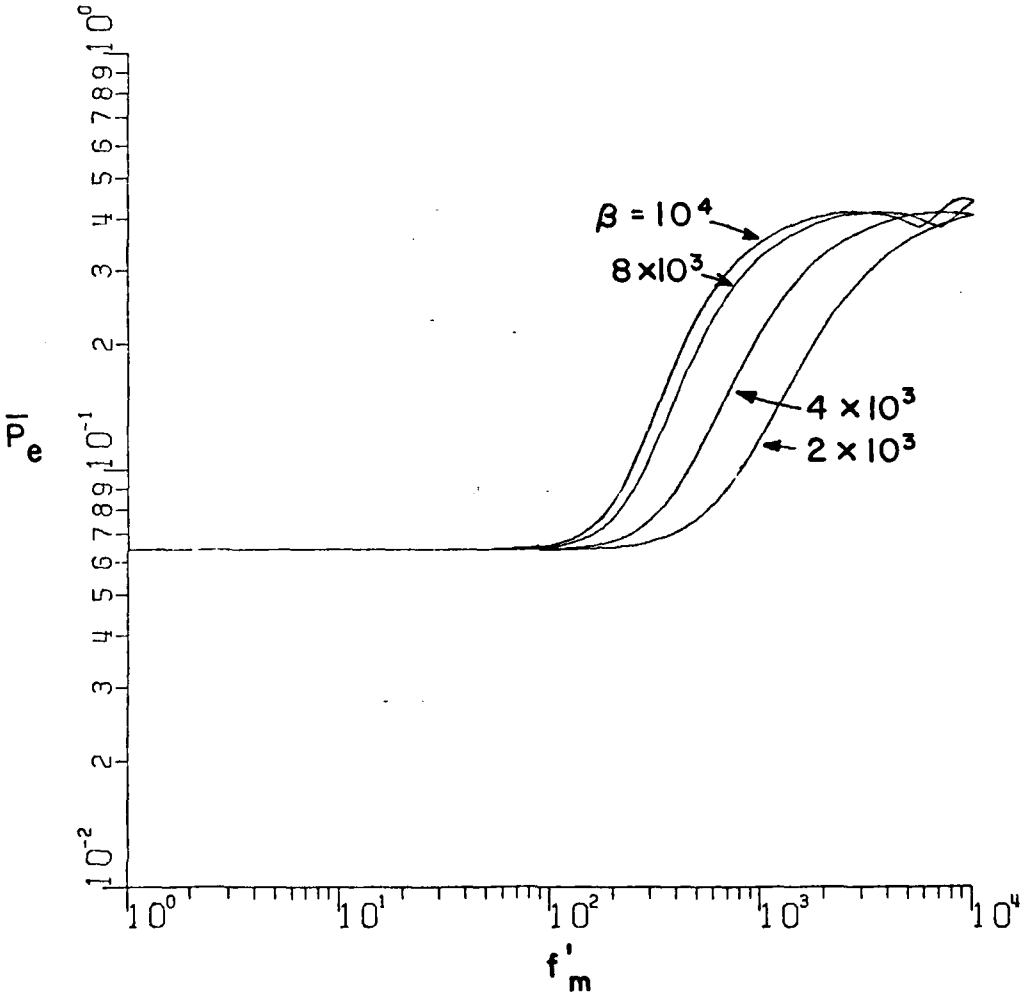

Figure 13. Bit error probability versus f_m'.
θ_d=30°, θ_i=45°, ξ_d=10 dB,
ξ_i=30 dB, $f_o'=10^8$.

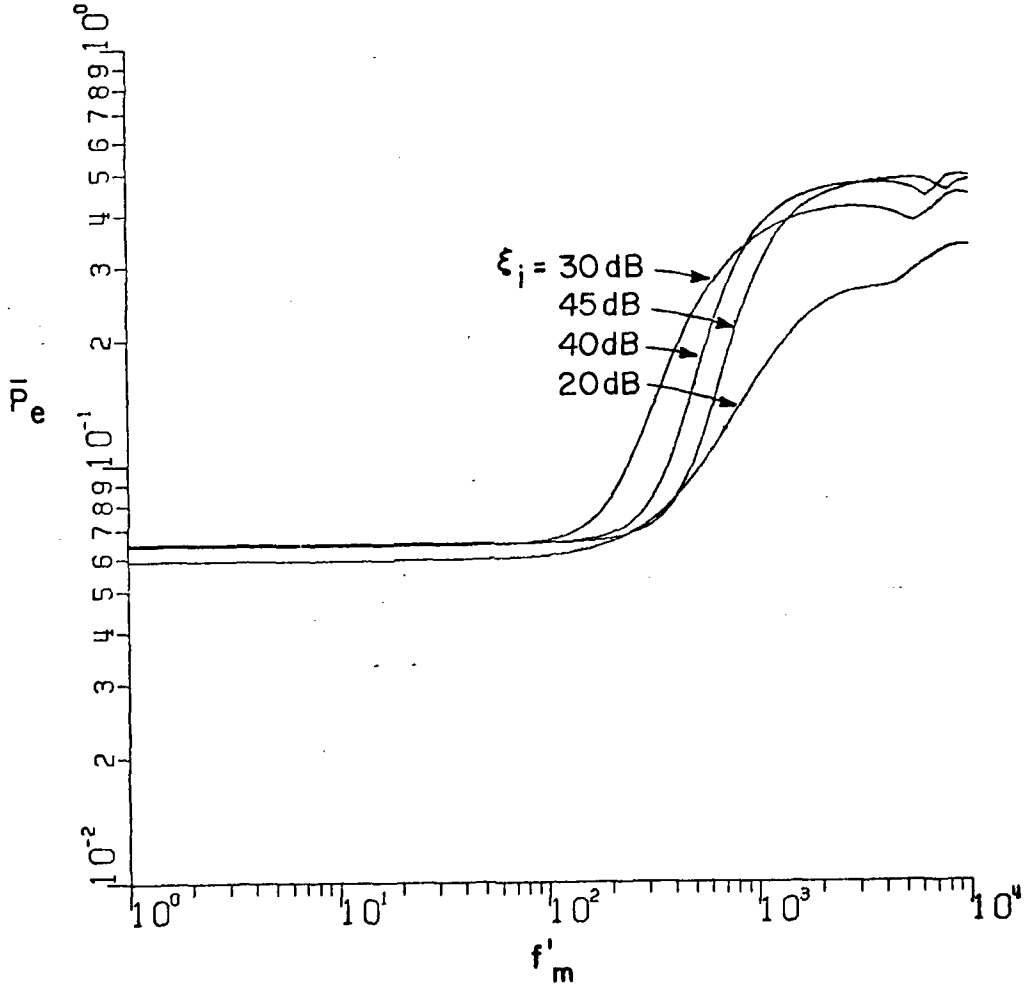

Figure 14. Bit error probability versus f'_m.

θ_d=30°, θ_i=45°, ξ_d=10 dB,

$\beta=10^4$, $f'_o=10^8$.

The envelope modulation is largest when the interference arrives close to the desired signal in space. The envelope modulation increases with the bandwidth of the input interference signal. Phase modulated interference also causes the array output SINR to vary with time. When the array is used in a digital communication system, such SINR variation increases the bit error probability.

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