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THE AIRCRAFT AVAILABILITY MODEL:
CONCEPTUAL FRAMEWORK AND MATHEMATICS

June 1983

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20. ABSTRACT

→ model does, how it does it, and the underlying mathematics. A description of the U.S. Air Force application to the programming, budgeting, and allocation of resources for recoverable spares is included. ↗

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ACKNOWLEDGEMENT

There have been many contributors to the development of the Aircraft Availability Model. The original impetus came from the work of W. B. Fisher, J. E. Heller, and J. W. Smith. Credit for the core of the model, in fact, its very existence, is owed to them.

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PREFACE

This report provides a complete description of the LMI Aircraft Availability Model: what the model does, how it does it, and why it works. It is written for readers with a reasonable background in probability and statistics; familiarity with multi-echelon inventory theory is helpful but not necessary. Material which is particularly technical and can logically be separated from the main development is included in appendices. Much of this material is available in the literature but is included for completeness. Numerical citations in the text of the form "[]" refer to the bibliography.

component (failure rates, repair times, assets on hand and on order, and so on). In addition, the availability computation requires configuration data for each aircraft type; these are obtained by restructuring a part of the D041 data base.

Aircraft population and flying hour programs are supplied from Air Force planning documents, from either the AFLC K004 (Projected Programs) System or the PA (Aerospace Vehicles and Flying Hour Programs) System.

PRODUCTS

The AAM produces curves of cost versus availability rate for each aircraft type. Each point on the curves corresponds to an optimum procurement/depot repair plan. Those curves enable logistics planners to see the consequences of various allocations of available funds for procurement and repair among different aircraft types and to make informed tradeoffs among those allocations. The curves may be used in two different, but complimentary, ways. Given established availability targets by aircraft type, the funds necessary to achieve those targets can be read from the curves. Conversely, the availability rates resulting from a specific allocation of funds to aircraft types can be obtained.

Associated with the model are reports of various formats and automated interactive programs, which make the AAM a true decision support system. Advisory shopping lists and repair plans, by item for a specific allocation, can be generated.

VALIDATION

Several extensive and carefully monitored tests of the AAM have been conducted, and they have demonstrated that the model does provide a valid way of relating an inventory of recoverable component spares to the availability rates of the aircraft types which those spares support. In 1973

The AAM uses a marginal analysis technique, i.e., it ranks the candidates for procurement and repair in decreasing order of benefit per cost to form an ordered "shopping list." Buying and repairing from this list in the order indicated assures that items which give the greater increase in availability rate per dollar (of procurement cost or repair cost, as appropriate) will be acquired earlier. Thus, the AAM optimizes aircraft availability for any funding constraint and produces optimum shopping lists and optimum repair strategies, by component, for each funding level. In doing so, it addresses a rich variety of real world complications which are often ignored by other models. For instance:

- Common components are treated in such a way that the effects of spares on the aircraft types which they support are explicitly computed.
- For POM and Budget formulation, the computations require simultaneous analysis of more than 90,000 recoverable components, applied to approximately 40 aircraft types (A-10, B-52, F-15,...) with more than 100 different subtypes (B-52H, F-15A, F-4G...).
- The AAM is a multi-echelon model. Two echelons of supply (depot and base) and their interactions are analyzed simultaneously.
- The AAM is a multi-indenture-level model. The effect of levels of indenture (i.e., components within larger components within subsystems, etc.), is explicitly calculated.
- Engines, which traditionally have been treated as end items, may be treated as components (Line Replaceable Units) of the aircraft which they power.

DATA REQUIREMENTS

For application to Air Staff POM and Budget formulations, the data inputs to the AAM are almost identical to those used by the AFLC in its recoverable spares requirements computation. In fact, the AAM was designed to use existing data bases to avoid the effort required to construct specially tailored data bases. The data used by the AAM are found in the AFLC D041 depot data bank and include the standard supply data for each recoverable

component (failure rates, repair times, assets on hand and on order, and so on). In addition, the availability computation requires configuration data for each aircraft type; these are obtained by restructuring a part of the D041 data base.

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LMI conducted a test which demonstrated the feasibility of the basic concepts and algorithms of the AAM. In 1977, LMI and AFLC conducted a joint test to find whether the AAM could accurately forecast aircraft availability rates. The conclusion reached by AFLC was that "...the model is a valid means of computing the probability that an aircraft will not be missing a recoverable part" and "...the LMI model can be used to determine a reasonable indication of actual Aircraft Availability." [13]

OTHER USES

The power and flexibility of the AAM allow it to be used for many purposes other than POM and Budget analyses. A good example is its recent use to support an AFLC study of LOGAIR, the Air Force airlift logistics resupply system. By appropriate modification to the input data, the model produced cost versus availability curves which reflected lengthened transportation times between base and depot, from degraded LOGAIR operations. The curves were then compared with the cost versus availability curves produced for the "baseline" with no degradation in LOGAIR.

The model's versatility is further demonstrated by these recent uses:

- An analysis of F100 engine support during a wartime scenario
- A determination of spares support required for transport aircraft (C-5, C-130, and C-141) for six different levels of wartime surge activity
- Analyses of the effect on availability rates and funding requirements of changes in projected flying hour programs during POM and Budget reviews at HQ USAF.

OSD and the Military Departments now recognize the benefits to be gained by replacing the commodity orientation of the supply and maintenance system with a weapon system orientation. The AAM is an example of the powerful and flexible tools that can be developed to support management by weapon system.

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CHAPTER 1. INTRODUCTION

WHY AVAILABILITY RATE?

The Aircraft Availability Model (AAM) uses a quantity called aircraft availability rate to measure the performance of the supply system. An aircraft is defined to be available if it is not missing a reparable component, e.g., a fuel pump, an altimeter, a fire control computer, or one of the many other reparable components used on aircraft. The availability rate for an aircraft type is then the percentage of aircraft available over a specified time period.

The definition of availability looks only at supply of reparable spares; it does not consider such actions as on-aircraft maintenance, scheduled or unscheduled, or the effect of shortages of consumables. Thus, availability rate is not identical to mission capable rate; it is, in effect, the reparable supply component of mission capable rate.

Performance of supply systems has typically been evaluated by using supply-oriented measures. Fill rate (the percentage of demands filled upon receipt over some time period) and backorders (the number of outstanding demands at a point in time) are two widely used measures. A serious drawback of such measures is that they do not look beyond the supply system to determine the impact of supply on the aircraft or other end items being supported. A policy which attempts to maximize fill rate will tend to concentrate on low-cost, high-demand items, and will accept infrequent, but long lasting, backorders on expensive, low-demand items. This is not a good strategy to maximize aircraft readiness.

It is unclear, also, just what constitutes an acceptable fill rate or backorder level. Considering the great differences in complexity among

various aircraft types, indiscriminate use of fill rate or backorder targets can have negative effects on aircraft support, even though a purely supply-oriented measure of performance would not reveal these effects. The use of availability rate has a significant advantage over the use of a fill rate criterion to support aircraft, in that using a fill rate criterion tends to penalize complex aircraft types.

Figure 1-1 shows a simplified, hypothetical example. It assumes three different aircraft types, all at the same location, differing only in how many reparable components each uses. All components are identical, with a pipeline of 5.¹ If enough spares are stocked so that the fill rate for each aircraft type is 92 percent, the availability rates for the more complex aircraft are lower than those for the simpler aircraft, as seen in the figure. This happens even though the investment, being equal for each component, is four times as great for aircraft C as for aircraft A.

FIGURE 1-1. FILL RATE PENALIZES COMPLEX AIRCRAFT

<u>AIRCRAFT A</u>	<u>AIRCRAFT B</u>	<u>AIRCRAFT C</u>
100 COMPONENTS	200 COMPONENTS	400 COMPONENTS
AVAILABILITY RATE	AVAILABILITY RATE	AVAILABILITY RATE
88 %	78 %	61 %
COST = \$X	COST = \$2X	COST = \$4X

¹The pipeline is the average number of units of the component in resupply, i.e., in repair or shipment. See Appendix B for more detail on this topic.

In actuality, the situation is more complicated, but the bias remains. We will see later that availability rate is defined as a product of probabilities--the probability the aircraft is not missing its first component times the probability the aircraft is not missing its second component, and so on. An aircraft with more components has more factors in the product, and, since each probability is less than 1.0, the product will tend to be smaller

Thus, the use of a fill rate criterion leads to a bias in favor of the less complex aircraft types. The use of a backorder criterion is more defensible if properly done, e.g., setting a target in terms of backorders per aircraft. As we will show, availability rate can be viewed as a measure that goes a step beyond backorders. There is a close relationship between availability rate and backorders per aircraft, which is discussed in Appendix A.

THE ENVIRONMENT

The AAM is a worldwide, steady-state model, portraying an entire system of supply and maintenance supporting flying activity. Demand activity is assumed to be steady,² though not necessarily constant, thus reflecting peacetime operations rather than wartime surge operations.³ The remainder of this section describes in detail the activity modelled by the AAM.

We consider an environment consisting of a number of aircraft or other end items stationed at several geographically distributed operating locations or bases. There are a number of different aircraft types, Mission Design (MD) or Type Mission (TM), such as the F-4, B-52, or F-14. These types can be

²The underlying failure processes are random but stationary. See Appendix B.

³The AAM, suitably modified, can be, and has been, used to model worldwide wartime surge activity, although that is not a common application of the model.

further classified into subtypes, Mission Design Series (MDS) or Type Mission Series (TMS), such as the F-4G, B-52H, or F-14A.

These aircraft are supported by an inventory of spare reparable (or recoverable) components stocked at each of the bases, as well as at a higher echelon of supply, or depot. (For the remainder of this report, we shall simply use the word component to mean reparable component and spare to mean spare reparable component.) The bases have a limited repair capability, while the depots are essentially industrial facilities, with an extensive repair and overhaul capability.

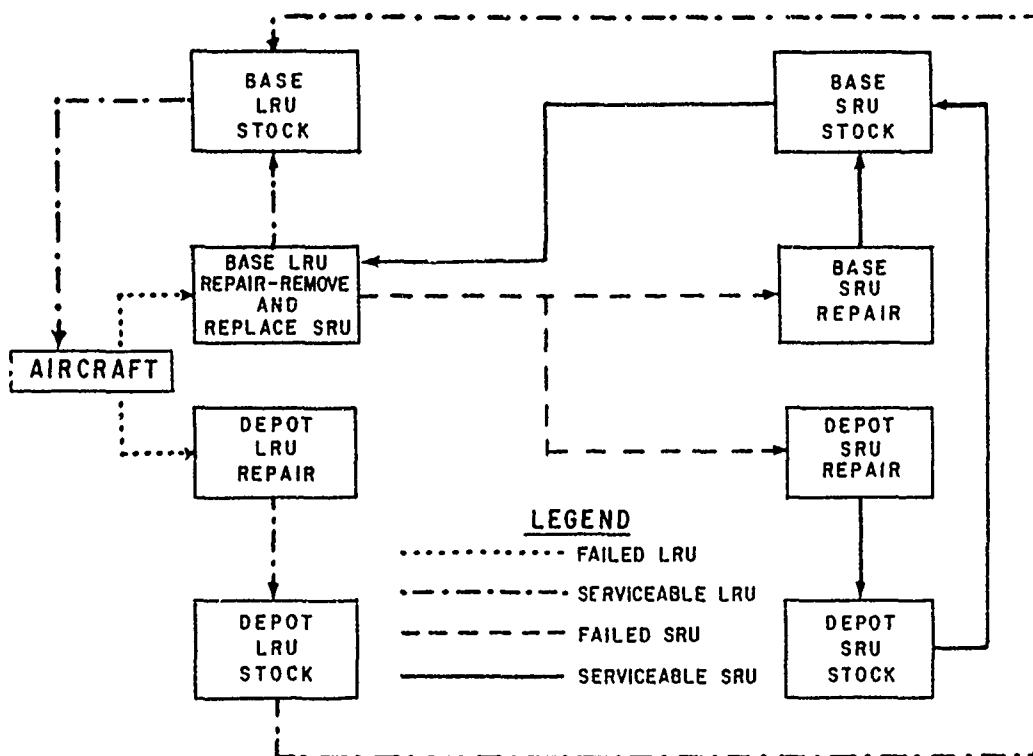
When a failure occurs on an aircraft, the failure is isolated to a single component, which is then removed from the aircraft and replaced with a serviceable spare from base supply as soon as one is available. Since this action takes place on the flight line, such components are called Line Replaceable Units (LRUs). They are also called first indenture level items.

The failed LRU then enters base maintenance, where it is repaired or classified as beyond the capability of base maintenance. In such a case, the failed LRU is shipped to a depot for repair, and a requisition is placed upon the depot for a serviceable unit to be shipped to the base.

Base level repair of the failed LRU typically consists of isolation of the problem to a failed subassembly, a Shop Replaceable Unit (SRU) or second indenture level item, and removal of this faulty subassembly. The SRU-LRU relationship is analogous to the LRU-aircraft relationship. The failed SRU is removed from the LRU and replaced with a serviceable spare from base supply if one is available. The failed SRU is repaired at base level if possible (typically by removal and replacement of a failed third indenture level item) or sent to the higher echelon for repair. Figure 1-2 shows the flow of serviceable and unserviceable units for the first two levels of indenture.

Figure 1-2 shows that the effect of LRU shortages and SRU shortages are quite different. If there is no spare available for a failed LRU, the unfilled demand (backorder) causes a "hole" on the aircraft. An aircraft in such a state is unavailable by the AAM definition. Lack of a spare SRU delays the repair of the LRU. If serviceable spares of the next higher assembly (NHA) are available, there will be no direct effect on the aircraft of the SRU backorder.

FIGURE 1-2. FLOW OF SERVICEABLES AND UNSERVICEABLES



The hierarchical structure continues through many levels of indenture, but the analogy with the first two levels holds. Lower indenture level

backorders will be generated by demands during the repair of the component's NHA and will serve only to delay the repair of that NHA.

In practice, of course, the situation is not so simple as depicted. Sometimes a subassembly of an LRU can be removed directly from an aircraft without intervening removal of the higher assembly. Sometimes more than one SRU must be removed from an LRU during maintenance. The portrayal is a simplification of actual operations but contains enough aspects of the true situation to yield an acceptable degree of accuracy, while not making the mathematical treatment intractable or the data requirements excessive.

CHAPTER 2. CALCULATION OF AIRCRAFT AVAILABILITY

The AAM calculation of availability makes use of the traditional supply measure of backorders, i.e., time weighted unfilled end-user demands. Methods of computing expected backorders (EBOs) for a component in multi-echelon, multi-indenture system are well known, and the AAM uses techniques derived from the METRIC model [14] and the MOD-METRIC model [10] to make these computations. However, to ensure adequate support of end items, it is necessary to go beyond consideration of component oriented measures and to calculate the probable effect of these component shortages upon aircraft. The definition of an available aircraft as one which is not waiting for a repairable component to be repaired or shipped to it may be rephrased as follows: an available aircraft is one with no LRU backorders outstanding.

The AAM computes the availability rate resulting from a given inventory of spares in a two-step process. First, it computes the expected backorders for each component on the aircraft. Second, it computes the probability of one or more of those expected backorders occurring on an aircraft. The remainder of this section discusses how those calculations are made.

For ease of exposition, let us temporarily assume that:

1. All components are first level of indenture
2. No components are applied to more than one aircraft type.

By setting the treatment of levels of indenture and commonality aside for the moment, it is possible to give a streamlined development, displaying the crucial ideas clearly. We will then show how levels of indenture and commonality may be incorporated into the basic framework.

The first step is to compute the number of expected backorders for each component and for the projected level of spares for the component if no further procurements are made. Typical applications of any supply model involve projections into the future of at least an item's procurement lead-time, as no buy decisions can have an impact before then. Thus, starting spares levels must be projected to a future point in time called the impact point, considering amount currently on hand and on order, expected condemnations, etc.

In this single indenture level case, the AAM uses essentially the mathematics developed in METRIC [14]. It computes the number of expected backorders for the component and finds the optimum distribution of spares between base and depot for every spares level.

This computation requires the following data for each component:

- total daily demand rate
- number of using locations (number of bases to which the aircraft type using the component is deployed)
- percentage of demands which are not repaired at base level
- average base repair time
- average depot repair time (includes retrograde time, the time to ship a failed component from base to depot)
- average order and ship time (from depot to base).

Using this mathematics, the AAM computes for component i with projected spares level n , the number of expected backorders, $EBO_{i,n}$. Factors entering this computation and the expected backorders resulting are those projected to the impact point. A detailed outline of this technique is included in Appendix B.

Now suppose for aircraft MD h , composed of several subtypes or MDSs, $h(k)$, $k = 1, 2, \dots, K(h)$, that $a(h(k), i)$ is the quantity per application (QPA) of component i on MDS $h(k)$, and $b(h(k), i)$ is the application percentage. Thus,

if $a(h(k),i) = 2$ and $b(h(k),i) = .8$, 80 percent of the aircraft of MDS $h(k)$ have 2 units of component i installed, while 20 percent have none. T_i is the total number of units of component i installed on aircraft of MD h , or the total number of "slots" on aircraft which should contain a functioning unit of component i . A backorder for component i results in an empty slot--a "hole" on the airplane. With n spare units of component i in the system, the probability that any slot is backordered is $EBO_{i,n}/T_i$, assuming that backorders are uniformly distributed among the slots. The probability that a slot is not waiting for a spare is $1 - EBO_{i,n}/T_i$, and, for an aircraft with $QPA = a$, the probability that the aircraft is not waiting for a spare is

$$(1 - EBO_{i,n}/T_i)^a.$$

Thus, for MDS $h(k)$,

$$\begin{aligned} q_{h(k),i,n} &= \text{the probability that an aircraft of MDS } h(k) \text{ is not missing} \\ &\quad \text{unit of component } i \text{ with } n \text{ spare units of component } i \text{ in} \\ &\quad \text{the system} \\ &= (1 - b(h(k),i)) + \\ &\quad + b(h(k),i) \cdot (1 - EBO_{i,n}/T_i)^{a(h(k),i)}. \end{aligned} \tag{2.1}$$

The probability that an aircraft of the MD will not be missing a unit of component i is calculated by taking a weighted average of the probability for each MDS. Thus, if MDS $h(k)$ has $N(h(k))$ aircraft, and the MD has a total of $N(h)$ aircraft,

$$\begin{aligned} q_{h,i,n} &= \text{the probability that a random aircraft of MD } h \text{ is not waiting} \\ &\quad \text{for a spare unit of component } i \text{ with } n \text{ spare units of} \\ &\quad \text{component } i \text{ in the system.} \\ &= \sum_{k=1}^{K(h)} \frac{N(h(k))}{N(h)} \cdot q_{h(k),i,n}. \end{aligned} \tag{2.2}$$

The above formula assumes that backorders are uniformly distributed among slots of different MDSs, i.e., a backorder for component i is as likely to occur on a slot of one MDS as on another. Clearly, this is an approximation. Differences in flying activity and mission profiles among MDSs will lead to differences in failure rates and resulting backorders. A reasonable approach to modelling this aspect of the situation is to weight the term $EBO_{i,n}/T_i$ by a factor reflecting the differences in failure rates among MDSs. Since failure rates by MDS are seldom available, the AAM uses a surrogate weighting by component usage, as follows.

If flying hours were equal for all aircraft in MD h , then the probability that a random unit of component i will be out of service due to supply with n spares in the system is simply $EBO_{i,n}/T_i$, independent of MDS. However, we would expect that a unit installed on an aircraft which flies, say, twice the average number of hours would be twice as likely to be out of service as a unit which received average use.¹ To account for this, we introduce a factor called the use index, which measures the use received by a unit installed on an aircraft of MDS h_k .

If $F_{h(k)}$ is the flying hour program for MDS $h(k)$ (in hundreds of hours per quarter), then the number of flying hours accumulated by units of component i on that MDS is $a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}$. If IP is the total item program, i.e., the number of flying hours accumulated by units of component i over MD h , then

$$IP = \sum_{k=1}^{K(h)} a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}.$$

¹At least this would be true for an item whose demands generated on a flying hour basis. If demands were generated on another basis, e.g., sorties or inventory months, a flying hour adjustment would be inappropriate, though a similar adjustment based on the pertinent program unit might be indicated.

The average hours of use received by a unit of component i is then IP/T_i . The average hours of use of a unit installed on MDS $h(k)$ is $F_{h(k)}/T_{h(k),i}$, where $T_{h(k),i} = a(h(k),i) \cdot b(h(k),i) \cdot N(h(k))$ is the number of units of component i installed on aircraft of MDS $h(k)$. The use factor for component i on MDS $h(k)$, $U_{h(k),i}$, is defined to be the ratio of its use on the MDS to its average use over the MD,

$$U_{h(k),i} = \frac{(F_{h(k)}/T_{h(k),i})}{IP/T_i}. \quad (2.3)$$

The use factor is then used to refine the expression derived earlier for the probability that an aircraft of MDS $h(k)$ is missing a unit of component i , giving

$$q_{h(k),i,n} = (1 - b(h(k),i)) + \quad (2.4)$$

$$+ b(h(k),i) \cdot 1 - \frac{U_{h(k),i} \cdot EBO_{i,n}}{T_i} \cdot a(h(k),i).$$

Incorporating this refined definition of $q_{h(k),i,n}$ into the definition of $q_{h,i,n}$ (Equation 2.2), it is possible to consider the impact on the availability rate of aircraft type h of any inventory of spares, allowing for differences in flying hour program and configuration within the aircraft type. Using this logic, the probability that the aircraft is not waiting for a serviceable unit of component i to be repaired or shipped to it with $n(i)$ spares of component i in the system is given by $q_{h,i,n(i)}$. This probability is called the component aircraft availability.

Assuming independence of backorders between components, the probability that a random aircraft of MD h is not missing any of its reparable components

is the product of all the individual component probabilities. Denoting this probability by A_h , then

$$A_h = \prod_i q_{h,i,n(i)} \quad (2.5)$$

Since an available aircraft is defined to be one which is not missing any reparable component, A_h is also the probability that the aircraft is available. Considering the entire aircraft type, A_h is the fraction of that aircraft type expected to be available, the availability rate for the aircraft type.

The assumption of independence of backorders between components is a simplifying assumption, and, of course, is not strictly true. A failure of one component will sometimes cause a failure of another (or at least make such failures more likely). Serviceable units are sometimes removed (cannibalized) from grounded airplanes to make other airplanes available. These actions and others affect the accuracy of the independence assumption, and the magnitude of that effect is discussed in Chapter 7. Although consideration of the effects of cannibalization is appropriate when using the AAM in an attempt to project actually occurring availability rates, it would be inappropriate to do so in computing requirements for peacetime operating stock (POS) without also considering the costs of that cannibalization. (Similarly, when the AAM is used to determine POS requirements, it ignores existing war reserve stocks, although those stocks are in fact used to support peacetime operations.)

The development to this point has provided us with what might be called an evaluative capability--the capability to calculate the availability rate for an aircraft type--given a spares level for all of the components. In the next chapter, we address the optimization question: given an amount of money, what spares should be procured to attain the best possible availability rate?

CHAPTER 3. THE AIRCRAFT AVAILABILITY CURVE AND THE SHOPPING LIST

This chapter outlines the AAM's optimization capability. The model actually provides the answer to a set of optimization questions, one for each MD:

With a given amount of money, what spare items should be procured to achieve the highest possible availability rate?

The AAM answers this question by producing an ordered "shopping list." Buying from the list in the order indicated yields the maximum availability rate for the dollars expended.

THE OPTIMIZATION PROCEDURE

The optimization procedure used is a marginal analysis technique. Candidate units for procurement are ranked in terms of decreasing benefit per unit cost, where the benefit is defined in terms of the increase in availability rate which would occur if that spare unit were added to the inventory. We shall outline the technique here, relegating the detailed mathematical justification to Appendix C.

Recall from Chapter 2, that for each component, the number of expected backorders for any spares level is given by $EBO_{i,n}$, $n = 0, 1, 2, \dots$. If we project a starting spares level of $n(j)$ for component j at the impact point, then the procurement of the first additional spare unit reduces the expected backorders from $EBO_{j,n(j)}$ to $EBO_{j,n(j)+1}$. It increases the probability that an aircraft of type h is not missing a unit of component i from $q_{h,j,n(j)}$ to $q_{h,j,n(j)+1}$.

The availability rate of aircraft type h, before procurement of the first additional unit, is

$$A = \prod_i q_{h,i,n(i)} \\ = \left[\prod_{i \neq j} q_{h,i,n(i)} \right] \cdot q_{h,j,n(j)}.$$

The availability rate after the first additional unit of component j (spare unit $n(j) + 1$) is procured is

$$A' = \left[\prod_{i \neq j} q_{h,i,n(i)} \right] \cdot q_{h,j,n(j)+1}.$$

Thus, the ratio of the new to the old availability rates,

$$A'/A = q_{h,j,n(j)+1}/q_{h,j,n(j)},$$

depends only on the spares level of component j.¹

We call this ratio $I_{h,j,n(j)+1}$, the improvement factor due to unit $n(j)+1$ of component j. In general,

$$I_{h,i,n} = q_{h,i,n}/q_{h,i,n-1}. \quad (3.1)$$

If C_i is the procurement cost of component i, define the sort value of the nth unit of component i, $S_{h,i,n}$, to be

$$S_{h,i,n} = \ln(I_{h,i,n})/C_i \\ = \ln(q_{h,i,n}/q_{h,i,n-1})/C_i. \quad (3.2)$$

As might be expected from the nomenclature, the sort value is the measure of benefit per cost that is used to sort the candidate units for procurement.

¹If A' , and hence, A , equals 0, the ratio is indeterminate, and the ratio is defined to be 1. In practical applications, such a situation rarely occurs.

The natural logarithm function in the expression of benefit arises because of the multiplicative nature of availability, as expressed in the product formula (Equation 2.5). It ensures the mathematical accuracy of the optimization. Further discussion of this topic is found in Appendix C.

Calculations to this point comprise an array for each component i and for all spares levels $n = n(i), n(i) + 1, n(i) + 2, \dots$, summarized in Table 3-1.

TABLE 3-1. COMPONENT INFORMATION

<u>Number of Spares</u>	<u>Expected BackOrders</u>	<u>Component Aircraft Availability</u>	<u>Improvement Factor</u>	<u>Sort Value</u>
$n(i)$	$EBO_{i,n(i)}$	$q_{h,i,n(i)}$	-	-
$n(i)+1$	$EBO_{i,n(i)+1}$	$q_{h,i,n(i)+1}$	$I_{h,i,n(i)+1}$	$S_{h,i,n(i)+1}$
$n(i)+2$	$EBO_{i,n(i)+2}$	$q_{h,i,n(i)+2}$	$I_{h,i,n(i)+2}$	$S_{h,i,n(i)+2}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

The "starting availability rate," A_s , for each aircraft type is calculated by Equation 2.5. Thus

$$A_s = \prod_i q_{h,i,n(i)}$$

where the index i includes all components applied to the aircraft type and where $n(i)$ is the projected spares level for component i if no further procurements are made. The first unit on the shopping list will be that with

the highest sort value, say unit $n(j) + 1$ of component j . Since $S_{h,j,n(j)+1} = \ln(q_{h,j,n(j)+1}/q_{h,j,n(j)})/C_j$, the availability rate after this unit is procured is given by

$$\begin{aligned}
 A &= A_s \cdot \exp(C_j \cdot S_{h,j,n(j)+1}) \\
 &= \left[\prod_i q_{h,i,n(i)} \right] \cdot \exp(\ln(q_{h,j,n(j)+1}/q_{h,j,n(j)})) \\
 &= \left[\prod_i q_{h,i,n(i)} \right] \cdot (q_{h,j,n(j)+1}/q_{h,j,n(j)}) \\
 &= \left[\prod_{i \neq j} q_{h,i,n(i)} \right] \cdot q_{h,j,n(j)} \cdot (q_{h,j,n(j)+1}/q_{h,j,n(j)}) \\
 &= \left[\prod_{i \neq j} q_{h,i,n(i)} \right] \cdot q_{h,j,n(j)+1}
 \end{aligned}$$

This is the product of the item availabilities reflecting the new spares levels. We now add the next item, the one with the second highest sort value to the shopping list. The general form of the above relation is then used to calculate the availability rate after this unit and each subsequent unit are added to the list.

$$A_{NEW} = A_{OLD} \cdot \exp(S_{h,j,n} \cdot C_j), \quad (3.3)$$

when the n th unit of item j is added to the list. Continuing in this way, we obtain a shopping list of which Table 3-2 is a hypothetical example. As demonstrated in Appendix C, this shopping list contains only optimal solutions. For any amount of money, buying in order from this list until funds are exhausted attains the highest possible availability rate.² For an

²Of course, it's likely that no point on the list will exactly correspond to the amount available. This is of no practical consequence, though it can be viewed theoretically as a form of the knapsack problem.

expenditure of \$30,398, buying the first five entries in Table 3-2 (1 unit each of components A, C, and D, and 2 units of component B) to attain an availability rate of 66.78 percent is the best strategy.

TABLE 3-2. SHOPPING LIST

<u>STARTING POSITION: .6666 AVAILABILITY 5 UNITS OF A and 1 OF C ON HAND</u>					
<u>NO. OF UNITS</u> <u>COMPONENT</u>	<u>UNIT</u> <u>COST</u>	<u>CUMULATIVE</u> <u>COST</u>	<u>AVAILABILITY</u> <u>RATE</u>	<u>AVAILABLE A/C</u> <u>GAINED PER \$</u>	
6 TH A	\$1,598	\$1,598	.6667	.0000388	
1 ST B	2,300	3,898	.6669	.0000352	
2 ND C	10,400	14,298	.6674	.0000312	
2 ND B	2,300	16,598	.6676	.0000283	
1 ST D	13,800	30,398	.6678	.0000154	
7 TH A	1,598	31,996	.6679	.0000144	
o	o	o	o	o	
o	o	o	o	o	
o	o	o	o	o	

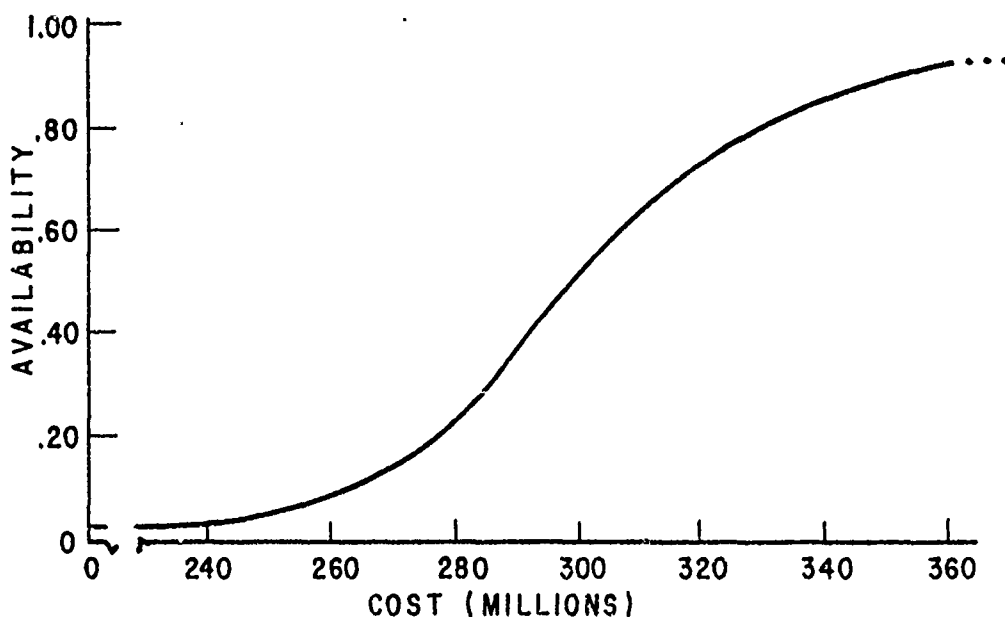
USE OF THE AAM AS A DECISION SUPPORT TOOL

Note that the optimization technique is MD-specific, i.e., a shopping list is constructed for each MD of interest.³ The optimization does not extend across MDs in the sense that it does not derive an "optimum" allocation of funds among different MDs. The problem of determining the proper balance

³Recall that we are assuming for the moment that no components are applied to more than one aircraft type. Complications due to the fact that some components are common and should appear on more than one aircraft shopping list will be considered later.

of support among MDs is more than a mathematical optimization problem; it requires the informed judgement of military planners. The AAM does not allocate funds to, say, the B-52 rather than the F-4, but it does show the consequences of such allocations. In particular, an extract from the shopping list, the aircraft availability curve, can greatly aid the planning necessary to determine the allocation of funds by aircraft type. This curve gives the funding required (for each MD) to reach any availability rate. It is simply taken from the two appropriate columns of the shopping list (the third and fourth columns of Table 3-2, for example). Figure 3-1 is a sample of such a curve.

FIGURE 3-1. COST vs. AVAILABILITY



Using a set of such curves, military planners can make informed decisions concerning aircraft support, considering both military priorities and economic

efficiencies. If availability targets by aircraft type are established, considering aircraft missions and military essentiality, then the investment required for each MD to reach those targets can be easily determined. Conversely, for a fixed overall funding level different allocations of that funding to MDs can be examined. The resulting mix of availability rates can be read from the curves; if the mix is unacceptable, the allocation can be modified. Because funding is always constrained, it is likely that every possible allocation will have some unattractive consequences, but the AAM allows planners to examine the options and make reasoned choices.

The AAM includes a number of automated interactive programs to facilitate this decision making. Perhaps the most obviously useful of these is one which automatically allocates given total funding to MDs to minimize the deviations of the resulting MD availability rates from a set of user-specified MD availability rate targets. (See Chapter 8 for a detailed description of some of these decision tools.)

CONSTRUCTION OF THE SHOPPING LIST AND AIRCRAFT CURVES - THE ALGORITHM

An outstanding feature of the AAM is its ability to process large amounts of data and to produce a large family of solutions. Models similar to the AAM typically process only a small number of components and/or produce only a few of the feasible solutions. The large scope of the AAM Air Force application is illustrated in Table 3-3. Considering each point on the availability curve as a solution to the optimality problem, the AAM produces literally hundreds of thousands of solutions. It is a very large scale model indeed.

The AAM achieves this capability by extensive use of mass storage to hold the results of intermediate calculations. This "architecture" is of the utmost importance in the treatment of levels of indenture. The following describes the processing for the single indenture case and indicates how it is applied recursively in the multiple indenture case in Chapter 6.

TABLE 3-3. SCOPE OF APPLICATION

SIMULTANEOUS CONSIDERATION OF ALL:

• AIRCRAFT

- 40 TYPES (F-16, B-52, ETC)
- 250 SUBTYPES (F-16A, F-16B, B-52G, B-52H, ETC)

• COMPONENTS

- 92,000 TOTAL
- 25% COMMON TO 2 OR MORE TYPES
- 90% COMMON TO 2 OR MORE SUBTYPES

The first⁴ step of processing requires data inputs consisting of:

- component logistics data (for the EBO calculations), such as failure rates, repair times (or pipelines), unit cost, NRTS rate
- application data (to identify which aircraft types each component is on, the quantity per application and application percentage)
- aircraft program data, such as aircraft inventory and flying hour programs (for use in the availability calculation), and the number of bases to which each aircraft type is deployed (for use in the EBO calculations).

Components are processed individually to avoid the inevitable core limitations encountered if simultaneous processing is attempted. For a given component i , an array of expected backorders is generated-- $EBO_{i,n}$, $n = n(i), n(i)+1, \dots$, where $n(i)$ is the component's projected asset level if no procurement is made.

⁴A certain amount of preprocessing is required to obtain and format the input data properly.

In some applications, management decisions may be made to buy enough spare units to fill the pipeline or a specified percentage of the pipeline or to buy additive quantities before beginning marginal analysis. Sometimes, especially when projecting far into the future, projected asset levels are negative, due to condemnations in the intervening time period. In such cases, enough spares must be procured to reach a zero level. These procurements are made automatically, the starting spares level suitably adjusted, and the cost accumulated into a "sunk cost" for the appropriate aircraft type.

Using the application data, each component's contribution to aircraft availability, $q_{h,i,n}$, is computed for each spares level n , as is the sort value, $S_{h,i,n}$, for the n th spare unit. The starting component availability, $q_{h,i,n(i)}$, contributes to a running computation of the starting availability rate of the appropriate aircraft type. When processing for component i is complete, component summary data are written into a file and saved for later processing. These data contain a header identifying the component, its sunk cost, starting EBO and component availability, unit cost, and records for each additional spare consisting of sort value $S_{h,i,n}$ and cost C_i for $n = n(i)+1, n(i)+2, \dots$ ⁵ For each additional spare unit of the component, a record identifying the component, its cost, and the sort value of that spare unit is written into a sort value file and saved.

When processing for all components is complete, the starting availability rate and sunk cost for each MD and the other summary data are written into a file. The sort value file is then sorted by MD major, sort value ($S_{h,i,n}$) minor. After sorting, all records pertaining to a given MD fall together, in order of descending sort value just as in the shopping list.

⁵If spares are procured in multiples (see Appendix D), the cost is the appropriate multiple of unit cost.

The (sorted) sort value file and the summary output file from the main program are now processed to construct the availability curves. The first record on the sort value file identifies the first MD. The sunk cost and starting availability for that MD are obtained from the summary file and form the first point on the curve. As each record is read from the sort value file, the cost is accumulated, and the availability, A , resulting from that procurement is calculated according to Equation 3.3. After each⁶ record from the sort value file is processed, a record consisting of the sort value $S_{h,i,n}$ for that spare, the corresponding availability attained, and the cumulative cost is written to an aircraft availability curve file. With the appearances of the first sort value record for a new MD, processing for the old MD is complete, and the curve file contains a table of the cost versus availability curve for the old MD.

To obtain the component information, i.e., the shopping list corresponding to a point on the MD curve, the data in the component summary data file is used. Given a cost or availability rate to identify a point on the curve, the curve file contains a sort value, S_h , associated with that point. Every spare unit on that MD which had a sort value, $S_{h,i,n}$, greater than S_h must be procured to attain that availability rate. These can be determined from the data on the component summary data file.

To determine the shopping list associated with a particular allocation of funds across all MDs, the cumulative cost for each MD, C_h , the resulting availability rate, A_h , and the associated sort value, S_h , from the curve file is used. Information on the component summary data file identifies the MD application of each component. Spare units from the component's sort value

⁶In practice, records are written to the availability curve file periodically, e.g., every fiftieth record, depending on the application.

array are accumulated as long as the unit's sort value is not less than the corresponding MD's curve sort value, $S_{h,i,n} \geq S_h$. Adding the number of units of the component bought as part of the sunk cost yields the total buy quantity for the component.

Thus, we see that the shopping list of Table 3-2 exists only conceptually. The AAM develops availability curves using only part of that information. For planning and allocation decisions, it uses the curves, not the detailed component information. However, the AAM can easily produce actual shopping lists (buy quantities for each component) when desired.

CHAPTER 4. COMMON COMPONENTS

In this chapter, we drop our assumption of non-commonality of components, recognizing that some (in fact, many) components are shared by more than one aircraft type. Proper consideration of commonality is crucial if results are to be meaningful, simply because there is so much of it. Table 4-1 is a matrix indicating the degree of commonality between several pairs of different Air Force MDs. The dollar magnitude is demonstrated by the fact that Air Force funding allocation to the common component category are typically about a quarter of the total reparable funding, larger than the allocation to any MD's peculiar components.¹

Assuming that component i is utilized by several aircraft types, the benefit due to procurement of a spare unit will not be monopolized by a single aircraft type, but will be shared by all aircraft types using that component, first come, first served. In this chapter, we develop a method for measuring this benefit for all aircraft types. In the notation of Chapter 2, we derive the formula for $q_{h,i,n}$ --the probability that an aircraft of MD h is not missing a unit of component i with n spare units of component i in the system--where component i is now shared by several aircraft types and where the effect of different configurations and flying hour programs is properly measured.

The formulas derived in this chapter are straightforward extensions of those in Chapter 2. Note first that $EBO_{i,n}$, the number of expected backorders for component i with n spares in the system, does not depend on whether the component is common to more than one MD. The demand rates and other factors

¹AFLC manages common components as separate categories (System Management Code or SMC 999-) rather than tying them to aircraft types.

TABLE 4-1. NUMBER OF COMMON COMPONENTS

	A007	A010	B052	B111	C005	C130	C135	C141	E003	E004	F004	F015	F016	F111	H001	H003	H053	T037	T038	T039
A007	2129	517	108	210	88	316	121	108	49	33	642	375	495	587	136	136	228	40	90	109
A010	517	1898	124	87	84	218	106	132	97	27	618	404	525	440	91	104	125	40	91	101
B052	108	124	5996	201	131	580	1119	219	180	90	129	86	111	121	89	115	166	89	141	118
B111	210	87	201	3179	90	100	165	87	100	53	198	72	184	2324	79	87	85	23	89	63
C005	88	84	131	90	4673	197	180	277	111	32	64	75	84	96	79	82	152	46	75	78
C130	316	218	580	100	197	4250	630	423	128	206	429	77	130	173	194	219	387	126	145	185
C135	121	106	1119	165	180	630	4011	403	304	673	82	54	93	97	130	152	225	74	122	160
C141	108	132	219	87	277	423	403	1969	365	42	87	70	112	110	103	123	194	53	119	133
E003	49	97	180	100	111	128	304	365	3728	70	68	71	85	68	40	37	68	21	60	61
E004	33	27	90	53	32	206	673	42	70	961	24	19	21	29	24	13	16	24	4	30
F004	642	618	129	198	64	429	82	87	68	24	5384	405	559	917	68	80	128	37	66	68
F015	375	404	86	72	75	77	54	70	71	19	405	3258	868	400	48	63	64	83	115	51
F016	495	525	111	184	84	130	93	112	85	21	559	868	3296	476	77	88	106	25	76	90
F111	587	440	121	2324	96	173	97	110	68	29	917	400	476	5972	87	108	114	24	100	74
H001	136	91	69	79	79	194	130	103	40	24	68	48	77	87	630	191	171	66	85	124
H003	136	104	115	87	82	219	152	123	37	13	80	63	88	108	191	645	301	60	114	144
H053	228	125	166	85	152	387	225	194	68	16	128	64	106	114	171	301	1158	52	102	126
T037	40	40	89	23	46	126	74	53	21	24	37	83	25	24	66	60	52	467	213	104
T038	90	91	141	89	75	145	122	119	60	4	66	115	76	100	85	114	102	213	707	120
T039	109	101	118	63	78	185	160	133	61	30	68	51	90	74	124	144	126	104	120	724

Intersection of row and column shows number of repairable components common to indicated MDs.

Diagonal entries give total number of repairable components applied to MD.

SOURCE: AFLC D041 Data Base, 30 September 1982.

used in the EBO computation are aggregated over the entire system, and the resulting $EBO_{i,n}$ is also system-wide rather than being in relation to one particular MD. This "pooling" effect is one of the benefits of commonality. (It is well known that the expected backorder total is lower if all spares are pooled than if portions of the component's spares inventory are segregated to satisfy only demands from certain sources.) As in Chapter 2, $EBO_{i,n}/T_i$ is the probability that a random slot will be missing a unit of component i . T_i , the total installed, or total number of slots, is obtained by summing over all MDs to which component i is applied. We must also allow for different levels of usage across MDs as we did in Chapter 2 across MDSs with a single MD. The logic is identical, and we give here a compressed treatment because of the similarity.

Suppose we have a number of aircraft types, MD h , $h = 1, 2, \dots, H$; within each MD there are MDSs h_k , $k = 1, 2, \dots, K(h)$. Let $a(h(k), i)$ be the QPA of component i on MDS $h(k)$, and let $b(h(k), i)$ be the application percentage. If $N(h(k))$ is the number of aircraft in MDS $h(k)$, and $N(h)$ is the number in MD h , and if we assume (temporarily) that backorders are uniformly (randomly) distributed across all slots, we obtain formulas formally identical to Equations 2.1 and 2.2.

$$\begin{aligned}
 q_{h(k), i, n} &= \text{the probability that an aircraft of MDS } h(k) \text{ is not missing} \\
 &\quad \text{a unit of component } i \text{ if there are } n \text{ spare units of} \\
 &\quad \text{component } i \text{ in the system} \\
 &= (1 - b(h(k), i)) + \\
 &\quad + b(h(k), i) \cdot (1 - EBO_{i,n}/T_i)^{a(h(k), i)} \quad (4.1)
 \end{aligned}$$

$q_{h,i,n}$ = the probability that a random aircraft of MD h is not waiting for a spare unit of component i if there are n spare units of component i in the system

$$= \sum_{k=1}^{K(h)} \frac{N(h(k))}{N} q_{h(k),i,n} \quad (4.2)$$

As in Chapter 2, the assumption of uniform distribution of backorders is an approximation, more so when dealing with different MDs instead of different MDSs within one MD. To refine this assumption, we again apply the use index of Chapter 2, in this case averaging component use across all aircraft types.

If $F_{h(k)}$ is the flying hour program for MDS $h(k)$, and IP is the total item program, i.e., the number of flying hours accumulated by units of component i over all MDs, then

$$IP = \sum_{h=1}^H \sum_{k=1}^{K(h)} a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}.$$

The average hours of use received by a unit of component i is then IP/T_i , and the use factor for a unit installed on an aircraft of MDS $h(k)$ is given by

$$U_{h(k),i} = \frac{F_{h(k)}/T_{h(k),i}}{IP/T_i}$$

where $T_{h(k),i}$ is the number of units installed in aircraft of MDS $h(k)$.

Equations 4.1 and 4.2 can now be refined as

$$q_{h(k),i,n} = 1 - b(h(k),i) + b(h(k),i) \cdot \left(1 - \frac{U_{h(k),i} \cdot EBO_{i,n}}{T_i} \right)^{a(h(k),i)} \quad (4.3)$$

$$q_{h,i,n} = \sum_{k=1}^{K(h)} \frac{N(h(k))}{N} q_{h(k),i,n} \quad (4.4)$$

These expressions are identical with those derived in Chapter 2 for a component dedicated to a single type. The fact that the component is shared has indeed entered into the computations, both in the value of T_i and in the computation of average use. In addition, the EBO computation considered total usage of the component over all aircraft types, whether the demands were generated by one aircraft type or by many. Since $q_{h,i,n}$ is related to a single type of aircraft, one should not be surprised at a similarity between this and the case of a peculiar component.

The above derivation provides the necessary machinery to calculate the effect of common components upon aircraft availability rates. It is also necessary to modify the marginal analysis, i.e., the formation of availability curves and shopping lists. The complication here, of course, is that a spare unit of a common component will appear on more than one shopping list.

The first adjustment required is to prorate cost. If we were to use the full cost of a common component in computing the benefit per cost of a spare to an aircraft type, we would be neglecting the benefit from sharing the spare among several aircraft types. To avoid that we define a prorating

factor, $V_{h,i}$, which is the proportion of "use" of component i by aircraft type h . Then an appropriate definition of sort value for aircraft type is

$$\text{sort value} = \frac{\ln q_{h,i,n+1} - \ln q_{h,i,n}}{V_{h,i} C_i} \quad (4.5)$$

where C_i is the cost of a unit of component i . A suitable measure of "use" is the ratio of total component hours flown per time period by aircraft of type h to total component hours flown per time period by all aircraft.² Defining $V_{h,i}$ as the proportion of the total item flying hour program generated by aircraft of MD h ,

$$V_{h,i} = \frac{\sum_{k=1}^{K(h)} a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}}{IP} \quad (4.6)$$

Thus, a spare unit of a common component has a sort value for each aircraft type. Sunk costs are prorated to each of the using aircraft types, and the component's contribution to each aircraft's starting availability rate is accumulated. The component summary data file includes information indicating each aircraft type h using the component and each prorating factor $V_{h,i}$. Instead of one array of sort values, there is one array per MD for the component. Records written to the sort value file also include records for each MD along with the sort value and the prorated cost.

The construction of the availability curves proceeds as before, with one important difference--the curves are no longer independent. The contribution from a spare unit of a common component appears in several curves, reflecting

²For items whose demands generate on other than a flying hour basis, $V_{h,i}$ is set to the proportion of the total item program generated by MD h , where the program is computed appropriately, e.g., on an inventory month basis.

the availability improvement that the unit's procurement would bring to several aircraft types.

This lack of independence is reflected in the formation of the shopping list corresponding to an allocation across aircraft types. Suppose we have for each MD h , a cumulative cost, C_h , an associated availability, Q_h , and sort value, S_h , from the availability curves file, and we wish to derive the shopping list corresponding to this allocation. The buy quantity for a peculiar component is computed as before. If component i is applied to MD h and MD k with allocations corresponding to sort values S_h and S_k , then it will have two arrays of sort values, one for each MD. We proceed down the array for MD h "buying" the indicated units as long as the component sort value, $S_{h,i,n}$, is greater than or equal to S_h . Adding the units bought for sunk cost gives a buy of B_h units, which is reflected in the cost/availability pair for MD h . Proceeding similarly for MD k , we obtain a buy B_k , reflected in the cost/availability of MD h . Since a given unit of component i may yield different availability improvements to the two MDs, and since the cost prorating factors may be unequal, the sort values for a given unit, $S_{h,i,n}$ and $S_{k,i,n}$, will generally not be equal and may be vastly different. In addition, the allocation may support the two MDs to different levels. Thus, it is likely that $B_h \neq B_k$. Under these circumstances, the AAM will buy the larger of B_h and B_k . This decision is arbitrary, but not unreasonable. It allows the aircraft type which "wants" the component most to drive the decision.

If $B_h > B_k$, then the position on the MD k availability curve is only an approximation. It does not reflect the benefit to MD k availability of the additional $B_h - B_k$ components which will appear on the shopping list, nor does it reflect the prorated cost to MD k of the additional units. To reconcile these imbalances and obtain precise results, the buys on the shopping list can

be added to each component's projected starting assets and the resulting availability rates calculated. The shopping list program itself produces the reconciled cost figures by MD.

Thus, the existence of common components causes a lack of independence between the availability curves, reflected in the fact that an allocation across aircraft types must be treated as an approximation. Reconciliation of these imbalances increases availability and costs. The magnitude of these increases depends on the allocation (e.g., an allocation which prescribes a high availability rate for the F-111 and a low availability rate for the similar B-111 will result in a large change in the B-111 during the reconciliation because of the great degree of commonality between the two types). With a reasonably balanced allocation, funding adjustments during reconciliation are typically on the order of three to five percent.

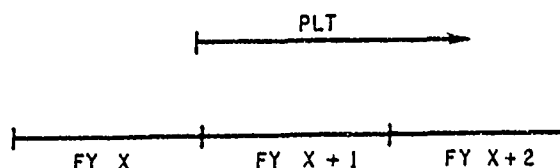
CHAPTER 5. THE REPAIR OPTION

Discussion of the AAM to this point has been concerned only with procurement of components. This chapter addresses a second dimension--that of component repair. The AAM may be run with a repair option, which gives it the capability to trade off depot repair with procurement of reparable items. Incorporation of this capability into the model involves several difficulties. The difference in leadtimes between procurement and repair is a crucial one, because repair actions affect availability much more quickly than do procurement actions. It is difficult to trade off the benefits of money spent now for repair, with essentially immediate benefits, and the same money spent for procurement, with benefits on the average of 18 months away.

In analyzing the expenditure of FY X funds for spares procurements, we assume that the benefit from those procurements will be realized an average procurement leadtime past the end of that fiscal year. In Figure 5-1, PLT represents that average leadtime, which recently has been 18 months for Air Force reparable components. Thus, availability rates are projected to the middle of FY X+2; procurement of spare units is based on their contribution to availability at that point relative to cost. The repair option of the AAM develops a repair strategy for FY X consistent with the procurement strategy. It determines the optimum mix of repair and procurement funding for FY X, optimum in the sense of maximizing availability rates in FY X+2.

Typically, AAM input data are current as of a given date (the asset cutoff date). Assets, projected demand rates, repair times and other logistics data are a combination of a snapshot of the system as of that date and the knowledge possessed on that date concerning future activity. Suppose

FIGURE 5-1. AAM TIMELINE



the spares level for component i at the asset cutoff date is n_0 . This quantity is obtained by identifying units of component i in various statuses--serviceable spares, carcasses in repair (base and depot), waiting for induction to depot repair, in transit--and then subtracting out "holes" on aircraft, i.e., backorders.

The AAM then projects a spares level for component i at its "buy point" (a procurement leadtime past the end of FY X) if no further procurements are made. If D is the projected number of total demands between the asset cutoff date and the buy point, R is the number of those demands which are repaired (base and depot) in that same period, and O is the number of units on order at the asset cutoff date, then the projected number of spares is given by $n(i) = n_0 + O + R - D$.

When the AAM is run without the repair option, $n(i)$ is the starting spares level for component i , and the starting availability rate for an aircraft type is given by the product formula

$$Q_h = \prod_i q_{h,i,n(i)}.$$

The repair option identifies which depot repairs are possible in the period of interest--FY X in our example. This quantity includes units uninducted at the depot at the asset cutoff date, plus all demands resulting in a carcass being sent to depot repair, minus those expected to be condemned.

Call this quantity $M(i)$; it is the maximum number of repairs of component i the depot can be expected to perform during FY X. Repairs already in progress at the beginning of FY X are not included in M , but are considered as occurring during FY X-1. We reduce the projected spares level $n(i)$, of each component by $M(i)$ to obtain a starting spares level, $nr(i)$, if none of those repairs are made.

As in the case without the repair option, if $nr(i)$ is negative we repair as many of $M(i)$ as necessary to raise $nr(i)$ to zero. If $M(i)$ is exhausted before a zero level is reached, then we procure the remainder of the spare units needed to reach a zero level. Depending on management decisions for a particular model exercise, we may buy or repair to fill the pipeline or a specified percentage of the pipeline.

The question we wish to answer is the following:

With a given amount of money, what is the optimum way to spend it on depot repair and procurement in FY X to achieve the best possible availability rate in FY X+2?

Just as in the consideration of procurement alone, we compute an expected backorder and availability improvement factor array for each component. Common components, of course, will have a set of improvement factors for several aircraft types. The array begins at a spares level $nr(i)$, assuming no repairs are made. For component i , the first additional $M(i)$ spares can be obtained by depot repairs at the cost of a repair RC_i . Only after the $M(i)$ are exhausted do procurements become necessary; additional units are then obtained at the full procurement cost C_i . Thus, the expected backorder and sort value array for component i will be of the form of Table 5-1 (compare with Table 3-1).

TABLE 5-1. COMPONENT INFORMATION - REPAIR OPTION

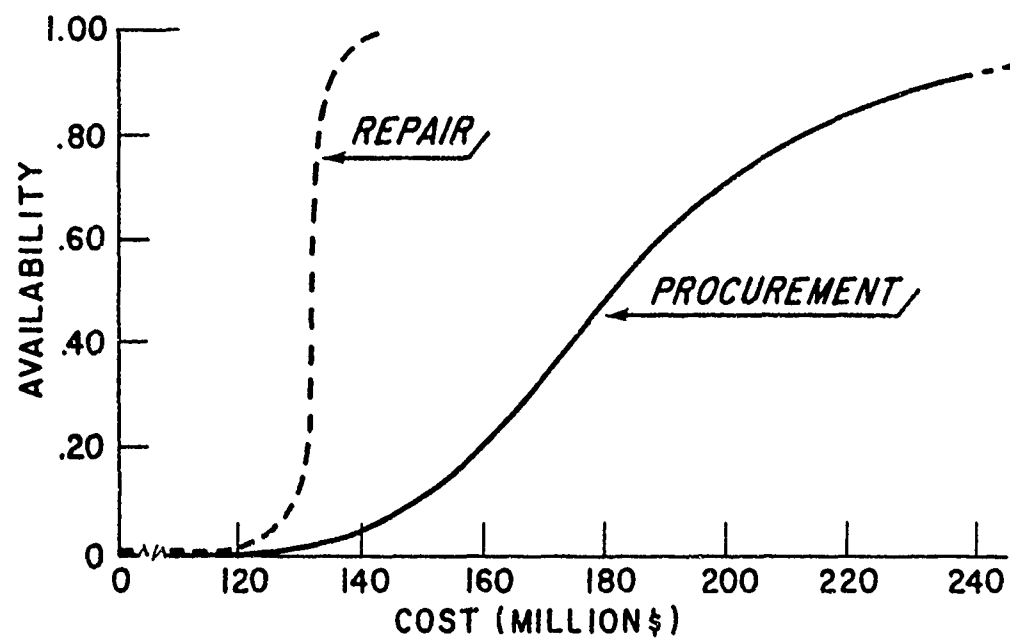
<u>Number of Spares</u>	<u>EBO</u>	<u>Component Aircraft Availability</u>	<u>Improvement Factor (IF)</u>	<u>Sort Value</u>
nr(i)	$EBO_{nr(i)}$	$q_{h,i,nr(i)}$	-	-
nr(i)+1	$EBO_{nr(i)+1}$	$q_{h,i,nr(i)+1}$	$\frac{q_{h,i,nr(i)+1}}{q_{h,nr(i)}}$	$\frac{\ln(IF)}{RC_i}$
nr(i)+2	$EBO_{nr(i)+2}$	$q_{h,i,nr(i)+2}$	$\frac{q_{h,i,nr(i)+2}}{q_{h,i,nr(i)+1}}$	$\frac{\ln(IF)}{RC_i}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$n(i)=nr(i)+M(i)$	$EBO_{n(i)}$	$q_{h,i,n(i)}$	$\frac{q_{h,i,n(i)}}{q_{h,i,n(i)-1}}$	$\frac{\ln(IF)}{RC_i}$
$n(i)+1$	$EBO_{n(i)+1}$	$q_{h,i,n(i)+1}$	$\frac{q_{h,i,n(i)+1}}{q_{h,i,n(i)}}$	$\frac{\ln(IF)}{C_i}$
$n(i)+2$	$EBO_{n(i)+2}$	$q_{h,i,n(i)+2}$	$\frac{q_{h,i,n(i)+2}}{q_{h,i,n(i)+1}}$	$\frac{\ln(IF)}{C_i}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Spare units are ranked in terms of sort value, just as before. Formation of the shopping lists and the availability curves is performed as discussed in Chapters 3 and 4 with the sole difference of identifying and accumulating repair costs and procurement costs separately.

Thus, a given availability rate for a particular aircraft corresponds to a total cost for procurement and a total cost for depot repair, their sum representing the smallest total cost needed to reach that availability rate. The shopping list corresponding to that availability rate indicates how many depot repairs of each component should be made in FY X and how many should be procured. Note that no spares of a component will be obtained through procurement until all candidates for repair have been exhausted, since $RC(i) < C(i)$ and the improvement factors of the first $M(i)$ additional spares are greater than those of the subsequent ones. A given component may be in any one of several states. It may not have exhausted its repair candidates, indicating that its projected asset position was more than adequate and could safely be reduced by skipping some repairs. It may have exhausted all its repair candidates, yet not have been valuable enough to warrant any procurements. (Since C_i is typically on the order of 5 times as large as RC_i , the first candidate for procurement will have a sort value about one-fifth that of the last candidate for repair, and these two will be widely separated on the shopping list.) Finally, it may have exhausted all its repairs and required procurement as well.

Figure 5-2 is a hypothetical graph of availability rate versus procurement and repair costs. Reading across from a given availability rate yields the optimum mix repair and procurement costs required to reach that availability rate.

FIGURE 5-2. PROCUREMENT AND REPAIR FUNDING vs. AVAILABILITY



CHAPTER 6. LEVELS OF INDENTURE

In the preceding chapters, we have assumed that all components are on the first level of indenture, i.e., that they are applied directly to the aircraft. The actual situation is more complicated, as shown in Table 6-1. That table is a partial display of the hierarchical relationship of components on the F016A. Component A indented below component B indicates that A is a subassembly of B.¹

TABLE 6-1. APPLICATION LEVELS REPORT FOR MDS - F016A

ITEM & NAME						QTY	FAP	PLC
LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6			
5865010450837EW	AMP DALR69					1	1.00	NR
	5865010564216EW					1	1.00	NR
5865010450848EW	IND ALR69					1	1.00	NR
	5865010610448EW	TUBE ALR69				1	1.00	NR
	5865010610544EW	CCA ALR69				1	1.00	NR
	58650114715LEW	CNT CALR69				1	1.00	NR
	58650114716LEW	CCB ALR69				1	1.00	NR
5865010450982EW	CHADSPP					2	1.00	NR
5865010489024EW	REC ALR69					1	1.00	NR
	5865010592957EW	REC ALR69				1	1.00	NR
	5865010595952EW	REC ALR69				1	1.00	NR
	5865010595953EW	REC ALR69				1	1.00	NR
	5865010595954EW	REC ALR69				1	1.00	NR
	5865010597283EW	REC ALR69				1	1.00	NR
	5865010599147EW	REC ALR69				1	1.00	NR
	5865010599455EW	REC ALR69				1	1.00	NR
	5865010599456EW	REC ALR69				1	1.00	NR
	5865010599457EW	REC ALR69				1	1.00	NR
	5865010606746EW	REC ALR69				1	1.00	NR
5865010491178EW	TU AUX 69					1	1.00	NR
5865010535296EW	IMU ALR69					1	1.00	NR
	5865010490354EW	CCA ALR69				1	1.00	NR
	5865010499843EW	CCA ALR69				1	1.00	NR
	5865010546126EW	PWR DALR69				1	1.00	NR
5865010540013FW	CHAFFMILE40					1	1.00	NR
5895005291911	RECEIVER					1	1.00	NR
	5895003397006	CFCT CD AT				1	1.00	NR
	5895005585844	PWR SUPPL				1	1.00	NR
	5895005585862	CIRCUIT CD				1	1.00	NR
	5895005585850	CNT CD AT				1	1.00	NR
	5895005585851	ARMY AMPL				1	1.00	NR
	5895005585852	LSBAND IRC				1	1.00	NR
	5895005585857	DVPC SWITCH				1	1.00	NR
	5895005585864	CKTD ASDY				1	1.00	NR
	5895005642041	DUAL RECVP				1	1.00	NR
	5895005585847	LAYOUT 1PF				1	1.00	NR
	5895005585846	LAYOUT 1PF				1	1.00	NR

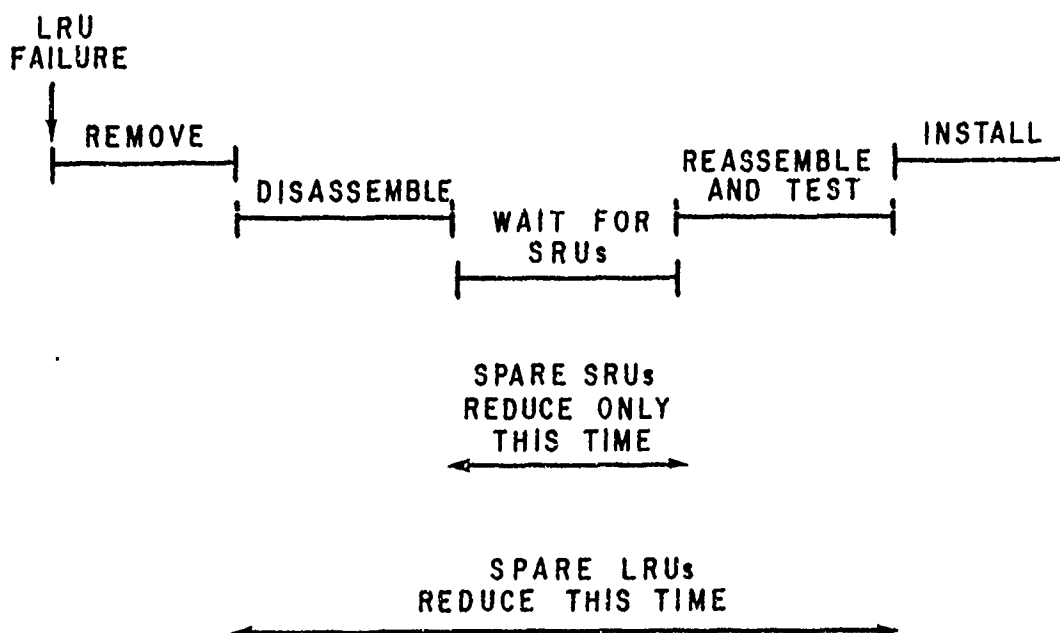
¹Reports in this form inspired the use of the term levels of indenture for the hierarchical relationship between components.

The maintenance philosophy for a first indenture level component, or LRU, is to remove a failed unit from the aircraft (or other end item), replace it with a serviceable spare, if one is available, and send the failed unit to maintenance.

Repair of an LRU typically concentrates on the isolation and removal of failed subcomponents, the second indenture level items, or SRUs. If spares for the failed SRU are available, the LRU can be returned to serviceable condition with no delay.

The impact of LRU backorders affects aircraft. The impact of SRU backorders is to lengthen LRU repair times, lowering the probability of an LRU spare being in serviceable condition when needed, as depicted in Figure 6-1. Thus, spares for LRUs have a direct impact on aircraft availability, while spare SRUs have an indirect effect. Of course, since SRUs typically are lower cost than LRUs, they may still be an attractive buy. The AAM considers these

FIGURE 6-1. LEVELS OF INDENTURE



complex relationships in its computation of aircraft availability, using a recursive treatment which allows the analysis of any number of levels of indenture.

Components are classified by level of indenture. All components on the lowest² level of indenture are processed first. The next higher level of indenture is then processed, considering the effect of the lower indenture level. In effect, each component on the higher level has a curve of cost versus lower level support, where the lower support is measured in terms of total EBOs for the subassemblies of the component. Note the analogy between such a curve and the cost versus aircraft availability curve produced by the AAM. Investment in spare units of the higher level component is traded off against an equal investment in its subassemblies to minimize expected backorders of the higher level component. The process is then repeated at the next higher indenture level, and so on. The details of this computation may be found in Appendix E.

As a result of this computation, we obtain an expected backorder array for each LRU. This array is similar to that discussed in Chapter 3 (Table 3-1) with one critical difference--some entries of the array correspond, not to buying an additional spare unit of the LRU, but to investing an amount of money equal to the cost of the LRU into the component's subassemblies. We refer to this cost as an "Lsworth" (LRU's worth) and speak of buying n Lsworths of SRUs, rather than using the equivalent phrase "buying from the SRU shopping list until the cumulative cost equals n times the LRU unit cost." The formation of the SRU shopping list ensures that the investment in SRUs produces the minimum SRU total EBO. A typical array is shown in Table 6-2. The algorithm guarantees that for the given amount of investment,

²In the Air Force application, the AAM treats five levels of indenture.

the resulting backorder level, $EBO_{i,n}$ (for component i , with an investment of n times the cost of component i) is the minimum for that investment.

TABLE 6-2. LRU EXPECTED BACKORDER ARRAY

<u>INVESTMENT</u>	<u>EBO</u>	<u>LRUs</u>	<u>DEPOT LRUs</u>	<u>SRUs</u>
0	39.51	0	0	0
1	38.14	0	0	1
2	37.06	0	0	2
3	36.06	1	1	2
4	35.06	2	1	2
5	34.06	3	1	2
6	33.06	4	1	2
7	32.07	5	2	2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Contrast the contents of Table 6-2 with the case of an item with no subassemblies. $EBO_{i,n}$ results from procurement of n spare units of component i . The only tradeoff concerns distribution of units between base and depot. When the item has subassemblies, $EBO_{i,n}$ must also reflect a tradeoff between buying units of component i and investing in its lower indenture items, themselves optimally distributed between base and depot.

We now construct curves of cost versus availability rate just as outlined previously. $q_{h,i,n}$ is defined as before, n is now the number of Lsworths invested in the family of component i --itself, its subassemblies, and

sub-subassemblies, etc. The product formula for aircraft availability (Equation 2.5) is as before, except that only the terms for LRUs are used. The effect of the SRUs is already included in the expected backorder total for the LRUs and their corresponding LRUs contribution to availability rate.

Once the aircraft availability curves are constructed, allocations of funds and resulting availability rates by aircraft type may be investigated. Funds required to reach specified availability rates for each aircraft type may be calculated and options may be examined by military planners just as outlined previously.

Shopping lists corresponding to an allocation by aircraft type may be generated. This is a recursive top-down procedure. As in the non-levels case, the sort values for each MD corresponding to the funding/availability rate are read from the curves. This information is used to determine how many Lsworths to invest in each LRU, by moving down the array in the component summary data file until the component sort value drops below the aircraft sort value. As before, conflicts between aircraft types for common components are resolved by going to the highest investment level. Records on the component summary data file contain information on how many of the LRUs that investment actually buys as well as a key to indicate the corresponding investment in SRUs. More details on this procedure are found in Appendix E.

If the repair option is exercised, a further complication arises. When trading off investment in an LRU with investment in its family of SRUs before the LRU's repair candidates are exhausted, we aggregate the SRU curve into increments of the LRU's repair cost, as we are trading off SRU investment against LRU repairs. We speak then of buying an Rsworth of SRUs (although the SRU curve will generally include a mix of repair and procurement). When the LRU's repair candidates are exhausted, the SRU curve is reaggregated into

Lsworths, and the tradeoff is then in terms of the LRU's procurement cost. The remainder of the procedure continues as before, with the obvious modifications needed to track procurement and repair costs and quantities separately.

CHAPTER 7. VALIDATION OF THE AAM

In 1977-1978, the AAM was tested jointly by LMI and AFLC. LMI's responsibility during the test was to determine the accuracy of the product formula (Equation 2.5) as an expression of availability rate. The AFLC portion of the test [13] covered the entirety of the model, the backorder computation together with the resulting availability projection. Both portions of the test produced positive results. This chapter concentrates on the results of the test of the product formula. Since the method of expected backorder computation in the AAM was (and is) widely accepted, it was the product formulation of availability that had the greater need of validation.

The test was performed using maintenance and depot requisition data from Air Force systems D056B and D165A for three time periods: 1 December 1975 - 19 January 1976; 1 March - 19 April 1976; 1 July - 19 August 1976 (see Figure 7-1 for flow chart). The Air Force G033B system was used to develop a file of valid aircraft serial numbers (or tail numbers) for the MDSs considered and also to determine a percentage of the time that airplanes of each MDS were in an inactive status (e.g., in depot overhaul). These aircraft are not considered in the application of the product formula.

A master test file was constructed which includes the start and stop times by date and shift of every LRU backorder incurred by one of the valid tail numbers. The test was run using only LRUs as these components are precisely those for which backorders affect an aircraft directly.

The master test file was sorted in MDS/tail number order, and the number of shifts for which a given tail number was unavailable, i.e., had one or more outstanding requisitions was recorded.

FIGURE 7-1. PRODUCT MODEL TEST

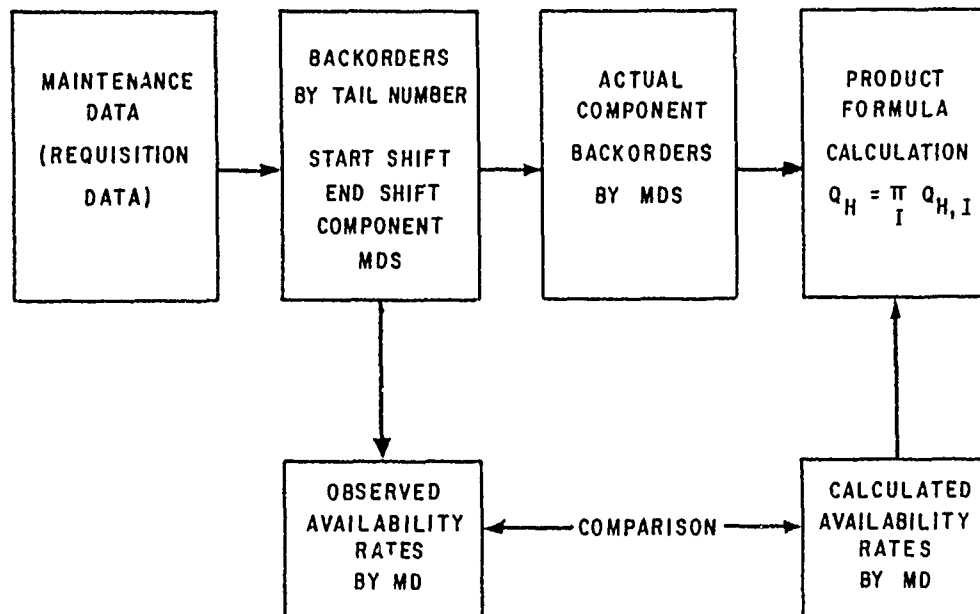


Table 7-1 shows some of the data collected. These data are extracted from the record for a single C005A aircraft, tail number 680002111, for the 50 day December-January time period. Each line represents a component that was missing from the aircraft at some time during that period. The block at the right shows when the component was missing, one column for each day. An entry in the nth column indicates the component was missing from the aircraft the nth day of the test period. Thus, the second line from the bottom indicates that component 6680004858638 was missing in days 11 through 18. A code for the entries indicates the type of maintenance action or effect on the aircraft. For example, the "U" on day 18 for component 5826004360760 indicates the missing condition was removed by a cannibalization. Reading down each column shows whether the aircraft was available that day. Reading across gives observed backorders for the component.

TABLE 7-1. TEST DATA

MDS	TAIL NO:	COMPONENT	DAY				
			1234567891	1234567892	1234567893	1234567894	1234567895
C005A	68000211	1560003128243		PZQ			
C005A	68000211	1560003128243		PZQ			
C005A	68000211	1560003128243		PQ			
C005A	68000211	1560003128243		PQ			
C005A	68000211	1560004954535				PQ	
C005A	68000211	1560008559405		PZQ			
C005A	68000211	1680001794139		P+Q			
C005A	68000211	1680002202806		PZQ			
C005A	68000211	4820005720645				Q	
C005A	68000211	4920000782418					PQ
C005A	68000211	4920004265486					Q
C005A	68000211	5821000658406					PQ
C005A	68000211	5821000658406					PQ
C005A	68000211	5821000704475			Q		
C005A	68000211	5826000613080		PQ			
C005A	68000211	5826001628452					PQ
C005A	68000211	5826004360760	BBBBBBBBBBBBBBBBUU				
C005A	68000211	6680004858638		PZZZZZZQ			
C005A	68000211	1560003128243					

Results for one time period are graphed in Figure 7-2. The sample correlation coefficient ρ , adjusted for the mean, of the calculated rates against the observed rates was 0.974, and the coefficient of determination $r^2 (= \rho^2)$ was 0.948. This represents an exceptionally good correlation; similar correlations were derived in the other time periods.

Figure 7-2 implies, however, that the calculated availabilities are biased. The points do indeed give a good linear fit, but not to the line $y = x$, as they would if the AAM were a perfect predictor. The high value of r^2 indicates that a simple linear correction could remove most of this bias, and Figure 7-3 shows that such an adjustment does indeed give good results. The adjustment¹ used was Adjusted Rate = 0.34 + 0.66 (Unadjusted Rate) which was derived for the December-January data; this adjustment also worked well for the other time periods. Table 7-2 summarizes the data on the graphs.

¹The AAM has sometimes used a refined estimate, Adjusted Rate = 0.31 + 0.69 (Unadjusted Rate) to adjust availability projections.

FIGURE 7-2. MAINTENANCE DATA (D056)
FOR JULY - AUGUST 1976

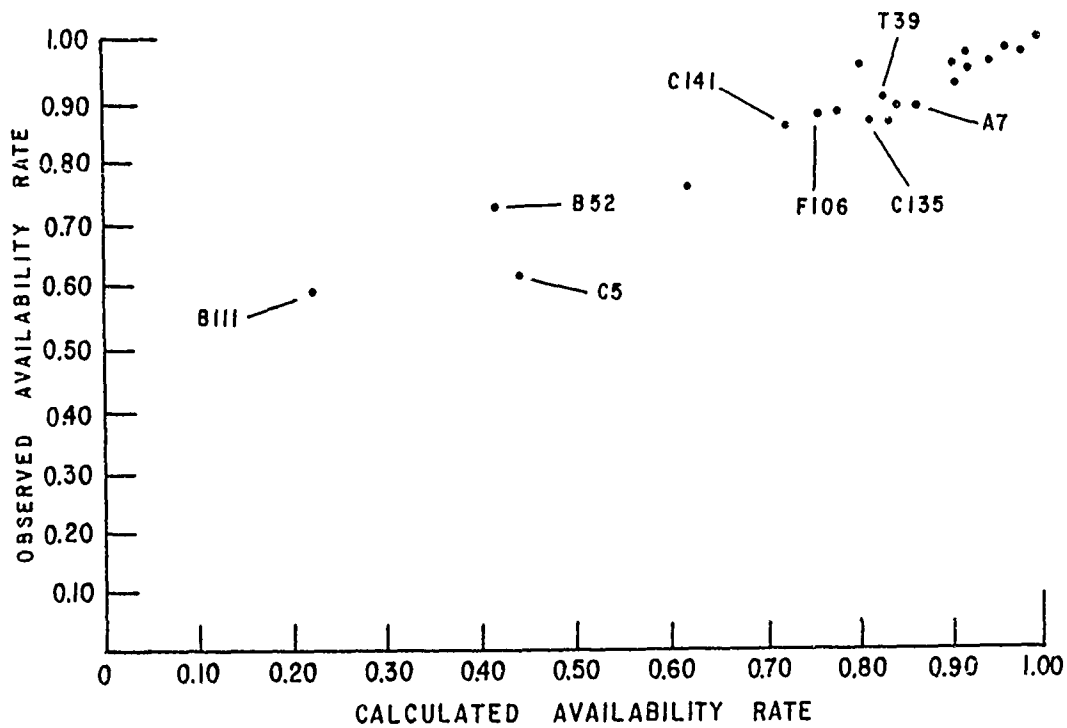


FIGURE 7-3. ADJUSTED MAINTENANCE DATA (D056)
FOR JULY - AUGUST 1976

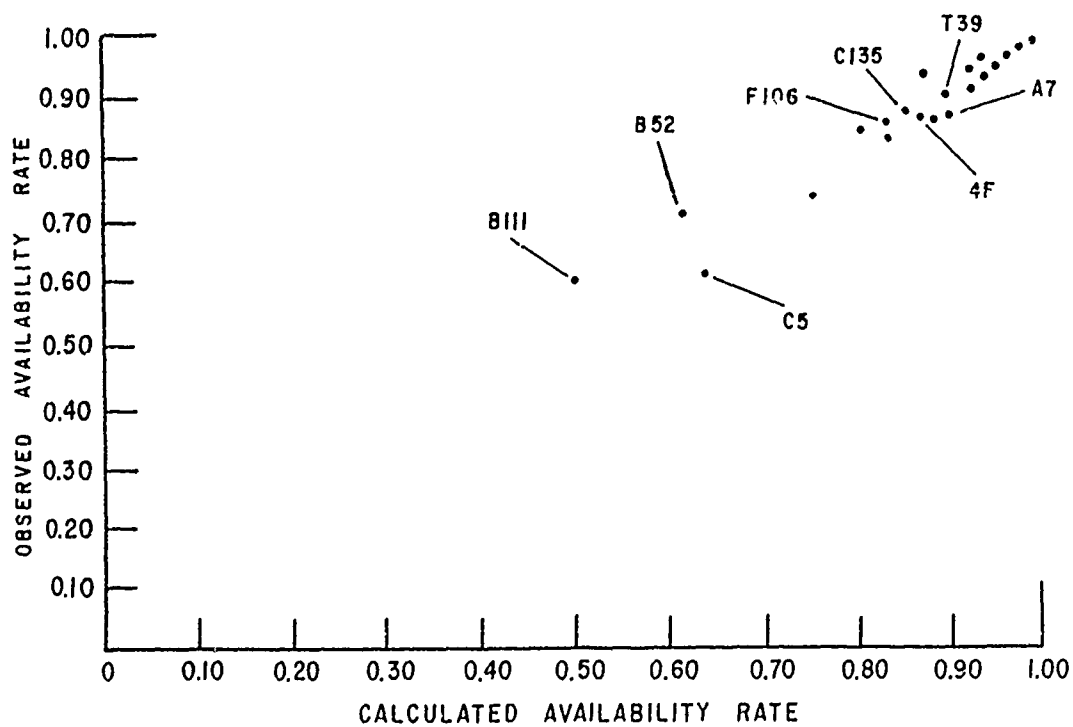


TABLE 7-2. MAINTENANCE DATA (D056) FOR JULY-AUGUST 1976

<u>MD</u>	<u>Calculated Availability Rate (Unadjusted)</u>	<u>Calculated Availability Rate (Adjusted)</u>	<u>Observed Availability Rate</u>
A007	0.858	0.908	0.893
B052	0.419	0.619	0.715
B111	0.231	0.495	0.596
C005	0.444	0.635	0.609
C097	0.938	0.961	0.951
C123	0.925	0.952	0.950
C130	0.779	0.856	0.865
C131	0.963	0.977	0.970
C135	0.815	0.880	0.883
C141	0.725	0.821	0.854
F004	0.837	0.894	0.878
F005	0.905	0.939	0.917
F100	0.969	0.981	0.974
F101	0.995	0.998	0.994
F106	0.764	0.846	0.869
H003	0.812	0.879	0.926
H053	0.629	0.757	0.745
T033	0.917	0.947	0.946
T037	0.968	0.981	0.973
T038	0.913	0.944	0.938
T039	0.844	0.899	0.900
V010	0.919	0.948	0.937

It is clear that, in addition to the inevitable statistical variability, the observed availability rates are generally higher than the calculated rates--in some instances significantly so. This bias was not unexpected. Several factors tend to concentrate backorders on a small number of aircraft rather than to allow a random distribution of the backorders as the AAM posits.

One of these factors is cannibalization. If we think of the process of removing serviceable components from an unavailable aircraft in order to make another aircraft available as the reverse of the process of moving backorders from the second aircraft to the already unavailable aircraft, we see that this would indeed produce higher observed availability rates. Further, on aircraft types with relatively low availability, one would expect extra effort to be expended to improve that situation; in particular, one would expect more cannibalization on aircraft types with low availability and a corresponding wider relative margin between calculated and observed availability rates. This is, in fact, borne out by the data.

Two other contributory factors are scheduled maintenance inspections and aircrew writeups. Various types of aircraft undergo periodic or phased inspections during which maintenance personnel inspect or functionally check large numbers of components and systems of the aircraft. These inspections typically result in the generation of many demands upon the inventory system, some of which are associated with the replacement or repair of components whose unserviceable condition had been known prior to the inspection but whose repair or replacement had been postponed until the inspection (when the aircraft would be scheduled for, perhaps, several days of maintenance). The second factor is a threshold effect of sorts in the behavior of aircrews reporting discrepancies on aircraft. They tend either to declare the aircraft as having no discrepancy or they report several discrepancies from a single flight. This, too, induces multiple demands on the inventory system.

None of the above processes is explicitly considered in the AAM. However, it has always been recognized that it was not feasible to include in the model the effect of every aspect of worldwide aircraft operations. The test data indicated that a simple linear adjustment could satisfactorily account

for some variables. In fact, the values of the coefficient of determination show that this adjustment function removes close to 95 percent of the discrepancy between calculated and observed availability rates.

TEST BY AFLC

The AFLC test of the AAM used the same data bases, but calculated EBOs from component asset and pipeline data, rather than observing actual back-orders as in the test of the product formula. Results for this test were again positive, although the inclusion of the link from asset and pipeline data to EBOs introduced another source of variability, resulting in somewhat lower correlations ($r^2 = .825$). As in the product formula, the innate conservatism of the availability formula resulted in calculated availability rates generally being lower than observed rates. Application of a linear correction was found appropriate and removed over 80 percent of the difference between the observed and the calculated availability rates. Table 7-3 is a summary of the AFLC results.

TABLE 7-3. AFLC TEST RESULTS

<u>MD</u>	<u>Active A/C</u>	<u>Calculated Availability Rate (Unadjusted)</u>	<u>Calculated Availability Rate (Adjusted)</u>	<u>Observed Availability Rate</u>
A007	364	0.700	0.872	0.888
A037	127	1.000	0.984	0.973
B052	301	0.198	0.684	0.701
B111	59	0.246	0.680	0.702
C005	57	0.065	0.635	0.552
C097	73	0.727	0.882	0.923
C123	61	0.848	0.927	0.962
C130	623	0.771	0.899	0.820
C131	49	0.598	0.834	0.892
C135	690	0.646	0.852	0.864
C141	232	0.344	0.739	0.763
F004	1581	0.772	0.899	0.894
F005	56	0.926	0.957	0.940
F015	49	0.361	0.745	0.712
F100	351	0.862	0.933	0.948
F101	138	0.679	0.864	0.915
F105	175	0.622	0.843	0.795
F106	178	0.202	0.686	0.759
H001	150	0.939	0.961	0.939
H003	73	0.863	0.933	0.862
H053	49	0.428	0.770	0.722
O002	247	0.967	0.972	0.991
T033	155	0.676	0.863	0.917
T037	442	0.976	0.975	0.986
T038	680	0.983	0.978	0.945
T039	110	0.564	0.821	0.773
V010	77	0.571	0.824	0.920

CHAPTER 8. THE AIR FORCE APPLICATION

The AAM is being used by two Air Force organizations. The Air Force Logistics Command (AFLC) is using the AAM as part of an interim procedure to allocate budgets for reparable spares (Budget Program 1500) and intends to fully incorporate availability methods into the Air Force Recoverable Consumption Item Requirements System (D041). The Air Staff, HQ USAF/LE, is using the AAM to evaluate the BP-15 and Depot Purchased Equipment Maintenance (DPEM) portion of POMs and Budgets.

This chapter describes how the AAM has been implemented and some of the special capabilities developed to make the model useful during the programming and budgeting process.

INPUT DATA

Input data are derived from several Air Force systems, predominantly the D041 system [1]. Updated quarterly, the D041 data base contains all the individual item logistics data used by the AAM--demand rates, repair times, unit cost, etc. The application data in the D041 "50" file is augmented by other files received from AFLC which contain information on engine and PE (Program Element) code applications to MDSs.

Aircraft program data are obtained from the Air Force PA (Aerospace Vehicles and Flying Hour Programs) System. Table 8-1 shows an extract from the PA.

Information from the D041 and the PA are used to construct the actual input to the AAM:

- an application file, containing the indenture application information for each component
- a component logistics data file containing component pipelines, cost, projected asset level, etc.
- a file of aircraft inventory and flying hours at the impact point.

TABLE 8-1. PA843 AIRCRAFT PROGRAM DATA

MD	1984		1985		1986		1987	
	AC	FHP	AC	FHP	AC	FHP	AC	FHP
A007	325	796	325	796	325	796	325	796
A010	581	2341	582	2349	583	2354	583	2360
A037	118	348	118	348	118	348	118	348
B001	2	0	3	1	7	26	33	135
B052	273	1168	269	1153	255	1137	241	1081
B111	57	191	57	204	57	213	56	223
C005	65	600	67	613	73	748	90	831
C130	695	3793	695	3846	683	3829	688	3832
C135	703	2602	703	2585	700	2951	698	2946
C140	14	92	14	92	14	92	14	92
C141	254	2940	254	2932	254	2920	254	2944
E003	28	312	30	332	30	428	29	524
E004	4	17	4	17	3	16	3	16
E111	19	66	30	104	36	132	36	132
F004	1459	3677	1474	3796	1468	3672	1378	3415
F005	103	310	103	311	103	311	103	311
F015	597	1809	613	1937	644	2030	707	2248
F016	591	1930	662	2257	768	2627	856	2898
F100	0	0	0	0	0	0	0	0
F102	0	0	0	0	0	0	0	0
F106	158	499	135	402	112	342	72	181
F111	276	792	264	786	260	797	257	802
H001	127	488	126	488	126	488	125	496
H003	77	274	74	264	71	258	71	259
H053	43	151	43	151	38	132	38	126
H060	10	44	10	44	10	45	11	57
R001	10	92	14	100	16	196	16	244
T033	144	548	144	552	144	552	144	552
T037	645	3487	661	3518	670	3686	662	3822
T038	814	3951	831	4032	833	4239	828	4160
T039	130	864	130	864	130	864	130	864
V010	73	318	73	318	73	318	73	318
OTHER	272	1402	277	1466	283	1515	305	1728
TOTAL	8667	35902	8785	36658	8887	38060	8944	38741

AC = Number of Primary Aircraft Inventory (PAI)

FHP = Flying Hour Program in Hundreds of Hours

D041 data are based upon the force activity levels projected in the PA document. Typically, formation of the D041 lags the PA, i.e., by the time the construction of the D041 data base is complete, the PA upon which it was based has been changed. In such cases, the AAM has the capability to modify the D041 to reflect the new projected force activity levels and to use the updated data as input [11].

DATA BASE ADJUSTMENTS

The Air Force BP-15 requirement consists of more than a distillation of the individual item requirements from D041 (regardless of whether the requirement is computed by fill rate criterion, as in the past, or by an availability criterion, as it will be in the future). It also includes requirements for items and programs that need special management attention--component modification programs, with requirements not yet defined at an item-specific level; consequences of management decisions, e.g., the uptrim of the F100 engine in FY83 and FY84 to improve acceleration will result in higher temperatures and increased parts consumption; non-recurring additives due to special requirements, e.g., a special Eddy Current Inspection of TF-34 fan blades to determine life remaining, expected to result in a 1 percent condemnation rate. These costs must be added to the item-specific costs determined by the AAM to obtain a realistic projection of required funding.

These adjustments (or "scrub" data) are derived from the P-18 Peacetime Deficit Adjustment submitted by each Air Logistics Center (ALC). Table 8-2 is a sample of adjustment data. The AAM P-18 subsystem processes these data, allocating identifiable costs to the appropriate MD and prorating the rest. Table 8-3 is a sample result of this process. These adjustments are then incorporated into the aircraft availability curves, as shown in Figure 8-1, to obtain appropriate availability projections.

TABLE 8-2. P-18 ADJUSTMENTS

31 MARCH 1982 P-18

ADJUSTMENTS (\$000)

<u>FY82</u>	<u>FY83</u>	<u>FY84</u>	
0	7150	2600	F100 UPTRIM
38313	45253	89303	T038 WING REPLACEMENT
- 382	0	0	F111 RECLASSIFIED FROM WRSK
- 178	178	0	F111 DELAY PENDING TEST
-18118	-11794	-14243	FINAL DESTINATION TRANSPORTATION
.	.	.	.
.	.	.	.
.	.	.	.

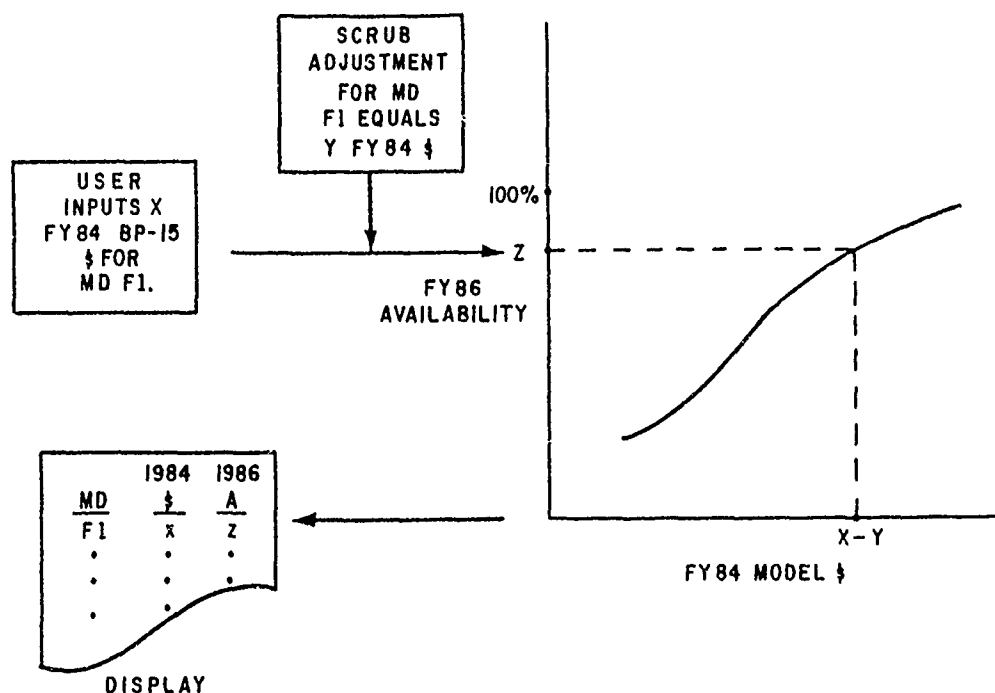
TABLE 8-3. P-18 MD ADJUSTMENTS

31 MARCH 1982 P-18

ADJUSTMENTS (\$ MILLIONS)

<u>MD</u>	<u>FY82</u>	<u>FY83</u>	<u>FY84</u>
A010	- 3.2	7.1	58.7
B052	- 54.0	24.7	13.4
B111	- 15.5	31.4	39.1
C005	22.5	21.8	17.9
F004	6.5	26.8	22.8
.	.	.	.
.	.	.	.
.	.	.	.
TOTAL	284.3	740.1	1046.2

FIGURE 8-1. AVAILABILITY PROJECTIONS WITH
DATA BASE ADJUSTMENTS



In addition to the ALC adjustments, AFLC and HQ USAF make adjustments to the requirements. These adjustments are similarly incorporated into AAM projections.

FISCAL YEAR BREAKOUT

At any given time during the Planning, Programming, and Budgeting System (PPBS) process, funding for several years is being examined. While FY X funds are being allocated to AFLC System Managers for execution, the FY X+1 Budget is being prepared for submission or considered for enactment, the FY X+2 POM is being formulated, and planning is taking place for years beyond that. Program changes are continual, as are refinements and updates of requirements estimates. Decisions for any of these years can affect the others. Cost

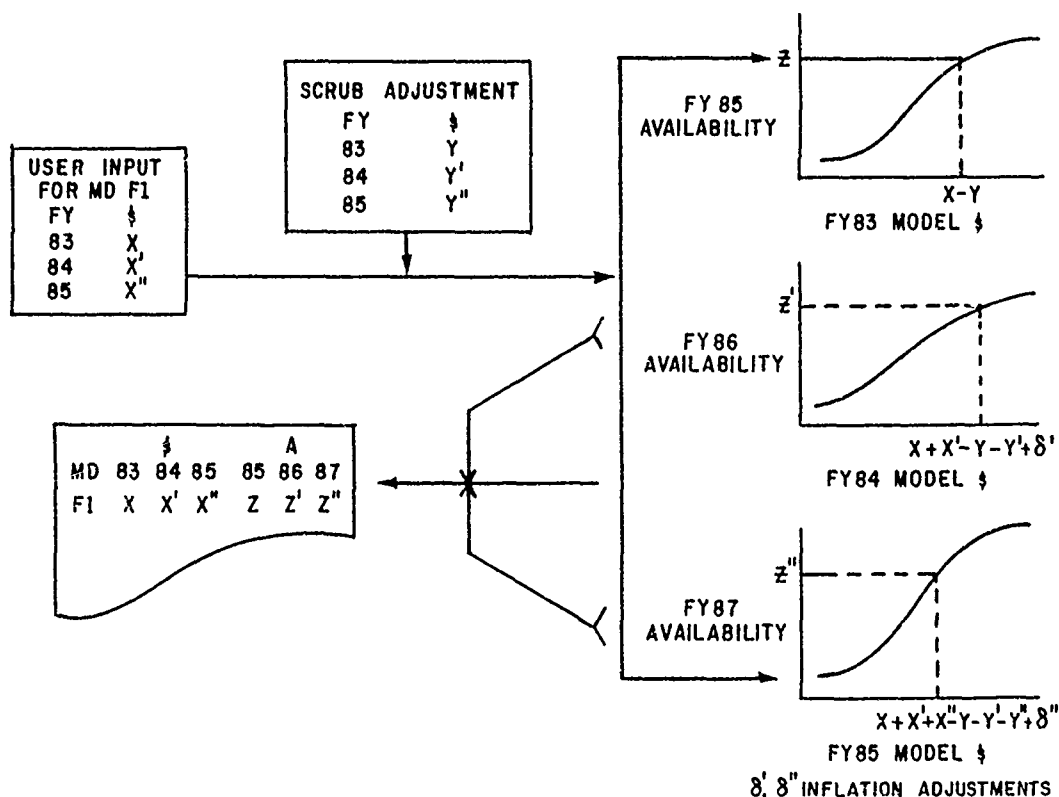
growth in FY X, if not offset by supplemental funding, must be carried over into FY X+1 and perhaps further.

To deal with this dynamic process, LMI has developed a "Breakout" subsystem to the AAM. Each AAM run covers a period of time from the D041 asset cutoff date through the end of a given program year (and to an impact point 6 quarters beyond the end of that program year). Typically, with a D041 having an asset cutoff date of 31 March FY X, the AAM will produce:

- A set of curves for six quarters past the end of FY X. Availability rates will depend on funding through the second half (31 March - 30 September) of FY X.
- A set of curves for six quarters past the end of FY X+1. Availability rates will depend on funding in FY X+1 and in the second half of FY X. FY X+1 funding is determined by subtracting funding for the second half of FY X, suitably inflated, from the cumulative funding.
- A set of curves for six quarters past the end of FY X+2. Availability rates depend on funding through FY X+2. FY X+2 funding is determined by subtracting previous years funding, suitably inflated, from the cumulative funding.
- (Sometimes) A set of curves for six quarters past the end of FY X+3. FY X+3 funding is determined similarly to that of FY X+2.

The AAM breakout system establishes a funding profile for all the fiscal years under consideration, incorporating data base adjustments from the P-18s and from AFLC. The user can specify dollar allocations by MD by fiscal year and receive a report on the availability consequences. The user may input availability targets by MD and aggregate funding by fiscal year. The AAM will automatically allocate funding among MDs to minimize deviations from the targets. The user may also combine the approaches, e.g., specify dollar allocations to reflect decisions made for FY X and FY X+1 and have the AAM spread money by MD targets for FY X+2. Figure 8-2 illustrates the Breakout process.

FIGURE 8-2. MULTI-YEAR AVAILABILITY PROJECTIONS



PROCUREMENT/REPAIR NOMOGRAMS

The repair option of the AAM determines an optimal division of available funds between procurement and depot repair of reparable components. However, procurement is funded under Budget Program 1500 (BP-15), repair is funded under DPEM (Depot Purchased Equipment Maintenance), and funds are not interchangeable between these programs. Thus, it is usually not possible to obtain an optimal mix of funds between procurement and repair and suboptimal solutions must be found. The Repair Nomograms were constructed for this purpose.

Nomograms are derived from the aircraft availability curves and represent a method for optimizing repair and procurement separately. Table 3-4 is a sample nomogram for the C-5. Each row line represents an optimal mix of

TABLE 8-4. PROCUREMENT-REPAIR NOMOGRAM
MD TYPE C-5

#A/C	AVAIL	ADJUSTED AVAIL	PCOST	QP	RCOST	QR
63.7	99.6	99.7	772556696.00	0.998	202991746.00	0.998
63.1	98.5	99.0	715749312.00	0.987	202991746.00	0.998
62.4	97.5	98.3	678166336.00	0.977	202973858.00	0.998
61.7	96.4	97.3	648409688.00	0.966	202940226.00	0.998
61.1	95.4	96.2	624971592.00	0.956	202893576.00	0.998
60.4	94.3	95.1	604098440.00	0.945	202870788.00	0.998
59.7	93.3	95.4	586758776.00	0.935	202835948.00	0.998
59.0	92.2	94.6	571365944.00	0.924	202773132.00	0.998
58.1	90.8	93.6	552558424.00	0.910	202726906.00	0.998
57.3	89.6	92.8	538740504.00	0.898	202556592.00	0.998
56.6	88.5	92.0	527623448.00	0.887	202530786.00	0.998
56.0	87.4	91.3	517962312.00	0.877	202429086.00	0.998
55.2	86.3	90.6	508331300.00	0.865	202301176.00	0.997
54.6	85.3	89.9	500076724.00	0.855	202194302.00	0.997
53.5	83.5	88.6	486851912.00	0.838	202154488.00	0.997
52.8	82.5	87.4	479277440.00	0.827	202016398.00	0.997
52.0	81.3	87.1	471764820.00	0.816	201796758.00	0.997
51.3	80.2	85.3	464938496.00	0.805	201698594.00	0.996
50.7	79.1	85.6	458886756.00	0.795	201504884.00	0.996
49.9	78.0	84.8	452351552.00	0.783	201445524.00	0.996
49.2	76.9	84.0	446310084.00	0.772	201386430.00	0.996
48.5	75.9	83.3	441182472.00	0.762	201236610.00	0.995
47.8	74.7	82.6	435696572.00	0.751	201102974.00	0.995
47.1	73.6	81.8	430641000.00	0.740	201074728.00	0.995
46.4	72.5	81.0	425381032.00	0.729	200951354.00	0.994
45.7	71.4	80.3	420704688.00	0.719	200732862.00	0.994
45.0	70.4	79.6	416216952.00	0.708	200698724.00	0.994
44.2	69.0	78.6	410660068.00	0.695	200636822.00	0.993
43.5	68.0	77.9	406370044.00	0.684	200536492.00	0.993
42.8	66.9	77.2	402304608.00	0.674	200402498.00	0.993
42.1	65.8	76.4	398404860.00	0.664	200276464.00	0.992
41.5	64.8	75.7	394920572.00	0.653	200238724.00	0.992
40.8	63.7	74.9	391078808.00	0.642	200124868.00	0.991
40.1	62.5	74.2	387714600.00	0.632	199956342.00	0.991
39.3	61.5	73.4	383945652.00	0.621	199844678.00	0.990
38.5	60.1	72.5	379734856.00	0.608	199749400.00	0.990
37.7	58.9	71.7	376082948.00	0.596	199643864.00	0.989
36.9	57.6	70.8	372346980.00	0.583	199471874.00	0.988
36.1	56.4	69.9	368619688.00	0.570	199437576.00	0.988
35.2	55.0	69.0	365007680.00	0.558	199314296.00	0.987
34.3	53.6	68.0	361042544.00	0.543	199162166.00	0.986
33.6	52.5	67.2	358143788.00	0.532	199071578.00	0.986
32.8	51.2	66.4	354987080.00	0.520	198972894.00	0.985
32.1	50.2	65.6	352474364.00	0.510	198782436.00	0.983
31.4	49.1	64.9	349681712.00	0.499	198718400.00	0.983
30.5	47.7	63.9	346244492.00	0.485	198670456.00	0.983
29.7	46.4	63.0	343261888.00	0.473	198470352.00	0.981
28.9	45.1	62.1	340292872.00	0.460	198346606.00	0.980
28.1	43.9	61.3	337587676.00	0.449	198196994.00	0.978
27.4	42.9	60.6	335191488.00	0.438	198126150.00	0.978

procurement and repair funds. Thus, a 78 percent availability can be obtained with procurement funding (PCOST) of \$452,351,552 and repair funding (RCOST) of \$201,445,524. The associated QP and QR (0.783 and 0.996), respectively, represent the contribution of procurement and repair funding to the availability

rate. The availability rate is the product of QP and QR, $0.783 \times 0.996 = 0.7799$. Choosing a mix of procurement and repair funding independently from each column will yield a non-optimal mix, but the resulting availability rate may still be determined by multiplying the appropriate QP and QR. For example, procurement funding of \$458,886,756 (QP = 0.795) and repair funding of \$198,470,352 (QR = 0.981), gives the same availability rate, $0.795 \times 0.981 = 0.7799$, but the total cost is \$657,357,108 compared to \$653,797,076 for the optimal mix.

Construction of the nomogram proceeds by working backwards through the shopping list, reversing the logic in the formation of the availability curve. If Q_0 = highest availability rate attained, set the corresponding QP = QR = $\sqrt{Q_0}$. If the last item on the shopping list was a procurement with cost C and sort value S, set the new value of QP equal to $QP / (\exp(S \cdot C))$ and reduce the cumulative procurement cost by C, reversing the application of Equation 2.5 in forming the curve. If the last item was a repair, we modify QR and cumulative repair cost similarly. Continuing this process produces a nomogram, with one record for each item on the shopping list. Only a small number of these records are actually saved and displayed. In fact, actual nomogram construction is an approximation of this technique using the availability curve rather than the shopping list.

The nomogram produces realistic estimates of projected availability for unbalanced funding. For serious imbalances, the technique breaks down logically, as it can implicitly procure an item before repairing all the carcasses of that item. Recent experience has shown that, although the mix of repair and procurement funding is not optimum, imbalances tend to be slight, and hence the nomogram is an appropriate tool for projecting availability.

ALTERNATE FLYING HOUR PROGRAMS

The relationship between costs and availability rates is dependent upon the planned Air Force flying hour program. During the programming and budgeting process, changes in the projected Air Force flying hour program are continually being made, and hypothetical changes are continually being suggested. The Air Force typically establishes a cost per flying hour (CPFH) for each MD and uses this cost to determine funding requirements in the POM and to investigate excursions from the baseline flying hour program. If A-10 flying hours increase by 10 percent in 1986, the CPFH methodology would compute a 10 percent increase in the A-10 FY84 BP-15 requirement. The two-year difference reflects the effect of procurement leadtimes; expenditures in FY84 will result, to a large extent, in spares entering the inventory in FY86.

This CPFH methodology has the virtue of simplicity and is appropriate for use in spreading costs among Program Decision Packages (PDPs) and Program Elements (PEs) in the Five Year Defense Plan (FYDP). However, the methodology has some severe shortcomings. An increase in FY84 flying hours, for instance, would result in higher component condemnations rates during FY84 and a corresponding increase in requirement for FY83 and, perhaps, FY84. But the methodology would look only at FY85 flying hours for FY83 funding and FY86 flying hours for FY84 funding, and is thus insensitive to the flying hour change in FY84. Nor does the methodology address the degradation in capability occurring in FY84 itself. In fact, it is not possible to relate CPFH factors to capability in any way.

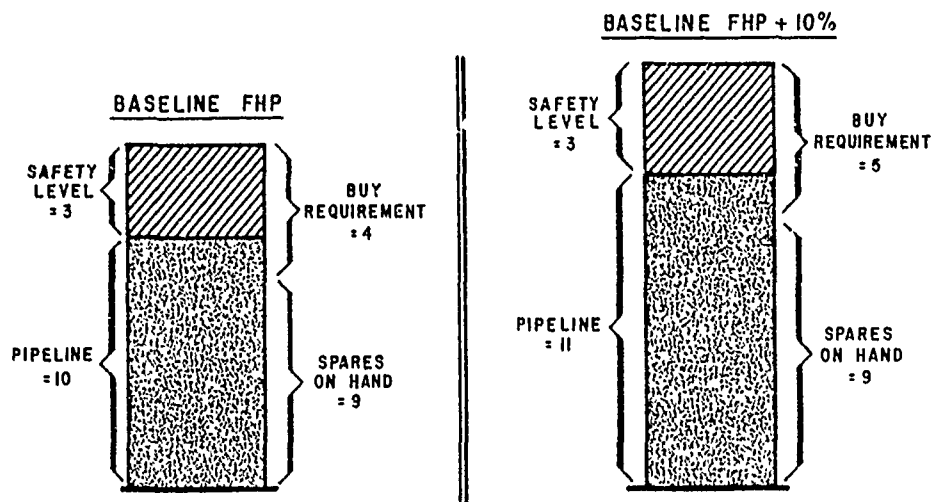
Another shortcoming of the method is that it fails to consider the marginal nature of the change in requirement. While it may be reasonable to assume that the total spares inventory needed to support a flying hour program

varies linearly with the program,¹ the funding requirement is based on the difference between total inventory required and total inventory now on hand (and on order). The relationship between this difference and the difference in the flying hour program may be far from linear.

Figure 8-3 illustrates this relationship for a single component. Suppose the component's projected pipeline (average number of units in resupply) with the baseline flying hour program is 10 units and the projected asset level is 9 units. If the required safety level is 3 units, the buy requirement is $10 + 3 - 9 = 4$. A 10 percent increase in flying hours increases the projected pipeline by 10 percent to 11 units. If the safety level remains unchanged, the new buy requirement is $11 + 3 - 9 = 5$. Thus, a 10 percent increase in program results in a 25 percent increase in the buy requirement. By varying the projected asset level for the component, we can see how sensitive the percent increase is to stock on hand. If the asset level were projected at 14 or above, there would be a buy requirement of 0 with either program. With projected assets of 13, there would be a buy requirement of 0 under the old program and 1 under the increased program, an undefined percent increase. Considering the many reparable components on any given MD, it is reasonable to expect changes by component to vary widely and the resulting aggregated relationship at the MD level to be quite complex.

¹We have accepted the current Air Force assumption that item failures are linearly related to flying hours. It is well known that this is not true for many classes of components, although a recent study of failure models by AFLC/XRS [2,3] concludes that there is no better determinant overall for recoverable item line replaceable unit requirements than flying hours. For items which AFLC classifies as non-flying-hour driven (e.g., items whose requirements generate on an inventory month basis, Program Select Code 3---), we use the appropriate item program. Should the Air Force begin computing requirements based on different item programs, e.g., sorties or cycles, the same methodology could be applied to those programs as to the flying hour program.

FIGURE 8-3. FIXED COST PER FLYING HOUR UNDERESTIMATES CHANGE IN FUNDING REQUIRED FOR AN INCREASE IN FLYING HOUR PROGRAM



● $\frac{\text{COST OF NEW REQUIREMENT}}{\text{COST OF OLD REQUIREMENT}} = \frac{5}{4} = 1.25$

● 10% INCREASE IN FHP $\Rightarrow$ > 25% INCREASE IN BUY REQUIREMENT

The AAM has the capability to compute changes in funding requirements due to flying hour changes from the (correct) marginal perspective. This analysis is made component by component, aggregating the results to produce accurate projections by MD. The logic is similar to that used to update the D041 data base to reflect a current PA, and a detailed discussion may be found in [11].

APPENDIX A

AVAILABILITY RATE AND EXPECTED BACKORDERS PER AIRCRAFT

Since the definition of aircraft availability involves expected backorders, it is not surprising that there is a fairly simple relationship between projected availability (A) and expected backorders per aircraft (EBO/AC). In fact, for each MD, it is approximately true that $EBO/AC = -\ln A$ where EBO is the sum of all LRU expected backorders for the MD. Note that the relationship does not depend on fleet size or aircraft complexity.

To see the mathematical justification for this relationship, let EBO_i be the number of expected backorders for component i, let QPA_i be its quantity per application on the aircraft type, and let AC be the number of aircraft. Recall that the availability rate is given by:

$$A = \prod_i \left(1 - \frac{EBO_i}{AC \cdot QPA_i} \right)^{QPA_i}$$

where the product is taken over all first indenture level items on the aircraft. We are assuming, for purposes of the demonstration, that all application percentages are 1.0 and that commonality considerations may be ignored.

The power series for $\exp(-EBO_i/(AC \cdot QPA_i))$ is

$$1 - \frac{EBO_i}{AC \cdot QPA_i} + \frac{1}{2!} \left(\frac{EBO_i}{AC \cdot QPA_i} \right)^2 - \frac{1}{3!} \left(\frac{EBO_i}{AC \cdot QPA_i} \right)^3 + \dots$$

Since

$$\frac{EBO_i}{AC \cdot QPA_i}$$

is typically small, we may ignore the higher order terms and write

$$\exp(-EBO_i/AC \cdot QPA_i) = 1 - \frac{EBO_i}{AC \cdot QPA_i}.$$

Then

$$\begin{aligned} A &= \prod_i \left(1 - \frac{EBO_i}{AC \cdot QPA_i} \right)^{QPA_i} \\ &= \prod_i [\exp(-EBO_i/(AC \cdot QPA_i))]^{QPA_i} \\ &= \prod_i \exp(-EBO_i/AC) \\ &= \exp\left(\sum_i -EBO_i/AC\right) \\ &= \exp(-EBO/AC). \end{aligned}$$

So we have $A = \exp(-EBO/AC)$ or, equivalently, $EBO/AC = -\ln A$.

Empirical evidence for the relation appears in the results of a comparison of the AFLC Variable Safety Level (VSL) model and the AAM [9]. (The VSL model is a single-indenture level model, which minimizes expected backorders to a fill-rate target.) Figures A-1, A-2, and A-3 contain a graph of the function $EBO/AC = -\ln A$, together with plots of data points from the test. The goodness of the fit provides the empirical evidence for the relation.

FIGURE A-1. EBO/AC VS. "A" WITH AAM STOCK LEVELS AND COST PER AIRCRAFT TYPE TO VSL COMPUTED REQUIREMENTS

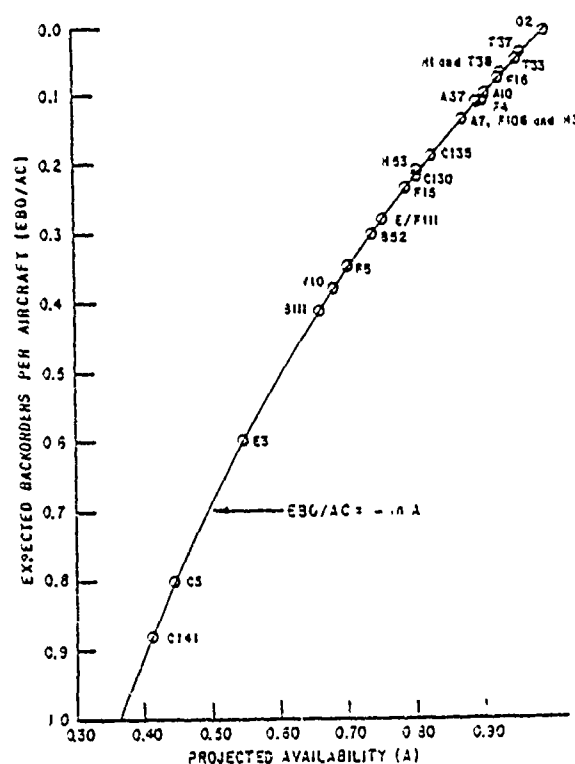


FIGURE A-2. EBO/AC VS. "A" WITH VSL STOCK LEVELS (AIRCRAFT WITH "A" GREATER THAN 0.35)

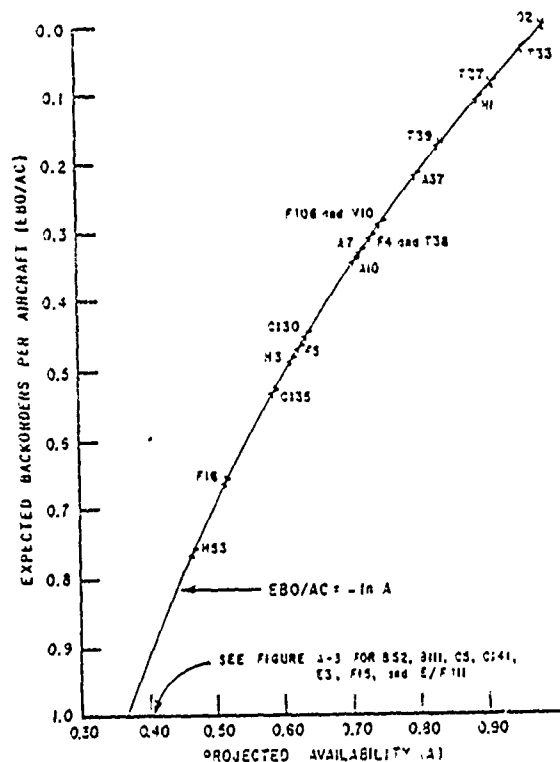


FIGURE A-3. EBO/AC VS. "A" WITH VSL STOCK LEVELS
(COMPLEX AIRCRAFT WITH "A" LESS THAN 0.35)

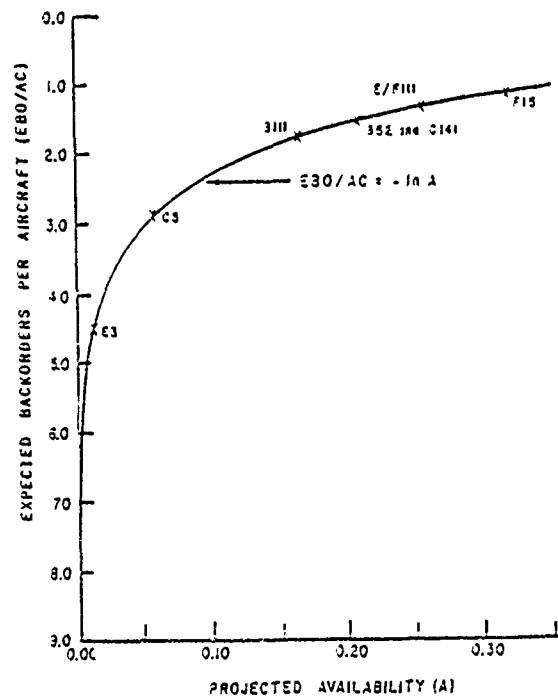


Figure A-1 shows data for optimal AAM stock levels when overall funds for each MD are set equal to the VSL computed requirement. The relationship holds, as well, for non-optimal stock levels computed by the VSL model, as shown in Figures A-2 and A-3. Of course, a given MD has higher A value and lower EBO/AC value for the equal-cost AAM solution. This may be seen by comparison of Figure A-1 with Figures A-2 and A-3. An extreme case is the E-3 aircraft. The VSL computed stock level gives a one percent projected availability rate with a value of 4.50 for EBO/AC (see Figure A-3). The equal-cost AAM solution gives a 73.9 percent projected availability rate, with an EBO value of 0.60 (see Figure A-1). Yet, in both instances, the relation still holds.

APPENDIX B

THE EBO MODEL

INTRODUCTION

At the heart of the computation of aircraft availability is the computation of component expected backorders (EBOs), derived from Sherbrooke's Multi-Echelon Technique for Recoverable Item Control (METRIC) [14]. This EBO model is widely accepted by the logistics community. Some variation of it is used in almost every model of multi-echelon component resupply in the Military and in academia.

The model portrays a multi-echelon resupply system which employs a $(s-1, s)$ continuous review stockage policy. This policy assigns to each stockage location a spares level (s) for each component. s is defined as the number of serviceable spare units on hand, plus the number due in (from base repair or depot resupply or other resupply sources) minus the number due out (to satisfy unfilled demands). When a demand is received at a location, it is either filled, reducing the number on hand by 1, or backordered, raising the number due out by 1. In either case, the spares level drops to $s-1$ (the reorder point) and action is immediately taken to raise the spares level back to s . A carcass may be inducted into repair or a spare may be ordered from the next higher echelon. In either case, the number of spares due in rises by 1 restoring the spares level to s .

INPUTS AND OUTPUTS

The model computes, one at a time, component EBOs using the following input (for each component):

- The expected number of units in base repair (the base repair pipeline)

- The expected number of units in depot repair (the depot repair pipeline)
- The expected number of units in transit from the depot to a base (the order and ship pipeline)
- The number of bases at which demands for the component occur (number of users).

The model produces an array of the component EBOs as a function of the spares level.

ASSUMPTIONS

Component demands are assumed to be generated by a Poisson process. The model is built around a theorem of Palm [12] which states that, for a Poisson demand process coupled with a resupply process (such as base repair), if the resupply time is independent of demand then the distribution of the number of items in resupply will be Poisson, depending only on the average resupply time and not the distribution of the resupply time. The EBO model invokes Palm's Theorem in every resupply situation including depot resupply to a base.

TECHNIQUE

The pipelines used in the EBO model are computed by multiplying the appropriate demand rate by the corresponding resupply time. For example, the base repair pipeline equals the base repair daily demand rate times the base repair time in days (base repair pipeline = Base DDR $\cdot$ BRT). The Base DDR equals the total DDR times (1 - NRTS) where NRTS is the Not Repairable This Station percentage (i.e., the percentage of repairs which are beyond the capability of the base repair shop). Similarly, the depot repair pipeline = Total DDR $\cdot$ NRTS $\cdot$ DRT, and the order and ship pipeline = Total DDR $\cdot$ NRTS $\cdot$ OST, where DRT is the depot repair time and OST is the order and ship time. Palm's Theorem then implies that the distribution of the number of units in resupply in any particular resupply segment is then given by a Poisson distribution whose mean is the corresponding pipeline.

The EBO model can allow for uncertainty in the mean demand rate when the demand process is Poisson but the mean demand rate is not exactly known. This is appropriate for AAM applications since component demand rates can change over time so projections of demand rates in the distant future are of limited accuracy. The EBO model uses a Gamma distribution to describe the probability distribution of the mean of the demand rate. Combining this with Palm's Theorem gives a negative binomial distribution for the number in any particular resupply pipeline. With complete certainty of demand rate, the Gamma reduces to a point distribution, and the negative binomial reduces to the Poisson. (See Appendix D for more detail.)

The formula for expected backorders for a particular component at a particular site is:

$$EBO = \sum_{x>s} (x-s)p(x) \quad (B.1)$$

where,

s is the stock level

x is the number of units in resupply for that site (including in repair at the site, on order from another site, and all other forms of due in to that site)

p(x) is the probability of having x units in resupply (a Poisson or negative binomial probability distribution).

The component EBO produced by the model is the worldwide total of the component's EBOs at all the bases. The depot EBO is coupled to this total by its impact on the number in resupply at the bases.

For a given component, the model computes the total worldwide EBO for many different total worldwide asset levels. For each worldwide asset level, the model considers every possible way to distribute those assets between base and depot and selects the distribution with the lowest total EBO.

The model first computes the EBO at the depot for each spares level at the depot using Equation B.1. The distribution used is a negative binomial whose mean is the depot repair pipeline.

The computation of EBOs at a base is similar to the computation of depot EBOs, and depends on the calculated depot EBO. From a base's perspective, the depot EBO is a resupply pipeline. The model views a backorder at the depot as a unit in resupply to a base in the "depot delay pipeline." The total base resupply pipeline equals the base repair pipeline plus the order and ship pipeline plus the depot delay pipeline. (Total base resupply pipeline = base repair pipeline + order and ship pipeline + depot EBO). The mean of the distribution of the number in resupply at a particular base equals the total base resupply pipeline divided by the number of users. The EBO model makes an important simplifying assumption here: all users have equal average demand rates (the uniform base assumption). The model then need only consider allocations of assets where each base gets the same number of spares (i.e., if there are three bases, the total number of spares allocated to the bases can only be 0,3,6,9.... As each of the first 3 spares gives the same EBO reduction as it is placed at each base, the EBO for 1 or 2 spares at the bases can be obtained by a linear interpolation between the EBO total for 0 spares at base level and the EBO total for 3 spares at base level). The total worldwide EBO equals the EBO at 1 base, as computed by Equation B.1, times the number of bases. Thus, a given spares level at the depot determines the base resupply pipeline and the resulting total EBO for all spares levels at the bases. The optimum distribution of spares between bases and depot is determined by comparison. As the resulting EBO for a given distribution of spares is calculated, it is compared with other EBOs for the same total (base and depot) spares levels. The optimum distribution is that which yields the least EBO for the given level.

APPENDIX C

MARGINAL ANALYSIS IN THE AAM

INTRODUCTION

As described in Chapter 3, marginal analysis in the AAM is used to solve the optimization problem: for a given amount of money, what spares should be procured to attain the highest possible availability rate? The purpose of this appendix is to present a proof that the solutions found by marginal analysis are in fact optimal.

THE UNDERLYING ARGUMENT

To understand why marginal analysis works, we will look first at the underlying mathematical argument.¹ Then we'll apply the argument to the specific problems addressed by the model, using the notation of Chapter 3, The Optimization Procedure.

Let $n(i)$ for $i=1, \dots, m$ denote a particular array of non-negative integers, which we'll call the initial level. In relation to the initial level, define the set $\{(s_i)\}$ of all integer arrays (s_i) with the property that $s_i \geq n(i)$ for each i . We'll call each such array a level. Let (c_i) for $i=1, \dots, m$ denote a set of positive real numbers, which we'll call costs. For any given level (s_i) , we can define its total cost C , in relation to the initial level, by the equation:

$$C = \sum_i (s_i - n(i))c_i.$$

¹The underlying mathematics of marginal analysis have been described in many places in the operations research literature. Selected references for both the general argument and the application to spares inventory problems may be found in [5,6,7,8,16].

Next, suppose we have functions f_i such that $f_i(s_i)$ is defined for all possible values of s_i and such that the difference functions:

$$\partial_i(s_i) = f_i(s_i) - f_i(s_i - 1)$$

are all positive and satisfy the diminishing difference relation:

$$\partial_i(n + 1) \leq \partial_i(n)$$

for all arguments n .

For all i and all n , define sort values $\{v_i(n)\}$ by:

$$v_i(n) = \partial_i(n)/c_i.$$

Form the ordered list L consisting of the $v_i(n)$ in descending order. Let L_C denote any initial section of the list L , where C is the sum of the costs c_i that appear in the section. Define the level (s_i) by $s_i = m_i$, where m_i is the maximum value of n appearing in the sort values $v_i(n)$ in the sublist L_C .

Assertion

The level (s_i) , as defined above, has total cost C and has the property that if (s_i') denotes any other level with total cost equal to or less than C , then

$$\sum_i f_i(s_i') \leq \sum_i f_i(s_i). \quad (C.1)$$

In other words, for the various possible total costs defined by the initial sections of the list L, the levels (s_i) represent undominated solutions to the problem of maximizing the sum

$$\sum_i f_i(s_i)$$

for cost C.

Proof:

The diminishing difference relation and the ordering of L (and therefore L_C) ensure that $v_i(j)$ is included in L_C for each j such that

$$n(i) + 1 \leq j \leq m_i.$$

This means for each i there are exactly $m_i - n(i)$ elements in the list L_C , and therefore

$$\sum_i (s_i - n(i))c_i = \sum_i (m_i - n(i))c_i = C.$$

So the level (s_i) does have total cost C.

Now let (s_i') be any other level with total cost

$$\sum_i (s_i' - n(i))c_i \leq C.$$

Let A_C denote the set of sort values associated with the level (s_i') defined as follows:

$$A_C = \{v_i(k) \mid n(i) + 1 \leq k \leq s_i', i = 1, \dots, m\}.$$

From the way L_C and A_C are constructed, it follows that:

$$\sum_i [f_i(s_i) - f_i(n(i))] = \sum_{L_C} v_i(j) c_i \quad (C.2)$$

and

$$\sum_i [f_i(s_i') - f_i(n(i))] = \sum_{A_C} v_i(k) c_i. \quad (C.3)$$

We can rewrite

$$\sum_{A_C} v_i(k) c_i$$

as:

$$\sum_{A_C} v_i(k) c_i = \sum_{A'_C} v_i(k) c_i + \sum_{A''_C} v_i(k) c_i,$$

where A''_C is the set of sort values common to A_C and L_C , while A'_C is the set of sort values in A_C but not in L_C . Since L_C is an initial section of the list L , it follows that:

$$\max_{A'_C} v_i(k) \leq \min_{L_C} v_i(j).$$

Thus we have:

$$\sum_{A_C} v_i(k) c_i \leq \min_{L_C} v_i(j) \sum_{A'_C} c_i + \sum_{A''_C} v_i(k) c_i. \quad (C.4)$$

Since the total cost of (s_i') is less than or equal to C , it follows that

$$\sum_{A'_C} c_i \leq \sum_{L'_C} c_i, \quad (C.5)$$

where L'_C is the set of sort values which are in L_C but not in A_C . Thus, Equation C.4 becomes:

$$\begin{aligned}
 \sum_{A_C} v_i(k) c_i &\leq \min_{L_C} v_i(j) \sum_{L'_C} c_i + \sum_{A''_C} v_i(k) c_i \\
 &= \sum_{L'_C} \min_{L_C} v_i(j) c_i + \sum_{A''_C} v_i(k) c_i \\
 &\leq \sum_{L'_C} v_i(j) c_i + \sum_{A''_C} v_i(k) c_i \\
 &= \sum_{L_C} v_i(j) c_i.
 \end{aligned}$$

Applying this last inequality to Equations C.2 and C.3 yields Equation C.1:

$$\sum_i f_i(s_i') \leq \sum_i f_i(s_i),$$

completing the proof.

There are two additional points to be made. First, it follows from the preceding argument that not only are marginal analysis solutions optimal in the sense of maximum availability for a given cost, they are also optimal in terms of identifying minimum cost to achieve a given availability. To see this, note that for any level (s_i') with total cost strictly less than C , the inequality in Equation C.5 would be strict. This makes the inequality in Equation C.1 strict, which means the cost C is minimal.

The second corollary has to do with the problem of minimizing a sum, subject to a cost constraint. For this application, we require the difference

functions $\partial_i(s_i)$ to be negative, and the diminishing difference relation to be:

$$\partial_i(n+1) \geq \partial_i(n).$$

The problem becomes the following: for a given total cost C , defined by some initial section on the ordered list of sort values, find the level (s_i) with the property that:

$$\sum_i f_i(s_i) \leq \sum_i f_i(s_i'),$$

where s_i' denotes any other level with total cost $\leq C$. By taking sort values defined by:

$$v_i(n) = -\partial_i(n)/c_i.$$

The argument goes through as before, but with the appropriate sign changes.

As a final point, it should be noted that the marginal analysis method produces solutions that are optimal, but, strictly speaking, it does not produce all possible optimal solutions. The total costs defined by the initial sections on the ordered lists L represent a discrete set of possible values, and for these values the marginal analysis method yields optimal solutions. For intervening cost values, however, marginal analysis does not produce solutions. In practical terms, the applications in the AAM are not affected. The set of solutions defined by the initial sections of the sort value lists is sufficiently rich to cover the full range of possible costs, expected backorder levels, and aircraft availability rates.

APPLICATIONS IN THE AAM

All versions of the AAM use marginal analysis to compute optimal cost versus availability curves. In the levels-of-indenture version of the model

(see Chapter 6 and Appendix E) marginal analysis is also used to compute minimal expected backorder (EBO) versus cost results for items below the first indenture level. We'll discuss the EBO application first.

As described in Chapter 6 and Appendix E, in the levels-of-indenture model we are interested in minimizing a sum of EBOs for a given cost. The initial level corresponds to the projected spares level for a set of SRUs on a given LRU at the impact point. In most applications, the initial level will also include some percentage of the pipeline plus levels to satisfy insurance, numeric stockage objectives (NSO) and negotiated requirements. Costs are unit costs. Total costs C correspond to the difference between sunk costs underlying the initial level and total dollars available. The functions $f_j(s_j)$ are the expected backorder functions $EBO(j, n_j)$ in the notation of Appendix E. The difference functions $\partial_j(n_j) = EBO(j, n_j) - EBO(j, n_j - 1)$ measure the reduction in EBOs when the n_j th spare is added. The sort values $v_i(n) = \partial_j(n)/c_i$ measure the reduction in EBOs per dollar achieved with the addition of the n th spare of component i . They are negative, and therefore the sum

$$\sum_j EBO(j, n_j)$$

is minimized as desired. It can happen that the EBO reduction for the j th spare of some component is greater than that for the $(j-1)$ th. (A base/depot redistribution phenomenon affecting EBO levels, known as "flushout," [4] can cause this situation.) To ensure that the diminishing difference relation holds, therefore, sequences of spares for a given component may be grouped together as necessary and assigned an appropriate average EBO reduction value. In prior model documentation, this process has been referred to as "convexification" or "glumping." It is equivalent to replacing each summand EBO_j in the objective function with its convex hull.

The application of marginal analysis to the cost versus availability problem involves maximization, but with an important additional facet. In the case of availability, we are interested in maximizing a product rather than a sum. Using the notation of Chapter 3, we want to maximize the product:

$$\prod_i q_{h,i,s_i}$$

where h denotes aircraft type, i is the component index, and (s_i) is some spares level (above the initial level $(n(i))$) having total cost C . We convert the problem to that of maximizing a sum by taking the natural logarithm of the product and maximizing:

$$\sum_i \ln q_{h,i,s_i}$$

for the cost C .

In the availability application, therefore, the functions $f_i(s_i)$ are defined by:

$$f_i(s_i) = \ln q_{h,i,s_i} \text{ (in the notation of Chapter 3),}$$

and the sort values

$$v_i(n) = \partial_i(n)/c_i = \frac{\ln(q_{h,i,n}/q_{h,i,n-1})}{c_i}.$$

The difference functions measure the natural log of the multiplicative improvement in availability per dollar when the n th spare of component i is added to the inventory. The difference functions $\partial_i(n) = \ln(q_{h,i,n}/q_{h,i,n-1})$ are all positive so the sum above is maximized, as desired. As in the EBO case, the diminishing difference relation can fail to hold. That is, the improvement in availability (measured by the log of the improvement factor)

may be greater for the $(n+1)$ th spare of some component than for the n th. As necessary, therefore, to ensure that the diminishing difference relation holds, spares for a given component may be grouped together and the appropriate geometric mean improvement factor assigned to each unit in the group.

APPENDIX D

TREATMENT OF UNCERTAINTY

INTRODUCTION

The variance-to-mean ratio (VMR) is a parameter that is used to represent uncertainty as a function of time in the availability projections. It is the ratio of the variance to the mean of the resupply pipeline probability distribution that is used in the expected backorder calculations at the heart of the AAM. A VMR of 1.0 yields a pure Poisson process, while greater VMRs yield negative binomial distributions that are more skewed toward the origin by flattening the distribution and increasing the length of the tail. The net effect of increasing the VMR is to decrease the backorder reduction effect of adding spares.

In purely statistical terms, a Bayesian technique is used to model uncertainty about projected pipeline sizes: assume that the initial estimate of a pipeline size, μ , is the mean of a Poisson resupply pipeline distribution, and assume that μ is the value of a random variable Λ that has a gamma distribution. The prior gamma distribution completely determines the resulting randomized process which is a negative binomial distribution. The negative binomial distribution is parametrized by its mean and VMR and computed recursively in the AAM.

In essence, the VMR expresses uncertainty about the accuracy of the resupply pipeline projections, and hence the accuracy of the component data base. The VMR is also used to express uncertainty about factors that are not accurately reflected in the model: component modification, component redesign, and technical surprises.

In this section, we will derive the negative binomial distribution as a Poisson process whose mean is itself a random variable, parametrize the negative binomial by its mean and VMR, derive the recursive algorithm for computing the negative binomial distribution, and show how the VMR is used to express uncertainty.

THE POISSON DISTRIBUTION

The Poisson distribution is a discrete one parameter probability density function (pdf) defined by the equation

$$P(x|\mu) = \frac{e^{-\mu} \mu^x}{x!} \quad (D.1)$$

$$\text{for } \mu > 0 \\ x = 0, 1, 2, \dots$$

$$\text{The pdf has mean } \mu_p = \mu$$

$$\text{and variance } \sigma_p^2 = \mu.$$

Note that the distribution has VMR

$$\sigma_p^2 / \mu_p = 1.$$

THE GAMMA DISTRIBUTION

The Gamma distribution is a continuous one parameter pdf defined by the equation

$$\gamma(y|n) = \frac{e^{-y} y^{n-1}}{\Gamma(n)} \quad (D.2)$$

where $n > 0$, $y \geq 0$, and $\Gamma(n)$ is the gamma function which satisfies the recurrence relation $\Gamma(n+1) = n\Gamma(n) = n!$ The gamma is commonly represented as

a continuous two parameter pdf which is obtained from Equation D.2 by a change of variables argument:

Let $y = zs$
then $z = y/s$ and $dy/dz = s$.

Applying this to the gamma pdf, we obtain:

$$\gamma(z|n, s) \equiv \gamma(zs|n) = \frac{e^{-zs} (zs)^{n-1}}{\Gamma(n)} s \quad (D.3)$$

for $0 < s < 1$, $n > 0$, and $z \geq 0$.

The pdf has mean $\mu_y = n/s$,

variance $\sigma_y^2 = n/s^2$

and variance-to-mean ratio $\sigma_y^2 / \mu_y = 1/s$.

THE POISSON PROCESS WITH AN UNCERTAIN MEAN

Assume that the initial estimate of a Poisson mean μ is the value of a random variable Λ that has a gamma distribution. Then the randomized Poisson process is defined by the marginal probability distribution

$$\begin{aligned} m(x) &= \int_0^{\infty} P(x|z) \gamma(z|n, q/p) dz && \text{for } q = 1 - p \\ &= \int_0^{\infty} \left[\frac{e^{-z} z^x}{x!} \right] \left[\frac{e^{-qz/p} (qz/p)^{n-1}}{\Gamma(n)} (q/p) \right] dz \\ &= \int_0^{\infty} \left(\frac{1}{x! \Gamma(n)} e^{-z-qz/p} q^n z^x (z/p)^{n-1} \frac{1}{p} \right) dz \\ &= \frac{\Gamma(x+n)}{x! \Gamma(n)} p^x q^n \int_0^{\infty} \left[\frac{e^{-z/p} (z/p)^{x+n-1}}{\Gamma(x+n)} \frac{1}{p} \right] dz \end{aligned}$$

$$\begin{aligned}
&= \frac{\Gamma(x+n)}{x!\Gamma(n)} p^x q^n \int_0^w \frac{e^{-u} u^{x+n-1}}{\Gamma(x+n)} du \quad \text{for } u = z/p \\
&= \frac{\Gamma(x+n)}{x!\Gamma(n)} p^x q^n \equiv N_b(x|n,p)
\end{aligned} \tag{D.4}$$

a negative binomial pdf with parameters

$$0 < p < 1, \quad q = 1 - p, \quad n > 0 \text{ and } x = 0, 1, 2, \dots$$

with mean $\mu_N = np/(1-p)$

$$\text{variance } \sigma_N^2 = np/(1-p)^2$$

and variance-to-mean ratio

$$\sigma_N^2 / \mu_N = 1/(1-p).$$

Note that the negative binomial pdf satisfies the following recursive relation

$$N_b(0|n,p) = (1-p)^n$$

and for $x \geq 1$

$$\begin{aligned}
N_b(x|n,p) &= \frac{\Gamma(x+n)}{x!\Gamma(n)} p^x (1-p)^n \\
&= \left[\frac{\Gamma(x+n-1)}{(x-1)!\Gamma(n)} p^{x-1} (1-p)^n \right] \left(\frac{x+n-1}{x} p \right) \\
&= N_b(x-1|n,p) \left(\frac{x+n-1}{x} p \right)
\end{aligned} \tag{D.5}$$

PARAMETRIZATION OF THE NEGATIVE BINOMIAL

Given a Poisson process with mean μ , and an arbitrary constant $Q > 1.0$, we wish to construct a negative binomial distribution with mean μ and variance-to-mean ratio Q .

The negative binomial pdf is given by:

$$N_b(x|n,p) = \frac{(x+n-1)!}{x! (n-1)!} p^n (1-p)^x$$

which has mean $\mu_N = \frac{np}{1-p}$

variance $\sigma_N^2 = \frac{np}{(1-p)^2}$

and variance-to-mean ratio $\sigma_N^2 / \mu_N = 1/(1-p)$.

Given our parameters μ and Q for the mean and VMR, we obtain the following relationships:

$$\mu = \mu_N = \frac{np}{(1-p)}$$

$$Q = \sigma_N^2 / \mu_N = \frac{1}{(1-p)}$$

and hence

$$q = (1-p) = \frac{1}{Q} \tag{D.6}$$

$$p = 1 - \frac{1}{Q} = \frac{Q-1}{Q}$$

$$\mu = \frac{np}{1-p} = \frac{n \left(\frac{Q-1}{Q} \right)}{1/Q} = n \left(\frac{Q-1}{Q} \right) Q = n(Q-1) \tag{D.7}$$

$$\text{so } n = \mu / (Q-1) \text{ and } p = \frac{Q-1}{Q}.$$

Note that $q/p = \frac{1}{Q-1}$ and $p/q = Q-1$.

Thus, the negative binomial pdf has the form

$$\begin{aligned} N_B(x|\mu, Q) &\equiv N_b(x|n, p) = N_b\left(x|\frac{\mu}{Q-1}, \frac{Q-1}{Q}\right) \\ &= \frac{\Gamma\left(x+\frac{\mu}{Q-1}\right)}{x! \Gamma\left(\frac{\mu}{Q-1}\right)} \left(\frac{Q-1}{Q}\right)^x \left(\frac{1}{Q}\right)^{\frac{\mu}{Q-1}} \end{aligned} \quad (D.8)$$

with mean $\mu_N = \mu$

variance $\sigma_N^2 = \mu Q$

and variance-to-mean ratio $\sigma_N^2 / \mu_N = Q$.

OBSERVATIONS

Note that the prior gamma distribution has the following characteristics:

$$\gamma(z|n, s) = \gamma\left(z|\frac{\mu}{Q-1}, \frac{1}{Q-1}\right)$$

$$\mu_Y = \frac{np}{p} = \frac{\mu}{Q-1} \frac{Q-1}{Q} Q = \mu$$

$$\sigma_Y^2 = \frac{np^2}{q^2} = \frac{\mu Q^2}{q-1} \frac{(Q-1)^2}{Q^2} = \mu(Q-1)$$

$$\sigma_Y^2 / \mu_Y = p/q = Q-1$$

and that the following relationships hold between the negative binomial and the prior gamma distributions

$$\sigma_Y^2 = \mu(Q-1) = \mu Q - \mu = \sigma_N^2 - \mu$$

$$\mu_Y = \mu = \mu_N$$

$$\begin{aligned} \left(\frac{\sigma_Y^2}{\mu_Y} \right) &= Q-1 = \left(\frac{\sigma_N^2}{\mu_N} \right) - 1 = \left(\frac{\sigma_N^2}{\mu_N} \right) - \left(\frac{\sigma_P^2}{\mu_P} \right) \\ &= \frac{\sigma_N^2 - \sigma_P^2}{\mu} \end{aligned}$$

NEGATIVE BINOMIAL RECURSION

The negative binomial satisfies the following recursive relationship shown in Equation D.5 with parameters μ and Q

$$\begin{aligned} N_B(0|\mu, Q) &= N_b\left(0 \mid \frac{\mu}{Q-1}, \frac{Q-1}{Q}\right) = \left(1 - \frac{Q-1}{Q}\right)^{\frac{\mu}{Q-1}} = \left(\frac{Q+1}{Q}\right)^{\frac{\mu}{Q-1}} \quad (D.9) \\ N_B(x|\mu, Q) &= N_b\left(x \mid \frac{\mu}{Q-1}, \frac{Q-1}{Q}\right) \\ &= N_b\left(x-1 \mid \frac{\mu}{Q-1}, \frac{Q-1}{Q}\right) \left(\frac{x + \frac{\mu}{Q-1} - 1}{x} \frac{Q-1}{Q} \right) \\ &= N_B(x-1|\mu, Q) \left(\frac{x + \frac{\mu}{Q-1} - 1}{x} \frac{Q-1}{Q} \right). \end{aligned}$$

EXAMPLE

The effect of increasing the VMR while keeping the mean constant is shown in Figure D-1.

Increasing the VMR skews the distribution toward the origin, flattening the distribution and increasing the length of the tail. The lower VMRs have peaked distributions with narrower support. The minimum VMR of 1.0 represents a pure Poisson process. In practical application, a VMR of 1.01 is used to approximate a Poisson distribution. Very large means result in near normal probability distributions for both the Poisson and the negative binomial distributions, as in Figure D-2.

FIGURE D-1. NEGATIVE BINOMIAL DISTRIBUTIONS WITH A MEAN OF 5

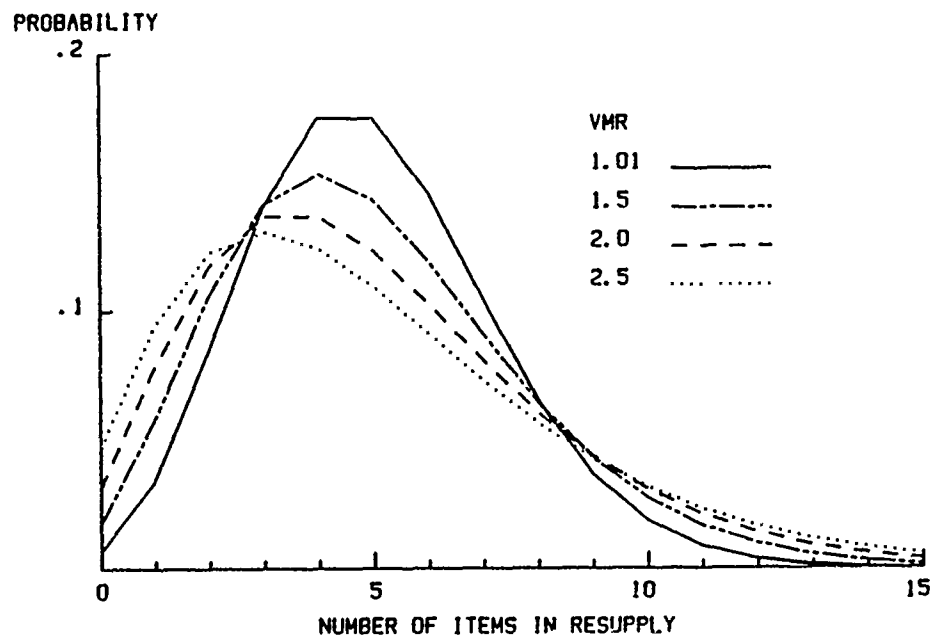
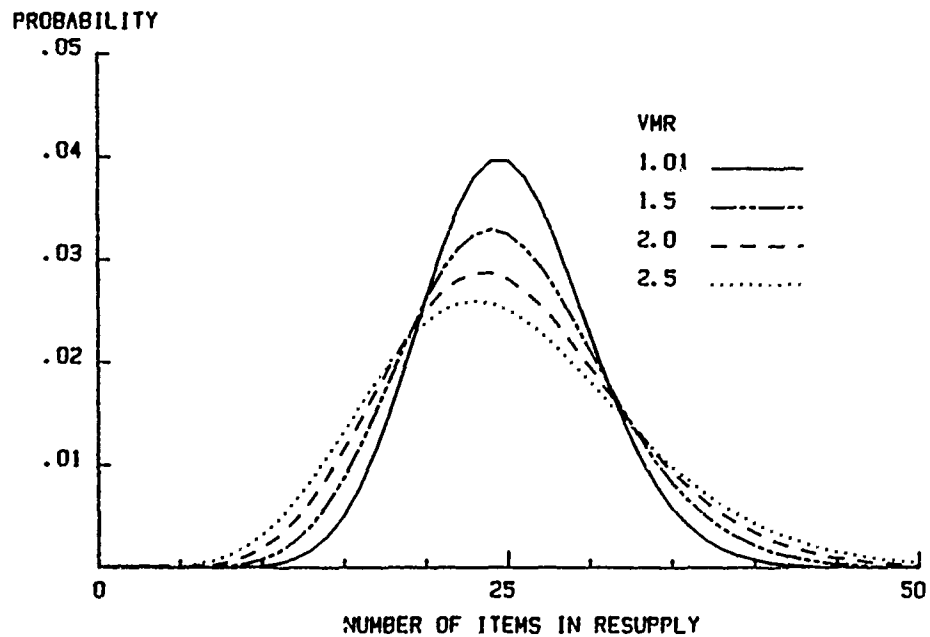


FIGURE D-2. NEGATIVE BINOMIAL DISTRIBUTIONS WITH A MEAN OF 25

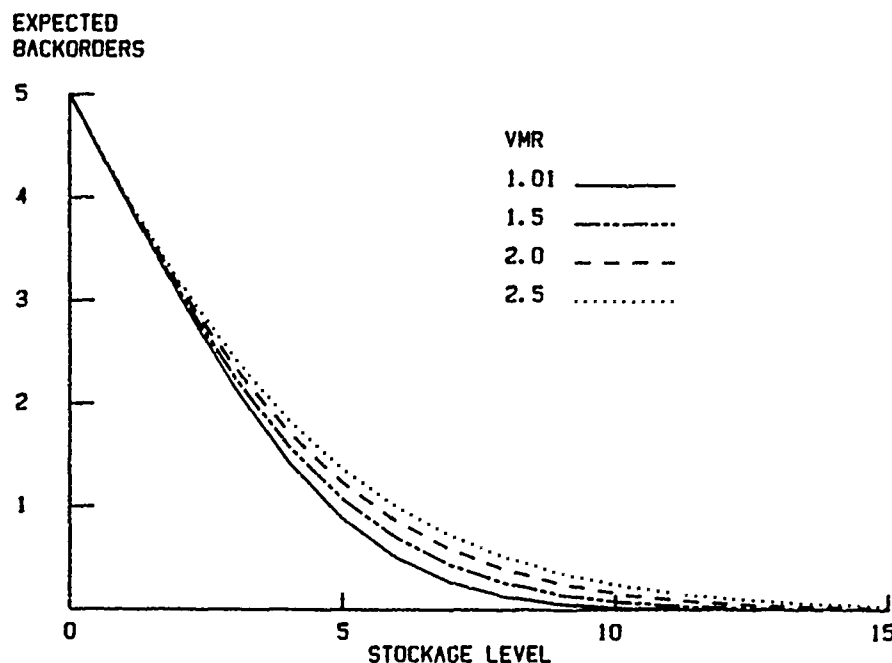


Given the probability distribution, the ultimate application of the theory involves the calculation of expected backorders defined by the following equation

$$B(s|\mu, Q) = \sum_{x>s} (x-s) N_B(x|\mu, Q). \quad (D.10)$$

The effect of increasing the VMR is shown in Figure D-3 for several negative binomial distributions with the same mean.

FIGURE D-3. EXPECTED BACKORDERS WITH A MEAN OF 5.



Higher VMRs result in higher initial probabilities and longer tails with a slower decline in the expected backorder level as a function of the spares stockage level. Lower VMRs result in peaked distributions with narrower support and a rapid decrease in expected backorders as spares are added. Raising the VMR increases the number of expected backorders at all levels beyond the

initial point; increases the number of spares required to reduce expected backorders to a negligible level; and decreases the backorder reduction effect of adding spares.

AAM RESULTS

The VMR is used to express uncertainty in the availability projections produced by the AAM. An initial estimate of a starting VMR is selected for the first model year, and the VMR is increased by a constant for subsequent model years.

The selection of the VMR is a subjective process that is used to reflect uncertainty about the pipeline parameters, the pipeline projections, technical surprises, the flying hour program, and a variety of other causes that relate to the performance of the resupply system in future years.

APPENDIX E

LEVELS OF INDENTURE

Suppose we have an LRU i with a family of SRUs $j = 1, 2, \dots, n$. For simplicity of exposition, we suppose the SRUs have no reparable subassemblies. (As we'll see, the extension to more levels of indenture requires a simple recursion.) We need now to make a tradeoff, not only between putting LRUs at bases or at the depot, but also between investing in spare units of SRUs, optimally distributed between base and depot. Since only LRU backorders affect aircraft availability directly, we need to solve a preliminary optimization problem within this LRU family--for a given investment, what allocation of that investment between spare units of the LRUs and its SRUs will yield the smallest possible SRU EBO? When this problem is solved, we can then tradeoff investment among LRUs to maximize aircraft availability.

We proceed by first looking at the SRU level. As in Appendix B we can compute EBO_j , the expected backorder total for each SRU j , given its spares levels and finding the optimum base/depot distribution of these spares. Without considering the effect of these backorders, the average system-wide base level resupply pipeline of the LRU is given by

$$P = \lambda \cdot RTS \cdot BRT + \lambda \cdot NRTS \cdot OST + EBO_D(ND) \quad (E.1)$$

where

λ = daily demand rate for LRU i

RTS = percentage reparable this station (at base level)

BRT = base repair time
 NRTS = 1 - RTS
 OST = order and ship time from depot to base
 $EBO_D(ND)$ = depot requisitions being delayed at the depot (depot expected backorders, which depends on depot stock level ND of LRU 2.)

In Appendix B, the three segments above were called the base repair pipeline, the order and ship pipeline, and the depot delay pipeline. As in [10], we assume that each of the SRU backorders is delaying the base repair of an LRU. Thus, the pipeline is more accurately expressed as

$$P = \lambda \cdot RTS \cdot BRT + \sum_j EBO_j + \lambda \cdot NRTS \cdot OST + EBO_D(ND). \quad (E.2)$$

As in the EBO model of Appendix B, the expected backorders at a base, with s spares of LRU_j at that base, are given by

$$\sum_{x>s} (x-s) p(x) P \quad (E.3)$$

where $p(x)$ is the distribution of the number of LRUs in resupply. $p(x)$ has mean p which includes the effect of SRU EBOs. The system-wide expected backorder total is obtained by accumulating over all bases.

Equation E.3 shows that, as in the single indenture level case, expected backorders may be reduced by putting spare LRUs at the bases (increasing s) or by reducing the resupply pipeline P . Equation E.2 shows that P can be reduced by reducing the depot delay with more spare LRUs at the depot or by eliminating some of the base delay by reducing SRU expected backorders. Thus, SRU spares and spare LRUs at the depot have a similar effect on LRU backorders.

We can compute for each SRU_j , the total system backorders for each level of spares $n_j = 0, 1, 2, \dots$. We denote this by $EBO(j, n_j)$. We sort each spare unit of all the these SRUs in decreasing order of backorder reduction divided by cost. Thus, the (n_j+1) th spare unit of SRU_j would have a "sort value" of

$$\frac{EBO(j, n_j) - EBO(j, n_j + 1)}{C_j}$$

where C_j is the unit cost of SRU_j . With the proper precautions taken for possible nonconvexities, buying in such an order minimizes the sum of the SRU backorders for the appropriate cumulative cost (see Appendix C). We thus obtain a curve of cumulative cost versus total SRU backorders. It is convenient to "digitize" this curve in cost increments of an LRU unit cost. We refer to this cost as an "Lsworth" and shall speak of buying NS Lsworths of SRUs to mean buying from the SRU shopping list until the cumulative cost equals NS times the LRU unit cost. We denote by $EBOs(NS)$ the total SRU EBOs with an investment of NS Lsworths.

Since the AAM deals with a large portion of the total Air Force budget for reparable components, the minor inaccuracies introduced by this digitizing are of little consequence. For smaller scale applications, such aggregating would be bypassed. The curve is, after all, computed on a unit-by-unit basis, and the tradeoffs may be made on such a basis if desired.

We now compute the depot backorders for the LRU with ND LRU spares at the depot, $EBO_D(ND)$, for $ND = 0, 1, 2, \dots$ and backorder reduction sort values as before (a single echelon problem only). At this stage, it is necessary to resort to a search procedure. Let ND denote the number of depot LRUs, NB the number of base LRUs, and NS the number of Lsworths of SRUs, distributed

optimally between base and depot. For a given triple (NS, ND, NB) the system expected backorder total of the LRU may be computed. We need only compare this to the expected backorder total for all other triples with the same total expense, (NB + ND + NS) times the LRU cost, to find the optimum expected backorder total and the corresponding allocation of expenditures. Recognizing that both depot LRUs and Lsworths of SRUs have the same effect on LRU EBOs, many of the comparisons in this three-way tradeoff can be eliminated without actually performing the EBO calculation. If

$$EBO_S(NS_1) + EBO_D(ND_1) < EBO_S(NS_2) + EBO_D(ND_2),$$

Equations E.2 and E.3 imply that the LRU EBO will be lower for (NS₁, ND₁, NB) than for (NS₂, ND₂, NB) for any value of NB. Thus, the EBO calculation for (NS₂, ND₂, NB) need not be made.

The actual procedure starts at the lowest level of indenture and cascades upward recursively. Thus, information at a given level incorporates the effect of optimum results of tradeoffs at all lower levels. There is no need, once computations have been made at a particular level, to return to that level.

As processing for an SRU is completed, the results of the computations are written to the component summary data file as described in Chapter 3. There is now, however, such a file for each level of indenture. In addition, the sort value used for SRUs is now in terms of reduction in expected backorders divided by cost, as the objective for SRUs is expected backorder minimization rather than aircraft availability maximization. Information identifying the items next higher assemblies (NHAs) is also included. The SRU expected backorders are prorated (on the basis of usage, $V_{h,i}$, as in Chapter 4) to each of the NHAs, as is the cost of the SRU. Commonality at

this level does not require a different sort value for each application, as it does on the LRU level, as the factors for cost and EBO proration are the same and simply cancel,

$$(V_{h,i} \cdot \Delta \text{EBO}) / (V_{h,i} \cdot C_i) = \Delta \text{EBO} / C_i.$$

Records indicating the SRU, NHA, cost, and sort value for each additional spare unit are written to a sort value file for that level. In addition, header records are written for each NHA indicating the prorating factor V_h and the starting prorated EBOs for the SRU. The header records for each NHA are formatted

$$\text{NHA}, V_h \cdot 10^{20}; \text{EBO} \cdot V_h; \text{SRU}.$$

On completion of SRU processing, the sort value file is sorted in NHA major, sort value minor, order.

Since $V_h \cdot 10^{20}$ is larger than any possible sort value, the header records for a given LRU will sort together at the top of the list and can be used to obtain total SRU EBOs for the LRU. Using a recursive relationship similar to Equation 3.3, we can compute the total SRU EBOs as SRU spares are added:

$$\text{NEW EBO} = \text{OLD EBO} - \text{sort value} \cdot \text{cost} \cdot V_h. \quad (\text{E.4})$$

Thus, we can construct a curve of SRU EBOs versus cost for the LRU's family of SRUs. As the LRUs are processed, this curve is used to trade off SRU investment against investment in the LRU itself and the optimal mix determined.

When the LRU records are written to the component summary data file, a field is added to indicate the optimum SRU investment for the indicated mix. This incorporates the information in Table 6-2, but in a different format. In

addition to header information concerning aircraft applications and corresponding factors, and sunk cost information, the file contains, for each MD application of the LRU, an array of records of the form

Sort Value = $S_{h,i,n}$; number of LRUs bought; Prorated Cost; SVP.

SVP (or sort value prime) indicates the sort value on the item's SRU shopping list corresponding to that investment, i.e., the optimum mix is obtained by buying the indicated number of the LRU itself and buying from the SRU shopping list until SVP is reached. SVP is analogous to the sort value on the aircraft availability curves, indicating how far down a shopping list one must buy to attain the indicated result (availability in the case of the aircraft, SRU total EBOs in the case of the LRU).

In the construction of a shopping list from allocation of funds to aircraft types, SVP is used to determine buy quantities for SRUs. Thus, if an LRU is applied to an aircraft type h , with allocated funding C_h , resulting availability rate Q_h , and aircraft sort value S_h , the program to form the shopping list travels down the LRU's sort value array in the component summary data file "buying" as long as $S_{h,i,n} \geq S_h$. The number of LRUs bought is read off directly, or a list of the SRUs applied to the LRU, and the corresponding SVP are stored in a file. (In case of imbalances with common component buys, the MD that "wants" the most of the LRU is allowed to drive the decision.) When LRU processing is complete, SRUs are then processed. The file of SRUs and SVPs is sorted in SRU order. For SRUs common to more than one LRU, the lowest SVP associated with the SRU is used to determine buys of the SRU. When an SRU is processed, it is identified to one (or more) LRUs and spare units are added to the shopping list as long as the sort value is above SVP.

We have illustrated this processing with two levels of indenture only. In fact, the procedure is recursive--a level-two SRU can have a family of

level-three SRUs as subcomponents and will thus have an SRU EBO curve below it just as an LRU does. Extensions to the processing are straightforward. The present AAM is configured for five levels of indenture.

AAM Levels of Indenture processing requires that an item be on one and only one level of indenture. It is in fact common for an item to be, say, at the first level in one MDS and at the second level in another. A portrayal of a typical situation is:

```
F-4  F-15
|    |
A    B
|
B
```

B is a subcomponent of component A on the F-4 but applied directly to the F-15. It is an LRU on the F-15, but an SRU on the F-4. AAM logic does not allow for component B to be processed twice. Instead, in the formation of the application file, a dummy link is inserted between B and the F-15 so that the relationship now looks like:

```
F-15
|
B'
|
B
```

B is processed with the other level-two items. Its EBOs are prorated to its two NHAs, A and B'. When level one is processed, the EBOs for B that were prorated to A are used to increase A's base repair pipeline. The remaining EBOs for B are passed directly through B' to have a direct effect on the F-15. This introduction of dummy links, coupled with the extensive common component logic of the model, permits a faithful portrayal of even the most complex indenture relationships.

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