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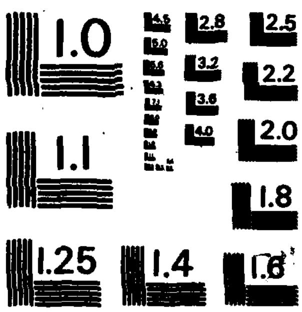

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Modeling of linear stochastic systems leads to the stochastic realization problem. In this project ~~we~~ develop[s] [the author] a comprehensive theory of stochastic realization. Such a theory should be the centerpiece of stochastic systems theory. First ~~we study~~ [He studies] the problem from a coordinate-dependent point of view. Secondly ~~we~~ [he] develop[s] a geometric theory of Markovian representation, which also accomodates infinite-dimensional systems.

In this framework we provide a unified theory of smoothing, [is provided] [over]

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the basic idea being to embed the given stochastic system in a class of similar systems all having the same output process and the same Kalman-Bucy filter. This approach provides stochastic interpretations of many important smoothing procedures.

The factorizations of the matrix Riccati equation underlying fast (non-Riccati) algortihms are analized in the context of Hamiltonian systems, and certain aspects of the algebraic Riccati equation are studied, as is the concept of invariant directions of the matrix Riccati equation.

We take A unified approach to the partial realization problem is taken, incorporating ideas from numerical linear algebra. We also study the Studied are questions of stability of partial realizations.

Finally, a statistical approach to stochastic optimization is presented, and convergence results for algorithms based on stationary data are obtained.

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A. INTRODUCTION

In this research project we have studied the following topics in Control and Systems Theory.

1. Stochastic realization theory
2. Smoothing
3. Theory for matrix Riccati equations
4. The partial realization problem
5. Stochastic optimization and control.

A summary of the results is given in Section B, a list of publications in Section C, and abstracts of the papers in Section D.

The following research personnel has been paid by project money.

Principal Investigator:

Anders Lindquist, Professor

Research Assistants:

Michele Pavon, graduate student; PhD in May 1979
Faris Badawi, graduate student; PhD in May 1980
Francisco Solis, graduate student; PhD in May 1981

Consultants:

Giorgio Picci, Professor at the University of Padova, Italy
Guy Ruchebusch, PhD, Ecole Polytechnique, Palaiseau, France.

In addition, some other researchers have indirectly contributed to the project through co-authorship, as can be seen from the list of publications.

A list of oral presentations of project results is provided in Section E.

B. SUMMARY OF RESEARCH RESULTS

1. Stochastic Realization Theory

Loosely speaking, the stochastic realization problem is the inverse problem of determining a stochastic system (or, more precisely, a class of such systems) having a given stochastic process as its output process. This is the fundamental problem in stochastic systems theory. Our study of this problem provides a framework for other investigations, such as smoothing (§2) and invariant directions (§3). Moreover, it provides conceptual insight into the structure of linear (and nonlinear) stochastic systems, which will be helpful in modeling practical systems.

Reference 1 considers the stochastic realization problems from a concrete (coordinate-dependent) point of view. The basic problem is to characterize and classify all linear stochastic systems having a given vector process as its output; such a model is called a *realization*. We distinguish between *internal* realizations which are completely determined by the given output process and *external* realizations which are not. It is shown that the state process of any internal realization can be expressed in terms of two steady-state Kalman filters, one evolving forward in time over the infinite past of the given process and one backward over the infinite future. Connections with the spectral factorization problem are investigated and a non-Riccati algorithm for generating families of stochastic realizations (or spectral factors), totally ordered with respect to their state covariances, is presented. It is shown that, to each realization, there corresponds a (strict sense) backward realization. This is a useful result in many applications, notably in smoothing theory (§2). Ref. 1 deals with continuous-time processes. The corresponding discrete-time results can be found in Ref. 6.

Both these papers deal with realization of stationary processes. Some results for the corresponding *nonstationary* problem are presented in References 9 and 11 (Chapter III), the latter of which contains, as a by-product, a natural stochastic interpretation of an algorithm due to Cleget for the minimum and maximum variance realizations.

In References 2, 3, 4 and 5 we develop a geometric state space theory for stochastic processes based on the concept of *splitting subspace*. Among other things this theory clarifies the roles of observability and controllability in stochastic systems theory and provides stochastic interpretations for and insight into filtering algorithms. In Ref. 2, given a stationary Gaussian stochastic process y with rational spectral density, we determine all Markovian families of minimal splitting subspaces of H(y), where H(y) is the Hilbert space spanned by the process y; the splitting subspaces act as state spaces. Each such family gives rise to an equivalence class of linear state space models with white noise inputs and y as the output, systems of the type used in modeling engineereing problems. In fact, we obtain *all* such models which can be constructed in terms of the given process (internal realizations). In Ref. 3 we develop a more general theoretical framework which will allow us to obtain a similar theory for non-stationary processes and to obtain also the models which cannot be constructed in terms of y (external realizations). In Ref. 4 the powerful theory of Hardy spaces is applied to the problem, yielding abstract solutions also for infinite-dimensional systems; this will enable us to apply the theory to time-delay systems and distributed parameter systems. Finally, in ref. 5, this Hardy space theory is generalized to the multivariate case. A new approach to the basic geometric problem is presented, more suitable for the multivariate setting. It is shown that each state space is the isomorphic image of the closure of a certain Hankel operator, which facilitates the application of the infinite-dimensional deterministic realization theory of Brockett, Baras and Fuhrmann to stochastic systems. However, this has to be done with some care, as we shall se below (Reference 24).

In Reference 8 we introduce a new mathematical framework for the basic geometric problem described above which we preceive as more natural and conceptually sounder than our previous one. The basic concept is that of *perpendicularly intersecting subspaces*. By relating minimality of stochastic realizations to the factorization of certain Hankel operators, the connections to deterministic realization theory is further clarified. Furthermore, spectral domain criteria for minimality, observability and constructibility are presented. The last part of the paper is devoted to a procedure to obtain differential equation representations for all state spaces which are finite dimensional.

Reference 13 contains a comprehensive presentation of the geometric theory. These results are then carried over to the spectral domain and described in terms of Hardy functions. Each state space is uniquely characterized by its structural function, an inner (or, in engineering language, all-pass) function which contains all the systems theoretical characteristics of the corresponding realizations. (This part of the theory has strong connection to Lax-Phillips scattering theory.) Observability and constructibility is described in terms of coprimeness of certain inner functions characterizing the splitting subspace, one of which is the structural function. From this, differential-equation-type representations are derived. However, this part of the paper contains an error, and the reader is referred to Ref. 24 for the correct version. (These difficulties are caused by the continuous-time assumption. They do not occur in discrete time.) To each splitting subspace there correspond two realizations, one evolving forward and one backward in time. It is shown that the forward realization is observable in the usual sense of deterministic systems theory if and only if the splitting subspace is observable in the geometric sense described in the beginning of the paper. In the same way, constructibility of the splitting subspace corresponds to observability (or, more precisely, constructibility) of the backward representation. (Both realizations are always reachable.) Finally, coordinates are introduced and concrete dynamical representations are derived. The coordinatization provides further insights into the connections between forward and backward Markovian representations, which have been the topic of several studies in the smoothing literature.

In Ref. 16 we settle a mathematical question which was long somewhat of a bottleneck in this area of research. Given a stationary Gaussian vector process y, consider a Markovian splitting subspace X contained in the frame space which is either observable or constructible. Such an X is called *reduced*. Our main result is that a Markovian splitting subspace is minimal if and only if it is reduced. If y is a scalar process, this is not hard to prove, because then all reduced Markovian splitting subspaces have the same structural function (an inner function which contains all the systems-theoretical information about the splitting subspace). In the vector case the key is our main lemma, which states that all reduced X have quasi-equivalent structural functions. This also leads to a Jordan model for the minimal X.

In Ref. 18 we give a comprehensive survey of our stochastic realization theory for discrete-time processes and provide some new ideas. In order to make the basic ideas the main thing and not unnecessarily obscure the mathematics, we have chosen the simplest possible formulation which still contains the main features of the problem. (Hence we consider a multivariate output process rather than a scalar one.) The construction of stochastic realizations consists of three steps: (i) determining all state spaces, (ii) analysis and characterization of the state spaces in terms of Hardy functions, and (iii) construction of dynamical systems (models). Step (i) is a geometric theory in the Hilbert space of random variables generated by the (given) output process; step (ii) is similar to the construction in Lax-Phillips scattering theory (although we do not yet fully understand the physical significance of this analogy); in step (iii), finally, we apply some of the ideas of Ref. 16 and analyze the models from a systems-theoretical point of view. Some new ideas concerning the question of degeneracy discussed in Ref. 21 are also presented.

Ref. 22 is a comprehensive treatment of the discrete-time problem. There are some decidedly nontrivial problems which are unique to the discrete-time setting (whereas, on the other hand, some subtle difficulties of the continuous-time formulation are no longer present). These include such considerations as the definition of past and future, different choices creating different classes of models, and in particular, certain degeneracies which do not occur in the continuous-time case and which are also observed in deterministic systems. One type of degeneracy is related to singularity of the transition function, another to the concept of invariant (or constant) directions. Criteria for degeneracy are first described geometrically in the basic Hilbert space of random variables. These results are then carried over to the spectral domain and described in terms of the same inner functions which occur in the above mentioned criteria for observability and constructibility. Then it is shown how degeneracy can be detected directly from the spectral density. Finally we consider the connections between the classes of models corresponding to two different choices of past and future. One leads to models without observation noise and one to models with such a noise term. It is shown that the connection between these classes depends on the occurance of degeneracy.

In the geometric theory of stochastic realization described above the output process is assumed to be Gaussian. Therefore all stochastic models are linear. This linear theory can be applied to non-Gaussian processes also, but then we need to give up the requirement that the state process is Markovian and (in continuous time) that the system is generated by a Wiener process, replacing these concepts by "wide sense Markov" (in the sense of Doob) and "orthogonal increment process" respectively. If we are not willing to do so, a nonlinear stochastic realization theory is needed. This is the topic of Ref. 17.

In this paper we apply Wiener's theory of homogeneous chaos to the nonlinear stochastic realization problem. This enables us to carry over the basic results of the linear theory, appropriately modified, to the nonlinear setting. The state spaces are taken to be direct sums of subspaces, one in each chaos. The component X of the first chaos is a Markovian splitting subspace determined as in the linear component; the component X_n of the n:th chaos is isomorphic to an n-fold tensor product of X. Therefore the elements of X_n can be represented by multiple Wiener integrals of order n.

The purpose of the paper is to investigate the structural aspects of the nonlinear stochastic realization problem and to clarify basic concepts. This is a first step toward a nonlinear realization theory. Hence we have not concerned ourselves with algorithmic aspects of the problem, and our analysis is based on the availability of an innovation representation, the actual determination of which is a nontrivial problem in itself. Our interest in the nonlinear realization problem emanates from its potential value as a conceptual framework for certain classes of nonlinear filtering problems.

Ref. 25, finally, deals with realization of continuous-time processes with stationary increments. This leads to a theory of forward and backward semimartingales. See Section D for further details.

2. Smoothing

In References 7 and 9 we develop a theory of smoothing for finite dimensional linear stochastic systems in the context of stochastic realization theory. The basic idea is to embed the given stochastic system in a class of similar systems all having the same output process and the same Kalman-Bucy filter. This class has a lattice structure with a smallest and a largest element; these two elements completely determine the smoothing estimates. This approach enables us to obtain stochastic interpretations of many important smoothing formulas and to explain the relationship between them. One of our main objectives was to provide a stochastic interpretation of Mayne-Fraser two-filter formula, a problem which previously has been approached (unsuccessfully) by many workers in the field. We show that the so-called "backward filter" is actually a *forward* filter, namely the maximum-variance realization. In my opinion, one will never obtain a clear picture of the merits of the various smoothing algorithms without investigating (as we have now done) their stochastic meanings.

In Chapter II of Ref. 11 a smoothing theory for discrete-time nonstationary systems is developed. Just as in continuous time, it is shown that the smoothing estimate is contained in a finite-dimensional space $H_t^{\square}$, the frame space. However, unlike the situation in continuous time, in the discrete setting, $H_t^{\square}$ is *not* of constant dimension, contributing to the fact that this problem is not merely a trivial modification of its continuous-time counterpart. In the continuous-time setting, the invertibility of certain covariance matrices is essential. In discrete time, this invertibility does not hold on the whole interval. Hence the generalized Moore-Penrose pseudoinverse is applied, thereby introducing some further structure in the theory. A two-filter formula of the Mayne-Fraser type is derived for the smoothing estimate. In the final section of Chapter II we use a different technique to derive the smoothing formulas of Bryson and Frazier and Rauch, Tung and Striebel, which does not employ the frame space.

The purpose of Ref. 14 (by Pavon) is to illustrate the role played by the conjugate process in continuous time stochastic realization theory. In particular, it is shown that the input of every internal realization can be generated by an anticausal whitening filter driven by the conjugate process. In this framework some new smoothing formulas phrased in terms of the conjugate process are derived. The optimal bilateral predictor is shown to be the natural discrete-time counterpart of the smoothing estimate of the observation signal, a fact overlooked in previous smoothing literature. Several explicit expressions for the bilateral prediction are given which represent a considerable improvement on the results of Salehi based on von Neumann's alternating projection theorem. In discrete time, stationary processes are considered, in continuous time, processes with stationary increments are studied.

3. Theory for matrix Riccati equations

In Ref. 12 the factorizations of the matrix Riccati equation underlying the fast (non-Riccati) algorithms are derived in the context of Hamiltonian systems and the variational problem corresponding to the matrix Riccati equation. The primary purpose of the paper is to place these factorization results in a natural theoretical framework, useful for further analysis. Previous derivations of these results, in their most general formulations, are based on *ad hoc* matrix manipulations which give very little insight into what is really going on. Although we are primarily interested in discrete-time Riccati equations, for comparison, the corresponding continuous-time result is also briefly discussed. This is also one of the topics of Chapter I in Ref. 11.

In the same chapter, (wide sense) stochastic realization theory for stationary processes with rational spectral densities is studied both in discrete and continuous time. The set $\mathcal{P}$ of all solutions of the Quadratic Matrix Inequality is considered, as is the subset P_o of P consisting of the solutions of the corresponding Algebraic Riccati Equation. A parametric representation for P is given and its boundary points are characterized. Then it is shown that the elements of P_o are extreme points of P. (Although this seems to be a well-known result, we were unable to find a proof of this anywhere in the literature. However, since then, we have discovered a similar proof [illegible] a recently published book by Faurre, Clerget and Germain.) [illegible]ever, the converse of this statement, namely that all extreme [illegible]ints of P belong to P_o, is false. This we show in Ref. 23.

In Ref. 6 Pavon studies *invariant directions* for the [illegible]trix Riccati equations of Kalman filtering and describes the connections to stochastic realization theory and filtering. The stochastic realization setting turns out to be the right framework for properly understanding the phenomenon of invariant directions, yielding a rather elegant geometric theory. In fact, the invariant directions are invariant over the whole class of stochastic realizations.

4. The partial realization problem

In References 19 and 20 we consider the *partial realization problem*, which is of central importance in (deterministic) systems theory. A scalar version of this problem can be described in the following way. Given a finite sequence $\gamma := \{\gamma_1, \gamma_2, \ldots, \gamma_N\}$ of real numbers, find a triplet $\Sigma = (A,B,C)$ of matrices in $\mathbb{R}^{n\times n}$, $\mathbb{R}^{n\times 1}$ and $\mathbb{R}^{1\times n}$ respectively so that

$$CA^{i-1}B = \gamma_i \quad \text{for} \quad i = 1,2,\ldots,N \tag{1}$$

and so that n is as small as possible. Such a triplet is called a *realization* of γ.

The importance of the partial realization problem in systems theory emanates from the fact that Σ corresponds to a *linear system*

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

$(t = 0,1,2,\ldots)$, where $x(t) \in \mathbb{R}^n$ is called the *state*, $y(t) \in \mathbb{R}$ the *output*, and $u(t) \in \mathbb{R}$ the *input* at time t. Then the sequence γ is a section of the *impulse response* of the system (2) in the sense that, if we choose $u(0) = 1$, $u(1) = u(2) = \ldots = u(N) = 0$, and $x(0) = 0$, we obtain the output $y(t) = \gamma_t$ for $t = 1,2,\ldots,N$. In other words, the partial realization problem is the *inverse problem* of determining a system (2) from its partial impulse response γ. A complete impulse response $\{\gamma_1,\gamma_2,\gamma_3,\ldots\}$ is usually not available from data, and therefore we must content ourselves with a partial realization. The same partial realization problem is obtained if (2) is replaced by the continuous-time system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{3}$$

In Ref. 19 we take a unified approach to the partial realization problem in which we seek to incorporate ideas from numerical linear algebra, most of which were originally developed in other contexts. We approach the partial realization problem from several different angles and explore the connections to such topics as factorization of Hankel matrices, block tridiagonalization, generalizations of the Lanczos process for biorthogonalization, the Euclidean algorithm and the principal part continued fractions of Arne Magnus, the Padé table, and the Berlekamp-Massey algorithm. In this way we are able to clarify some previous results by Rissanen, Kalman and others and place them in a broader context. This leads to several results and concepts which we think are new. Our analysis is restricted to the scalar case, but some definitions and formulations have been rigged to facilitate an extension to the matrix case. Finally, we discuss briefly partial realization of covariance sequences.

A finite or infinite sequence γ (of real numbers) is said to be *stable* if it admits minimal realization by a stable linear system. (System (2) is said to be stable if all eigenvalues of A are in the open unit disc, and (3) is stable if they are in the open half plane.) In Ref. 20 it is shown that the preservation of stability as such a sequence is truncated or extended is not a generic property even among stable sequences. This is of interest for identification of linear systems from partial data and for partial realization of random systems, in which cases it constitutes a negative result and answers in the negative a question raised by Kalman. Certain finite sequences have infinitely many minimal realizations each having different stability properties. In this case, a graphical criterion in the spirit of the Nyquist criterion is derived to exploit this lack of uniqueness in order to determine whether one can achieve a stable pole placement by a judicious choice of partial realization.

5. Stochastic estimation and control

In Reference 15 F. Solis considers stochastic optimization problems formulated as the minimization of expectation functionals. This research consists of two parts. In the first, sufficient conditions for these functionals to be lower semi-continuous, differentiable and inf-compact are presented. The subdifferentiability properties of the functionals are investigated, and these results are used in obtaining the sufficient conditions for differentiability and to settle certain questions of Lipschitz domination. These problems are usually solved assuming knowledge of the underlying probability measure. In most cases, this measure is an approximation based on a sample, and the larger the sample the better the approximation. In the second part, sufficient conditions are obtained for the convergence of the solutions based on these approximations to the solution of the actual problem, and conditions under which we can estimate the rate of convergence are identified. Finally, an algorithm is proposed which estimates the rate of convergence and some problems if implementation are discussed.

Reference 10 is merely a short note to point out an error which is very common in the literature. This error, which can be found in many of the early papers and books on the subject and therefore is constantly carried on to more recent works, is caused by a misunderstanding of the effects of feedback in stochastic systems.

C. PUBLICATIONS

1. A. Lindquist and G. Picci, On the stochastic realization problem, *SIAM J.Control and Optimization* 17 (May 1979), 365-389.

2. A. Lindquist and G. Picci, A state space theory for stationary stochastic processes, *Proc. 21st Midwest Symposium on Circuits and Systems*, August 1978, 108-113 (invited paper).

3. A. Lindquist, G. Picci and G. Ruckebusch, On minimal splitting subspaces and Markovian representations, *Mathematical Systems Theory* 12 (May 1979), 271-279.

4. A. Lindquist and G, Picci, A Hardy space approach to the stochastic realization problem, *Proc. 1978 Conf. Decision and Control*, San Diego, 933-939 (invited paper).

5. A. Lindquist and G. Picci, Realization theory for multivariate stationary Gaussian processes I: State space construction, *Proc. 4th Intern. Symp. Math. Theory of Networks and Systems*, July 1979, Delft, Holland (invited paper).

6. M. Pavon, Stochastic realization and invariant directions of the matrix Riccati equation, *SIAM J. Control and Optimization*, 18 (1980), 155-180.

7. F. Badawi, A. Lindquist and M. Pavon, A stochastic realization approach to the smoothing problem, *IEEE Transactions on Automatic Control*, AC-24, Dec. 1979, 878-888.

8. A. Lindquist and G. Picci, Realization theory for multivariate Gaussian processes II: State space theory revisited and dynamical representations of finite dimensional state spaces, *Proceedings of the Second International Conference on Information Sciences and Systems*, Patras, Greece, June 9-14, 1979, Reidel Publishing Co., Dordrecht-Holland, 1980.

9. F. Badawi. A. Lindquist and M. Pavon, On the Mayne-Fraser smoothing formula and stochastic realization theory for nonstationary linear stochastic systems, *Proc. 1979 Conference on Decision and Control*, Fort Lauderdale, Florida, Dec. 1979, 505-511.

10. A. Lindquist, Comments on 'A simple proof of the separation theorem for linear stochastic systems with time delays', *IEEE Transactions on Automatic Control* (correspondence item), AC-25 (April 1980) 274-275.

11. F. Badawi, Structures and Algorithms in Stochastic Realization Theory and the Smoothing Problem, PhD thesis, Department of Mathematics, University of Kentucky, January 1980 (140 pages).

12. F. Badawi and A. Lindquist, A Hamilton approach to the factorization of the matrix Riccati equation, *Math. Programming Studies* 18 (1982), 27-38. (Special conference issue on numerical methods in systems engineering, R. Wets, ed.).

13. A. Lindquist and G. Picci, State space models for Gaussian stochastic processes, *Stochastic Systems: The Mathematics for Filtering and Identification and Applications*, K. Hazewinkel and J.C. Willems, eds., Reidel Publ. Co., 1981, 169-204 (invited paper).

14. M. Pavon, The conjugate process in stochastic realization theory, *Math. Programming Studies* 1981 (same issue as ref. 12).

15. F. Solis, Stochastic Optimization Problems: A Statistical Approach, PhD thesis, Univ. of Kentucky, 1981.

16. A. Lindquist and G. Picci, On a condition for minimality of Markovian splitting subspaces, *Systems & Control Letters* 1 (1982), 264-269.

17. A. Lindquist, S. Mitter, and G. Picci, Toward a theory of nonlinear stochastic realization, *Feedback and Synthesis of Linear and Nonlinear Systems*, D. Henrichsen and A. Isidori (eds.), Springer Verlag, 175-189.

18. A. Lindquist, M. Pavon and G. Picci, Recent trends in stochastic realization theory, *Harmonic Analysis and Prediction Theory, the Pesi Masani Volume*, V. Mandrekar and H. Salehi (eds.), North-Holland Publ. Co.

19. W.B. Gragg and A. Lindquist, On the partial realization problem, *Linear Algebra & Appl.*, special issue edited by R. Brockett and P. Fuhrmann. (61 pages; to appear).

20. C.I. Byrnes and A. Lindquist, Stability and instability of partial realizations, *Systems & Control Letters* 2 (1982), 99-105.

21. A. Lindquist and M. Pavon, Markovian representation of discrete-time stationary stochastic vector processes, *Proc. 1981, IEEE Decision and Control Conference*, San Diego, Calif., Dec. 1981.

22. A. Lindquist and M. Pavon, On the structure of state space models for discrete-time stochastic vector processes, *IEEE Trans. Automatic Control*, AC-29 (1984), (to be published).

23. A. Lindquist, C. Martin and G. Picci, Extreme points of Riccati inequalitites, (note submitted for publication).

24. A. Lindquist and G. Picci, Infinite dimensional stochastic realizations of continuous-time stationary vector processes, (submitted for publication).

25. A. Lindquist and G. Picci, Forward and backward semimartingale representations of stationary increment processes (submitted for publication).

D. ABSTRACTS OF PUBLICATIONS

1. On the stochastic realization problem

Given a mean-square continuous stochastic vector process y with stationary increments and a rational density Φ such that $\Phi(\infty)$ is finite and nonsingular, consider the problem of finding all minimal Gauss-Markov representations (stochastic realizations) of y. All such realizations are characterized and classified with respect to deterministic as well as probabilistic properties. It is shown that only certain realizations (internal stochastic realizations) can be determined from the given output process y. All others (external stochastic realizations) require that the probability space be extended with an exogeneous random component. A complete characterization of the sets of internal and external stochastic realizations is provided. It is shown that the state process of any internal stochastic realization can be expressed in terms of two steady-state Kalman-Bucy filters, one evolving forward in time over the infinite past and one backward over the infinite future. An algorithm is presented which generates families of external realizations defined on the same probability space and totally ordered with respect to state covariances.

2. A state space theory for stationary stochastic processes

Consider a stationary Gaussian stochastic process $\{y(t);t \in R\}$ with a rational spectral density, and let H(y) be the Hilbert space spanned by it. The problem of determining all stationary and purely nondeterministic families of minimal splitting subspaces of H(y) is considered; the splitting subspaces constitute state spaces for the process y. It is shown that some of these families are Markovian, and they lead to internal stochastic realizations. A complete characterization of all Markovian and non-Markovian families of minimal splitting subspaces is provided. Many of the basic results hold without the assumption of rational spectral density.

3. On minimal splitting subspaces and Markovian representations

Given a Hilbert space H, let H_1 and H_2 be two artibrary subspaces.

The problem of finding all minimal splitting subspaces of H with respect to H_1 and H_2 is solved. This result is applied to the stochastic realization problem. Each minimal stochastic realization of a given vector process y defines a family of state spaces. It is shown that these families are precisely those families of minimal splitting subspaces (with respect to the past and the future of y) which satisfy a certain growth condition.

4. A Hardy space approach to the stochastic realization problem

Given a purely nondeterministic mean-square continuous Gaussian stationary stochastic process we consider the problem of characterizing all minimal splitting subspaces X which evolve in time in a Markovian fashion. Let $H^{+/-}$ and $H^{-/+}$ be the projections of the future of the given process onto the past and the past onto the future respectively. It is shown that the family {X} of minimal Markovian splitting subspaces can be isomorphically described as a partially ordered family of subspaces of the form $X \stackrel{\sim}{=} j\mathfrak{X}^*$ where $\mathfrak{X}^* \stackrel{\sim}{=} H^{-/+}$ and j ranges over the family of all inner divisors of a fixed inner function j_* uniquely defined by $H^{+/-}$. The procedure is illustrated with an application to a process with a rational spectral density.

5. Realization theory for multivariate stationary Gaussian processes I: State space construction

A new approach to the problem of determining all internal minimal Markovian splitting subspaces of a given stationary process y is presented which is particularly suitable for solving the multivariate stochastic realization problem. Under the assumption that y is strictly noncyclic it is shown that each such state space can be expressed in terms of two subspaces spanned by the pasts of a corresponding pair of Wiener processes, one being a forward and the other a backward innovation process for the state space. This makes it possible to solve the problem in the Hardy space H_2^+, and each state space is seen to be the isomorphic image of the closure of a certain Hankel operator in H_2^+. This result will facilitate the application of existing infinite-dimensional deterministic realization theory to obtain explicit Markovian representations; this is the topic of Part II.

6. Stochastic realization and invariant directions of the matrix Riccati equation

Invariant directions of the Riccati difference equation of Kalman filtering are shown to occur in a large class of prediction problems and to be related to a certain invariant subspace of the transpose of the feedback matrix. The discrete time stochastic realization problem is studied in its deterministic as well as probabilistic aspects. In particular a new derivation of the classification of the minimal Markovian representations of the given process z is presented which is based on a certain backward filter of the innovations. For each Markovian representation which can be determined from z the space of invariant directions is decomposed into two subspaces, one on which it is possible to predict the state process without error forward in time and one on which this can be done backward in time.

7. A stochastic realization approach to the smoothing problem

The purpose of this paper is to develop a theory of smoothing for finite dimensional linear stochastic systems in the context of stochastic realization theory. The basic idea is to embed the given stochastic system in a class of similar systems all having the same output process and the same Kalman-Bucy filter. This class has a lattice structure with a smallest and a largest element; these two elements completely determine the smoothing estimates. This approach enables us to obtain stochastic interpretations of many important smoothing formulas and to explain the relationship between them.

8. Realization theory for multivariate Gaussian processes II: State space theory revisited and dynamical representations of finite dimensional state spaces

The purpose of this paper is twofold. First, some of the results of Part I are generalized and clarified, reformulated in a mathematical framework which we now perceive as more natural and conceptually sounder, the basic concept being perpendicularly intersecting subspaces. By relating minimality of stochastic realizations to the factorization of certain Hankel operators, the connections to deterministic realization theory is further clarified. Furthermore, spectral domain criteria for minimality, observability and constructibility are presented. Secondly, stochastic differential equation representations are presented for all state spaces which are finite dimensional. The infinite dimensional case will be the topic of Part III of this sequence of papers.

9. On the Mayne-Fraser smoothing formula and stochastic realization theory for nonstationary linear stochastic systems

This paper is a shortened version of Ref. 7, its basic purpose being to provide an easily accessible introduction to the results of Ref. 7, many of which are presented here without proofs. However, we have tried to rearrange the material of Ref. 7, changing the logical order in which various topics are introduced, and occasionally we regard the results from a somewhat different angle. This has been done to increase the present paper's usefulness as a complement to Ref. 7.

The work reported here is aimed at providing a theory of smoothing in the context of stochastic realization theory. This approach enables us to obtain stochastic interpretations of many important smoothing formulas and to explain the relationship between them. In this paper, however, we shall only consider one such formula, namely the Mayne-Fraser two-filter formula, which has a very natural interpretation in the stochastic realization setting; we refer the reader to Ref. 7 for further results. As a by-product, we also obtain certain results on the stochastic realization problem itself which cannot be found in Ref. 7.

12. A Hamilton approach to the factorization of the matrix Riccati equation

In this note the theory of Hamiltonian systems and an idea due to L.E.Zachrisson is used to obtain the factorizations of the matrix Riccati difference equation on which the usual derivations of fast (non-Riccati) algorithms are based. Although we are primarily interested in discrete-time Riccati equations, for comparison, the corresponding continuous-time result is briefly discussed.

13. State space models for Gaussian stochastic processes

A comprehensive theory of stochastic realization for multivariate stationary Gaussian processes is presented. It is coordinate-free in nature, starting out with an abstract state space theory in Hilbert space, based on the concept of splitting subspace. These results are then carried over to the spectral domain and described in terms of Hardy functions. Each state space is uniquely characterized by its structural function, an inner function which contains all the systems theoretical characteristics of the corresponding realizations. Finally coordinates are introduced and concrete differential-equation-type representations are obtained. This paper is an abridged version of a forthcoming paper, whichin turn summarizes and considerably extends results which have previously been presented in a series of preliminary conference papers.

16. On a condition for minimality of Markovian splitting subspaces

Given a stationary Gaussian vector process, consider a Markovian splitting subspace X contained in the frame space which is either observable or constructible. Such an X will be called *reduced*. In this paper we show that a Markovian splitting subspace is minimal if and only if it is reduced. This was claimed in some earlier papers but there are nontrivial gaps in the proofs presented there. The proof is based on a lemma stating that all reduced X have quasi-equivalent structural functions. This property is also important in isomorphism theory for minimal splitting subspaces.

18. Recent trends in stochastic realization theory

This is a survey of some recent work on Markovian representation of multivariate stationary stochastic processes. First a geometric theory in Hilbert space is developed. Next these results are translated into a Hardy space setting, and a complete set of Markov models are constructed for the given process. These models are then analyzed from a systems theoretical point of view.

19. On the partial realization problem

In this paper we take a unified approach to the partial realization problem in which we seek to incorporate ideas from numerical linear algebra, most of which were originally developed in other contexts. We approach the partial realization problem from several different angles and explore the connections to such topics as factorization of Hankel matrices, block tridiagonalization, generalizations of the Lanczos process for biorthogonalization, the Euclidean algorithm and the principal part continued fractions of Arne Magnus, the Padé table, and the Berlekamp-Massey algorithm. In this way we are able to clarify some previous results by Rissanen, Kalman and others and place them in a broader context. This leads to several results and concepts which we think are new. Our analysis is restricted to the scalar case, but some definitions and formulations have been rigged to facilitate an extension to the matrix case.

20. Stability and instability of partial realizations

A finite or infinite sequence of real numbers is said to be stable if it admits minimal realization by a stable linear system. It is shown that the preservation of stability as such a sequence is truncated or extended is not a generic property even among stable sequences. This is of interest for identification of linear systems from partial data and for partial realization of random systems, in which cases it constitutes a negative result. Certain finite sequences have infinitely many minimal realizations each having different stability properties. In this case, a graphical criterion in the spirit of the Nyquist criterion is derived to exploit this lack of uniqueness in order to determine whether one can achieve a stable pole placement by a judicious choice of partial realization.

22. On the structure of state space models for discrete-time stochastic vector processes

From a conceptual point of view, structural properties of linear stochastic systems are best understood in a geometric formulation which factors out the effects of the choice of coordinates. In this paper we study the structure of discrete-time linear systems with stationary inputs in the geometric framework of splitting subspaces set up in the work by Lindquist and Picci. In addition to modifying some of the realization results of this work to the discrete-time setting, we consider some problems which are unique to the discrete-time setting. These include the relations between models with and without noise in the observation channel, and certain degeneracies which do not occur in the continuous-time case. One type of degeneracy is related to the singularity of the transfer function, another to the rank of the observation noise and invariant directions of the matrix Riccati equation of Kalman filtering. We determine to what extent these degeneracies are properties of the output process. The geometric framework also accomodates infinite-dimensional state spaces, and therefore the analysis is not limited to finite-dimensional systems.

23. Extreme points of Riccati inequalities

The purpose of this note is to clarify some of the relations that exist between solutions of the algebraic Riccati equation and the quadratic matrix inequalities. In particular the main result of the note is to establish that there are *extreme points* in the solution set of quadratic matrix inequalities that are *not* solutions of the algebraic Riccati equations. The history of this is typical of many results involving Riccati equations in the engineering literature-total confusion. It has been part of the folklore for many years that the solutions of the algebraic Riccati equation are extreme points of the quadratic matrix inequalitis. In Badawi's thesis (Ref. 11) a very elegant proof is given: however, in review it was discovered that there had in fact appeared a proof in the literature. In the book FCG there is indeed a proof and a footnote to the effect that there are extreme points other than the solutions of the algebraic Riccati equation.

Thus it seems to be known that there are extreme points that are not solutions of the algebraic Riccati equation. However, we have been unable to find a proof in the literature or in the folklore. In this note we present a class of examples that establishes that there are other extreme points.

FCG Faurre, P., Clerget, M. and Germain, F., "Opèraturs Rationel Postifs", Dunod, 1979.

24. Infinite dimensional stochastic realizations of continuous-time stationary vector processes

In this paper we consider the problem of representing a given stationary Gaussian process with nonrational spectral density and continuous time as the output of a stochastic dynamical system. Since the spectral density is not rational, the dynamical system must be infinite-dimensional, and therefore the continuous-time assumption leads to certain mathematical difficulties which require the use of Hilbert spaces of distributions. (This is not the case in discrete time.) We show that, under certain conditions, there corresponds to each proper Markovian splitting subspace, two standard realizations, one evolving forward and one evolving backward in time.

25. Forward and backward semimartingale representations of stationary increment processes

Let $\{y(t)\}$ be a p.n.d. Gaussian stationary increments process and $\{S_t\}$, $\{\bar{S}_t\}$ two families of subspaces (or σ-algebras) such that (i) S_t is increasing and $\bar{S}_t$ is decreasing with time (ii) the future (past) increments of y at the instant t are S_t (resp. $\bar{S}_t$) measurable (iii) the pair $(S_t, \bar{S}_t)$ is perpendicularly intersecting for each t. The process y has a *forward semimartingale representation with rcspect to* $\{S_t\}$ if

$$dy(t) = z(t)dt + du(t) \qquad (*)$$

where z(t) is an S_t-measurable (stationary) process and u(t) is an S_t-martingale. In this representation, $\{x(t)\}$ is often called the forward conditional derivative of y w.r. to $\{S_t\}$.

We study conditions under which the existence of a forward representation of the type (*) implies that y also admits a semimartingale representation w.r. to the companion "backward" family

$\bar{S}_t$ i.e. one of the form

$$dy(t) = \bar{z}(t)dt + d\bar{u}(t)$$

where $\bar{z}(t)$ is $\bar{S}_t$-measurable and $\bar{u}(t)$ is a (backward) $\bar{S}_t$-martingale. As it turns out, the theory of *Stochastic Realization* together with the concepts of (stochastic) observability and constructability provides natural tools for attacking the problem.

E. INTERACTIONS

Results obtained within the project have been presented on the following occations.

Professional Meetings

1. Workshop on Current Topics in Communications, Washington University, St. Louis, March 6-7, 1978 (A. Lindquist, invited speaker).

2. Workshop on Fast and Square-Root Algorithms, Université Catholique de Louvain, Belgium, June 1978 (A. Lindquist, invited speaker).

3. 21st Midwest Symposium on Circuits and Systems, Ames, Iowa, August 14-15, 1978 (A. Lindquist, invited speaker).

4. 17th Conference on Decision and Control, San Diego, January 10-12, 1979 (A. Lindquist; SIAM-paper, invited).

5. Special Session on Integral Equations with Emphasis on Fredholm and Hammerstein Equations, 85th annual meeting of the AMS, Biloxi, January 24-28, 1979 (A. Lindquist, invited speaker; could not attend due to illness).

6. IEEE International Symposium of Information Theory, Grigano, Italy, June 25-29, 1979 (A. Lindquist or G. Picci).

7. 4th International Symposium on Mathematical Theory of Networks and Systems, Delft, Holland, July 3-6, 1979 (A. Lindquist and G. Picci, invited speakers).

8. Special Session of Stochastic Realization Theory (organized by A. Lindquist), 1979 International Conference on Information Sciences and Systems, Patras, Greece, July 9-13, 1979 (G. Picci).

9. 1979 Conference on Decision and Control, Fort Lauerdale, Florida, Dec 12-14, 1979 (F. Badawi).

10. Miniconference in Mathematical System Theory, Ohio State University, February 23, 1980 (A. Lindquist, invited speaker).

11. Numerical Techniques in System Engineering Workshop, University of Kentucky, June 16-20, 1980 (G. Picci, invited speaker).

12. NATO Advanced Studies Institute Meeting on Stochastic Systems, Les Arcs, France, June 22-July 5, 1980 (G. Picci, invited speaker).

13. Short Course on Diffusion Processes, Clemson University, (A. Lindquist, invited speaker but unable to attend).

14. Workshop on Feedback and Synthesis of Linear and Nonlinear Systems, Rome, Italy, June 29-July 3, 1981 (A. Lindquist, invited speaker).

15. 1981 International Symposium on the Mathematical Theory of Networks and Systems (MTNS), Santa Monica, California, August 5-7, (A. Lindquist, invited speaker).

16. European Conference on Circuit Theory and Design, The Hague, The Netherlands, August 25-28, 1981 (A. Lindquist, invited speaker, unable to attend).

17. Workshop on Rational Approximation for Systems, Leuven, Belgium, August 31-September 1, 1981 (A. Lindquist, invited speaker, unable to attend).

18. The 1981 IEEE Decision and Control Conference, San Diego, Dec. 1981 (was unable to attend, but Ref. 21 in proceedings).

19. NATO Advanced Studies Institute of Workshop on Nonlinear Stochastic Problems, Algarve, Portugal, May 16-28, 1982 (A. Lindquist, invited speaker).

20. The Second Bad Honnef Conference of Stochastic Differential Systems, Bonn, West Germany, June 27-July 3, 1982 (A. Lindquist invited speaker).

21. 1983 International Symposium on the Mathematical Theory of Networks and Systems, Beer Sheva, Israel, June 20-24, 1983 (A. Lindquist, invited to organize a special session on stochastic realization theory: talk by G. Picci).

22. Workshop on Linear Operator Theory, Weizmann Institute, Rehovot, Israel, June 13-18, 1983; organized by Harry Dym and Israel Gohberg (A. Lindquist, invited speaker).

Colloquia and Seminars

1. Washington University, November 1978 (G. Picci)

2. M.I.T., November 1978 (G. Picci)

3. M.I.T., November 1978 (M. Pavon)

4. Toronto, April 9, 1979 (A. Lindquist)

5. Brown University, April 10-11, 1979 (A. Lindquist)

6. M.I.T., April 12, 1979 (A. Lindquist)

7. University of Maryland, February 23, 1980 (A. Lindquist)

8. Johns Hopkins University, Baltimore, Febrauary 29, 1980 (A. Lindquist)

9. Series of Lectures and Colloquium at University of Maryland, College Park, November 10-14, 1980 (A. Lindquist)

10. Series of Lectures and Colloquium at M.I.T., November 19-22, 1980 (A. Lindquist)

11. Colloquium at McGill University, Montreal, Canada, November 24, 1980 (A. Lindquist)

12. Colloquium at Washington University, St. Louis, December 2, 1980 (A. Lindquist)

13. Seminar at U.C.L.A., December 4, 1980 (A. Lindquist)

14. Colloquium at University of California, San Diego, December 5, 1980 (A. Lindquist)

15. Seminar at University of California, Santa Barbara, December 8, 1980 (A, Lindquist)

16. Colloquium at University of Florence, Italy, June 1981 (A. Lindquist)

17. Colloquium at Schlumberger-Doll Research Center, Ridgefield, Connecticut, Nov. 18, 1982 (A. Lindquist)

18. Colloquium at University of Maryland, April 5, 1983 (A. Lind-quist).

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