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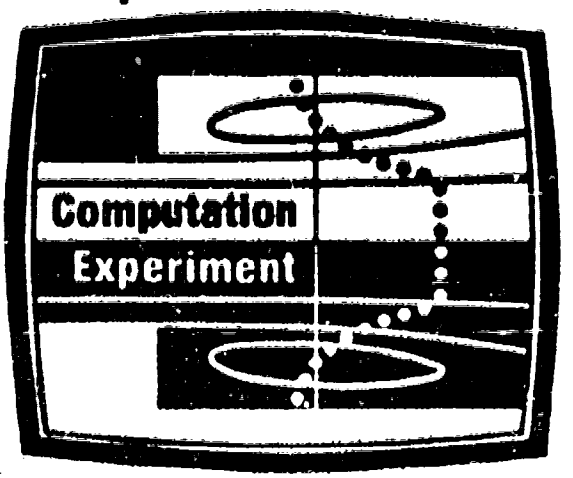
Complex Turbulent Flows



Volume II

Taxonomies, Reporters' Summaries, Evaluation, and Conclusions

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Edited by
S. J. Kline,
B. J. Cantwell,
G. M. Lilley

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COMPLEX TURBULENT FLOWS
EXPERIMENTAL DATA
ATTACHED BOUNDARY LAYERS
SEPARATED FLOWS
TWO-DIMENSIONAL FLOW
THREE-DIMENSIONAL FLOW

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Volumes II and III together give an overview of the state of the art in Computing Complex Turbulent Flows in 1981 using the data base established for this purpose in Volume I. The materials are intended to be comprehensive in the sense of providing all elements necessary for understanding the state of the art. Thus, Volumes II and III include: (i) taxonomies that organize the flows, methods of modeling, numerics; (ii) comments by non-computors (the reporters) on results for each class of flows; (iii) discussions carried through to closure; and carefully edited (see Discussion Procedures in

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Volume I); (iv) samples of recent high-level research computations that are currently beginning to provide information of aid to turbulence modelers; (v) an overall evaluation of the state of the art by a distinguished committee of nine workers in the field; (vi) an opinion by the leading editor on the question of "universality" of turbulence models; potential roads toward further progress, and discussion thereof by others; (vii) all computer output compared with data, case by case; (viii) comments by the computor groups on experiences and problems; (ix) several cross indexes to aid readers.

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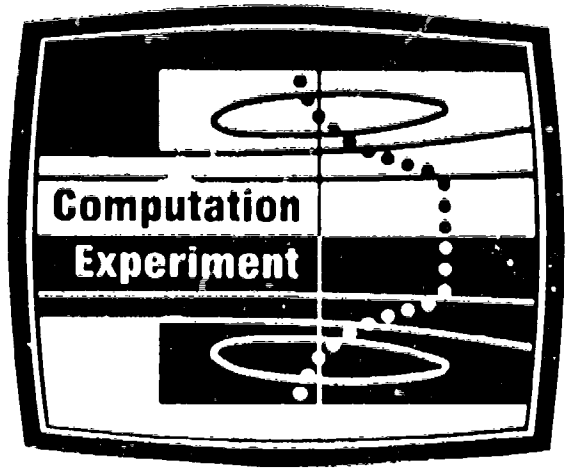

THE 1980-81 AFOSR-HTTM-STANFORD CONFERENCE ON COMPLEX TURBULENT FLOWS: COMPARISON OF COMPUTATION AND EXPERIMENT

VOLUME II

TAXONOMIES, REPORTERS' SUMMARIES, EVALUATION, AND CONCLUSIONS

Proceedings of the 1981 Conference

Stanford University, Stanford, California

September 14-18, 1981

Edited by S. J. Kline, B. J. Cantwell, and G. M. Lilley

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ACKNOWLEDGMENTS

The 1981 Conference, together with the 1980 Conference, is best viewed as a cooperative learning process within the research community. To achieve the aims of the Conference with impartiality, the fullest cooperation of the scientific and engineering communities was sought and generously given. Our special thanks are extended to all the Computors and Computor Groups, Technical Recorders, whose names appear in the List of Participants to the 1981 Conference in this volume. The 1981 Conference was attended by approximately 190 invited participants from 18 countries (Australia, Brazil, Canada, Chile, England, France, Hong Kong, India, Iran, Israel, Italy, Japan, Netherlands, South Africa, Switzerland, United States of America, West Germany, and Yugoslavia).

The principal financial sponsorship for the work of this conference was supplied by the U.S. Air Force Office of Scientific Research under contract AF F49620-80-C-0027 and a predecessor grant. Added support for work on data processing of compressible flow cases was supplied by the NASA Ames Research Center under Grant NAG 2-79. Contributions were also made by the Langley and Lewis Research Centers of NASA, by the U.S. Office of Naval Research and by the National Science Foundation. The specific assistance of Dr. Morris Rubesin and Dr. Dennis Bushnell of NASA with regard to funding is gratefully acknowledged. The steadfast support of Dr. James Wilson of AFOSR was critical to the success of the Conference. When the volume of cases found to be useful grew far beyond the initial ertimates, with a resulting and considerable increase in cost, Dr. Wilson spent much effort to organize the concerned government agencies and thereby secure the funds to complete the work. Funds for some special purposes were also supplied from the Heat Transfer and Turbulence Mechanics (HTTM) Program of the Industrial Affiliates Program of the Stanford Thermosciences Division. Funds to cover an overdraft in the base contract were generously supplied by the Stanford School of Engineering.

Special thanks are accorded the Organizing Committee: S. J. Kline (Chairman, P. Bradshaw, B. J. Cantwell, B. E. Launder, E. Reshotko, M. W. Rubesin, and G. Sovran, who worked long and thoughtfully beginning in 1977 on the plans, organization, and personnel problems. The continuing confidence and enthusiasm of the members of this committee were important ingredients in the success of the Conference.

The unenviable task of judging the comparison between data and computer output fell to the Evaluation Committee: H. W. Emmons (Chairman), D. R. Chapman, P. G. Hill, G. M. Lilley, M. Lubert, M. V. Morkovin, W. C. Reynolds, P. J. Roache, and J. Steger. Professor Emmons had also been the Chairman of the Evaluation Committee for the 1968 AFOSR-IFP-Stanford Conference on the Computation of Turbulent Boundary Layers. The

work of the Evaluation Committee was greatly appreciated and their conclusions provided important guidelines for further effort in work on Complex Turbulent Flows.

The critical task of summarizing results of computation in each class of flows was assumed by the eleven "Reporters" shown in the program. This allowed the attendees and the Evaluation Committee to obtain a clear picture of the large, heterogeneous mass of results that otherwise would not have been possible within a few days. The reporters did this task on very short time scales in a highly responsible manner and with great clarity. None of the eleven persons asked to be a reporter, declined the task, and hence each represents the expert best qualified to discuss the given class of flows in the opinion of the Organizing Committee.

Responsibility for the important task of creating, ad novo, a taxonomy of turbulence modeling methods and also of numerical procedures was assumed by Prof. J.H. Ferziger. He was very ably assisted by Jorge Bardina on turbulence modeling and by Gary Allen on numerical procedures.

Special thanks are also due to Brian Launder and Wolfgang Rodi for several tasks including: organizing groups of computors thus giving a wide comparison of related k-ε methods for various numerical methods; carrying out full computations on predictive cases; organizing the computors during the meeting to insure maximum learning from their experiences; and much effort with K. Hanjalić on studying and describing all the methods employed.

No conference of this sort would have been held without the willing help of the Host Committee, who made all the local arrangements, and the many Stanford graduate students who served as Technical Recorders and Aides; to all these we add our grateful thanks. Special appreciation is given to Profs. J. P. Johnston and J. H. Ferziger for important assistance in studying data and closing many special gaps. Roger Strawn and Ranga Jayaranan organized the aides and supervised all physical arrangements in an excellent fashion. Professors W. C. Reynolds and R. J. Moffat lent the support of the Thermosciences Division and the Department of Mechanical Engineering at important points. Professor M. V. Morkovin acted as a senior advisor on many issues.

The arduous and critical task of transferring the output from the computor groups on to the poster boards was expertly handled by a dedicated cadre of Stanford graduate students and others working under the direction of B. J. Cantwell. They included: Jorge Bardina, Juan Bardina, Jalal Ashjee, Ken Schultz, Jim Talleghani, and Priscilla Spolyar. Special thanks are due to Juan Bardina and Jalal Ashjee for many late nights in the computer center writing plot routines to make the standard plotting windows with experimental results displayed.

The Conference was fortunate in having the aid of a very able secretarial staff. Particular praise is needed for both Ditter Peschcke-Koedt, who was responsible for all typing and manuscript organization, and to Pauline Polen, the Conference Secretary, who organized the paperwork system, including finances, travel, filing, and programs in a highly independent and responsible fashion. Thanks are also due to Priscilla Spolyar, Ruth Korb, and Diana Thompson for various secretarial duties and for their enthusiasm and support. Assistance in arranging the off-day tour was provided by Ed and Pauline Polen.

PREFACE

This is the second of three volumes of Proceedings of the 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows: Comparison of Computation and Experiment.

The Conference has three related goals:

1. To reach consensus in the research community on trustworthy data sets that can be used as input for modeling of turbulence in complex flows and as the basis for standard "trials" for checking output of computations.
2. The creation of a "data library" on magnetic tape. This library will hold the data selected as trustworthy in standard normalized form. The data will be computer-readable and widely accessible at a moderate fee.
3. Comparison of the output of current methods of computation for turbulent flows for a set of "basic test cases" covering a broad range of flows.

The work of the first goal occupied the 1980 meeting of the Conference and is reported in Volume I of these proceedings.

The work of the second goal is embodied in a magnetic tape. Description and order forms can be obtained from

Complex Turbulent Flows
Dept. Mechanical Engineering, Stanford University
Stanford, CA 94305 USA

The work of the third goal occupied the 1981 meeting and is reported in this and the third volume of the Proceedings. In this volume the proceedings and discussions of the 1981 meeting and taxonomies of methods and numerics are presented along with the discussions of the meeting. A "Reader's Guide" (pp. xx below) suggests various parts of this volume that are particularly useful for a variety of purposes. Volume III will provide: (i) the comprehensive results of the computation output; (ii) their comparison with the experimental data; (iii) commentaries of Computor Groups on experiences in performing the computations.

In each volume an attempt is made to make the material complete by including discussion of important questions, appropriate position papers, taxonomies, description of procedures and criteria for selections and judgements.

The general position papers including the Introduction are contained in Volume I and are not repeated in this volume. The discussions concerning methods, the report of the Evaluation Committee, and opinions concerning future developments are contained in this volume; see Table of Contents and Reader's Guide.

A conclusion of the 1980-81 Stanford Conference is that the search for a "Universal Model of Turbulence," which can provide answers of an engineering accuracy for

a wide range of turbulent homogeneous and shear flows is premature and may remain illusory. As discussed herein in an opinion by S.J. Kline on "Universal or Zonal Modeling," the models of turbulence for each flow case need to reflect the physical structure of that flow. When this approach is followed, zonal models appear to have a far greater chance of success than the approaches used as a foundation for the 1980-81 Stanford Conference. As a result, it is the strong recommendation of the Conference, which can be gained from the report of the Evaluation Committee and the opinion on "Zonal Modeling" by S.J. Kline referred to above, that the same data base, using the Data Library, should be used for a comparison with flow models on a flow-by-flow basis. As far as possible, these flow models should reflect to an appropriate level the actual turbulent flow structure present, and the sensitivity of the output should be checked in terms of changes to the flow model via changes in its descriptors. It was also a recommendation of the Organizing Committee that this further work should be the subject of a follow-on conference in the next few years; such a conference is being actively planned. Questions concerning the follow-on conference should be addressed to Prof. G.M. Lilley.

The editors will welcome notification concerning any errors in this or other volumes. Such notices can be sent to any of the editors at the addresses listed.

S. J. Kline
B. J. Cantwell
G. M. Lilley

July 1982

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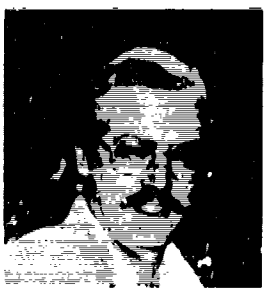

B.J. Cantwell

B.E. Launder

S.J. Kline

E. Reshotko

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P. Bradshaw

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Peter Bradshaw, Imperial College, London
Brian Cantwell, Stanford University
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Gino Sovran, General Motors Research Laboratories

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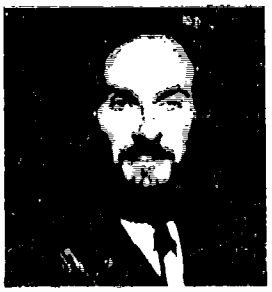

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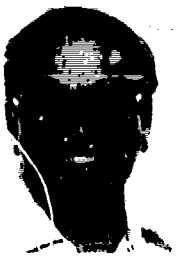

J.H. Ferziger

S.K. Kline

J.K. Eaton

R.J. Moffat

Evaluation Committee, 1981 Meeting

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M. Lubert, General Electric, KAPL
M. V. Morkovin, Illinois Institute of Technology
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Host Committee at Stanford

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James P. Johnston
Stephen J. Kline
Robert J. Moffat
William C. Reynolds

B.J. Cantwell

W.C. Reynolds

THE 1980-81 AFOSR-HTTM-STANFORD CONFERENCE ON COMPLEX TURBULENT FLOWS: COMPARISON OF COMPUTATION AND EXPERIMENT

8:30-8:45 INTRODUCTION--S.J. Kline*

SESSION I 8:45-10:00 am, Monday, September 14, 1981

TAXONOMY OF METHODS I

Chairman: P. Bradshaw
Technical Recorders: S. Deiwert, R. Subbarao

8:45-9:05 (1) Overview of Taxonomy--J. Ferziger/J.Bardina/G. Allen

9:05-9:25 (2) Integral Techniques--J. Cousteix

9:25-10:00 (3) Velocity and Length Scales--K. Hanjalić

10:00-10:30 COFFEE & REFRESHMENTS

SESSION II 10:30-12:00 noon, Monday, September 14, 1981

TAXONOMY OF METHODS II

Chairman: P. Bradshaw
Technical Recorders: F. Gessner, A. Strawa

10:30-10:45 (1) Stress/Strain Relations in Differential Methods--W. Rodi

10:45-11:00 (2) Turbulence Modeling in the Vicinity of a Wall--B. Launder

11:00-11:15 (3) Complex Strain Fields--P. Bradshaw

11:15-11:30 (4) Compressibility Effects in Turbulence Modeling--M. Rubesin

11:30-12:00 (5) Discussion on Taxonomy and Methods

12:00-2:00 LUNCH

SESSION III 2:00-3:30 pm, Monday, September 14, 1981

INCOMPRESSIBLE FREE SHEAR FLOWS

Chairman: A. Roshko
Technical Reporter: R. Chevray
Technical Recorders: L. Smits, S. Caruso

2:00-2:45 (1) Cases 0311, 0381, 0471 - Simple Strains
Cases 0331, 0382 - Extra Strains

2:45-3:30 (2) DISCUSSION

3:30-4:00 REFRESHMENTS

SESSION IV 4:00-5:30 pm, Monday, September 14, 1981

INCOMPRESSIBLE WALL-BOUNDED FLOWS I

Chairman: P. Klebanoff
Technical Reporter: R. Simpson
Technical Recorders: D. Driver, A. Cutler

4:00-4:45 (1) Cases 0141, 0211, 0261, 0612 - Boundary Layers
Cases 0142, 0143 - Diffusers

4:45-5:30 (2) DISCUSSION

*See Preface.

SESSION V 8:30-10:00 am, Tuesday, September 15, 1981

INCOMPRESSIBLE WALL-BOUNDED FLOWS II

Chairman: B. Newman
Technical Reporter: H. Nagib
Technical Recorders: S. Honami, J. Simonich

8:30-9:15	(1)	Cases 0231, 0232, 0233 - Curved Boundary Layers
		Cases 0241, 0242, 0244 - Suction/Blowing
		Case 0263 - Wall Jet
9:15-10:00	(2)	DISCUSSION

10:00-10:30 COFFEE & REFRESHMENTS

SESSION VI 10:30-12:00 noon, Tuesday, September 15, 1981

HOMOGENEOUS FLOWS

Chairman: R. Bleckwelder
Technical Reporter: J. Lumley
Technical Recorders: G. Settles, D. Ota

10:30-11:15	(1)	Cases 0371, 0372, 0373, 0374, 0375, 0376 - Homogeneous Flows
11:15-12:00	(2)	DISCUSSION

12:00-2:00 LUNCH

SESSION VII 2:00-3:30 pm, Tuesday, September 15, 1981

TRANSONIC FLOWS

Chairman: P. Sockol
Technical Reporter: P. Kutler
Technical Recorders: H. Legner, A. Ortega

2:00-2:45	(1)	Cases 8601, 8621 - Simple Strains
		Cases 8623, 8611, 8612, 8691 - Extra Strains
2:45-3:30	(2)	DISCUSSION

3:30-4:00 REFRESHMENTS

SESSION VIII 4:00-5:30 pm, Tuesday, September 15, 1981

SUPERSONIC FLOWS I

Chairman: H. Fernholz
Technical Reporter: D. Bushnell
Technical Recorders: J. Viegas, S. Kale

4:00-4:35	(1)	Cases 8101, 8201, 8403, 8411, 8501, 8301 - Simple Strains
4:35-5:10	(2)	DISCUSSION
5:10-5:20	(3)	Relaminarization--W. M. Kays - Case 0281
5:20-5:30	(4)	DISCUSSION

SESSION IX 8:30-10:00 am, Wednesday, September 16, 1981

NUMERICS AND ADVANCED METHODS

Chairman: W. Ballhaus
Technical Recorders: A. Leonard, P. Lowery

8:30-9:10 (1) Comparison of Numerics for Specified k-ε Model--B. Launder)

9:10-9:30 (2) Direct Simulation of Homogeneous Turbulence--R. Rogallo

9:30-10:00 (3) Large-Scale Numerical Simulation of Wall-Bounded Turbulent Shear Flows--P. Moin and J. Kim

10:00-10:30 COFFEE & REFRESHMENTS

SESSION X 10:30-12:00 noon, Wednesday, September 16, 1981

INCOMPRESSIBLE SEPARATED FLOWS

Chairman: J. Johnston
Technical Recorders: R. Luxton, P. Eibeck

EXTERNAL FLOWS

Technical Reporter: W. McCroskey

10:30-10:45 (1) Case 0441

10:45-10:55 (2) DISCUSSION

INTERNAL FLOWS

Technical Reporter: J. Eaton

10:55-11:30 (3) Cases 0421, 0431, P2, P3, P4--Backward-Facing Step and Predictive Cases

11:30-12:00 (4) DISCUSSION

12:00-2:00 LUNCH

SESSION XI 2:00-3:30 pm, Wednesday, September 16, 1981

INCOMPRESSIBLE DUCT FLOWS

Chairman: B. Cantwell
Technical Reporter: J. Jones
Technical Recorders: O. McMillan, M. Lee

2:00-2:45 (1) Cases 0111, 0112, 0511, 0512, P1--Internal Secondary Flows

2:45-3:30 (2) DISCUSSION

3:30-4:00 REFRESHMENTS

SESSION XII 4:00-5:30 p.m., Wednesday, September 16, 1981

SUPERSONIC FLOWS II

Chairman: D. Coles
Technical Reporter: J. Marvin
Technical Recorders: F. Pierce, R. Moser

4:00-4:45 (1) Cases 8631, 8632, 8651, 8661, 8663, 8671, 8641 - Extra Strains

4:45-5:30 (2) DISCUSSION

THURSDAY, SEPTEMBER 17, 1981

TOUR OF THE WINE COUNTRY
OR
FREE DAY

SESSION XIII 8:30-10:00 am, Friday, September 18, 1981

REPORTS

Chairman: E. Reshotko
Technical Recorders: I. Castro, S. Pronchick

Session Reports III to XII

10:00-10:30 COFFEE & REFRESHMENTS

SESSION XIV 10:30-12:00 noon, Friday, September 18, 1981

Chairman: E. Reshotko
Technical Recorders: T. Simon, G. Allen

(1) Ad-Hoc Committee Reports

(2) User's Viewpoints--G. Sovran, E. Tjonneland

12:00-2:00 LUNCH

SESSION XV 2:00-3:30 pm, Friday, September 18, 1981

Chairman: G. Sovran
Technical Recorders: W. Feiereisen, E. Adams

(1) Evaluation Committee Report--H. W. Emmons

(2) Discussion of Report

3:30-4:00 REFRESHMENTS

SESSION XVI 4:00-5:00 pm, Friday, September 18, 1981

Chairman: S. Bogdonoff
Technical Recorders: S. Birch, B. Afshari, G. Lilley

Universal or Zonal Modeling--The Road Ahead

A Personal Opinion (S. J. Kline)

GENERAL OPEN DISCUSSION

D. Peschcke-Koedt

Pauline Polen

GENERAL NOMENCLATURE

Symbol: Computer	Symbol: Conventional	Meaning	S.I. Units
BETA	β	$(dp/dx)\ \delta^*/\tau_w$	-
DEL	δ_{995}	Boundary-layer thickness to 0.995 U_e	m
DELS	δ^*	Displacement thickness = $\int_0^\delta \left(1 - \frac{\rho U}{\rho_e U_e}\right) dy$	m
ENTH	δ^{**}	Energy thickness = $\int_0^\delta \frac{\rho U}{\rho_e U_e}\left(1 - \frac{U^2}{U_e^2}\right) dy$	m
CLTH	Δ	Clauser thickness = $\int_0^\delta \frac{\rho_e U_e - \rho U}{\rho_w U_*} dy$	m
EPSILON	ε	Dissipation function	$m^2\ sec^{-3}$
THETA	θ	Momentum thickness = $\int_0^\delta \frac{\rho U}{\rho_e U_e}\left(1 - \frac{\eta}{U_e}\right) dy$	m
XNU	ν	Kinematic viscosity	$m^2\ sec^{-1}$
RO	ρ	Density	$kg\ m^{-3}$
TAU	τ	Shear stress	$N\ m^{-2}$
PHIL	ϕ_L	Left-hand side of momentum integral equation balance	-
PHIR	ϕ_R	Right-hand side of momentum integral equation balance	-
CD	C_D	Drag coefficient	-
CL	C_L	Lift coefficient	-
CF	C_f	Skin-friction coefficient = $\tau_w/(\frac{1}{2}\rho_e U_e^2)$	-
CFE	C_f	C_f as reported by originator	-
CFLT	C_f	C_f according to Ludwieg-Tillmann formula	-
CFPT	C_f	Measured using Preston tube	-
Cp	C_p	Pressure coefficient	-
G	G	Equilibrium shape factor = $\int_0^\delta \left(\frac{\rho_e U_e - \rho U}{\rho_w U_*}\right)^2 d(y/\Delta)$	-
H	H	Shape factor = δ^*/θ	-
HS	H^*	δ^{**}/θ	-
KAY	K	$(\overline{u^2} + \overline{v^2} + \overline{w^2})$	-
-	k	Turbulence kinetic energy (K/2)	
LREF	L_{ref}	Reference length	m
XM	M	Mach number	-

Symbol: Computer	Symbol: Conventional	Meaning	S.I. Units
XMREF	M_{ref}	Reference Mach number	-
P	p	Pressure	$N\ m^{-2}$
PR	Pr	Prandtl number	-
PREF	p_{ref}	Reference pressure	$N\ m^{-2}$
PIU1	$\overline{pu}$	Pressure-velocity covariance	-
QREF	q_{ref}	Reference dynamic pressure	$N\ m^{-2}$
RE	Re	Reynolds number based on reference values	
		$Re = \frac{U_{ref}\ L_{ref}}{\nu_{ref}}$	-
RDELS	R_{δ^*}	Reynolds number $= U_e \delta^* / \nu$	-
ROIU1	$\overline{\rho u}$	Density-velocity covariance	-
RTHETA	R_θ	Reynolds number $= U_e \theta / \nu$	-
ST	St	Stanton number	-
STR	Str	Strouhal number	-
-	T	Temperature	
-	t	Time	
TENTH	t^+	Thermal energy thickness $\frac{\sqrt{C_f/2}}{St}\left(\frac{T_w - T}{T_w - T_e}\right)$	m
XS	s	Coordinate tangent to an arc	m
XN	n	Coordinate normal to an arc	m
U	U	Mean streamwise velocity	$m\ sec^{-1}$
V	V	Mean transverse velocity	$m\ sec^{-1}$
W	W	Mean spanwise velocity	$m\ sec^{-1}$
UDEF	-	Defect velocity $= (U_e - U)/U_\star$	$m\ sec^{-1}$
UE	U_e	Velocity external to boundary layer	$m\ sec^{-1}$
UI	U_∞	Free-stream velocity	$m\ sec^{-1}$
UREF	U_{ref}	Reference velocity	$m\ sec^{-1}$
US	$U_\star$	Wall shear velocity $= \sqrt{\tau_w/\rho_w}$	$m\ sec^{-1}$
UPLUS	U^+	$U/U_\star$	-
U2	$\overline{u^2}$	Reynolds stress	$m^2\ sec^{-2}$
V2	$\overline{v^2}$	Reynolds stress	$m^2\ sec^{-2}$
W2	$\overline{w^2}$	Reynolds stress	$m^2\ sec^{-2}$

Symbol			
Computer	Conventional	Meaning	S.I. Units
U1V1	$\overline{uv}$	Reynolds shear stress	$m^2\ sec^{-2}$
U1W1	$\overline{uw}$	Reynolds shear stress	$m^2\ sec^{-2}$
V1W1	$\overline{vw}$	Reynolds shear stress	$m^2\ sec^{-2}$
UnVm	$\overline{u^n v^m}$	Higher-order velocity covariance	-
	ω	Frequency (also ϵ/k)	sec^{-1}
X	x	Streamwise coordinate	m
Y	y	Transverse coordinate	m
Z	z	Spanwise coordinate	m
X	x or s	Streamwise coordinate on curved surface	m
Y	y or n	Direction normal to curved surface	m
Z	z	Spanwise coordinate	m
YPLUS	y^+	yU_*/ν	-

Subscript "w" denotes wall value.

Subscript "e" denotes conditions external to boundary layer.

$$\phi_L(x) = \frac{U_e^2\theta}{(U_e^2\theta)_o} - 1 + \frac{1}{2}\int_{x_o}^{x} \frac{\delta^*}{\theta_o} d\left(\frac{U_e^2}{(U_e^2)_o}\right) .$$

$$\phi_R(x) = \int_{x_o}^{x} \left(\frac{U_*}{(U_e)_o}\right)^2 d\left(\frac{x}{\theta_o}\right) .$$

A READER'S GUIDE TO VOLUMES II AND III

Volumes II and III together attempt to give an overview of the state of the art in Computing Complex Turublent Flows in 1981 using the data base established for this purpose in Volume I. The materials are intended to be complete in the sense of providing all elements necessary for understanding the state of the art. Thus, Volumes II and III include:

(i) taxonomies that organize [illegible] flows, methods of modeling, numerics;

(ii) comments by non-computors (the reporters) on results for each class of flows;

(iii) discussions carried through to closure and carefully edited (see Discussion Procedures in Volume I);

(iv) samples of recent high-level research computations that are currently beginning to provide information of aid to turbulence modelers;

(v) an overall evaluation of the state of the art by a distinguished committee of nine workers in the field;

(vi) an opinion by the leading editor on the question of "universality" of turbulence models, potential roads toward further progress, and discussion thereof by others;

(vii) all computer output compared with data, case by case;

(viii) comments by the computor groups on experiences and problems;

(ix) several cross indexes to aid readers.

Since some of these documents are of an unusual kind and the materials are quite extensive, it seems appropriate to provide some guidance to readers about the uses of the two volumes.

The reader who wants a quick summary of the overall state of the art will probably want to read first the report of the Evaluation Committee (p. 979), and may also want to read the OPINION on Universality of Modeling (p. 991) with its comments (p. 999) and Closure (p. 1007).

The reader interested in what can be done for a particular class of flows will probably want to look at the Reporter's remarks for the class closest to the applications of concern and to examine in detail the output for these classes in Volume III. The reader will probably also want to read some Computors' comments on experiences and problems in these flows (these can be found through the indexes in Volume III).

The reader interested in the underlying structure of the subject of Computational Fluid Dynamics will probably want to look at the taxonomies of the methods and numerics that have been used to date (p. 634) and may also want to read the INTRODUCTION in

Volume I, since it includes a short history and an exploration of the nature of the problem.

The reader interested in the range and accuracy of a given type of modeling can track the output of models of a given type by first studying the taxonomic descriptors (p. 641) and then turning to the indexes in Vol. III to find what flows have been computed by the given methods. The same general procedure can be followed by a reader interested in what can be done currently in predicting a given class of flow.

The indexes are formulated in three forms as follows:

1. Index by group number including Computor Coordinator and members of Computor Group
2. Index by methods
3. Index by flow cases numerically

The reader can find further useful information on the experimental data available for each flow case from reference to the PICTORIAL SUMMARIES presented in Volumes I and III in numerical order of "Flows", the plots presented in Volume I and by reference to the DATA LIBRARY on magnetic tape. Computer-readable tapes containing the Data Library can be ordered from*

Complex Turbulent Flows
Dept. Mechanical Engineering
Stanford University, CA
U.S.A. 94305

*Volume I, p. 58, gives addresses at Stanford University and in Europe from which the Data Tape can be ordered. As of September 1982 no copies of the Master Tape have been released from Stanford University to other agencies. Copies of the Data Tape should, therefore, in the first instance, be requested from the address given above.

SESSIONS I and II

Chairman: P. Bradshaw

Technical Recorders: (I)

S. Deiwert
R. Subbarao

Technical Recorders: (II)

F. Gessner
A. Strawa

OVERVIEW OF TAXONOMY: MORPHOLOGY OF THE FLOWS AND COMPUTATIONAL METHODS — J. H. Ferziger, J. Bardina, and G. Allen

INTEGRAL TECHNIQUES — J. Cousteix

VELOCITY AND LENGTH SCALES IN TURBULENT FLOWS — A REVIEW OF APPROACHES — K. Hanjalić

STRESS/STRAIN RELATIONS IN DIFFERENTIAL METHODS FOR TURBULENT FLOWS — W. Rodi

TURBULENCE MODELING IN THE VICINITY OF A WALL — B. E. Launder

COMPLEX STRAIN FIELDS — P. Bradshaw

COMPRESSIBILITY EFFECTS ON TURBULENCE MODELING — M. W. Rubesin

DISCUSSION ON TAXONOMY AND METHODS

J.H. Ferziger

OVERVIEW O. TAXONOMY: MORPHOLOGY OF THE FLOWS AND COMPUTATIONAL METHODS

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Department of Mechanical Engineering
Stanford University, Stanford, CA 94305, U.S.A.

J. Bardina

1. Introduction

Comparisons between the present conference and the well known 1968 conference (Kline et al., 1968) and other meetings that have occurred in the interval are inevitable. The 1968 meeting was devoted to just one type of flow (boundary layers) and computational methods were then in their infancy or, perhaps, early childhood. Categorization of computational methods and models for turbulent flows was a new issue, and the morphology paper of that meeting (Reynolds, 1968) was critical to setting the other presentations into context; the job was handled in an outstanding fashion and Reynolds' paper has become a standard reference.

In the thirteen-year interval since that meeting, there has been a continuing effort on the morphology of turbulence models. The influence of this work has been so strong that an author presenting a new method will normally describe how the new method relates to the established categories. The existence of an evolving morphology has made it much easier for users of turbulence models to understand the relationships between the various methods and thus to make intelligent choices. This continuing morphology work has made the task of the authors of this paper far simpler. Without it, the job would have been nearly impossible, and for it, we are indebted to individuals too numerous to mention by name.

The most difficult task we face comes from the breadth of range of flows that challenge the computors in the current meeting. This breadth is itself a measure of how far the field has come in the interval. The changes are so great that a new morphology of the flows is required. This morphology will be reviewed in Section 2 of this paper.

In any computation of a turbulent flow, two sets of approximations have to be introduced. First, the nonlinearity of the equations and the enormous range of length and time scales in turbulent flows demand that only some average set of quantities be computed. The equations invariably contain averages of products, some or all of which must be approximated by what we call a turbulence model in order to effect closure. Second, the resulting set of equations has to be mathematically approximated to permit solution on a computer; this is the numerical method used in the calculation. A related issue is that only a finite geometrical region can be included in the calculation; boundary conditions are required to represent the interaction with the domain

exterior to the region considered in the calculation. These boundary conditions can have a considerable effect on the results.

Turbulence models and numerical methods are usually considered independent pieces of the total method. Certainly, within broad limits, one can solve any set of model equations with any numerical method. However, when the agreement between computation and experiment is less than the computor desires, it is difficult to decide whether the errors arise from the turbulence model or the numerical procedures. Assignment of the blame is an issue that will occupy much of the discussion at this meeting and has already occupied much of the attention of the Organizing Committee. There is no totally satisfactory solution at the present time, and we strongly recommend a continuing effort directed at finding one. For purposes of morphology, we will deal with turbulence modeling and numerical methods separately, as tradition dictates, but the difficulties should be kept in mind. The model morphology is given in Section 3 and is followed by the numerical method morphology in Section 4.

A further issue concerns engineering applications of computation. For instance in aeronautical applications only the lift, drag, side-force and moments are needed for design. In order to assure that these quantities are accurately calculated, it is necessary to compute more detailed information. How much "excess baggage" has to be carried in order to assure that the necessary items are given to some required accuracy does not seem to have had as much attention as it deserves until now.

2. Flow Morphology

The classification scheme for the flows considered at this conference is shown in Fig. 1. The classification is based primarily on a series of binary subclassifications. Most of these are standard and well known. We shall comment only on the aspects that may be unfamiliar to some readers, and shall consider them in the order in which they appear in the figure.

a. Homogeneous/Inhomogeneous: A homogeneous flow is one in which the statistical state of the fluid is the same everywhere at a given time; for purposes of this conference, the homogeneous flows are treated as a single flow although there are six cases. Inhomogeneous flows are all others.

b. Compressible/Incompressible: Compressible flows are defined as those in which $M > .3$ somewhere in the flow.

c. Free-Shear Flows/Wall-Bounded Flows: This is a well known distinction; however it can become a little unclear in separated and reattaching flows.

d. Simple Strains/Extra Strains: We have used Bradshaw's (1973) idea that a simple flow is one in which there is simple shear (a velocity

gradient in a direction normal to the principal streamline). An extra rate of strain is then any additional effect such as acceleration/deceleration, blowing/suction, curvature, rotation, etc.

e. Near Field/Far Field: This classification, which has been applied only to free-shear flows in this conference,* is the well-known distinction between the early, transitional state of the flow and the later flow in which an asymptotic (and, usually, self-similar) state is reached.

f. Attached/Separated: This applies only to wall-bounded flows.

g. Internal Flow/External Flow: Obviously, internal flows are a subclass of wall-bounded flows.

h. In a few cases, in which there were a large number of flows, a further division by flow type has been made.

3. Turbulence Model Morphology

a. General

The primary classification of treating turbulent flows is given in Table 1 which is an extension of a scheme presented by Kline et al. (1978). The bulk of the submissions to this conference are at Levels 2 and 3 with Level 3 having the large majority. No methods at Level 1 have been presented; only two entries at Level 4 have been submitted and Levels 5 and 6 are each represented by one state-of-the-art paper. The terms used in Table 1 are probably familiar and will not be elaborated.

The bulk of the presentations are at Levels 2 and 3; hence further classification schemes for these two levels were developed. Since it is impossible to anticipate turbulence models that will be developed in the future, the morphology is designed to cover only the methods presented. We hope that it is broad enough to contain models which will be developed in the near future. The classification of each method is given by an alphanumeric descriptor which denotes its principal properties. As only a few integral methods were submitted, a two-character descriptor proved sufficient. For the one-point closure schemes, a four-character descriptor was necessary. This also allows distinction between Level 2 and 3 methods to be made very easily.

b. Integral equations

The morphology and symbols for integral methods is shown in Table 2. All of the methods use the momentum integral equation so there is no need for the morphology to denote this fact. The further classification specifies what type of auxiliary equation is used and, in the case of entrainment methods, the type of entrainment correlation employed and whether lag is utilized or not.

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*No wall-bounded flows are described by this classification.

c. One-Point Closure Methods

The morphology and symbols for one-point closure methods are given in Table 3. At this level the methods are based on the time averaged Navier-Stokes equations. The major issue in modeling is the treatment of the Reynolds stresses that occur in these equations, i.e., the closure approximation used. The first one (or two) symbols denote the approximation applied to the Reynolds stresses.

The simplest and oldest method is to assume the Reynolds stresses are proportional to the mean strain rate or velocity gradients. These methods are called Boussinesq (B) or eddy-viscosity models. The eddy viscosity can be prescribed in terms of the mean velocity field or defined in terms of quantities derived from partial differential equations. By far the largest number of entries received employ the Boussinesq model.

In algebraic models (A) the Reynolds stresses are found by solving a system of algebraic equations. The Reynolds stress is not proportional to the mean strain rate. Partial differential equations may or may not be required for various terms which appear in the algebraic equations for the Reynolds stress.

In differential models (D) the Reynolds stresses are obtained by solving partial differential equation(s), but the number of equations solved is smaller than the number of relevant Reynolds stresses.

In Reynolds-stress models (RS) all or the equations for the relevant Reynolds stresses are modeled. These models may employ extra differential equations for a length scale or rate of dissipation.

In one-point closure models, the central two symbols of the four-character descriptor denote which partial differential equations are used (these symbols have different significance for Reynolds-stress models). A wide variety of equations have been used in the one-point closure models. The most popular single method is based on the use of two partial differential equations.

The final character in the descriptor for one-point closures is related to the wall treatment. Many computors do not treat the viscous sublayer near the wall explicitly in their calculations. Instead, they place the first mesh point far enough from the solid boundary so that it lies in the buffer or logarithmic region of the flow and use the law of the wall to provide a relationship between the wall shear stress and the velocity at the first mesh point. Other computors compute to the wall and use the exact no-slip boundary condition. When computation is carried to the wall, it is usually necessary to modify the turbulence model near the wall; typically a damping factor is introduced into the eddy viscosity.

d. Two-Point Closure

Table 4 lists the two two-point closure models that have been submitted. These models are based on spectral theories of turbulence and are just beginning to be

investigated at the present time. In the entries at this meeting they have been applied only to some of the simpler flows. These methods appear to have considerable promise for the future, and may see extensive development in the next ten years.

Finally, a review of large-eddy simulation is presented by P. Moin and a review of recent work in full simulation of turbulence is presented by R. Rogallo.

4. Numerical Methods

The numerical method taxonomy is given in Table 5. The information has been summarized in two categories. The method used for discretizing the equations is defined by a four-character code and the method used in solving the discretized equation is represented by a five-character code. In principle, the results should depend only on the discretizing method and not on the method used to solve the equations, but this may not always be the case.

Unfortunately, one critical piece of information is missing from this taxonomy. The number and placement of grid points within the flow field can have enormous influence on the quality of the results produced. Since this feature is different for each case done by a given method (indeed, an author may have used more than one mesh arrangement on a single case), it is impossible to represent it by the kind of symbolic notation used in this morphology. This is a serious shortcoming and is a major obstacle to trying to compare the various methods objectively. To deal with this difficulty, computor groups are asked to supply the number of grid points used ($N \times M$) for each flow as part of the work of the Conference. In addition, during the 1981 meeting, the computor groups were asked to provide information concerning location of the grid points nearest the wall. Where available, information on these points is included in the results of Volume III. See also comments in Appendix A below.

The discretizing method has been characterized as finite difference, finite volume or finite element. The distinctions among these are not always clear. In most cases, a regular grid is used. The staggered grid is used by some authors, mainly in incompressible calculations, nearly always with a finite-volume type of method.

The treatment of the convective terms in nearly all of the methods is by central or upward differencing; many programs use a hybrid of the two based on the cell Reynolds number. Although the taxonomy does not display it, most of the programs use second-order difference approximations; some of the upwind codes are first order. Even fewer codes use methods of higher order than second. It is important that the flow field produced by a numerical simulation globally conserve mass, momentum, and energy. There is controversy about whether or not it is necessary to use a computational method which is explicitly conservative of these properties. Perhaps this conference will shed some light on this issue.

None of the flows in this conference contain boundary geometries of high complexity; the backward-facing step and some of its variations and the two-dimensional airfoils are the most difficult flows in this respect. The treatment of arbitrarily shaped boundaries is a difficult issue; many new methods are still being developed. The common procedures at present include: boundary-fitted coordinate systems (most finite-element methods are of this type); transformation to a new coordinate system such that the computational domain becomes a rectangle; use of a Cartesian mesh with irregular stars at the boundary. The use of boundary-fitted coordinates is by far the most popular method at this conference. Coordinate system transformations are a distant second. Irregular stars are used in only one entry.

Next, we turn to consideration of the solution methods. The first issue is overall strategy. Once-through methods can be used when the flow is treated as parabolic. Parabolic methods have been applied to some nominally "elliptic" flows. Most other methods for the elliptic flows use either relaxation techniques or some variation of them. Some particular time-like relaxation methods are indicated by the taxonomy.

On every iteration of a relaxation method each of the variables must be updated. This can be done either simultaneously or sequentially; the second character of the solution descriptor indicates which method is used. The two types of methods are nearly equally represented in the entries with the sequential approach seeming to be a bit more popular.

In compressible-flow calculations, the pressure is either determined by a specific equation for it or, more commonly, from an equation of state and other variables which are calculated explicitly. Thus, compressible codes have an "E" as the third character of the solution descriptor. Incompressible-flow codes, on the other hand, must determine the pressure from some kind of elliptic equation. This can be done by solving a Poisson equation for the pressure (by a relaxation method) or by a more direct iterative technique which drives the velocity divergence to zero; both methods are used with the Poisson equation having a slight edge in numbers.

The iteration method usually linearizes the equations at each step. The linearized equations are then solved by an inner iteration technique. Some of the possibilities for the latter are indicated by the fourth character of the solution taxonomy. These are: point and line iteration, alternating direction implicit (ADI), and direct solution. No point methods have been submitted, and the others are approximately equally represented.

Finally, the methods may use over- or under-relaxation. This is indicated by the final character. Under-relaxation is the most common method.

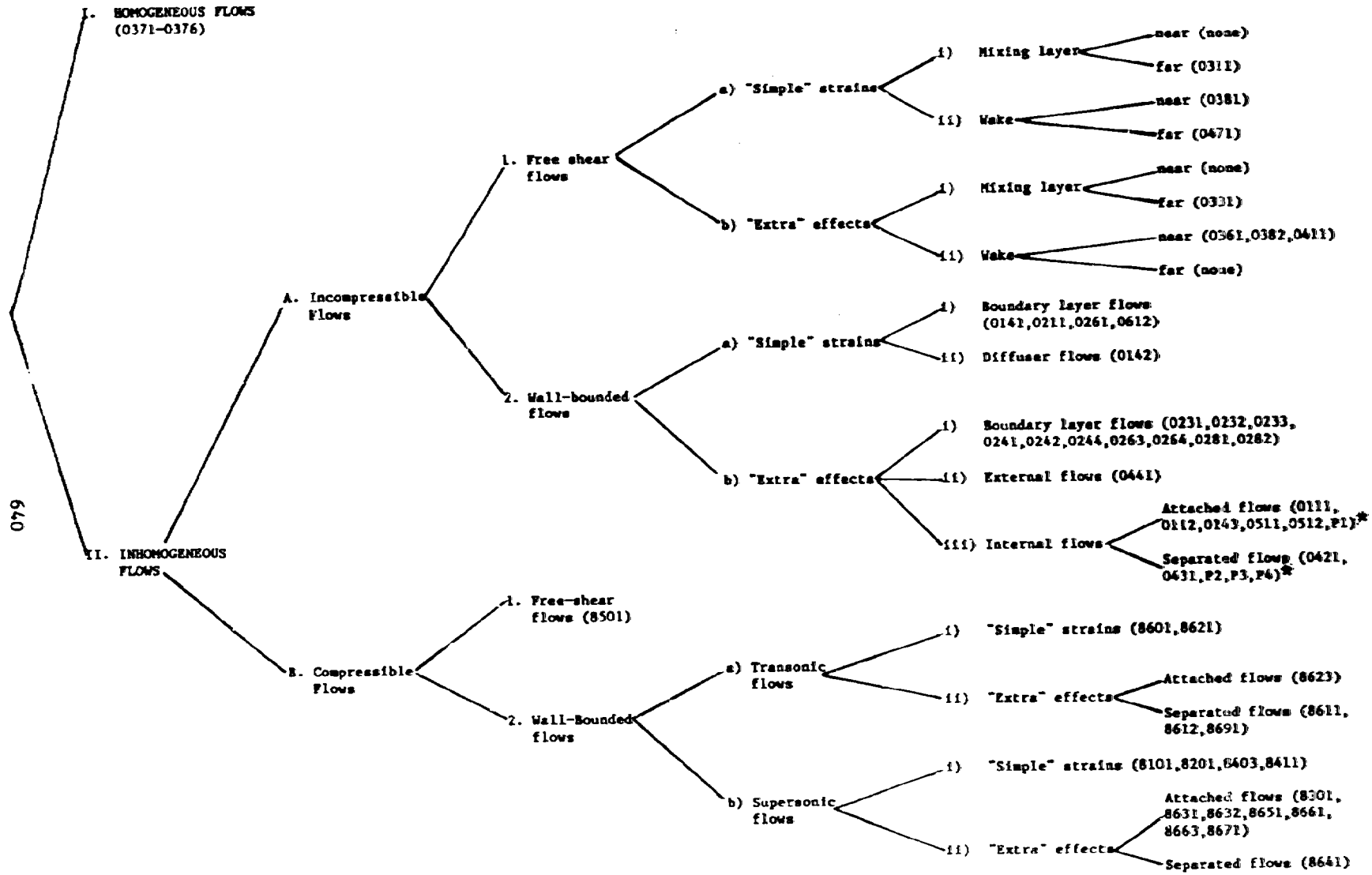


Figure 1. Classification of turbulent flows.

*Flows designated P1, etc., were predictive cases. Following the 1980 meeting, these flows were assigned permanent numbers as follows: P1 = 0113; P2 = 0422; P3 = 0423; P4 = 0424.

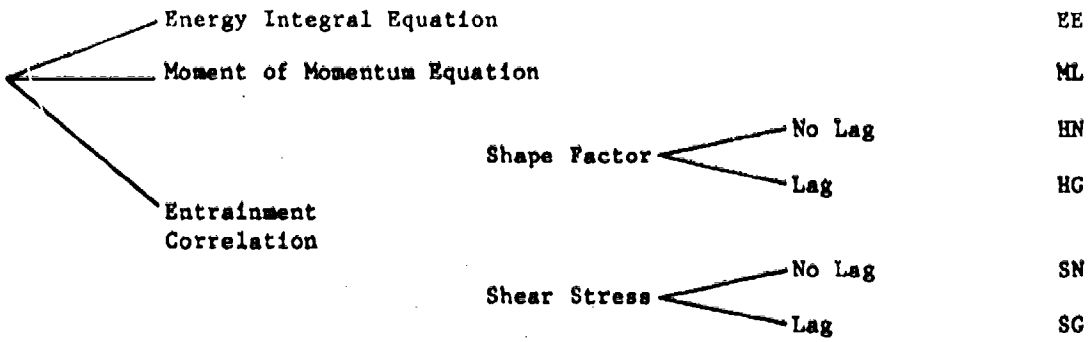
TABLE 1

Classification of Methods for Computing Turbulent Flows

Level	Method
1	Correlations
2	Integral Methods
3	One-Point Closures
4	Two-Point Closures
5	Large-Eddy Simulation
6	Full Simulation

TABLE 2

Classification of Integral Methods



Examples

a. The method used by Whitfield (Group 12) is EE which means that an energy integral equation is used.

b. A method used by Ferziger (Group 45) is HN which denotes that it is an entrainment/shape-factor method without lag.

TABLE 3

Classification of One-Point Closures

i	Treatment of Reynolds Stresses		
	Boussinesq (eddy viscosity)	B	___
	Algebraic	A	___
	Differential	D	___
	Reynolds stress	RS	___
	Other	O	___

TABLE 3 cont.

TABLE 3 (cont.)

ii Sub-Classifications for Classes B and A Above

Prescribed	OP	___
ℓ from ODE	OL	___
k equation, ℓ prescribed	1K	___
k equation, ℓ from ODE	1L	___
ℓ prescribed, ω equation	1W	___
Two-equation model:		
k, ε	KE	___
k, ω	KW	___
k, ℓ	KL	___
$k^2\ell$, ε	QE	___

iii Sub-Classifications for Class D Above

k, ε, $\overline{uv}$, $\overline{vw}$	4E	___
ε, $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$	5E	___
$k^2\ell$, $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$	5Q	___

iv Sub-Classification for Class RS Above

No additional PDE	T	___
Additional PDE for ε	E	___
Additional PDE for ℓ	L	___

v Special Effects

1. Treatment Near Solid Walls

No slip	Explicit damping	X	___
	No explicit damping	N	___
Law of the Wall	Explicit damping	Y	___
	No explicit damping	Z	___
Not applicable (free shear flows only)		C	___
Other		O	___

Examples

(a) The method used by Bailey (Group 50) is BOPX and therefore has:

B Boussinesq eddy viscosity
OP Prescribed values of the eddy viscosity
X No-slip damped-wall eddy viscosity

(b) The method used by Chow (Group 23) is AKEZ and therefore has:

A Algebraic treatment of Reynolds stress
KE Two-equation k,ε solver for Reynolds stress
Z No special wall treatment

(c) The descriptor RS4E denotes

RS Reynolds-stress equation used for Reynolds stress
4E Four equations used involving k, ε, $\overline{uv}$, $\overline{vw}$.

TABLE 4

Two-Point Closures

Direct Interaction Approximation	2DI
Eddy Damped Quasi-Normal Hypothesis	2ED

TABLE 5

Numerical Method Taxonomy

Discretizing	Solving
Method employed	Strategy
R - finite difference--regular grid S - staggered grid V - finite volume-regular grid E - finite element	O - once through (take precedence if others were checked) I - iterative, not time like B - backward implicit N - implicit Crank-Nicolson F - implicit, other (combination of the above)
Treatment of convective terms	Dependent variables
C - central differencing U - upwind differencing D - artificial diffusivity H - hybrid (combination of any of the above) Q - quadratic upwind K - skew upwind	J - dependent variable solved for simultaneously K - dependent variable solved for separately
Conserved quantities	Pressure determined by
F - mass, momentum (and/or) energy N - non-conservative or mass only conserved	Q - Poisson equation C - corrective algorithm equivalent to Poisson equation E - equation of state
Treatment of difficult boundary geometries	Iteration method
X - asymmetric "stars" at the boundary B - boundary-fitted coordinate system (may include others) T - coordinate transformations	P - point substitution L - line substitution M - matrix inversion A - ADI
	Relaxation
	R - over relax U - under relax

APPENDIX A

QUESTIONNAIRE EMPLOYED TO COMPILE COMPUTATION METHODS AND NUMERICS

Comment

The material on morphology presented above was constructed iteratively by the authors. An initial scheme was prepared from preprints and submitted to computors. Based on the computors' comments an improved scheme was proposed as a questionnaire. The questionnaire was circulated during the 1981 meeting, and the results used to describe each turbulence method and numerical scheme.

A copy of the questionnaire follows. Notice that the questionnaire includes information of some types not covered in the morphology described above.

Group # ___ Method Name _____

TAXONOMY QUESTIONNAIRE

Please check descriptions which apply to your program. If none of the categories fits your method, write a description in the place provided at the bottom.

Table A.1

Check appropriate level at left and go to table indicated at right.

CLASSIFICATION OF METHODS FOR COMPUTING TURBULENT FLOWS

Level	Method
___ 1	Correlations
___ 2	Integral Methods (Go to Table 2)
___ 3	One-Point Closures (Go to Table 3)
___ 4	Two-Point Closures (Go to Table 4)
___ 5	Large Eddy Simulation
___ 6	Full Simulation

Table A.2

CLASSIFICATION OF INTEGRAL METHODS

(Two-character descriptor; check one)

Energy Integral Equation		EE	___
Moment of Momentum Equation		ML	___
Shape Factor	No Lag	HN	___
	Lag	HG	___
Entrainment Correlation			
Shear Stress	No Lag	SN	___
	Lag	SG	___
Other (describe)			___

Table A.3

CLASSIFICATION OF ONE-POINT CLOSURES

A. Treatment of Reynolds Stresses (check one and proceed to appropriate section)

Boussinesq (eddy viscosity)	B	___	(Go to Section B)
Algebraic	A	___	(Go to Section B)
Differential	D	___	(Go to Section C)
Reynolds stress	RS	___	(Go to Section D)
Other (Give as complete a description as possible)	O	___	

B. Sub-Classifications for Classes B, and A Above (check appropriate one and proceed to Section E)

Prescribed	O	___
ℓ from ODE	O	___
k equation, ℓ prescribed	1K	___
k equation, ℓ from ODE	1L	___
ℓ prescribed, ω	1W	___
two-equation model:		
k, ϵ	KE	___
k, ω	KW	___
k, ℓ	KL	___
$k^2\ell$, ϵ	QE	___

C. Subclassifications for Class D Above

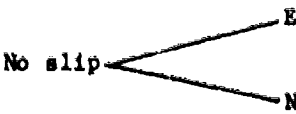
k, ϵ, $\overline{uv}$, $\overline{vw}$	4E	___
ϵ, $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$	5E	___
$k^2\ell$, $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$	5Q	___

D. Classification for Reynolds Stress Models RS Above (check one and proceed to Section E)

No additional PDE	T	___
Additional PDE for ϵ	E	___
Additional PDE for ℓ	L	___

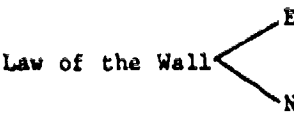
E. Special Effects

1. Treatment Near Solid Walls



X ___

N ___



Y ___

Z ___

Not applicable (free shear flows only) C ___

Other (please describe) O ___

2. Extra Rates of Strain

Are there extra terms (other than those derived directly from the Navier-Stokes equations) that are designed to account for:

Curvature ___
Lateral divergence ___
Rotation ___
Roughness ___
Blowing ___
Suction ___
Pressure gradient ___
Other (please list) ___

Table A.4

TWO-POINT CLOSURES

Direct Interaction Approximation	2DI
Eddy Damped Quasi-Normal Hypothesis	2ED

Table A.5

NUMERICAL METHOD TAXONOMY

A. Overall Strategy

Zonal (separate methods in different regions; matched or patched)	Z	___
Global (single method for entire flow)	G	___

Note: If zonal, we shall assume that you have a turbulent region and a potential flow region. If this is so, complete the remainder for each region, carefully denoting the region to which the description applies.

B. PDE Type

Parabolic (equations solved by marching in streamwise direction)	P	___
Elliptic (iterative method required)	E	___
Hyperbolic	Y	___
Mixed (specify)	M	___

C. Type of Discretization

Finite difference--regular grid	R	___
Finite difference--staggered grid	T	___
Finite volume--regular grid	V	___
Finite volume--staggered grid	S	___
Finite element	L	___
Hybrid	H	___
Other (specify)	O	___

D. Differencing of Convective Terms

Central	C	___
Upwind standard	U	___
Upwind--skewed	K	___
Rotated	A	___
Explicit artificial viscosity	D	___
Hybrid (specify)	H	___
Other (describe)	O	___

E. Formal Order of Accuracy (convective terms)

First	1	___
Second	2	___
Third	3	___
Fourth	4	___
Mixed (specify)	M	___
Other (specify)	O	___

F. Formally Conserved Quantities

Mass only	Q	___
Mass and momentum	N	___
Mass, momentum, and kinetic energy	J	___
Mass, momentum, and total energy	W	___
Other (specify)	O	___

G. Treatment of Difficult Boundary Geometries

Asymmetric stars	X	___
Boundary-fitted coordinates	B	___
Coordinate transformations	F	___
Non-applicable (simple geometry only)	*	___
Other (specify)	O	___

H. Flow Type

Compressible	C	___
Incompressible	I	___

I. Strategy

Explicit, time-like	X	___
Explicit, not time-like	E	___
Implicit, time-like	T	___
Implicit, not time-like	M	___
Semi-implicit	S	___
Other (specify)	O	___

J. Tactics

Point substitution	P	___
Line substitution	L	___
Direct matrix inversion	D	___
Split (ADI-like)	A	___

IF METHOD IS FOR COMPRESSIBLE FLOWS, SKIP TO SECTION M.

K. Pressure (Incompressible only)

Poisson equation	Q	___
Artificial compressibility	F	___
Other form of pressure correction	O	___

L. Relaxation (Incompressible only)

Under-relaxation throughout	U	___
Over-relaxation throughout	R	___
Variable relaxation	V	___
Other (specify)	O	___

- - - -

INTEGRAL TECHNIQUES

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INTRODUCTION

Among the variety of calculation methods for turbulent flows, none is able to give universal answers to the wide range of problems to be solved. Conceptually, the solution of the Navier-Stokes equations should give the solution of all problems. However, their use is not well adapted to practical situations, or even to physical understanding of phenomena. Therefore, every calculation method deserves to be considered. One of the aims of this meeting is to define the approximate field of application of various methods.

In addition, for engineering calculations, the important features of a method are its speed and ease of use for the required accuracy. Hence, it is not paradoxical to observe that on one hand, people are trying to develop sophisticated methods such as large-eddy simulation, and at the same time others (or the same people) are working on simple methods such as integral procedures. This is not a contradiction. On the contrary, the simpler and the more sophisticated methods are complementary.

Let us remark that the conclusions of the Evaluation Committee in the 1968 AFOSR-IFP-Stanford Conference specifically say that well-developed integral procedures for boundary layers are just as accurate as differential methods. It is true that most differential methods presented in 1968 were in their first stages of development. There is a tendency to assume, however, that differential methods are automatically more accurate than integral procedures, since differential procedures involve more complex modeling and have more flexibility to compute different turbulent quantities. What is sometimes forgotten is that the differential procedures often demand as input to the modeling data that are difficult to obtain, and hence scarce and often inaccurate. The integral procedures, on the contrary, can be built on data that are easier to obtain, far more abundant and often subject to smaller uncertainties. Hence a degree of "fine tuning" in the modeling is possible in integral procedures that is difficult to imagine in differential procedures. It is true that this fine tuning restricts the range of application of any particular procedure. However, we must remember that the universality of procedures is definitely in doubt. If it turns out that we must ultimately use various procedures for various classes of problems in

order to achieve the requisite engineering accuracy, then this restriction of range may be more illusory than real.*

Since the 1968 AFOSR-IFP-Stanford meeting (Kline et al., 1968), all the methods have undergone modifications, improvements or extensions. This is true for turbulence modeling as well as for integral techniques. It can even be said that integral methods have been improved thanks to the progress of turbulence modeling.

In the present paper, we shall try to show how authors have constructed various integral techniques, and how some of them have connected these methods to the physics of the phenomena, through turbulence modeling. In particular, it will be shown that these methods have been extended and improved to take into account effects such as: wall curvature, free-stream turbulence, compressibility effects. Some integral methods are now able to treat three-dimensional boundary layers.

Another aspect of computational fluid dynamics has developed rather recently: the so-called viscous-inviscid interaction. This is a very important aspect for integral methods since they are readily included in an interaction process and give very efficient procedures. Calculations of the flow around airfoils have been performed for both incompressible and transonic flow using integral methods with excellent results.†

BASIC EQUATIONS

The applicability of integral methods, though restricted to thin shear layers (wall boundary layers, wakes, ...), is not limited to two-dimensional steady boundary layers. Three-dimensional thin shear layers and unsteady forced flows can be computed by integral procedures. However, such cases lie outside the scope of the meeting, and hence these problems will not be discussed. Effects of compressibility and extra rate of strain have been included in some integral methods, but for the sake of clarity, the basic ideas will be presented for incompressible flow.

The governing equations of an incompressible turbulent flow are the continuity equation and the Navier-Stokes equations. When these equations are averaged and the boundary-layer hypotheses are incorporated, one obtains

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\nu \frac{\partial U}{\partial y} - \overline{uv} \right) \tag{2}$$

$$0 = \frac{\partial P}{\partial y} \tag{3}$$

---

*[Ed.: This remark is prescient; see discussion beginning on p. 990.]

†[Ed.: See discussion by Melnik, p. 806.]

These simplified equations are somewhat restrictive because, for example, the normal stress terms have been dropped, and the normal pressure gradient has been assumed to be negligible. Such hypotheses are doubtful, in particular near regions of flow separation or for flows along curved walls. However, these effects, especially the effects of curvature on turbulence, are known to be important, and are accounted for in several integral methods.

The first problem that faces the developer of an integral method is the choice of equations. This problem is not specific to these techniques, and is encountered in a wider class of methods called methods of weighted residuals.

In an integral method, the partial differential equations are not solved as such. Instead simpler equations are obtained by multiplying Eqs. 1 and 2 by some function of U or y, and integrating with respect to y.

Nearly all methods use the von Kármán equation, which is the momentum equation integrated between $y = 0$ (the wall) and $y = \delta$ (the boundary-layer edge):

$$\frac{C_f}{2} = \frac{d\theta}{dx} + \theta \frac{H + 2}{U_e} \frac{dU_e}{dx} \tag{4}$$

Starting with a given distribution of U_e, the early methods solved Eq. 4 by assuming closure relationships for the skin-friction and the shape parameter.

A correlation for skin friction, such as the Ludwieg-Tillmann relation, has been often used:

$$C_f = 0.246\, R_\theta^{-0.268}\, 10^{-0.678H} \tag{5}$$

It is more difficult to give a law for the shape parameter. Obviously, a constant value of H is not correct. A relationship between H and the pressure gradient does not work very well because the shape parameter does not respond quickly to changes in the pressure gradient. To account for such history effects, most authors use an additional equation which, though sometimes called an auxiliary equation, is as important as the primary equation. Several options have been devised for this equation. The most obvious equation is the integrated form of the continuity equation:

$$\frac{d\delta}{dx} - \frac{V_e}{U_e} = \frac{1}{U_e} \frac{d}{dx} U_e(\delta - \delta^*) \tag{6}$$

This equation is often called the entrainment equation because it describes the rate at which the outer fluid is entrained into the boundary layer. Physically, this process is very important because it controls the growth of the boundary layer to a large extent.

The entrainment parameter $C_E = d\delta/dx - V_e/U_e$ requires implicitly a certain turbulence modeling. Michel et al. (1968) have shown that C_E is related to the ratio

$(\partial\tau/\partial y)/(\partial U/\partial y)$ at the edge of the boundary layer. To show this, let m be the slope of a line U/U_e = constant. Along such a line, the momentum equation is

$$\left(\frac{V}{U_e} - \frac{U}{U_e}\,m\right)\frac{\partial}{\partial y}\left(\frac{U}{U_e}\right) + \frac{1}{U_e}\frac{dU_e}{dx}\left[\left(\frac{U}{U_e}\right)^2 - 1\right] = \frac{\partial}{\partial y}\left(\frac{\tau}{\rho U_e^2}\right) \tag{7}$$

If the boundary-layer edge (where $U = U_e$) can be defined at a finite distance, δ, then Eq. 7 can be written on the line $y = \delta$, as

$$\frac{d\delta}{dx} - \frac{V_e}{U_e} = -\left\{\frac{\partial}{\partial y}\left(\frac{\tau}{\rho_e U_e^2}\right) \Big/ \frac{\partial}{\partial y}\left(\frac{U}{U_e}\right)\right\}_{y=\delta} \tag{8}$$

One of the most common objections to using the entrainment equation is that the boundary-layer thickness δ is ill defined. Many authors prefer to use another additional equation. The mean energy integral equation is often employed. This is obtained by first multiplying the momentum equation by U, and then integrating the resulting equation to obtain

$$\frac{d}{dx}\left(U_e^3\delta^{**}\right) = 2D^* \tag{9}$$

where

$$D^* = \int_0^\delta \left(-\overline{uv} + \nu\frac{\partial U}{\partial y}\right)\frac{\partial U}{\partial y}\,dy\ .$$

Although the mean kinetic energy equation and the von Kármán equation are derived from the same equation, they are independent (because the integration process leads to a loss of information).

The shear-work integral (or dissipation integral) expresses the deformation work of the Reynolds stress $-\rho\overline{uv}$ and of the viscous stress $\mu(\partial U/\partial y)$. The second term in the integrand of D^* represents the viscous dissipation of mean energy and is positive definite. The first term is generally positive, and is therefore likely to decrease the mean energy. This term also occurs in the turbulent kinetic energy equation (with an opposite sign), where it is called the production term. Indeed, this term is responsible for exchange of energy between the mean flow and the fluctuating flow.

Other types of additional integral equations can be used in place of an entrainment equation or mean energy equation. They can be obtained by multiplying the momentum equation by y^n before integration (if $n = 1$), the moment of momentum integral equation is obtained). In this case, some turbulence model is required because such equations involve an integral of the turbulent shear stress.

CLOSURE RELATIONSHIPS

The set of global equations formed, for example by the von Kármán equation and the entrainment equation, or by the von Kármán equation and the mean energy integral equation, contains more unknowns than equations. Therefore, closure relationships are needed.

Three types of relationships can be considered:

i) relationships among integral thicknesses or shape parameters;

ii) a skin-friction law;

iii) entrainment equation, or dissipation integral, or some other shear-stress integral.

Integral Thicknesses

Several methods have been used to obtain relationships between integral thicknesses. Felsch et al. (1968) have chosen a purely empirical approach. This is not the easiest method, because it requires considerable knowledge of the basic parameters. This technique also needs data in a fairly wide range of experimental conditions. Felsch et al. use an empirical relationship for the shape parameter $H^* = \delta^{**}/\theta$, which is assumed to be a function of the shape parameter $H = \delta^*/\theta$ alone.

A second way of specifying the closure relationships for integral thicknesses is to model the mean velocity profiles. A classic representation of velocity profile is the power law:

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{(H-1)/2} \tag{10}$$

Equation 10 is a rather crude representation. More elaborate models have been proposed on the basis of a physical understanding of the boundary layer. These models are derived from the decomposition of the boundary layer into an inner region and an outer region, between which a logarithmic overlap region exists; this property is certainly the key to the success of such representations. As noted by Kline et al. (1969), all successful methods in the 1968 AFOSR-IFP-Stanford Conference use one "wall-wake correlation" or equivalent information. This approach is supported by the method of matched asymptotic solutions.

Let us recall that in the inner region, it is assumed that the velocity scales on the friction velocity $U_* = U_e(C_f/2)^{1/2}$ and that the velocity profile follows a universal law:

$$U^+ = f(y^+) \ , \quad U^+ = \frac{U}{U_*} \ , \quad y^+ = \frac{yU_*}{\nu} \tag{11}$$

whereas in the outer region, the velocity defect law is used:

$$\frac{U_e - U}{U_*} = F'(\eta) \ , \quad \eta = \frac{y}{\delta} \tag{12}$$

Compatibility of the behavior of the velocity profile in the two regions leads to a logarithmic form in the overlap region, which in terms of inner variables is

$$U^+ = \frac{1}{\chi} \ln y^+ + C_1 \tag{13}$$

where χ and C_1 are universal constants (for example, $\chi = 0.41$, $C_1 = 5$), and which in terms of outer variables is

$$\frac{U_e - U}{U_\tau} = -\frac{1}{\chi} \ln \eta + C_2 \tag{14}$$

where C_2 is a constant depending on the pressure gradient.

As noted above, the existence of a logarithmic region is a very important property of turbulent boundary layers. Recent experiments tend to prove that the law of the wall is even more tenacious than previously believed. The experiments of East and Sawyer (1979) indicate that the law of the wall remains valid over a wide range of pressure gradients, whereas the experiments of Purtell et al. (1981) show that the constants C_1 and χ are independent of Reynolds number even at low Reynolds numbers ($R_\theta = 480$) for a flat-plate boundary layer.

In terms of physical distances, the inner region is very thin (if the Reynolds number is high enough). Therefore, a representation of the velocity defect law is sufficient to calculate the integral thicknesses. One of the well-known representations has been proposed by Coles (1956). This is based on Eqs. 12 and 14 as

$$F'(\eta) = -\frac{1}{\chi} \ln \eta + \frac{B}{\chi}\left(2 - \omega(\eta)\right) \tag{15}$$

The wake function, ω, can be approximated by

$$\omega = 1 - \cos(\pi\eta) \tag{16}$$

and B is a constant for equilibrium layers, but varies in non-equilibrium situations.

The form of function ω does not give zero slope at the edge of the boundary layer. Hence, other formulae have been proposed in order to insure that $(\partial U/\partial y) = 0$ at $y = \delta$. For this, an additional function must be introduced for representing the law of the wake (see, for example, Meier, 1976).

Other analytical forms have been proposed to represent the velocity profiles. A rather sophisticated formulation has been given by Whitfield (1980), which is valid over the whole thickness of the boundary layer. The representation is the sum of two transcendental functions, one expressed in terms of the inner variable y^+, and the other in terms of an outer variable y/θ. The resulting formulae can be used for computing attached and separated boundary-layer velocity profiles as well. In this representation, the velocity profiles depend on three parameters: the skin-friction coefficient, the Reynolds number based on the momentum thickness and the shape parameter. Nevertheless, in the method proposed by Whitfield, only two parameters are kept independent because a skin-friction law of the form $C_f = C_f(H, R_\theta)$ is used. Figure 1 shows the ability of the Whitfield profiles to represent a great variety of experimental velocity profiles.

Finally, a third method consists of analyzing the properties of a family of self-similar solutions (equilibrium boundary layers). Such solutions have been studied by Mellor and Gibson (1963). Later, Michel et al. (1968) used them in a systematic way to develop an integral method. These methods use the idea that any such relationships between certain integral thicknesses are more applicable in equilibrium boundary layers.

The principle of these solutions is to assume that for equilibrium flows the velocity-defect profiles are a function of η alone, and not of η and x. This allows the partial differential equations to be transformed into ordinary differential equations. In addition, the assumption of high Reynolds number leads one to consider the skin friction as a small parameter. The momentum equation then becomes

$$\frac{\tau}{\tau_w} = 1 - \frac{F}{F(1)} + P\,\eta\,F' \tag{17}$$

where

$$F' = (U_e - U)/U_*$$

$$F(\eta) = \int_0^{\eta} F'(\bar{\eta})\, d\bar{\eta}$$

$$P = 2\bar{\beta} + 1/F(1)$$

$$\bar{\beta} = -\frac{\delta}{U_*}\frac{dU_e}{dx}$$

If a turbulence model is provided (Michel et al. (1968) used a mixing-length model), Eq. 17 can be solved for a given value of the pressure gradient parameter $\bar{\beta}$. This produces a one-parameter family of velocity profiles which can be used in an integral method.

Instead of characterizing each velocity profile by the value of the pressure-gradient parameter, Michel et al. preferred to use the Clauser shape parameter G defined as

$$G = \frac{\int_0^1 F'^2\, d\eta}{\int_0^1 F'\, d\eta} \tag{18}$$

In this method, it is assumed that even for non-equilibrium boundary layers, a velocity-defect profile can be characterized by the value of its Clauser parameter. In other words, it is assumed that for a given value of G, a velocity-defect profile has the same form in equilibrium or non-equilibrium flows.

From the definitions of G and F(1), a relationship for $H^{**} = (\delta - \delta^*)/\theta$ is deduced:

$$H^{**} = H\left(\frac{G}{F(1)}\frac{H}{H-1} - 1\right) \tag{19}$$

where F(1) can be determined as a function of G from the solution of Eq. 17.

Yet another way of obtaining closure relationships is to analyze the properties of results obtained from a numerical solution of the partial differential equations. If a turbulence model is sufficiently accurate, it could be used to generate solutions for certain flows, in equilibrium or not, and their properties could be introduced in integral methods. This approach does not seem to have been investigated thoroughly.

Skin-Friction Law

The skin-friction law is obtained either empirically or deduced from the velocity-profiles representation. The most famous empirical skin-friction law is due to Ludwieg-Tillmann (Eq. 5). This law is valid for a wide range of attached boundary layers. Alternative forms of this law have also been proposed. For example, Felsch et al. (1968) use

$$C_f = 0.058\ R_\theta^{-0.268}\ (0.93 - 1.95\ \log_{10} H)^{1.705} \tag{20}$$

which gives $C_f = 0$ for $H = 3$. Many other laws are available (i.e., Green et al., 1972; Whitfield et al., 1980).

Another approach for obtaining a skin-friction law is to employ a velocity-profile representation. From definition, the skin friction should be deduced from the slope of the velocity profiles at the wall. This procedure, however, needs a very accurate representation. In addition, most of the models assume a universal form near the wall $U^+ = f(y^+)$ which reduces to $U^+ = y^+$ at the wall; this is an identity which cannot give the skin friction. Instead, the skin friction is derived from the overlap between the law of the wall and the velocity-defect law. Compatibility of Eqs. 14 and 13 leads to:

$$\left(\frac{C_f}{2}\right)^{-1/2} = \frac{1}{\chi} \ln \frac{U_e\ \delta}{\nu}\left(\frac{C_f}{2}\right)^{1/2} + C_1 + C_2 \tag{21}$$

The constant C_2 can be specified as a function of some shape parameter if the law of the wake is known. For example, Coles' law of the wake gives $C_2 = 2B/\chi$ (Eq. 15 with $n \to 0$), and B can be related to δ^*/δ by integrating Eq. 15 between $n = 0$ and $n = 1$:

$$\frac{\delta^*}{\delta}\left(\frac{C_f}{2}\right)^{-1/2} = \frac{1}{\chi} + \frac{B}{\chi} \tag{22}$$

Substitution of Eq. 22 into Eq. 21 gives an implicit equation for the skin friction:

$$\left(\frac{C_f}{2}\right)^{-1/2} = \frac{1}{\chi} \ln R_{\delta^*} + C_1 + \frac{1}{\chi} \ln \frac{\delta}{\delta^*}\left(\frac{C_f}{2}\right)^{1/2} + 2\left[\frac{\delta^*}{\delta}\left(\frac{C_f}{2}\right)^{-1/2} - \frac{1}{\chi}\right] \tag{23}$$

with $C_1 = 5$ and $\chi = 0.41$.

In Eq. 23, δ^*/δ can be related to the classic shape parameter H by using Coles' velocity profile (Eq. 15). An analytical representation has been proposed in Bardina et al. (1981):

$$\frac{H-1}{H}\frac{\delta}{\delta^*} = 1.5 + 0.179\,\frac{(C_f/2)^{1/2}}{\chi}\,\frac{\delta}{\delta^*} + 0.321\left(\frac{(C_f/2)^{1/2}}{\chi}\,\frac{\delta}{\delta^*}\right)^2 \qquad (24)$$

In fact, Eqs. 23 and 24 constitute a skin-friction law of the form $C_f = C_f(R_\theta, H)$.

A similar approach has been used by Michel et al. (1968). Instead of using Coles' velocity profile, they use the solution of the self-similarity equation (Eq. 17).

A comparison between Ludwieg-Tillmann law and the law resulting from Eqs. 23 and 24 is shown in Fig. 2 for $R_\theta = 10^4$. The two laws give nearly the same results, except that Ludwieg-Tillmann law indicates a zero skin friction when H tends to infinity, whereas the other law gives $C_f = 0$ when $H = 4$.

In Fig. 3, Ludwieg-Tillmann law is compared with all skin-friction values available in Vol. II of the 1968 AFOSR-IFP-Stanford Conference. These values have been evaluated by using the Clauser plot technique. The least that can be said is that the correlation is very good. The few erratic points indicated S5 and S6 come from the Stratford experiments, and correspond to velocity profiles for which the standard law of the wall is not valid at all. The other points which are not correlated with the Ludwieg-Tillmann law correspond generally to low Reynolds numbers and high values of H. In these cases, it is suspected that transition has not been completed. A number of other skin-friction laws have been proposed. The refinements which have been carried out lead to a scatter less than that observed for the Ludwieg-Tillmann law (see, for example, Lyrio et al., 1981)

To conclude, it appears that a consensus has been reached for the skin-friction law. If efforts have to be made for improving integral methods, they do not concern the skin-friction law, except perhaps at low Reynolds number.

Entrainment, Dissipation Functions, History Effects

From definition, the dissipation function involves the evaluation of a shear-stress integral. From Eq. 8, the entrainment function is related to the behavior of the shear stress near the boundary-layer edge. Hence, these functions imply, implicitly or explicitly, the use of a turbulence model.

For example, in the method proposed by Whitfield (1980), the shear-work integral is calculated by dividing the boundary layer into three regions. In the inner part, the total shear stress is assumed to be constant and equal to its value at the wall; in the middle region, the turbulent shear stress is calculated by means of a mixing-length formula; and in the outer region, a constant-eddy viscosity of the form

suggested by Clauser is used:

$$\nu_T = 0.0168\ \delta^*\ U_e \tag{25}$$

In many methods, the entrainment coefficient C_E or the dissipation integral D^* are estimated with reference to equilibrium boundary layers. For such boundary layers, the x-dependence is eliminated, so the integral equations give algebraic relationships between C_E or D^*, and the boundary-layer characteristics. This process has been used by Michel et al. (1968) and by Green et al. (1972) to get an expression for the entrainment coefficient.

In certain circumstances, history effects can be very important, for example in flows which have first increasing and then decreasing positive pressure gradient. Such flows are rather difficult to calculate (whereas flows with positive and always increasing pressure gradients are easier).

It has been argued that the flow near the wall adjusts very rapidly to changes in some parameter, for example the pressure gradient (see Tani, 1968). On the other hand, the outer layer, which is dominated by large eddies, has a large inertia, and does not respond instantaneously to external variations. This physical idea led some investigators to the use of a lag-equation for calculating the entrainment coefficient or the dissipation integral. This has been done by Felsch et al. (1968), who derived an empirical relationship for D^* which takes into account the effects of history and of non-equilibrium.

These effects are introduced in the relationships connecting the Clauser parameter G and a pressure gradient parameter β:

$$\beta = -\frac{2\delta^*}{C_f}\frac{1}{U_e}\frac{dU_e}{dx}$$

For a given equilibrium boundary layer, each of these parameters is constant. Hence, for equilibrium boundary layers an equilibrium locus (a curve $G(\beta)$) can be drawn. In non-equilibrium flows, the representative point $G(\beta)$ is generally not on the equilibrium locus. This difference is used to characterize the deviation from equilibrium.

To model these effects, Green et al. have used as guidelines the kinetic-energy equation, as modeled by Bradshaw and Ferriss (1968), from which they inferred a lag-entrainment equation. Essentially, the effects of history are included through the difference between the actual pressure gradient and a fictitious equilibrium pressure gradient, which would give the same boundary-layer characteristics.

The use of a lag equation introduces problems in specifying initial conditions if a calculation is started in a zone deviating considerably from equilibrium. Nevertheless, this should not be a serious problem because the situation is very rare in practice.

Extensions--Extra Rate of Strain

One of the major philosophical differences between an integral method and a field method is that the former starts with a simple situation and tries to go to more complicated ones, whereas the latter is devised to deal with the more complex situations, and should work for the simpler ones. In fact, as discussed in the Introduction, this is not automatically true, and it is often observed that turbulence models need modifications to treat complex flows.

Several integral methods have been extended to calculate compressible flows with or without wall heat flux, and to calculate three-dimensional boundary layers. In addition, some of the extra rates of strain listed by Bradshaw (1973) have been incorporated in integral methods; in particular, the strong effects of wall curvature have been taken into account. The effects of free-stream turbulence have also been included.

Many of the integral methods are able to represent compressibility effects, at least for flows over adiabatic walls. For example, in the method developed by Whitfield et al. (1980), it is assumed that the velocity profiles can be described by the same representation as in incompressible flow. This means that the velocity profiles are characterized by incompressible integral thicknesses, and corresponding compressible integral thicknesses are obtained by using a relation between velocity and temperature. The compressible skin friction is related to the incompressible skin friction coefficient, and this has been done by Winter-Gaudet (1970). This latter coefficient is calculated by using an incompressible formula.

In the method proposed by Cousteix et al. (see for example Cousteix, 1981), the effects of compressibility, three-dimensionality, wall curvature, and free-stream turbulence have been taken into account in a systematic way using the properties of self-similar solutions. Generally, the self-similarity is only approximate in the sense that all the parameters which must be constant to achieve rigorous self-similarity, are not completely constant. Nevertheless, these solutions are very useful in understanding the role of the principal parameters on the development of the boundary layer.

These solutions allow calculation of families of velocity (and enthalpy) profiles, the properties of which are used to form the closure relationships needed for the solution of global equations. To calculate these self-similar solutions, a mixing-length model is used; this scheme has been adapted to determine the effects of wall curvature and free-stream turbulence.

Often the effects of extra rate of strain can be incorporated by modifying the entrainment function or the dissipation integral. This is the case, for example, when dealing with wall-curvature effects. A first approximation consists of assuming that the velocity profiles can be represented by flat-wall velocity profiles. For this, it

is convenient to consider a velocity defect ($U_p - U$) where U_p is a boundary-layer potential velocity, defined as the extension of the external flow velocity within the boundary layer.

Indeed the main effects of wall curvature, at least in the case of convex walls, lie in the outer region of the boundary layer which affects the entrainment process and also the shear-stress distribution; it thus also affects the shear-work integral. These properties are corroborated by the self-similar solutions developed by Cousteix and Houdeville (1977). The entrainment coefficient was accordingly modified in their method. The skin-friction law is only slightly affected by the curvature. Figure 4 shows the variations of the entrainment coefficient which decreases with positive curvature (convex wall). Hence, the boundary layer extracts less energy from the external flow, and the general level of boundary-layer turbulence decreases; consequently the boundary layer becomes more sensitive to pressure gradients.

In the method of Green et al. (1972), the effects of wall curvature or of other extra rates of strain have been included by modifying the entrainment coefficient. The effects of these parameters have been derived through a modification of the dissipation length in the turbulent kinetic-energy equation, which affects the lag entrainment equation. Indeed, one of the main virtues of this procedure is the use of the turbulent kinetic-energy equation. It provides a guide to the modifications of turbulence structure which affect the entrainment coefficient. It should be remembered, however, that modifications of the dissipation length must be introduced in much the same way as the mixing-length model is modified by Cousteix and Houdeville (1977). In any case, this procedure certainly has advantages compared to a fully empirical approach.

Let us note that the same improvements can be introduced in the shear-work integral. In fact, it seems that the effects of extra rates of strain can be incorporated at least cost, by evaluating this integral by means of a flat-wall velocity-profile family and a modified turbulence model.

The effects of free-stream turbulence can be represented by a variation of the entrainment coefficient, which increases with the level of turbulence (Fig. 5). In fact, the level of turbulence is not the only parameter: the length scale of free-stream turbulence should also be taken into account. In the analysis proposed by Arnal et al. (1976), this length scale is implicitly assumed to be on the order of the boundary-layer thickness.

However, as discussed by Bradshaw (1980), it is very difficult to dissociate the effects of intensity and of length scale experimentally, because there is a strong tendency for intensity and length scale to vary together. In the usual range of experimental conditions, the length scale of free-stream turbulence is often on the order of the boundary-layer thickness (Cousteix, 1981).

As a first approximation, the skin-friction law is not significantly altered by the effects of free-stream turbulence. Figures 6 and 7 show a comparison of a classic skin-friction law $C_f = C_f(R_\theta, H)$, with experimental data obtained in the case of a boundary layer perturbed by free-stream turbulence. The correlation is satisfactory, even when the intensity of free-stream turbulence is as large as $(\overline{u^2})^{1/2}/U_e \approx 6.5\%$ (Charnay et al., 1971). At very high levels of free-stream turbulence (above 20% or so), Lisin et al. (1960) have shown that there is some effect on the skin-friction law. However, this work was not completed and the uncertainties are a bit high.

A last extension of integral methods should be mentioned for the calculation of wakes; this has been done, for example, by Green et al. (1972). Often, the hypotheses are rather crude, especially in the vicinity of the trailing edge; the effects of asymmetry are often represented in an approximate manner. These methods are very useful because airfoil calculations are more and more taking into account the effects of wakes. A method of wake calculation has also been extended to three-dimensional wakes (Cousteix et al., internal report).

Use of Integral Methods in Viscous-Inviscid Interaction Techniques

An important field of applications of integral methods lies in their use in viscous-inviscid interaction techniques. Several reasons can be given for such a choice. First, when an iteration process is involved, appreciable time can be gained compared to the use of a field method. Moreover, it should be remembered that, when designing for example an airfoil, a great number of calculations are performed. Second, the numerical techniques for integral methods are significantly simpler than those for field methods; this can be an advantage when a boundary-layer calculation is a part of a very complex numerical technique.

In any case, even if the ultimate goal is the use of a field method with a sophisticated turbulence model, the use of an integral method can be justified as a first step to elaborate the interaction technique and as a useful tool for the simpler cases. When the flow is really complex, the validity of interaction techniques can be questioned. At a certain level of sophistication of the approximation, it may be more reasonable to solve the full time-averaged Navier-Stokes equations.

The simplest viscous-inviscid interaction techniques consist of dissociating the viscous- and inviscid-flows calculations. Strictly speaking, the weak interaction involves an inviscid-flow calculation which gives the external conditions for the shear-flow calculation; the inviscid flow is then calculated again by using the concepts of displacement surface or wall transpiration. In the former case, a slip condition is applied along the body surface altered by the displacement thickness. In the latter case, the matching condition is applied at the wall where a normal velocity is prescribed as a boundary condition for the inviscid flow. This normal velocity is

$$V_w = \frac{d(U_e \delta^*)}{dx} \tag{26}$$

In both cases, an integral method can be used to give the inviscid-flow boundary condition. Sometimes, this technique needs improvements because the interaction is not really weak. Then, an iterative process is used. This is encountered when the boundary layer becomes relatively thick and, therefore, more sensitive to pressure gradients. The iterative process can then be unstable or very difficult to stabilize.

When separation occurs, additional problems arise. First, the equations become elliptic, hence disturbances are propagated upstream by the reverse flow and downstream by the main flow. Second, the roles of the viscous and inviscid flows are inverted in the sense that the pressure gradient is controlled by the boundary layer (a small change of pressure gradient corresponds to a large change of the displacement thickness). In addition, a mathematical problem can arise in such regions for integral methods as well as for partial differential equations.

To show this, let us consider an integral method based on the von Kármán equation, and on the entrainment equation. The system to be solved is:

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta \frac{H + 2}{U_e} \frac{dU_e}{dx} \tag{27}$$

$$\frac{d(\delta - \delta^*)}{dx} = C_E - \frac{(\delta - \delta^*)}{U_e} \frac{dU_e}{dx} \tag{28}$$

Let us assume that $H^{**} = (\delta - \delta^*)/\theta$ is a function of H alone (as this is done in the Green et al. method). Then Eqs. 27 and 28 become

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta \frac{H + 2}{U_e} \frac{dU_e}{dx} \tag{29}$$

$$\frac{dH^{**}}{dH} \frac{d\delta^*}{dx} + \left(H^{**} - H \frac{dH^{**}}{dH}\right) \frac{d\theta}{dx} = C_E - \frac{(\delta - \delta^*)}{U_e} \frac{dU_e}{dx} \tag{30}$$

which is a system of two equations for two unknowns $d\delta^*/dx$ and $d\theta/dx$.

This system can be solved, except when the determinant of the system is zero. Generally, this is encountered at a point $H = H_c$ defined by the condition $dH^{**}/dH = 0$. Then, the system is either impossible or indeterminate. In practice, the latter possibility is most improbable because the velocity distribution must be such that

$$H^{**} - H \frac{dH^{**}}{dH} = \left(C_E - \frac{\delta - \delta^*}{U_e} \frac{dU_e}{dx}\right) \Big/ \left(\frac{C_f}{2} - \theta \frac{H + 2}{U_e} \frac{dU_e}{dx}\right) \tag{31}$$

which is rarely encountered if the velocity distribution is prescribed.

Hence, the system is generally impossible in the sense that it leads to an infinite value of $d\delta^*/dx$. Moreover, a study of the nature of the equations in unsteady

flow clearly shows that when H becomes greater than H_c, the information is propagated both upstream and downstream. Therefore, it seems logical to associate the singular point with the point where the time-averaged skin friction is zero or $\overline{C_f} = 0$. This allows the system to be elliptic when the skin friction is negative. However, the beginning of downstream influence on the upstream is not defined by the point $\overline{C_f} = 0$. Indeed, upstream of the point $\overline{C_f} = 0$, the flow goes forward a certain fraction of the time. Therefore, as discussed by Kline et al. (1981a), the singularities should be associated with a point of incipient downstream to upstream influence, rather than with the point of full detachment defined by $\overline{C_f} = 0$.

Let us note that in an integral method working with the mean-energy equation and the von Kármán equation, the same problem can be encountered when $dH^*/dH = 0$.

Related to this mathematical singularity, there is a numerical difficulty. A small change in some parameter near the point $H = H_c$ leads to large changes in the solution. It is well known that the solution is very sensitive to pressure gradient for example, but also the numerical method can strongly affect the results.

In several integral methods, this problem is apparently removed because the singularity does not occur in the range $1 < H < \infty$ (Fig. 8). In fact, in all the cases, the slope dH^{**}/dH (or dH^*/dH) becomes very small when the shape parameter H increases, and numerical problems arise. When H becomes large, say 3 or 4, the slope dH/dx becomes very large if not infinite. Therefore, in practice, the problem is not avoided.

In this situation, a strong interaction with the inviscid flow is needed. This does not necessarily mean that the boundary-layer equations are no longer valid, but it does imply that rigorous mathematical and numerical compatibility of the boundary conditions applied to the viscous and inviscid flows is required (Le Balleur, 1980). Nevertheless, the concepts of displacement surface or of wall transpiration can still be used. Improvements can be incorporated in the calculation of the viscous flow by using equations for the difference between the velocity and an inviscid velocity, defined within the boundary layer (Le Balleur, 1980).

These techniques of strong interaction require an inverse procedure for solving the boundary-layer equations. In this case, the external velocity is no longer prescribed for the boundary-layer set of equations: U_e becomes an unknown of this system. The prescription of U_e is replaced by the coupling equation. This is equivalent to saying that the displacement thickness (or the wall transpiration) are prescribed data for the boundary-layer problem.

It can easily be checked that the system of equations in the inverse mode is not singular when $H = H_c$. Also, the numerical difficulties in the neighborhood of this point, or when dH^{**}/dH (or dH^*/dH) becomes small, are avoided.

Incidentally, this inverse mode sometimes is called the design mode because it is used for designing a body shape with an optimization condition. In this case, the

boundary-layer equations give the external velocity distribution, and the body shape results from an inverse inviscid-flow calculation.

Several techniques of strong interactions using integral methods have been developed, and have been used for this meeting. In the method developed by Le Balleur (1980), special care has been taken in the relaxation-type iterative procedure. In particular, the relaxation coefficients are calculated. This gives a great robustness to the method and leads to time saving. In addition, a special treatment for strong interaction has been elaborated in the neighborhood of shock waves, which is very important for transonic airfoils calculations.

Melnik (1980) has also worked on strong interaction with an integral method (Green et al., 1972), but in this case a special emphasis has been placed on the trailing edge strong interaction, which is treated in an elaborate manner.

The method developed by Moses et al. (1978) is based on the simultaneous solution of the inviscid flow and the boundary layers using successive line relaxation. The boundary layer is calculated with an integral technique using the momentum and kinetic-energy integral equations. Simple correlations based on the logarithmic velocity profiles are used for the skin friction, the energy-dissipation integral, and the shape parameter H^*. These correlations are extended to include approximations for separated flow, which result in the limiting case of a free-shear layer. The essential feature of the method is the simultaneous solution of the inviscid flow and the boundary layers, allowing to make stable calculations with separated regions.

CONCLUSION

The integral methods for calculating boundary layers were first designed for hand calculations, and very crude approximations were therefore usually introduced. Since that time, these methods have been substantially improved. The 1968 AFOSR-IFP-Stanford Conference demonstrated that a large number of practical situations can be computed with these techniques. Essentially, the test cases of the 1968 Conference dealt with the effects of pressure gradients. Since then, the methods have been improved by including the effects of history, compressibility, three-dimensionality, extra rates of strain, wall curvature, etc.

Undoubtedly, refinements in integral methods are still needed. However, paths are open for taking these effects into account. It seems therefore that these methods are well adapted to calculate a wide variety of boundary-layer flows. Over the last few years, an important effort has been devoted to developing sophisticated viscous-inviscid interaction techniques; in these techniques integral methods have been extensively employed because they are easy to use and little computation time is required.

It seems, therefore, that these methods will continue to play an important role in computation fluid dynamics and that, for many practical problems, they offer a solution of good accuracy and reasonable speed.

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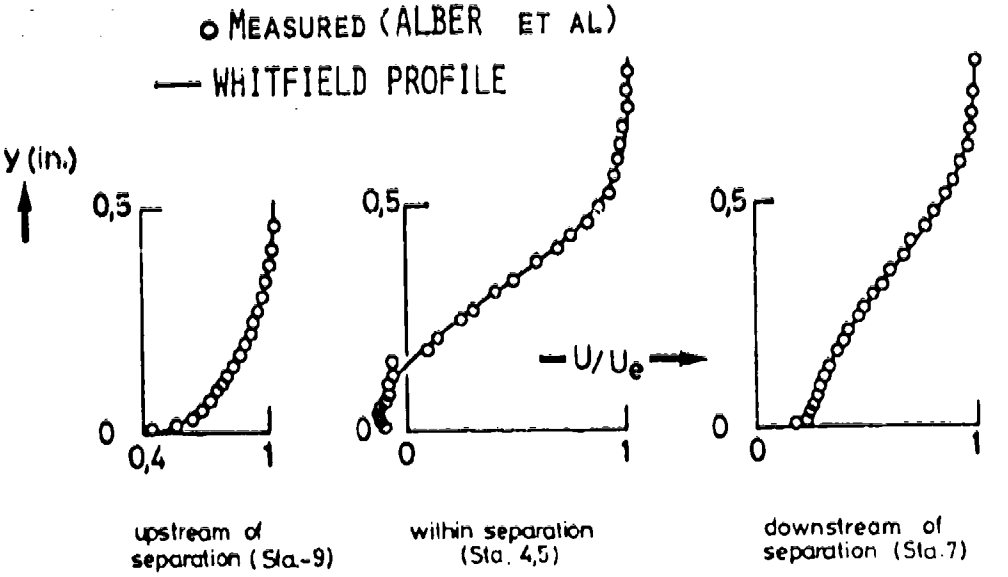

Figure 1. Comparison of measured attached and separated boundary-layer profiles with Whitfield et al. (1980) profiles.

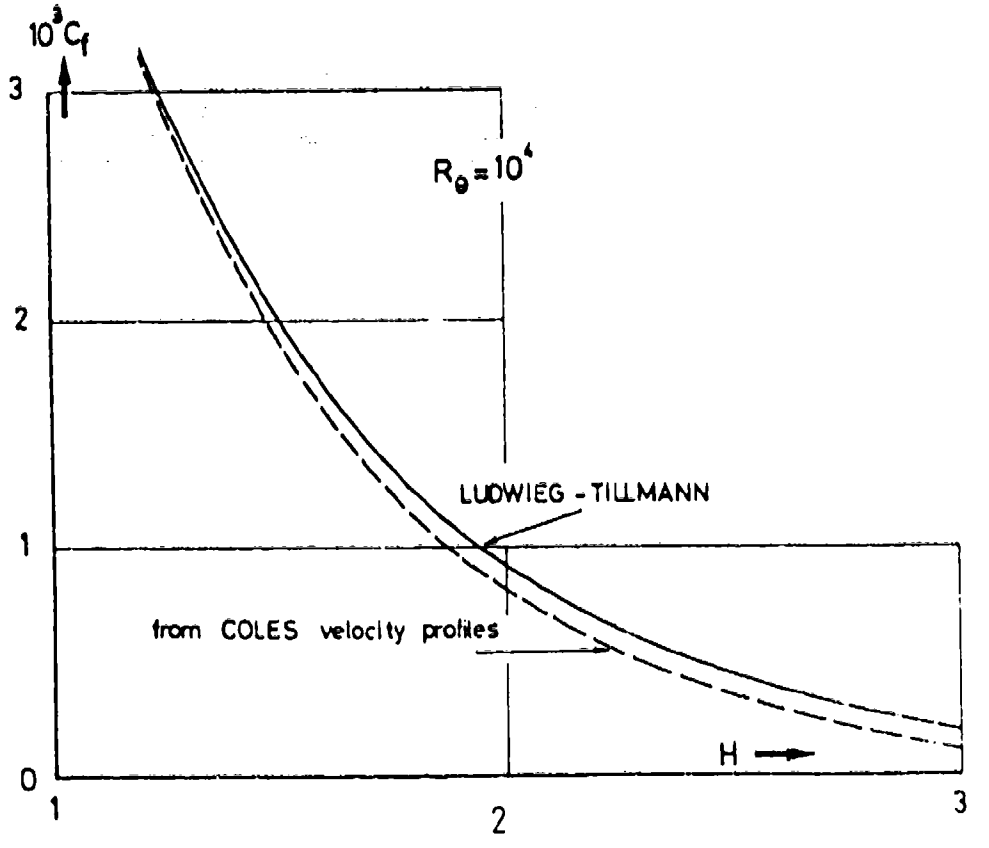

Figure 2. Comparison of two-skin friction laws.

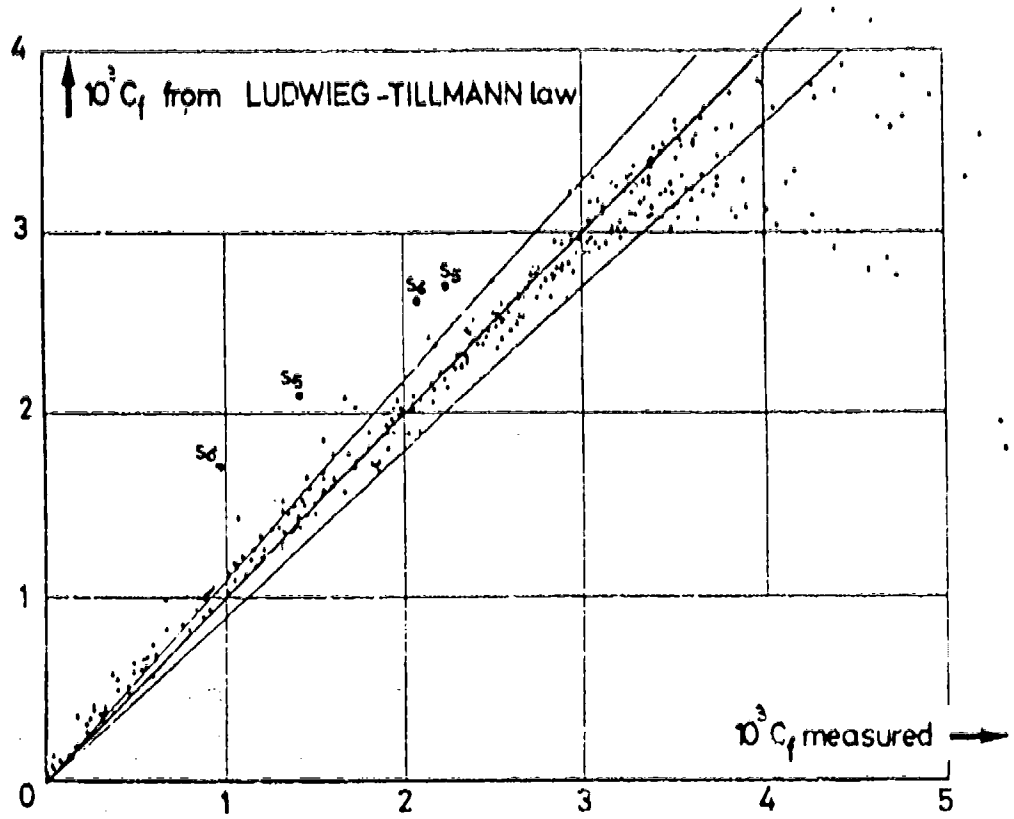

Figure 3. Comparison of Ludwieg-Tillmann skin-friction law with all the data of the 1968-AFOSR-IFP-Stanford Conf.

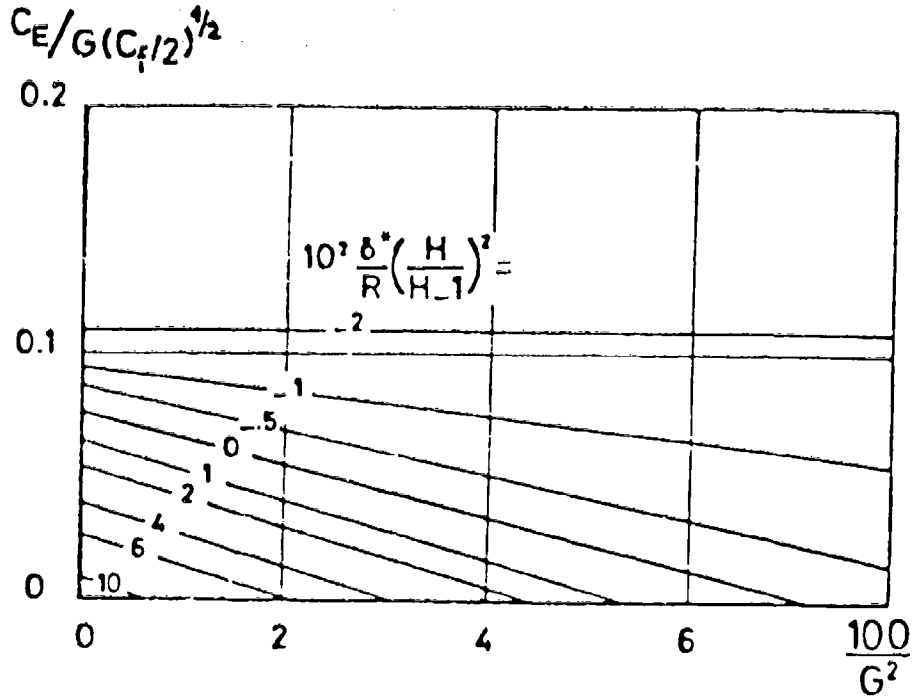

Figure 4. Effect of wall curvature on the entrainment coefficient (Cousteix and Houdeville, 1977). R = radius of curvature.

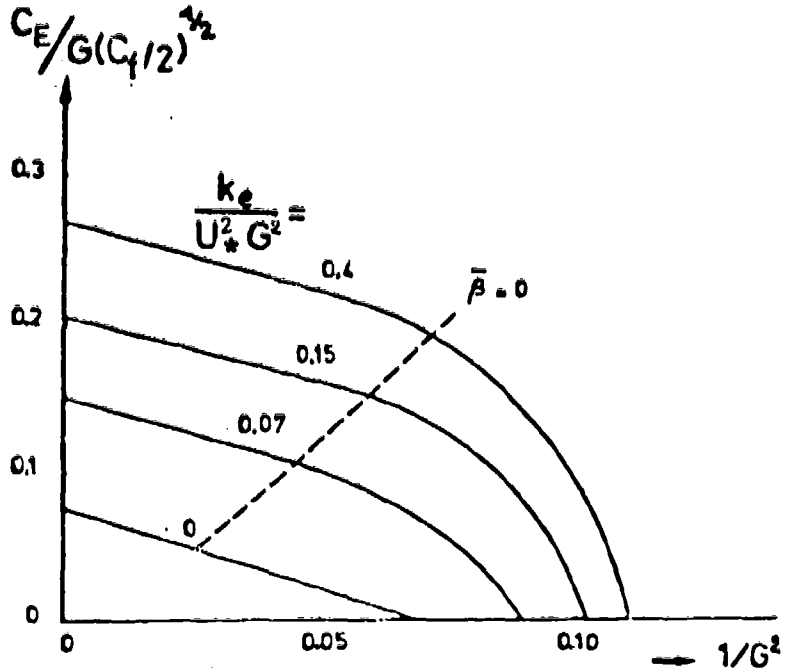

Figure 5. Influence of free-stream turbulence on the entrainment coefficient (Arnal et al., 1976).

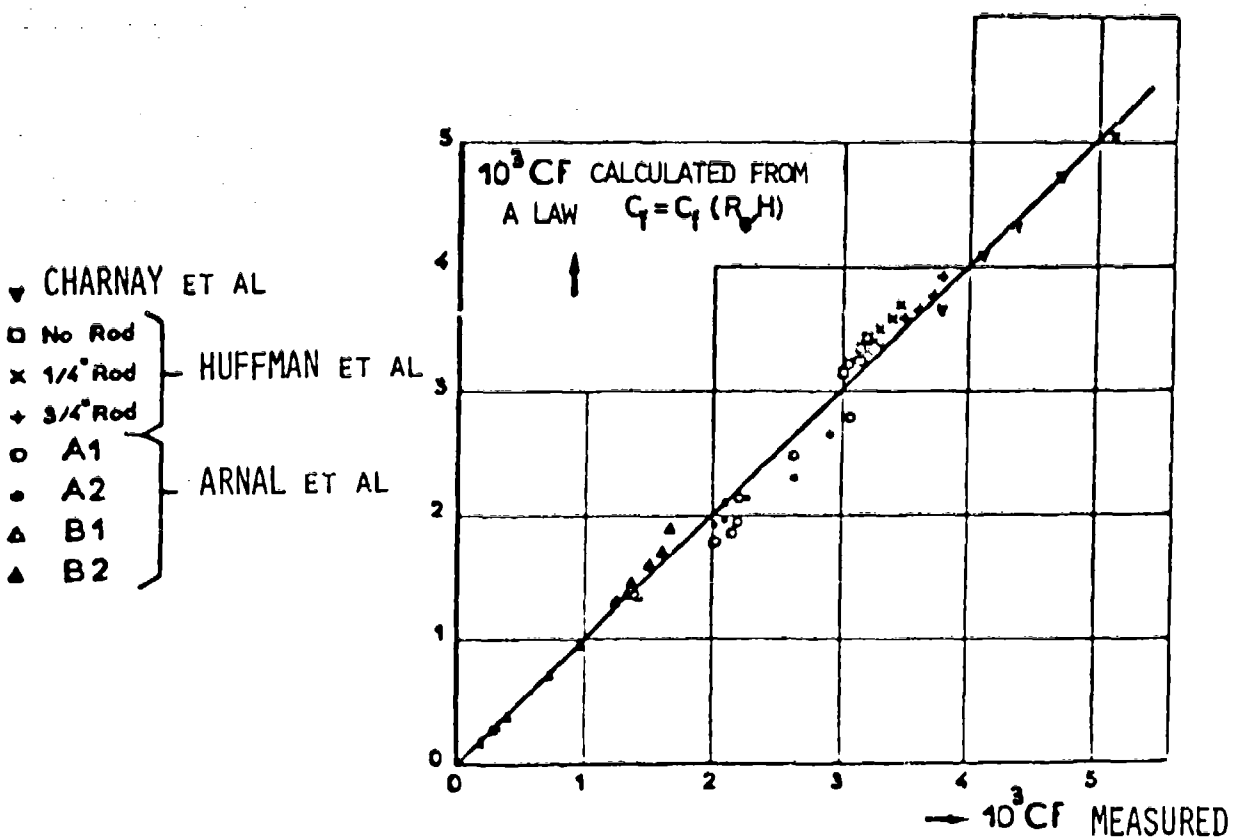

Figure 6. Verification of the skin-friction law for boundary layers with free-stream turbulence (Arnal et al., 1976; Cousteix, 1981).

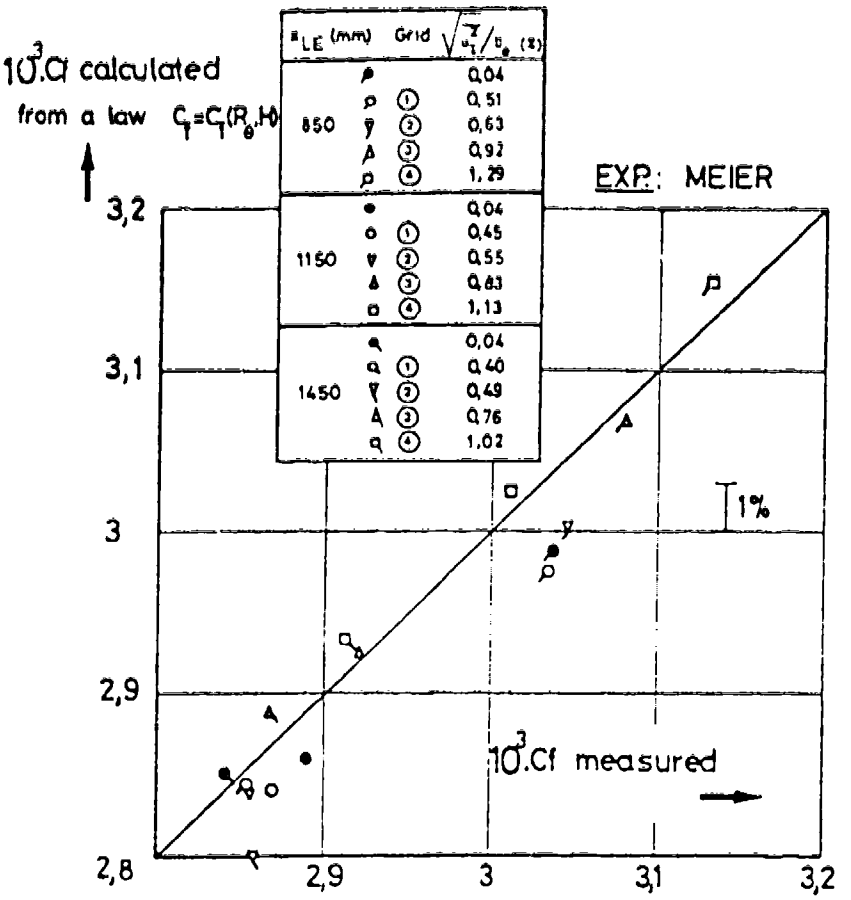

Figure 7. Verification of the skin-friction law. Boundary layer with free-stream turbulence (Arnal et al., 1976; Cousteix, 1981).

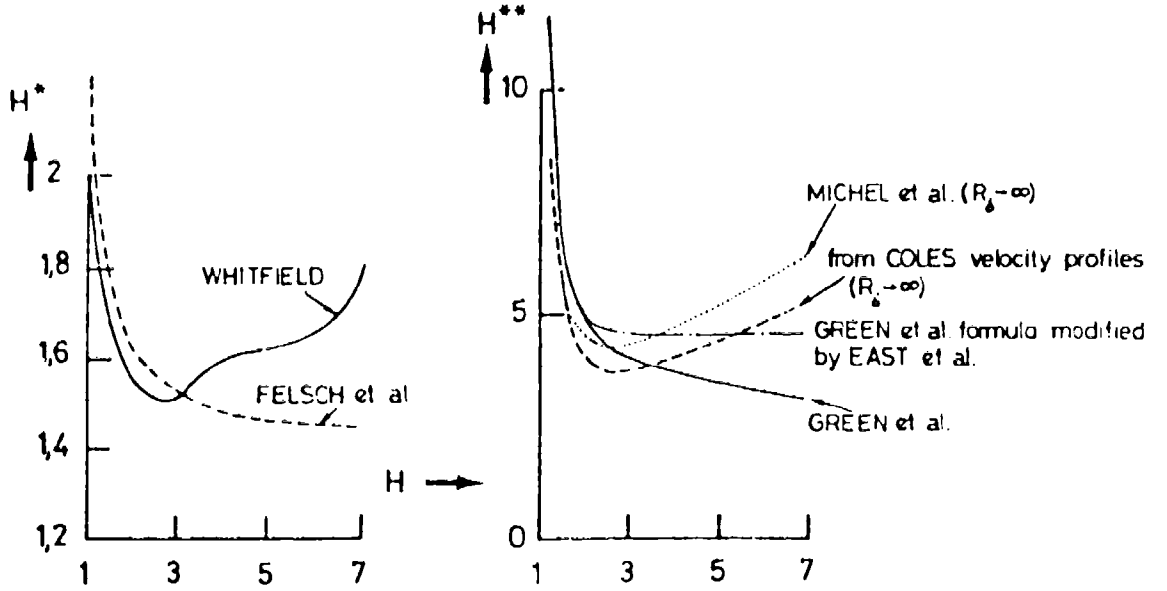

Figure 8. Behavior of the functions H^* (H) and H^{**} (H).

VELOCITY AND LENGTH SCALES IN TURBULENT FLOWS
- A REVIEW OF APPROACHES -

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INTRODUCTION

The dynamic processes occurring in turbulent flows proceed at characteristic time, length, and velocity scales. Their satisfactory specification is an inescapable requirement for mathematical simulation of the various turbulence interactions and, consequently, for the accurate calculations of the flow properties. In simple Boussinesq-type closure models, the characteristic scales are used only to define the turbulent mixing through the definition of eddy viscosity at a point in the flow,

$$\nu_t \sim L\,V \sim L^2/T$$

where $\sim$ denotes "has the dimensions of," V is the velocity scale, L the length scale, and T the time scale. In more elaborate closure schemes which solve separate transport equations for some characteristic turbulence properties, the scales are also utilized to characterize--in the course of modeling--various "superfluous" terms, representing different turbulent interactions.

Various turbulence interactions are known to occur at different rates, but most methods that have been used so far in fact employ only a single time and length scale for their description. These methods are referred to as <u>single-scale methods</u>.

There have been proposals in the past and more recently to introduce two or more characteristic time or length scales, in order to characterize independently the dynamics of different eddy sizes and associated turbulent processes. These methods are usually called <u>multiple- (or multi-) scale methods</u>.

Most of the differential [illegible] techniques submitted for the present Conference, that solve the time-averaged me[illegible]momentum equation in partial-differential form an calculate relevant local turbulent stresses, employ two variables with which one ca[illegible] define only one characteristic time and length, or alternatively, velocity scale. Hence all these models fall into the category of single-scale methods. Only one computor group (Cousteix et al., see Volume III) submitted calculations (albeit for homogeneous flows only) employing the two-scale method by way of supplying four characteristic turbulence parameters.

In order to define characteristic scales, some of the single-scale methods rely only on the <u>mean field</u> flow parameters and bulk flow dimensions. In this way they avoid the necessity for introducing any specific turbulence property which would, by nature of the process, need to be supplied from the solution of the appropriate dynamic differential equation.

Other methods introduce one turbulence property which is combined with some of the gross flow features--most often flow dimension--to produce a characteristic time and length scale. Highly popular seem to be the techniques that employ two turbulence properties which yield again only one characteristic turbulence time and length scale but without having to draw on any direct information about the mean flow features.

Some authors divide the flow into two or more regions and apply different scales to different parts of the flow. Two-layer approach is frequently applied to wall flows where the mean field model is used to calculate the flow within the viscous layer, while more advanced techniques are employed in the outer flow zones.

Here a brief comparative survey is presented of the turbulence scales, their definition, and method of prescription of each of the categories of single-scale methods mentioned above.* A short coverage of the two-scale method is also given at the end.

SINGLE-SCALE MEAN-FIELD METHODS

This class of methods usually employs the standard Prandtl mixing-length hypothesis though there are some minor variations in the way both the length and time scales are defined.

Because no specific turbulence parameter is supplied, and because the scales are used only to specify the eddy viscosity, the mean rate of strain or mean vorticity serve directly to define the characteristic time scales.† In the case of unidirectional shear such as in thin-shear flows $T = |\partial U_1/\partial x_2|^{-1}$ is used without exception. However, in more general cases three different forms have been used as follows:

$$T_1 = \left|\frac{U_1}{x_2} + \frac{U_2}{x_1}\right|^{-1} \qquad (1)$$

Only used for plane flow calculations

$$T_2 = \left|\left(\frac{\partial U_1}{\partial x_2}\right)^2 + \left(\frac{\partial U_2}{\partial x_1}\right)^2\right|^{-1/2} \qquad (2)$$

$$T_\Omega = \Omega^{-1} = \left|\epsilon_{ijk}\frac{\partial U_i}{\partial x_j}\right|^{-1} \qquad (3)$$

Used by Viegas et al. in three-dimensional flows

It is pertinent to note that none of the expressions incorporates normal strain. It is obvious that the three expressions give different values of T for all situations except in the case of unidirectional plane shear, when $(\partial U_2/\partial x_1) = 0$. The comparison

*A conventional classification of turbulence models according to a number of differential equations solved in addition to the mean flow equations is not followed since some computors employ a differential form of equations to specify the length scale, but solely in terms of mean flow parameters (i.e., Pletcher and Kwon, 1982; see Vol. III).

†Excluding the common use of U_* as the velocity scale in a wall region.

between the three expressions for the plane flow shown in Fig. 1 indicates that in the case of pure rotation, ($S_{ij} = 0$, $\Omega = 2\ \partial U_1/\partial x_2$), in the absence of strain and body forces only expression (1) yields the infinite time required for turbulence to be generated, in compliance with real physics.

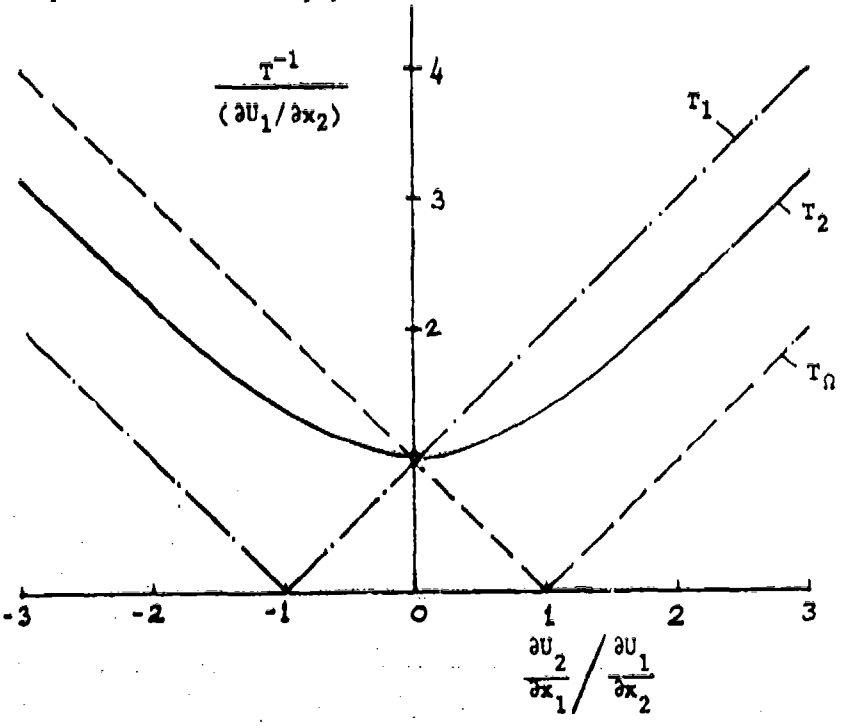

Figure 1.

Contrary to this, it follows from expression (2) that T_2 has finite and equal values both in the case of pure rotation ($S_{ij} = 0$, $\Omega = 2\ \partial U_1/\partial x_2$) and in the case of irrotational shear ($\Omega = 0$, $S = \partial U_1/\partial x_2$), an outcome that may produce the erroneous results in flows with predominant rotation. Expression (2) approaches asymptotically to (1) as the ratio $(\partial U_2/\partial x_1)/(\partial U_1/\partial x_2)$ increases in both negative or positive direction. (Note: $S_{ij} = (\partial U_i/\partial x_j + \partial U_j/\partial x_i)/2$ with $S = |S_{ij}|$.)

The time scale based on the absolute value of the mean vorticity seems even more inappropriate since in the case of irrotational shear it yields an infinite value of T and, consequently, zero eddy viscosity—although the flow may be highly sheared—but producing a finite T for the case of pure rotation.

Further insight into the behavior of the time scales defined through mean-flow parameters may be obtained by comparison with models that use the kinetic energy of turbulence, k, to define the velocity scales. It is readily seen that in the case of a homogeneous equilibrium flow where the turbulence energy production is balanced locally with the energy dissipation rates, the characteristic time scale, defined now as $T_k = L/k^{1/2}$ reduces to the expression

$$T_k = \left[\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}\right]^{-1/2} \qquad (4)$$

which, in the absence of normal straining, becomes equal to T_1, giving further support for expression (1).

The length scale used in the mean-field methods is in most cases a variant of the Prandtl mixing length, ℓ, defined in terms of flow dimensions. Some authors use a ramp form of the mixing length with separate specification of ℓ for the near-wall and far-from-the-wall flow zones (Pletcher, models 1 and 2, Vol. III), while others use a continuous expression, mostly in the form $\ell/\delta = \lambda \tanh(\kappa y/\lambda\delta)$, (Cousteix et al. k-ε model; Viegas et al., Vol. III), or other forms. For the flow remote from the wall several groups used the eddy-viscosity formula

$$(\nu_t)_{outer} = \frac{0.0168\, U_e\, \delta^*}{\left[(1. + 5.5\,(y/\delta)^6\right]} \qquad (5)$$

where the displacement thickness δ^* (or a fraction of it) served as the length scale, while the free-stream velocity U_e represents the characteristic scale, corrected by the non-dimensional wall distance (y/δ).

All of the authors who employ the mixing length hypothesis in the wall region solve the mean momentum equation throughout the viscous sublayer; hence the length scale is corrected by using the Van Driest form of damping function, $D = 1 - \exp(-VL/A\nu)$, though there is a variation of both the velocity scale V and the coefficient A. Most often here, $\sqrt{\tau/\rho}$ is used for V based either on the local- (Cousteix et al.; see Vol. III), or wall-shear stress (Ha Minh et al.; Viegas et al.; see Vol. III), but other variables were also used (e.g., $k^{1/2}$, Orlandi; $(\nu\, \partial U/\partial x)^{1/2}$, Pletcher, etc.; see Vol III). Mixing length ℓ or simply the wall distance is used for L. The standard value of $A = 26$ appears in most models, though some authors have introduced the effects of blowing/suction and pressure gradient by modification of A.

For flows in corners and other regions bound by two or more walls, several groups have used Buleev-type harmonic length formulations, obtained from the integration of the reciprocal of the wall distance over the plane or space angle (Cousteix et al.; Viegas et al.; see Vol III). Other forms of specification of the length scales have been used in more complex flow cases, e.g., Pletcher and Kwon, who, for the case of recirculating flow behind a backward-facing step made ℓ depend also on the distance of the dividing streamline and the position of maximum shear stress from the bottom wall.

All methods mentioned so far specify the length scale in the form of algebraic expressions containing mean-flow parameters. The exception is the model of Pletcher and Kwon where, for the outer region of an attached thin-shear flow, the length scale is obtained from the solution of an ordinary differential equation, but again in terms of only mean flow properties.

SINGLE SCALE METHOD WITH ONE TURBULENCE PARAMETER

These methods employ without exception the turbulence kinetic energy $k = \overline{u_i u_i}/2$ which proved to be the best defined and most readily obtainable turbulence parameter. Most often $\overline{u_i u_i}/2$ is supplied from the solution of its differential transport equation. Alternatively, k is calculated from equations governing each component of the turbulence normal stresses. Here $k^{1/2}$ serves as the characteristic velocity scale, while the length scale is specified in terms of gross flow dimensions mostly in algebraic form - in line with the mixing length hypothesis (with the exception of Pletcher's differential equation which was also used in connection with k-equation). The kinetic energy equation is used in its standard form with the production term modeled by means of an eddy viscosity and mean rate of strain while the dissipation rate is defined as $k^{3/2}/L$. The diffusive transport of k is modeled in all cases in gradient form. Various modifications to the k equations are employed to account for low Re number effects if the solution is carried out through the near wall viscous zone. The form of equation used by Orlandi has a term to account for the pressure diffusion, but the proposed term does not seem to possess a transport character.

SINGLE SCALE METHODS EMPLOYING TWO TURBULENT PARAMETERS

The majority of the differential techniques employed at the Conference use two turbulence properties by which a characteristic scale is defined. In all cases $k^{1/2}$ again plays the role of the velocity scale. For the second turbulence property, the majority of methods made use of the energy dissipation rate $\varepsilon = \nu \overline{(\partial u_i/\partial x_j)^2}$ which, in combination with k produces the turbulence time and length scales. There are, however, other approaches. Mellor and Celenligil (Vol. III) consider the product $(q^2 L)$, with $q^2 = 2k$. Viegas et al. (Vol. III) and Murphy (Vol. III) employ the Wilcox-Rubesin model* where the mean square of the fluctuating frequency ω^2 is used for the second turbulence parameter, while Donaldson et al. (Vol. III) solve the transport equation for the turbulence length scale (Λ in their notation). The characteristic turbulence scales for these two-parameter models are summarized in Table 1 below:

TABLE 1

Model	V	L	T
k-ε	$k^{1/2}$	$k^{3/2}/\varepsilon$	k/ε
k-kL	$k^{1/2}$	$(kL)/k$	$(kL)/k^{3/2}$
$k-\omega^2$	$k^{1/2}$	$k^{1/2}/\omega$	ω^{-1}
k-Λ	$k^{1/2}$	Λ	$\Lambda/k^{1/2}$

*[Ed.: The frequency, ω, used by Hanjalić reflects the frequency of the large-eddy structure. It has been defined by Rubesin, herein, as the dissipation rate per unit of kinetic energy.]

All of the equations have been derived empirically and have the same conservation form which includes the source and transport terms, but none bears much reliance on the physics of the process that they are supposed to represent.

The transport equation governing ε has a standard form

$$\frac{D\varepsilon}{Dt} = D^*_\varepsilon + (C_{\varepsilon 1} G - C_{\varepsilon 2} \varepsilon)/T \qquad (6)$$

where D^*_ε represents the diffusive transport, G is the generation term and T stands for the scale representing the evolvement of ε. There are, however, minor variations on how G is specified. Most groups use $P = -\overline{u_i u_j}\, \partial U_i/\partial x_j$ for G, $\overline{u_i u_j}$ being specified either in terms of eddy viscosity or obtained from the solution of turbulent stress equations. However, some groups (Demirdžić and Gosman, Launder et al., and Hanjalić et al., see Vol. III) use a recently proposed term of the form $C_\mu k^2/\varepsilon\, (\partial U_i/\partial x_j)^2$ at all levels of modeling (Hanjalić and Launder, to be published). Hah and Lakshmi: rayana avoid the mean velocity gradient completely by specifying G in terms of turbulent stress anisotropy, i.e., $G = C_\ell\, (\varepsilon/k)\, a_{ij} a_{ji}$, where $a_{ij} = (\overline{u_i u_j}/k - 2/3\, \delta_{ij})$.

Some groups have included an additional term in Eq. 6 to account for the curvature effects; Cousteix et al. use the Launder-Priddin-Sharma form (Launder et al., 1977), while Rodi et al. have employed curvilinear coordinates for calculating the curved flows with the curvature terms emerging in the course of derivation in both the k and ε equations.

Groups that solve the ε-equation within the viscous sublayer introduce a modification to account for the low Re number effects as discussed by B. E. Launder in his review (present volume).

It is worth recalling that the rate of change of ε is highly sensitive to the values chosen for the coefficients $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ (or rather to their difference). Yet a considerable variation of the values among the groups is noticeable, as seen from the selected sets in Table 2.

It should be pointed out that there are other differences among the models used by various groups which may justify the choice of different values of coefficients. These, however, should not apply to $C_{\varepsilon 2}$, which in all cases emerges from experimental evidence on the law of decay of isotropic grid turbulence. The test case 0371 corresponding to this situation adopts as the most reliable the data of Comte-Bellot and Corrsin which suggest $C_{\varepsilon 2} = 1.8$. However, most groups chose a value closer to 1.9 which may be regarded as within the experimental scatter. The value $C_{\varepsilon 2} = 2.0$ corresponding to the decay law $k \propto t^{-1}$ should, however, be avoided since it does not allow the turbulence Reynolds number to decay and, in so doing, to enter what is known as the "final period of decay".

TABLE 2

Constant	Computor Group*								
	Launder Group 17								
	G	Ha	Hu	L	C	R	MC†	V†	D†
$C_{\varepsilon 1}$	1.44	1.44	1.47	1.44	1.57	1.44	1.6	1.045	1.675
$C_{\varepsilon 2}$	1.92	1.90	1.92	1.92	2.0	1.92	2.0	1.83	1.8
$C_{\varepsilon 2}-C_{\varepsilon 1}$	0.48	0.46	0.45	0.48	0.43	0.48	0.4	0.785	0.125

*G = Gosman et al.; Ha = Hanjalić et al.; Hu = Humphrey et al.; L = Launder et al.; C = Cousteix et al.; R = Rodi et al.; MC = Mellor and Celenligil; V = Viegas et al.; D = Donaldson et al.

†Values obtained upon the conversion of equation for (q^2L), ω^2, and Λ, respectively into an equation for ε in a homogeneous flow.

A different approach is followed by Cousteix in his model RSEC 22B, where $C_{\varepsilon 1}$ is estimated on the basis of comparison of k-ε model for homogeneous flow with the equation of Lin and Wolfstein (1980) for the tensorial volume of turbulence $\hat{V}_{ij}$ and, supposedly, more exact dynamic equation for the product $(k\,\hat{V}_{ij})$. Here $C_{\varepsilon 1}$ appears as a function of the so-called dimensionless time, mean rate of strain and turbulence anisotropy.

Methods that use turbulence variables other than ε, such as q^2L, ω^2 or Λ entail the solution of the differential transport equation for these quantities with the same general form as the ε-equation. All authors express the diffusive transport in terms of gradients of the variable in question. The source is represented by the difference of the generation (expressed in all cases in terms of P) and sink terms, but each of these three equations contain a corrective term which does not have its counterpart in the standard form of ε-equation. In the (q^2L) equation of Mellor and Celenligil the corrective term operates only in near-wall flows. The ω^2-equation of Viegas et al. contains the square of the length scale derivative while the Λ equation of Donaldson includes a similar type of term with (qΛ) derivatives. Both terms will be generally non-zero which partially explains the need for a substantial difference in coefficients associated with the generation terms in the case of non-homogeneous flow.

However, for the case of homogeneous flows the equations for all four properties can be converted one into another, the only difference remaining in the values of the empirical coefficients, as indicated in Table 2. Comparison of the calculations of the homogeneous flows should thus serve as an indicator of the suitability of different second variables for two-parameters closure schemes.

It seems therefore that the Mellor (q^2L) equation, transferred to ε-form, should have the coefficients of similar values as those in the standard ε-equation if both

schemes claim a similar quality of predictions. This is because the corrective terms appear to be significant only in the wall vicinity region where the diffusive terms of two equations differ significantly due to the different behavior of the two properties in question. In fact, if a formal comparison of the equations is made in the local-equilibrium, ($P = \varepsilon$) constant-stress near-wall region, the large diffusive transport term in the ε-equation serves in full to compensate the non-zero source term $(C_{\varepsilon 1}P - C_{\varepsilon 2}\varepsilon)/T$. In the (q^2L) equation, due to the different and much slower variation of (q^2L) close to the wall, a satisfactory form of the equation is only achieved by additional terms. Similar comments can be made for the Λ and ω^2 equations except that now the corrective source terms are influential over most regions of the flow; this is presumably why the equivalents of the coefficients $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are significantly different than those employed in the ε equation.

Finally, when judging the suitability of the equations that supply the second turbulence parameter, one should bear in mind the problem of specifying the boundary conditions. From this point of view, (q^2L) and Λ equations seem to offer least uncertainties, though many users of the ε and ω^2 equation have succeeded in overcoming this problem.

TWO-SCALE METHODS

Coustcix et al. comprises the only participating group employing a two-scale model. Such schemes allow some account to be taken of the spectral energy transfer to turbulence. The essence of the model is the division of the turbulence energy spectrum into two parts roughly at the wave number above which no significant mean strain production occurs (Hanjalić et al., 1979). The two spectrum regions are expected to respond at different rates to the imposed outside conditions, the rate of change being characterized by different time scales. Two transport equations are then provided describing the rate of change of turbulence energy associated with each of the two spectrum regions, k_p and k_t, respectively. The closure of these equations is accomplished by defining ε_p as the rate of energy transfer out of the "production" range in the high wave number region of the spectrum, so that ε_p serves as the sink in k_p and as the source of k_t equation, while the dissipation rate ε defines the sink of k_t. Hence two more transport equations are solved, each characterizing the evolutions of the two energy transfer rates. The model uses four turbulence parameters; k_p, k_t, ε_p, and ε_t to provide two independent time (or length) scales

$$T_p = k_p/\varepsilon_p \quad \text{and} \quad T_t = k_t/\varepsilon$$

characterizing the turbulence interactions in two distinct wave number regions of the energy spectrums.

The model has been used in conjunction with the Reynolds-stress equation, and applied to the calculation of homogeneous flows.

CONCLUDING REMARKS

A wide variety of definitions of turbulence scales, employed by participating groups may serve as an indication of the diversity of views and beliefs in the current turbulence modeling practices. The approaches adopted span the range from the single mixing length hypothesis to multiple-scale models. However, it is obvious that the complexity of the flow considered plays a decisive role in selecting the level of closure, the correlation being, as expected, of reciprocal character; the more complex the flow, generally the simpler the modeling scheme that is adopted. This t i reflects sometimes a general distrust in the applicability of more advanced s in situations where numerics, specification of boundary and/or initial conditions and other aspects pose a high degree of uncertainty, and hence one cannot justify the need for the application of higher order schemes. Indeed, some groups have stated explicitly that their primary objective was to test the applicability of simpler models in more complex flows in order to establish clearer criteria on the need to move up in the closure hierarchy. This not only concerns the level of turbulence correlations where the closure is employed, but also the number of important turbulence scales that ought to be accounted for.

In spite of some evident deficiences in employing just a single time and length scale to characterize the important turbulence interactions, the single-scale approach will probably remain for some time as the most popular tool for closing the equation within the scope of Reynolds averaging techniques. However, the trend towards the use of two or more turbulence parameters instead of mean flow features seems indisputable.

Multiple-scale methods have some obvious physical appeal, but require much more extensive research before they reach the present level of development of the existing single-scale models and offer unchallenged advantages that would justify increasing the computing costs needed for calculations. Before this is achieved, the problem associated with the treatment of boundary conditions, in particular the wall boundaries, within the scope of the single-scale schemes, will have to be resolved in a more acceptable manner than it is at present, though this should not slow down parallel efforts in improving the multi-scale approach.

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STRESS/STRAIN RELATIONS IN DIFFERENTIAL METHODS FOR TURBULENT FLOWS

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INTRODUCTION

Differential methods solve the partial differential equations governing the flow field without introducing profile assumptions. Most of these methods employ time-averaged Navier-Stokes equations, the averaging time being long compared with the time scale of the turbulence so that the averaging is carried out over the whole spectrum of the turbulent fluctuations. In most general form (for the flows considered at this conference), the equations read:

$$\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij}) \tag{1}$$

where U_i is the mean velocity in direction x_i, P is the mean pressure and ρ the mean density. For incompressible flows the mean quantities are conventional averages and for compressible flows mass-weighted averages. τ_{ij} is the total averaged stress tensor, which is composed of a viscous stress and a turbulent stress $\tau_{ij,t}$:

$$\tau_{ij} = 2\mu\left(S_{ij} - \frac{1}{3}\frac{\partial U_k}{\partial x_k}\delta_{ij}\right) + \tau_{ij,t} \tag{2}$$

where μ is the molecular viscosity and the mean rate of strain tensor is defined by

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

The turbulent stresses, also called Reynolds stresses, are the following correlations between velocity fluctuations introduced by the averaging procedure:

$$\begin{aligned} \tau_{ij,t} &= -\overline{\tilde{\rho}\, u_i'' u_j''} \quad \text{for compressible flow} \\ \tau_{ij,t} &= -\rho\,\overline{u_i u_j} \quad \text{for incompressible flow} \end{aligned} \tag{4}$$

Here, u_i and u_i'' are the fluctuating velocity components for conventional and mass averaging, respectively, and $\tilde{\rho}$ is the instantaneous density. The Reynolds stresses represent the transport of momentum by the turbulent fluctuations in the averaged equations; hence the averaged equations can be solved only when a turbulence model is introduced that determines these stresses at each point in the flow. Some models solve a differential transport equation for $\tau_{ij,t}$ while others relate the stress

directly to the mean strain rate S_{ij}. The present paper summarizes the various stress/strain relations and Reynolds-stress equations used in calculations for the 1980-81 AFOSR-HTTM-Stanford Conference.

Other types of differential methods solve the time-dependent Navier-Stokes equations either by trying to resolve the turbulent motions at all scales (direct methods) or by introducing a sub-grid-scale model for the small-scale turbulent motion that cannot be resolved by the numerical scheme (large-eddy simulation). The present paper does not cover the assumptions made in sub-grid-scale modeling; it is restricted to turbulence models for the Reynolds stresses appearing in the time-averaged equations.

EDDY-VISCOSITY MODELS

Most turbulence models used in the calculations for the 1980-81 AFOSR-HTTM-Stanford Conference assume that the Reynolds stresses are proportional to the mean rate of strain, in analogy to the viscous stresses in laminar flow. This assumption is known as the Boussinesq eddy-viscosity concept, and hence the descriptors in these proceedings start with the letter B in all methods using this concept. For general situations, this concept reads

$$\tau_{ij,t} = 2\mu_t\left(S_{ij} - \frac{1}{3}\frac{\partial U_k}{\partial x_k}\delta_{ij}\right) - \frac{2}{3}\rho k\delta_{ij} \tag{5}$$

where μ_t is the turbulent or eddy viscosity, which is a scalar quantity. In contrast to the molecular viscosity, μ_t is not a fluid property; rather it depends on the state of turbulence and must also be determined by the turbulence model. The term with $\partial U_k/\partial x_k$ is zero for incompressible flows, and is introduced to force the trace of the first term to vanish for compressible flow also. The second term was added so that the trace of (5) yields the identity

$$\tau_{ii,t} = -2\rho k \tag{6}$$

which also defines the turbulent kinetic energy per unit mass, k. When Eq. 5 is used to eliminate $\tau_{ij,t}$ in the momentum equation (1), the second term can be absorbed by the pressure-gradient term so that in effect the static pressure is replaced as an unknown quantity by the modified pressure P + 2/3 k. Hence, the appearance of k in (5) does not necessitate the determination of k; it is the distribution of the eddy viscosity only that has to be determined.

For dimensional reasons, the eddy viscosity μ_t is proportional to a velocity scale $\hat{V}$ and a length scale L characterizing the turbulence motion,

$$\mu_t \propto \rho\hat{V}L \tag{7}$$

Most turbulence models employing the eddy-viscosity concept calculate μ_t from the distributions of velocity and length scales obtained in one way or another. Simple

models like the Prandtl mixing-length model relate the scales to the mean-flow field. One-equation models determine the velocity scale from a transport equation. Two-equation models, like the frequently used k-ε model, solve transport equations for both velocity and length scale. The different ways of determining these two scales are discussed in detail in the companion paper by Hanjalić in this volume.

For thin-shear layers, where only the velocity gradient $\partial U_1/\partial x_2$ is of importance, Eq. 5 yields the well-known relation for the shear stress τ_{12}

$$\tau_{12} = \mu_t \frac{\partial U_1}{\partial x_2} \tag{8}$$

while for the normal stresses it follows that $\tau_{11} = \tau_{22} = \tau_{33} = -2/3\ \rho k$, but in reality the turbulence is not isotropic. This points to the fact that an isotropic eddy viscosity cannot account for certain phenomena that create directional influences on the turbulence. The algebraic stress relations to be discussed shortly are also of the eddy-viscosity type, but they allow for non-isotropic eddy viscosities and can therefore account for directional influences.

For curved shear layers, the strain rate S_{12} in the curvilinear coordinate system of Fig. 1 is $1/2\ (\partial U_1/\partial x_2 - U_1/Rh)$ so that the shear stress expression (8) changes to

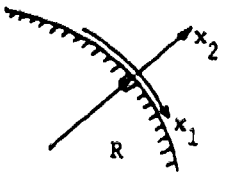


Figure 1. Curvilinear coordinate system.

$$\tau_{12} = \mu_t \frac{\partial U_1}{\partial x_2} (1 - R_f) \tag{9}$$

where $h = 1 + x_2/R$ and $R_f = \dfrac{U_1/Rh}{\partial U_1/\partial x_2}$ is the curvature Richardson number.

REYNOLDS-STRESS-EQUATION MODELS

The methods in these proceedings, for which descriptors start with RST, use differential transport equations to solve for all non-zero Reynolds-stress components. The equations are model forms of the following exact transport equations (given for incompressible flow):

$$\frac{D}{Dt}\overline{u_i u_j} - D\overline{u_i u_j} = P_{ij} + \pi_{ij} - \varepsilon_{ij} + \nu \nabla^2 \overline{u_i u_j} \tag{10}$$

where

$$\frac{D}{Dt} \equiv U_\ell \frac{\partial}{\partial x_\ell}$$

Turbulent Diffusion $$D_{\overline{u_i u_j}} = -\frac{\partial}{\partial x_\ell}\left(\overline{u_i u_j u_\ell} + \frac{\overline{pu_i}}{\rho}\delta_{j\ell} + \frac{\overline{pu_j}}{\rho}\delta_{i\ell}\right)$$

Production $$P_{ij} = -\overline{u_i u_\ell}\frac{\partial U_j}{\partial x_\ell} - \overline{u_j u_\ell}\frac{\partial U_i}{\partial x_\ell}$$

Pressure-Strain $$\pi_{ij} = \overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

Dissipation $$\varepsilon_{ij} = 2\nu\,\overline{\frac{\partial u_i}{\partial x_\ell}\frac{\partial u_j}{\partial x_\ell}}$$

The pressure-strain, dissipation, and turbulent diffusion terms require model approximations in order to eliminate the unknown correlations appearing in these terms. The model assumptions introduced in the following paragraphs are also of consequence to the algebraic stress relations discussed below because the algebraic methods employ truncated forms of the modeled stress equations in most cases.

All the RST models assume that local isotropy prevails so that the dissipation of $\overline{u_i u_j}$ can be approximated as

$$\varepsilon_{ij} = \frac{2}{3}\delta_{ij}\varepsilon \tag{11}$$

where ε is the rate of dissipation of turbulent kinetic energy k and is determined from a length-scale equation in the RST models (often from an equation for ε itself). For regions of low-Reynolds number the Donaldson et al. (see Volume III) model involves an additional term which allows for non-isotropic dissipation.

By eliminating the fluctuating pressure p from the pressure-strain correlation π_{ij} via a Poisson equation for p, it can be shown that two distinct processes contribute to this correlation, one arising from the interaction of fluctuating velocities only ($\pi_{ij,1}$), and one due to the interaction of mean strain and fluctuating velocities ($\pi_{ij,2}$), see e.g. Launder, Reece and Rodi (1975) (hereafter designated LRR). This analysis also indicates that both processes are influenced by the presence of a wall. All models use Rotta's (1972) proposal for the first part (sometimes called return-to-isotropy part):

$$\pi_{ij,1} = -c_1\frac{\varepsilon}{k}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}k\right) \tag{12}$$

It should be mentioned here that in their model RSEC 22, Cousteix et al. (see Vol. III) calculated k and ε separately for the low- and high-wave-number parts of the turbulence spectrum (multi-scale model of Hanjalić et al., 1979) and used the low-wave-number values of k and ε in (12). For the second part (sometimes called rapid part), LRR have proposed the following model (model 1 of LRR):

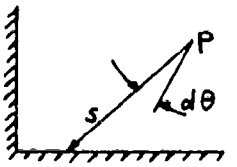
$$\pi_{ij,2} = -\frac{c_2 + 8}{11}\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right) - \frac{30c_2 - 2}{55}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)k - \frac{8c_2 - 2}{11}\left(D_{ij} - \frac{2}{3}\delta_{ij}P\right) \quad (13)$$

where

$$D_{ij} = -\left(\overline{u_i u_\ell}\frac{\partial U_\ell}{\partial x_j} + \overline{u_j u_\ell}\frac{\partial U_\ell}{\partial x_i}\right) \quad (14)$$

and P is the production of turbulent kinetic energy ($P = 1/2\ P_{ii}$). This model was used by Cousteix et al. (see Vol. III) and Hanjalić et al. (1979) for their calculations at this conference. LRR found that the first term in (13) is the dominant one, and in their model 2 only this term is retained (with a slightly different value for the constant c_2). LRR model 2 was used by Launder et al. (see Vol. III) and Rodi et al. (see Vol. III) for their calculations at this conference. Mellor and Celenligil (see Vol. III) retained only the second term in (13) with a rather small numerical constant so that $\pi_{ij,2}$ does not contribute very much to the pressure-strain correlation in their model. Donaldson et al. neglect $\pi_{ij,2}$ altogether and compensate for this by using a larger value for c_1, in relation (12) for $\pi_{ij,1}$.

The pressure-strain models introduced so far do not account for wall-proximity effects. These effects basically damp fluctuations normal to the wall and enhance fluctuations parallel to the wall. LRR proposed additive wall-proximity corrections to (12) and (13), which can be effected by making the numerical coefficients in front of the individual terms functions of the wall-damping parameter L/x_n, where L is the local length scale of turbulence (e.g. $k^{3/2}/\varepsilon$) and x_n is a characteristic wall distance. LRR proposed a linear function of L/x_n, but in their algebraic stress model for turbulence-driven secondary flows in ducts Rodi et al. used a quadratic function in order to reduce the influence of the wall-proximity correction in the center portion of channel flows (for details see Naot and Rodi, 1981). Further, Rodi et al. (see Vol. III) calculated the characteristic wall distance x_n in rectangular channels from the following equation:



$$\frac{1}{x_n^2} = \frac{2}{\pi}\int_0^{2\pi}\frac{d\theta}{s^2} \quad (15)$$

In situations with only one wall present, a different wall-proximity correction became more popular, which is partly due to Shir (1973) and partly due to Gibson and Launder (1978). This correction is effected by adding the following terms to the pressure-strain models (12) and (13) (only the first term is usually retained in the latter):

$$\pi'_{ij,1} = c'_1 \frac{\varepsilon}{k} \left(\overline{u_n^2}\, \delta_{ij} - \frac{3}{2} \overline{u_n u_i}\, \delta_{nj} - \frac{3}{2} \overline{u_n u_j}\, \delta_{ni} \right) f\left(\frac{L}{x_n}\right) \tag{16}$$

$$\pi'_{ij,2} = c'_2 \left(\pi_{nn,2}\, \delta_{ij} - \frac{3}{2} \pi_{ni,2}\, \delta_{nj} - \frac{3}{2} \pi_{nj,2}\, \delta_{ni} \right) f\left(\frac{L}{x_n}\right) \tag{17}$$

Here, n denotes the direction normal to the wall and $f(L/x_n)$ is the wall-damping function mentioned above. A linear relationship $f = k^{3/2}/(x_n \varepsilon)$ is used in models employing (16) and (17). Among these are the algebraic stress models of Launder et al. (see Vol. III) and Rodi et al. (see Vol. III) (in the latter case the model used for two-dimensional shear layers). The wall corrections (16) and (17) have the advantage over the LRR wall correction that they produce the experimentally observed influence of a wall on the turbulence fluctuations also under conditions of vanishing mean strain (in this case only $\pi_{ij,1}$ is left, and simply changing the value of the coefficient c_1 does not lead to a redistribution among the fluctuating components). On the other hand, (16) and (17) are difficult to apply to complex geometries like corner regions.

LRR neglected the turbulent diffusion of $\overline{u_i u_j}$ by pressure fluctuations and investigated the following models for diffusion by velocity fluctuations:

$$-\overline{u_i u_j u_k} = c_s \frac{k}{\varepsilon} \left[\overline{u_i u_\ell} \frac{\partial \overline{u_j u_k}}{\partial x_\ell} + \overline{u_j u_\ell} \frac{\partial \overline{u_k u_i}}{\partial x_\ell} + \overline{u_k u_\ell} \frac{\partial \overline{u_i u_j}}{\partial x_\ell} \right] \tag{18}$$

$$-\overline{u_i u_j u_k} = c'_s \frac{k}{\varepsilon} \overline{u_k u_\ell} \frac{\partial \overline{u_i u_j}}{\partial x_\ell} \tag{19}$$

LRR found that, for the shear flows investigated by them, the simpler form (19), which is not compatible in its symmetry properties, gave better results than the more complex, tensor-invariant form (18). Lumley (1980) suggested that this may be due to the absorption in (19) of some pressure diffusion, which is not symmetrical and which was not modeled explicitly. Most of the RST calculations for the 1980-81 AFOSR-HTTM-Stanford Conference were carried out only for the homogeneous flow cases where the diffusion term is absent. Donaldson et al. (herein) employed a modified version of (18) in which the individual stresses in front of the derivatives were replaced by k and tensor invariance was enforced. Mellor and Celenligil (see Vol. III) used a diffusion model that corresponds to Eq. 18.

There is one further aspect of these authors' RST calculations that needs to be mentioned. They solved the time-dependent equations to approach an asymptotic steady state. However, in some cases they obtained cyclic solutions, indicating unsteady vortex shedding. For the comparison with the experiments, the results were averaged

over the cycles (angle bracket below), and the quantities $\langle(U_i - \langle U_i\rangle)(U_j - \langle U_j\rangle)\rangle$ were added to the stresses $\langle\overline{u_i u_j}\rangle$ to obtain the total average stresses.

Methods where a Stanford descriptor starts with D do not solve transport equations for all non-zero Reynolds-stress components but relate some components directly to others. The method of Bradshaw et al. (1967) is a classic example where, for two-dimensional boundary layers, an equation is solved for the shear stress τ_{12} while the normal stresses are assumed proportional to τ_{12}. However, to the author's knowledge, this model was not used in calculations for the 1980-81 AFOSR-HTTM-Stanford Conference.

ALGEBRAIC STRESS MODELS

Because RST models are rather complex and computationally expensive, proposals were made to simplify the Reynolds-stress equations such that they reduce to algebraic expressions but still retain many of their basic features. Methods using such algebraic stress relations have Stanford descriptors starting with A. The reduction of the differential stress equations to algebraic expressions is achieved by eliminating the gradients of $\overline{u_i u_j}$ in the rate of change, convection, and diffusion terms by model approximations. All effects that enter through the source terms of Eq. 10 are thereby retained. The simplest model is to neglect the terms in question, that is to assume local equilibrium for the Reynolds stresses. For consistency, the production P of turbulent kinetic energy appearing in the modeled forms of the pressure-strain term in Eq. 10 must be set equal to the dissipation rate ε. Such local equilibrium models (as far as the algebraic stress relations are concerned) have been used by Cousteix et al. (see Vol. III), Nakayama et al. (see Vol. III), and Rodi et al. (see Vol. III) to calculate the duct-flow test cases with turbulence-driven secondary motions. The former two basically adopted Launder and Ying's (1973) model which originates from the stress-equation model of Hanjalić and Launder (1972) and neglects all gradients of the secondary velocity components compared with those of the longitudinal velocity. Rodi et al. (see Vol. III) derived their model by simplifying the LRR model 1, retaining the secondary velocity gradients. The proposal of Hanjalić and Launder (1972) for the rapid part of the pressure-strain term differs somewhat from Eq. 13 used in LRR model 1, and these authors did not introduce a wall-proximity correction. However, when the secondary velocity gradients in Rodi et al.'s (see Vol. III) model are also neglected, the resulting algebraic expressions for the stresses appearing in the momentum equations governing the secondary flow are identical, except for the numerical coefficient. In Rodi et al.'s model the coefficient is between 10 (near the channel center) and 18 (near walls) times that in Launder and Ying's (1973) model. On the other hand, Rodi et al. found that the calculated secondary motion is quite sensitive to the inclusion of the secondary velocity gradients. It should further be mentioned

that all methods applied to duct flow with turbulence-driven secondary motions use algebraic stress modeling only for the stresses appearing in the momentum equations for the secondary flow but calculate the shear stresses in the longitudinal momentum equation by a standard eddy-viscosity model.

Rodi (1976) suggested a model that does not neglect the rate of change, convection, and diffusion of $\overline{u_i u_j}$ but relates these terms to the corresponding ones for the turbulence kinetic energy k:

$$\frac{D\,\overline{u_i u_j}}{Dt} = D_{\overline{u_i u_j}} = \frac{\overline{u_i u_j}}{k}\left(\frac{Dk}{Dt} - D_k\right) = \frac{\overline{u_i u_j}}{k}(P - \varepsilon) \tag{20}$$

The second equality follows from the trace of Eq. 10 with $k = \frac{1}{2}\overline{u_i u_i}$, $P = \frac{1}{2}P_{ii}$, $\varepsilon = \frac{1}{2}\varepsilon_{ii}$. Equation 20 is based on the assumption that the temporal and spatial changes of $\overline{u_i u_j}/k$ are small compared with the change of $\overline{u_i u_j}$ itself. As an example, the algebraic stress relation is given that follows when the LRR model 2 equations are simplified with the aid of (20):

$$\overline{u_i u_j} = k\left(\frac{2}{3}\delta_{ij} + \frac{(1-\gamma)\left(\frac{P_{ij}}{\varepsilon} - \frac{2}{3}\delta_{ij}\frac{P}{\varepsilon}\right)}{c_1 + \frac{P}{\varepsilon} - 1}\right) \tag{21}$$

Here, γ is the numerical coefficient in the first term in Eq. 13 (the only term in LRR model 2). k and ε appearing in (21) are usually calculated from transport equations for these quantities (see the companion paper of Hanjalić). Both Launder et al. (see Vol. III) and Rodi et al. (see Vol. III) used Eq. 20 to simplify stress equations based on LRR model 2 with the wall-proximity corrections (16) and (17) added, but in their model for two-dimensional shear layers Rodi et al. applied (20) only to normal stresses and neglected convective and diffusive transport of the shear stress (for flows remote from walls Eq. 21 still applies but with P/ε-1 absent). When the stress equations are first transformed into the curvilinear coordinate system of Fig. 1 and then simplified to algebraic expressions, the influence of curvature on the individual stresses enters automatically. For curved shear layers, the expressions are given in the summary paper of Rodi et al. (see Vol. III). The resulting relation for the shear stress τ_{12} is of the eddy-viscosity form (9), but it predicts the dependence of the eddy viscosity on the Richardson number R_f characterizing the curvature (see Fig. 6) and on the wall-damping function f.

Hah and Lakshminarayana (see Vol. III) relate the convection and diffusion of $\overline{u_i u_j}$ to the production P_{ij}, but the details of this relationship are not given. They also take LRR model 2 as the starting point of their derivation.

Finally, methods employing the Wilcox-Rubesin (1980) turbulence model employ a stress/strain relation that can also be classified as an algebraic stress model but

was not derived by simplification of Reynolds-stress equations. Because the eddy-viscosity relation (5) implies that the stresses are aligned with the mean rates of strain, Wilcox and Rubesin (1980) added the term

$$\frac{8}{9}\rho k \frac{S_{im}\Omega_{mj} + S_{jm}\Omega_{mi}}{\beta^* \omega^2 + 2S_{mn}S_{nm}}$$

to Eq. 5 in order to allow for situations where the stresses are not aligned. The derivation was guided by Saffman's (1976) constitutive stress relation and the requirement "to satisfy some boundary layer needs." The vorticity vector appearing in the additional term is

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \tag{22}$$

and ω is defined by

$$\varepsilon = \beta^* \omega k \tag{23}$$

where β^* is an empirical constant (0.09). For thin-shear layers, the extra term is absent for the shear stress τ_{12} so that the standard relation (8) results, and for the normal stresses under local equilibrium conditions ($P = \varepsilon$) there follows

$$\overline{u_1^2}/k = \frac{8}{9} \quad , \quad \overline{u_2^2}/k = \frac{4}{9} \quad , \quad \overline{u_3^2}/k = \frac{6}{9} \tag{24}$$

Equation 24 is in much better agreement with experiments than are the isotropic normal stresses resulting from the eddy-viscosity relation (5). Here it should be mentioned that the algebraic stress models introduced above yield similar results for equilibrium shear layers.

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TURBULENCE MODELING IN THE VICINITY OF A WALL

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1. Introduction

The condition of no slip at a rigid boundary ensures that over some region of a turbulent wall layer, however thin, viscous effects on the transport processes must be large. In round terms, molecular influences will be influential over a region extending from the surface to where the local "eddy" Reynolds number, based on a typical eddy dimension normal to the wall and the intensity of fluctuation in that direction, is of order 10^2. Any mathematical model of turbulence for this region must address the problem of appropriately incorporating viscous effects into the system of equations employed. The region in question is thin (even in low-speed laboratory studies rarely extending over more than 2 mm), the processes are highly complex and the acquisition of accurate experimental data is greatly complicated not just by the thinness of this sublayer but by wall-proximity influences of various kinds on the instruments themselves. It is thus no wonder that our knowledge of this important region of flow is still sketchy.

Yet, the very thinness of this viscosity-dependent region also brings some compensating simplification. For, streamwise convective transport within this viscous sublayer is frequently so small compared with diffusive or (in the case of properties of the turbulent field source or sink processes) that it may be neglected. In cases where surface transpiration is absent and where the influence of force fields (including pressure gradients) across the sublayer is negligible, the flow-field properties, suitably normalized, are then functions of only a normal-distance Reynolds number. The resultant distribution of mean velocity is known as the Law of the Wall. If the distribution of mean velocity and turbulence properties of interest is determined by experiment these profiles may be used to provide near-wall boundary conditions for the momentum and turbulent transport equations thus avoiding the need to extend numerical computations to the surface itself. This approach is especially advantageous if the matching with experimental conditions is applied outside of the viscosity-dependent region.† For then the turbulence model used for the numerical computations does not have to account for viscous effects. Morever, one escapes the need for the especially

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†But still close enough to the wall for streamwise transport to be negligible.

fine mesh that is inevitably required to resolve the viscous region because there the curvature of both the mean velocity and the turbulence properties is so high.

The present article provides a short survey of the main approaches to treating the wall region within the framework of one-point closures. Section 2 considers schemes which incorporate viscous effects into the turbulence model and thus carry numerical computations to the wall itself--an approach referred to as a "low-Reynolds-number treatment." Section 3 examines approaches based on the law-of-the-wall and its extension or generalization, that are commonly referred to in the literature as "wall-function treatments."

2. Low-Reynolds-Number Treatments

Table 1 summarizes the various ways on which viscous effects have been included into turbulence models. The simpler the closure level, the easier is the task of incorporating viscous influences. Thus, with the mixing length hypothesis, all computors follow broadly Van Driest's (1956) proposal that the near-wall mixing length should be multiplied by a damping factor $(1 - D^*)$, where D^* has the exponential form,

$$D^* = \exp\,(-\,Re_t^*)$$

Van Driest recommended

$$Re_t^* = \frac{y\sqrt{\tau_w/\rho}}{26\nu} \tag{1}$$

More than a dozen variants of this form have been employed in subsequent research studies. All these forms reduce in the limit, where there is negligible variation of shear stress across the viscous region, to Van Driest's original form. When the shear stress increases across the sublayer (as in a boundary layer approaching separation or where there is blowing through the wall), the increase of Re_t^* with y is more rapid than Eq. 1 suggests. Conversely, for a shear stress falling rapidly with distance from the wall (as in accelerated boundary layers or low-Reynolds-number pipe flows), experiments conclusively show that Eq. (1) gives too large values for Re_t^*. The subsequent amendments to Van Driest's original form have in part been designed to remove this weakness. An initial step, perhaps employed first by Patankar (1966), was to replace the wall shear stress by τ, the local stress. This modification has generally been held to produce an insufficient effect of shear stress variations. Consequently, alternatives of the form

$$Re_t^* = \frac{y\sqrt{\tau_w/\rho}}{26\nu}\left(\frac{\tau}{\tau_w}\right)^n \tag{2}$$

have been proposed--with n ranging from 1 to 2 (see Kays and Moffat, 1975). Other schemes have involved replacing the Van Driest constant, 26, by a function of dimensionless pressure-gradient and mass-transfer parameters (Launder and Jones, 1969; Launder and Priddin, 1973).

TABLE 1
Low-Reynolds-Number Treatments

No.	Model Type	Typical Sort(s) of Modification	Computers at 1980-81 Stanford Conference	Other References
1	Mixing-length hypothesis	$\ell_m = \kappa y(1-D^*)$ D^*—viscous damping function	Pletcher | Nagamatsu Cousteix | Rakich Viegas | Deiwert Ha Minh/Vandromme	Kays and Moffat, 1975. Review: Launder and Priddin, 1973.
2	Boussinesq models based on k transport	$\nu_t = C_\mu k^{1/2}\ell$; ℓ and/or C_μ dependent on Re_t. Molecular transport and Re_t-dependent dissipation in k equation	Orlandi Murphy	Glushko, 1965. Wolfshtein, 1967.
3	Boussinesq models based on 2 turbulent transport equations	$\nu_t = C_\mu k^2/\varepsilon$; $C_\mu(Re_t)$ • Molecular transport in k equation • Add Re_t-dependent terms to ε equation	Rodi (Lam and Bremhorst, 1978) Wilcox, Viegas (Wilcox and Rubesin, 1980) Cousteix, Ha Minh/Chassaing (Jones and Launder, 1972)	Comparative assessment: Patel, Rodi, and Scheurer, 1981
4	Reynolds-stress transport with prescribed length scale	• Re_t-dependent terms in $\overline{u_i u_j}$ equations • $\ell = \ell(Re_t)$	Donaldson	Donaldson, 1969
5	Reynolds-stress transport with ε transport eq.	• Re_t-dependent processes in $\overline{u_i u_j}$ equations. • Add Re_t-dependent terms to ε equation	-	Hanjalić and Launder, 1976

At the one-equation level, one again needs to modify the prescribed length scale in a similar fashion to the adaptation of the mixing length. Now, the Reynolds number, Re_t^* is usually replaced by the turbulence Reynolds number, $Re_t \equiv \rho k^{1/2} y/\mu$ where k is the turbulent kinetic energy. Moreover, in the transport equation for turbulence energy, transport due to molecular action must now be included while the term representing the dissipation rate of turbulence energy will normally require multiplication by a Reynolds-number-dependent function. In Orlandi's scheme a further viscosity-dependent term is added. Ostensibly, this is to account for kinetic energy diffusion by pressure fluctuations; however, the model of this process is not of a diffusive character.

At the two-equation level, in addition to the adaptations to the viscosity formula and the kinetic energy transport equation, substantial viscous effects must be introduced to the equation for the "second variable" whether this variable be the energy-dissipation rate, ε, the mean square turbulence "frequency," ω^2, or some other quantity whose determination (along with k) allows the turbulence length and time scales to be deduced. Patel et al. (1981) have recently completed a comparison of the available low-Reynolds-number forms of two-equation models. It was from this comparison that the Lam-Bremhorst adaptation of the Jones-Launder k-ε model was used by Rodi's group for the laminarizing and transpired boundary layers.

Here it should be mentioned there is no need to use the same closure level across the viscosity-affected region as in the remainder of the boundary layer. Cousteix's group, in some of their calculations employ a mixing-length treatment in the low-Reynolds-number region and a k-ε Boussinesq treatment beyond it. At the matching point the kinetic energy for the outer region is taken as $C_\mu^{1/2}$ times the local shear stress, and ε is chosen to ensure continuity of turbulent viscosity at the matching point. In principle, a neater inner/outer matching would result from using a one-equation treatment in place of the mixing-length hypothesis over the viscous region. Since k would then appear as a dependent variable in both regions, the local equilibrium approximation between shear stress and turbulence energy would not be employed.

At the Reynolds-stress-transport level the modeler must attempt to represent how the individual processes in the stress equation, namely stress diffusion, dissipation and pressure interactions, are affected by viscosity. At this conference, Donaldson's group appears to be the only one to have attempted a low-Reynolds-number closure at this level. When, as in their scheme, the length scale is prescribed algebraically, viscous damping is normally applied to drive the length scale to zero as the wall is approached more rapidly than would be achieved by the linear variation alone.*

*In fact, the only papers from Donaldson's group available to the writer, do not include a specific Reynolds-number-dependence on the length scale.

Alternatively, if a transport equation for ε is provided, Reynolds-number-dependent terms must be added in a similar fashion to that in a two-equation Boussinesq model.

As a footnote, it may be observed that, scarcely without exception, Reynolds-number influence is expressed through functions of exponential type. These forms allow appropriate representation of the rapid changeover from viscous-to-turbulent dominance that occurs in the low-Reynolds-number region. Yet the evaluation of exponential functions is time-consuming for the computor. Piecewise linear fits could be chosen to give essentially the same physical model at a fraction of the computing cost.

3. Wall Function Approaches

The neglect of streamwise transport from the momentum and turbulence transport equations produces an equation set that is a function of just one independent variable, the coordinate normal to the wall, y. Solution of these ordinary differential equations over a region near the wall would give the variation of velocity, turbulence energy, etc., in terms of y and flow parameters such as wall shear stress, blowing rate and pressure gradient. By suitable rearrangement the results can be grouped in dimensionless form--"wall functions"--in a way that facilitates the application of boundary conditions. Any of the turbulence models mentioned in Section 2 could form the basis for generating such wall functions. In the late 1960s Spalding and his then students (Runchal, 1968; Spalding, 1967a,b; and Wolfshtein, 1967) worked out various systems of wall-functions based on one-equation Boussinesq models. Yet, these formally-generated wall functions never became particularly popular partly because, as generated, they were numerical rather than analytic functions to which analytical approximations had to be devised. The resulting algebraic complexity obscured the physics. Consequently, when situations arose where the physics of the underlying turbulence model were inadequate, the tendency was to shift to simpler formulae that, with a little insight, the user could adapt in the light of known experimental behavior. The wall-function treatments employed by computors at this conference are broadly of such simpler types, and are summarized below.

Four formulae have been used to provide the connection between mean velocity and wall shear stress; these are summarized in Table 2. In the case of a uniform-stress wall layer in local equilibrium, the proposals are all equivalent. Differences among the proposals only become significant as separation is approached. Mellor's form is based on integration of the streamwise momentum equation (with x-transport neglected); it assumes the mixing length to be unaffected by streamwise pressure gradients (Mellor, 1966). Here it might be added that from an extensive series of experiments of equilibrium wall boundary layers in adverse pressure gradients, East and Sawyer (1979) conclude that the dissipation length scale $k^{3/2}/\varepsilon$ rather than the mixing length

TABLE 2

The Near-Wall Velocity-Shear Stress Connection

No.	Form	Computor Groups	Literature Source
1	$\frac{U}{\sqrt{\tau_w/\rho}} = \frac{1}{\kappa} \ln\left(\frac{E\, y \sqrt{\tau_w/\rho}}{\nu}\right)$ $\kappa \approx .41$, $E \approx 9.0$	Rodi (boundary layers)	
2	$\frac{U}{\sqrt{\lvert\tau_w/\rho\rvert}} = f\left(\frac{y\sqrt{\tau_w/\rho}}{\nu},\ \tau/\tau_w\right)$	Mellor	Mellor (1966)
3	$\frac{Uk^{1/2}}{\tau_w/\rho} = \frac{1}{\kappa^*} \ln E^* \frac{yk^{1/2}}{\nu}$ $\kappa^* \approx .23$, $E^* \approx 5.0$	Mansour, Chow, Humphrey | Pollard, Spalding, Rodi	Launder & Spalding (1974); Rodi
4a	$\frac{Uk_v^{1/2}}{\tau_w/\rho} = \frac{1}{\kappa^*} \ln E^* \frac{yk_v^{1/2}}{\nu}$	Launder, Ha Minh	Chieng & Launder (1980)
4b	$\frac{Uk_v^{1/2}}{\tau_w/\rho} = \left[\left(\frac{E^* yk_v^{1/2}}{\nu}\right)^{v_w^+/\kappa^*} - 1\right] \Big/ v_w^+$, $v_w^+ \equiv v_w/k_v^{1/2}$	Hanjalić	-

is the scale that most closely retains a universal near-wall form as separation is approached. All oth treatments employ the form proposed by Launder and Spalding (1974) or the variant of Chieng and Launder (1980). In these versions, the characteristic turbulent velocity scale is $k^{1/2}$ rather than $(\tau_w/\rho)^{1/2}$; it is the form that naturally emerges from applying a one-equation Boussinesq model outside of the viscous sublayer. The use of a constant value of E implies that the edge of the viscous layer occurs at a fixed value of $yk^{1/2}/\nu$ (approximately 20).

Hanjalić's group appears to be the only one to have applied the wall-function approach to flows involving transpiration. The power-law velocity distribution that he uses results from integrating the momentum equation. The formulation employs a one-equation Boussinesq model and assumes that the edge of the viscous sublayer, y_v, is given by $y_v k_v^{1/2}/\nu = 20$, irrespective of blowing rate. This assumption is known not to hold for large rates of blowing and suction (and, indeed, for large adverse or favorable streamwise pressure gradients). Current work by R. W. Johnson at University of Manchester, Inst. of Science and Technology suggests that, on the basis of heat transfer in separated flows, the sublayer Reynolds-number variation is roughly:

$$\left(\frac{y_v\, k_v^{1/2}}{\nu}\right) = 20\left(1 - 10\Delta\tau/k_v^{1/2}\right) \qquad (3)$$

where $\Delta\tau$ is the increase in total shear stress across the viscous sublayer. While this formula clearly gives the correct trends, it should not be assumed as adequate for representing the effects of mass transfer or favorable pressure until much more testing has been undertaken.

In cases of three-dimensional flow, it seems that computors have applied their velocity/shear-stress relation to the resultant velocity component parallel to the wall. For the 90° bend, however, where secondary flows are substantial, Rodi's group modifies the secondary friction by an extrapolation of the streamwise and secondary velocities at the first two nodes away from the wall (see Eq. 9 of the summary of Rodi's group in Vol. III).

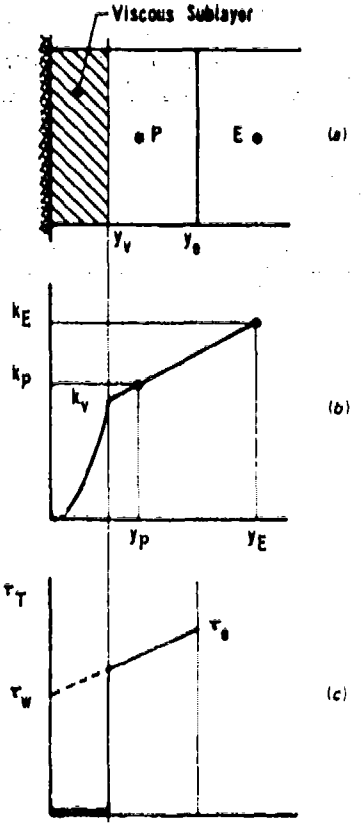

Figure 1. Near-wall physical model: (a) wall-adjacent cell, (b) assumed distribution of turbulence energy, and (c) assumed distribution of turbulent shear stress.

The near-wall treatments of the stresses are not reported in sufficient detail to be certain of precisely the practices of different groups. Diffusion of the Reynolds stresses to the wall is uniformly neglected, and thus the kinetic energy "floats" at a level imposed predominantly by the generation and dissipation rates in the near-wall region. Thus, it is the way in which these latter processes are represented that largely determines the level of k in the near-wall region. For example, in finite-volume elliptic procedures, the mean value of the turbulence energy production over the near-wall cell, $\overline{P}$, is, in the nomenclature of Fig. 1, usually represented as

$$\overline{P} = \tau_w U_e / y_e \qquad (4)$$

while, from the Chieng-Launder treatment,

$$\overline{P} = \tau_w \frac{(U_e - U_v)}{y_e} + \frac{\tau_w(\tau_e - \tau_w)}{\rho\kappa^* k_v^{1/2} y_e}\left(1 - \frac{y_v}{y_e}\right) \qquad (5)$$

The second term in Eq. (5) accounts for shear-stress variation across the near-wall cell. In the uniform stress limit, the values of $\overline{P}$ emerging from (5) are typically only one-third of those given by (4), since the latter treatment incorrectly takes velocity change across the viscous sublayer as generating turbulence energy. An alternative approach, which is

sometimes employed, is to assume the local value of production at the near-wall node, P_s to be equal to the average over the cell. The mean-velocity gradient is then determined from the logarithmic law, e.g.:

$$\overline{P} = \tau_w \left.\frac{\partial U}{\partial y}\right|_p = \frac{\tau_w U_p}{y_p} \frac{1}{\kappa \ln(E\, y_p^+)} \qquad (6)$$

The near-wall energy dissipation rate has usually been obtained by assuming a universal dissipation length scale $\ell_\varepsilon = C_\mu^{-3/4}\kappa y$, and thus $\varepsilon = k^{3/2}/(C_\mu^{-3/4}\kappa y)$. In a near-wall flow in local equilibrium, this choice gives a mixing length equal to κy. A variation of this approach has been used by Mansour and Morel (see Vol. III), who assume a linear variation in ℓ_ε from the wall to the second node away from the wall. The level at the second node is then obtained from the ε and k values emerging from the finite-difference solution. This practice leads to lower levels of ε near separation than would be given by a fixed length scale. This difference is perhaps the main reason that these workers report the shortest reattachment length of the five groups tackling case 0421 with the k-ε Boussinesq model.

The question arises whether a point or a space average value of ε should be used. Strictly, if the production rate is an average value over the cell, the dissipation rate should also be so averaged. The usual practice in the codes employed by Pollard, Humphrey, Spalding, Chow (see Vol. III) and others is to use the value at node P as representative of that over the cell. The Chieng-Launder (1980) formulation, used by a few groups, employs an integral treatment, and includes a contribution associated with turbulent energy dissipation in the viscous sublayer.

As it happens, the widely different practices in handling the wall-adjacent region appear to have had no substantial effect on the predictions of the external flow. They will, however, lead to differences in the level of wall friction near separation. At present there is an insufficient number of cases providing data of C_f where separation occurs (or is imminent) to allow an assessment of the relative merits of these near-wall treatments.

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COMPLEX STRAIN FIELDS

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INTRODUCTION

The 1968 AFOSR-IFP-Stanford Meeting dealt with flows in which, with one or two exceptions, the only significant rate-of-strain component was $\partial U/\partial y$. These flows will hereafter be called "simple shear layers"; jets, wakes and so on are included, as well as boundary layers. Since 1968 it has gradually been realized that extra rates of strain, additional to $\partial U/\partial y$, can have a surprisingly large effect on turbulence--that is, calculation methods and mental concepts based entirely on measurements in simple shear layers may significantly underestimate the effect of extra rates of strain. Even three-dimensionality of the mean flow, which adds an extra mean shear component $\partial W/\partial y$, can defeat calculation methods based on extension of two-dimensional methods; for example, the eddy viscosities defined for the x-component and z-component shear stresses can be markedly different. In two-dimensional flows, several possible extra strain rates exist, and even $\partial V/\partial x$ (i.e. streamline curvature) has a large effect on turbulence, surprisingly since it is merely an addition to the rate of shear strain.

Once the failure of simple concepts and simple calculation methods to predict the effects of extra strain rates became clear, modified calculations began to appear. At first, the modifications were merely ad-hoc correlations of experimental data, generally based on the obvious dimensionless parameter, the ratio of the extra strain rate to the mean shear $\partial U/\partial y$. More recently, methods based on the Reynolds-stress-transport equations have been devised which predict at least some of the effects of extra strain rates. Of course, the Navier-Stokes equations and exact equations derived from them contain the effects of extra strain rates automatically, but the results of the present meeting show that even the most sophisticated one-point transport-equation models do not give entirely satisfactory predictions of curved flows. A physical discussion of the effect of extra strain rates should provide a useful link between the proceedings of the 1968 meeting and those of the present conference on more advanced flows.

In addition to "extra rates of strain" as defined above, the title of this paper can be held to include abnormal distributions of $\partial U/\partial y$--notably, changes of sign--and fluctuating rates of strain in the form of an external turbulence field. These topics are discussed more briefly, with references to other reviews. A topic which is nominally excluded by the title but will nevertheless be included because of its relevance is the effect of body forces (buoyancy or Coriolis for instance) on turbulent flows; the effect of body forces on turbulence can also be very much larger than would be expected from simple consideration of time-averaged transport equations.

Nearly all turbulent flows in which Reynolds-stress gradients have a significant effect on the mean acceleration are identifiable as shear layers, perhaps strongly distorted ones. That is, the Reynolds shear-stress gradient has a larger effect on the streamwise acceleration than any normal-stress gradients. In strongly distorted flows, more or less by definition, pressure gradients greatly exceed Reynolds-stress gradients over short distances; Reynolds-stress gradients are therefore locally unimportant, but the influence of the distortion on the Reynolds stresses may be remembered far downstream, and therefore needs to be represented in calculation methods. Except in a very few cases such as flow near the stagnation point of an impinging jet, the ratio of any extra strain rate, such as $\partial V/\partial x$ or $\partial U/\partial x$, to the simple shear $\partial U/\partial y$ will seldom exceed about 0.1; a flow which is still dominated by the simple shear, although other velocity gradients are not small enough to justify the application of the thin-shear-layer approximation (which, roughly speaking, requires extra strain rates not to exceed about 0.01 $\partial U/\partial y$) may be called a "fairly thin shear layer". We can therefore distinguish

1. Simple shear layers in which any extra strain rate e is so much smaller than $\partial U/\partial y$ that it does not affect the turbulence structure, say $e/(\partial U/\partial y) < 0.001$.
2. Thin shear layers in which $e/\partial U/\partial y$ is sufficiently small, say 0.01, that the thin-shear-layer approximation may be applied, but not necessarily small enough for the effect of extra rates of strain on the turbulence to be negligible.
3. Fairly thin shear layers, $0.01 < e/\partial U/\partial y < 0.1$.
4. Strongly distorted flows, $e/\partial U/\partial y > 0.1$.

These classifications, although rough, will be found useful in the discussion below; note that, alas, it is necessary to distinguish (1) and (2), since effects of extra rates of strain can be quite strong even in "thin" shear layers. In the case of body forces, a quantity having the same dimensions as strain rate can be defined and used in the same way as e; examples include the use of the angular velocity of rotation in the case of Coriolis force, and the Brunt-Väisälä frequency N in the case of buoyant flows.

The discussion below will be based on the exact transport equations for Reynolds stress, obtainable by simple but tedious algebra from the time-averaged Navier-Stokes equations. This is not to be taken as an authoritative statement that only calculation methods based explicitly on Reynolds-stress-transport equations can possibly deal with complex strain fields (although such is the author's opinion), but since these equations are the only exact ones describing the evolution of the Reynolds stress it may be said that any turbulence model is in some sense a simplified representation of them. If we write the Reynolds-stress-transport equation in words as

(transport by mean flow) = (generation by interaction with mean flow)
- (redistribution by pressure-strain fluctuations)
- (transport by turbulent fluctuations)
- (viscous dissipation)

where the pressure-strain term sums to zero in the turbulent-energy equation and the viscous-dissipation term is negligible in shear-stress equations, we can see that the explicit effect of extra strain rates appears only in the generation terms. (In flows with body forces, extra generation terms appear.) The unexpected feature of the influence of many extra strain rates and body forces on turbulent flow is that the change in the Reynolds stresses is considerably larger than would be expected merely from the size of these explicit extra terms; it follows that extra strain rates must implicitly affect the other main terms in the equation, notably the pressure-strain term or the dissipation (the effect on the turbulent-transport terms can also be significant, but, obviously, affects only the distribution of stress rather than its general level).

EXTRA STRAIN RATES IN TWO-DIMENSIONAL OR AXISYMMETRIC FLOW

The extra strain rates that are known to produce surprisingly large effects on turbulent boundary layers (say) are

(a) curvature, $e = \partial V/\partial x$

(b) lateral divergence, $e = \partial W/\partial z$

(c) bulk compression, $e = -\mathrm{div}\underset{\sim}{U}$

where the definition of positive strain rate has been chosen so that it implies an increase in turbulent intensity and/or shear stress in a boundary layer with conventional axes. Effects of extra strain rates on free shear layers are less well documented than effects on boundary layers, but there are enough data to indicate that the effects are again large. Launder (1980) points out that a careful consideration of the generation term in the shear-stress equation, and the coupling between the different Reynolds-stress-transport equations, indicates a rather larger direct effect of the extra generation terms than would appear at first sight, at least for the effect of curvature, however, this comment certainly does not apply to all of the Reynolds-stress-transport equations and all of the cases--for example, lateral divergence markedly increases turbulent kinetic energy, as well as shear stress, but the extra turbulent energy production rate is negative. The work of Keffer (1965, 1967) shows that rather large changes of eddy structure occur when lateral convergence or divergence is applied to a wake, and there is a good deal of evidence of large changes in turbulence structure for both stabilizing and destabilizing longitudinal curvature. Bradshaw (1973) reviews most of the experimental work on the effects of extra strain rate done up to that time, and an update with special reference to stabilizing

curvature is provided by Gillis and Johnston (1980). There is little doubt that a real problem still exists, and that further experimental and theoretical work is needed before calculation methods can be used to predict the effects of extra strain rate with reasonably quantitative accuracy.

Qualitatively, the effects of longitudinal curvature of the streamlines in a flow with fixed boundaries are very similar to the effects of Coriolis force in flow rotating about a spanwise axis, not unreasonably since a rotating flow has curved streamlines as seen by a fixed observer. Structural changes observed in buoyant flows are also qualitatively similar to those seen in curved flows; crudely, buoyancy forces due to density changes in a gravitational field are equivalent to "centrifugal force". Quantitative analogies between buoyant flows and curved flows can now be regarded as superseded by the increasing experimental knowledge of curved flows as such. There is ample evidence that unstable curvature produces a tendency to longitudinal vortex motions, which appear (Hoffmann and Bradshaw, 1981) to be effectively steady patterns rather than modifications of the large eddy structure of the turbulence proper. Stabilizing curvature produces a marked reduction in turbulence intensity in the outer part of the shear layer, but again the large eddy structure appears to be qualitatively the same. Quantitative changes, in stress/intensity ratios and dimensionless triple products, can of course be quite large.

The effect of lateral divergence can be attributed to the intensification of spanwise vorticity in the large eddy motions; Keffer's measurement and flow visualization in a wake show this rather well, although compared with the amount of attention given to streamline curvature effects the effect of lateral divergence has not been very well covered by experimenters. A hint that large changes in large eddy structure may occur in general is given by the measurements of Smits et al. (1979) who found that the flow over a body of revolution in which concave longitudinal curvature was followed by lateral divergence did not develop the longitudinal vortices that would have been expected if such curvature had occurred without lateral divergence.

The effect of compression or dilatation of turbulent shear layers is again not very well documented, but Rubesin (1977) has shown that at least two of the current transport equation turbulence models for compressible flow will reproduce bulk compression effects at least qualitatively. However, the mechanism of compression effects (J. E. Green, private communication) appears to be the reduction of cross-sectional area, in the x,y plane, of fluid elements, leading to an increase in their spanwise vorticity, so one might expect the effects on eddy structure to be much the same as in lateral divergence. The best documented effect of bulk compression is certainly surprising, being the tendency for skin-friction coefficient to increase, and shape parameter to decrease, in strong adverse pressure gradients at high supersonic speeds. The effects are very much larger than would be expected merely from the

Mach number effect on these parameters. The work of Narasimha and Viswanath (1975) on centered expansions at sharp convex corners showed a marked reduction in turbulent intensity, partly attributable to convex curvature but apparently owing something to the dilatation as such. Very large rates of dilatation occur in combustion processes, and flow visualization pictures of turbulent combustion flame fronts seem to show much less wrinkling than one would expect in a non-combusting flow at a similar Reynolds number, suggesting that the fine structure, at least, may be partly suppressed by dilatation--but this is a subjective judgment, and the effects of extra strain rate must be accounted among the minor problems of modeling turbulent combustion! The table below sets out recent experimental work on extra strain rates.

TABLE 1

Recent Experimental Work on Complex Strain Fields

(a) Boundary layers on curved surfaces (0231-0233)*

Brinich and Graham, 1977; Crane and Winoto, 1980; Ellis and Joubert, 1974; Gillis and Johnston, 1980; Hunt and Joubert, 1979; Mayle et al., 1979; Mayle et al., 1977; Ramaprian and Shivaprasad, 1977; Ramaprian and Shivaprasad, 1978; Shizawa et al., 1980; Shivaprasad and Shivaprasad, 1978; Smits et al., 1979; Sundarasiva, 1980.

(b) Wall jets on curved surfaces (0263, 0331)

Alcaraz et al., 1977; Guitton and Newman, 1977; Wilson and Goldstein, 1976.

(c) Rotating flows (0372)

Furuya et al., 1977; Ibbetson and Tritton, 1975; Koschmieder, 1979; Koyama et al., 1979; Kuzay and Scott, 1977; Lezius and Johnston, 1976; Rothe and Johnston, 1975; Rothe and Johnston, 1979.

(d) Vortices and swirling flows

Baker et al., 1974; Escudier et al., 1980; Panton et al., 1980; Pullin and Perry, 1980; Uberoi, 1979; Young and Rao, 1978.

(e) Lateral convergence/divergence (0142, 0143)

Keffer et al., 1978; Patel and Lee, 1978, Patel et al., 1974; Robinson, 1974; Smits et al., 1979; Tanaka and Tanaka, 1976; Tanaka and Tanaka, 1977; Witze and Dwyer, 1976.

(f) Compressible flow in pressure gradient (8403, 8411, 8631)

Fernholz and Finley, 1980; Laderman, 1980.

*See also the references for the test case flows whose case numbers are given in parentheses.

BODY FORCES

As commented above, the two main forces, buoyancy forces and Coriolis forces, both produce qualitatively the same effects as surface curvature. Judging by the success of conventional turbulence models in predicting vertical plumes and vertical free convection boundary layers (e.g. Cebeci and Khattab, 1975) longitudinal body forces do not have a very large effect on turbulence. However, body forces acting normal to the plane of a shear layer produce large effects which are a matter of everyday meteorological experience. Large bodies of meteorological data exist (e.g. the book edited by McBean, 1979), but at this point it should be commented, without rancor, that standards of accuracy in meteorological prediction methods are poorer than those to which engineers aspire, so that turbulence models which are acceptable for meteorological purposes ought not to be taken over for engineering purposes without careful scrutiny.* Qualitative meteorological data and concepts have proved extremely valuable in forming our ideas about the effects of extra strain rates; evidence for the presence of longitudinal vortex rolls is a case in point, and the prevalence of internal waves in meterological flows should encourage one to seek them in curved or rotating flows also.

The lateral body forces found in shallow layers of rotating fluid can produce various spectacular effects (Turner, 1973) including the generation of hurricanes, tornados and their laboratory equivalents, but there appears to be little information about changes in turbulence structure, and designers of rotating machinery generally regard these effects as being submerged in the general difficulties of predicting three-dimensional shear layers, the effects of roughness, and transient conditions.

THREE-DIMENSIONAL FLOWS

Most unfortunately it has not proved possible at this Conference to present basic test cases for three-dimensional boundary layers on swept wings, and other mildly three-dimensional flows. The results of the Trondheim meeting reported by East (1975) showed remarkably poor agreement between all available boundary-layer-calculation methods and the three-dimensional separating boundary layer investigated by van den Berg and Elsenaar (1975). Other data sets produced disagreements which were qualitatively similar but less spectacular; and the general consensus is that current calculation methods of all types overestimate the spanwise component of eddy viscosity (that is, the ratio $\overline{vw}/(\partial W/\partial y)$ in conventional axes). Rotta (1976, 1979) and Schneider (1977) have attempted to correlate the differences in eddy viscosity, or its equivalent, in terms of the difference in direction between the shear-stress vector and the velocity or velocity-gradient vector. These attempts have not been wholly

*[Ed.: See also the remarks by J. Wyngaard, Vol. I, pp. 314-316.]

satisfactory, and it certainly does not seem that $\partial W/\partial y$ can be regarded as an "extra strain rate" in the sense discussed above (it should be noted that in most three-dimensional boundary layers in wind tunnel experiments, $\partial W/\partial y$ is given quite accurately, in the outer part of the layer, by the inviscid secondary-flow formula based on turning angle, and it may be that the rate of change of flow angle, roughly $\partial^2 W/\partial x \partial y$ or possibly just $\partial W/\partial x$, may be the relevant "extra strain rate"). The van den Berg experiment has been repeated by Pontikos (1982).

This writer, at least, feels that investigation and modeling of mildly three-dimensional boundary layers should have a high priority in turbulence studies, if only because persons inexpert in the subject (at least!) are likely to doubt the community's ability to predict strongly-three-dimensional flows with imbedded vortices, while the swept wings that have been in service for the last thirty years or so still present us with such fundamental problems.

ABNORMAL DISTRIBUTIONS OF $\partial U/\partial y$

Eddy-viscosity-based methods, whether they employ transport equations or simply algebraic formulae, fail rather obviously in flows where $\partial U/\partial y$ changes sign (except for symmetrical flows where eddy viscosity is well behaved at a velocity peak although mixing length is not). In general, the shear stress is non-zero at the point of zero $\partial U/\partial y$ so that the eddy viscosity is, or should be, infinite. In most simple asymmetric shear layers, such as wall jets, the distance between the points of zero shear stress and zero $\partial U/\partial y$ is small, and the overall results of eddy-viscosity methods are not too seriously in error. However, this shortcoming of eddy-viscosity methods may be responsible for a large part of the error in predictions of recirculating flows—although stress-transport models are not very successful in this case either.

The reason for the separation of the points of zero shear stress and zero velocity gradient is the turbulent transport of shear stress in the y-direction; models which represent this adequately seem to have no trouble in simple asymmetric shear layers (or, presumably, in shear layers with negative $\partial U/\partial y$ imposed in the free stream).

The boundary layer and wake of a multi-element airfoil (fitted with a leading-edge slat and one or more slotted trailing-edge flaps) form a severe test case for turbulence models, since the wake of one element merges with the boundary layer on one surface of the next element and the velocity profile at the final trailing edge can have several reversals of gradient. Also, small separated flow regions are usually present, even at design conditions.

FREE-STREAM "TURBULENCE"

The quotes are advisable because, particularly in turbomachinery, free-stream disturbances may include periodic and/or irrotational fluctuations as well as true

turbulence. The test case for this meeting, nominally-isotropic grid turbulence, is a somewhat idealized one, but few systematic investigations of real-life sources of free-stream disturbances exist. Hancock (1980) to whom the test case of Flow 0211 is originally due, reviews measurements in shear layers with grid turbulence at some length. The work of Walker (1972) on the intersection of (moving) blade wakes with the following row of (stationary) blades in an axial turbomachine, which is being extended by Schultz at Oxford, would be an ideal subject for modern conditional sampling techniques.

MODELING

The essential question in modeling flows in complex strain fields is, of course, whether the empirical constants in the model become functions of rate-of-strain parameters like $e/(\partial U/\partial y)$ or, better, eL/q where L is a turbulence length scale and q is a velocity scale such as $(\overline{u_i^2})^{1/2}$. With the exception of $e \equiv -\text{div } \underset{\sim}{U}$, the appearance of e-dependent parameters destroys the rotational invariance of the model, and since the use of rotational invariance to establish relations between modeling constants is common practice, this is an embarrassment. However, Launder (1980) has successfully added non-invariant terms (proportional to production by normal stresses) to his dissipation transport equation in order to improve predictions for thin shear layers subjected to a range of values of $\partial V/\partial y$; although $\partial V/\partial y$ is, of course, nominally an extra strain rate, it is not generally thought to have "surprisingly large effects". Therefore, non-invariant allowances for the mean extra strain rates of interest should not be regarded as illegal, immoral or even inconvenient. As indicated above, most flows that are controlled by Reynolds-stress gradients are identifiable shear layers, and the axes to which the model equations are referred can be aligned with the direction of the shear layer, perhaps automatically by the computer.

The loss of rotational invariance remains an embarrassment, and several authors have obtained good agreement between invariant model predictions and some experiments on flows with extra strain rates. For example, Gibson (1981) compared predictions of a $\overline{u_i u_j}$ -ε model (essentially Launder's) with the data of Castro and Bradshaw (1976; flow 0331 of this conference) in a strongly-stabilized curved mixing layer. The effects of curvature were reproduced to an accuracy not much poorer than that of the data. However, results as good as those of a full transport model were obtained by assuming a length-scale correction factor $1 - 2.1R_f \approx 1 - 4.2e/(\partial U/\partial y)$ where R_f is a curvature parameter analogous to the flux Richardson number. The empirical constant 2.1 was chosen to reproduce the length-scale variation calculated by the full transport model, but corresponds closely to a best-straight-line fit through the plot of L/L_o against R_f given by Castro and Bradshaw. It is much smaller than the factor of 5 to 7 needed to reproduce <u>small</u> curvature effects. The flow is so highly curved that a

linearized correction is certainly not strictly applicable; curvature effects in the experiment are limited by nonlinear effects—such as the non-negativity of dissipation rate—so that the linear correction successfully used by Gibson probably represents an approximation to a curve in the R_f, L/L_o plane which starts as $L/L_o = 1 - 5R_f$ (say) for small R_f and necessarily limits as $L/L_o \to 0$ as $R_f \to \infty$. Similar comments apply to the correlation for strongly stabilized boundary layers on convex surfaces developed by Gillis and Johnston (1980). Irwin and Smith (1974) show that their version of Launder's model (Hanjalić and Launder, 1972) reproduces curvature effects in the outer layer but not in the inner layer (the reason being that the model response to curvature depends strongly on $\overline{v^2}/\overline{u^2}$).

Rubesin (1977) shows that the transport-equation model of Donaldson and the eddy-viscosity-transport model of Wilcox reproduce the surprising rise in skin-friction coefficient in retarded supersonic boundary layers. The question of how they do it is meaningful only if one finds the rise surprising, but certainly simpler eddy-viscosity or stress-equation models do not show a rise in c_f. Wilcox's model also gives good results in curved flows, evidently because $\overline{v^2}$, which receives the extra production rate $-\overline{uv}\,\partial V/\partial x$, is used as a velocity scale in preference to the more usual $\overline{u_1^2}$.

The writer is not aware of any successful computations of laterally-diverging flows by models without e-dependent correction terms; prediction of the radial wall jet, which has about the same $d\delta/dx$ as the plane wall jet and therefore twice the entrainment rate, would be a suitable test case. As mentioned above, fully-three-dimensional flows present basic problems.

Simple corrections for extra strain effects are generally implemented in the specification of length scale. In length-scale or dissipation-transport equations, e naturally appears in the form

$$\frac{DL}{Dt} = c_1\, e\, L + \ldots$$

$$\text{or} \quad \frac{D\varepsilon}{Dt} = -\, c_2\, e\, \varepsilon + \ldots$$

and if we neglect coupling with other equations this form predicts exponential growth or decay, whereas what really happens in response to—say—a small step change in e is that L changes, with a time constant of order $10\delta/U$, to values which are roughly those predicted by $L/L_o = 1 + 10e/(\partial U/\partial y)$. It seems unlikely that coupling will remedy the implausible behavior of the transport equation because the e-dependent terms do not contain the dependent variables of other transport equations.

Recently several authors (e.g. Townsend, 1980) have used rapid-distortion theory to calculate Reynolds stresses, or stress ratios, in complex strain fields. Now rapid-distortion theory is strictly applicable only if the extra terms introduced with

the Reynolds-stress-transport equations by the distortion (say, the extra production in the turbulent-energy equation) are large compared with all existing terms (say, the dissipation rate). This is very nearly the same as requiring $e/(\partial U/\partial y) \gg 1$, which is so far from being satisfied in most cases of practical importance that this writer believes the quantitative results of rapid-distortion theory to be untrustworthy. In brief, the theory supplies a model of the pressure-strain term which is valid in the limit $e/(\partial U/\partial y) \to \infty$ but is at best only a qualitative guide for $e/(\partial U/\partial y) < 0.1$.

The present position, therefore, is a not unfamiliar one in turbulence research; some calculation methods reproduce some of the experimental results, but the process by which they do it is unclear or unrigorous. In assessing the calculations submitted to this conference, the question of whether, and how, the models simulate the effects of extra rate of strain stands out as the basic question to be asked about our progress since 1968.

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COMPRESSIBILITY EFFECTS IN TURBULENCE MODELING

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INTRODUCTION

Traditionally, developments in turbulence modeling have been made in analyses which take advantage of the considerable simplifications that result from the assumption of constant-fluid properties. Modelers dealing with problems involving chemistry or high-speed aerodynamics have had to modify such models to account for the variations in the real-fluid properties. In keeping within the spirit of the present conference, where fluid mechanics is emphasized relative to heat and mass transfer, only those property variations due to compressibility in an adiabatic flow will be treated here. The fluid will be treated as a single species.

The manner of accounting for compressibility depends to a large extent on the basic type of model being modified. The present discussion will distinguish between models that are used with integral quantities represented by ordinary differential equations or those that are used with partial differential conservation equations. The modifications to the latter models will be classified according to whether they employ an eddy viscosity or deal directly with Reynolds stresses. In addition, the eddy-viscosity models are distinguished between algebraic models or models that use two transport equations to define the local intensity and scale of turbulence. Finally, a brief description is given of the effects of compressibility on turbulence that were found in a numerical simulation of low-Reynolds-number, homogeneous shear flow.

COMPRESSIBILITY CORRECTIONS IN BOUNDARY-LAYER INTEGRAL METHODS

The boundary-layer lag entrainment method of Green et al. (1973) will be used to illustrate how compressibility corrections are introduced into integral methods. The salient points in the compressibility modifications are outlined in Fig. 1. The local mean density is used in the definitions of the displacement and momentum thicknesses, and, consequently, in the shape factor H. An additional shape factor, $\bar{H}$, is introduced which in its numerator accounts for the mean density across the boundary layer as well as the displacement. For incompressible flow this shape factor reduces to the usual H. The entrainment factor, also containing local density, is related empirically to the $\bar{H}$, in the same manner as are their incompressible counterparts. The von Kármán integral relationship in its compressible form contains an additional Mach number squared term in the group containing the shape factor. Additionally, the local skin-friction coefficient on a flat plate, used as a reference quantity for the actual skin friction, is found from its compressible relationship employing the Reynolds

number based on momentum thickness but where the variables are stretched to account for compressibility in the manner introduced by Spalding and Chi (1964). Incidentally, the particular stretching functions used by Green et al. (1973) for introducing compressibility differ considerably from the Spalding and Chi forms, or those forms that result from reference temperature methods (Sommer and Short, 1955) or transformations such as Van Driest's (1956) or Coles (1962). The local skin-friction coefficient in the presence of a pressure gradient is related to its corresponding flat-plate quantity through the local $\bar{H}$ and an $\bar{H}_0$, the latter being consistent with the flat-plate skin-friction coefficient and the local Mach number. The local shape factor, H, is related to $\bar{H}$ by use of an adiabatic, Crocco relationship. The entrainment equation utilizes compressible forms of the shape factors and the local skin-friction coefficient. The lag equation, which is based on the Bradshaw and Ferriss version of the turbulence kinetic energy equation (1971), is modified to account for compressibility through the use of local densities and of compressible versions of the parameters defined earlier.

COMPRESSIBILITY CORRECTIONS TO MODELS FOR USE WITH PARTIAL DIFFERENTIAL CONSERVATIONS EQUATIONS

Averaging Processes

Preliminary to correcting turbulence models for use in the solution of the partial differential conservation equations governing the flow of a compressible fluid, it is necessary to adopt a particular set of dependent variables in which to express the flow field. Two approaches are indicated in Figs. 2a and 2b. Both approaches begin with the instantaneous partial differential equations for the conservation of mass and momentum, and an algebraic relationship representing the conservation of energy in an adiabatic flow. The first method is based on expanding each dependent variable as the sum of a mean and fluctuating quantity. Then the equations are averaged in the Reynolds sense, where the average of an averaged quantity remains unaltered and that of a single fluctuating quantity vanishes. Only moments of fluctuating quantities result in nonzero average turbulence quantities. The averaging process transforms the conservation and energy equations to those indicated in the lower half of Fig. 2a. Unlike the case of incompressible flow, where this expansion and averaging process introduce only a single moment tensor (the Reynolds stresses), now many additional moments occur that are dependent explicitly on the density fluctuations or mass flow fluctuations as indicated by the starred quantity. For steady-state boundary-layer flows, these additional terms do not introduce significant difficulties. Here $\overline{\rho' u_1'}$ is clearly negligible relative to $\overline{\rho u}_1$, and this allows the evaluation through continuity of the entire group $\overline{\rho u}_2 + \overline{\rho' u_2'}$. Also, the only significant Reynolds stress occurs in the term where $j = 2$, i.e., normal to the surface, and

there the $\overline{u_2 \rho' u_1'}$ term is of smaller order than $\overline{\rho u_1'' u_2''}$. With the neglect of third-order correlations, and use of these order-of-magnitude arguments, it was not necessary to model the density fluctuation terms. When the boundary-layer order-of-magnitude arguments break down, as in the region near separation or in recirculating flows, the principal density fluctuation terms cannot be argued away and must be modeled. A model that does this will be described briefly later.

An alternative means of deriving statistical conservation equations for turbulent flow is to use the mass-weighted dependent variables introduced by Favre (1978). In this method, the velocity components (and temperature and enthalpy) are resolved into the sum of a mass-weighted mean plus a corresponding fluctuation. In Fig. 2b these are represented with superscript tilde and double primes, respectively. The rules for averaging the mass-weighted quantities are indicated on the figure. The mean of a mass-weighted mean quantity is itself. The mean of the mass-weighted fluctuating quantity is not zero; but terms containing single mass-weighted fluctuations can be made to vanish by mass-weighting them with the density before averaging, i.e., $\overline{\rho A''} = 0$. The conservation equations that result from this operation, indicated in Fig. 2b, contain no more terms than their incompressible counterparts. In addition, the Reynolds-stress components retain their symmetry, unlike the starred quantity in Fig. 2a.

From a computational viewpoint, with an eddy-viscosity model these equations are in the same conservation form and possess the same dependent variables as the equations governing laminar flow. The same solution algorithms can be applied to both types of flow without alteration, and this alone has made the use of mass-weighted variables very popular. From a modeling viewpoint, there is an apparent simplification in that the effects of turbulence are felt in the conservation equations primarily through the Reynolds stresses. It will be indicated later that in modeling these Reynolds stresses with field equations, questions about density fluctuations will reappear. The effects of viscosity fluctuations in $\overline{\tau_{ij}}$ are usually small.

Eddy-Viscosity Models

The mass-weighted Reynolds stress can be expressed in terms of an eddy viscosity with the tensorial constitutive relationship indicated at the top of Fig. 3. This relationship differs in form from its incompressible counterpart in that the mean rate-of-strain term is modified by a term containing the mean divergence, which in compressible flow is not equal to zero. This modification allows the trace of the Reynolds-stress tensor to reduce to twice the kinetic energy, as it should.

The lower part of Fig. 3 shows an eddy-viscosity model that is currently popular for use in the solution of attached compressible boundary layers or flows with small regions of separation. The forms shown are identified with Cebeci and Smith (1974)

but are characteristic of the extensions to compressibility used in several other models (e.g., Maise and McDonald, 1968; and Price and Harris, 1972). The model divides the boundary layer into an inner and outer region, reflecting the concept of the wall and wake regions of incompressible flow (Coles, 1956). In the inner region, other than in the use of the compressible form of strain, compressibility is introduced in this model by the local density in the definition of the eddy viscosity and by modifying the Van Driest (1956) damping relationship for the mixing length to utilize the physical properties at the wall. Away from the wall, the mixing length remains as in incompressible flow. In the outer region, the eddy viscosity again depends on the local density. The length scale, as in incompressible flow, is set proportional to the kinematic displacement thickness or to the overall boundary-layer thickness and is, therefore, only implicitly dependent on the compressibility.

The aforementioned algebraic models imply an equilibrium between the mean motion of the fluid and its local turbulence. When changes occur rapidly in a flow field, the turbulence may lag the mean motion and, to allow for this out-of-equilibrium state, models have been developed that utilize transport equations for the turbulence itself. The simpler of such models represent the transport of variables used in defining an eddy viscosity. The eddy viscosity, then, can be out of equilibrium with the mean motion. The Reynolds stresses, however, being tied to the mean motion through constitutive relationships, react immediately to changes in the mean motion, even though the magnitude of this reaction is mitigated to the extent of the lag of the eddy viscosity. Thus, eddy-viscosity models are still tightly coupled to the mean motion even though differential equations are used for the turbulence quantities. Typical of such models is the Jones/Launder "k-ε" model shown in Fig. 4. In this model, the eddy viscosity is found from the local turbulence kinetic energy and dissipation rate. This model has been extended to compressible flow by computational methods (e.g., Viegas and Horstman, 1978), where the kinetic energy and dissipation rate have been interpreted as mass-weighted quantities. The equations defining these quantities are identical, term by term, with their incompressible counterparts except that the local density applies and the nonzero divergence of the mean motion is taken into consideration. Earlier analyses (Rubesin, 1976; Wilcox and Alber, 1972; and Wilcox and Traci, 1976) showed that applying mass-weighted averaging to the turbulence equations introduced additional terms that were dependent directly on the compressibility and had no counterpart in the corresponding incompressible equations. Mass-weighting alone does not account entirely for all the effects of compressibility. These additional terms where density fluctuations appear explicity were modeled (Rubesin, 1976; Wilcox and Alber, 1972; and Wilcox and Traci, 1976) but subsequent calculations with these modeled terms (Coakley and Viegas, 1977) showed them to be relatively unimportant in a variety of flow fields at Mach numbers below 5. If these extra terms are neglected,

the equations for the kinetic energy and the dissipation rate shown in Fig. 4 are identical in form with their incompressible versions. The effect of using the local mean density in these equations can be illustrated by considering their behavior in the logarithmic region of a compressible boundary layer. If the mean transformed velocity is expressed in logarithmic form, and the local shear is related to the wall-shear and mass-weighted kinetic energy as indicated in Fig. 4, the resulting density scaling within the turbulent transport equations is indicated in the bottom of the figure. The use of the local mean density in the transport equations then provides an eddy viscosity in the log region of a boundary layer that is identical with the one resulting from the mixing-length model shown in Fig. 3.

ynolds-Stress Models

Models for the transport of each of the Reynolds stresses in compressible flow have been developed in both primitive and mass-weighted dependent variables. The most complete model in primitive variables is due to Donaldson and his colleagues (e.g., Donaldson and Sullivan, 1972; and Sullivan, 1978). It is expressed in a set of partial differential equations for the variables indicated in the upper part of Fig. 5. The length scale in this model, Λ, can be prescribed algebraically or with a differential equation that evolved from an equation representing two-point correlations. Note that the first moments involving density fluctuations are part of the set of modeling equations. To close these equations, tensor invariant models for 41 other second- and third-order moments are developed. Many of these are necessary to account explicitly for the density fluctuations.

Reynolds-stress transport models have also been developed in mass-weighted dependent variables (Dussauge, 1981; and Wilcox and Rubesin, 1980). Again, as in the two-equation models, the effects of density fluctuations are largely implicit. This reduces the modeling required to close the turbulence-transport equations to about 12 moments. The important terms representing the correlation of the fluctuations of pressure and rate of strain are treated by forms identical to their incompressible counterparts, but with the mean rate of strain modified by the mean dilatation. The assumption leading to this approach is that the mass-weighted turbulence is solenoidal in form, so that even in a compressible flow the trace of the pressure rate of strain terms vanish. This approach is consistent with the form of the mass-weighted kinetic-energy equation indicated in Fig. 4. The length-scale parameter, either the dissipation rate, ε, or the dissipation rate per unit of kinetic energy, ω, is again found from an equation identical to that used in the corresponding two-equation model.

COMPRESSIBILITY SIMULATION

The effect of compressibility on turbulence has recently been studied in a numerical simulation of a homogeneous shear flow (Feiereisen et al., 1981). Although the

computation is restricted to low Reynolds number and zero mean dilatation, some of the effects of compressibility observed in the computations are quite interesting and may apply to a broader range of conditions. The contributions of compressibility can be delineated quite clearly in wave number space by resolving the Fourier transforms of the velocity vectors into components perpendicular and parallel to the wave number vector, i.e., the solenoidal and "compressible" components, respectively. The contributions of each in real-space statistical correlations is then evaluated. It was found that the contributions of the "compressible" parts of the Reynolds stresses and dissipation were very small. Special care also was taken to include the effects of density fluctuations on the fluctuating pressure in the pressure rate-of-strain correlations. Incidentally, this effect was neglected in the Poisson equation for pressure used in the Reynolds-stress models described earlier. For the high turbulence Mach number of the calculations, it was found that these density-fluctuation terms contributed as much as the normally retained "slow" and "fast" terms. Where the turbulence Mach number is reduced to that likely to be encountered in aerodynamic environments, however, even at $M = 5$, the contribution to the pressure rate of strain is negligible and gives some degree of justification to the neglect of the contribution of density fluctuating terms in the Reynolds-stress models discussed earlier.

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REF. GREEN'S LAG ENTRAINMENT METHOD

- USE LOCAL MEAN DENSITY IN DEFINITIONS OF δ^*, θ, AND H
- ADDITIONAL SHAPE FACTOR: $\bar{H} = 1/\theta \int_0^{\delta} \frac{\rho}{\rho_e}\left(1 - \frac{U}{U_e}\right) dy$
- ENTRAINMENT FACTOR $H_1 = 1/\theta \int_0^{\delta} \frac{\rho U}{\rho_e U_e} = \frac{\delta - \delta^*}{\theta} = \text{EMP}(\bar{H})^*$
- COMPRESSIBLE MOMENTUM EQ. (i.e. INCLUDES M^2 TERM)
- MODIFIED INCOMPRESSIBLE FLAT PLATE SKIN FRICTION – $F_c C_{f_o} = f_1(F_R R_\theta)$
- LOCAL SKIN FRICTION – $C_f = f_2(C_{f_o}, \bar{H}_o, \bar{H})$ $\bar{H}_o = f_3(C_{f_o}, M)$

LOCAL SHAPE FACTOR – $H = f_4(\bar{H})$, ADIABATIC RELATIONSHIP

- ENTRAINMENT EQ. – UTILIZES COMPRESSIBLE SHAPE FACTORS, C_f
- LAG EQUATION (C_E) – BRADSHAW FERRISS τ/ρ EQ. FOR $(\tau/\rho)_{MAX}$
  $\bar{H}$ = CONST. DEFINES EQUILIBRIUM

$$\theta \frac{dC_E}{dx} = f(C_1, \bar{H}, M)$$

*(An empirical function of $\bar{H}$).

Figure 1. Compressibility corrections in integral methods.

CONTINUITY $\rho_{,t} + (\rho u_j)_{,j} = 0$

MOMENTUM $(\rho u_i)_{,t} + (\rho u_j u_i + \delta_{ji} p - \tau_{ij})_{,j} = 0,\ \tau_{ij} = \mu[(u_{i,j} + u_{j,i}) - \frac{2}{3}\delta_{ij} u_{k,k}]$

ENERGY $\frac{\gamma}{\gamma - 1} p + \frac{1}{2}\rho u_i u_i = \rho C_p T_{tot}$

REYNOLDS AVERAGING $u_i = \bar{u}_i + u_i',\ \rho = \bar{\rho} + \rho'$ $\quad \bar{\bar{A}} = \bar{A},\ \overline{A'} = 0$

$$\bar{\rho}_{,t} + (\bar{\rho}\bar{u}_j + \overline{\rho' u_j'})_{,j} = 0 \tag{2a}$$

$$(\bar{\rho}\bar{u}_i + \overline{\rho' u_i'})_{,t} + \left[(\bar{\rho}\bar{u}_j + \overline{\rho' u_j'})\,\bar{u}_i + \delta_{ji}\,\bar{p} - \bar{\tau}_{ij} + (\bar{\rho}\overline{u_i' u_j'} + \bar{u}_j\,\overline{\rho' u_i'} + \overline{\rho' u_i' u_j'}^{\,*})\right]_{,j} = 0$$

$$\frac{\gamma}{\gamma - 1}\,\frac{\bar{p}}{\bar{\rho}} + \frac{1}{2\bar{\rho}}\left|\bar{\rho}\bar{u}_i\bar{u}_i + 2\bar{u}_i\,(\overline{\rho' u_i'}) + \bar{\rho}\overline{u_i' u_i'} + \overline{\rho' u_i' u_i'}\right| = C_p T_{tot} = \text{CONST.}$$

WHERE $^* = \overline{(\rho u_j)' u_i'}$

AND $\bar{\tau}_{ij} = 2\bar{\mu}\bar{S}_{ij} + 2\overline{\mu' S_{ij}}$ WITH $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij} u_{k,k}$

MASS WEIGHTED AVERAGING

$u_i = \tilde{u}_i + u_i'',\ \rho = \bar{\rho} + \rho'$

$\bar{\tilde{A}} = \tilde{A},\ \overline{A'} = 0,\ \overline{A''} \neq 0$ but $\overline{\rho A''} = 0$

$$\bar{\rho}_{,t} + (\bar{\rho}\tilde{u}_j)_{,j} = 0$$

$$(\bar{\rho}\tilde{u}_i)_{,t} + \left|\bar{\rho}\tilde{u}_j\tilde{u}_i + \delta_{ij}\bar{p} - \bar{\tau}_{ij} + \overline{\rho u_i'' u_j''}\right|_{,j} = 0 \tag{2b}$$

$$\frac{\gamma}{\gamma - 1}\,\frac{\bar{p}}{\bar{\rho}} + \frac{1}{2\bar{\rho}}\left|\bar{\rho}\tilde{u}_i\tilde{u}_i + \overline{\rho u_i'' u_i''}\right| = C_p T_{tot} = \text{CONST.}$$

WHERE

$\bar{\tau}_{ij} = 2\tilde{\mu}\tilde{S}_{ij} + 2\widetilde{\mu' S_{ij}} + 2\bar{\mu}''\tilde{S}_{ij} + 2\overline{\mu'' S_{ij}''}$

WITH $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij} u_{k,k}$

Figure 2. Averaging processes in compressible flows.

CONSTITUTIVE RELATIONSHIP

$$-\overline{\rho u_i'' u_j''} = -\delta_{ij}\, 2/3\, \bar{\rho}\, \tilde{k} + 2\mu_t \left[\tilde{S}_{ij}\right] \text{ WHERE } \tilde{S}_{ij} = 1/2\,(\tilde{u}_{i,j} + \tilde{u}_{j,i}) - 1/3\delta_{ij}\tilde{u}_{k,k}$$

BOUNDARY LAYER MODEL

INNER LAYER

$$\mu_t = \bar{\rho}\, \ell^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$$

$$\ell = \kappa y \left[1 - \exp\left(\frac{-y\sqrt{\tau_w/\rho_w}}{\nu_w A^+} \right)\right]$$

OUTER LAYER

$$\mu_t = 0.0168\, \bar{\rho}\, \tilde{u}_e\, \delta_k^* \Big/ \left|1 + 5.5\,(y/\delta)^6\right|$$

$$\delta_k^* = \int_0^{\delta} \left(1 - \frac{\tilde{u}}{\tilde{u}_e}\right) dy$$

OR

$$\ell = 0.09\,\delta$$

Figure 3. Compressibility corrections to eddy-viscosity models; algebraic models.

e.g. JONES-LAUNDER

$$\mu_t = C_\mu\, \bar{\rho}\, \frac{\tilde{k}^2}{\epsilon}$$

$$\bar{\rho}\frac{D\tilde{k}}{Dt} = P - \bar{\rho}\epsilon + \left(\frac{\mu_t}{\sigma_k}\,\tilde{k}_{,j}\right)_{,j}$$

$$\bar{\rho}\frac{D\epsilon}{Dt} = C_1\,\frac{\epsilon}{\tilde{k}}\,P - C_2\,\bar{\rho}\,\frac{\epsilon^2}{\tilde{k}} + \left(\frac{\mu_t}{\sigma_\epsilon}\,\epsilon_{,j}\right)_{,j}$$

$$P = \left[-\delta_{ij}\, 2/3\, \bar{\rho}\, k + 2\mu_t\,(\tilde{S}_{ij})\right]\tilde{u}_{i,j}$$

IN LOG REGION OF COMPRESSIBLE BOUNDARY LAYER, WHERE

$$\frac{U}{U_*} = \frac{1}{\kappa}\ln\left(\frac{yU_*}{\nu_w}\right) + B,\quad U = \int_0^U \sqrt{\frac{\rho}{\rho_w}}\, du,\ \&\ \tau \cong \tau_w = \bar{\rho}_w U_*^2$$

$$\frac{\bar{\rho}}{\mu}\,\tilde{k} = \frac{U_*^2}{\sqrt{C_\mu}}\ ;\ \epsilon\left(\frac{\bar{\rho}}{\rho_w}\right)^{3/2} = \frac{U_*^3}{\kappa y}\ ;\ \mu_t \approx \bar{\rho}\,\frac{U_*\kappa y}{\sqrt{\frac{\bar{\rho}}{\rho_w}}}$$

Figure 4. Compressibility corrections to eddy-viscosity models; two-equation models.

PRIMITIVE DEPENDENT VARIABLES (e.g. Donaldson and Sullivan)

P.D.E. FOR $\bar{u}_i$, $\bar{h}$, $\bar{\rho}$, $\overline{u_i'u_j'}$, $\overline{h'u_i'}$, $\overline{h'h'}$, $\overline{\rho'u_i'}$, Λ (22 Equations)

MODELS FOR 41 MOMENTS: e.g. $\overline{u_i'u_j'u_k'}$, $\overline{u_i'\rho'\rho'}$, $\overline{u_{i,m}'u_{j,m}'}$

$\overline{\rho'u_{\ell,\ell i}'}$, $\overline{u_{i,j}'u_{\ell,\ell}'}$, ...

MASS WEIGHTED DEPENDENT VARIABLES (e.g. DUSSAUGE, WILCOX)

P.D.E. FOR $\tilde{u}_i$, $\tilde{h}$, $\bar{\rho}$, $\widetilde{u_i''u_j''}$, $\widetilde{u_j''h''}$, ω (15 Equations)

MODELS FOR 12 MOMENTS: e.g. $\widetilde{u_i''u_j''u_k''}$, $\overline{u_k''p'}$, $\overline{p'(u_{k,i}''+u_{i,k}'')}$,

$\overline{u_i''\tau_{ij}}$, $\overline{u_i''}$, ...

Figure 5. Compressibility corrections to Reynolds-stress transport models.

(FEIEREISEN, REYNOLDS AND FERZIGER)

SCOPE

HOMOGENOUS SHEAR FLOW

UNIFORM MEAN PRESSURE AND TEMPERATURE

FINDINGS

- COMPRESSIBILITY CONTRIBUTES LITTLE TO REYNOLDS STRESSES AND DISSIPATION
- ALTHOUGH COMPRESSIBILITY CONTRIBUTION TO PRESSURE, RATE OF STRAIN CORRELATION WAS SHOWN TO BE SIGNIFICANT, FOR A BOUNDARY LAYER AT $M < 5$ EFFECT IS NEGLIGIBLE

Figure 6. Compressibility effects observed in turbulence simulation.

DISCUSSION

SESSIONS I AND II

The discussion on these sessions focused on the "Taxonomy" paper of J. Ferziger, J. Bardina, and G. Allen. Most of the detailed comments related to numerical methods rather than turbulence modeling. All relevant points were noted by the authors and have been incorporated in the final version of the paper above.

INTRODUCTION TO SUMMARIES BY TECHNICAL REPORTERS
(Sessions III through XII)

An important feature of the 1981 Conference was the presentations made by the Technical Reporters summarizing the results obtained from the Computor Groups for various classes of flow cases.

This was a difficult task not only because of the short time available between the receipt of the computer plots and the date of the Conference, but also owing to the vast amount of outputs in many of the flow cases. The Technical Reporters were given a free hand in their presentations to the Conference. They were asked to highlight the significant results in the comparison between data and computer output, and were given the option whether to include all the results presented to the Conference or to make an illustrative selection of results.

As a result, the presentations at the Conference varied considerably from one Technical Reporter to another. Some preferred to rely on a verbal presentation, others preferred a visual presentation with carefully prepared viewgraphs. In all cases these presentations were of critical value to the conference participants since they were having their first introduction to what the Computor Groups had achieved in this comparison between data and computer output. These presentations, therefore, were an important step towards achieving a full and active discussion in the relevant sessions and in the after-dinner discussions. All the discussions were carefully reported by the Technical Recorders and were edited during the meeting by participants and finally by the Session Chairman. The edited summaries of the Technical Reporters have been reproduced in these proceedings where the Editors considered the information provided in them added to the information which could be obtained from:

(i) the computors' summary reports, in Volume III;

(ii) the plots for each flow case; see Plates 1-187 in Volume III;

(iii) the session discussions, in this volume.

Where reproduction of the Technical Reporters' presentations would require a duplication of many plates* presented in Volume III, the Editors' decision has been to omit the material in this volume. This reduced the possibility of errors arising from amendments to the output and flow descriptors or from omissions in Reporters' figures owing to late submissions of plots.

It is recommended that study of the flow cases should begin through the information given in this volume under the relevant session, including the details given

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*The term "plate" in these proceedings refers only to the output or computations compared to data in Volume III; other illustrations are called figures.

under the Technical Reporter's summary and the discussion, with reference to the plates and Computors' summaries in Volume III, as appropriate.

In reference to the plates and figures in the Reporters' remarks it is important to note that data are marked by open diamond-shaped symbols (as in Volume I); the data are plotted directly from the Data Library Tape. Computer output is presented generally as continuous full lines with no symbols (or as filled-in symbols, crosses, or X marks).

The plots are labeled according to the turbulence model classification described in the taxonomy section of this volume.

SESSION III

Chairman: A. Roshko

Technical Recorders:

L. Smits
S. Caruso

Technical Reporter

R. Chevray

INCOMPRESSIBLE FREE-SHEAR FLOWS

Cases 0311, 0381, 0471 — Simple Strains
0331, 0382 — Extra Strains

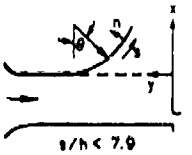
PICTORIAL SUMMARY

Flow 0310. Data Evaluator: S. Birch. "Planar Mixing Layer."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0311 Data combined from several sources.		L/θ_i vs x/θ_i											Initial development of the $U_2 = 0$ plane mixing layer.

PICTORIAL SUMMARY

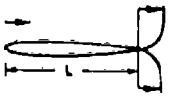
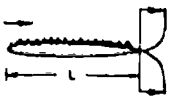
Flow 0330. Data Evaluator: P. Bradshaw. "Free-Shear Layer with Streamwise Curvature."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0331 I. Castro P. Bradshaw			17*	17*	17*	17*	17*	17*	17* $\overline{q^2 u}$ $\overline{q^2 v}$ $\overline{u^2 v}$, $\overline{uv^2}$ Intermittency		$2.5_5 \times 10^5$ (based on nozzle opening h)	Turbulence level <0.01%	Tabulated data are in a mix of x, y, or r,θ coordinates. Digital processing techniques are used.

*Results are plotted in semi-curvilinear (s,n) coordinates; the n = 0 line being a nominal curved centerline of the mixing layer.

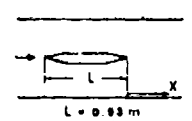
PICTORIAL SUMMARY

Flow 0360. Data Evaluator: V. C. Patel. "Wakes of Two-Dimensional Bodies."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0361 J. Andreopoulos			10		4	4	-	4	Triple correlations	-	6.8×10^4 (based on L)	Free-stream turb. < 0.05%	Symmetrical Case. Conditional averages to study the mixing in the central part of wake.
Case 0362 J. Andreopoulos			6		4	4	-	4	Triple correlations	-	6.8×10^4 (based on L)	Free-stream turb. < 0.05%	Asymmetrical Case. Conditional averages to study the mixing in the central part of wake.

PICTORIAL SUMMARY

Flow 0470. Data Evaluator: P. Dreacher. "Flow over the Trailing Edge of Blades and Airfoils."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 0471 P. Viswanath J. Cleary W. Seegmiller C. Horstman		Surface and wake pressure	9	-	7	7	-	7	-	-	2.4 to 3.7×10^7 (based on chord)	0.4 and 0.7	Subsonic trailing edge flow, variable flap angle. Symmetric and unsymmetric cases.

Methods Used--Session III

Method[†]	Case 0311	0381	0471	0331	0382
AKEZ	•	•		•	•
BKEZ	•	•	•	•	•
BKEX		•	•		•
AKEC				•	
BKEC				•	
BKWX			•		
RSTC	•				
RSEC		•			•
DSQZ				•	
AKWX	•				
Plate No. Vol. III	52	97, 98	117-120	53-58	99, 100

[†]The flow descriptors with the final letter X and Z refer to the flow model used in the upstream domain, where the flow is wall-bounded. In a free-shear flow such as a wake or mixing region, the final letter in the flow descriptor should read C.

INCOMPRESSIBLE FREE-SHEAR FLOWS

Simple Strains, Cases 0311, 0381, 0471 and Extra Strains, Cases 0331, 0382

Technical Report

by

R. Chevray*

Data for six incompressible free-shear flows were available to computors. These were divided into two broad classes. Flows involving simple strain were comprised of the planar mixing layer (Case 0311), the wake of the flat plate with symmetric initial boundary conditions (Case 0381), and the trailing edge of blades and airfoils (Case 0471). The other class included flows with extra strains: the free-shear layer with streamwise curvature (Case 0331), the axisymmetric wake (Case 0361), and the wake of the flat plate with asymmetric initial boundary conditions (Case 0382).

All computational methods used here were one-point closure type, classified according to the treatment of the Reynolds stresses. For the simple strain cases, ten methods were of the Boussinesq type, three of the algebraic type, and two of the Reynolds-stress type. For extra strain cases, again the Boussinesq type was the most used (six methods) followed by five of the algebraic type, one of the Reynolds-stress type, and two using a five-equation model. For each of the cases tackled, a different method was used. All cases were very similar in the methods of attack except for the axisymmetric wake (Case 0361) in which an inverse (interactive) solution procedure was suggested. Computations had to be started well ahead of the separation point by using data of Patel et al. (1978) provided at $x/L = -0.34$. However, no computor attempted this case.

Most of the reported results were obtained using k-ε, two-equation models. All methods used to predict different flows were satisfactory. The mean velocity was predicted very well. The turbulent intensity and shear stress were predicted well, although slightly less accurately than the mean velocity. For the higher-order terms to be computed, departures from the experimental data were noticeable; in general, the higher the order of the term to be computed, the harder it was to predict. There was little difference in the results obtained when different methods were used; all predicted the cases proposed quite well. The only exceptions to this statement were for cases in which there exists strong pressure gradients. None of the methods used worked as well in situations with strong pressure gradients.

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DISCUSSION

SESSION III

INCOMPRESSIBLE FREE-SHEAR FLOWS

Case 0311, Simple Free-Shear Layer

Three methods (Groups 07, 17, and 24) all used parabolic k-ε models but the constant in the production term was 1.40 for Group 24, and 1.44 for Groups 07 and 17. A value of 1.40 gave a 12% higher spreading rate. B. Launder noted that the choice of layer width (based on distance between the points at which $U = \sqrt{0.9}U_1$ and $\sqrt{0.1}U_1$) was the main reason for the underprediction of rates of spread by computors. The differences between computations and data would be much less, if the 90% velocity point was chosen, since the k-ε model gives a poor prediction of velocity shape near the high-velocity edge. S. Birch pointed out that 1.40 represents a compromise to bias the predictions toward the near-field calculations for jets, and hence its use here. To capture the apparent overshoot in the slope of the data given in Plate 52, Vol. III, B. Launder suggested that the source term in the dissipation equation may require some anisotropic modeling.

Case 0381, Wake Behind a Flat Plate

V.C. Patel emphasized that these are results for the near wake only (x = 400 mm is equivalent to 7δ) and that asymptotic values should have been calculated to enable comparison with other data (but they were not required). For example, though the calculations for the near wake show good agreement, significant discrepancies with the accepted far-wake growth rate were found by Group 07. This was confirmed by S. Birch (Group 24). D. Vandromme (Group 35) found discrepancies up to 15% between different models; however, a k-ω^2 model behaved slightly better than a k-ε model.

V.C. Patel stated that δ^* should have been requested; R. Melnik concurred, and, in addition, asked for the centerline velocity U_c, adding that Andreopoulos showed analytically a logarithmic variation of U_c with distance. This could be used for comparison with computation. B. Launder noted that in his calculation (Group 17), $\overline{v^2}/u^2$ blows up in the near wake, which is inconsistent with (Group 35) Vandromme's calculations using the model RSEC. Further discussion of this point, at a later time, was encouraged.

Case 0471, Symmetric Airfoil Wake

C. Horstman pointed out that contrary to his expectations, there were considerable differences between the calculations presented and asked why. The ensuing discussion did not ascertain any particular reason. In particular, it was not clear whether the differences at higher M (= 0.7) are due to compressibility or whether,

because of the large trailing edge angle, interaction effects are also playing a role. It was felt this point requires further consideration. It was generally agreed that k-ε models do not perform well in strong adverse pressure gradients and $k-\omega^2$ models are perhaps better in this aspect.

Flow 0331, Curved Shear Layer

I. Castro stated that, due to high turbulence levels, the $\overline{uv}$ data were probably only accurate to within 15-20%, and are always lower than the true value. Further information after the meeting indicated possible errors in the original data specification. This remains to be clarified. Some discussion concerned the need for elliptic methods in this case. B. Launder felt that this was probably so. N. Mansour reported that the bleed flow rate had a strong effect on his results, but that the boundary conditions on the wall were not important. N. Mansour, B. Launder, I. Castro, and M. Celenligil all commented that the details of the numerics are important in this case, but further discussion was postponed until after the presentation by Launder in Session IX. It was also noted that (Group 07) Scheuerer/Rodi and (Group 19) Hah/Lakshminarayana utilize apparently similar models, yet obtain different results. On closer examination it was seen that the source term in the dissipation equation and the convection and diffusion effects in the Reynolds transport equation are handled somewhat differently in these two methods.

Case 0361, Axisymmetric Wake

R. Chevray asked why no computations were presented for this flow case. W. Rodi stated that this case really required an elliptic method with a non-rectangular, non-uniform grid; no suitable numerical method was available for this situation.

Case 0382, Asymmetric Wake

The general opinion (advanced by B. Launder) was that the asymmetry of this case did not introduce any complexities additional to those already encountered in Case 0381. V.C. Patel asked for a physical explanation of why the agreement in the near wake was so good, especially with the peaks in the $\overline{uv}$ distribution. M. Morkovin suggested that the mechanism by which these peaks are produced in the calculation ought to be traced and W. Rodi agreed that this should be done. Later discussion suggested that the effect is connected with the relative fullness of the velocity profiles on the two sides. In effect, an internal shear layer is produced downstream of the trailing edge.

SESSION IV

Chairman: P. S. Klebanoff

Technical Recorders:

D. Driver
A. Cutler

Technical Reporter

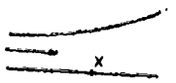
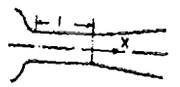
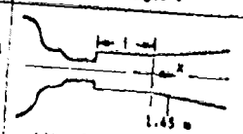
R. L. Simpson

INCOMPRESSIBLE WALL-BOUNDED FLOWS — I

Cases 0141, 0211, 0261, 0612 — Boundary Layers
0142, 0143 — Diffusers

PICTORIAL SUMMARY

Flow 0140. Data Evaluator: R. Simpson. "Diffuser Flows (unseparated)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0141 A. Samuel P. Joubert		$\frac{dp}{dx} > 0$, $\frac{d^2p}{dx^2} > 0$	12	-	6	6	6	6		Floating balance, Preston tube, Clauser plots	1.76×10^6 per m	Turbulence Intensity 0.3%	Two-dimensionality was checked.
Case 0142 R. Possorini	 l/d = 4.5; Area Ratio 4; Cone Angle 4°	> 0	12	12	12	12	12	12	-	Clauser plots, Preston tube	2.54×10^5 (based on inlet pipe radius)	Turbulence Intensity 0.3%	Conical diffuser —core (low turbulence).
Case 0143 R. Possorini	 1.45 m l/d = 7.25; Area Ratio 4; Cone Angle 5°	> 0	6	6	6	6	6	6	-	Clauser plots, Preston tube	1.22×10^5 (based on inlet pipe radius)	Turbulence Intensity 1%	Conical diffuser —core (high turbulence).

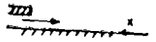
PICTORIAL SUMMARY

Flow 0210. Data Evaluator: P. Bradshaw. "Effect of Free-Stream Turbulence on Boundary Layers."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0211 P. Hancock and others		Smoothed curve of $\frac{\Delta C_f}{C_{f_o}}$ vs $\left(\frac{u'}{U}\right)_e \Big/ \left(\frac{U_e}{C_{f_{995}}} + 2\right)$ given											

PICTORIAL SUMMARY

Flow 0260. Data Evaluators: B. Launder and W. Rodi. "Turbulent Wall Jet."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0261 Various data takers		> 0	Yes	-	Yes	Yes	Yes	Yes	Inter-mittency. Triple, quad-ruple velocity correlations	Razor blade	4×10^3 to 5×10^4		Equilibrium wall jet in adverse pressure gradient.

PICTORIAL SUMMARY

P. ow 0610. Data Evaluator: D. E. Coles. "Attached Boundary Layers ('68 Conference)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity: U	Mean Velocity: V or W	Turbulence Profiles: $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0612 K. Wieghardt		0	23	-	-	-	-	-	-	Clauser plots	2.2 × 10^6 per ft	Turbulence intensity < 0.25%	Data include computed values of boundary layer integral parameters.

Methods Used—Session IV

Method	Case 0141	0142	0143	0211	0261	0612
AKEZ		●	●		●	●
BKEZ	●	●	●	●		●
BKEY	●	●	●			●
BKEX	●			●	●	●
AKWX	●	●	●			●
BKWX	●					●
BOPX	●	●	●			●
BOLX	●					●
BIKX	●	●	●			●
HN	●	●	●	●		●
EE	●	●				●
SG	●					●
HG	●					
RSEN	●					●
RSTN	●					●
Plate No. Vol. III	9–18	19–22	23–26	27	48, 49	133–138

INCOMPRESSIBLE WALL-BOUNDED FLOWS -- I

Boundary-Layer Flows, Cases 0612, 0141, 0211, 0261 and Diffusers, Cases 0142, 0143

Technical Report

by

R. L. Simpson*

Editor's Comment

R. Simpson rated each flow and showed ratings with figures of output. Because these detailed ratings were not done by other reporters, only a summary of R. Simpson's ratings are shown in Table 1 below, indicating the number of ratings in various categories for each flow. The reader should refer to the pages shown under each flow in the table or methods used above to examine details of output.

A good prediction (G) produced a curve with a shape similar to the data and only a few percent numerical deviation. A fair (F) prediction produced a curve with greater than a few percent deviation or a slightly different shaped curve. A poor (P) prediction shows significant deviations in the curve shape and magnitudes.

Table 1

Case†	Total Entries	Number Rated					
		E	G	F^+	F	F^-	P
0612	21	0	8	8	5	0	0
0141	22	0	3	4	6	9	0
0142	11	0	1*	4	1	3	2
0143	10	0	1*	0	5	0	4

†R. Simpson did not comment on Flow Cases 0211, 0261.

*Rated G^-.

Some general comment on the results of R. Simpson's ratings follows.

1. As Simpson remarked, a degree of subjectivity does exist in the ratings. Despite this, the ratings clearly do distinguish good from poor results and indicate a clear picture of the overall state of the art in 1981 for the flows considered.
2. Table 1 shows that even in the simplest case (flat plate), results of various methods are very mixed. No result is rated excellent for any flow.
3. The accuracy of prediction decreases sharply as the flow complexity increases.

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4. Cross-comparison with methods showed no correlation or obvious trends between type or sophistication of method and accuracy of output. This point remains true through the body of results reported in the 1981 meeting (see report of Evaluation Committee herein).
5. No single method stands out as best; the best rated method for one flow is not the same as the best rated for other dissimilar flows.

COMMENTS BY R. SIMPSON

Flow 0612

As pointed out by Coles* in his description of the Wieghardt flat-plate zero-pressure-gradient flow, the integral parameters deduced from the data are strongly affected by the interpolation scheme used in integration because few data points were obtained near the wall. Consequently, the Ludwieg-Tillmann C_f values are more uncertain near the beginning of the flow. Also, since the flow was tripped by a blunt leading edge and a trip wire, the first profile was probably at about the minimum Reynolds number for turbulent flow. The two types of C_f values shown in Plates 135 and 136 in Vol. III do not agree until downstream of 1 m.

The predictions of U/U_e vs y (Plates 133 and 134 in Vol. III), C_f vs X (Plates 135 and 136 in Vol. III), and H vs X (Plates 137 and 138 in Vol. III) were examined downstream of 1 m for the 20 methods whose results were received before the 1981 Conference. None of the results are considered as excellent because of deviations in the shapes and numerical values of predicted curves as compared to the data.

Flow Case 0141

This case is a strong test of prediction methods for plane turbulent boundary layers subjected to strong adverse pressure gradients. The C_f data are very well documented with good agreement among results from independent measurements. The initial conditions and the turbulence structure are well documented.

With these broad general criteria, Table 1 summarizes the subjective rating of the 22 different predictions received before the 1981 Conference. Since C_f and U/U_e distributions are of most interest to engineers, these quantities were weighed heavier than $-\overline{uv}/U_e^2$ profiles in determining the overall rank. Since most criteria for the onset of flow detachment in the separation process are related to C_f as it approaches zero, it is very important that C_f be well predicted.

Only one of several methods could be regarded as good. The remaining methods do not predict C_f very well near the end of the test length where a strong adverse pressure gradient exists. The poorer methods overestimate C_f by 100% or more in this

*D.E. Coles and E.A. Hirst, Proc. Computation of Turbulent Boundary Layers—1968 AFOSR-IFP-Stanford Conference, Vol. II.

region. Several of the methods that predict C_f and U/U_e only fairly or poorly still show good $-\overline{uv}/U_e^2$ profiles near the end of the test length.

Flow Cases 0142, 0143

These cases provide strong tests of axisymmetric diffuser prediction methods in the presence of low core turbulence (0142) and high turublence (0143). The experimental data including initial conditions and turbulence structure are well documented.

None of the methods produce excellent results because of deviations in the shapes and numerical values of one or more predicted curves as compared to the data. That is, none of the prediction methods is extremely good. However, in both cases 0142 and 0143, AKEZ-18 does the best job predicting C_f and C_{pw}, which are the parameters of most engineering interest. The other methods are at best fair.

DISCUSSION

SESSION IV

General Comments on Universality of Models

P. Klebanoff asked: "Can we say that we have made a substantial advance in modeling for simple cases, such as the flat-plate boundary layer or the boundary layer with adverse pressure gradient, since 1968?" And, "Can we state unequivocably that PDE methods are better than integral methods for such flows?"

There was general acceptance that for simple flows of this sort, the situation had not improved significantly with the use of two-equation models over integral methods. However, our requirements have now become more stringent, and PDE methods can handle more complex situations and provide a basis for the development of more universal models.

M. Rubesin gave as an example the calculation of the flow over a blunt trailing edge. In this application, two-equation models can calculate the flow "straightaway," whereas a mixing-length model would require new assumptions.

M. Rubesin further asked, "Isn't the objective to compile a set of standard flows that can be used to create a universal model?"

S. Kline commented that it is possible to be misled by the idea of universality if it is used as a holy grail. We have accurate simple models for given kinds of flow structure, and it should therefore be possible to use "fine-tuned" models in various zones characterized by different flow structures.

E. Reshotko commented that we should seek flexibility and power rather than universality. G. Sovran and M. Lubert added that in industry it is important to have programs that solve specific problems reliably and economically. M. Lubert also commented that industry is using two-equation models already, and they are quite cheap to run—so why not?

K. Hanjalić noted that it is easy to make a fix for one specific flow, but it may not be sufficiently general.

S. Kline replied that he saw nothing wrong with the fine tuning for a specific zone, provided that the tuning is not done arbitrarily but is related to knowledge about the physics, that is, flow structure. We do know that different types of flow have different structures.*

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*See the following Postscript to Session IV Discussion by S. Kline, and its extensions in Session XV beginning on p. 990.

P. Klebanoff commented that he saw no reason why the procedure advocated by Kline is not okay for engineering purposes, particularly since it is questionable whether Reynolds averaging reflects the appropriate physics, and since the differences in various geometries are associated with the physics of the large-scale structure. It may not be realistic therefore, at least for the present, to search for universality.

S. Kline noted that the difference in the large eddies may well be the reason for the difference in tuning constants from one case to another. When one looks at a large number of flow cases, as in this Conference, there appears to be a trade-off between range of domain and accuracy of output.

K. Hanjalić said that modelers working with differential methods are not interested in tuning constants for one flow, since this may interfere with results in another flow for which it is used. However, he did admit there is room for much improvement.

P. Klebanoff commented that he could understand how fine tuning for one flow may interfere with the results for another flow configuration. However, he felt intuitively that if fine tuning for a given complex flow resulted in the same model not giving good results for a simpler aspect of the same flow configuration then there is something drastically wrong with the modeling.

M. Rubesin commented that universal modeling will require large-eddy simulation. In response to Klebanoff's query, whether this is feasible for engineering with present-day computers, Rubesin replied that computers have developed and will continue to develop to meet the needs. In addition, he noted that we should continue to search for improved closure methods to give a wider range of application.

Modeling Problems

P. Klebanoff asked, "Are there any problems that computors would like to discuss that would affect the outcome of the models?"

P. Orlandi asked, "Is the Falkner-Skan (Levy-Lees) transformation a desirable way to initiate a boundary-layer calculation?"

There were pro's and con's expressed on this question by several modelers. For example, D. Wilcox said that the Levy-Lees transformation is not convenient for internal flows. Several others indicated that it is often convenient to use Levy-Lees when one starts with a laminar flow, and it seems to make little difference after the boundary layer becomes turbulent. G. Scheuerer (Group 07) said that his group is using a Patankar-Spalding type transformation.

A discussion on the number of grid points needed followed. Various models use widely different numbers of grid points in y, ranging from as low as 7 in some instances of (Group 01) John Moore's method to several hundred in the methods of Wilcox (Group 37) and others. P. Orlandi expressed incredulity that as few as 7 points could

be used. D. Wilcox emphasized that the number of grid points needed is "method-dependent." He gave three rules of thumb for the location of the grid point closest to the wall in order to achieve grid independence. These rules are:

		Location of Grid Point Nearest to Wall
(i)	Using parabolic marching and integrating	$y^+ \approx 1$
(ii)	Parabolic marching with wall function	$y^+ \approx 20$
(iii)	Time marching with wall functions	$y^+ \approx 50$

M. Leschziner (Group 17[3]) added that when using wall functions one needs to take the closest point in the logarithmic region, that is, $y^+ \geq 50$. G. Scheuerer added that for Group 07's method the nearest grid point should be in the range $30 \leq y^+ \leq 150$.

M. Lubert relayed a request from the Evaluation Committee that each computor group tell the number of grid points, the initial conditions and the numerics used by each method. He asked this be done in order to provide possible explanations for why methods with the same taxonomic descriptors give different results.* On this point, K. Hanjalić (Group 18) reinforced Lubert's suggestion, and expressed concern about the specifications of initial conditions, wall treatments, inclusion or omission of normal strains, and the form of the dissipation equation and the values of its coefficients. Hanjalić also said that there is no unified view on a number of these questions even within groups using the same general type of methods, nor is there a unified view regarding the sources of these differences in output.

On the matter of evaluating ε, P. Orlandi (Croup 05) remarked that it is difficult to initialize calculations in k-ε methods because adequate data for ε do not exist. Moreover, Orlandi said it is difficult in general to adjust constants in the ε-model equations for boundary layers owing to this lack of data. P. Klebanoff added that this is a very important point because the experimentalists have not provided complete information on ε, and hence local isotropy is assumed in many models to evaluate the dissipation. However, experiments indicated that the validity of local isotropy is open to serious question particularly in the region close to the wall.

P. Orlandi felt that the flat-plate layer (Case 0612) must be used in calibrating turbulence models in the vicinity of the wall because for this flow there are experimental data owing to Klebanoff and to Launder on the turbulent energy balance. Orlandi also asked if the k-ε models give a dissipation that is larger than production in the viscous region.

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*This request led S. Kline, as General Chairman of the Conference, to arrange meetings during lunchtimes between modelers and subgroups of the Evaluation Committee to clarify these issues.

Case 0612. Flat-Plate Boundary Layer (Wieghardt)

It was noted by several computors that differential methods appear to consistentently underestimate the shape factor $H = \delta^*/\theta$. It was felt that this usually arises from poor integration of the profile over near-wall points.

E. Reshotko noted that methods employing integral schemes calculate H better. There was consensus that H is not a very useful criterion for assessing adequacy of differential models since the deficiencies are numerical.

P. Klebanoff asked why outputs so often appears inadequate near the leading edge. This question was not resolved and led to the establishment of an ad-hoc committee on starting/marching conditions.

Case 0141. Boundary Layer with Increasingly Adverse Pressure Gradient (Samuel and Joubert)

P. Klebanoff initiated the discussion by remarking that many k-ε methods do not provide the appropriate values of C_f in this flow. He asked if reasons for this are known.

R. Pletcher noted that this is connected with the fact that k-ε models do not predict sufficiently small length scales near detachment. He said the Pletcher model BOLX-30 captures this particular effect correctly. D. Wilcox commented that he could not use p(x) to get $\partial p/\partial x$ and the measured gradients were different from those he obtained.

R. Simpson remarked that he selected this flow because pressure gradient was measured independently and was not taken by differentiating pressure data.

B. Launder commented that the dissipation-rate equation gives length scales that are too large in the region approaching detachment. W. Rodi said they got the same result.

J. Cousteix remarked that downstream of the point of inflection in $U_e(x)$ his method had difficulty.

D. Wilcox remarked to J. Cousteix that his wall functions could be the source of these problems.

Cases 0142 and 0143. Low- and High-Core Turbulence Axisymmetric Diffuser Flows (Pozzorini)

P. Klebanoff began the discussion by asking the computors: "How important are initial conditions?" and "How were initial conditions for ε obtained, particularly for the high-core turbulence case?" The concerns expressed above on lack of data on ε were reiterated.

J. Moore asked whether parabolic or elliptic methods were used on these cases. He was particularly concerned that elliptic effects might be present near the initializing station. All computors present used parabolic methods except H. Moses (Group 27). Moses used both an elliptic and a parabolic method and found no significant

when starting the calculations with a uniform static pressure at the initializing station.

M. Leschziner and K. Hanjalić agreed that differences in the initial conditions used were important in the output particularly for the high-core turbulence case.

Case 0211. Effect of Free-Stream Turbulence on C_f (Correlation by Bradshaw)

In the afternoon session W. Rodi asked P. Bradshaw why the data to be predicted were only a single line, not a function of the initial condition chosen. P. Bradshaw explained that he had wanted the computors to provide only the results of a computation after the effect of the choice of poorly defined initial conditions had disappeared, i.e., far downstream.

P. Klebanoff asked in the dinner session why agreement of the computations with the data was so poor. G. Scheuerer mentioned that large length scales in reality do not interact with the boundary layer but that many models predict an interaction. M. Rubesin pointed out that all the k-ε models predicted much the same results; low in comparison with the data plot.

E. Reshotko noted that the scatter is more apparent than real owing to the very large scale on this plot.

S. Kline remarked that the uncertainty bounds need to be addded to this plot, and they will show that the scatter in the experiments is considerable.

P. Klebanoff remarked that measurements of free-stream turbulence may include potential fluctuations and this may lead to lack of correlation between intensity and scale.

Case 0261

There was no discussion of this case.

Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

P. Bradshaw pointed out that the dissipation ε may not always be the "real" viscous dissipation, being a property of the energy transfer from large to small scales, and therefore it is not easily measured.

D. Wilcox agreed with P. Klebanoff's general consensus figure of $y^+ = 50$ for the first near-wall node, only if the modified wall functions suggested in his forthcoming AIAA paper were used.

Postcript to the Discussion of Session IV

by S.J. Kline

In the opening remarks to this Conference I suggested that one central problem was the question of whether there exists a single method that gives sufficient accuracy over the entire domain of applications. The discussions in this meeting, particularly on Monday evening, when taken together with the comparison of computation and data provide considerable evidence on this point. Together they suggest a somewhat different view than has been articulated clearly before. Hence I set forth this view in this postcript as a basis for discussion.

It is central to this viewpoint that we distinguish flow zones from flow cases. Flow zones are regions within a single flow. Flow cases are realizations of a given geometry. There is a manifold of infinities of flow cases and an even larger manifold of infinities of geometries that are pertinent to industrial applications, including but not limited to the flow cases of this Conference. The situation in flow zones is quite different, but to see that a preliminary remark is needed. A flow zone is a portion of a flow field that has a particular kind of flow structure. I, therefore, take as a premise that flow zones should be selected based on what we know about flow structure—that is the underlying physics of the flows. In those terms, I believe there are only a very limited class of what I will call "structural flow zones." My first rough cut shows 20, when I exclude hypersonic flows and more phenomena. The list of 20 paradigmatic structural flow zones is attached to this postcript as Appendix I. Suggestions on additions or combinations from members of this Conference are solicited.

Some comments concerning Appendix I are needed. First, I have not included what Peter Bradshaw calls extra strains. Such extra strains are a set of necessary subdimensions for some classes of structural flow zones. They may or may not be needed depending on the applications and the flow geometry, but they can be treated as subdimensions added into the modeling for some of the structural flow zones.

Second, some method already presented in this Conference seems to do quite well for each structural flow zone when taken separately. For example, the adverse-pressure-gradient boundary layers are modeled quite well by method HN-45 of Ferziger et al. This same method does not do at all well when used on a flow with high free-stream turbulence since that violates an assumption in the method.

Within the class of attached and separating boundary layers, method HN-45 does remarkably well for the quantities it produces and even more remarkably when one considers its simplicity. One reason HN-45 performs as it does is because the model is shifted, within the method automatically, when the point of incipient detachment is reached. The field is taken to have three structural flow zones: (1) inviscid,

(ii) attached boundary layer, and (iii) detaching and detached layers rather than the two zones used in classic Prandtl boundary-layer analysis. We moved to three zones specifically because we observed, based on considerable experience, that different constants were needed for the attached layer on the one hand and detaching and detached layers on the other.

This seems very consistent with what Dave Wilcox reported in a general session. When he altered his model constants to fit free-shear layers better, the fit for attached layers was degraded. Another example seems to be the intrepid and remarkably good computation of the square channel with cruciform case 0113 (P1), by Rodi et al. (Group 07), using AKEZ-07A. The same method does not do nearly so well for fully separated flows as in the step cases. This at least suggests that this method is well adjusted for wall-dominated flows, but not so well for free-shear layers and recirculating zones.

It is these and other similar observations that suggest that a trade-off between range of domain and accuracy does seem to exist in even the most sophisticated models currently available.

All the remarks above and Appendix I suggest that there are not two levels of generality, but rather several. At the lowest level of generality, there are tailored methods such as HN-45 that work well over a quite limited class of problems but do not extend easily. At a second level there are in this meeting a variety of models of much greater range of domain, but none of them have as accurate results for the particular cases that are well handled by HN-45 and for which it was designed. All the methods so far available seem to suffer in accuracy when an attempt is made to make them universal, that is, to apply a single method with one turbulence model to as broad a class of flows as that given for this Conference.

I want to emphasize that the particular set of structured flow zones shown in Appendix I is based on my own experiences with study of flow structure, and is merely a first cut at setting forth such zones. I do want to add, that in my view, the detaching and reattaching flow zones are like each other in structure but unlike attached layers far from detachment. The basis for this belief is considerable recent and older researches in these flow zones. Much of that experience is summarized in a recent paper by Kline, Bardina and Strawn, AIAA preprint 81-1220 from June 1981 Palo Alto meeting.* Similar remarks apply to recirculating zones, but the information is less complete. See for example Chui and Kline, Mech. Engrg. Dept. Report MD-19.

I also want to emphasize that even though Appendix I is my own personal view of the proper set of structured flow zones, it is nevertheless anchored in my understanding of flow structure, and is not arbitrary in that sense. I have a strong

*Scheduled for publication in AIAA Jou. Dec. 1982 or Jan. 1983.

prejudice about flow structure; I think structure is important, and that modeling needs to account for it, not in all detail but to some appropriate level. The current AIAA paper just cited is convincing evidence to me on this point. One major conclusion of that paper is that the entire community of both experimentalists and computors have erroneously taken incipient detachment to be full detachment throughout the entire literature, and that this misconception was rooted in a lack of understanding of the flow structure. The same remark applies to the long history of statistically averaged measurements in the near-wall zone. Those measurements for decades missed the important structures so beautifully illustrated in this Conference by the recent results of Moin and Kim using large-eddy simulation. Nor are these examples isolated. This suggests that models tied to structure in a general way may be more profitable than a search for a single set of closure constants that has for so long preoccupied nearly all the attention of the research community. I think there is little doubt that iteration between experiment and model for a given type of structural flow zone can certainly illuminate both the structure and the model faster than either alone. In the process of such iteration between modeling and experiments for a number of zones one should also certainly learn more about the proper choice of structural flow zones and thus an improved Appendix I. For example, one might find that attaching/detaching zones are the same problem structurally as the shock-boundary-layer interaction, as I suggested by implication in the general session. Secondary flow of the first type and three-dimensional boundary layers may be only one class. And so on.

Still another example of the utility of tailored methods is the results for transonic airfoils, Case 8601. Here again integral methods that were specifically tailored for the class of flows do better than available differential procedures.

I do not think these examples suggest that integral methods are better than differential methods. I do think they suggest a distinct lack of correlation between sophistication of modeling and accuracy of output, and that again suggests the trade-off between range of domain and accuracy.

We can probably agree that range of domain is an important criterion. The question is how to achieve range of domain without loss of accuracy. One way can be visualized from Appendix I. Suppose one or more groups created a well-adjusted model for each of the 20 structural flow zones of Appendix I, or some other small finite set of structural flow zones that represent an improved version of Appendix I. Taken over the whole community, or even over the group of modelers connected with B. Launder and W. Rodi, this is not an impossible task. One might for example use a k-ε or ASM model with different constants for the different structural flow zones and an ASM or RST model for another. The models for the structural flow zones then become subroutines within a program. The computer can easily keep track of the type of zone at a given

point; hence a mechod can be constructed that had models for say three or four structural flow zones which together cover a very wide set of applications and geometries. It would be very rare that one would need anything approaching the full 20 models in a given class of flow cases.

At the present moment, and for some years forward such an approach may be the only way to achieve a very large range of domain, reasonable running times, and good accuracy within a single program.

None of this is intended to suggest that efforts aimed at finding more sophisticated models should stop. The experience of the work on large-eddy simulation presented by P. Moin and J. Kim in this Conference make the utility of such methods very clear. They are important research tools already and may in time become useful methods for practice. But they are not useful for practice today, nor will they be in the near future, and it is not clear whether they ever will be cheap enough to employ in the repeated design passes that are essential in industrial designs.

In sum, my present thinking goes in the following direction. I would not favor the construction of ad-hoc models based on flow cases; that approach will not close; there are simply too many cases. Moreover, it would be bad science since it would not connect the flow physics to the modeling. On the other hand I can see considerable potential for practical solutions to computations of complex turbulent flow fields based on models connected to and adjusted for small numbers of structural flow zones of the sort implied in Appendix I. The use of zonal modeling based on such ideas has been largely limited to integral methods thus far, but I see no reason why they cannot be applied with great profit to the differential methods as well.

APPENDIX I

KNOWN STRUCTURAL FLOW ZONES OMITTING HYPERSONICS AND WAVE PHENOMENA

1. Inviscid flow
2. Two-dimensional attached boundary layers
3. Three dimensional attached boundary layers
4. Reattaching/detaching zones
5. Mixing layer
6. Axisymmetric wake
7. Plane jet
8. Axisymmetric wake
9. Plane wake
10. Recirculation zone (fully stalled zone)
11. Shock/boundary-layer interaction 2D
12. Shock/boundary-layer interaction 3D
13. Mach No. effects on items 3-7 above
14. Secondary flow, 1st type
15. Secondary flow, 2nd type
16. Laminar boundary layers
17. Transition
18. Homogeneous flows
19. Trailing-edge interactions
20. Large-scale vortical motion

Editors' Note:

An extended version of this discussion, including a revised version of Appendix I was written by S. J. Kline shortly after the 1981 meeting and circulated to all attendees of the meeting with the statement that written comments from attendants would be published. Five such comments were received, and a closure written. The extended version, comments, and closure appear under Session XV below; this includes revisions of Appendix I above.

DISCUSSION ON POSTSCRIPT

S. Birch: We cannot view complex flows from the point of view of geometry. We must go to zonal modeling. There are more than 20 modules. I think the effort to move to more general models is an effort to reduce the number of modules by applying more general concepts.

S. Kline: I also disagree with the idea of basing on geometry. I agree that you just cannot look at flow cases, that is bad physics. The physics is in the flow structure, not in whether one is looking at an airplane or an automobile as some comments suggested during earlier discussions.

P. Bradshaw: We have to go further. We have to make the adjustable constants functions of dimensionless parameters, perhaps more than 20.

S. Kline: There are some subdivisions within these 20 modules. The point is that the list is not very long.

A. Savill: Things are not so desperate; there is a degree of structural uniformity in different flows.

S. Kline: I agree.

D. Bushnell: The problem is there is a relaxation of one of these flows into another.

S. Kline: Yes, that will have to be looked at.

M. Rubesin: A good model is one that is good enough; this depends on the user. It is amazing that the k-ε model with six constants can show the qualitative features of an enormous amount of flows. If we are interested only in the qualitative features, we have the universal model. It is only when we need accuracy for particular cases that we need to do the fine tuning.

S. Kline: I do not know why one would be unwilling to adjust the constants of the k-ε model when going from one flow to another; this is not arbitrary. It is adjusting the model to the physics of the flow in a repeatable fashion.

D. Coles: It is frightening that there are 20 factorial junctions to make between one flow module to another.

S. Kline: It is not a problem; we know how to do the merges already, i.e., potential flow boundary-layer interaction. I am not contemplating 20 factorial; you cannot find such a flow. Junctions occur primarily in pairs. Hence, the maximum number would be 20!/18! = 380; many of these are either trivial or unnecessary. We will, however, need to sort these into a manageable number of classes again based on common physical features.

B. Launder: It is more convenient not to change the constants; it is like a supermarket where you can do all your shopping in one place.

SESSION V

Chairman: B. G. Newman

Technical Recorders:

S. Honami
J. Simonich

Technical Reporter

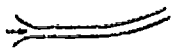
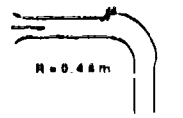
H. Nagib

INCOMPRESSIBLE WALL-BOUNDED FLOWS -- II

Cases 0231, 0232, 0233 -- Curved Boundary Layers
0241, 0242, 0244 -- Suction/Blowing
0263 -- Wall Jet

PICTORIAL SUMMARY

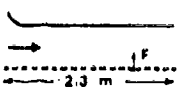
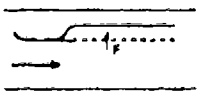
Flow 0230. Data Evaluators: T. Sinon and S. Honami. "Boundary Layer Flows with Streamwise Curvature."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0231-convex Case 0232-concave P. Hoffmann & P. Bradshaw		12	5	-	5	5	-	5	$\overline{u^3}$=5 $\overline{u^2v}$=5 $\overline{uv^2}$=5 $\overline{v^3}$=5	5[a]	5.2×10^4 (based on δ at beginning of curve)	δ/R = 0.01	Mild Curvature.
Case 0233 J. Gillis & J. Johnston		24	11	-	12	12	12	12		24[a]	4.6×10^4 (based on δ at beginning of curve)	0.10	Strong Curvature with Recovery.

[a] Clauser Plot Method.

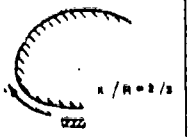
PICTORIAL SUMMARY

Flow 0240. Data Evaluator: L. Squire. "Turbulent Boundary Layers with Suction or Blowing."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0241 P. Andersen W. Kays R. Moffat	 Blowing V = 0.004 0.05 < x < 2.286 m	0	9	-	-	-	-	-	-	9	6.7×10^5 per m	M=0	No turbulence data.
Case 0242 P. Andersen W. Kays R. Moffat	As Case 0241 Suction V = -0.004 0.05 < x < 2.286 m	Adverse $U_\infty \sim x^{-0.15}$		9	-	1	1	1	-	9	6.7×10^5 per m	0	Turbulence data available at initial station.
Case 0244 A. Favre R. Dumas E. Verollet M. Coantic	 Suction -0.014 < V -0.05 < x < 0.4 m	0	1	-	1	1	-	1	-		7.3×10^5 per m	0	Data available for 4 suction rates: V = -0.002, -0.0053, -0.0100, -0.0144. Velocity and turbulence data restricted to one streamwise station.

PICTORIAL SUMMARY

Flow 0260. Data Evaluators: B. Launder and W. Rodi. "Turbulent Wall Jet."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity: U	Mean Velocity: V or W	Turbulence Profiles: $\overline{u^2}$	Turbulence Profiles: $\overline{v^2}$	Turbulence Profiles: $\overline{w^2}$	Turbulence Profiles: $\overline{uv}$	Turbulence Profiles: Others	C_f	Re	Initial Condition	Other Notes
Case 0263 D. Quitton B. Newman		-	Yes	-	Yes	Yes	-	Yes	R		7×10^3 to 5×10^4 (based on slot height)		Self-preserving wall jet on logarithmic spiral surface.

Methods Used--Session V

Method	Case 0231	0232	0233	0241	0242	0244	0263
BIKX				●	●	●	
BOLX				●	●		
BKEX	●	●	●	●	●	●	
BKEZ	●	●	●				
AKEZ	●	●	●				●
AKEX	●	●	●				
AKWX				●	●	●	
BOPX	●	●	●				
RSEN				●		●	
RSTN				●	●	●	
HN	●	●	●				
SG	●	●	●				
Plate No. Volume III	28-32	33-37	38-41	42,43	44,45	46,47	50

INCOMPRESSIBLE WALL-BOUNDED FLOWS - II

Curved Boundary-Layer Flows: Cases 0231, 0232, 0233

Boundary Layers with Suction and Blowing: Cases 0241, 0242, 0244

Wall Jet: Case 0263

Technical Report

by

H. Nagib*

The presentation of H. Nagib was in the form of a pictorial presentation. A summary of the methods used in these flow cases is given.

For each flow case, reference was made to the Plates in Vol. III. The reader will need to see the appropriate plates in Volume III as listed in the table entitled "Methods Used" above in order to follow the discussions on p. 755.

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A NOTE FROM THE EVALUATION COMMITTEE

Introduction

The following note was prepared after a meeting of the Evaluation Committee on the first day of the Conference and was presented at the beginning of Session V. Its contents display the attempt made to achieve a correctness in the Proceedings of the Conference as well as a consensus of agreement in its conclusions.

As a result of the experiences gained at the session on Monday afternoon (9/14), the Evaluation Committee would wish to make the following brief comments.

1. Taxonomy of Flow Models (Flow model descriptors--e.g., BKEX-64)

For the Evaluation Committee to make a valid evaluation of the computor outputs presented to this meeting we need the assurance that

(a) A flow model of a given Computor Group, e.g., BKEX-64, remains fixed in all its parameters and constants when used for more than one flow case, and the numerics remain fixed also.

(b) When a flow model of a given Computor Group (e.g., BKEX) has been changed, by changing constants or the numerics, it should have been given a new descriptor, e.g., BKEX-64A.

(c) The broad nature of the flow model and essential features of its numerics should be available in the paper, "Remarks on Taxonomies" by Jorge Bardina. This paper gives the important summary of the flow models used and their numerics. Would Computor Groups check and provide update information to Jorge Bardina as soon as possible, so that the Evaluation Committee will have a correct picture of what each computor output means.

We need to have it confirmed that when fine-tuning of a flow model or its numerics has been made to get a better fit with data on one flow case, the same model, in all respects, is then used on another flow case and must be labeled accordingly.

The meeting is indebted to those Computor Groups who have submitted outputs for a number of flow cases using the same flow model and numerics. What we ask is that these flow models be correctly labeled and details of the model be accurately given in the paper by Jorge Bardina.

2. Taxonomy of Numerics

Discussions relating to numerics have led us to believe that the Meeting cannot do otherwise than accept that the outputs from Computor Groups are meaningful and accurate. Later in the meeting we will receive the paper by Brian Launder on his experience relating to the sensitivity of the

Numerics. At this stage we ask the Meeting to help us in clarification of why, within a given flow model category

e.g., (i) integral methods
(ii) one-point closure B or A
(iii) one-point closure RS
(iv) two-point closure 2(DI/ED)

some outputs are better (worse) in agreement with the data than others. Is it because in the flow model used

(a) the constants or parameters have been specially chosen, or

(b) the initial conditions have been changed, or

(c) the numerical method has been improved?

Would Computor Groups please help in establishing such clarifications and get the necessary information into the paper by Jorge Bardina as soon as possible, as well as by informing the Meeting during Discussion of the particular Flow Cases. It is important to the Evaluation Committee and the Meeting that where a Computor Group has found good agreement with data as a result of improving and optimizing the numerics and that their prior numerics were not so successful, that we be informed of this experience.*

3. Questions Relating to Data

We believe that questioning of the data at this meeting should be carefully handled and will best be left to a special ad-hoc subcommittee, rather than to devote a large part of any discussion period in any session for that purpose. Any Computor Group who wishes to challenge the data in a given Flow Case should inform Prof. Kline. Thus suitable arrangements can be made for the setting up of an Ad-Hoc Committee for such issues to seek clarification.†

*[Ed.: The information thus passed to Dr. Bardina has been incorporated into the descriptors and tables of these Proceedings.]

†[Ed.: No such formal challenge arose for a data set selected by the 1980 meeting and used in the 1981 meeting.]

DISCUSSION

SESSION V (Morning)

CURVED-WALL BOUNDARY LAYERS

Integral Methods

P. Smith (Group 32) said his method was based on Green's lag-entrainment method, which treats curvature as a correction. There are arbitrary limits set up in the program which were met. He admits his computation was therefore in error for Case 0233.

J. Cousteix (Group 22) remarked that for his two curves for C_f for Case 0231, the initial values for skin friction and shape factor are incompatible. He claimed in all methods, that if the initial values of H were used, C_f would be lower in all cases.

In Case 0232, the trend of C_f is good but the level is not.

Methods BKEX 22A and BKEX 22B used the same values of constants. In method A, the ε-equation is modified by a similar method proposed by Launder et al. In Method B, the eddy viscosity coefficient, C_μ, is modified.

Differential Methods

W. Rodi (Group 07) stated that all the flows were calculated with the same method (i.e., an algebraic stress model), which does not use wall functions. This is method AKEX; it automatically turns into BKEX when curvature is absent. For the curved flows additional calculations were done with AKEZ.

B. Launder commented he found difficulty in computing Cases 0231 and 0232.

SUCTION AND BLOWING

Differential Methods

D. Wilcox (Group 37) commented that the expanded scales for C_f for the strong suction case made the results seem worse than they were. (Note: C_f is not plotted for Case 0244.)

He experienced a hard time starting the flow from the given initial conditions.

There seems to be some discrepancy between Wilcox' results and RSTN 21 in regard to momentum thickness calculated from the velocity profiles.

In regard to starting conditions, Wilcox used the prescribed velocity profile for the mean flow. Initial profiles of k and ω are computed by assuming that the length scale computed by these quantities is similar to the mixing-length profile. The amplitude of k could be varied by a factor of 4.

P. Orlandi (Group 05) said he had replotted the experimental values of $Q^{1/2}/U_e$ for comparison with his calculated values of $Q^{1/2}/U_e$.

For the blowing and suction flows, Cases 0241, 0242, and 0244, Orlandi started his calculation without blowing or suction and introduced blowing or suction a short distance downstream.

K. Hanjalić (Group 18) stated the reason he had poor agreement in Case 0244 was because he used a wall function, which did not allow for low Reynolds number effects, and strong suction tends to make the flow relaminarize.

In response to a question by K. Hanjalić, R. Sullivan (Group 21) said he computed all the way to the wall without using any damped-eddy viscosity. The scale was algebraically defined. For high Reynolds numbers, the dissipation was isotropic. Near the wall, the dissipation was nonisotropic. For calculation method RSTN 21, typically 30-50 mesh points were used.

In response to a question from G. Schuerer, regarding an 80% underprediction of shear stress for the no-suction case of Case 0244, D. Wilcox (Group 37) replied that this underprediction is common for many flat-plate computations, although the overall profile is good.

WALL JET

W. Rodi stated that this is a difficult case for an eddy-viscosity model because the shear stress does not go to zero at the location of the maximum velocity. A curvature coefficient, which is a function of Richardson number, goes to infinity at this location.

His results indicate the proper trend in regard to the spreading rate.

W. Rodi commented that Gibson presented an RSE model at the Davis Turbulent Shear Flow Conference which works well.

GENERAL DISCUSSION

P. Bradshaw said he wanted to know who used curvature-dependent constants for the curvature cases.

SG32--Smith. See comment above.

HN 22C, BKEX 22A, BKEX 22B--Cousteix. J. Cousteix stated he used an entrainment coefficient, which is a function of (δ^*/R), and is the same for both concave and convex walls.

AKEX 07, AKEZ 07--Rodi. W. Rodi stated that coefficient C_μ is a function of the curvature Richardson number and is an outcome of the use of the algebraic stress model.

AKEZ 17B--Launder. B. Launder said he had no curvature effects, since his method used a pure algebraic stress model.

BOPX 04--Nagamatsu. No comment.

P. Bradshaw asked if any features of the methods were functions of the transpiration velocity near the wall.

AKWX 37--Wilcox. No!

RSTN 21--Sullivan. No!

BOLX 30--Pletcher. Our method modifies the Van Driest function in a semi-empirical way as described in Pletcher (1974).

BKEX 07--Rodi. No!

BIKX 05--Orlandi. No!

J. Cousteix (Group 22). In response to questions by Lakshminarayana and by Orlandi, Cousteix stated he had used no curvature terms in his mean-momentum equations for the computations of Cases 0231 and 0233.

J. Moore asked if anyone had computed the presence of Taylor-Görtler vortices. No one replied.

W. Rodi noted that for Case 0233, the curvature terms in the mean-momentum equation were quite important.

SESSION V (Discussion from Dinner Meeting)

TAXONOMY

W. Rodi (Group 07) stated that his method for curvature reduced to the same model for blowing and suction. When there was no curvature C_μ is a function of Richardson number for curved flows, but $C_\mu = 0.09$ for no curvature. The same method is used for both concave and convex flows.

D. Wilcox (Group 37) stated his method for suction and blowing used what he termed an algebraic stress model, which was passive; that is, it had no effect on Reynolds shear stress.

Houdeville (Group 22) reported that in their method designated 22A there is no curvature correction to the mean-momentum equation. Curvature is handled by an equation for C_{ε_2} :

$$C_{\varepsilon_2} = C'_{\varepsilon_2}\left(1 - \alpha \frac{k^2}{\varepsilon^2} \frac{U}{r^2} \frac{\partial rU}{\partial r}\right)$$

Their method BKEX 22B modifies the eddy-viscosity coefficient:

$$-\overline{uv} = C_\mu\left(1 - \beta \frac{k^2}{\varepsilon^2} \frac{U}{r^2} \frac{\partial rU}{\partial r}\right) \frac{k^2}{\varepsilon} \frac{\partial U}{\partial y}$$

G. Scheuerer stated that curvature terms should be included in the mean-momentum equation for flows with curvature.

D. Wilcox agreed with Cousteix's group that curvature terms in the momentum equation

have a small effect and cites his computations on So and Mellor's data.

H. Nagib observed that for convex-curvature computations, all methods showed damped shear stress far from the wall.

M. Houdeville remarked that in their integral method for curved flows, curvature effects were modeled by a modified entrainment coefficient. Self-similar solutions were calculated by a modified mixing-length model.

W. Rodi said he was still not sure of the differences in Cousteix's methods 22A and 22B.

H. Nagib summarized Orlandi's results for Case 0244. Orlandi had plotted computed $K^{1/2}/U_e$ vs experimental $\overline{u^2}^{1/2}/U_e$ on his graph (See Plate 46, Vol. III).

D. Wilcox cautioned computors that wall functions should be compatible with the law-of-the-wall.

M. Gibson said that in his opinion wall functions are valid and that the law of the wall holds. For calculating curved-wall flows algebraic stress models are less important. Transport equations for shear stress will be cheaper to compute. Gibson believes you need a three-equation model with k, ε, and $\overline{uv}$ as variables.

W. Rodi said most computors used C_p values from the wall data for Case 0231. There was some difficulty expressed in fitting this data set.

B. Newman noted that in all computations for the concave flow, Case 0232, three-dimensional effects were not included.

In response to a question regarding new data for the Hoffman & Bradshaw flow, S. Kline quoted Bradshaw by saying that there were no subsequent data taken.

M. Gibson. From Nagib's presentation of C_f and θ data, it appeared to Gibson that some calculations do not have an overall conservation of momentum. Gibson suggested this be taken account of by the Evaluation Committee in their assessment of the various computational methods.

TRANSPIRATION

Commenting on Hanjalić's method, D. Wilcox suggested that Hanjalić's peak values are incorrect because of his use of wall functions.

D. Wilcox said you cannot use the same wall functions for flat-plate flows as for cases with pressure gradient or blowing.

B. Launder suggested that K. Hanjalić had used a velocity law-of-the-wall and his treatment of wall functions was consistent. Hanjalić's method employed a universal sublayer thickness, $y\, k^{1/2}/\nu$ = constant.

D. Coles noted the law-of-the-wall is essentially empirical. We can handle the effects of roughness and polymers in the flow, but not mass transfer at the wall.

R. Pletcher (Group 30) said that in his model a Van Driest damping function is used for shear stress which is a function of the blowing rate.

W. Rodi said that in his method there are no special changes to deal with transpiration but only a wall-boundary condition.

P. Orlandi said that his method for transpiration was the same as for the flat-plate case, except for the v-component velocity at the surface.

D. Wilcox said he used a surface boundary condition on ω (which is related to 1/time-scale), that depends on blowing rate.

P. Orlandi said that both Wilcox and Rodi used a step function change in the wall condition for the suction case similar to his own initial conditions.

M. Gibson referred to his computation of a wall jet on a curved surface presented recently at the Shear Flow Meeting in Davis. He said he did not have any particular difficulty in calculating the flow.

DISCUSSION OF MODIFICATIONS

Modifications to Allow for Curvature Effects in Cases 0231, 0232, 0233--Curved Boundary Layers

W. Rodi--07: C_μ is made a function of curvature Richardson number, reverting to the value 0.09 for no curvature. Pressure changes across the flow are calculated and are particularly important for Case 0233. The method is the same for concave and convex flows.

J. Cousteix--22: In A and B there is no curvature correction to the mean-momentum equation.

--22A: The constant C_{ε_2} in the dissipation equation is made linearly dependent on

$$\frac{k^2}{\varepsilon^2}\frac{U}{r^2}\frac{\partial(rU)}{\partial r}$$

--22B: The eddy-viscosity coefficient C_μ is made linearly dependent on the same parameter.

--22C: The entrainment coefficient in this integral method is modified in the light of approximate self-similar solutions using the above differential methods.

It was again emphasized that the concave Case 0232 was a three-dimensional flow because of the existence of Taylor-Görtler vortices or related coherent structures, but was tackled, as instructed, by two-dimensional methods.

It was suggested that a method incorporating three transport equations for k, ε, and $\overline{uv}$ might be useful for solving two-dimensional curved flow. This method would lie between k-ε or k-ω methods and methods which incorporate transport equations for all components of the Reynolds stress tensor. In this connection it was generally agreed that turbulence transport equations were needed for predicting highly curved flows.

Modifications to Allow for Effects of Transpiration on Wall Boundary Conditions in Cases 0241, 0242, 0244--Boundary Layers with Blowing/Suction

All computors allowed for the purely kinematic effect of transpiration:

C. Donaldson (21), Rodi (07), and Orlandi (05) applied no other correction.

R. Pletcher (30) modified the Van Driest damping function empirically.

K. Hanjalić (18) assumed a universal, non-dimensional, sub-layer thickness using $k^{1/2}$ as velocity scale. More details of this procedure is given in Launder's review paper.

D. Wilcox (37): The boundary condition for ω is made to depend the on non-dimensional blowing rate.

A specific point was that P. Orlandi had plotted $K^{1/2}$, where $K = \overline{u_i u_i}$.

During the discussions Gibson made the interesting suggestion that the computed results might be judged, like the experiments, by the extent to which they confirmed an overall momentum balance.

The calculation of the curved wall j. Case 0263, was attempted by only one computor and proved to be extremely difficult.

Finally, some considerable difference of opinion arose between representatives of the k-ε and k-ω schools on the question of boundary conditions. The question was not resolved.

Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

J. Humphrey said that in his experience it was not enough, in curved-wall-flow prediction, to include a curvature correction on C_μ in k-ε models. Wall-pressure-fluctuation effects arising from the pressure-strain terms also have to be considered.

In reply to J. Johnston, B. Newman said that there had been no discussion of why predictions in the recovery region were poor. W. Reynolds pointed out that the shear-stress predictions were already too high at the start of the recovery, but W. Rodi said that in his predictions the overshoot only appeared after the curved section.

SESSION VI

Chairman: R. Blackwelder

Technical Recorders:

G. Settles
D. Ota

Technical Reporter

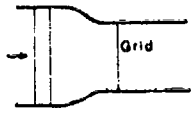
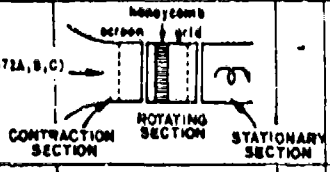
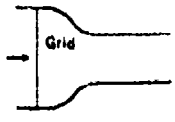
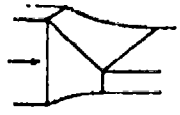
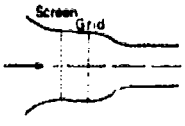
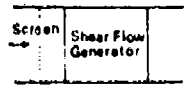
J. L. Lumley

HOMOGENEOUS FLOWS

Cases 0371, 0372, 0373, 0374, 0375, 0376

PICTORIAL SUMMARY

Flow 0370. Data Evaluator: J. Ferziger. "Homogeneous Turbulent Flows."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Conditions	Other Notes
Case 0371 G. Comte-Bellot S. Corrsin		zero	Yes	-	Yes	Yes	-	-	Auto-correlation Energy spectra	-	1600 3200 (based on M)	-	Homogeneous isotropic turbulence. Grid 3 cases
Case 0372 R. Vigeland M. Nagib (0372A,B,C)		-	Yes	-	Yes	Yes	-	-	-	-	-	-	Rotating homogeneous turbulence.
Case 0373 M. Uberoi (0373A,B,C,D) M. Tucker A. Reynolds (0373E)		zero	Yes	-	Yes	Yes	-	-	Spectra of $\overline{u^2}$, $\overline{v^2}$	-	3710 (based on M)	-	Return to isotropy. Contraction 3 cases.
Case 0374 A. Townsend (0374A) M. Tucker A. Reynolds (0374B)		-	Yes	-	Yes	Yes	Yes	-	Longitudinal spectra	-	-	-	Plane strained homogeneous turbulence.
Case 0375 J. Tan-atichat (0375A,B,C,D,E)		-	Yes	-	Yes	Yes	-	Yes	Spectrum cross-covariance	-	270 to 8100 (based on M)	-	Axisymmetric strained homogeneous turbulence. Grid 6 cases. Contraction 7 cases.
Case 0376 V. Champagne V. Harris S. Corrsin (0376A) V. Harris J. Graham S. Corrsin (0376B)		zero	Yes	-	Yes	Yes	Yes	Yes	Auto-correlation	-	-	-	Sheared homogeneous turbulence.

Methods Used--Session VI

Method	Flow Case					
	0371	0372	0373	0374	0375	0376
RSEC	•	•	•	•	•	•
RSLC	•		•	•	•	•
AKWC	•	•		•		•
AKEC				•	•	•
2ED	•	•	•	•	•	•
Plate No. Volume III	59	60-62	63-70	71-76	77-86	87-96

HOMOGENEOUS FLOWS

Cases 0371, 0372, 0373, 0374, 0375, 0376

Technical Report*

by

J. L. Lumley†

We identify in approximate order of complexity the methods used to calculate these flows:

AKWC 37	Wilcox	A two-equation mixed eddy-viscosity and algebraic-stress model.
RSLC 21	Donaldson et al.	No rapid term; algebraic equation for length scale.
AKEC 07	Rodi et al.	LRR2 reduced to algebraic form by taking transport of the normal stresses only proportional to the transport of energy.
AKEC 17A	Launder et al.	LRR2 reduced to algebraic form by taking transport of Reynolds-stress tensor proportional to the transport of energy.
RSEC 17A	Launder et al.	LRR2
RSEC 18	Hanjalić et al.	LRR1 with term Pε/k in dissipation equation replaced by $kU_{j,j}U_{i,j}$.
RSEC 22B	Cousteix et al.	LRR1 with $C_{\varepsilon 1}$ determined from Lin and Wolfstein.

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RSEC 22	Cousteix et al.	The Hanjalić, Launder and Schiestel multiscale model, which divides the spectrum in two parts with exchange between them.
2ED-52	Cambon et al.	Spherically averaged spectral model.
2ED-52A	Cambon et al.	Complete spectral model.

More complete descriptions of the various models can be found by referring to the computors' summaries indicated in the second column. For convenience, we compare and contrast here. The models differ in the way in which the Reynolds stresses are obtained, the way in which the "rapid" terms in the Reynolds-stress equation (the pressure-strain terms proportional to the mean velocity gradient) are treated, and the way in which the length scale is obtained. The Reynolds stress may be obtained from an eddy-viscosity assumption relating it directly to the mean-strain rate; it may be obtained from a simplification of the modeled Reynolds-stress equation permitting an algebraic solution; or it may be obtained from a complete solution of the modeled Reynolds-stress equation. The rapid terms may be omitted altogether; or either a simplified (LRR2), or more complete (LRR1) model proposed by Launder, Reece and Rodi may be used; the length scale may be specified algebraically, or obtained from a differential equation, and this equation may be for a dissipation rate, or for a frequency. In the equation for the dissipation, a number of different assumptions may be introduced, primarily concerning the way in which the coefficients depend on other variables. The turbulent spectrum may be assumed to be characterized entirely by the scales of the energy-containing eddies; or it may be broken into two parts separately scaled, with transfer between the two; or the equations for the spectrum may be solved directly, treating each wave-number separately. In the spectral treatment, all wave-numbers of a given magnitude can be lumped together, or a full treatment can be given.

We will discuss only a representative sample of the calculations: the isotropic turbulence of Comte-Bellot and Corrsin (0371); a single case of rotation (0372C) with $\Omega = 60\ sec^{-1}$; two cases of return to isotropy (0373A,E); the plane strains of Townsend and of Tucker and Reynolds (0374A,B); a single axisymmetric strain (0375E) with contraction ratio = 16; and both shears (0376A,B) with shears equal to 12.9 and 48 sec^{-1}.

Plate 59, Vol. III shows the results for the isotropic decay of Comte-Bellot and Corrsin. It is evident that all the models that attempted this (or a related) equation would be expected to do well here. It should also be mentioned that the spectral models have no adjustable constants--there is only one constant, associated with the Eddy-Damped Quasi-Normal Markovian approximation, and that is determined from a test-field model calculation. It is evident that several of the computors have tried different values of the initial dissipation, improving their results. This is perfectly legitimate, since the initial dissipation is not an independent parameter, but must be determined from the initial slope of the energy curve. The initial value

first specified for these calculations was in some cases in error. The calculations are, of course, quite sensitive to this value. Moreover, it is difficult to determine the value graphically unless a parabolic or better fit is made to the first few data points.

In the case of homogeneous rotating flow, reference to Plates 60, 61, and 62, Vol. III makes it evident that essentially everyone who tried the flow had difficulty. It is clearly easier to get the energy than it is to get the anisotropy, but even there the eddy-viscosity approach does badly. The algebraic stress model and the spectral models both get the energy right, but the anisotropy is not good, even for the spectral model. One problem here is the necessity of knowing the entire spectral density tensor as initial condition. Since this is never measured, it was necessary for this group to construct the tensor using various assumptions. The two curves that appear in Plates 61 and 62 correspond to two different assumptions regarding the form of the initial tensor. The failure of the algebraic stress model presumably indicates that the rotation is not properly parameterized in the rapid terms.

In the first return-to-isotropy case, reference to Plate 63, Vol. III makes it evident that (Group 21) Donaldson's programs do not do well. We can probably indirectly attribute this to the absence of the rapid terms, although these terms would be zero in this flow; however, the return-to-isotropy coefficient has probably been adjusted to give good results in shear flows, to compensate for the absence of the rapid terms, and is hence too large. Either the algebraic stress model or the full-Reynolds-stress model does reasonably well, with the latter doing better. The spectral model appears to do well.

The agreement with the second return-to-isotropy case (Plate 69, Vol. III) is less satisfactory. The first case corresponded to axisymmetric turbulence with one component smaller than the other two. This second case corresponds to one component low and one high, with the third intermediate. It is evident here that nearly everyone has trouble with the intermediate component; surprisingly, Donaldson's model now does satisfactorily on the other two components, suggesting that the return-to-isotropy term has been optimized for a particular type of anisotropy.

In the case of Townsend's plane strain (Plates 71, 72, and 73, Vol. III), it is again the intermediate component that gives essentially everyone trouble, suggesting again that these methods have probably been optimized for a different type of strain. The other two components are predicted more or less satisfactorily, although there is a tendency for most methods to feed too much energy to the component that is gaining energy. Here the best ones are Donaldson's models, lacking rapid terms. This might suggest that the parameterization of the rapid terms in those models that use them is inadequate, and that one may do better with a simpler model.

The second plane strain (Plates 74, 75, and 76, Vol. III), which is essentially twice the intensity, shows that one needs at least a full-Reynolds-stress model to do well on all three components. The simpler models seem to overpredict $\overline{v^2}$ and underpredict $\overline{u^2}$ consistently. Even the spectral models have some trouble with $\overline{v^2}$.

In the case of the weak shear (Plates 85 and 86, Vol. III), again it is clear that at least a full-Reynolds-stress model with rapid terms seems to be necessary. An algebraic stress model can give good results for the component energies, but not the Reynolds stress. In fact, the more evolved models of Cousteix also run into trouble; they very much underpredict the Reynolds stress. The spectral models do not do well either. The model of Hanjalić seems to be the best all around. We may note that everyone who has trouble with the Reynolds stress, underpredicts it and that this value of the strain rate is rather low relative to technologically interesting values.

In the case of the higher shear of Harris et al. (Plates 91 through 96, Vol. III), if we limit ourselves to the algebraic stress models and better, and if we concentrate on the Reynolds stress, we again find that the models of Cousteix do not do particularly well, nor does the spherically averaged spectral model. The latter can be explained by the spherically averaged spectral models' known failure under large total shearing deformation. We must assume that the elaboration of the models to include spectral scales, and to improve the length-scale equation, has done something unforeseen and physical. The remaining models do not do badly, but there is a collective tendency to overpredict the Reynolds stress. We note that the shear here is larger than technologically interesting values.

Finally, we have the axisymmetric strain of Tan-Atichat (Plates 77 through 86, Vol. III). If we concentrate on the low (axial) component, we see that the models without rapid terms take it down too low (overcompensating again?), while none of the others takes it down low enough. The simpler of the algebraic stress models seems competitive, while the more complex one seems inexplicably to be in trouble.

Summarizing these varied results, leaving aside for a moment the spectral models, I would say that a complexity at least equivalent to a full-Reynolds-stress model seems to be necessary for reasonable results in all circumstances. Even then, accuracy of prediction of diagonal components of the Reynolds stress appears to depend on type and intensity of anisotropy, suggesting that the return-to-isotropy term has not been modeled to take into account the fact that rates of return should differ for differing types and degrees of anisotropy. Off-diagonal terms tend to be underpredicted when the shear is low, and overpredicted when the shear is high, leading one to suspect that the rapid terms, in addition, have been optimized for a particular range of shears and level and type of anisotropy. The fact that the models do not respond well to large contractions suggests this also. The response to rotation has evidently not been properly modeled at all; this presumably also resides in the rapid terms.

The spectral models (see Plates 87 through 96, Vol. III) generally perform well, although there appear to be some difficulties. Some of these are attributable to the problem of generating adequate initial conditions, and some are associated with the known failure of the spherically averaged model under large shear deformation. When it is considered that there is only one constant, and that is determined from an isotropic calculation (of the test field model), the performance is remarkable.

As has frequently been pointed out at this meeting, it is evident that existing models have been optimized for particular types of flows and types and levels of anisotropy and shear. Hence, it probably does not make much sense at the present time, from a practical point of view, to calibrate a model intended for shear flows against homogeneous distortions, for example. This raises the general question of universality of models: should we hope for future models that will span increasingly large classes of flows, and will these come about because better physics has been built into the models, or simply because better and better curve-fitting schemes have been found? I believe it makes sense to search for models that cover as large a domain as possible on the basis of better physics; I base this belief on the success that has been experienced in the last decade in extending existing models on a physical basis to new situations: transport, return-to-isotropy, and buoyancy are now much better understood by some workers, and models for these terms are under development that have every appearance of being much more nearly universal. The last serious hold-outs are the rapid terms, which are still poorly understood. In addition, I feel that our ability to curve-fit is rapidly running out, and that if we are to extend the existing models, the only hope is by consideration of the physics. Finally, I believe that the effort to model turbulence has been enormously salutary for the fundamental study of turbulence, and has caused us to look at turbulence in a different way and ask new questions, the answers to which have shed considerable light. I would be sorry to see us abandon the search for universality, however limited, accent empiricism and lose this momentum.

DISCUSSION

SESSION VI*

HOMOGENEOUS FLOWS

Case 0371, Isotropic Turbulence (Comte-Bellot and Corrsin experiment)

1. The initial dissipation cannot be determined well from the experimental data, yet it is an important condition for a successful calculation. It is therefore legitimate for computors to adjust the initial dissipation, so as to meet the initial energy slope.
2. It should be recalled that the decay exponent is not unique, but is a function of Reynolds number, anisotropy, and spectrum shape.
3. More information on the initial turbulence spectrum is needed for level 4, 5, and 6 techniques.

Case 0372, Rotating Turbulence (Wigeland & Nagib experiment)

The group observed that none of the existing one-point models show any effect of pure (solid-body) rotation. Further, the existing two-point models do not handle this very satisfactorily.

Case 0373, Return to Isotropy (Uberoi experiment)

The rate of return to isotropy probably depends upon the structure of the turbulence, and therefore a single constant probably cannot describe the different rates of return to isotropy in different flows.

Case 0374, Plane Strain (Townsend and Tucker & Reynolds experiments)

1. The computor groups as a whole did better on the Tucker-Reynolds flow than on the Townsend flow.
2. Future data takers should be very careful to document the strain to which the turbulence is actually exposed, and the homogeneity of the turbulence. We have some concerns about these in the Townsend experiment.
3. The GENCE experiment (J. [illegible] Mech., 1979) should be called to the attention of future predictors.

Case 0375, Axisymmetric Strain (Tan-atichat experiment)

Estimates of the axial turbulent transport in these flow cases would be interesting, and might be significant.

Case 0376, Homogeneous Shear Flow (Champagne et al., and Harris et al. experiments)

1. The bulk of computors underpredicted the Reynolds shear stress in the low-shear case and overpredicted it in the high-shear case.

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*As consolidated and edited in the ensuing meeting following the session.

2. This discrepancy is likely due to effects omitted by the computors.

GENERAL COMMENTS

1. These flows have been criticized by some as not being technologically significant, but they are building-block flows which should be fundamental in the development of a model.
2. A proper test of universality of a turbulence model would be its ability to predict a broad range of flows including both homogeneous and inhomogeneous cases. (See discussion on Universal or Zonal Modeling p. 748 and pp. 991-1014.)

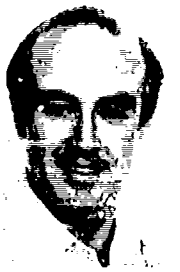
Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

H. Nagib pointed out that since experiments show that $\overline{u^2} \sim x^{-1.25}$ (and not x^{-1}), ε is not a constant. J. Ferziger added that the experimental range of x was too small in most experiments to get an uncertainty in ε better than $\pm$ 20%. W. Rodi emphasized that only the initial ε was required.

W. Reynolds and M. Morkovin agreed that the energy variations in the flow need to be used as a test of models, and the latter added that some experiments in this class are not as reliable as others.

SESSION VII



Chairman: P. Sockol

Technical Recorders:

M. Legner
A. Ortega

Technical Reporter

P. Kutler

TRANSONIC FLOWS

Cases 8601, 8621 — Simple Strains
8623, 8611, 8612, 8691 — Extra Strains

DISPLACEMENT EFFECTS IN TRANSONIC AIRFOIL FLOWS — R. E. Melnik

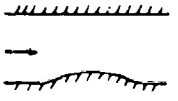
PICTORIAL SUMMARY

Flow 8600. Data Evaluators: M. Rubesin and C. Horstman. "Impinged Normal Shock Wave-Boundary Layer Interaction at Transonic Speeds."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8601 O. Mateer A. Brosh J. Viegas			8	-	-	-	-	8	$\overline{uv}$ -	Buried wire	3×10^5 to 6×10^6 (based on δ_o)	1.3 - 1.5	Axisymmetric profile data for one test condition. Skin friction data for several test conditions varying M and Re.

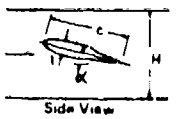
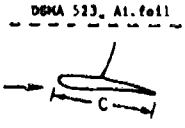
PICTORIAL SUMMARY

Flow 8610. Data Evaluators: M. Rubesin and C. Horstman. "Transonic Flow over a Bump."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8611 W. Bachalo D. Johnson			12	12	12	12	-	12	-	-	1.4×10^5 (based on δ_o)	0.875	Axisymmetric. Complete LDV data.
Case 8612 J. Delery P. Le Dimet			28	28	28	28	-	28	-	-	7×10^6 (based on δ)	M_{shock} 1.37	Complete LDV data. Top wall is important in solving problem.

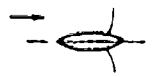
PICTORIAL SUMMARY

Flow 8620. Data Evaluator: R. E. Melnik: "Transonic Airfoils."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8621 F. Cook M. McDonald M. Firmin	RAE 2822 Airfoil 	Airfoil upper & lower surface pressure	9	-	-	-	-	-		Modified Law of the Wall	5.7 to 6.5 $\times 10^6$ (based on chord)	0.676 to 0.730	RAE 8×6 ft. transonic tunnel. Data include computed values of boundary-layer integral parameters.
Case 8623 F. Spaid L. Stivers, Jr.	DSMA 523 Airfoil 	Airfoil upper & lower surface pressure	7	-	-	-	-	-	p/p_o	Law of Wall by transformed velocity	2-4 $\times 10^6$ (based on chord)	0.6 to 0.8	NASA-Ames 2×2 ft. transonic tunnel.

PICTORIAL SUMMARY

Flow 8690. Data Evaluators: M. Rubesin and C. Horstman. "Non-Lifting, Transonic Airfoil with Shock-Separation."

Case / Data Taker	Test Rig Geometry	dp/c_w or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8671 J. McDevitt A. Okuno N. Seegmiller			?	?	?	?	-	?	Surface pressure	Buried wire	1.1×10^7 (based on chord)	0.785	Circular arc airfoil with large separation.

Methods VII

Method	Case 8601	8611	8612	8621	8623	8691
BKEX	●					
BKWX	●	●				
BØPX			●	●	●	●
AKWX				●	●	
HN				●		
SG				●	●	●
EE				●		
Plate No. Vol. III	152-154	155	156	157-165	166-168	178

TRANSONIC FLOWS

Simple Strains: Cases 8601, 8621 (Compressible Flow)

Extra Strains: Cases 8623, 8611, 8612, 8691 (Compressible Flow)

Technical Report

by

P. Kutler*

The presentation of P. Kutler is in the form of a pictorial presentation. A summary of the computors for these transonic flow cases is given in Fig. 1. The following figures are self-explanatory.†

Case 8601: Figs. 2 through 6.

Case 8611: Figs. 7 through 11.

Case 8621: Figs. 12 through 20.

Case 8623: Figs. 21 through 27.

Case 8691: Figs. 28 through 30.

SUMMARY OF CONTRIBUTIONS

	CONTRIBUTOR								
CASE	12	22C	32	36	37	39	42	47	50
8601[a]	–	–	–	●	–	–	–	–	–
8611	–	–	–	●	–	–	–	–	–
8621	●	●	●	–	●	●	●	●	●
8623	–	–	–	–	●	–	●	●	●
8691	–	–	–	–	–	–	●	–	●

[a]Plots were also received from Group 41 (McDonald) using Method BKEX for Case A only.

Figure 1.

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*NASA-Ames Research Center, Mail Stop 222-12A, Moffett Field, CA 94035.

†Technical Reporters were given the option of reviewing a sample of the computed flows. In this session P. Kutler did not comment on Flow Case 8612.

SHOCK-WAVE/TURBULENT BOUNDARY-LAYER INTERACTION AT TRANSONIC SPEEDS

CASE 8601

SUMMARY

FACILITY: NASA AMES, HIGH REYNOLDS NO.1 TUNNEL

MODEL: CONSTANT-DIAMETER, AXISYMMETRIC TEST SECTION
SHOCK GENERATED BY HOLLOW CYLINDRICAL CHOKE

MEASUREMENT TECHNIQUES: PRESSURE PROBES, PRESTON TUBE OR BURIED HOT WIRE, SUPPORTED CROSS-WIRE PROBE

TEST CONDITIONS: $1.32 < M_\infty < 1.48$
$8.5(10)^6 < Re\ 255(10)^6$

Figure 2.

WALL PRESSURE DISTRIBUTION

$M_\infty = 1.44, Re = 40(10)^6$
CASE 8601
TEST A

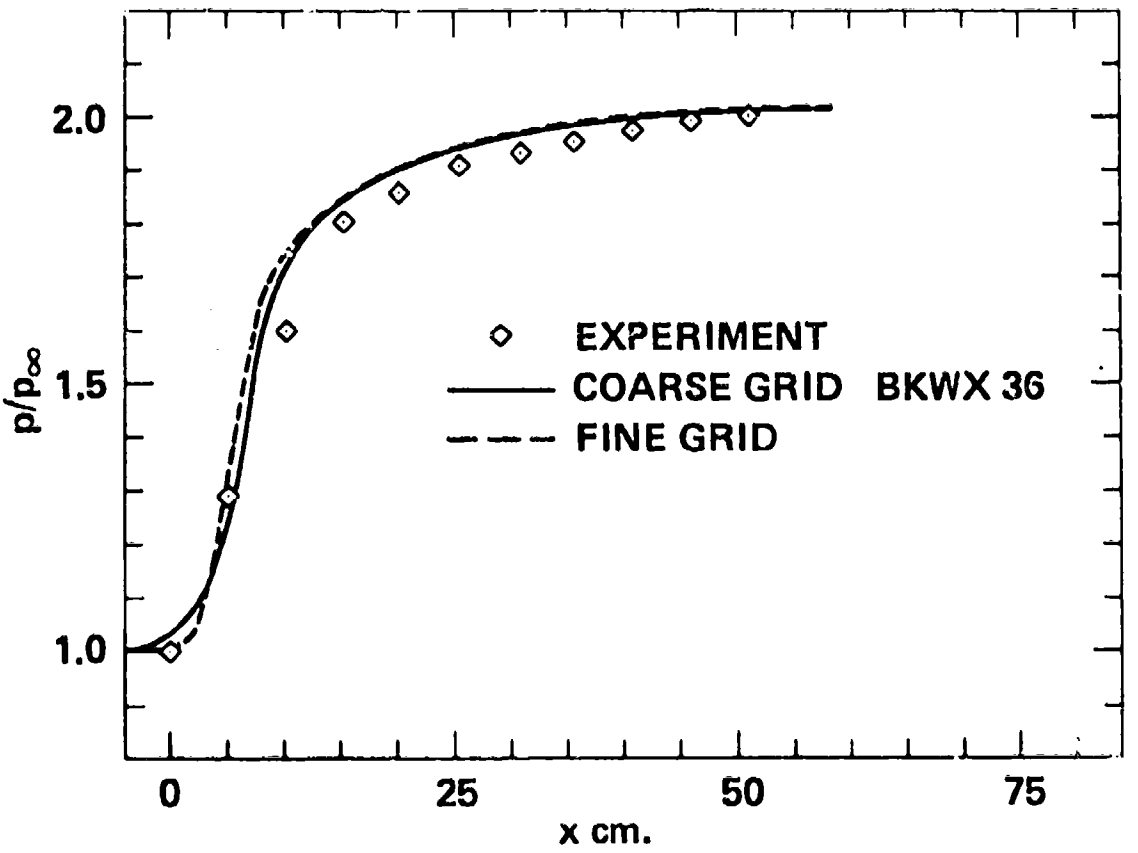


Figure 3.

SKIN FRICTION DISTRIBUTION

$M_\infty = 1.44$, $Re = 40(10)^6$
CASE 8601
TEST A

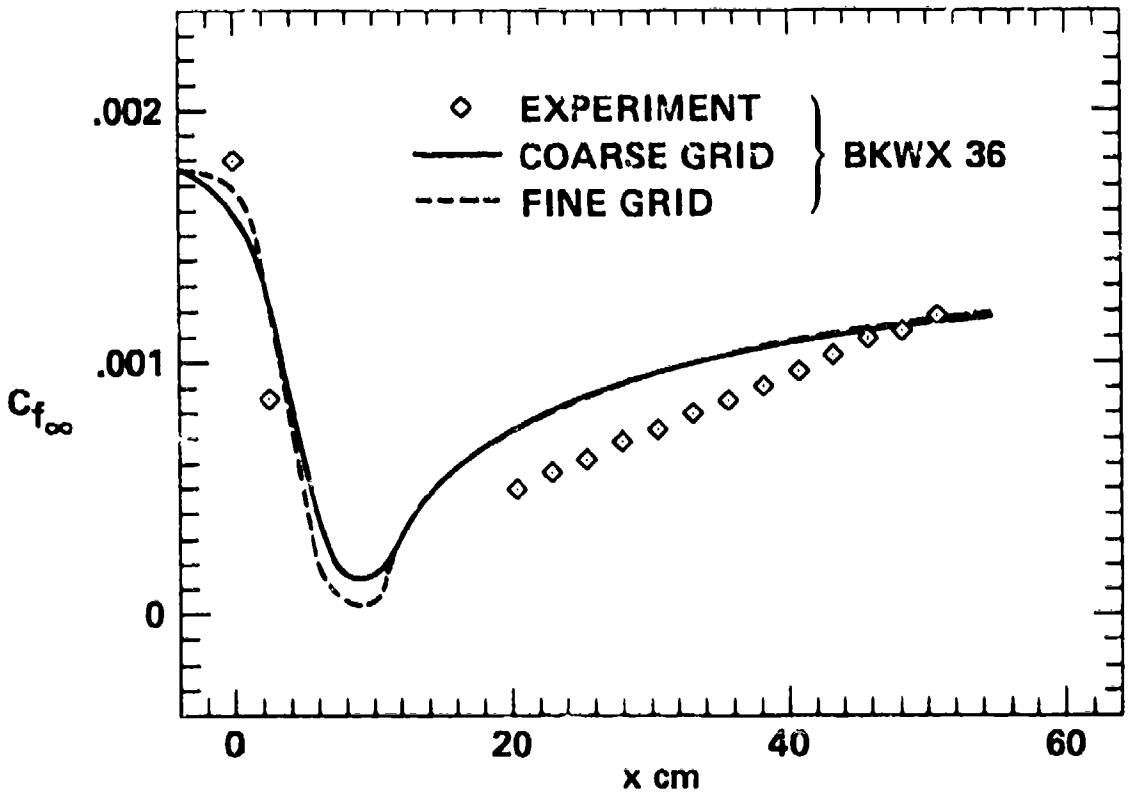
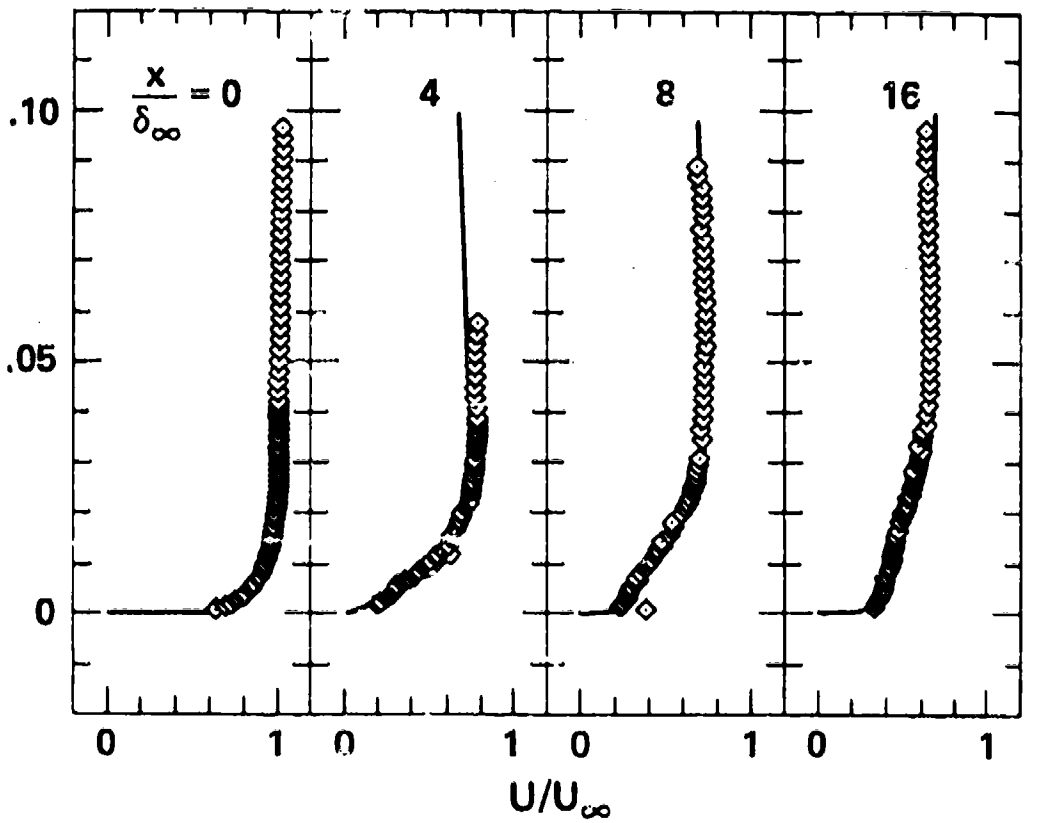


Figure 4.

VELOCITY PROFILES

$M_\infty = 1.44, Re = 40(10)^6$
CASE 3601
TEST A

◇ EXPERIMENT
—— BKWX 36



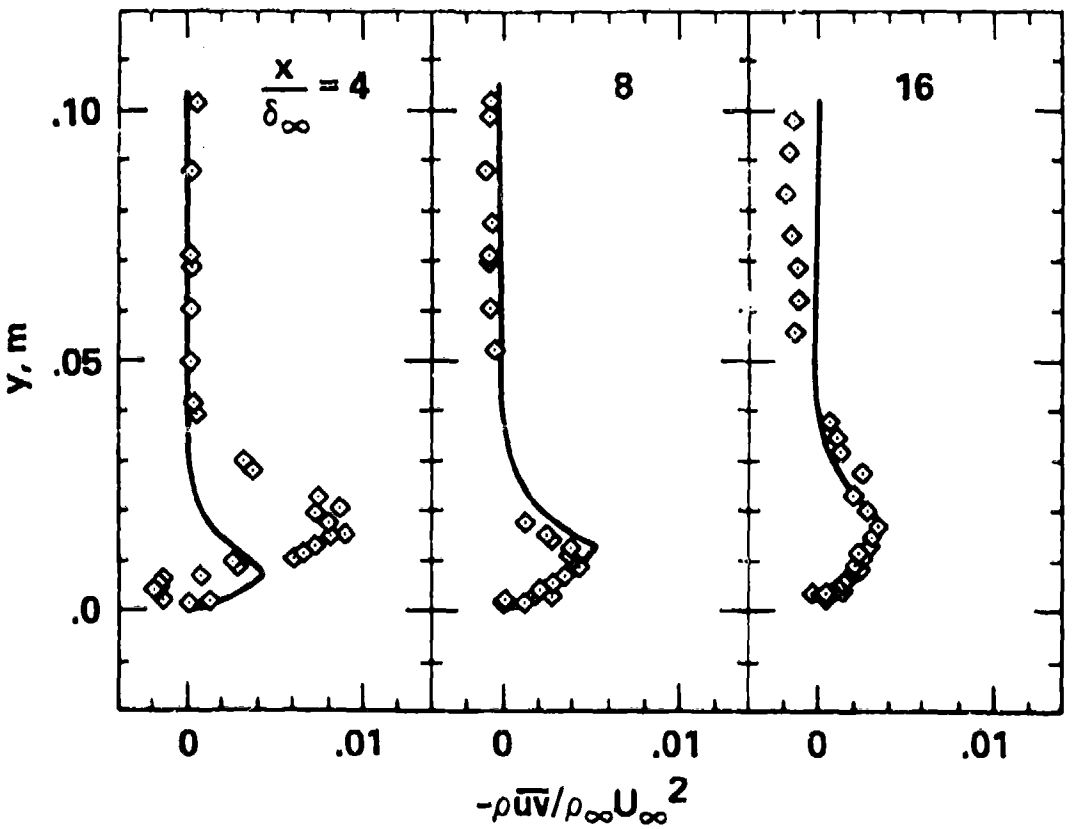
NOTE: COARSE AND FINE GRIDS GIVE IDENTICAL RESULTS

Figure 5.

SHEAR STRESS PROFILES

$M_\infty = 1.44$, $Re = 40(10)^6$
CASE 8601
TEST A

◇ EXPERIMENT
—— BKWX 36



NOTE: COARSE AND FINE GRIDS GIVE IDENTICAL RESULTS

Figure 6.

TRANSONIC TURBULENT BOUNDARY LAYER SEPARATION ON AN AXISYMMETRIC "BUMP"

CASE 8611

SUMMARY

FACILITY: NASA AMES, 2- BY 2-FOOT TRANSONIC WIND TUNNEL

MODEL: ANNULAR CIRCULAR ARC BUMP AFFIXED TO A CIRCULAR CYLINDER ALIGNED WITH THE FLOW DIRECTION

MEASUREMENT TECHNIQUES: LASER VELOCIMETER, OIL FLOW VISUALIZATIONS, CONVENTIONAL PRESSURE INSTRUMENTATION

TEST CONDITIONS: $M_\infty = 0.875$

$Re = 13.1(10)^6 /M$

Figure 7.

WALL PRESSURE DISTRIBUTION

$M_\infty = .875, Re = 13.1(10)^6/M$
CASE 8611

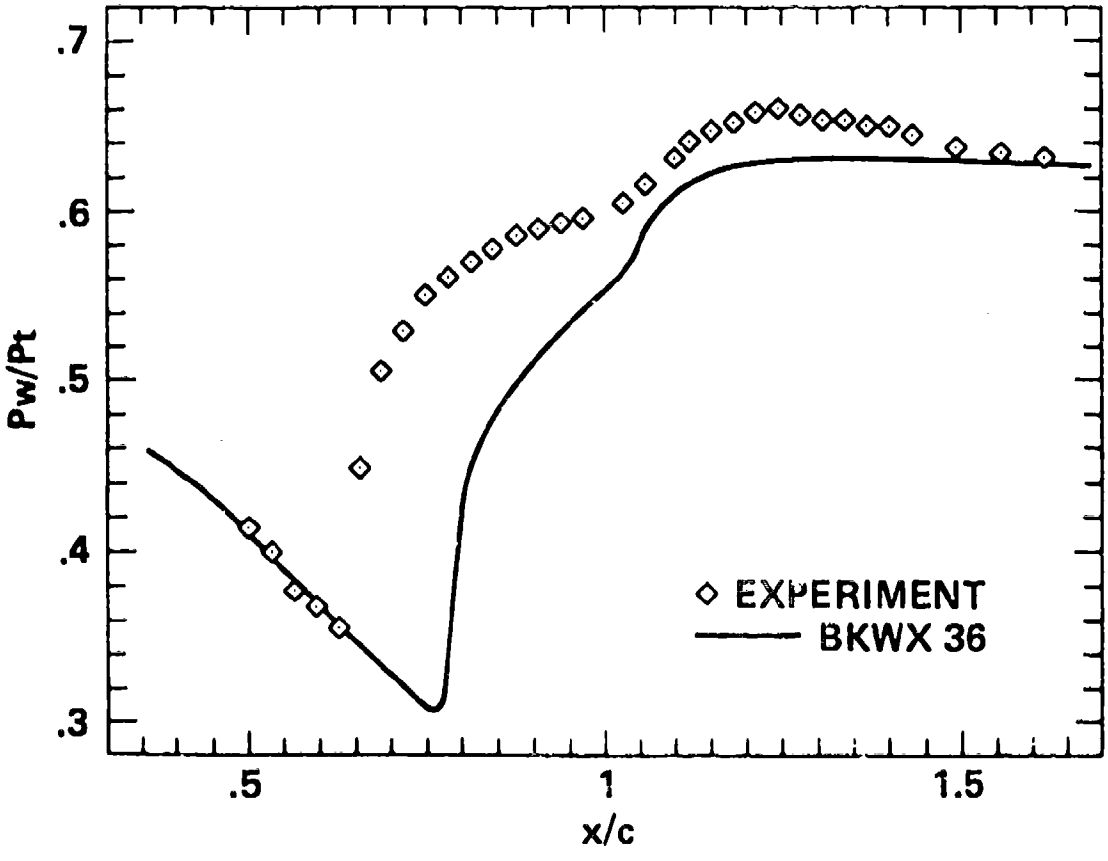


Figure 8.

VELOCITY PROFILES

$M_\infty = .875$, $Re = 13.1(10)^6/M$
CASE 8611

◇ EXPERIMENT
—— BKWX 36

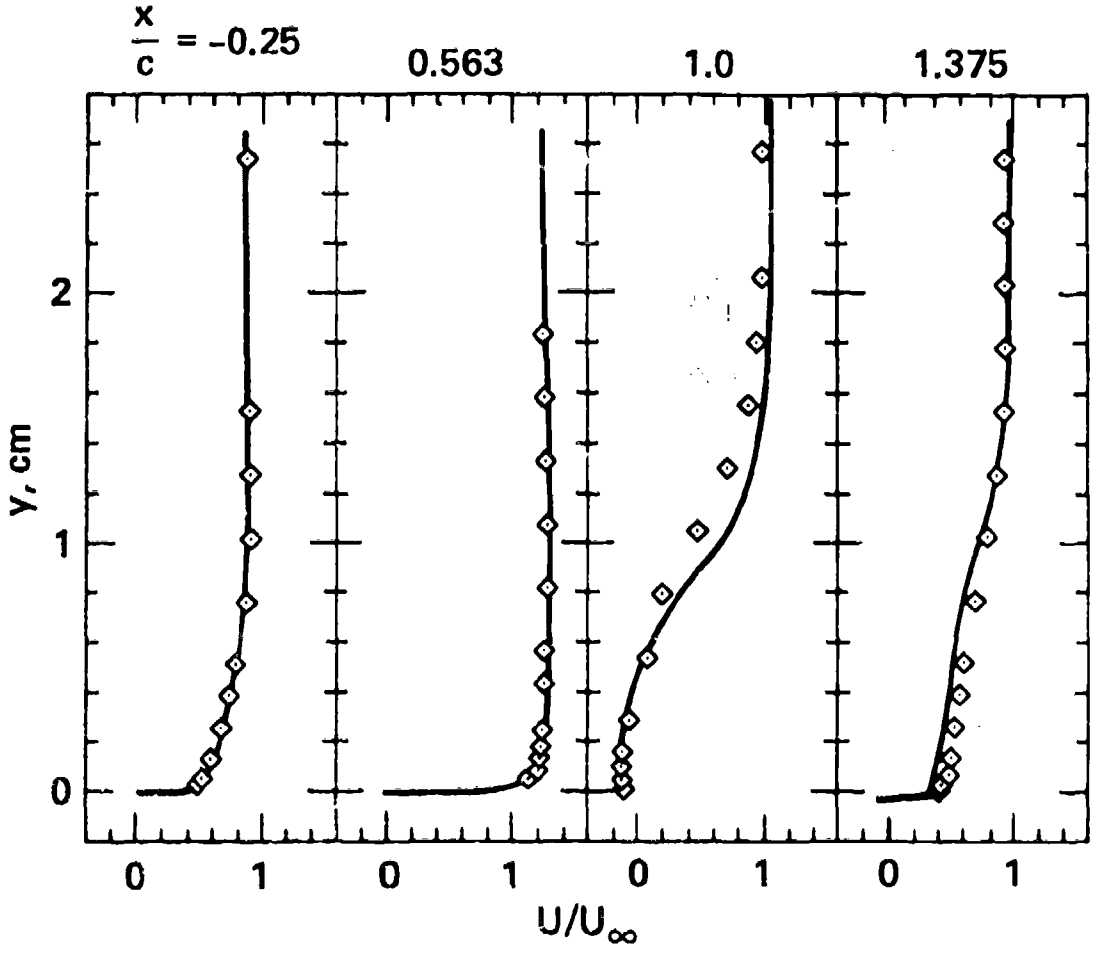


Figure 9.

TURBULENT KINETIC ENERGY PROFILES

$M_\infty = .875$, $Re = 13.1(10)^6/M$
CASE 8611

◇ EXPERIMENT
—— BKWX 36

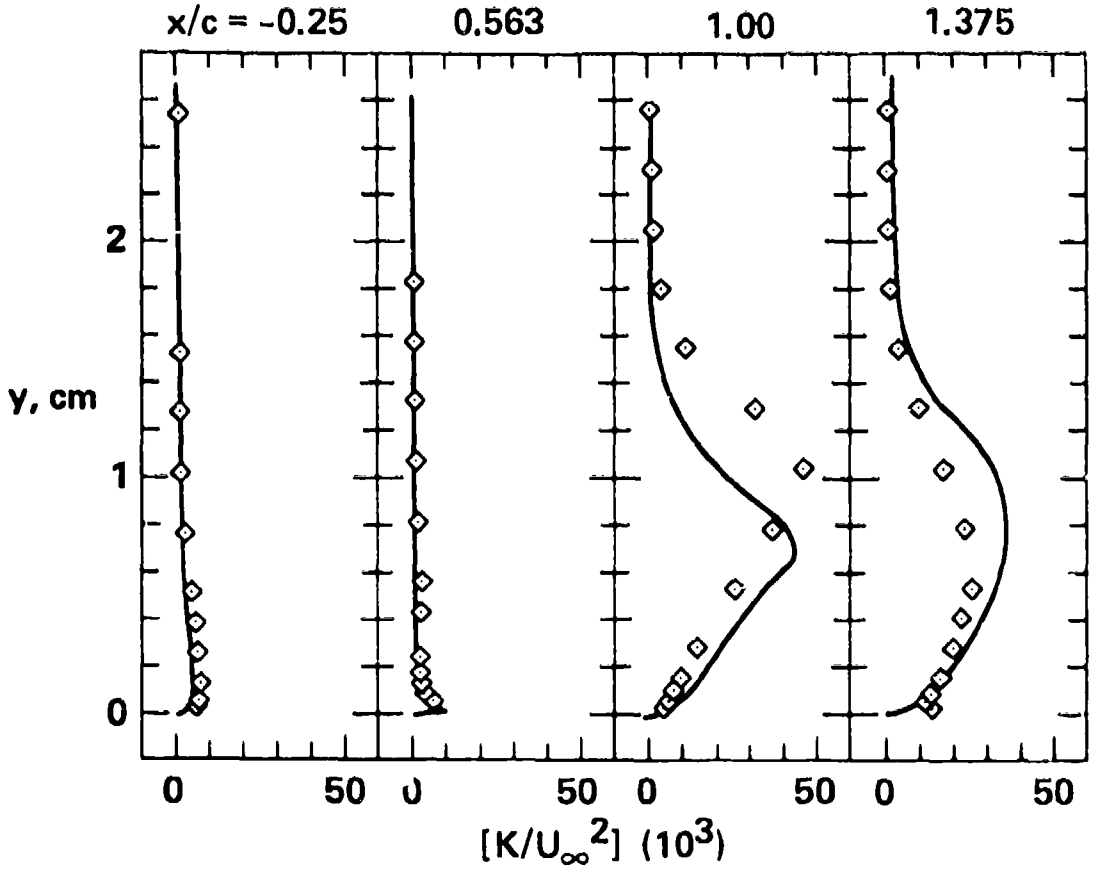


Figure 10.

REYNOLDS SHEAR STRESS PROFILES

$M_\infty = .875$, $Re = 13.1(10)^6/M$
CASE 8611

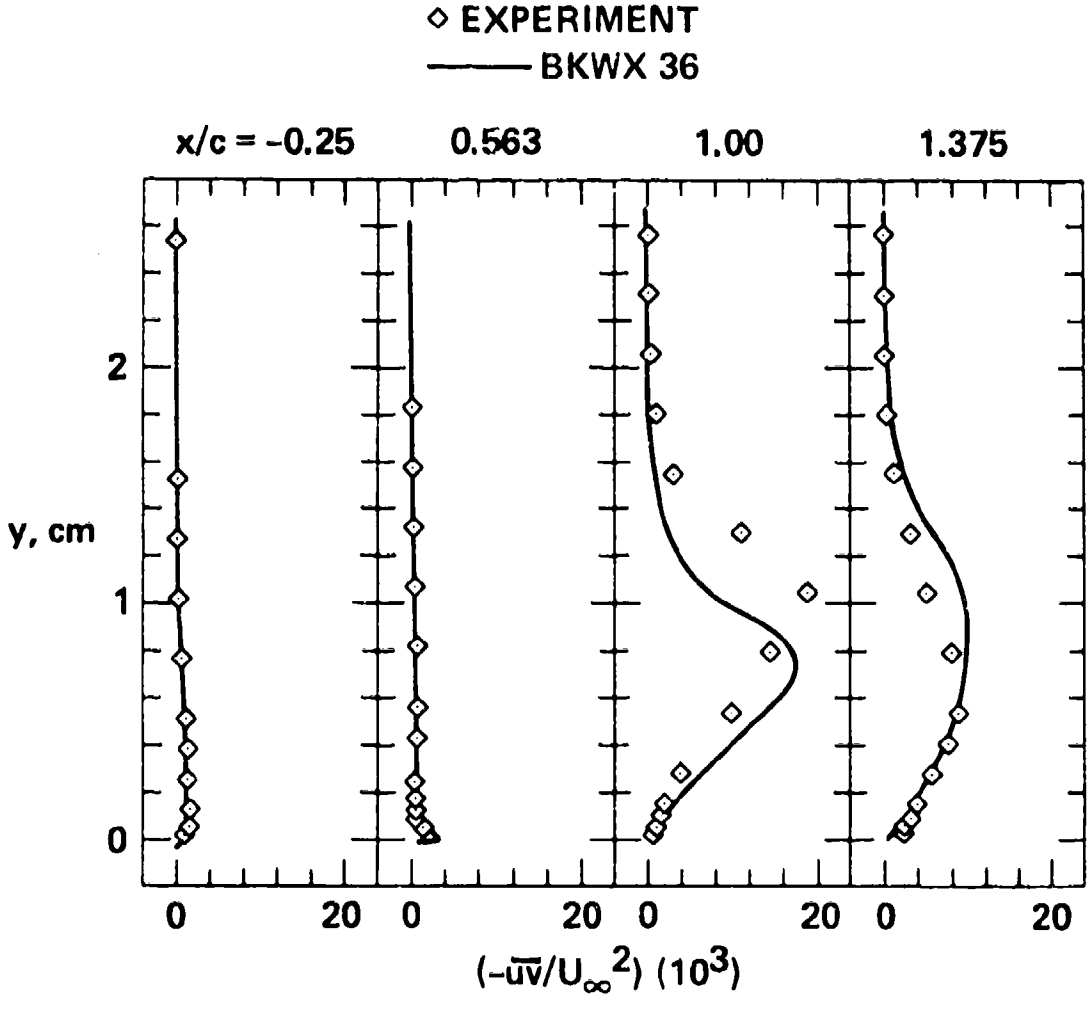


Figure 11.

TRANSONIC AIRFOILS

CASES 8621, 8623

SUMMARY

RAE 2822

AR = 3

MODERATELY REAR-LOADED

12.1% THICK SUPERCRITICAL

$.676 < M\infty < .730$

$2.40 < \alpha_g < 3.19$

$2.7(10)^6 < Re < 6.5(10)^6$

DSMA 523 s

AR = 4

SIGNIFICANT REAR-LOADED

MODIFIED WHITCOMB 11.0% THICK

$.6 < M\infty < .8$

$1.8 < \alpha_g < 2.6$

$2(10)^6 < Re < 4(10)^6$

[Ed.: See also the δ^* vs chord plots and the further comments of R. Melnik on these case, p. 806.]

Figure 12.

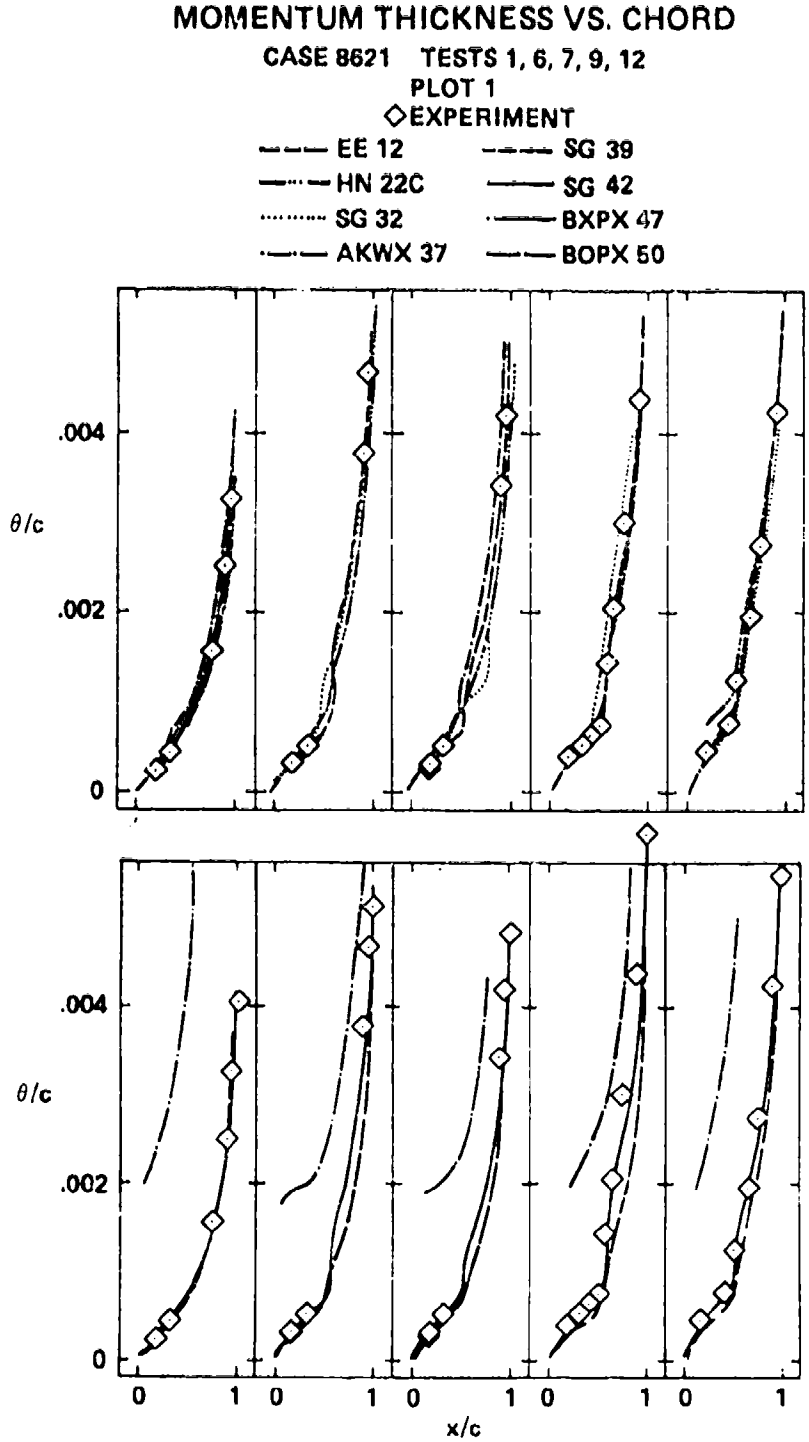

MOMENTUM THICKNESS VS. CHORD
CASE 8621 TESTS 1, 6, 7, 9, 12
PLOT 1
◇EXPERIMENT
EE 12
HN 22C
SG 32
AKWX 37
SG 39
SG 42
BXPX 47
BOPX 50
θ/c
.004
.002
0
0
1
x/c

Figure 13.

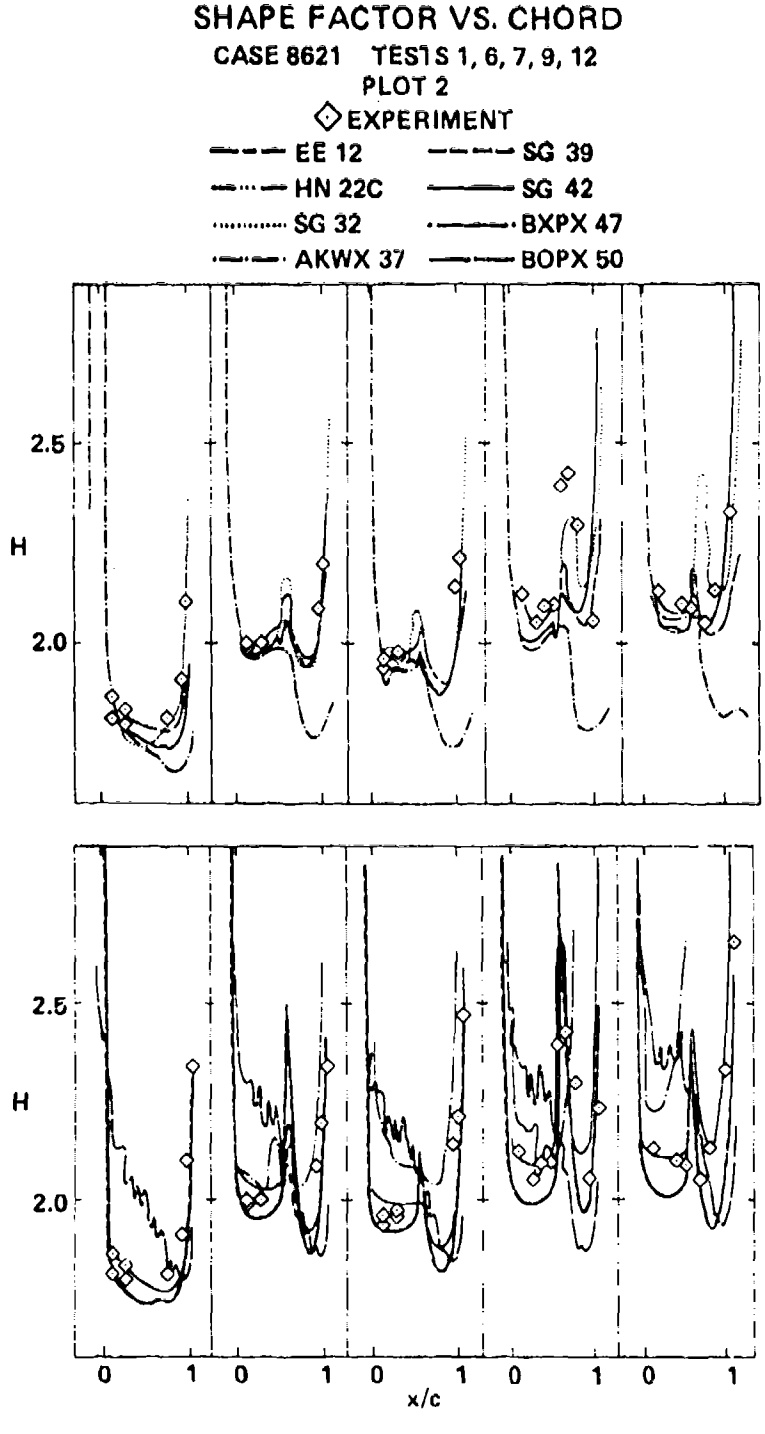

SHAPE FACTOR VS. CHORD
CASE 8621 TESTS 1, 6, 7, 9, 12
PLOT 2
◇ EXPERIMENT
EE 12
SG 39
HN 22C
SG 42
SG 32
BXPX 47
AKWX 37
BOPX 50
2.5
H
2.0
2.5
H
2.0
0
1
0
1
0
1
0
1
0
1
x/c

Figure 14.

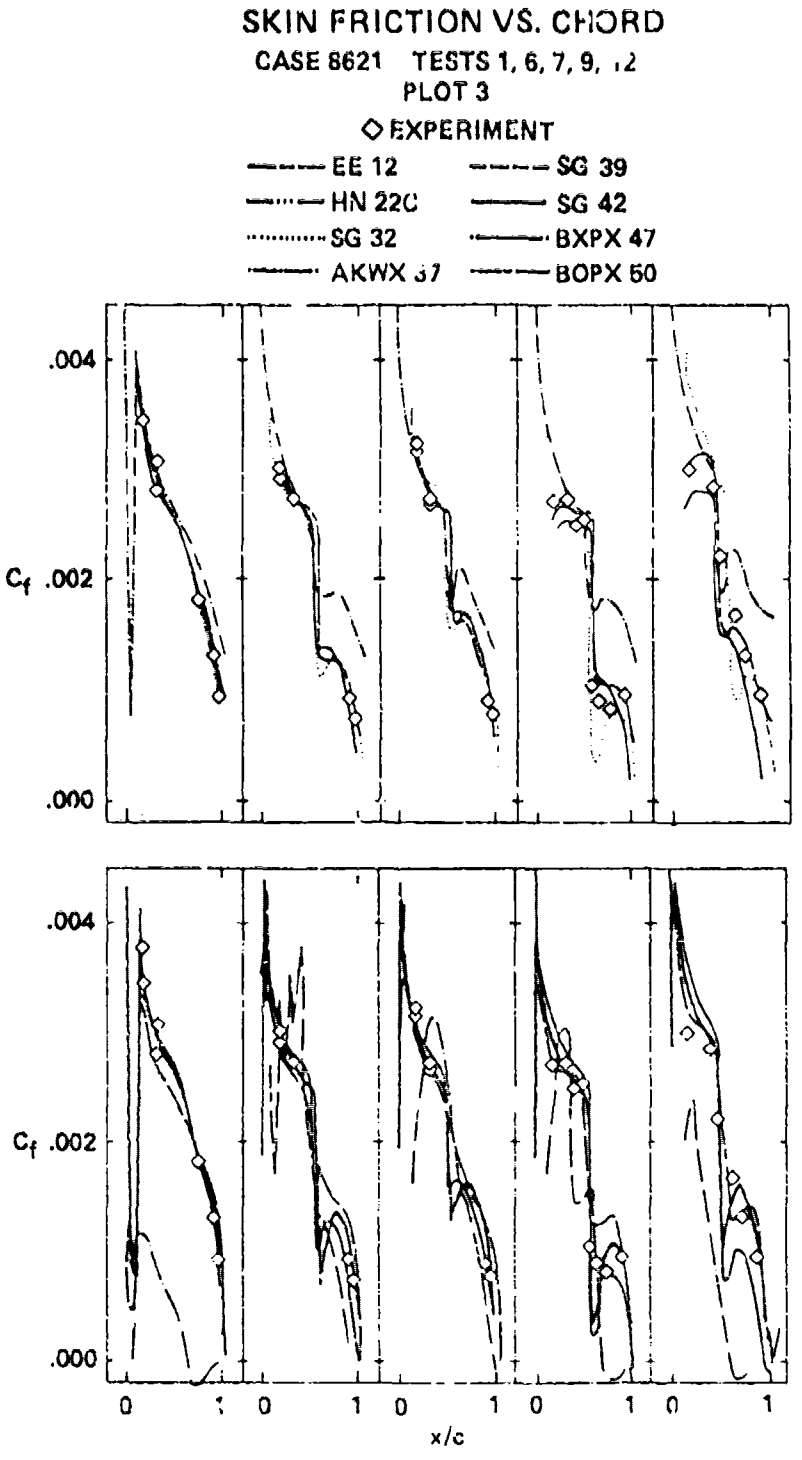

SKIN FRICTION VS. CHORD
CASE 8621 TESTS 1, 6, 7, 9, 12
PLOT 3
◇ EXPERIMENT
EE 12
SG 39
HN 22C
SG 42
SG 32
BXPX 47
AKWX 37
BOPX 50
.004
C_f .002
.000
.004
C_f .002
.000
0
1
0
1
0
1
0
1
0
1
x/c

Figure 15.

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION

CASE 8621 TEST 1
PLOT 4 FILE 4

$M_\infty = 0.676$, $\alpha_g = 2.40°$, $Re = 5.7(10)^6$, $x/c_{tr} = 0.11$

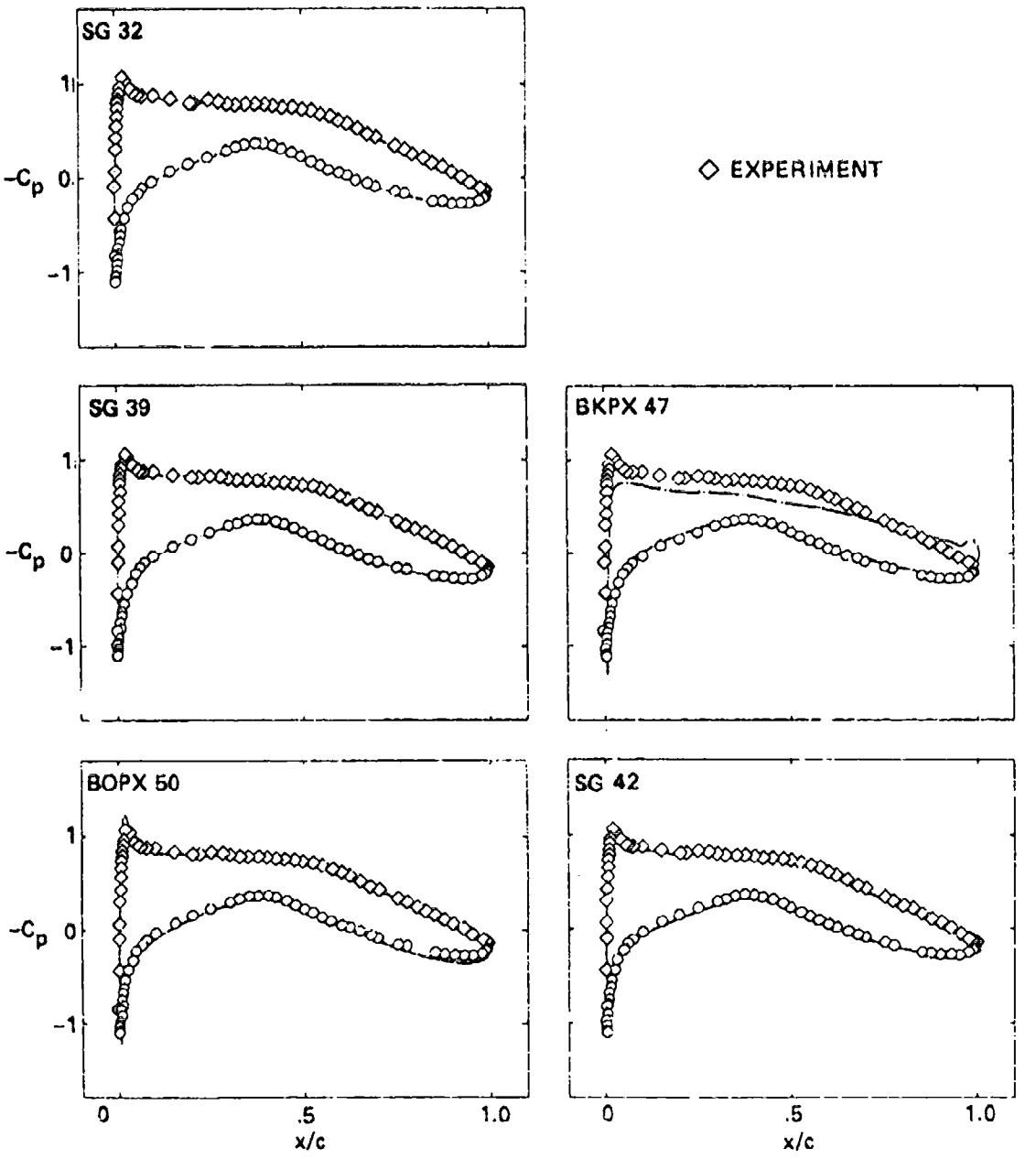


Figure 16.

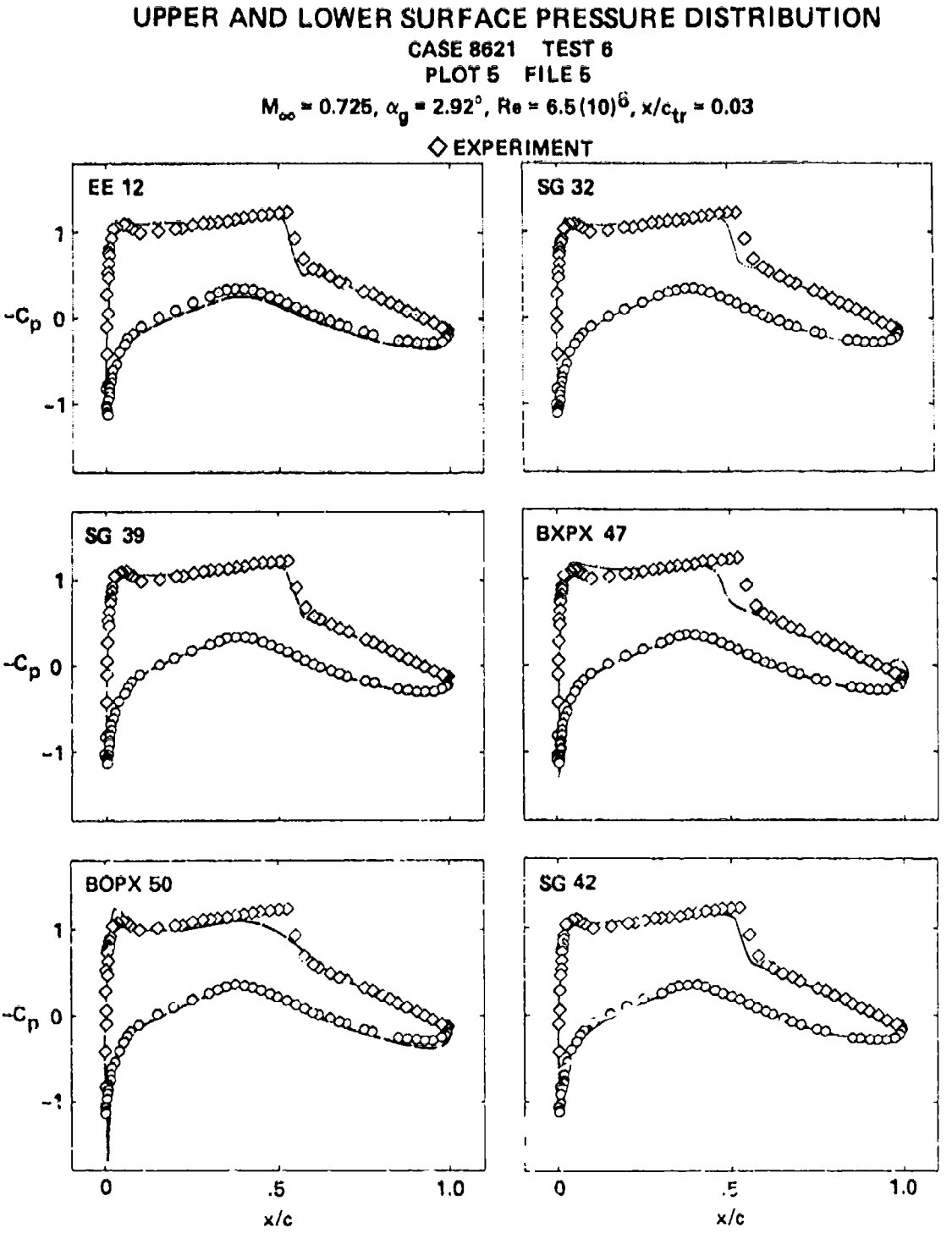

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION
CASE 8621 TEST 6
PLOT 5 FILE 5
M_∞ = 0.725, α_g = 2.92°, Re = 6.5(10)^6, x/c_{tr} = 0.03
◇ EXPERIMENT
EE 12
SG 32
SG 39
BXPX 47
BOPX 50
SG 42
-C_p
1
0
-1
0
.5
1.0
x/c

Figure 17.

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION

CASE 8621 TEST 7
PLOT 6 FILE 6

$M_\infty = 0.725$, $\alpha_g = 2.55$, $Re = 6.5(10)^6$, $x/c_{tr} = 0.03$

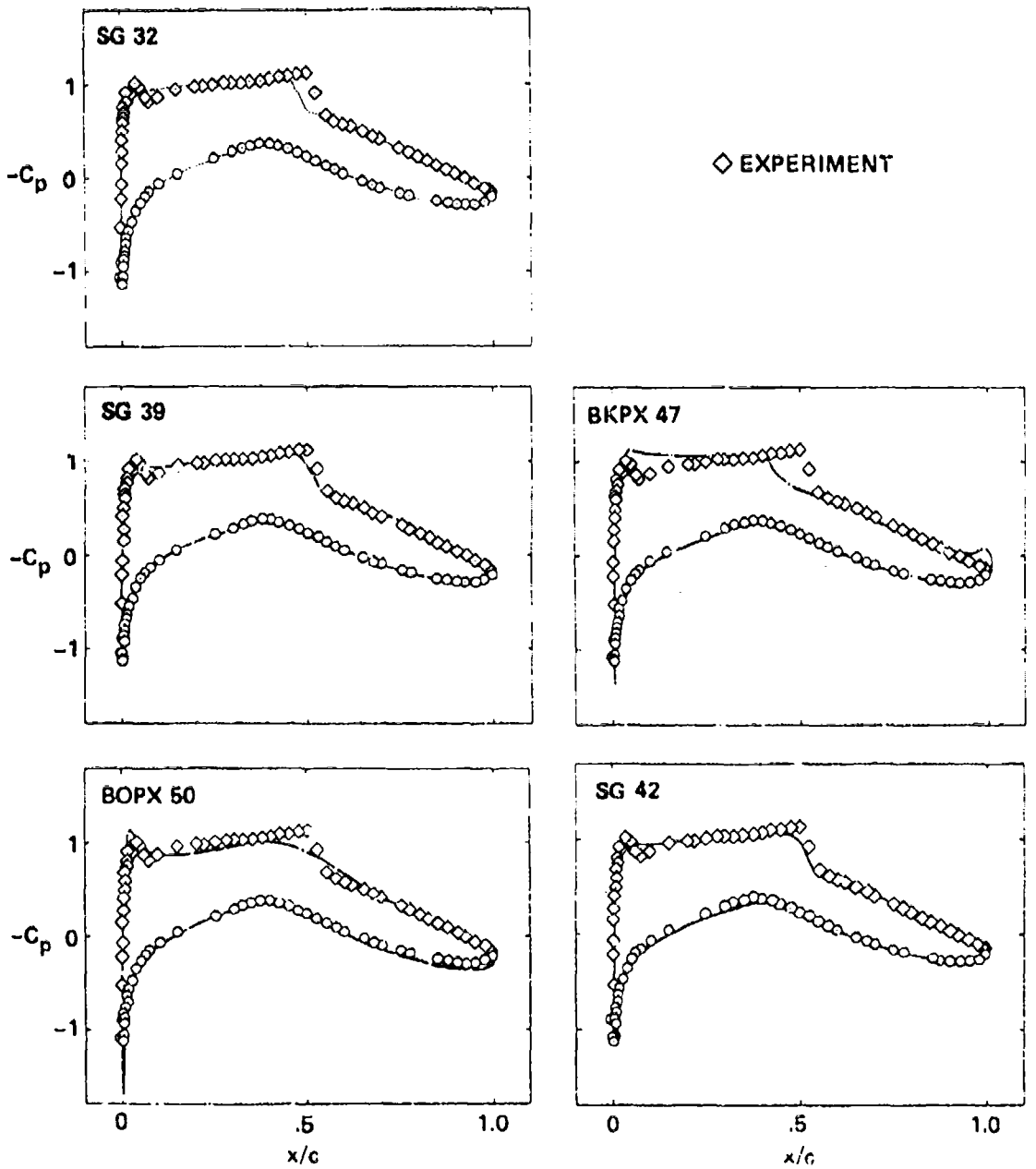


Figure 18.

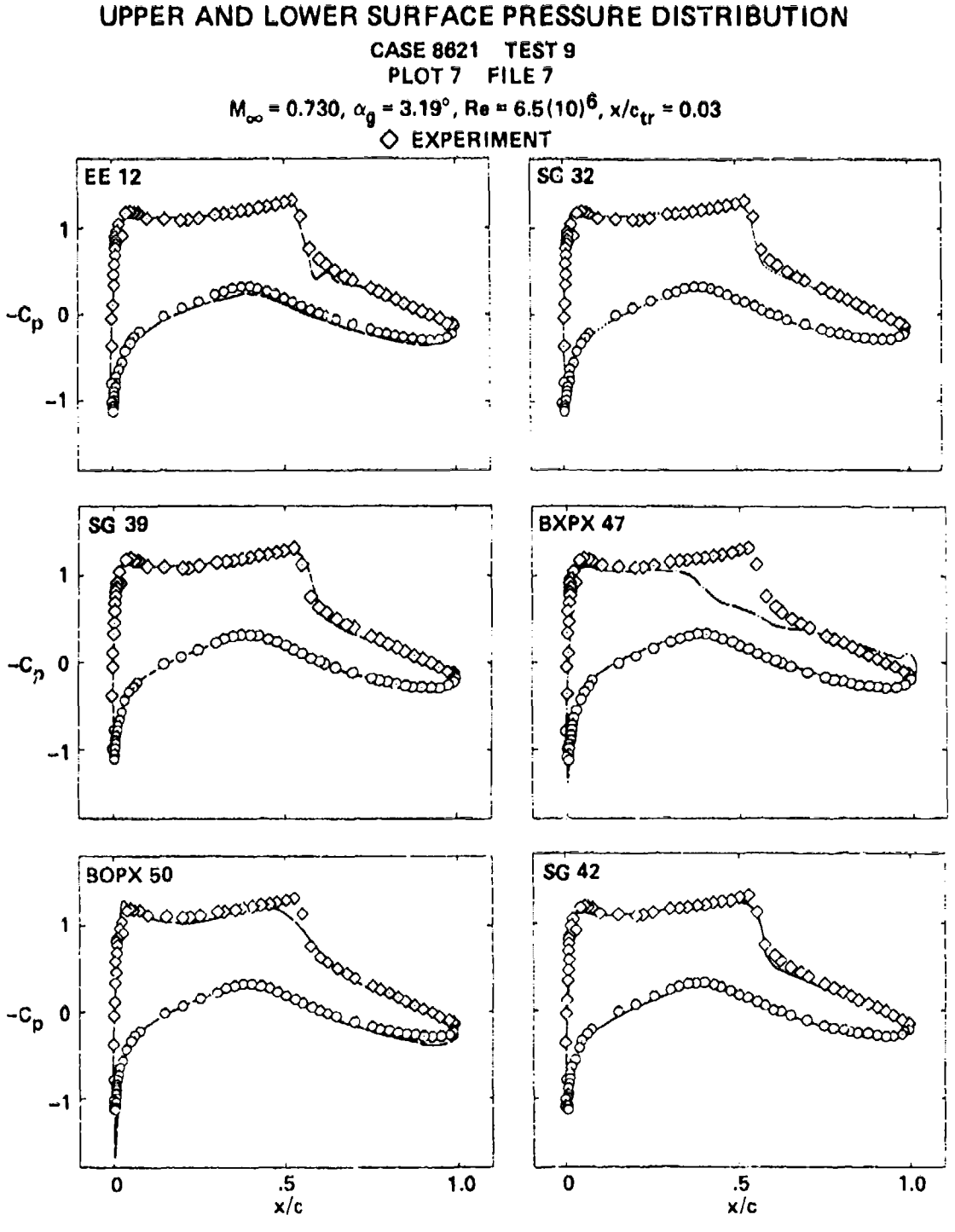

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION
CASE 8621 TEST 9
PLOT 7 FILE 7
$M_\infty = 0.730$, $\alpha_g = 3.19°$, $Re = 6.5(10)^6$, $x/c_{tr} = 0.03$
◇ EXPERIMENT
EE 12
SG 32
SG 39
BXPX 47
BOPX 50
SG 42
$-C_p$
1
0
-1
0
.5
1.0
x/c

Figure 19.

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION

CASE 8621 TEST 12

PLOT 8 FILE 8

$M_\infty = 0.73$, $\alpha_g = 3.19°$, $Re = 2.7\ (10)^6$, $x/c_{tr} = 0.03$

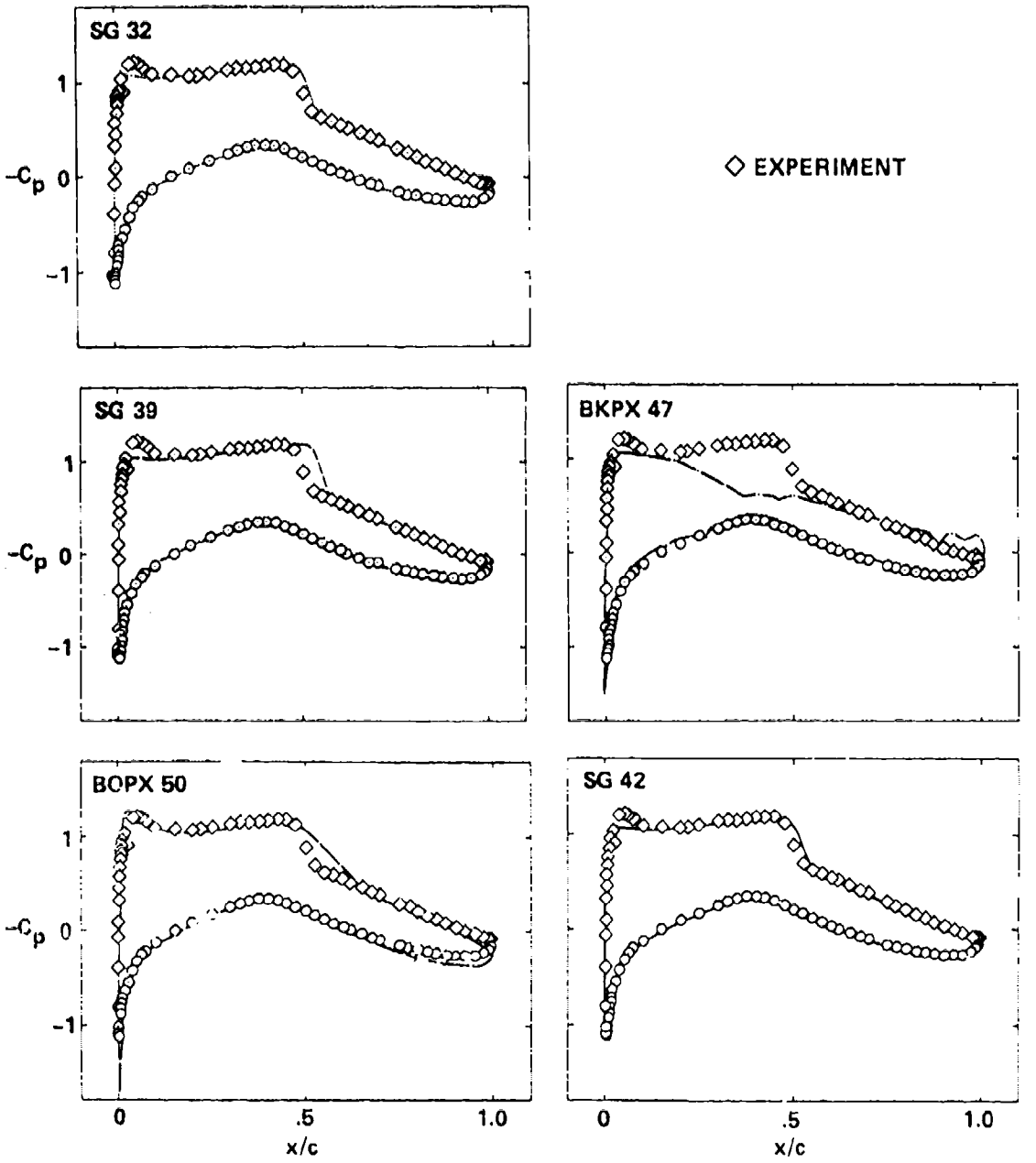


Figure 20.

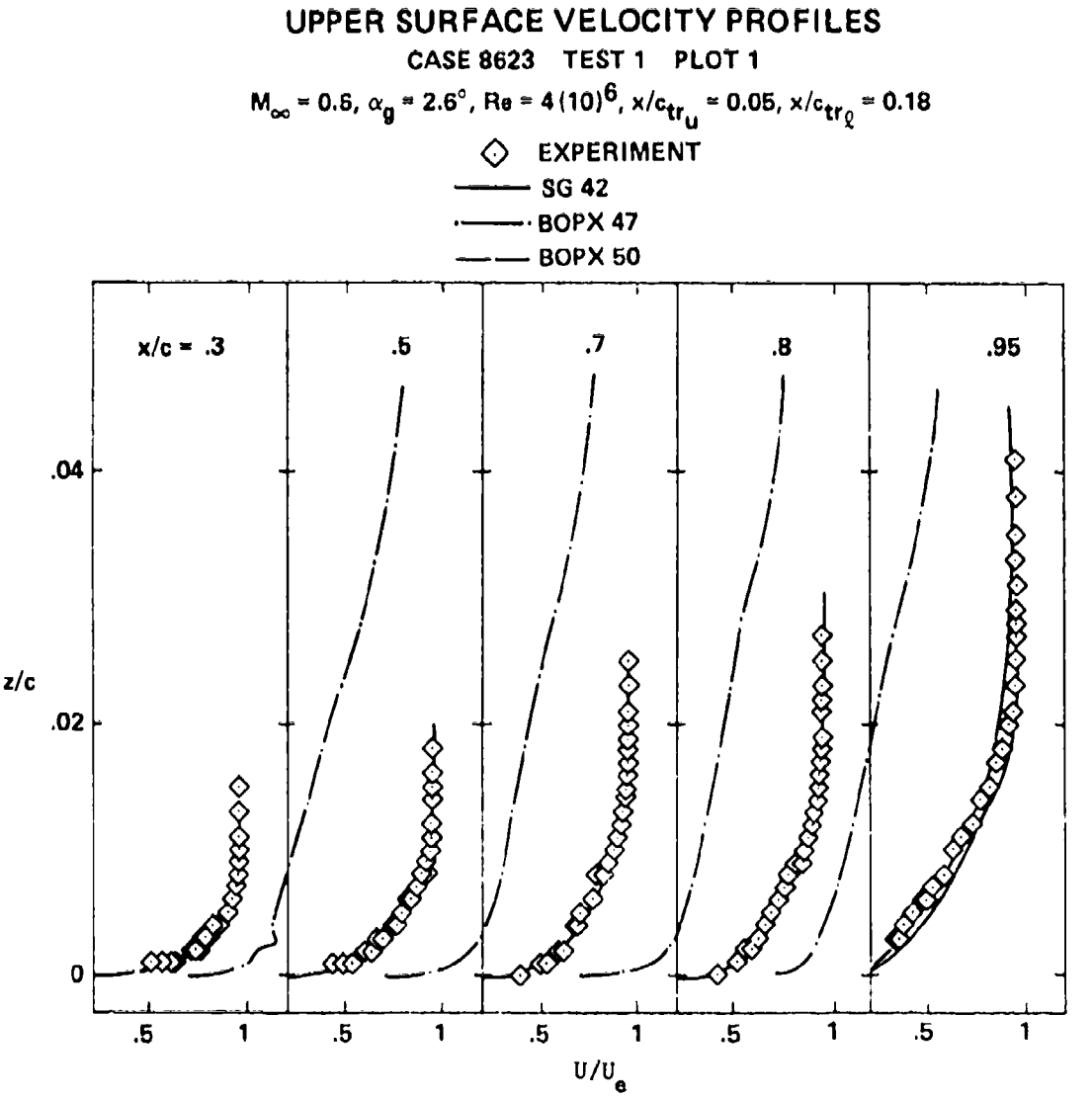

UPPER SURFACE VELOCITY PROFILES
CASE 8623 TEST 1 PLOT 1
M_∞ = 0.6, α_g = 2.6°, Re = 4 (10)^6, x/c_{tr_u} = 0.05, x/c_{tr_ℓ} = 0.18
EXPERIMENT
SG 42
BOPX 47
BOPX 50
x/c = .3
.5
.7
.8
.95
z/c
.04
.02
0
.5
1
U/U$_e$

Figure 21.

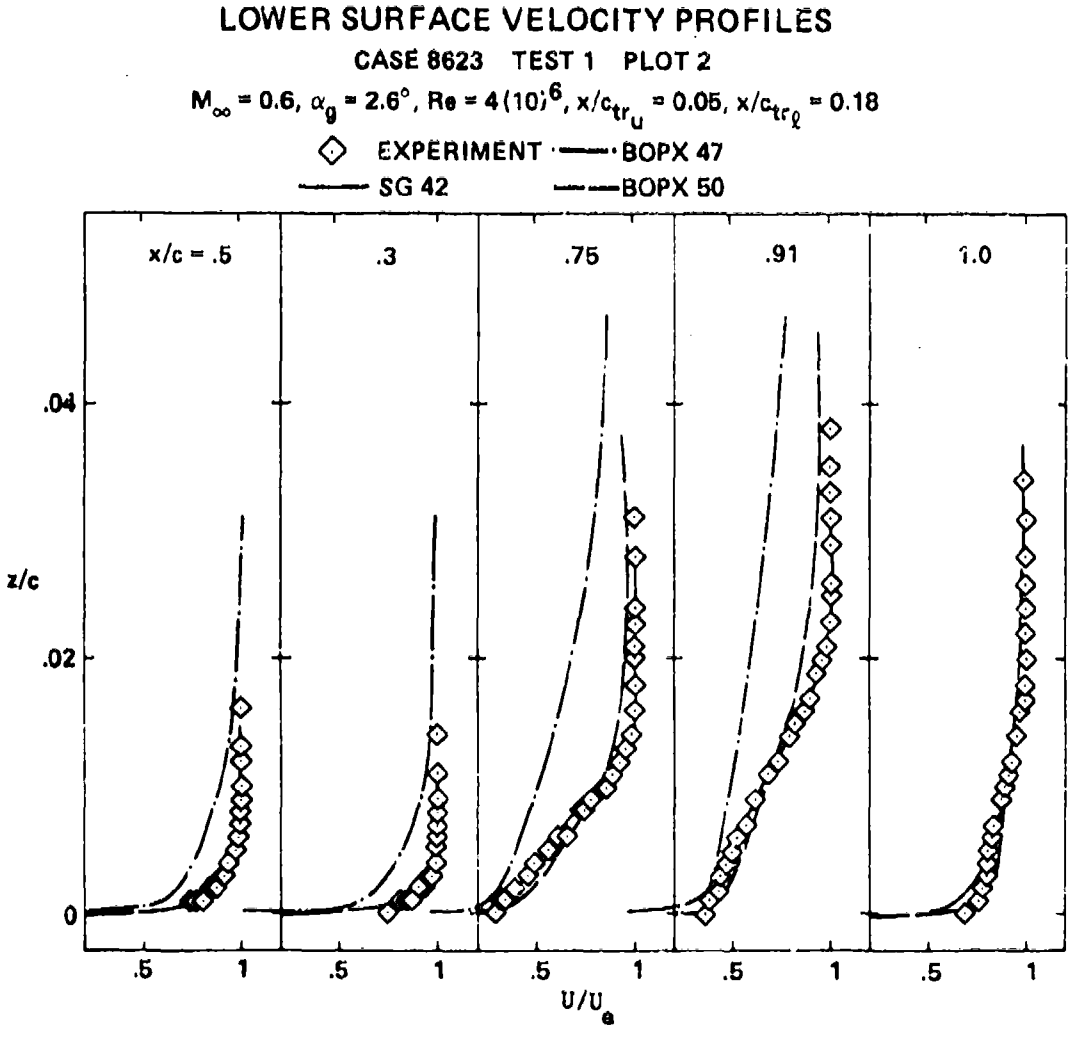

LOWER SURFACE VELOCITY PROFILES
CASE 8623 TEST 1 PLOT 2
$M_\infty = 0.6$, $\alpha_g = 2.6°$, $Re = 4(10)^6$, $x/c_{tr_u} = 0.05$, $x/c_{tr_\ell} = 0.18$
EXPERIMENT
BOPX 47
SG 42
BOPX 50
x/c = .5
.3
.75
.91
1.0
.04
.02
0
z/c
.5
1
U/U_e

Figure 22.

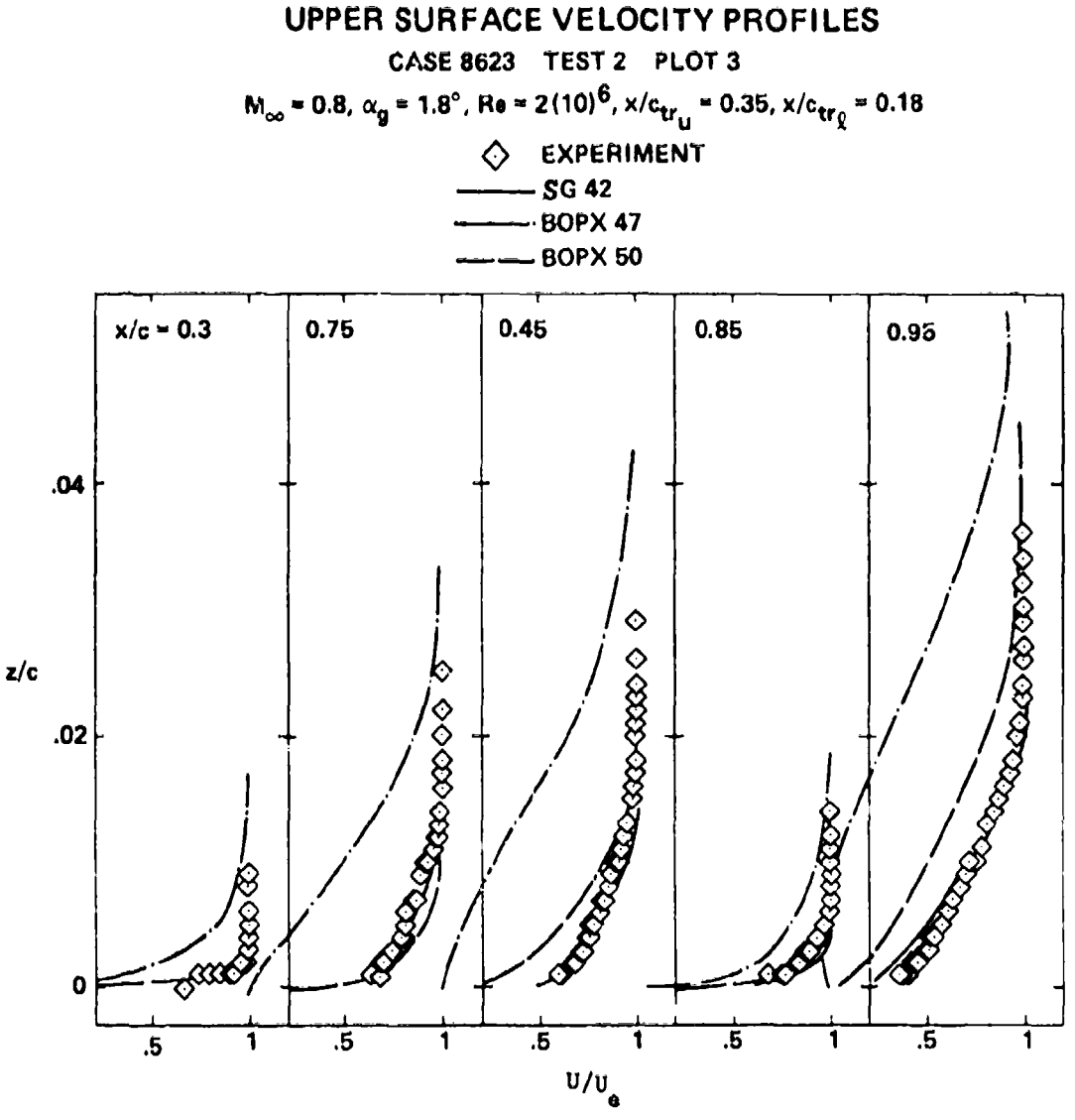

UPPER SURFACE VELOCITY PROFILES
CASE 8623 TEST 2 PLOT 3
$M_\infty = 0.8$, $\alpha_g = 1.8°$, $Re = 2(10)^6$, $x/c_{tr_u} = 0.35$, $x/c_{tr_\ell} = 0.18$
EXPERIMENT
SG 42
BOPX 47
BOPX 50
x/c = 0.3
0.75
0.45
0.85
0.95
z/c
.04
.02
0
.5
1
U/U_e

Figure 23.

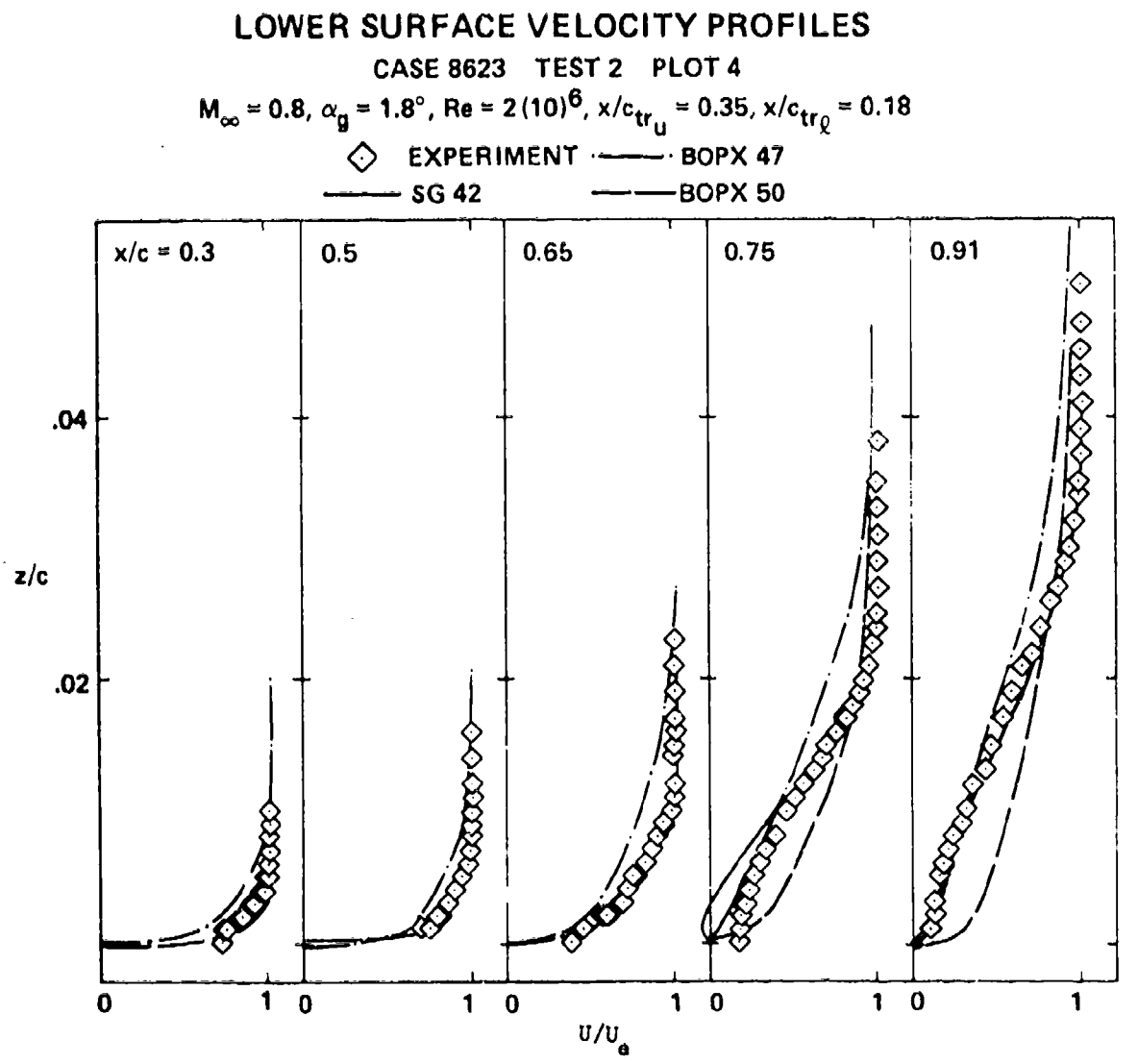

LOWER SURFACE VELOCITY PROFILES
CASE 8623 TEST 2 PLOT 4
$M_\infty = 0.8$, $\alpha_g = 1.8°$, $Re = 2(10)^6$, $x/c_{tr_u} = 0.35$, $x/c_{tr_\ell} = 0.18$
EXPERIMENT
BOPX 47
SG 42
BOPX 50
x/c = 0.3
0.5
0.65
0.75
0.91
.04
.02
z/c
0
1
U/U_e

Figure 24.

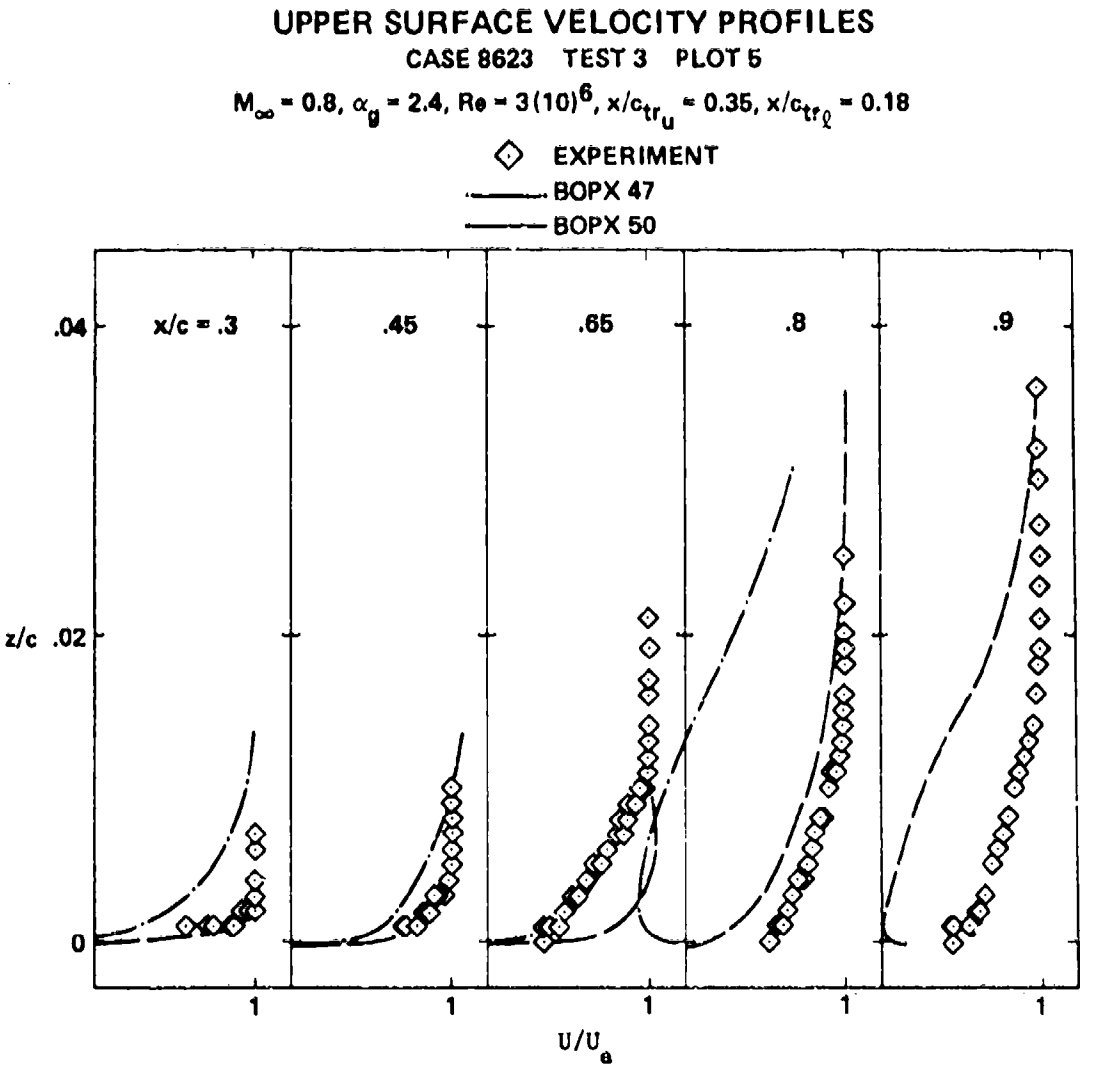

UPPER SURFACE VELOCITY PROFILES
CASE 8623 TEST 3 PLOT 5
M_∞ = 0.8, α_g = 2.4, Re = 3(10)^6, x/c_{tr_u} = 0.35, x/c_{tr_ℓ} = 0.18
EXPERIMENT
BOPX 47
BOPX 50
x/c = .3
.45
.65
.8
.9
.04
z/c .02
0
1
1
1
1
1
U/U$_e$

Figure 25.

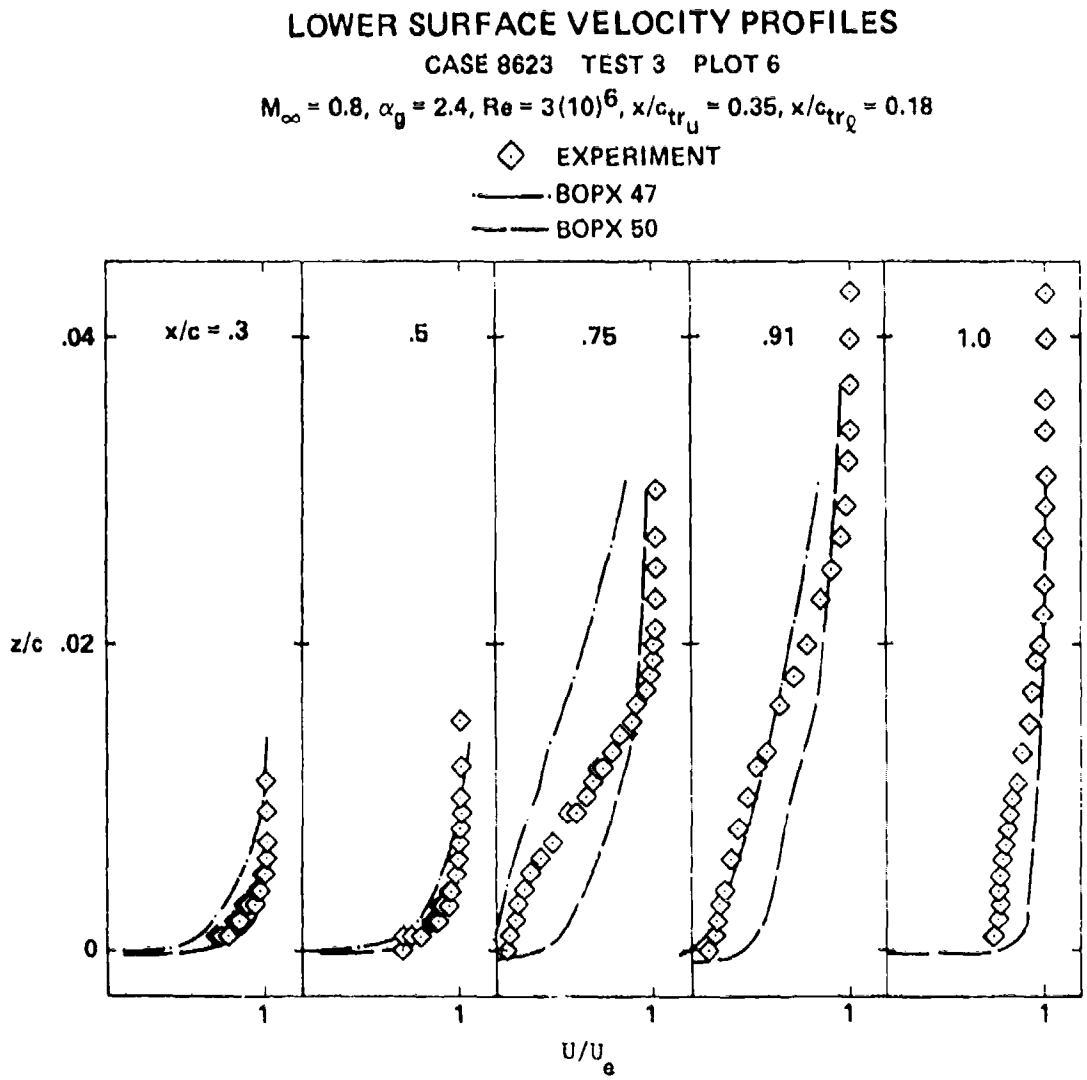

LOWER SURFACE VELOCITY PROFILES
CASE 8623 TEST 3 PLOT 6
M_∞ = 0.8, α_g = 2.4, Re = 3(10)^6, x/c_{tr_u} = 0.35, x/c_{tr_ℓ} = 0.18
EXPERIMENT
BOPX 47
BOPX 50
x/c = .3
.5
.75
.91
1.0
.04
.02
0
z/c
1
U/U$_e$

Figure 26.

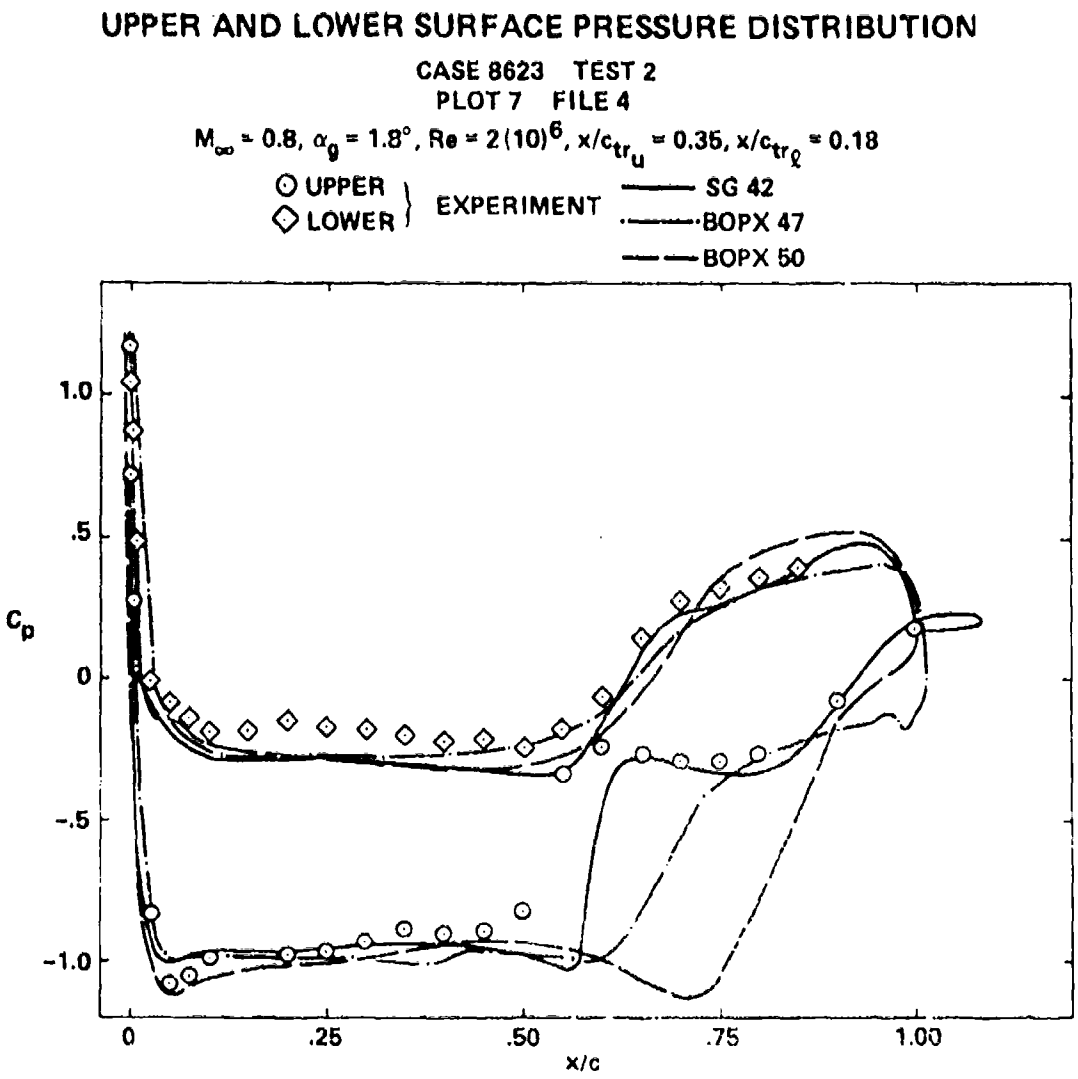

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION
CASE 8623 TEST 2
PLOT 7 FILE 4
$M_\infty = 0.8$, $\alpha_g = 1.8°$, $Re = 2(10)^6$, $x/c_{tr_u} = 0.35$, $x/c_{tr_\ell} = 0.18$
UPPER
LOWER
EXPERIMENT
SG 42
BOPX 47
BOPX 50
C_p
1.0
.5
0
-.5
-1.0
0
.25
.50
.75
1.00
x/c

Figure 27.

NON-LIFTING TRANSONIC AIRFOIL, SHOCK-SEPARATED FLOW

CASE 8691

SUMMARY

FACILITY: NASA AMES, HIGH REYNOLDS NUMBER TUNNEL

MODEL: THICK CIRCULAR ARC AIRFOIL

800

MEASUREMENT TECHNIQUES: PRESSURE TRANSDUCERS, HOT-WIRE FRICTION GAGES, TWO-COLOR FORWARD SCATTER LASER VELOCIMETER

TEST CONDITIONS:

$M_\infty = .785$

$Re_c = 1.1\ (10)^7$

ZERO INCIDENCE

Figure 28.

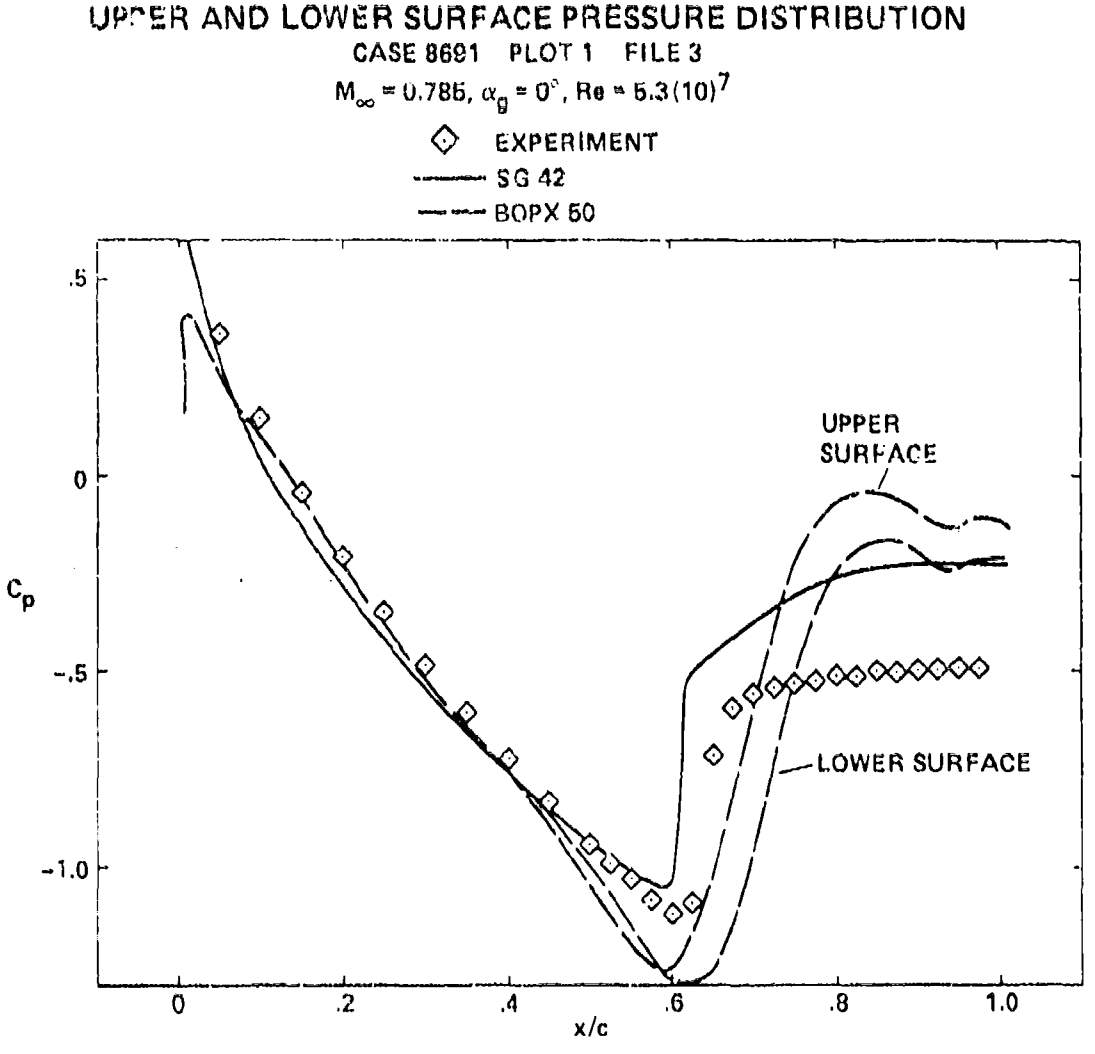

UPPER AND LOWER SURFACE PRESSURE DISTRIBUTION
CASE 8691 PLOT 1 FILE 3
$M_\infty = 0.785$, $\alpha_g = 0°$, $Re = 5.3(10)^7$
EXPERIMENT
SG 42
BOPX 50
.5
0
-.5
-1.0
C_p
UPPER SURFACE
LOWER SURFACE
0
.2
.4
.6
.8
1.0
x/c

Figure 29.

UPPER AND LOWER SURFACE VELOCITY PROFILES

CASE 8691 PLOT 2 FILES 7, 9, 11, 13

$M_\infty = 0.785$, $\alpha_g = 0°$, $Re = 5.3(10)^7$

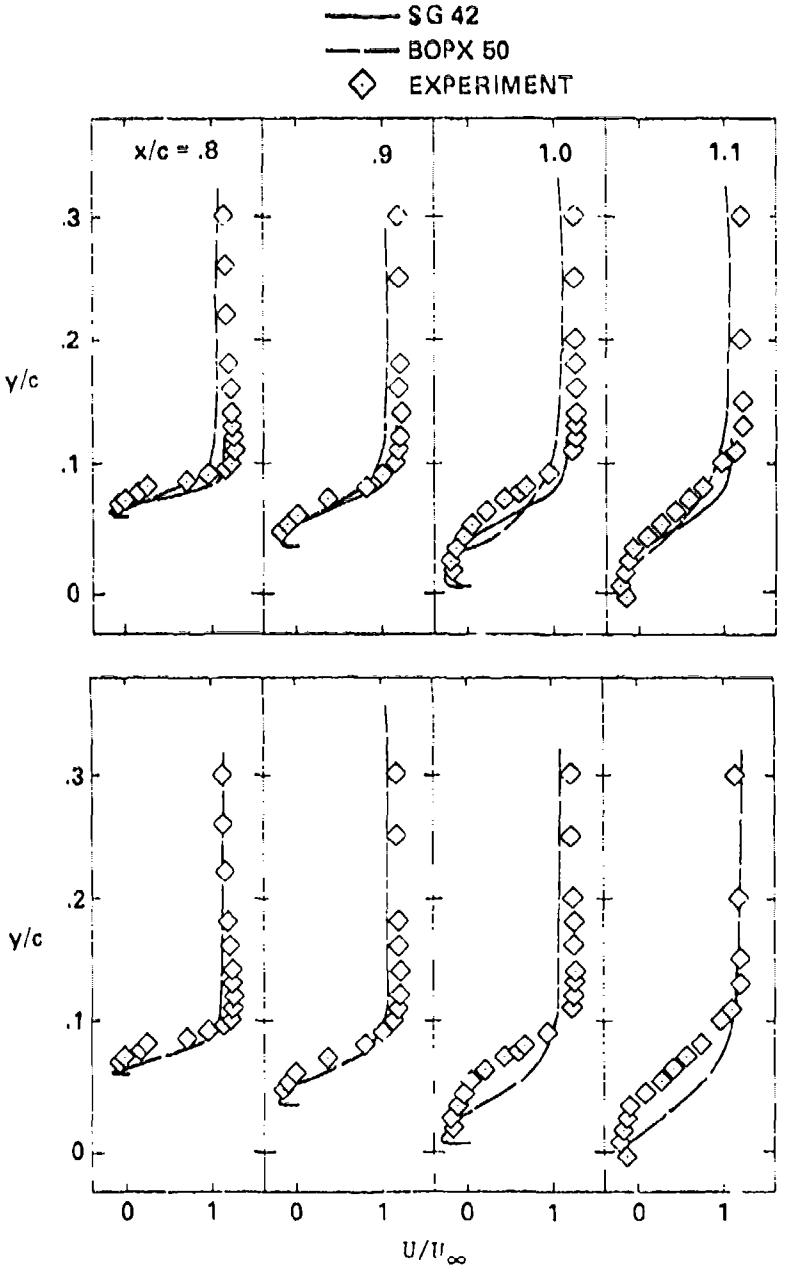


Figure 30.

DISCUSSION

SESSION VII

TRANSONIC FLOWS

General Conclusions

It is not possible to separate out the specific effects of three important factors in the transonic flow computations, namely

(a) Computational concerns (e.g., grid spacing)

(b) Turbulence modeling (e.g., is eddy viscosity good?)

(c) Experimental specification (e.g., p_o, C_L, α).

Displacement Thickness

δ^* is important in transonic airfoil flow calculations. R. Melnik proposed that eddy-viscosity approaches do not allow for good calculations of δ^* near trailing edges for the flows under consideration. C. Horstmann disagreed. He said that it is not possible to indict eddy-viscosity closure based upon our present experience. An overall consensus was not reached regarding the use of δ^* as an indicator of the validity of a particular turbulence closure scheme, i.e., integral versus finite-difference time-averaged Navier-Stokes techniques.

Experimental Specification

The computors of these cases used different criteria for setting the airfoil incidence. Groups 12, 32, 39, and 42 for Case 8621 imposed the measured lift coefficient on their calculations as per the instruction in the specification. Groups 47 and 50 ran at the geometric incidence. Group 09 ran at an angle of attack determined from the wall correction formula given in the original data report (available on the tapes). It was agreed that all computor groups should make sure that the imposed conditions (α or C_L) used in the calculations are clearly indicated in their documentation.

Numerical Dissipation

U. Mehta raised a point regarding numerical dissipation. He asked the time-averaged Navier-Stokes computors whether or not they varied their numerical dissipation (viscosity) in their calculations. Smoothing constants were the same for all cases computed by Group 50. G. Deiwert (Group 47) stated that that dissipation was looked at, and was not a problem.

Case 8691

This case does not simulate free-air conditions because the measured downstream pressure must be used in this calculation. The free-air conditions cannot be used. Wind-tunnel walls must be included to predict this case correctly (C. Horstman).

Case 8611

The problem with the shock location, as found in the work of Viegas et al. is due to the turbulence model used. C. Horstman said he had changed the shock location to bring it nearer the experimental location by lowering the eddy-viscosity by 50%.

C. Horstman stated that this case could very well be influenced by curvature terms.

Large Separations

(a) Large separation bubbles are defined as bubbles with a clear plateau pressure region present (R. Melnik).

(b) Problems were encountered with large separations with both inviscid-viscous interaction schemes and the numerical method. Integral methods are very dependent upon the turbulence model used when applied to large separated regions (J. LeBalleur).

Comments Presented in Session XIII

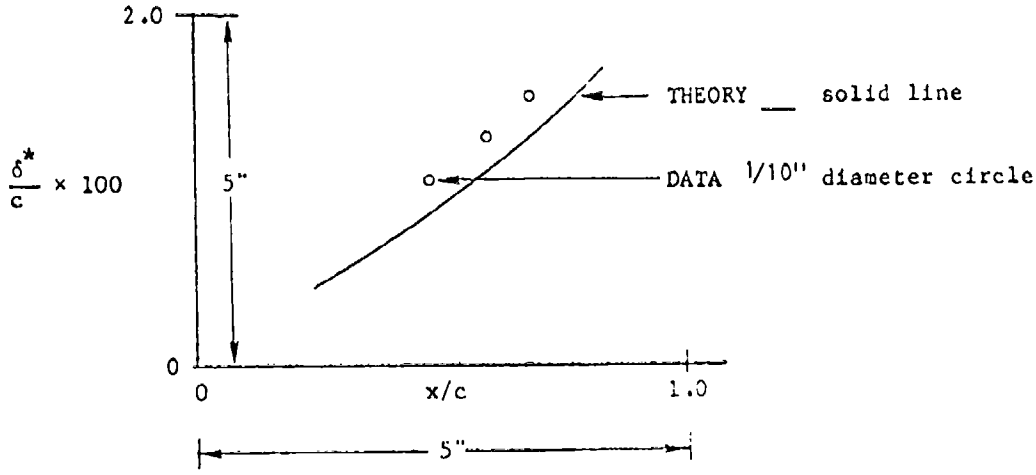
The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

Regarding the airfoil case, A. Roshko commented that some of the older methods (e.g. Lees) could do quite well in predicting the separation line. R. Melnik pointed out that they were supersonic calculations, which could be marched, whereas he used a fixed-point iteration method (for the subsonic calculation) which does not work, if the mesh spacing is too high.

TRANSONIC FLOW -- Case 8621

The specifications for this case did not request plots of δ^* vs x/c. Due to the sensitivity of δ^*, especially near the trailing edge, the Conference requested R. E. Melnik to collect information on δ^* vs x/c from the computor groups. The request was made as follows:

> I have been asked by Prof. Kline to arrange for the presentation of displacement thickness result in the final volume. I would appreciate your sending me plotted results of δ^*/c using the following format. Please use graph paper with nonreproducible lines, and δ^*/c determined from the H and θ/c results you sent in with your original results. You should send your plots to R. E. Melnik, Grumman Aerospace Corporation, Bethpage, NY 11714.



$0 \leq x/c \leq 1.2$

$0 \leq (\delta^*/c) \times 100 \leq 2$

The results of this inquiry are given in the report by R. E. Melnik below.

DISPLACEMENT EFFECTS IN TRANSONIC AIRFOIL FLOWS

by

R. E. Melnik*

It is well known that viscous effects can have a very strong influence on the section characteristics of rear-loaded supercritical airfoils even at the high Reynolds numbers typical of flight. The dominant viscous effect is due mainly to the rapid thickening of the boundary layer over the last 5-10% chord on the upper surface of the airfoil which in turn is induced by the large adverse pressure gradients appearing in this region of rear-loaded airfoils. The displacement effect is considerably amplified by the presence of shock waves, which produce nearly discontinuous increases of displacement thickness across the shock wave. This, in turn, acts to increase dramatically the displacement thickness approaching the trailing edge and thereby increase the decambering effect of the boundary layer. In supercritical conditions viscous effects can reduce the lift by as much as 50% of the inviscid value.

Because of the close relationship between displacement effects and section characteristics on airfoils it was thought to be important to examine how well the various theoretical methods predicted the displacement thickness. Unfortunately the computors of the transonic airfoil cases (8621, 8623) were not asked to submit their displacement-thickness results prior to the meeting and these results were, generally, not available at the meeting nor are they included with the theoretical predictions published elsewhere in this volume. Because of the importance attached to the predictions of displacement thickness, it was decided at the meeting to invite all computors of the transonic airfoil cases to submit their displacement-thickness results for the RAE 2822 airfoil (Case 8621) to this author for inclusion in the written proceedings.

The computations of the four groups who responded are compared with the experimental results in Figs. 1 and 2. The results in Fig. 1 (Case 1) are for a subcritical flow while those in Fig. 2 (Case 9) are for a supercritical flow The supercritical case contains a shock wave of moderate strength with a local Mach number in front of the shock of $M = 1.30$. Although the shock strength is relatively large, the data for skin friction indicated that the boundary layer remained fully attached.

The theoretical results submitted include three integral-boundary-layer (EE 12, SG 39, SG 42) solutions and one finite-difference parabolized time-averaged Navier-Stokes (BOPX 50) solution. All calculations were carried out at the experimental (uncorrected) Mach number. The angle of attack in the EE 12 and BOPX 50 computations

*Grumman Aerospace Corporation, Research & Development Center, Bethpage, NY 11714.

was set to the geometric incidence, while in the SG 39 and SG 42 calculations it was adjusted to match the experimental lift coefficient. The resulting differences in the incidence employed in the various computations is not expected to affect the overall trends and conclusions. The results indicate that all three integral methods do a credible job of predicting the displacement thickness in these flows and that the finite-difference time-averaged Navier-Stokes method does not. All of the integral methods correctly predict the nearly discontinuous increase across the shock wave and the large increase of δ^* approaching the trailing edge. It is not surprising then, that they also give very good predictions of the pressure distribution. The only significant differences between the three integral methods is in the wake and this is likely due to difference in the treatment of the wake coupling conditions.

The reason for the poor performance of the time-averaged Navier-Stokes method in these cases is not fully understood although, as in many other cases considered in the conference, it can likely be attributed to both numerical error and inadequacies of the eddy-viscosity model employed in the method. The time-averaged Navier-Stokes calculations employed about half the number of mesh points on the airfoil as were used in the integral methods and consequently suffer much higher truncation errors as is evidenced also by their poor predictions of surface pressure. This is very likely not the whole story as the predictions are equally poor for the subcritical case in Fig. 1, where the solution is smoother and the truncation errors can be expected to be much smaller. These results suggest that eddy-viscosity-type models for the Reynolds stress may be seriously inadequate in the steep pressure rises that occur near shock waves and trailing edges of rear-loaded airfoils. This conclusion is supported by similar results reported by Spaid and Stivers (1980). In this reference, finite-difference solutions of the boundary-layer equations using the Cebeci-Smith eddy viscosity model showed similar underpredictions of the displacement thickness near trailing edges. As in the present set of results, integral boundary-layer calculations reported in Spaid and Stivers (1980) showed reasonable agreement with the measurements. Cebeci and Meier (1979) showed that higher-order closure methods faired no better for airfoil-type flows. It was demonstraced for the Newman airfoil data that neither the Bradshaw (TKE) nor the Jones-Launder (k-ε) models improved the predictions of the standard Cebeci-Smith eddy-viscosity model and that all three grossly underpredicted the displacement thickness growth near the trailing edge.

The results above support two main conclusions: 1) integral methods can give very good predictions of viscoue flow over airfoils with strong viscid/inviscid interaction and 2) eddy-viscosity-type models for the Reynolds stress may lead to gross underpredictions of the boundary-layer growth on the upper surface of airfoils and may be inadequate for airfoil-type flows.

References

Cebeci, T., and H. G. Meier (1979). "Modeling requirements for the calculations of the turbulent flow around airfoils, wings and bodies of revolution," AGARD Conference Turbulent Boundary Layers - Experiments Theory and Modeling, AGARD-CP-271, The Hague, Netherlands.

Spaid, F. W., and L. S. Stivers, Jr. (1980). "Supercritical airfoil boundary layer measurements," Journal of Aircraft, 17:8.

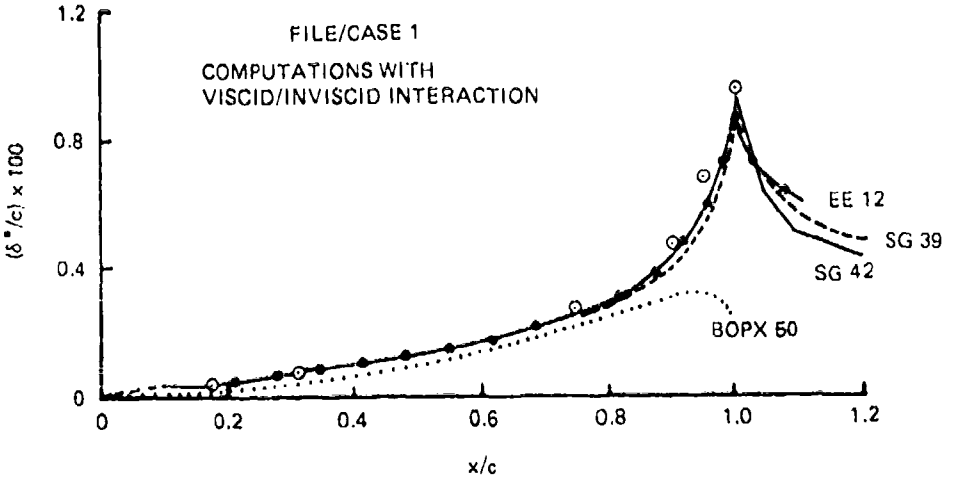


Figure 1. Displacement-thickness results for Case 8621 (transonic airfoils).

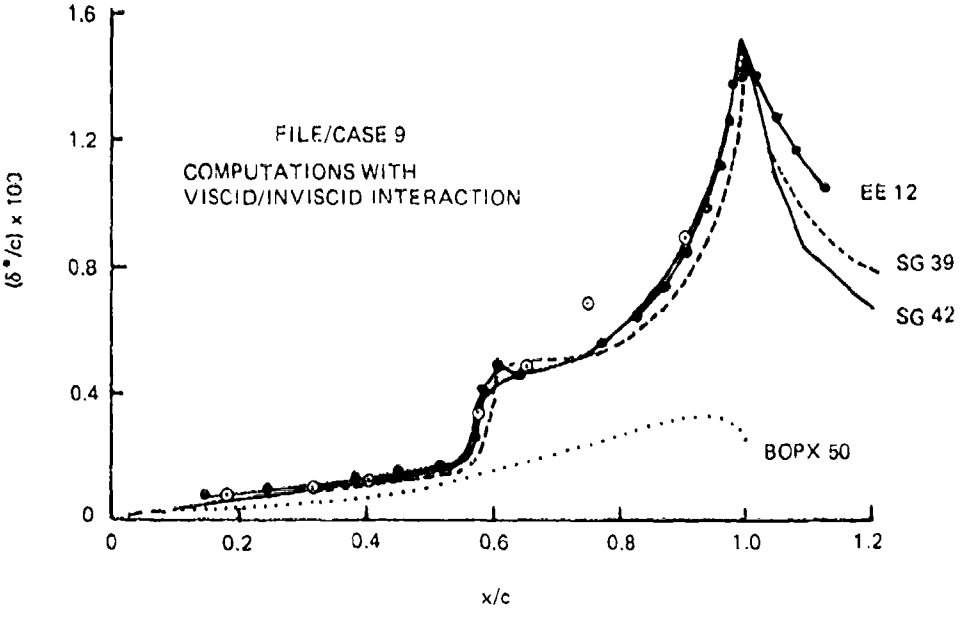


Figure 2. Displacement-thickness results for Case 8621 (transonic airfoils).

SESSION VIII

Chairman: H. H. Fernholz

Technical Recorders:

J. Viegas
S. Kale

Technical Reporter

D. M. Bushnell

SIMPLE COMPRESSIBLE STRAINS:

Cases 8101, 8201, 8301, 8403, 8411, 8501

RELAMINARIZATION — Case 0281

-- W. M. Kays

PICTORIAL SUMMARY

Flow 8100. Data Evaluators: M. Rubesin and C. Horstman. "Supersonic Flow Over a Flat Plate (Insulated Wall)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 8101 Various data takers		C_f/C_{f_0} vs. M											Effect of compressibility on skin friction.

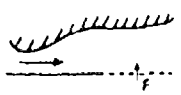
PICTORIAL SUMMARY

Flow 8200. Data Evaluators: M. Rubesin and C. Horstman. "Supersonic Flow Over a Flat Plate (Cooled Wall)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 8201 Various data takers		C_f/C_{f_0} vs. T_w/T_o											Effect of surface cooling on skin friction.

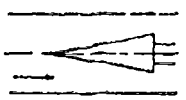
PICTORIAL SUMMARY

Flow 8300. Data Evaluator: L. Squire. "Turbulent Boundary Layers with Suction or Blowing at Supersonic Speeds."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 8301 G. Thomas	 Blowing $0.02 < y < 0.0028$ $0 < x < 0.127$ m	< 0	11	-	-	-	-	-	-	-	5×10^6 per m	M (2.53 to 2.67)	M, F, θ, δ^* vs x data available on file. No turbulence data.

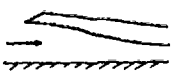
PICTORIAL SUMMARY

Flow 8400. Data Evaluators: M. Rubesin and C. Horstman. "Boundary Layers in an Adverse Pressure Gradient in an Axisymmetric Internal Flow."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8403 M. Kussoy C. Horstman M. Acharya		> 0	8	-	8	8	8	8	8	Buried wire	1.6×10^5 to 4.32×10^6 (based on δ_0)	2.3	Axisymmetric. Profile data for 2 pressure gradients. Skin friction data for 6 pressure gradients, each at 4 Reynolds numbers.

PICTORIAL SUMMARY

Flow 8410. Data Evaluators: M. Rubesin and C. Horstman. "Boundary Layers in an Adverse Pressure Gradient in 2-Dimensional Flow."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8411 F. Zwarts		> 0	6	-	-	-	-	-	-	Preston probe	3.5×10^4 (based on θ_0)	4	

PICTORIAL SUMMARY

Flow 8500. Data Evaluator: P. Bradshaw. "Compressible Effects in Free Shear Layers."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 8501 Various data takers		Smoothed curve of $d\delta/dx$ vs. M given.											Shear layer spreading rate vs Mach number.

Methods Used--Session VIII

Method	Flow Case 8101	8201	8301	8403	8411	8501
AKWX	•	•		•	•	
BKWX	•	•		•		
BOPX	•	•				
BKEX	•	•				
RSTN	•	•	•	•	•	
RSTC						•
AKWC						•
HN	•	•			•	
SC	•				•	
Plate No. Vol. III	139-141	142,143	144	145-148	149,150	151

SIMPLE COMPRESSIBLE STRAINS

Cases 8101, 8201, 8301, 8403, 8411, 8501

by

D. M. Bushnell*

These flows are essentially compressible counterparts to the incompressible turbulent boundary layers considered at the 1968 Stanford Meeting. Except for 8403 (and perhaps 8411), the normal pressure gradient is quite small and therefore these should be the simplest (numerically) and most accurate (turbulence-modeling-wise) of the compressible cases.

Cases 8101 and 8201

As indicated on Fig. 1, Cases 8101 and 8201 are zero-pressure-gradient compressible boundary layers at relatively high Reynolds number. Case 8101 is essentially the variation of skin friction with Mach number for the adiabatic case, whereas 8201 considers the dependence of skin friction upon wall cooling for Mach 5. Figures 2 and 3 provide an interesting historical background for Case 8101. Figure 2 indicates the state-of-the-art circa 1960 (Schubauer and Tchen, 1961), at which time there was a dearth of experimental information. As seen on the figure, the then-available theories predicted a tremendous range in skin-friction level. The reason for much of this theoretical "scatter" centered around the diverse treatments of the density

*NASA Langley Research Center, Hampton, VA 23665.

fluctuation terms in the Reynolds-stress expansion. By the 1970s (Fig. 3; Cary, 1969), a significant amount of data was available and the actual experimental variation of skin friction with Mach number was found to be closer to the lower bound of the 1960 vintage theories (Fig. 3, adiabatic wall data indicated by open symbols). Also, by the 1970s, the available "theories" (which were primarily data correlations) were in much closer agreement. The data indicate, somewhat surprisingly, that the density fluctuation terms do not have to be included directly into heuristic turbulence models; allowance for the local mean-density level appears to be sufficient.

There are two quite contrasting observations which one could draw from a comparison of Figs. 2 and 3. The first observation is quite comforting: by the 1970s, "theory" and experiment were in quite good "agreement," at least for the simple compressibility effects upon skin friction (Case 8101). The other observation is not quite so comforting; given a new physical situation (compressibility), and a lack of data, the turbulence modelers of the 1950s could not agree as to what constituted a valid prediction, and most were considerably in error. As a personal observation, this reporter is not sure that our present state of knowledge in turbulence precludes this situation from occurring again. The circa 1980 results (Fig. 4) from the predictors at the present conference are in "reasonable" agreement with the experimental trend and level. There are no obviously wild predictive results (Figs. 4 and 5), the excursions from the van Driest curve (suggested by the data evaluators) are of the same order as the experimental data "scatter" (Fig. 3). The effect of wall cooling upon skin friction (Figs. 6 and 7) is also relatively well predicted, with no obvious difficulties.

Case 8301

This flow involves the concurrent effects of favorable pressure gradient and normal injection from the wall (Fig. 8). Historically, there has been very little difficulty in computing either of these effects separately, and the one predictor who tried this case seems to have had considerable success (Figs. 9-11).

Case 8403

This case is the first of these "simple compressible strains" which has a "non-boundary" nature. The flow develops on the inside wall of an axisymmetric configuration and is subjected to a train of compression and expansion waves (Fig. 12). The finite inclination of these waves even at this low a Mach number, produces a normal pressure gradient (the data evaluators for this case indicate a maximum pressure change, from wall to edge, the order of 20%). Detailed predictions are shown for centerbody IV at the intermediate Reynolds number (Figs. 13-17). The skin-friction and mean-velocity profile comparisons (Figs. 13 and 14) indicate that RSTN 21 is relatively successful for this case, whereas the other two predictions are somewhat

less successful (but note that they agree with each other). However, the form factor results (Fig. 15) are somewhat confusing, as the only relatively successful prediction for H is not RSTN 21, but in fact one of the other results. The explanation of this inconsistency apparently lies in the relative ambiguity in the definition of form factor when the static pressure varies in the normal direction (Fernholz and Finley, 1980), i.e., form factor is evidently not always a suitable parameter for assessing the accuracy of a prediction technique.

Of further concern are the comparisons between predicted and measured turbulence quantities (Figs. 16 and 17). The calculations, particularly for Reynolds stress (which appears, of course, directly in the longitudinal momentum equation for mean velocity, indicate significant over-predictions. Therefore, although method RSTN 21 produced reasonably accurate mean flow results, the corresponding Reynolds-stress results are quite poor. The most obvious explanation for this state of affairs is that the flow is essentially pressure driven and therefore for this case mean-flow predictions may not constitute a valid test of the turbulence-modeling approach.

Case 8411

This case is an adiabatic wall adverse-pressure-gradient flow (Fig. 18); somewhat similar to Case 8403, except at higher Mach number, two-dimensional instead of axisymmetric and without the favorable pressure-gradient region. There is some concern for this case that the normal pressure gradient may not be negligible. The skin-friction predictions (Fig. 19) indicate that RSTN 21 and SG 32 have produced quite reasonable results. However, the momentum-thickness predictions are poor for all of the methods (Fig. 20). The apparent agreement (of three predictors!) with the form factor data (Fig. 21) should probably not be taken too seriously, in view of the discussion with respect to form factor on Case 8403.

Case 8501

This test case is the simplest compressible free-shear flow, a free-shear layer with essentially zero Mach number on the low speed side (Fig. 22). This case was the cause celèbre of the 1972 Langley Turbulent Free Mixing Meeting (NASA, 1972), at which time there were no satisfactory predictions forthcoming for the apparent large reduction in spreading rate with Mach number.

Since 1972, several further sets of data have become available, and all of them reinforce this "unpredictable" trend. In the present meeting, only two predictors attempted this case (Fig. 23). The only satisfactory result evidently necessitated an ad hoc "correction" to the Rotta constant as a function of fluctuating Mach number (Brian Launder, private communication, 1981). There are no detailed data in existence to determine whether or not this is a reasonable "fix." The physics of this simplest of the compressible free mixing cases is still essentially unknown.

References

Cary, A. M., Jr. (1969). "Turbulent boundary layer heat transfer and transition measurements with extreme surface cooling in hypersonic flow." Thesis, University of Virginia.

Fernholz, H. H., and P. J. Finley (1980). "A critical commentary on mean flow data for two-dimensional compressible turbulent boundary layers," AGARD-AG-253.

NASA SP-321 (1972). Free Turbulent Shear Flows, Conference Proceedings, Vol. I.

Schubauer, G. B., and C. M. Tchen (1961). Turbulent Flow, Princeton Aeronautical Paperbacks, Princeton University Press.

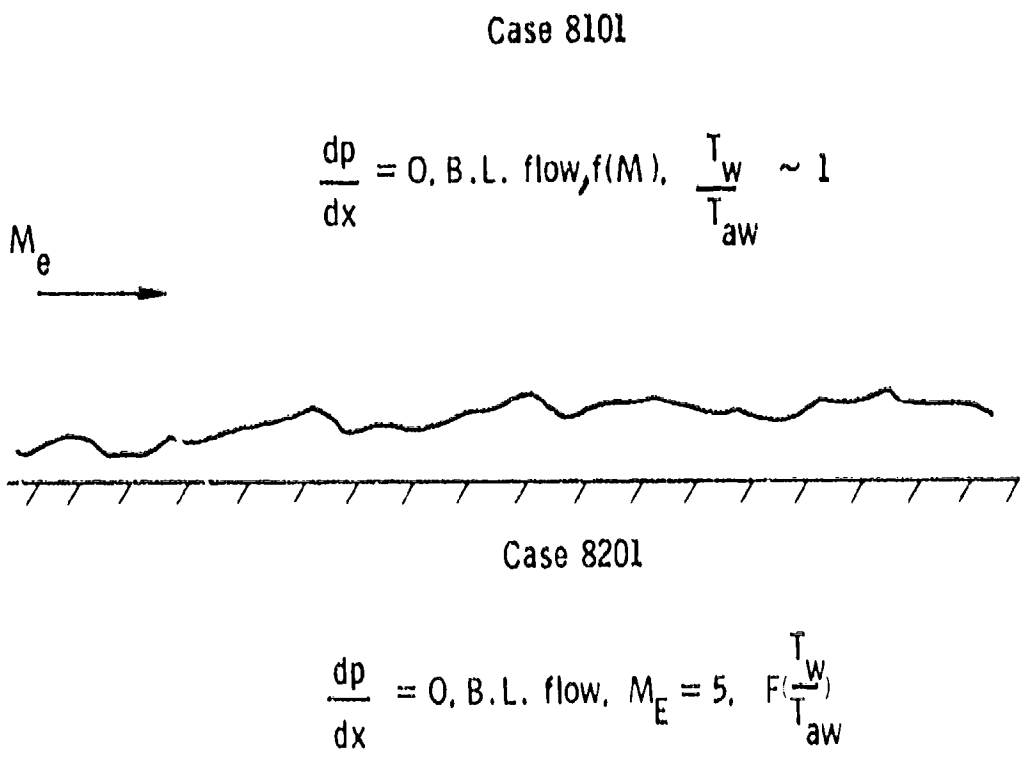


Figure 1. Simple compressible wall flow cases.

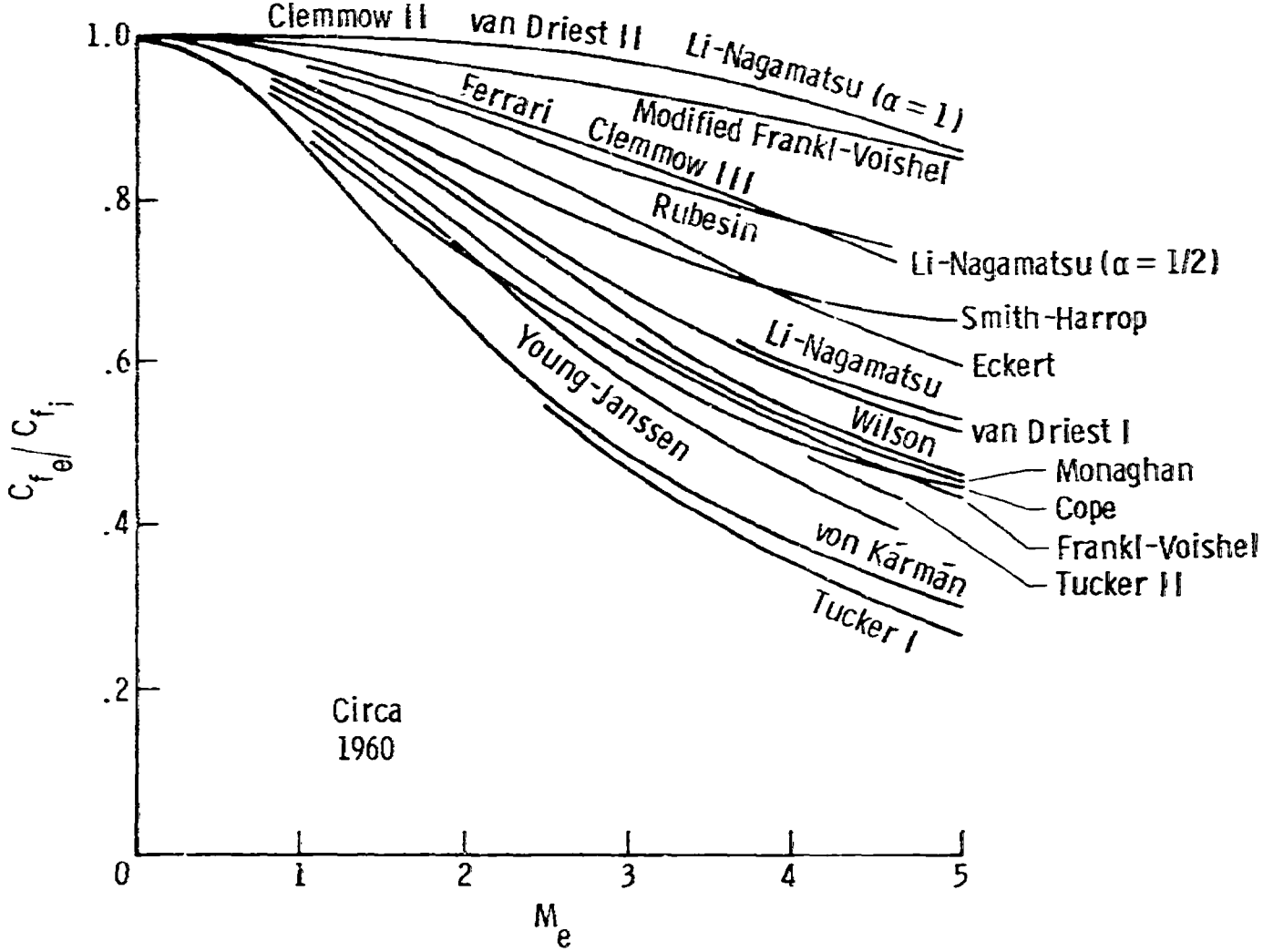


Figure 2. Variation of C_{f_e}/C_{f_i} with Mach number (insulated wall).

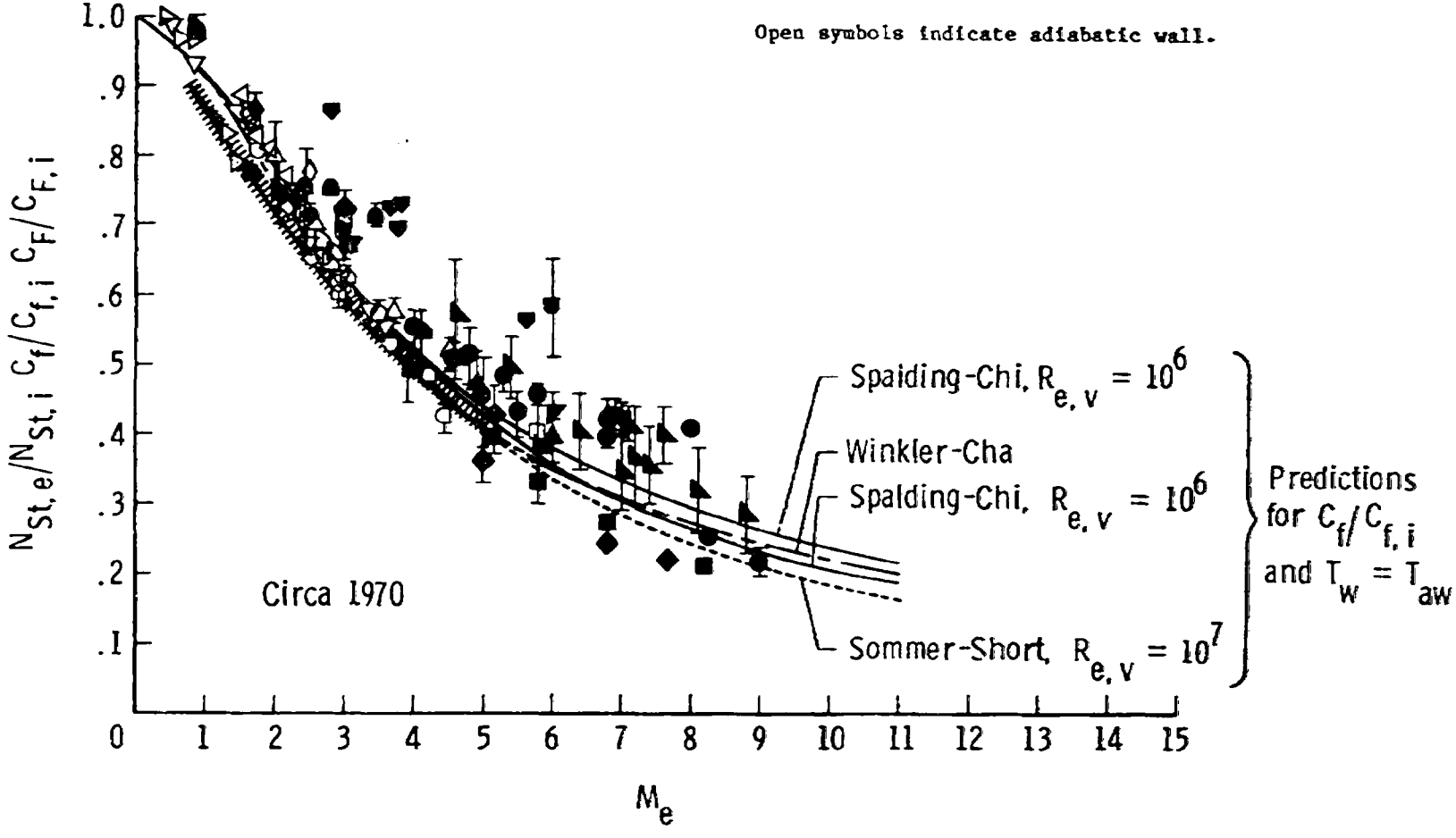


Figure 3. Variation of skin-friction coefficient and Nusselt number with Mach number.

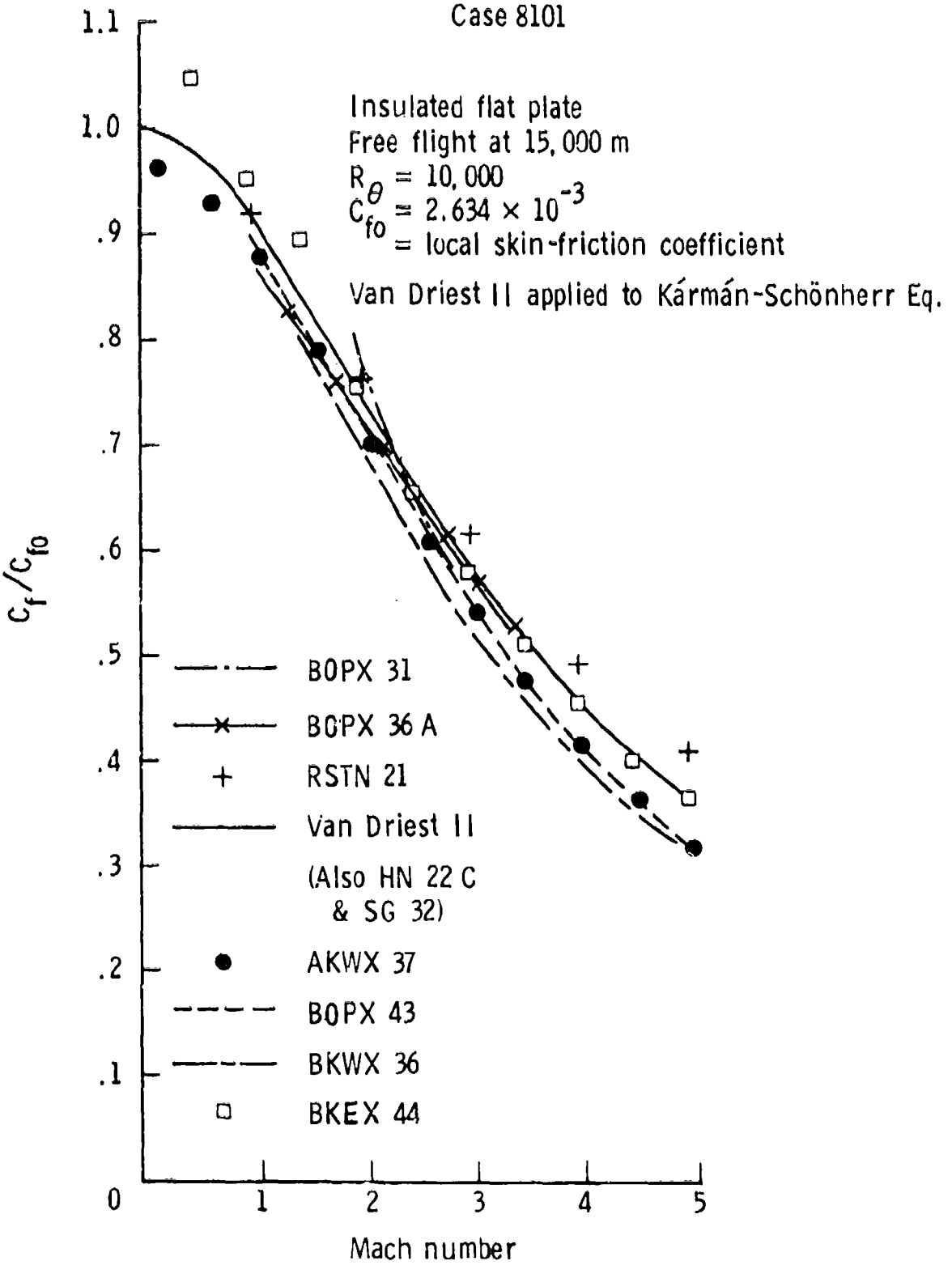

Case 8101
Insulated flat plate
Free flight at 15,000 m
R_θ = 10,000
C_{fo} = 2.634 × 10^{-3}
= local skin-friction coefficient
Van Driest II applied to Kármán-Schönherr Eq.
C_f/C_{fo}
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1
2
3
4
5
Mach number
BOPX 31
BOPX 36 A
RSTN 21
Van Driest II
(Also HN 22 C
& SG 32)
AKWX 37
BOPX 43
BKWX 36
BKEX 44

Figure 4.

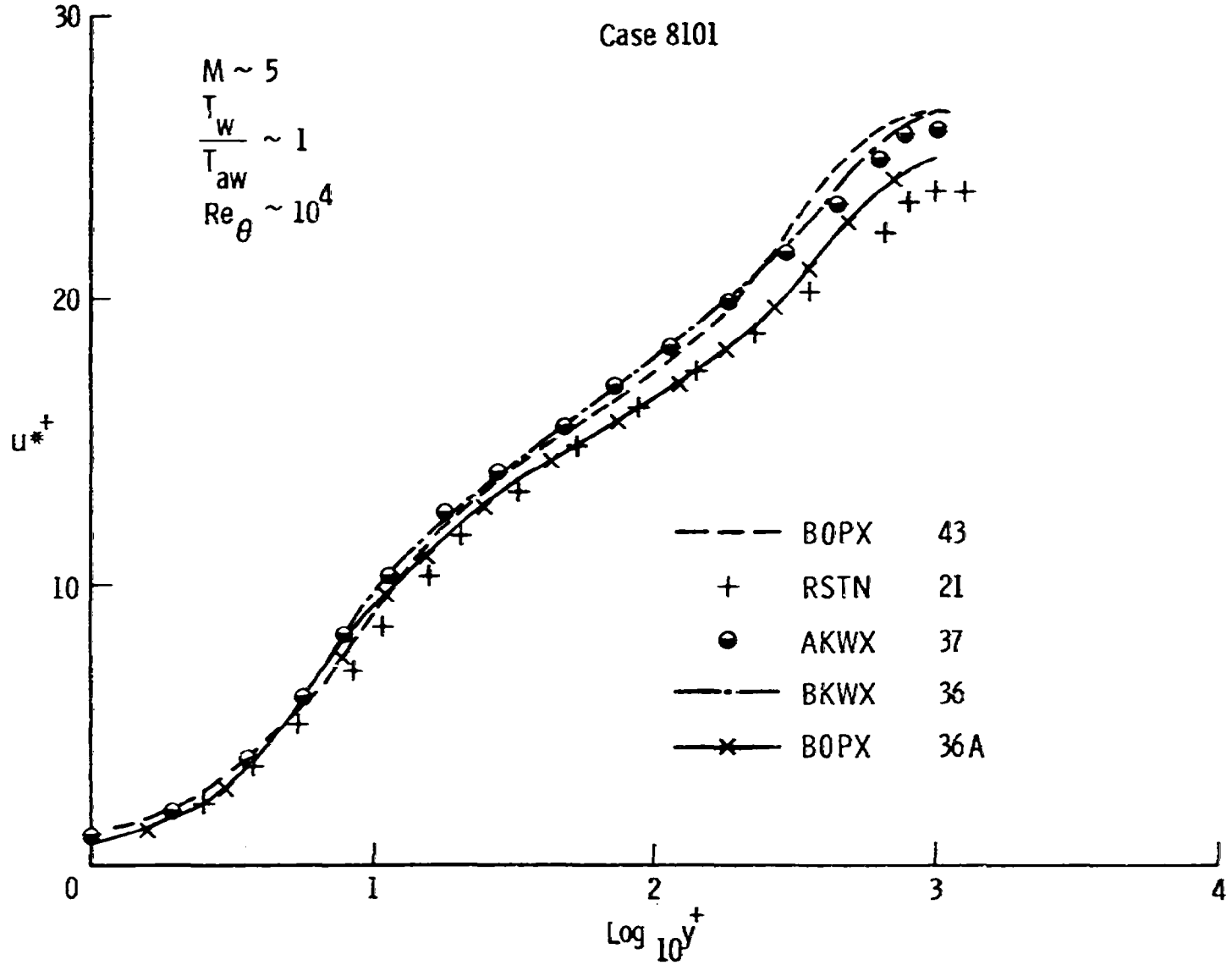


Figure 5. Law of the wall at M = 5.

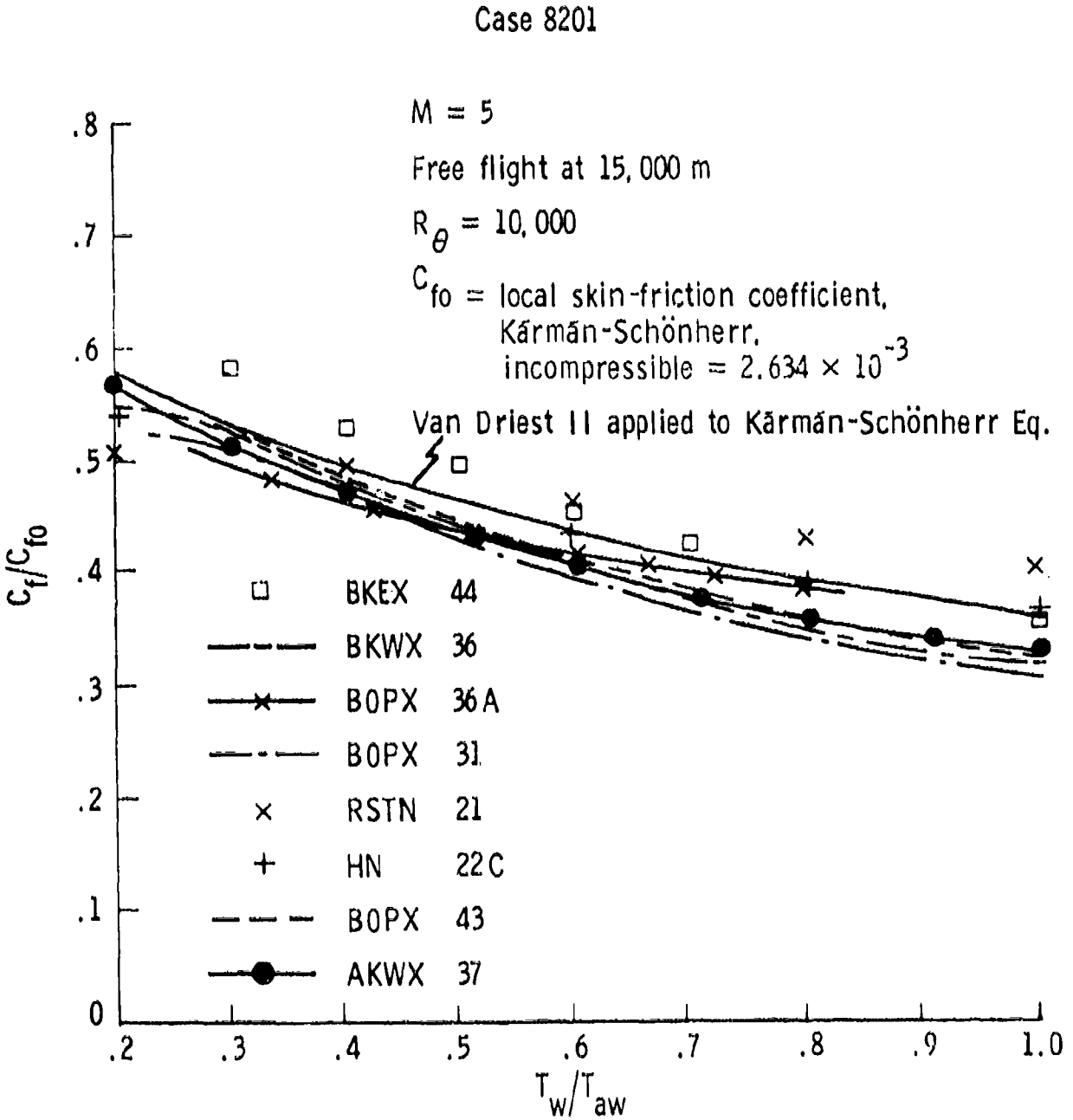
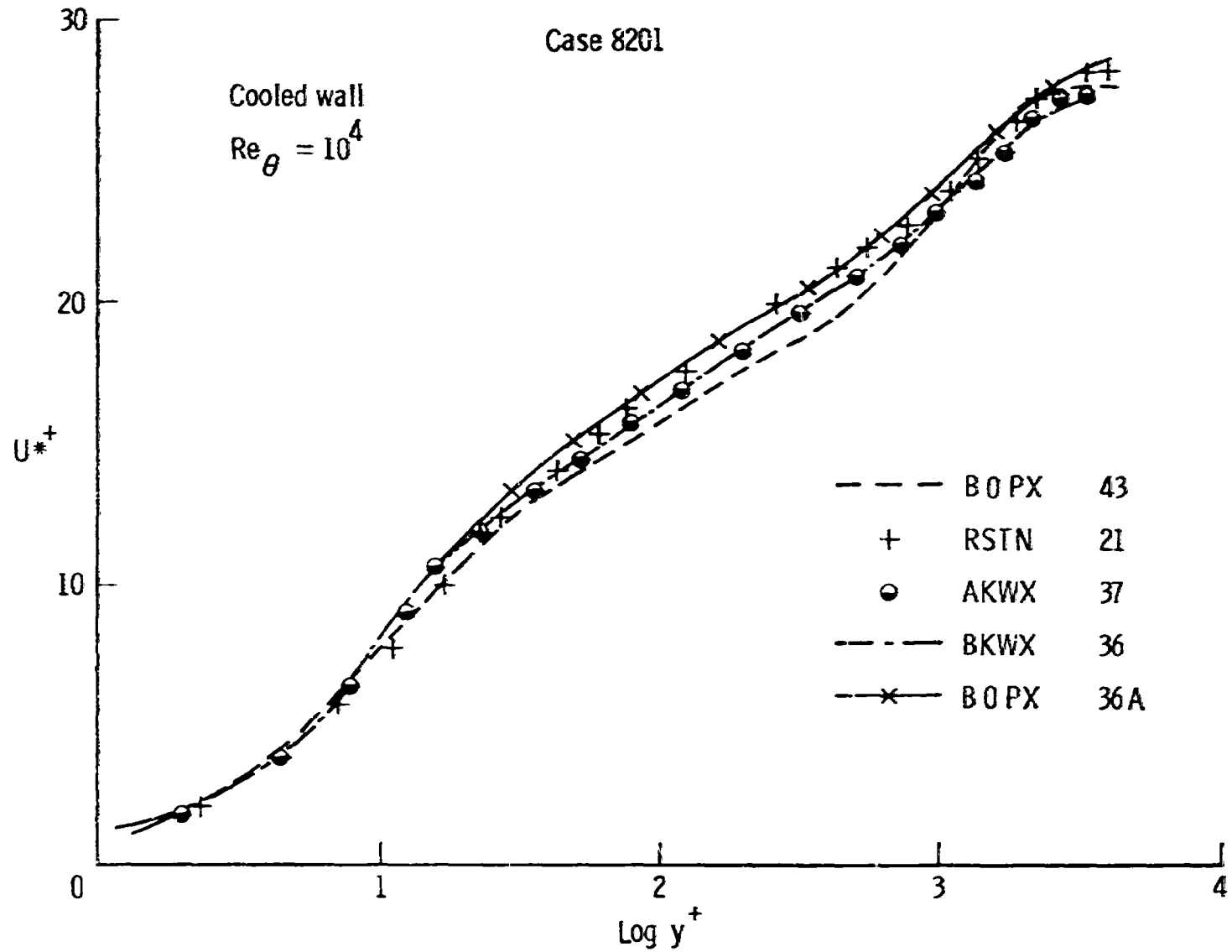


Figure 6. Variation of C_f with T_w/T_{aw} at M = 5.



821

Figure 7. Law of the wall at M = 5.

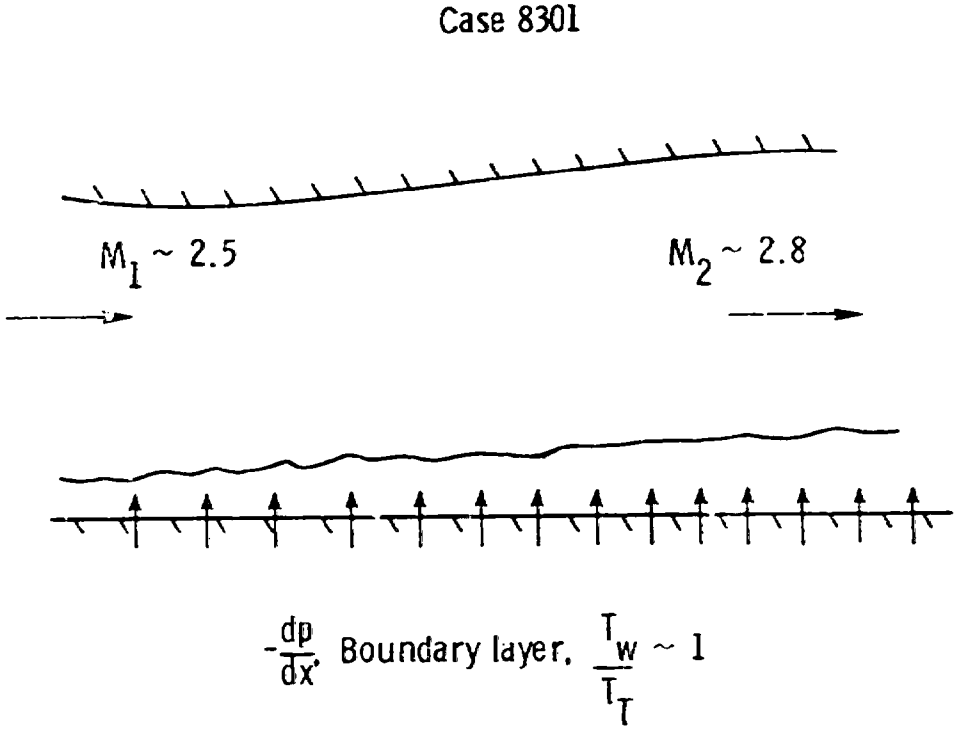


Figure 8. Supersonic boundary layer with favorable pressure gradient and injection.

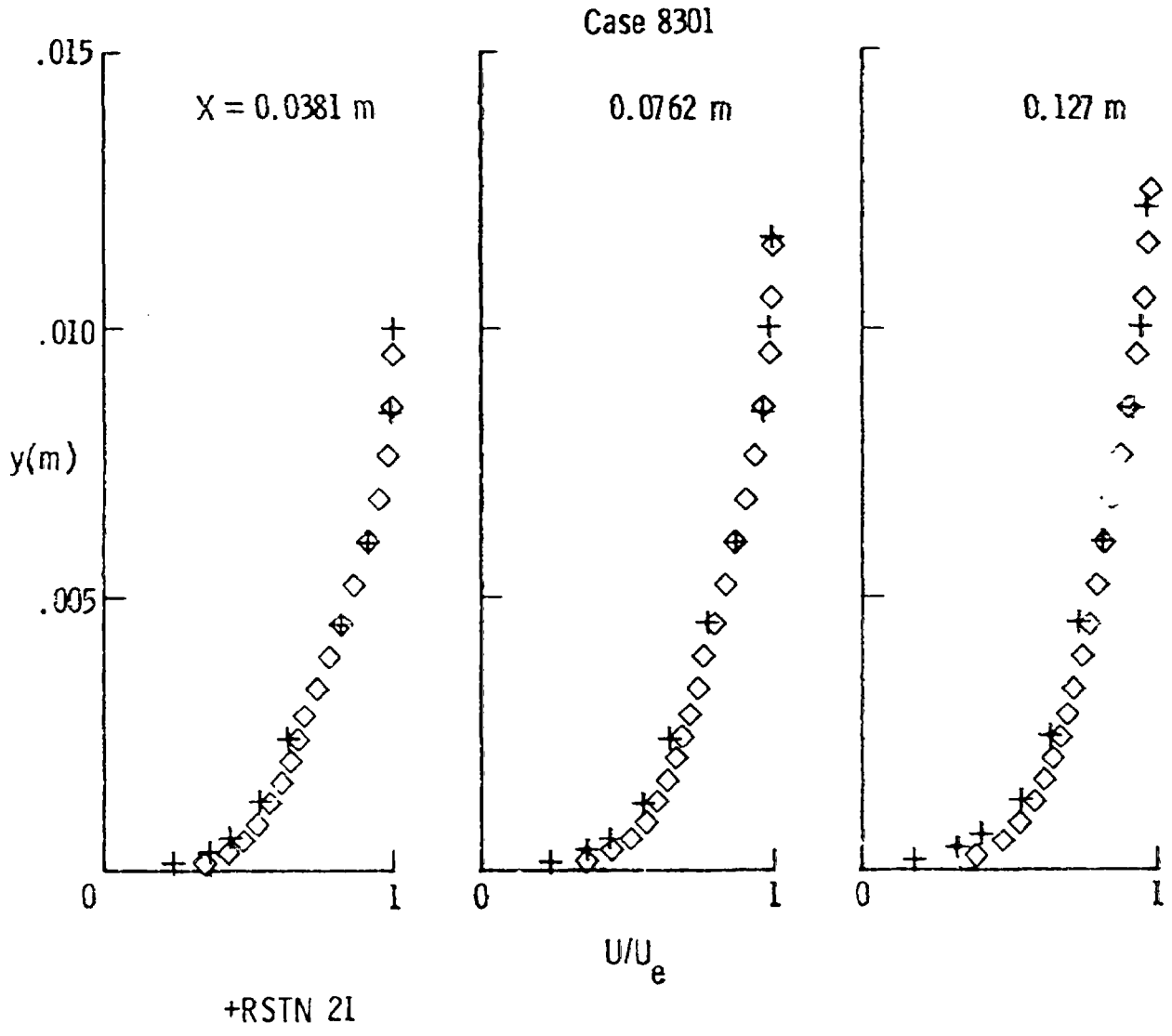


Figure 9. Variation of U/U_e with y at M ~ 2.5.

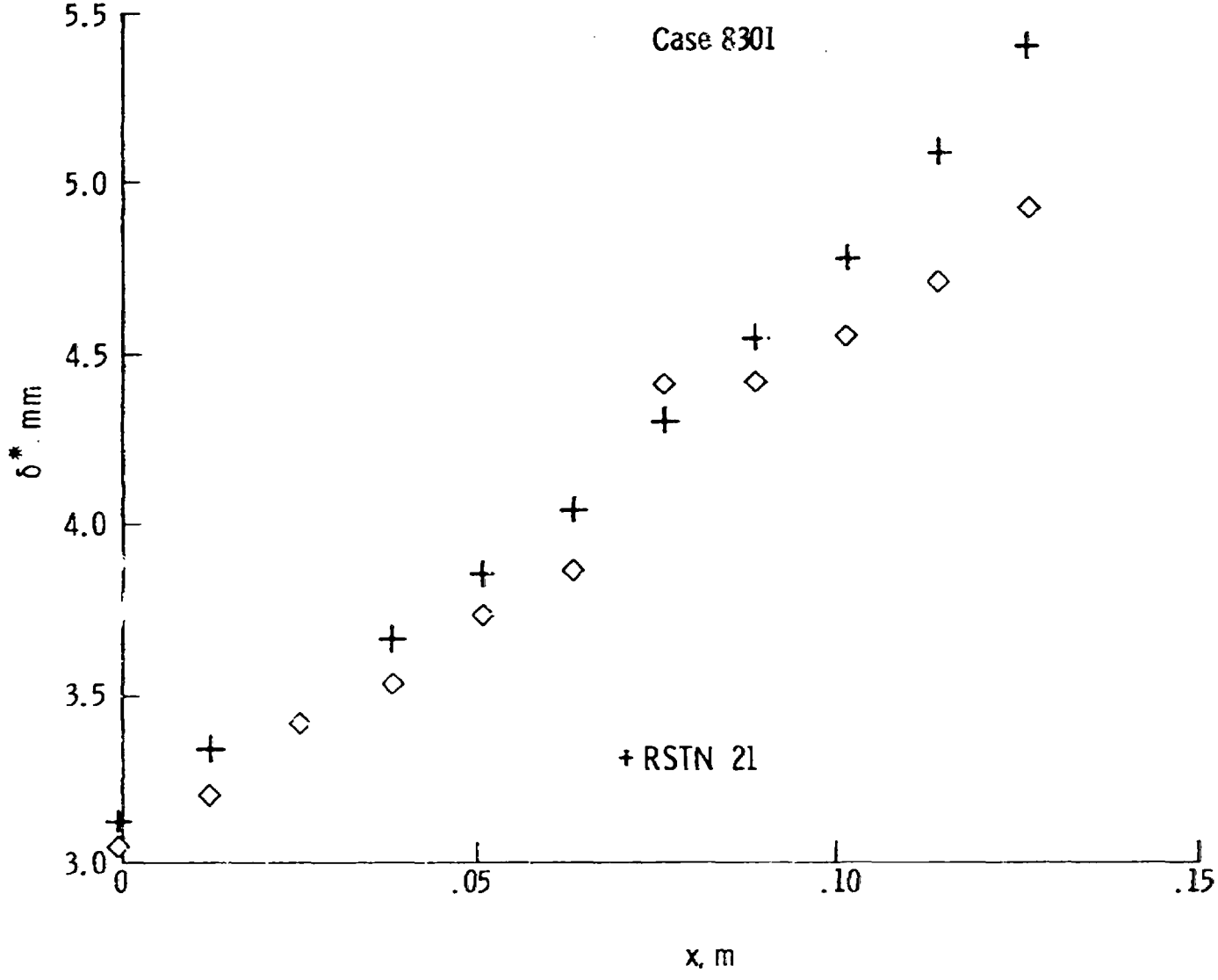


Figure 10. Variation of displacement thickness with x at M ~ 2.5.

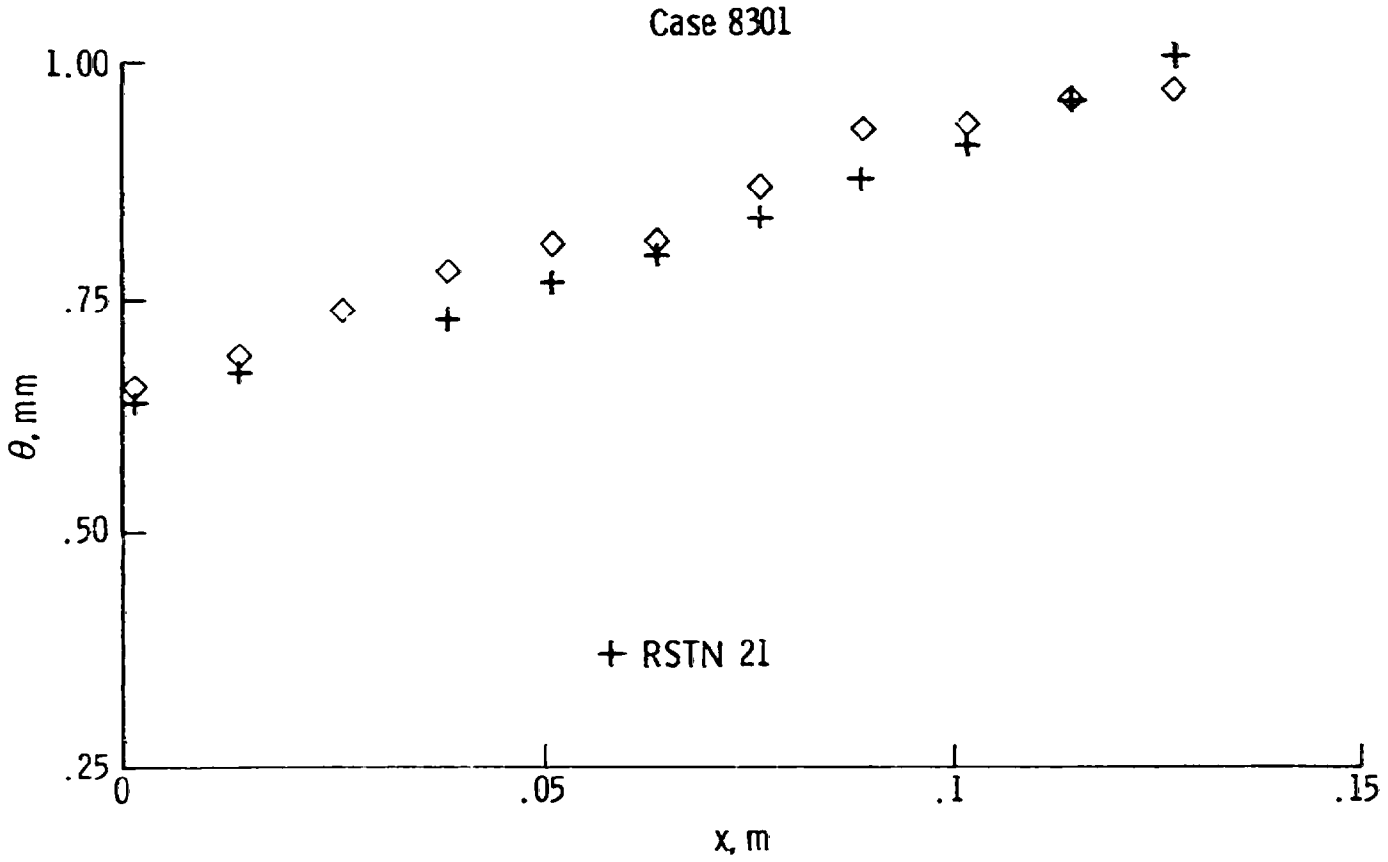


Figure 11. Variation of momentum thickness with x at M ~ 2.5.

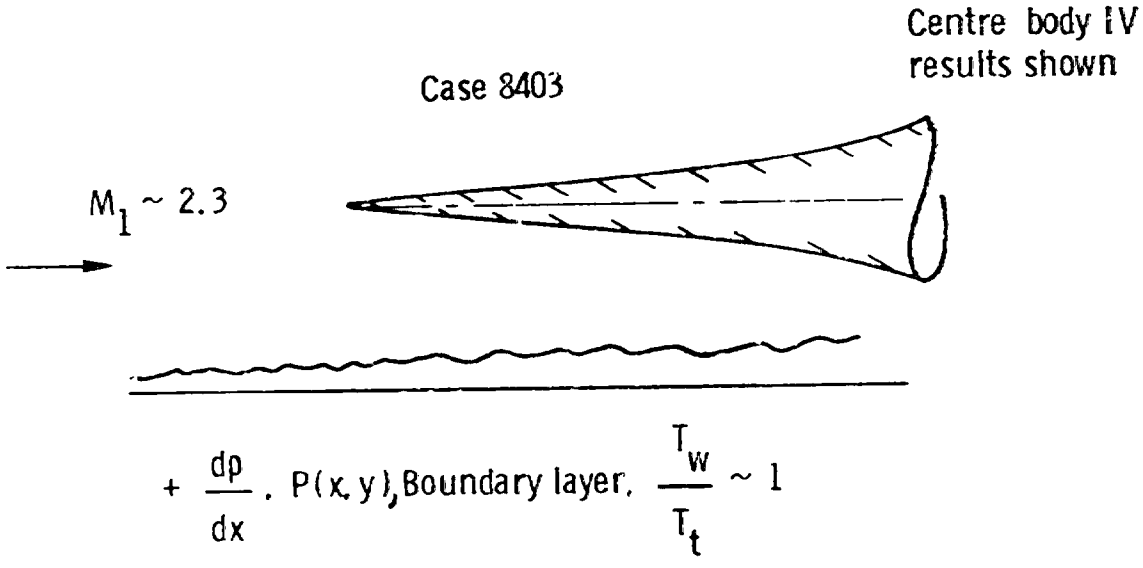


Figure 12. Axisymmetric (inside) supersonic adverse-pressure-gradient boundary layer.

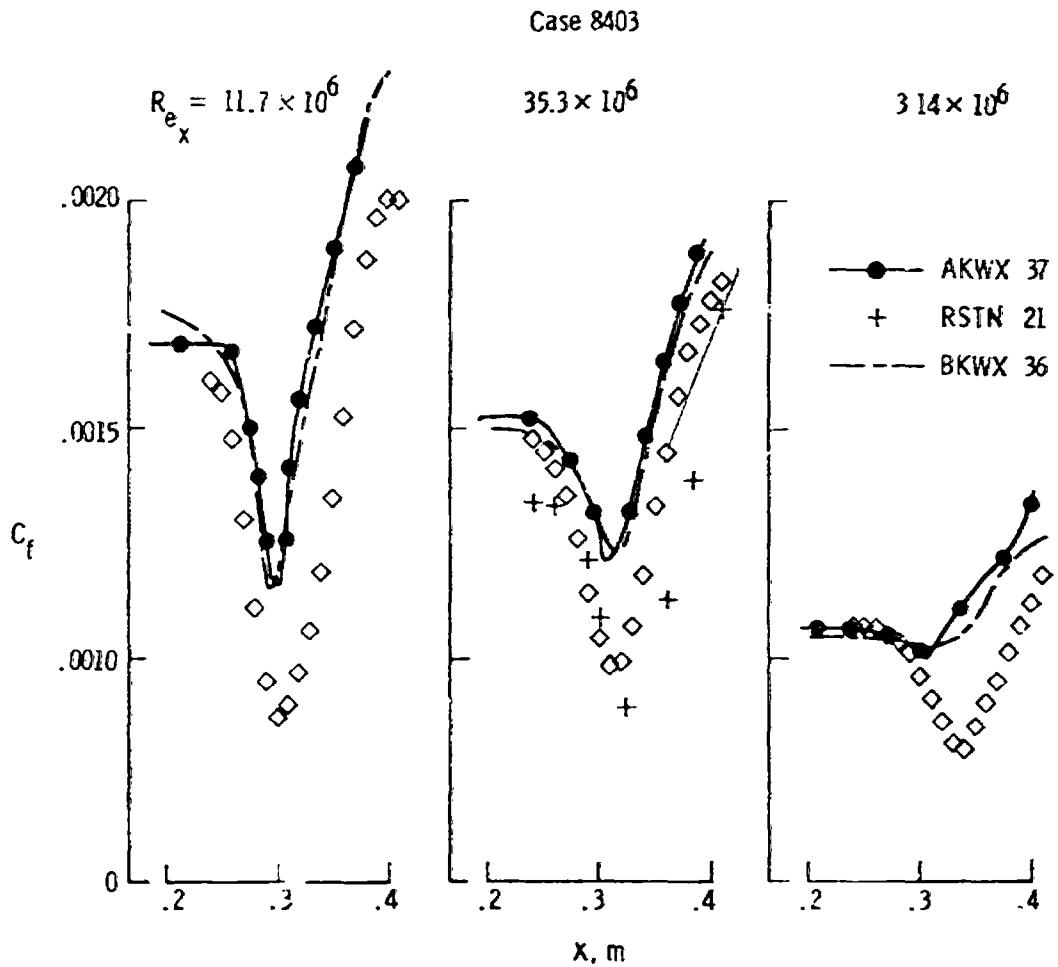


Figure 13. Variation of C_f with distance and Reynolds number for M = 2.3.

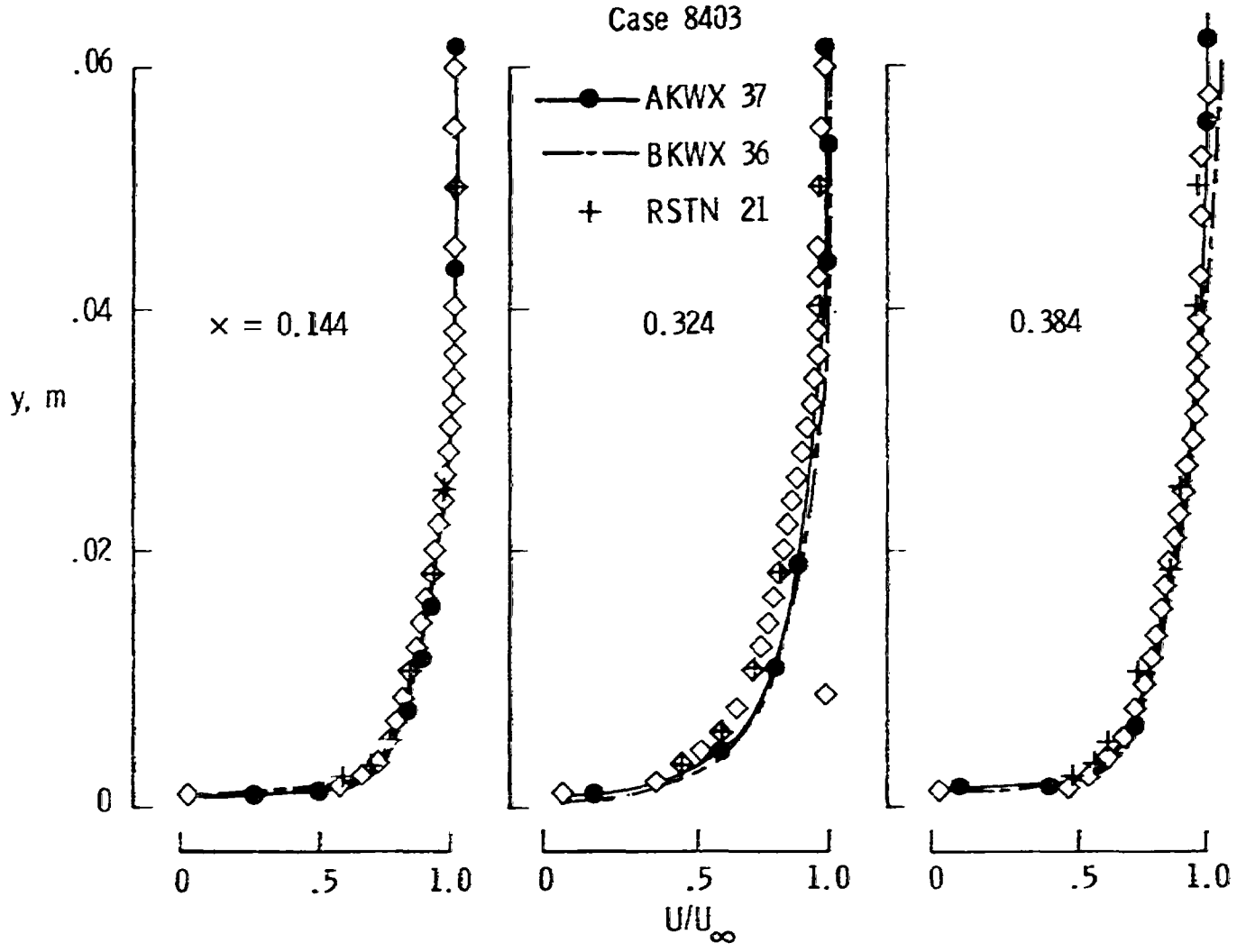


Figure 14. Variation of U/U_∞ with y and x for M = 2.3.

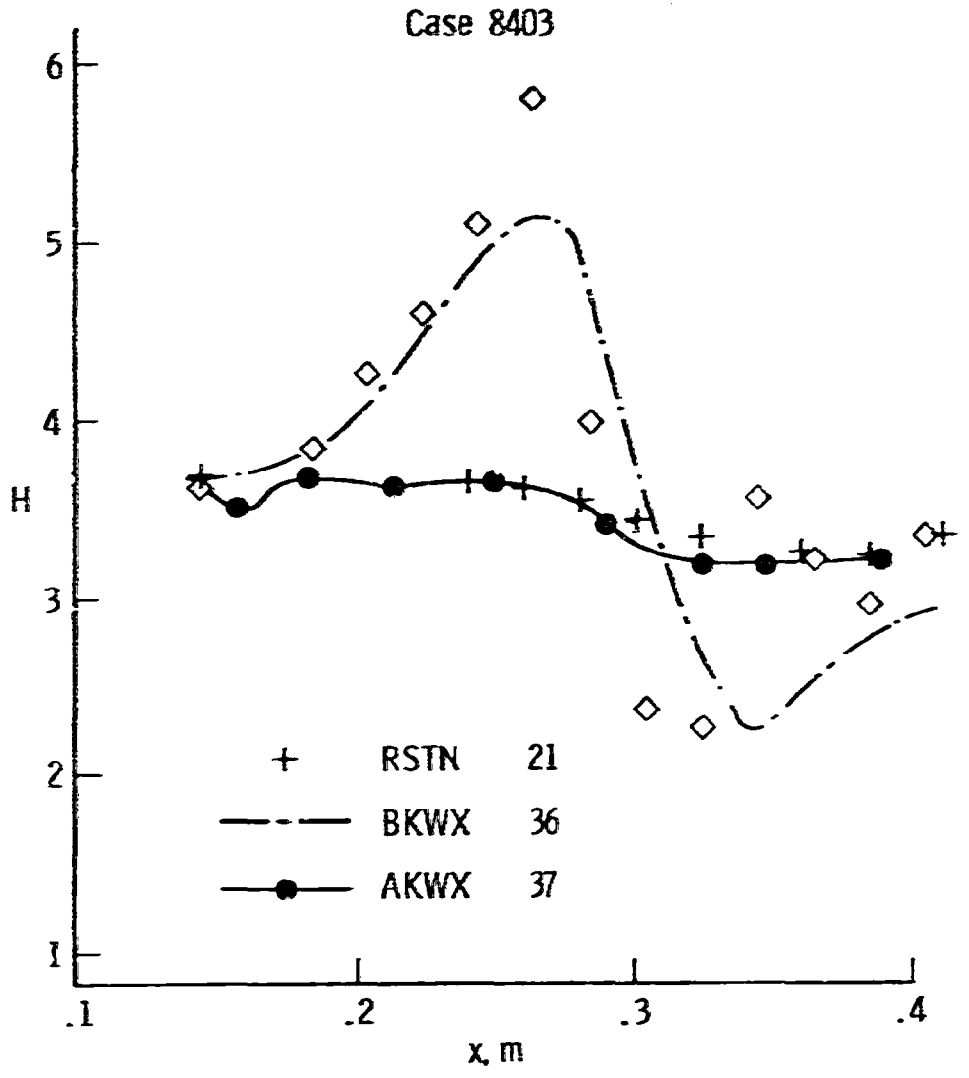


Figure 15. Variation of H with distance at M = 2.3.

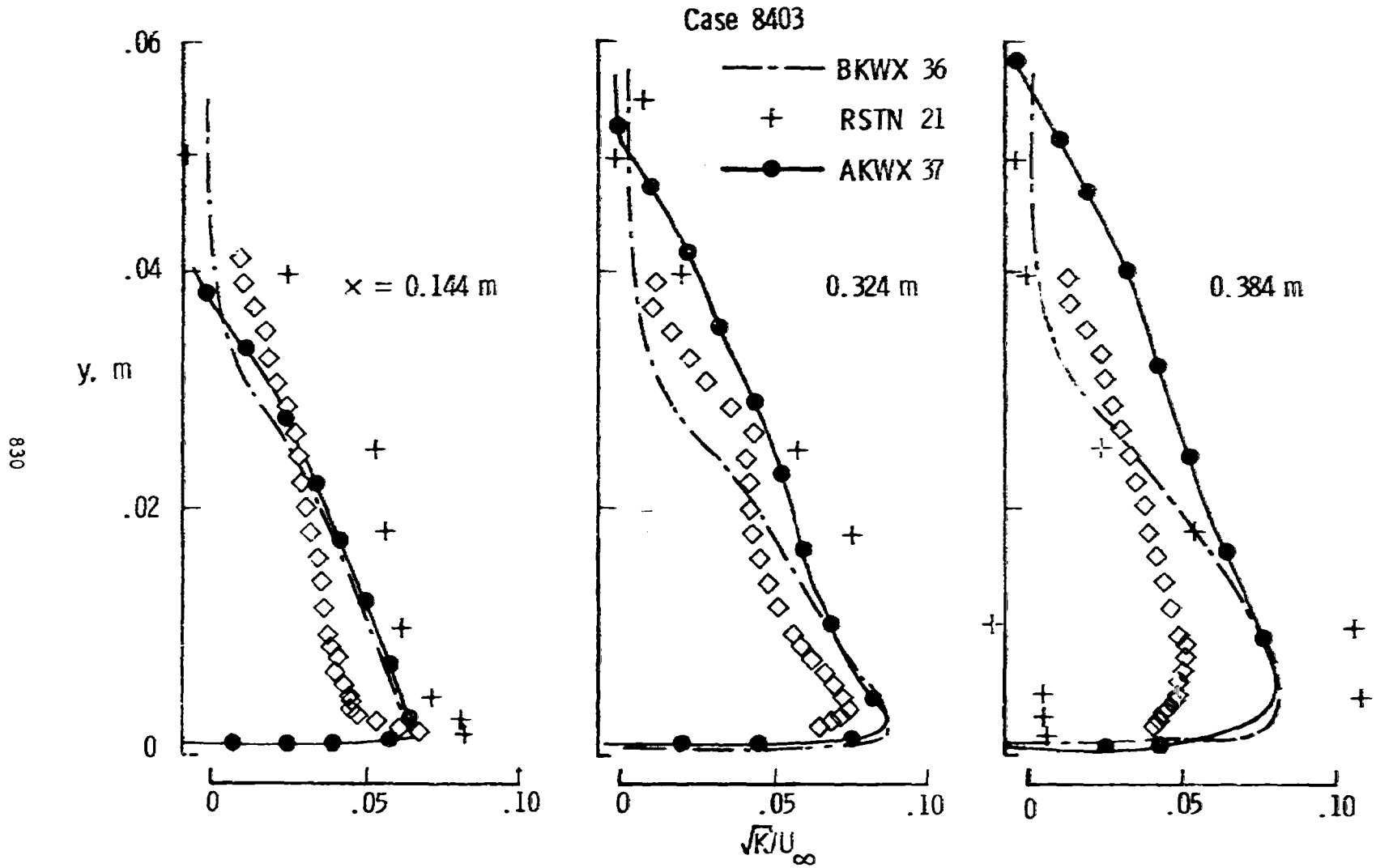


Figure 16. Variation of turbulent intensity with distance at M = 2.3.

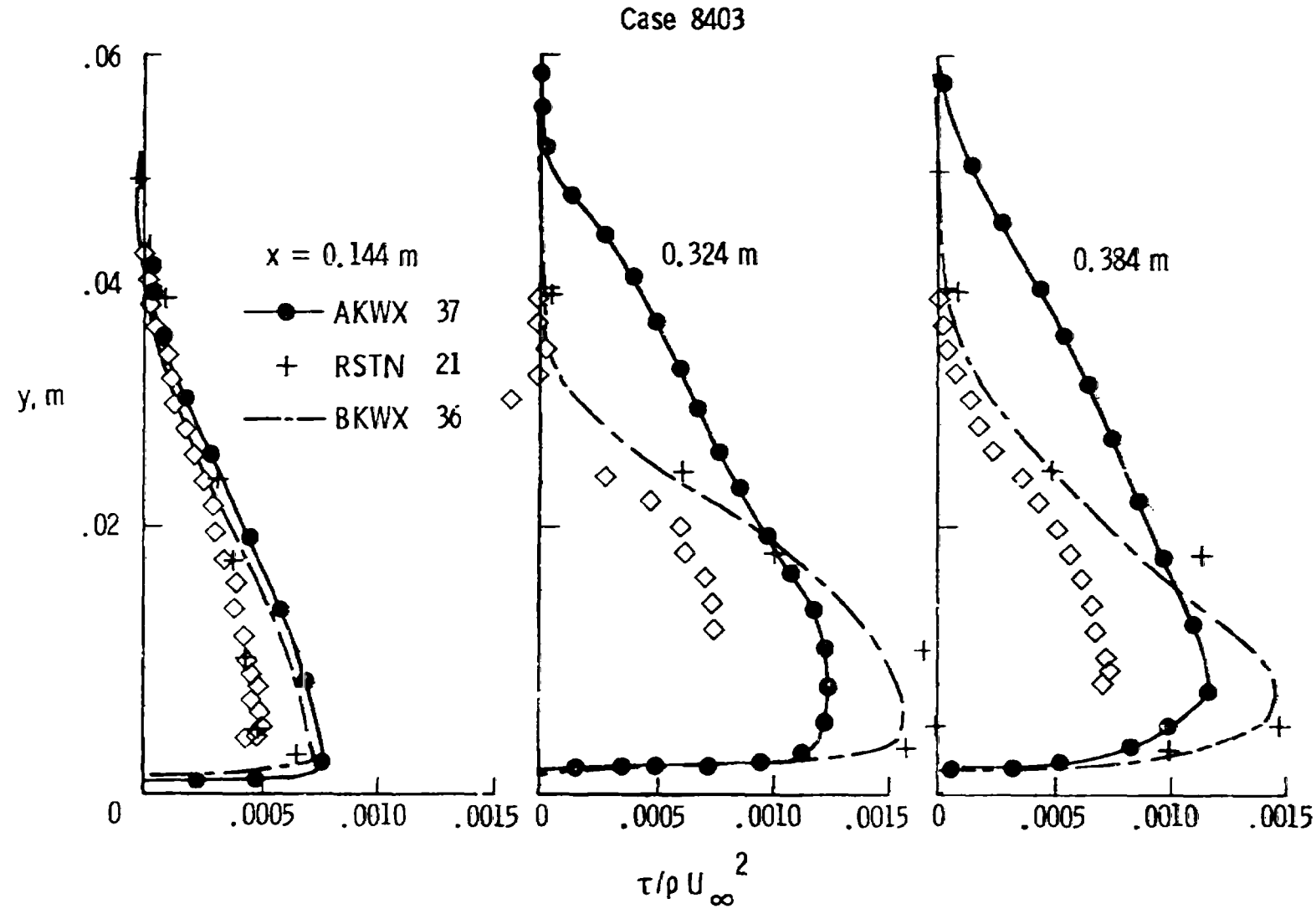


Figure 17. Variation of shear stress with distance at M = 2.3.

Case 8411

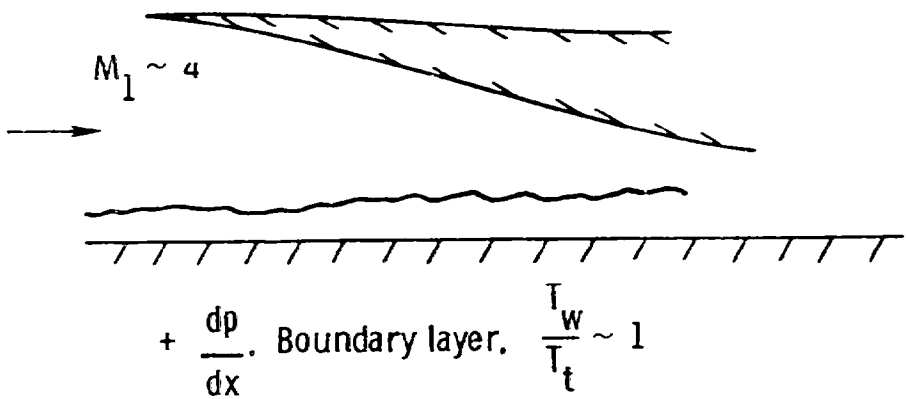


Figure 18. Two-dimensional adverse-pressure-gradient boundary layer.

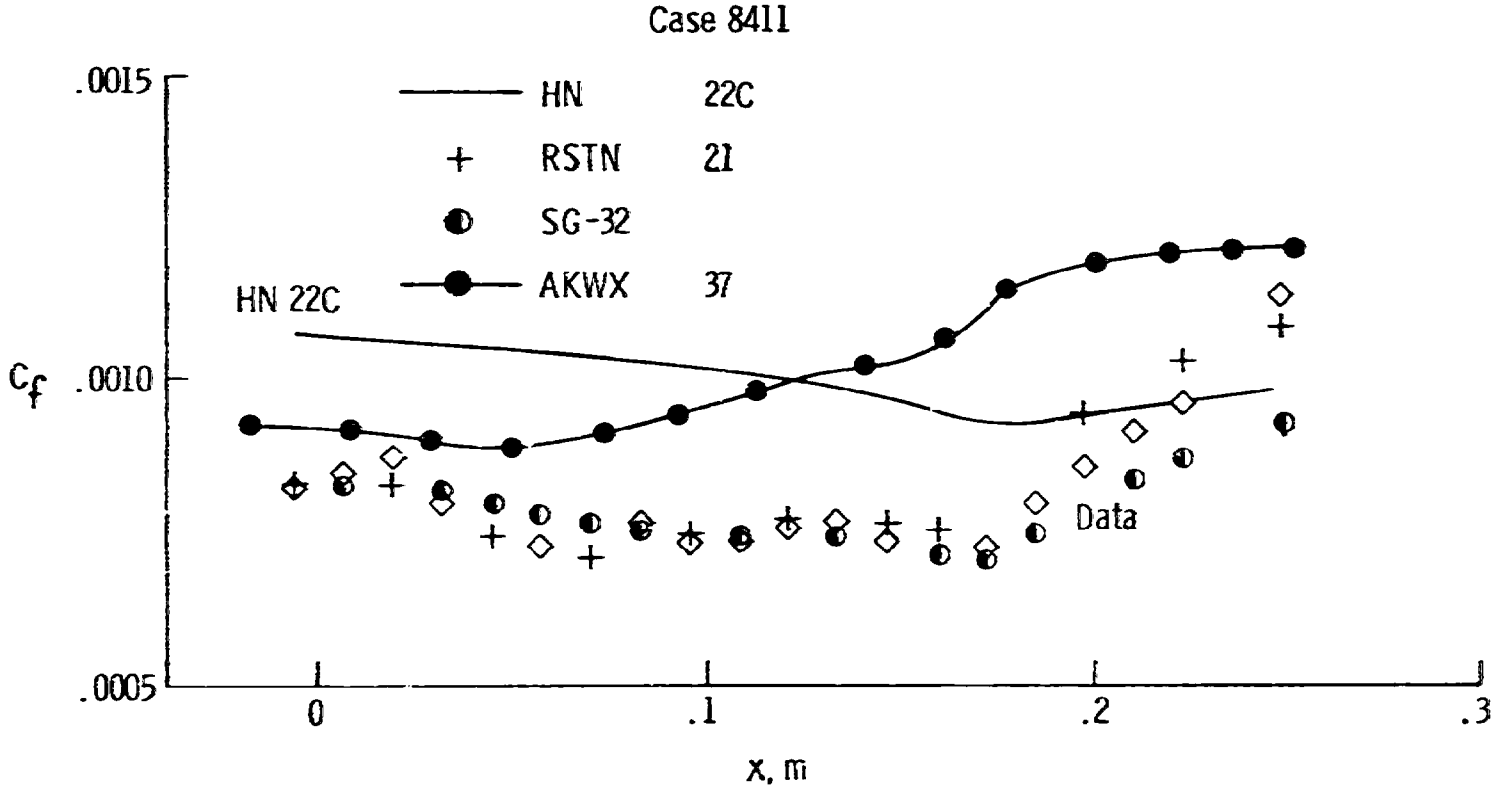


Figure 19. Variation of C_f with distance at M = 2.3.

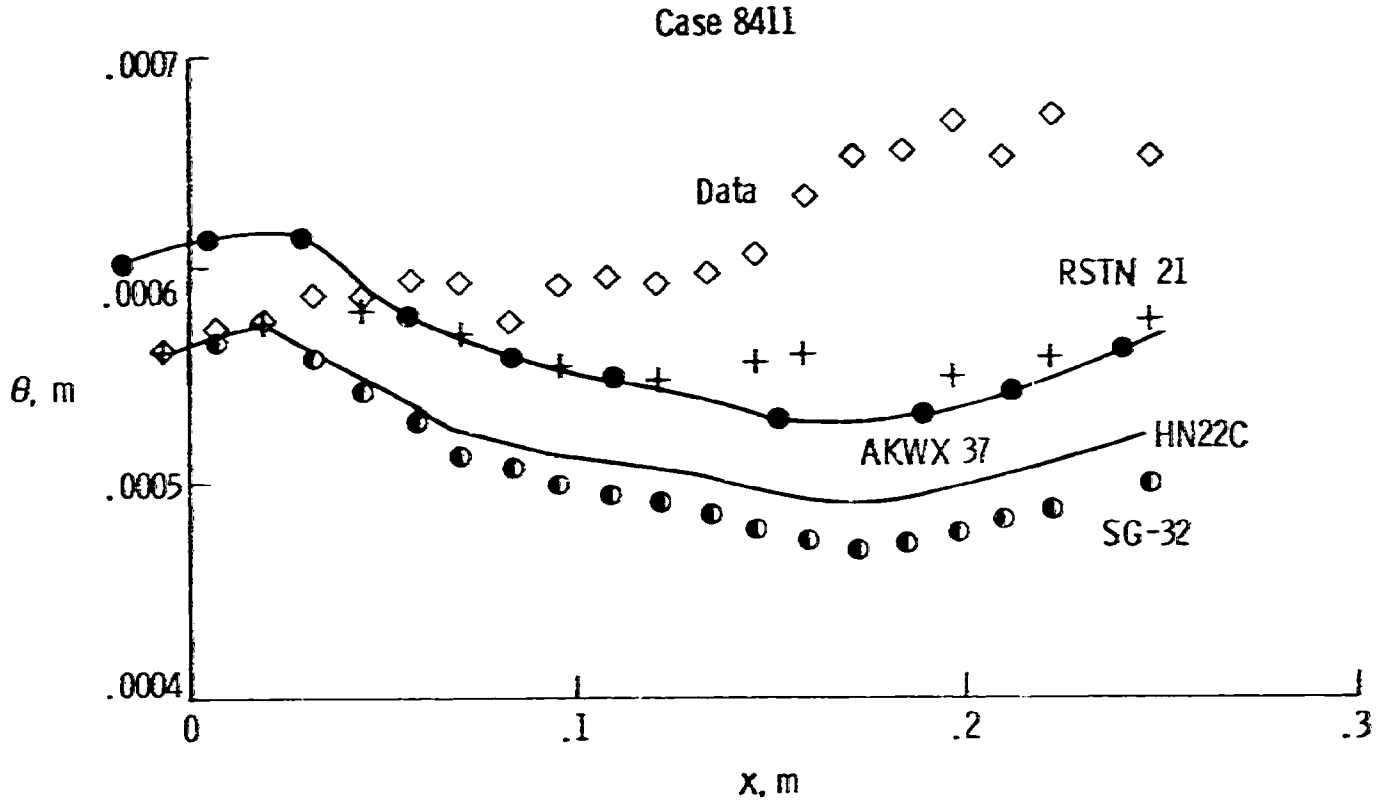


Figure 20. Variation of θ with distance at M = 2.3.

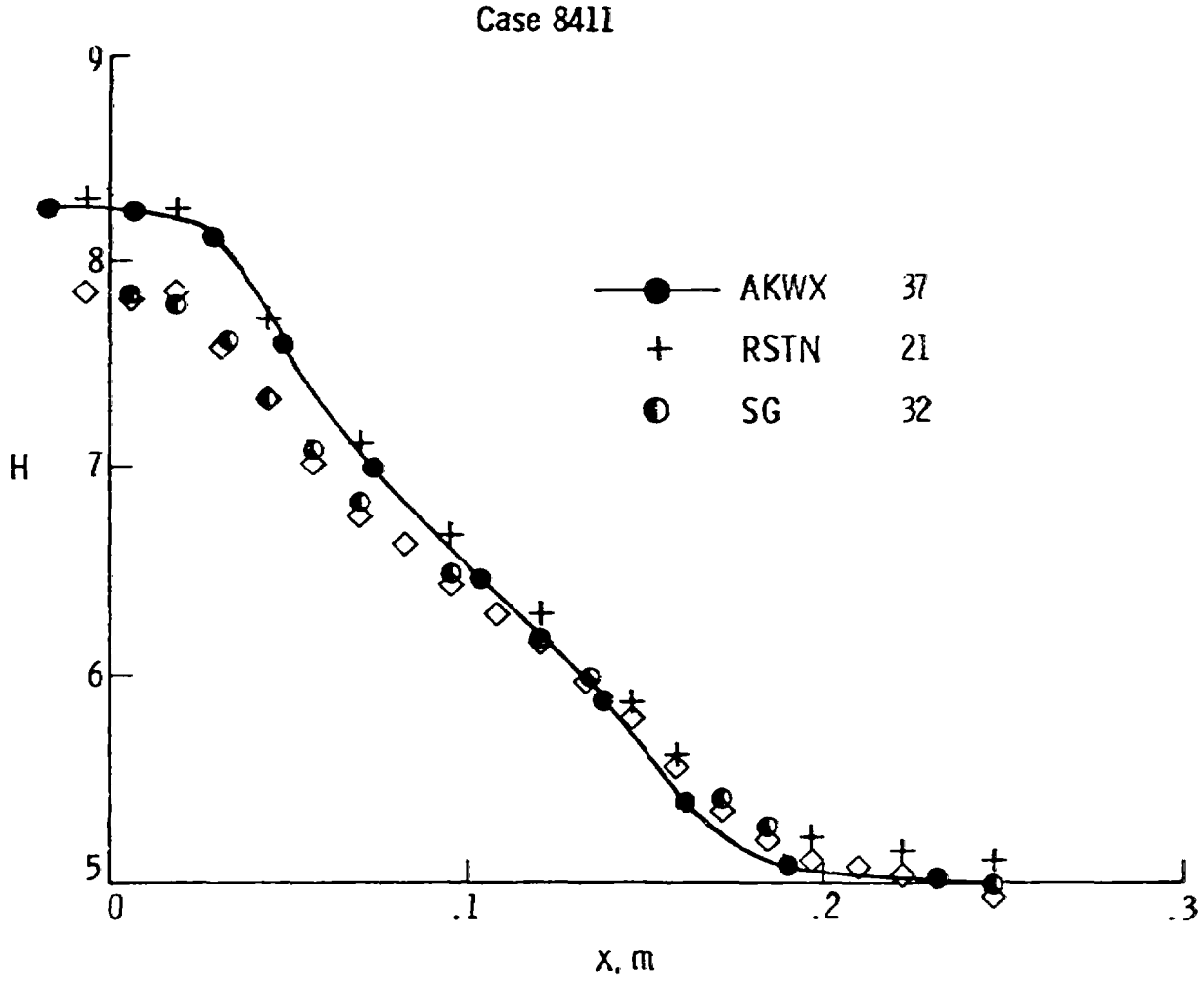
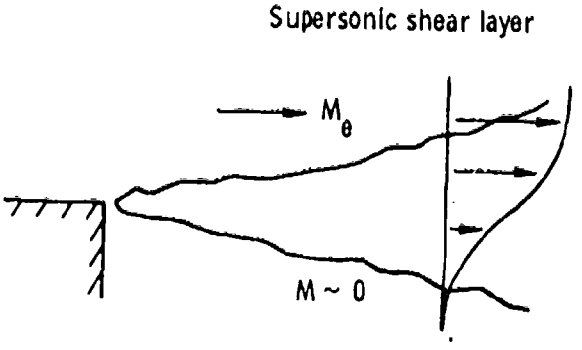


Figure 21. Variation of H with distance at M = 2.3.

Case 8501



$$\frac{dp}{dx} = 0, \text{ Thin shear layer, } f(M).$$

Figure 22. Case 8501, supersonic shear layer.

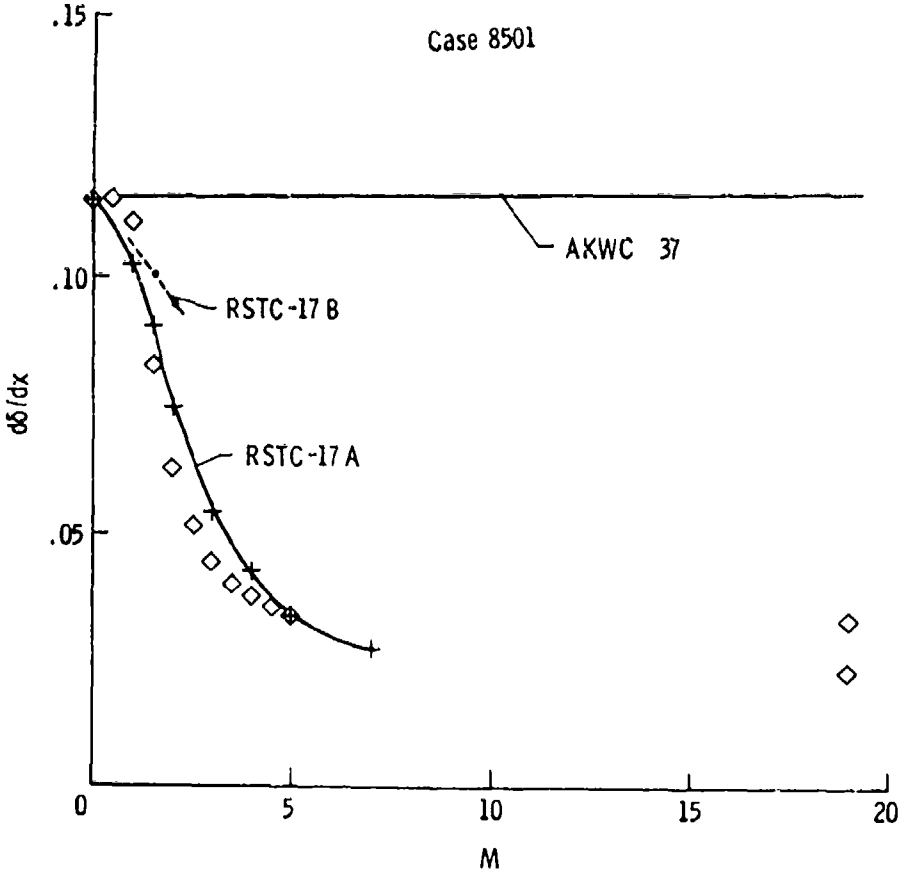


Figure 23. Variation of δ with distance and Mach number.

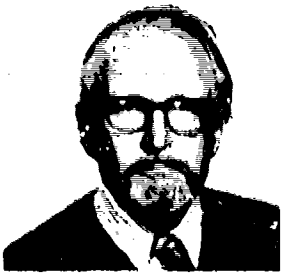
PICTORIAL SUMMARY

Flow 0280. Data Evaluator: K. R. Sreenivasan. "Relaminarising Flows."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0281 K. Simpson D. Wallace		-	15	-	15	-	-	-	-	15	2342 (based on δ^* at inlet)		Boundary layer developing under mild adverse pressure gradient upstream of contraction.

Method Used--Session VIII
(Relaminarization)

Method	Case 0281
BIKX	•
BKEX	•
Plate No. Vol. III	51



RELAMINARIZING FLOW

Case 0281

Technical Report

by

W. Kays*

Groups 05 and 07 attempted this flow using methods B1KX and BKEX respectively.

Although the calculated values of θ, H, and the velocity profiles are in good agreement with the data, this is not so for C_f. Group 07 comments that the poor agreement in C_f reflects the sensitivity to the specification of the free-stream velocity and consequently the pressure gradient. The initial values of C_f depend on the initial values of turbulence, and insufficient data are available from the data-takers to fix these values with certainty.

As noted in the discussion of these cases in Vol. I, relaminarization is one of the few clear limits of turbulent production processes. The process is not a sharp one, but rather represents a trend that must be incorporated in any flow involving strong acceleration of the external flow over a boundary layer. For both these reasons, relaminarizing flows are important checks and limits on any model of turbulent flows or turbulence production.

However, the existing data for relaminarizing flows are relatively old and sparse. Indeed, there is question regarding whether or not the changes in C_f are larger than the uncertainties in the data. Using modern instrumentation considerably better data should be achievable. Hence these flows represent a significant research area both for improved data and for a larger number of checks of existing turbulent flow models.

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*School of Engineering, Stanford University, Stanford, CA 94305.

DISCUSSION

SESSION VIII

The computors pointed out that clarifications are required with respect to the taxonomy of Groups 36 and 37. Both those groups have used the same model but interpreted it differently (BKWX and AKWX, respectively).*

Case 8101

Regarding the temperature-recovery factor, it was pointed out that they were computed to be 0.95 ± 5%, for the range of Mach Numbers considered, by different computors.

Cases 8101 and 8201

It was noted that Groups 36 and 37 used different methods and grids (and codes) based on the same model, and they found identical results. This suggests that the models were independent of the numerics.

Case 8301

No comment.

Case 8403

H. Fernholz suggested that inclusion of uncertainty limits on the experimental data would have been advantageous in making the comparisons between the data and the computed values.

D. Bushnell questioned the relative magnitude of the pressure gradient and the total shear-stress gradients in the two test cases (centerbodies 4 and 2). The results for centerbody 4, presented at the session, indicated strong effects due to pressure gradients since the pressure increased, leveled off, and then decreased. In contrast, the pressure distribution on centerbody 2, not presented at the session, rose monotonically and subsequently leveled off to a relatively long plateau. R. Sullivan (Group 21) presented this data set at the dinner; it showed an improved agreement between the calculated and measured shear-stress distributions.

It was agreed that the flow for centerbody 2 is less dominated by pressure gradients and would provide a better measure of present turbulence models.

J. Viegas (Group 36) pointed out that their mean-flow results were inferior to boundary-layer code results because their time-averaged Navier-Stokes code solved the entire flow field, which in this case yielded a quantitatively different surface-pressure distribution, when compared with the experimental data. However, the trends predicted by their code agreed with the experimentally obtained trend. It was also

*[Ed.: See "Summary of Methods and Numerics," Volume III.]

noted that D. Wilcox had used different constants in his k-ω model than those used by J. Viegas (Group 36).

H. Fernholz restated his earlier remark, that for flows with normal pressure gradients, parameter H as defined in the "normal" sense has no meaning. [Refer AGARDograph 253, Chapters 6 and 7.]

Case 8411

H. Fernholz withdrew his earlier remarks regarding no mention of three-dimensional effects in Zwarts' Ph.D. thesis [entry 7007, AGARDograph 223]: "Surface flow visualisation tests showed that there was considerable flow convergence in the adverse-pressure-gradient region as well as divergence in the zero-pressure-gradient regions. This was attributed to inflow/outflow from the tunnel sidewall driven by the significant pressure differences normal to the test surface. Pitot traverses and Preston-tube measurements were made at stations 38.1 mm to either side of the centerline, at streamwise intervals of 50.8 mm. C_f differed from the centerline value by up to 10%. Differences in integral thicknesses were up to 5%."

C. Horstman (Group 36) confirmed that his momentum balance on Zwarts' data yielded very good agreement and hence this flow merits serious consideration. However, there remains a disagreement with P. Bradshaw who did not get an accurate momentum balance for this experiment. It was questioned whether the parameter H is of any use in this flow, since a normal pressure gradient might exist.

The disagreement between the results of the integral methods used by Groups 32 and 22 for this flow was possibly related to their selection of which parameter to match.

It was pointed out that even a slight convergence in this flow might account for the increase in measured θ relative to calculations even though computed and measured values of H and C_f agree well.

Case 8501

D. Chapman pointed to the disagreement between results obtained with unaltered turbulence models and the experiments. He suggested that this implied the absence of some essential physics in the turbulence models for mixing layers in compressible flows. M. Morkovin agreed and suggested that it should be a matter for basic research to clarify this aspect.

Relaminarization, Case 0281

R. Simpson explained that this experiment was not a relaminarization of a turbulent boundary layer but rather a laminarescence of a turbulent boundary layer. (A sink flow in which the entrainment ceases, the turbulent bursts occur at constant frequencies, turbulence becomes frozen, and the mean profiles become laminar-like.)

In contrast to W. Kays' remarks, R. Simpson stated that the uncertainty range in the skin-friction measurements were approximately ± 15%. This would, however, not account for the 25% difference shown between calculated and measured skin friction. R. Simpson showed good momentum balances done independently by K. Sreenivasan for this experiment. The conflict with Kays' statements in the session thus remains unsettled.

It was unanimously decided to reaffirm support of this flow as a test case and V. C. Patel suggested that future computors should present the following quantities:

(i) Changes in velocity profiles in inner-layer coordinates.

(ii) The development of turbulent quantities in the streamwise direction.

SESSION IX

Chairman: W. Ballhaus

Technical Recorders:

A. Leonard
P. Lowry

NUMERICS AND ADVANCED METHODS

INFLUENCE OF NUMERICS AND COMPUTOR VARIANCE IN THE COMPUTATION OF COMPLEX TURBULENT FLOWS -- B. E. Launder

DIRECT SIMULATION OF HOMOGENEOUS TURBULENCE AT LOW REYNOLDS NUMBERS -- R. S. Rogallo

LARGE-SCALE NUMERICAL SIMULATION OF WALL-BOUNDED TURBULENT SHEAR FLOWS -- P. Moin and J. Kim

INFLUENCE OF NUMERICS AND COMPUTOR VARIANCE IN THE COMPUTATION OF COMPLEX TURBULENT FLOWS

Results contributed by

P. Chassaing, I. Demirdzic, A. D. Gosman, H. Ha Minh, R. A. Issa, M. A. Leschziner, N. N. Mansour, T. Morel, A. Pollard, A. K. Rastogi, W. Rodi, G. Scheuerer, M. Sindir, and D. D. Vandromme

Assembled and Compared by

B. E. Launder

1. Introduction

For the two flows designated "Central Test Cases" several computor groups provided numerical predictions with essentially the same turbulence model, a Boussinesq viscosity model based on transport equations for the kinetic energy, k, and its rate of dissipation, ε (Launder and Spalding, 1974). Some of these were contributed as part of the regular submission to the Conference while others have been made especially to assist this comparison. The aim of this effort has been to allow some assessment to be made of the seriousness of the variations in computed behaviour that can arise from

(i) differences in the forms of the mean-flow equations solved (e.g., the difference between the full elliptic transport equations and a parabolic approximation thereof).

(ii) differences in the numerical accuracy arising both from different numerical approximations and from different degrees of mesh refinement.

(iii) "Computor variance", into which category is swept all the very minor variations in practice between computors that cannot feasibly be identified except by a card-by-card check of different programs and also differences arising from round-off error by the computor.

This document summarizes the outcome of this collaborative exercise and draws some conclusions. Computations are identified by a letter code only, a degree of anonymity that is appropriate to the aims of the enquiry.

2. Case 0421—The Backward-Facing Step (see Fig. 1)

Of the six sets of computations summarized in Table 1, sets A-C have been obtained with versions of the Gosman-Pun TEACH code, set D also results from the same basic numerics as TEACH while sets E and F have been generated by time-marching treatments. These last two schemes employ a central-difference approximation of convection while the programs used for sets A-D adopt the hybrid central-upwind approximation according to whether the cell Peclet numer is greater than or less than 2. In practice the upwind option is invoked over most of the flow domain even with the very

fine mesh employed for set D. Most computors do not report the length of their computational domain though in all cases it appears to be close to 20. Figure 2, adapted from data provided in support of set B, seems to indicate that this is indeed a sufficient length to have negligible effect on the separated region.*

Table 1

Computations of Case 0421 Backward-Facing Step with same BKEZ Model

Set	Nodes X x Y	Mass Residuals	Treatment for Convection	Reattachment length
A	32 x 32	4×10^{-4} 6×10^{-5}	Upwind †	5.8 5.2(5.3)
B	42 x 42	6×10^{-4}	Upwind †	5.5
C	28 x 28	"tight tolerance"	Upwind †	5.6(5.8)
D	87 x 38	10^{-2}	Upwind †	5.9
E	39 x 30 59 x 30 39 x 58	?	Central	5.2 "neither change affected R_L"
F	40 x 40	10^{-3}	Central	6.2

†Central difference when cell Peclet number less than 2

A wide range of mesh sizes has been adopted: the computors submitting sets A and C, who employed the coarsest grids, acknowledge that their results may suffer a little from insufficient mesh refinement. Those contributing sets B, D and E believe their grid-refinement tests have established that their reported results are grid-independent. Figure 3 shows the dependence of reattachment length on number of nodes (provided in support of set B): the reattachment length increases as the number of nodes is raised because the magnitude of false diffusion is then progressively diminished. Figure 4 (again provided in connection with set B) shows for a 42 × 42 grid,

*In the calculations the length of the solution domain has been successively increased by adding extra columns of nodes leaving the existing columns of nodes at exactly the same points used for computations involving shorter domains.

the variation along the duct of the maximum normalized value of the false diffusion coefficient Γ_F at any station where Γ_F has been obtained from the formula of Mallison and de Vahl Davis (de Vahl Davis and Mallison, 1976):

$$\Gamma_F = \frac{\rho U \Delta x \Delta y \sin 2\theta}{4(\Delta y \sin^3\theta + \Delta x \cos^3\theta)}$$

where Δx and Δy are the cell dimensions and θ is the angle made by the resultant velocity with the grid lines.

The false diffusion coefficient thus defined is normalized by the local value of turbulent viscosity and multiplied by the ratio of the local-to-maximum shear stress at any station (the latter factor was introduced to avoid giving undue prominence to regions where the shear stress was low). The factor exceeds 0.2 in the region 1-3 step heights beyond the step. However, this represents the maximum error at any section, not the mean, and the computor's estimate that the effect of such an error on the computed reattachment length did not exceed 0.3H is probably reasonable.

In all contributed cases "convergence" of the iterative solution was signalled by the sum of the residual errors in satisfying the difference equations falling below some posted value. A value of the summed errors of 10^{-3} times the inlet value is a commonly adopted target although whether or not that is an appropriate value depends on the flow domain, the dependent variable in question, and several other factors. By accident, the group providing set A allowed computations to proceed many iterations beyond the usual termination point and found detectable changes in the computed profiles (Fig. 5). The normalized mass source appreciably decreased during the extended iteration sequence though the computors believe that the variation was principally due to the dissipation rate equation being insufficiently converged. Another computor whose solution extended to nearly 2500 iterations, reported oscillations in reattachment length until ε had become truly stationary. The similarity of these observations from independent sources strongly underlies the need to monitor carefully the convergence of the turbulence as well as the mean-flow equations. That less attention has been paid to them in the past can be attributed to the absence of any simple scale (such as the inlet mass flow provides for the continuity equation) for deciding whether numerical errors are "small" or significant. For the k and ε equations a reasonable scheme would be to compare the summed residual errors with the corresponding summed rate of generation of the dependent variable in question over the flow domain.

We note from Table 1 that the reported reattachment length varies, over the six sets of computations, by ± 0.5 step height about the mean of 5.7.* While it is not possible to attribute entirely the cause of the difference between one set and another, some at least can be linked to inlet conditions. While computors adopted the given mean velocity upstream of inlet, only two components of the turbulence energy were reported, and the location of measurement did not coincide with those of mean velocity. Accordingly, some estimation and interpolation was needed to obtain the kinetic energy profile. Group D, in fact, simply prescribed a uniform k. The situation is more serious for the dissipation-rate profile. Most computors have obtained ε from $k^{3/2}/\ell$ where the distribution of the length-scale ℓ is prescribed. Figure 6 shows a startling variation of choice for ℓ. One would expect computations made with larger inlet length scales (i.e. larger viscosities) to produce shorter reattachment lengths, but there is no clear trend in this direction. There are plenty of other possible sources of these variations quite apart from the big differences in computational methodology such as steady-state versus transient, central- versus upwind-differencing, etc. Two that may be mentioned are the detailed differences in applying, through "wall functions," the wall boundary conditions and the various arrangements for interpolating pressures at the desired position--a problem that arises for staggered non-uniform grids. Figures 7 and 8 compare the reported mean-velocity and shear-stress profiles for the different sets of computations. Clearly differences do exist among the reported results though to keep these in perspective it would be fair to say that if these were differences in experimental results we were comparing, the measure of agreement shown in these figures would be regarded as impressive. Set F which exhibits the longest reverse-flow region also shows the fastest rate of growth in the outer region. Consistently the maximum shear stress is the highest for this set. To give some counterbalance to any impression that this set should therfore be discarded, however, it needs to be restated that sets A-D are all founded on the same underlying discretization strategy; so it is, in principle, possible that all suffer from a common flaw. In this connection, we note that at the most downstream station, the velocity profile of set E, the other central-difference computation, also falls below the other computed distributions.

3. Case 0331--The Curved Mixing Layer (see Fig. 1)

The five sets of computations contributed for this case have emerged from quite different numerical treatments. Set G has used the parabolic thin-shear-flow equations, adopting the given "free stream line" as a reference surface and obtaining the

*This degree of variation is similar to that found in experiments involving repeat runs on the same apparatus. It is significantly short of the experimentally estimated length of 7.0 ± 0.5 step heights.

pressure variation normal to the flow by assuming radial equilibrium. The four other sets have been obtained from the solution of the discretized (elliptic) Reynolds equations. Sets H, I and J have been calculated with TEACH-based codes. The first of these employs conventional upwind differencing, set I incorporates a skew-upwind treatment while set J is a new code, still undergoing testing, that adopts a non-orthogonal curvilinear coordinate set; in its application to this problem one of the families of coordinate lines follows a curved path roughly--but not exactly--following that of the mean streamlines, Fig. 9. Finally, set K has resulted from a time-stepping scheme employing central differences.

The computations supplied in this last set were for a 50 × 50 grid; a further computation has subsequently been made by the same computors with a 61 × 59 mesh which, they report, exhibits negligibly different mean-velocity profiles but small variations in turbulence energy (the finer grid giving a maximum kinetic energy 5% larger at 30° from discharge). Set J adopted a grid of 37 × 49 nodes, the larger number in the direction of flow development. Sets H and I were generated by a coarser mesh (27 × 31) though a smaller x-dimension was covered and the wall boundaries were treated as frictionless, thus obviating the need to place a substantial number of nodes in the wall boundary-layer regions. By comparison, the thin-shear-flow treatment of set G has a grid which spreads to just cover the shear layer as it evolves with 40 cross-stream nodes and a forward step of about 0.02 times the shear-layer width. With this density of grid nodes, the computations would have suffered less from numerical error than any of the elliptic treatments. There remains, of course, the questions of whether the simplified representation of the strain field and the assumption of radial equilibrium which are endemic with parabolic treatments introduce significant errors of a different kind.

An extract from the computed development for the five sets is shown in Figs. 10 and 11 and in Table 2. Clearly, from Fig. 10, at the 90° position (nominally at the end of the curved portion) the Cartesian upwind computation (H) has produced a significantly broader velocity profile than the other results--a consequence of numerical diffusion arising from large grid-to-flow skewness, a feature known to be highly detrimental to upwind treatments (Leschziner, 1980). The other profiles are less easily compared because, for the elliptic cases a certain relative displacement of the profile can arise from different outflow treatments at the bleed slot (see Fig. 9). However, the maximum slope of the velocity profile at 90° is given in Table 2 for each of the sets of results, the values being normalized by the experimentally measured value. It should be said that the turbulence model used is known <u>not</u> to mimic the great sensitivity of turbulence to streamline curvature so one would expect the computed shear-layer width to be too large and, thus, the maximum slope to be too small. In this respect all computations conform with expectations. All three of the

alternative elliptic treatments lead to a considerably steeper velocity profile than with Cartesian-upwind differencing. Because the shear layer does not exactly follow the non-orthogonal mesh shown in Fig. 10 the upwind scheme used in set J will tend to produce a too rapid spread and thus a too low slope to the velocity profile. Certainly the other two elliptic schemes give steeper profiles. In principle the skew-upwind treatment should be free of errors associated with grid-to-flow skewness, but Leschziner (1980) has found for linear problems that "wiggles" can occur with this scheme under certain conditions. It seems at least possible, therefore, that it can generate too steep velocity gradients. The performance of the central-difference scheme adopted for set K is also somewhat uncertain in high Reynolds number flows, the treatment being ordinarily unstable unless used (as here) in a time-marching solution. The parabolic solution, set G, gives results intermediate between set J and sets I and K, though closer to the former.

Table 2

Computations of Case 0331, the Curved Mixing Layer
With the Same (BKEZ) Model of Turbulence

Set	Grid	Convection	Equation set	Coordinates	Relative max. slope of velocity at 90°
G	40 x .02δ δ = local shear layer width	"Upwind"	Parabolic	Weakly non orthogonal normalized stream function as cross stream variable	0.74
H	27 x 31	Upwind	Elliptic	Cartesian	0.50
I	27 x 31	Skew-Upwind	Elliptic	Cartesian	0.81
J	37 x 49	Upwind	Elliptic	General non orthogonal	0.70
K	50 x 50 (61 x 59)	Central	Elliptic	Cartesian	0.85

Differences among the computed results are more pronounced in the shear-stress profiles shown in Fig. 11. At 90° the maximum shear stress generated by the parabolic solution is 80% larger than that for the central-difference elliptic computation. The non-orthogonal elliptic treatment lies between these two and its shear-stress level in relation to that shown for the central-difference elliptic scheme is broadly what would be expected from a comparison of the mean-velocity profiles for the two cases. The extremely high shear-stress levels arising from the parabolic solution naturally raise the question of whether this code adopts a sufficiently complete approximation of the "minor" strains in a highly curved flow. At 60° the peak shear stress is twice as large as that given by the non-orthogonal elliptic scheme; it is difficult to conclude that such a discrepancy arises solely from numerical diffusion in the latter treatment.

Overall, this flow is numerically a more difficult one to compute accurately than Case 0421. If we discard, as we should, the Cartesian upwind results,* the differences among the remaining results may well be unimportant for many engineering purposes. Nevertheless, they do weaken our ability to reach conclusions about the turbulence model. Exactly which is, numerically, the most accurate of the contributed result for this case is not certain. There are two extremes of interpretation that seem possible:

(i) If we accept the boundary layer equations as a satisfactory <u>differential</u> model of the flow development, the odds must be heavily in favor of the parabolic solution being the correct one. The non-orthogonal upwind set would then emerge as the best of the elliptic solutions though somewhat impaired by numerical diffusion.

(ii) If, alternatively, we discount the possibility that numerical errors could cause <u>both</u> the skew-upwind and the fine-grid central-difference treatments to give a too steep velocity profile at 90° (noting that false diffusion generally acts to diminish gradients in dependent variables), one is led to accept the very similar results given by these two schemes as the most accurate. The non-orthogonal elliptic solution would then be seen as suffering fairly severely from false diffusion (though far less so than an upwind treatment with a Cartesian grid) due to the grid being insufficiently well aligned with the flow. One would conclude that the parabolic solution did not include a sufficiently complete representation of the strain field and/or that the assumption of radial equilibrium was inadequate.

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*To be fair to the computor generating the results, we should note they were only created as "cannon fodder."

The computors may be able to distinguish between these scenarios by a more rigorous processing of their output than has been possible merely from a comparison of rather small-scale graphs. It would of course not be unlikely if the truth lay somewhere between these extremes.

The downstream development of the mixing layer after the bend shows a gradual reduction in the differences between the various numerical solutions. Indeed, at $x = 0.556$, even the upwind-Cartesian elliptic treatment gives very similar results to the other schemes. This paradoxical result has arisen from a serious numerical diffusion of turbulence energy earlier in the flow development giving seriously too low levels of energy (and thus too small viscosity levels) at the end of the curved portion. The too small viscosities in turn lead to a too small rate of growth downstream of the bend. This helps to illustrate how an upwind treatment can sometimes fortuitously achieve far better agreement with the correct numerical solution than estimates based on the magnitude of false diffusion would lead one to expect.

4. Conclusions

Satisfactorily grid-dependent solutions for simple two-dimensional elliptic flows with upwind differencing can be obtained using meshes of about 40×40 provided flow lines cut at a narrow angle with the mesh and provided the flow pattern is fairly simple. This is the situation pertaining for Case 0421. Nevertheless there remains a significant amount of variation among the results for this flow whose origin cannot be firmly attributed. There are considerable differences in the prescribed inlet conditions adopted, and this seems likely to have made the greatest contribution. The possible influence of detailed differences in numerical practice has not been eliminated, however. In this respect, it may be significant that at the most downstream position ($X/H = 15.7$), the two independently formulated time-dependent treatments give profiles that stand apart from the other four sets of results all of which emply the SIMPLE algorithm.

In a flow, such as Case 0331, where the streamlines make a large angle with the coordinate lines (if a Cartesian grid is employed), conventional upwind differencing of convective transport cannot be used for accurate numerical work with a practical number of grid nodes. The three alternative elliptic treatments lead to a much slower rate of growth of the shear layer as it develops around a 90° arc—indicating, we believe, a more accurate representation of the convective processes.

For this case there is generally an encouraging degree of conformity among the results from the three preferred elliptic schemes and the parabolic computation. It is not possible to conclude, on present evidence, which solution has achieved the best numerical accuracy. For engineering purposes the differences in the mean field are rather small though one must acknowledge that they certainly cloud the issue of

turbulence modeling. It would probably be informative to retest the codes with the effective viscosity prescribed (perhaps uniform, as for laminar flow) rather than obtained from two turbulence scalars whose levels are themselves found from transport equations and which are thus also liable to numerical dispersion.

The above conclusions are in consonance with and have inevitably been influenced by three recent studies (Han et al., 1981; Leschziner, 1980; Leschziner and Rodi, 1981) on numerical accuracy in the computation of high Reynolds number elliptic flows.

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APPENDIX

THE DIFFERENTIAL FORM OF THE TURBULENCE MODEL EMPLOYED

The kinematic Reynolds stresses $\overline{u_i u_j}$ are obtained from

$$-\overline{u_i u_j} = \nu_T\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\,\delta_{ij}k$$

where the turbulent kinematic viscosity ν_T is given by

$$\nu_T = c_\mu k^2/\varepsilon$$

The two scalar properties of turbulence, k, the turbulence kinetic energy and ε, the energy dissipation rate, are obtained from the solution of the following transport equations:

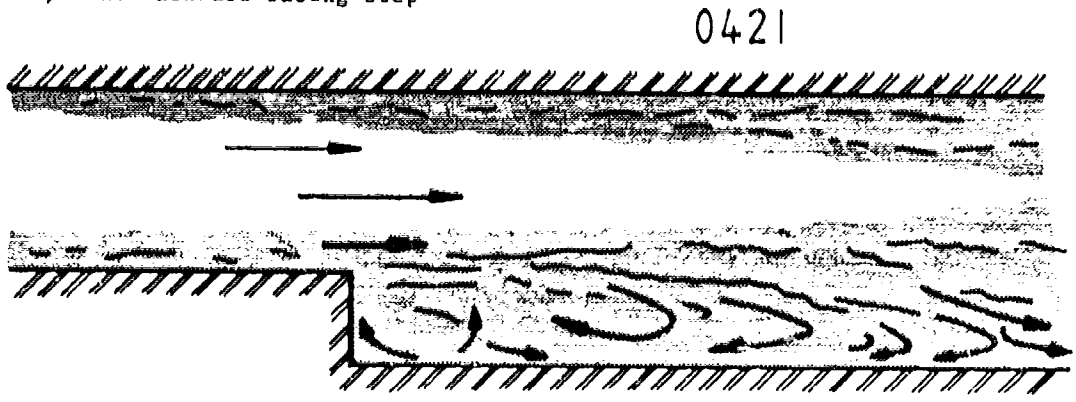
$$U_j\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_T}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + \nu_T\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \varepsilon$$

$$U_j\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + c_{\varepsilon_1}\frac{\nu_T\varepsilon}{k}\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - c_{\varepsilon_2}\frac{\varepsilon^2}{k}$$

The following values are assigned to the coefficients:

c_μ	c_{ε_1}	c_{ε_2}	σ_k	σ_ε
0.09	1.44	1.92	1.0	1.3

a) The backward-facing step



b) The curved mixing layer

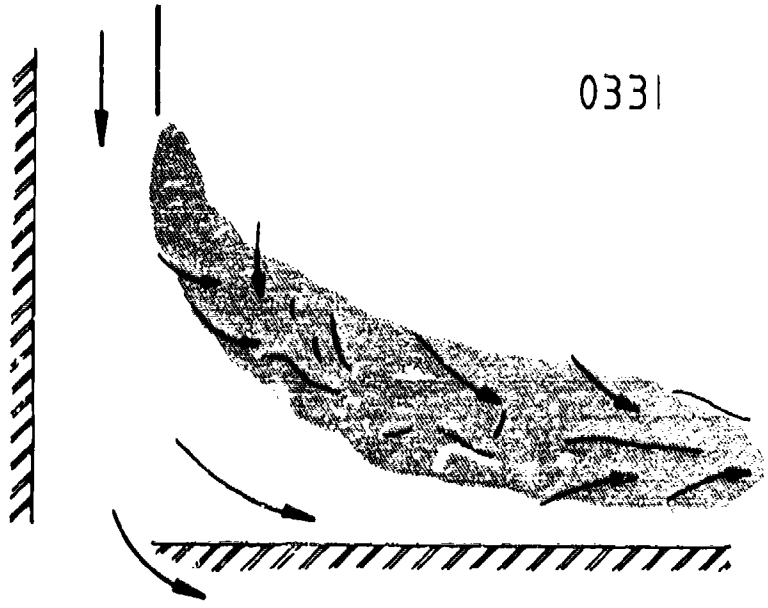


Figure 1. The central test cases.

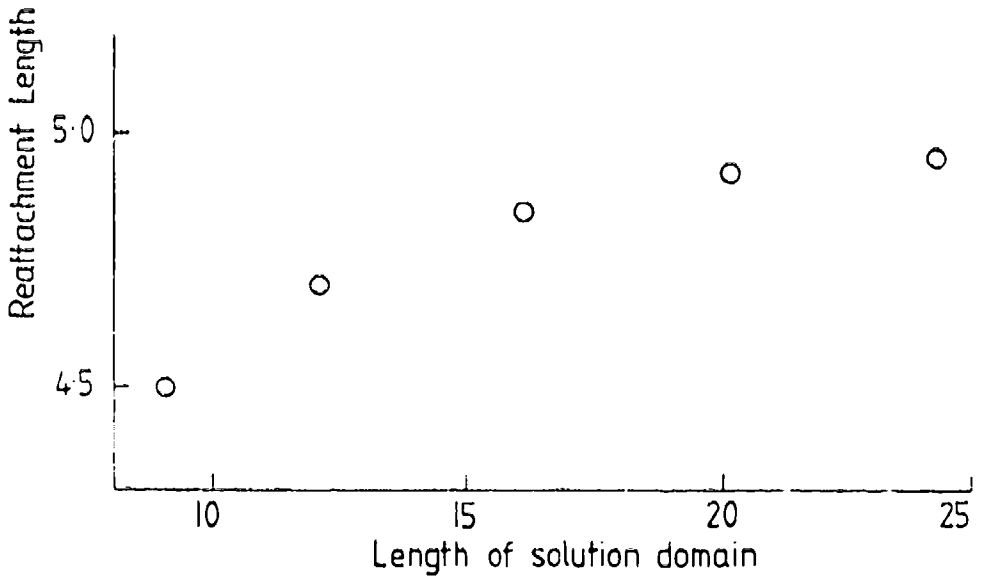


Figure 2. Dependence of reattachment length on length of solution domain.

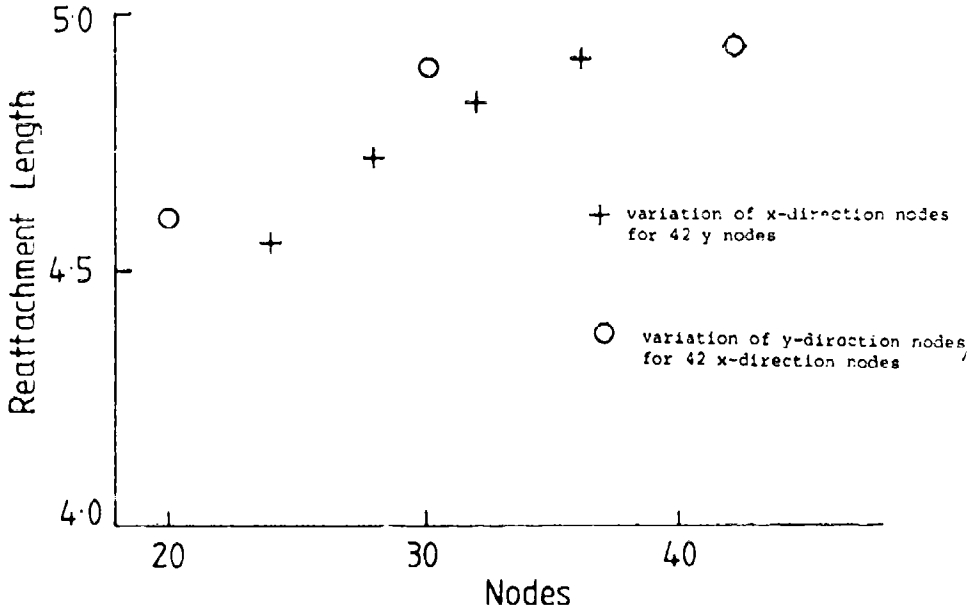


Figure 3. Dependence of reattachment length on number of nodes in x and y directions.

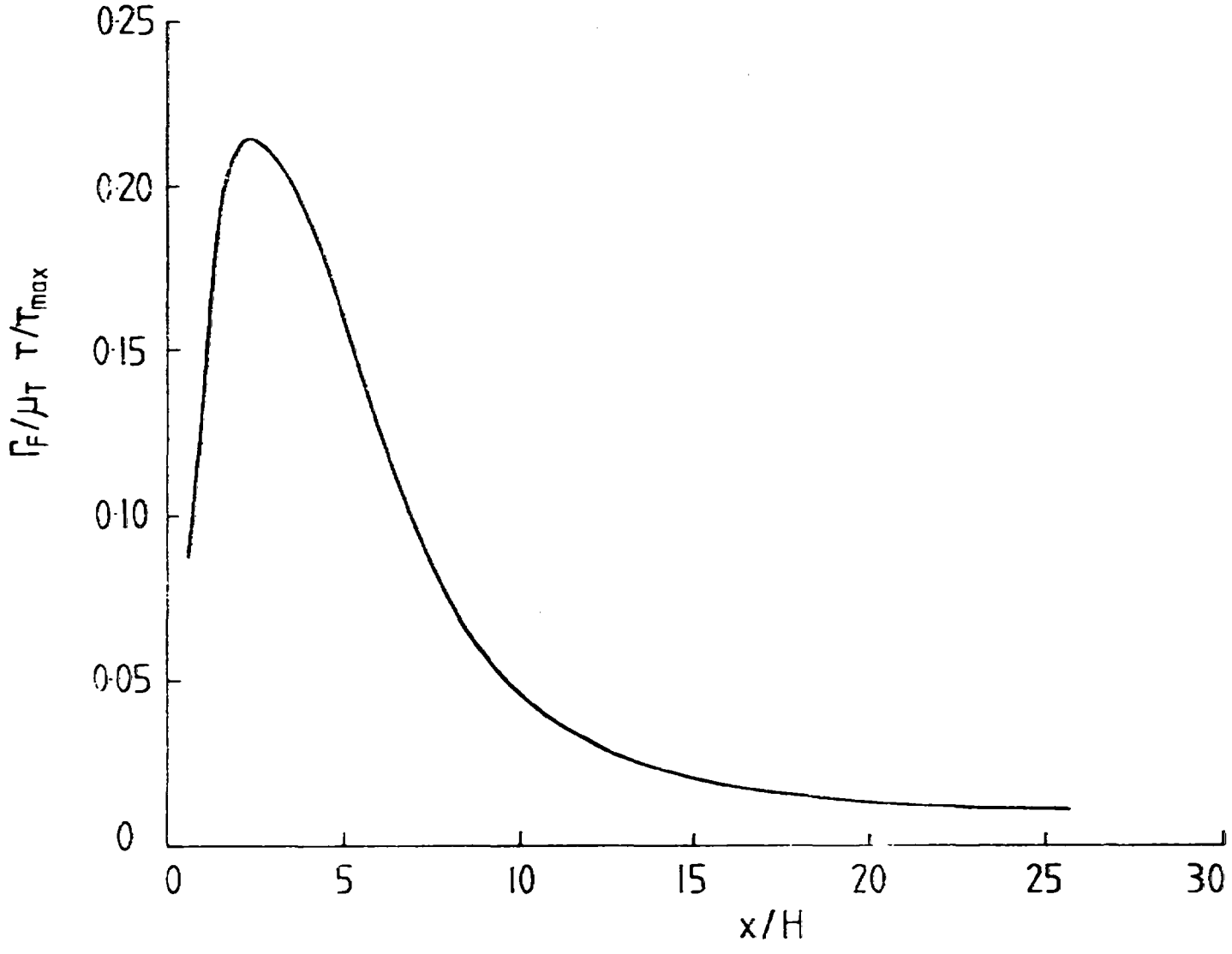


Figure 4. Maximum relative level of false diffusion coefficient at any section (Set B).

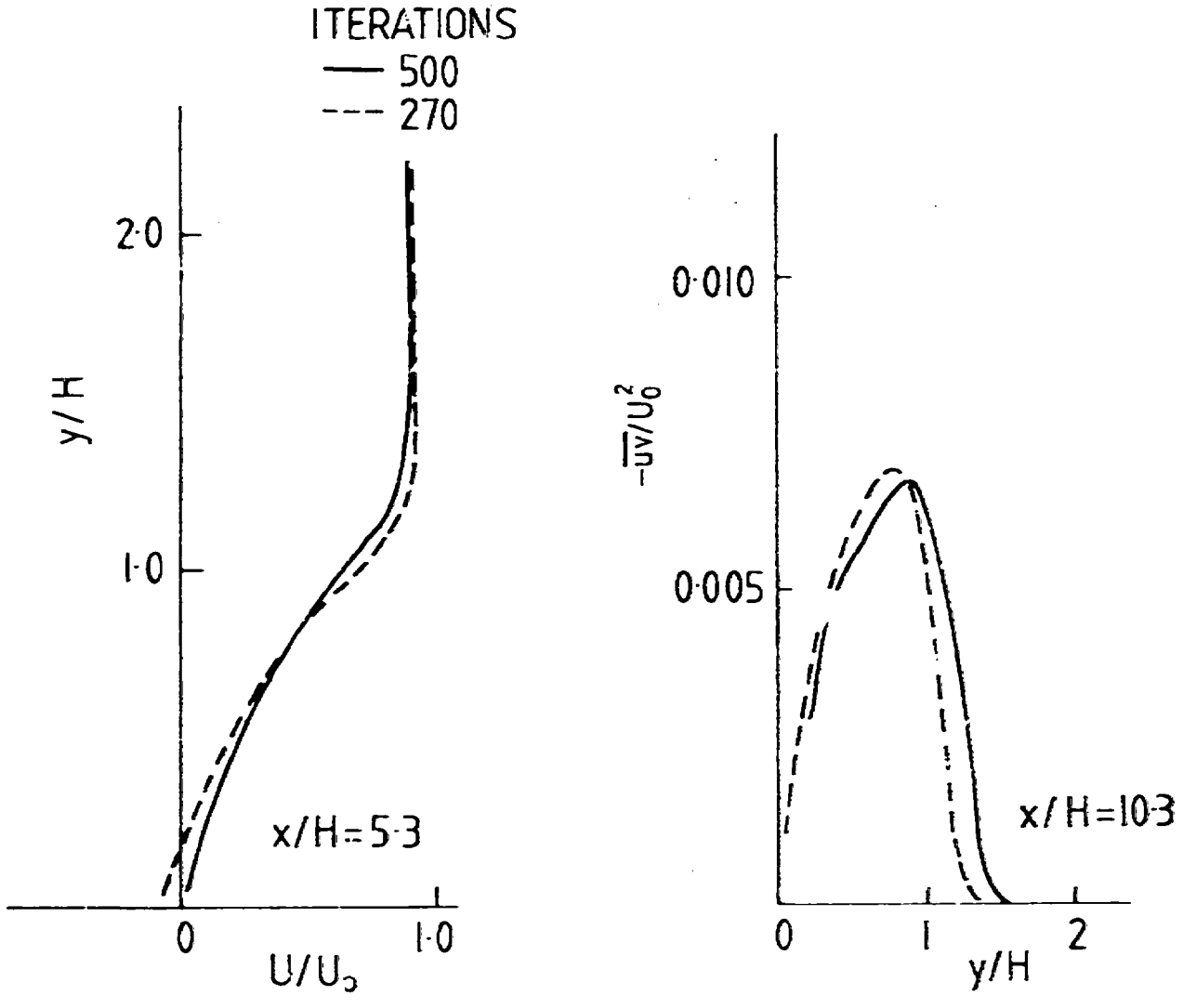


Figure 5. Example of incomplete convergence; a) mean velocity; b) shear stress.

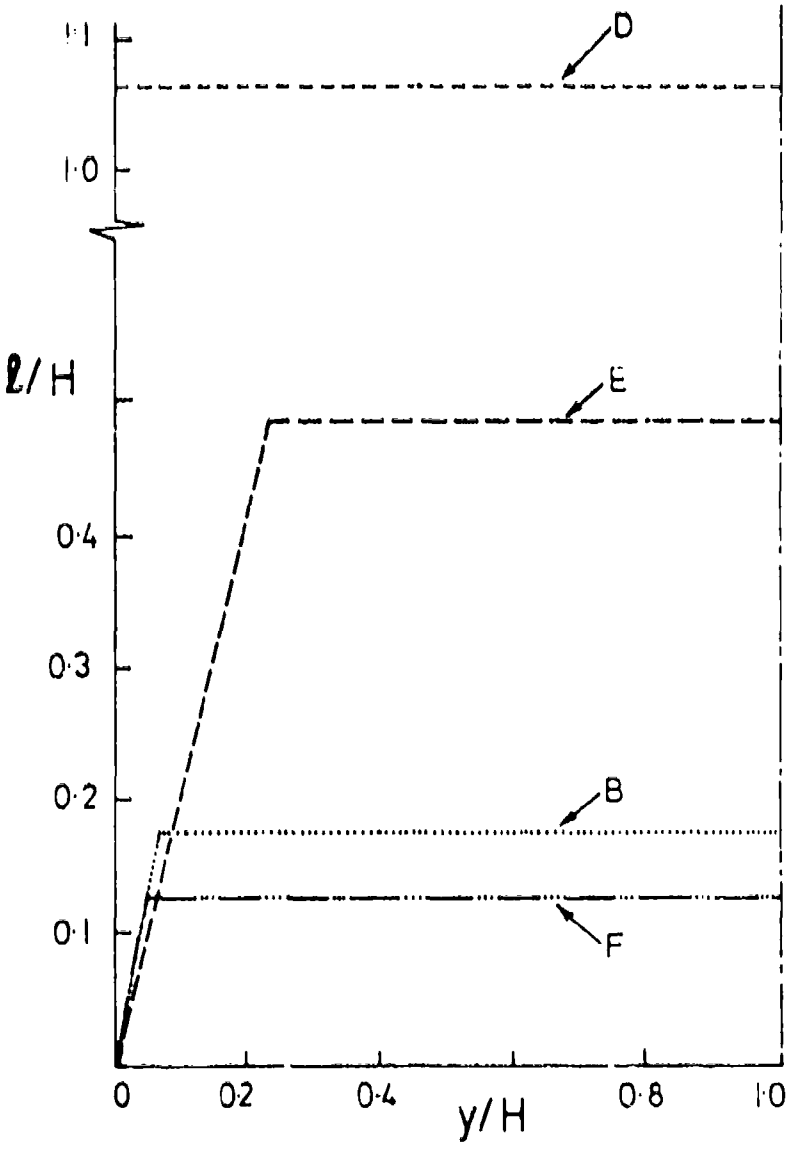


Figure 6. Variation in prescribed inlet length scale.

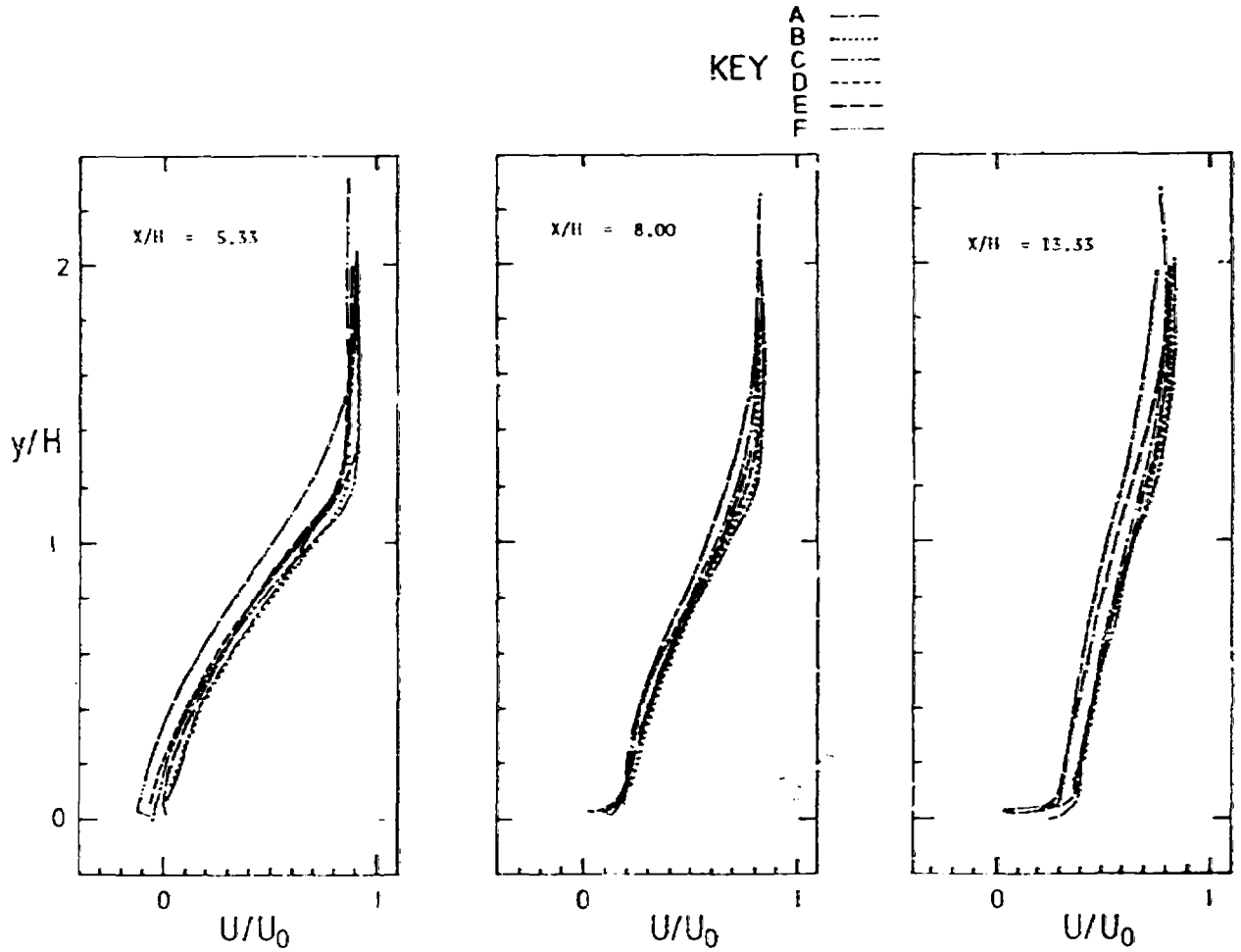


Figure 7. Mean-velocity profiles downstream from back step.

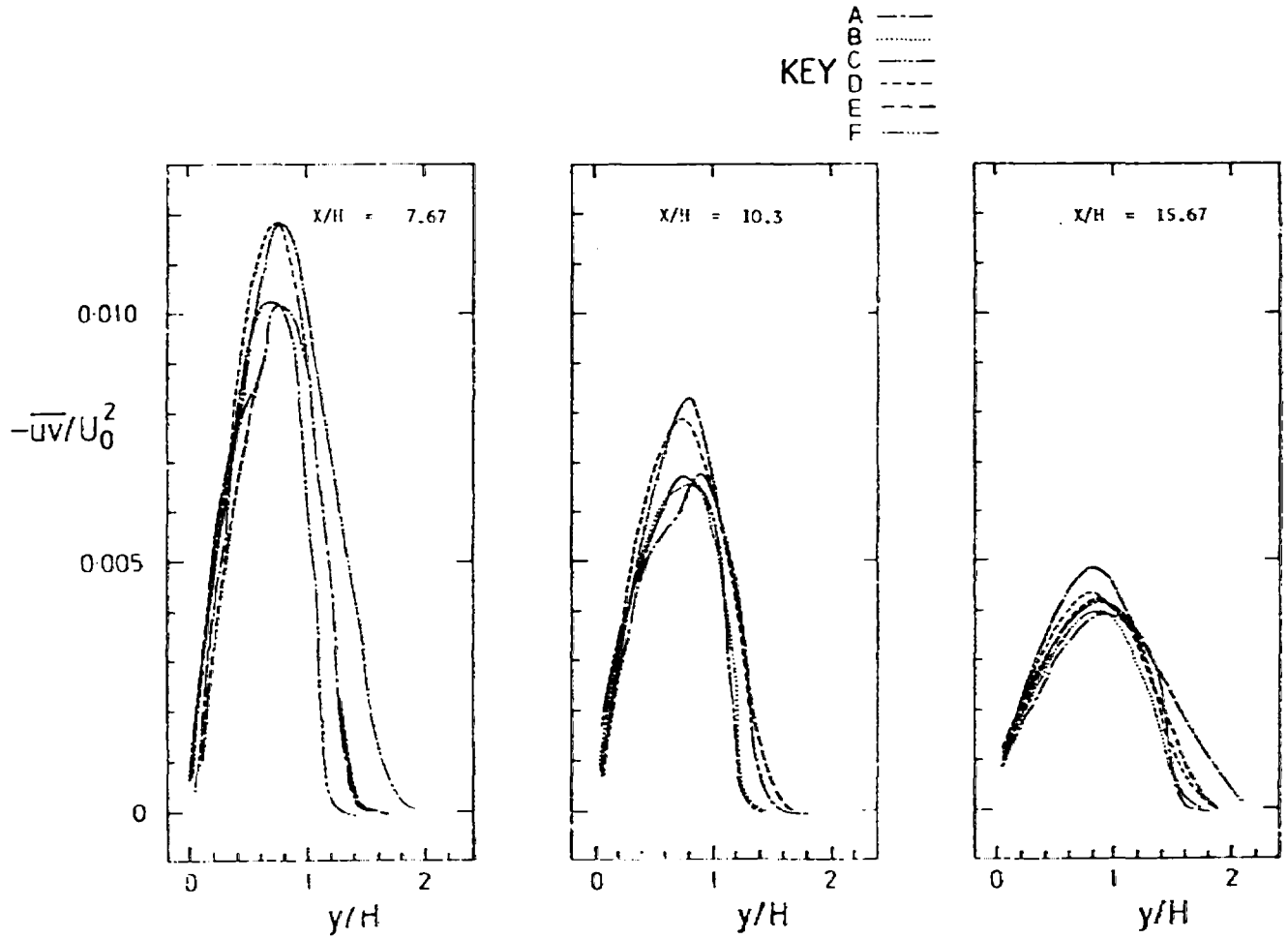


Figure 8. Shear-stress profiles downstream of back step.

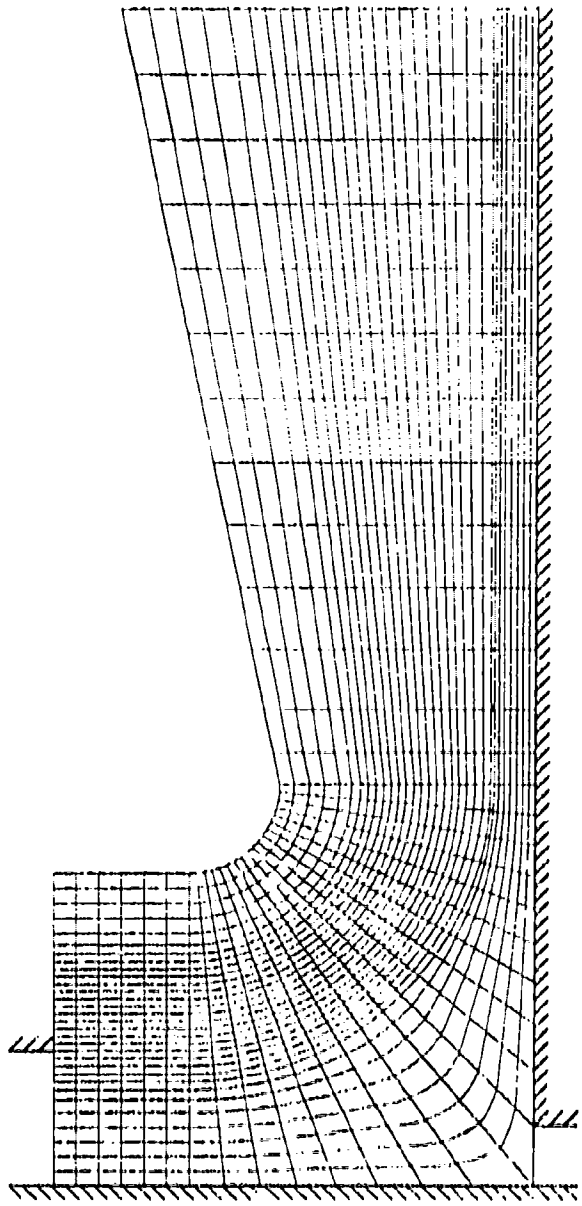


Figure 9. Non-orthogonal grid adopted for Set J.

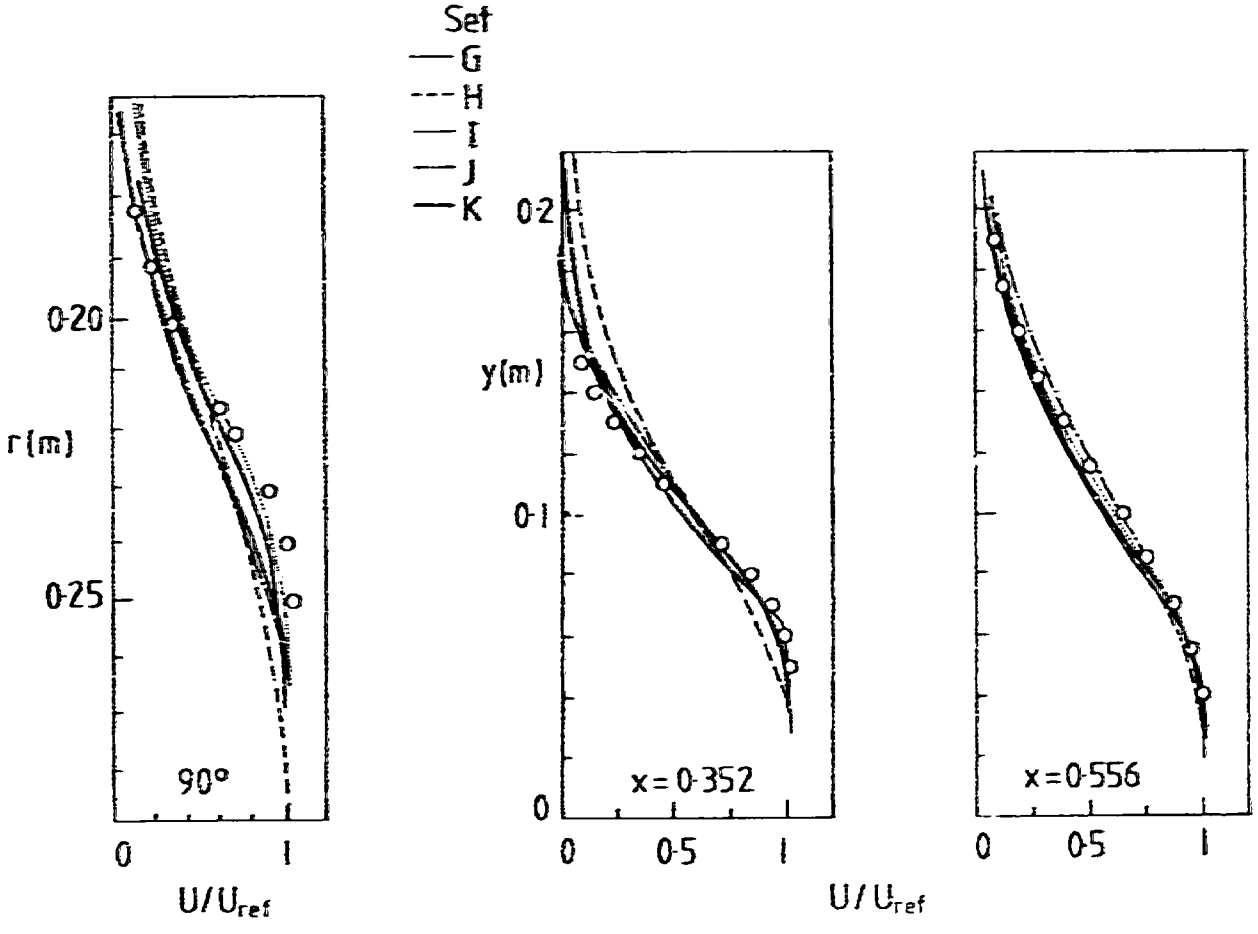


Figure 10. Mean-velocity distributions at end of curved section and downstream therefrom. o - experimental data.

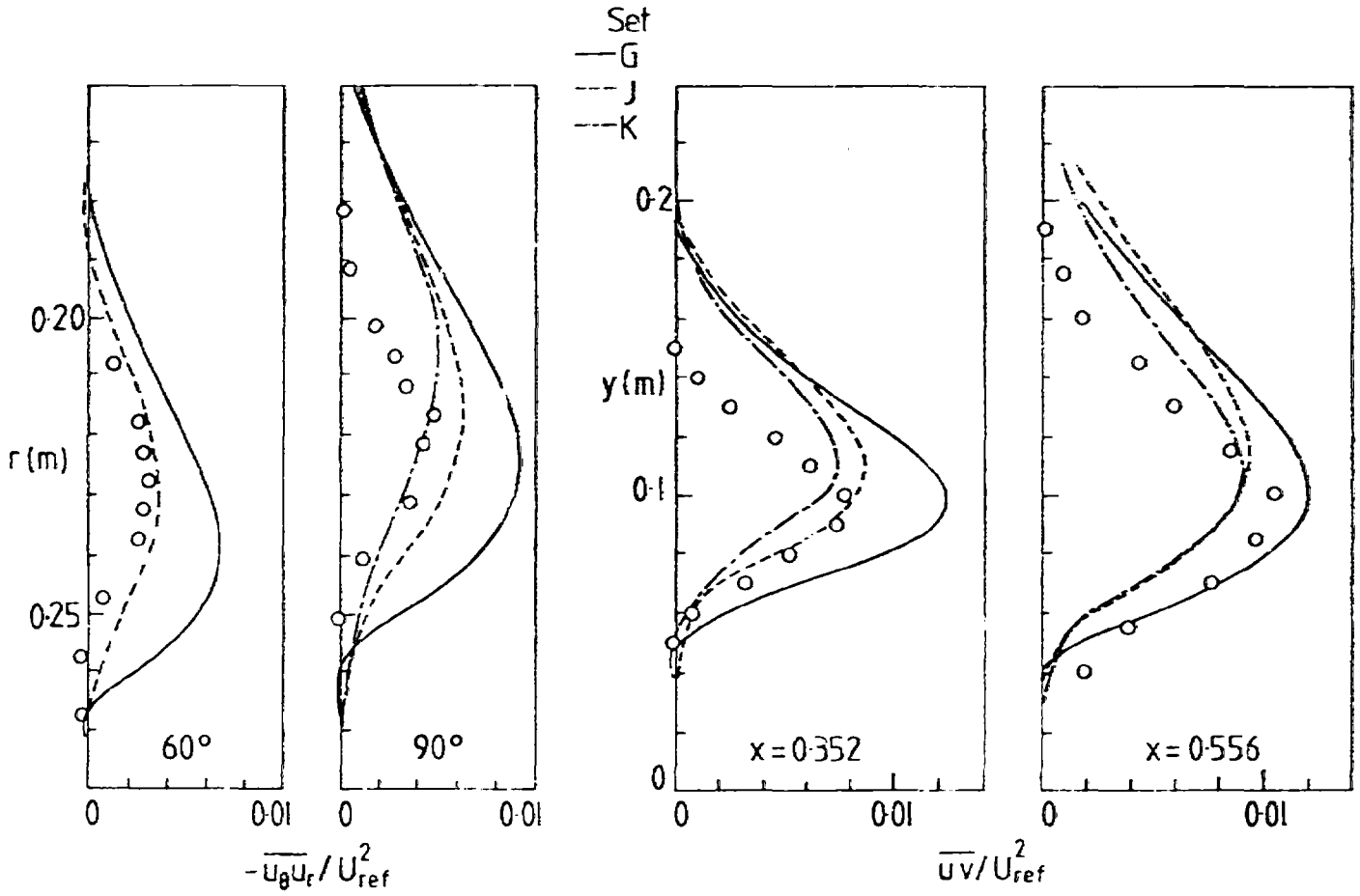


Figure 11. Shear-stress profiles in curved mixing layer.

DIRECT SIMULATION OF HOMOGENEOUS TURBULENCE AT LOW REYNOLDS NUMBERS

By

R. S. Rogallo*

In a set of direct numerical simulations of homogeneous turbulence, the velocity field is represented at every instant by a three-dimensional Fourier series in a coordinate system moving with the spatially linear mean flow. In these coordinates the full Navier-Stokes equations admit spatially periodic solutions with fixed period for all time. The representation of statistical homogeneity in space by strict periodicity requires that the computational period be much longer than any turbulence scale containing significant energy, or in other words, the simulation must provide an adequate statistical sample of the energy containing eddies. On the other hand, the mesh cell must be smaller than the turbulence dissipation scales if the simulation is to be faithful to the Navier-Stokes equations at all scales. Current computing power limits mesh size to about 2 million cells which in turn limits simulations to those cases having a rather small range (roughly one decade) of energetic turbulence scales. A low Reynolds number is usually required to limit the range of scales; this is particularly true for anisotropic cases where the mean strain leads to growth in both length and velocity scales (and hence Reynolds number) of the turbulence. For these cases the simulation breaks down when the growing range of scales can no longer be contained on the mesh, and this can occur at either the large scales (inadequate statistical sample) or the small scales (inadequate numerical resolution).

Results of direct simulations of several anisotropic turbulent fields are given in Rogallo (1981). The mean flows considered are (1) plane strain, (2) axisymmetric strain, (3) uniform shear, and (4) uniform rotation. Due to the mesh limitations, total strains were limited to 4 or less and total shears (t dU/dy) to 10 or so. Only qualitative agreement with experiment could be achieved for the strain cases due to lack of measured detail in the experiments, particularly length scales, but quantitative agreement was obtained for the more important shear flow where accurate and detailed experimental data are available. In the very interesting case of uniform rotation the comparison with experiment is hindered by a simulation input error resulting in a very low Reynolds number, and by the rather large anisotropy of the grid-generated turbulence of the experiment. Ironically, while this is the easiest case to simulate because rotation inhibits the energy cascade, and thus limits the range of scales, it is the most difficult case to handle experimentally.

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The primary results of the simulations for a variety of the turbulence fields are tabulated and provide a data base for the design and testing of models for some of the terms in the time-averaged Navier-Stokes equations. These data are used to compare measured values with values given by the Rotta and the Launder-Reece-Rodi models for terms in the Reynolds-stress equations for homogeneous turbulence.

This study represents the first attempt, known to the author, to provide fairly detailed computational results for anisotropic turbulence to the turbulence-modeling community. Even though the future will bring larger and faster computers, and more productive ways of utilizing them in the study of turbulence, there is already much to learn from the simulated turbulence fields in hand.

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LARGE-SCALE NUMERICAL SIMULATION OF WALL-BOUNDED TURBULENT SHEAR FLOWS

By

P. Moin and J. Kim*

In recent years the large-eddy simulation (LES) technique has shown considerable promise for the calculation of turbulent flows. With LES one obtains the large-scale turbulent eddies from direct numerical integration of the filtered three-dimensional time-dependent Navier-Stokes equations, whereas the small scale flow field is modeled. For a given flow, the dividing line between the small- and large-scale flow fields is determined by the computational grid resolution. When a large number of grid points is used, a large fraction of turbulent eddies is directly calculated and hence a smaller fraction has to be modeled. The portion of turbulence that is modeled has scales that are of the order of the computational grid volume which is small compared to the overall dimensions of the flow field. This is in contrast to phenomenological turbulence modeling where statistical correlations involving all of the turbulence are modeled; and to the direct numerical simulation approach where <u>all</u> the turbulence scales are resolved on the computational grid.

Since, in LES, the large eddies are obtained directly from the governing flow equations, in a typical calculation most of the information about the turbulence energy, the structure, and time-dependent characteristics of the flow are retained rather than lost in the averaging process. In addition, unlike the large eddies, small eddies are much more amenable to general modeling because they transport relatively little turbulent momentum and energy and tend to be more isotropic. Furthermore, in contrast to the direct numerical simulation approach, in LES one is not restricted to low Reynolds number flows. These considerations provide important inducements for the development of LES.

In the numerical simulation of homogeneous turbulent flows and free-shear flows one can generally obtain the computer power necessary to resolve the important large eddies. This is because these eddies do not vary greatly in size throughout the flow domain. In addition, the overall characteristics of the large eddy structures in these flows are rather insensitive to the Reynolds number. On the other hand, numerical simulation of wall-bounded turbulent flows imposes a great demand on computer speed and memory. Near the wall, <u>local</u> large eddies are typically several orders of magnitude smaller than those away from the wall. More importantly, these relatively small but locally big eddies near the wall, are responsible for most of the turbulent

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energy production for the entire flow field (Kim et al., 1971). Therefore, for a successful simulation, one has to be able to resolve all the large eddies in the neighborhood of the wall and at a distance from the wall. If this requirement is not met, due to computer limitations, the subgrid-scale (SGS) eddies take on a more active role and the choice of SGS model becomes more critical.

In the direction normal to the walls, one can distribute the grid points with non-uniform spacings. More points can be placed near the wall to resolve the viscous sublayer and the buffer layer. The real difficulty arises in the neighborhood of the wall due to the fine spacing of the streaks (Kline et al., 1967) in the spanwise, z, direction. The size of the computational box in the z-direction should be large enough that the artificialities of the boundary conditions do not influence the statistics of the flow field in an undesirable way. At the same time the computational grid resolution should be fine enough to resolve the streaks. For high Reynolds number flows ($Re > 10^5$) these requirements together with the similar (but less severe) considerations for the number of grid points in the streamwise and vertical directions place a demand on the required number of grid points ($> 2 \times 10^6$) that is beyond the capabilities of the present computers. However, for low Reynolds number flows such simulations are within reach of the presently available super computers. It should be emphasized that unlike the direct numerical simulation approach, in LES no attempt is made to resolve the Kolmogoroff length scale, η. For the case of plane Poiseuille flow, for example, η can be expressed in the wall units as

$$\eta^+ = \left(Re_\tau \frac{u_\tau}{U_m}\right)^{1/4}$$

where Re_τ is the Reynolds number based on the channel half width, δ, and shear velocity, u_τ, and U_m is the average mean velocity. For moderate Reynolds numbers ($Re = U_m\delta/\nu \approx 10^4$), the number of grid points required to resolve η^+ in all the space dimensions is about four orders of magnitude greater than that required to resolve the important large eddies throughout the flow field.

One approach for reducing the total number of grid points required for adequate resolution of sublayer and outerlayer large eddies is the use of grid embedding technique. Here, one would place a large number of grid points only in the vicinity of the walls and relatively few grid points in the outer region. The immediate problem with this approach lies in the numerical difficulties associated with its implementation. Another approach is that of Deardorff (1970) and Schumann and co-workers (see for example Grotzbach and Schumann, 1979), in which the dynamics of the flow in the vicinity of the walls is altogether neglected. The calculations are carried out only to a point in the logarithmic layer where semi-empirical boundary conditions are used to represent the near-wall turbulence. This approach is attractive in that at

high Reynolds numbers the number of grid points required is drastically reduced. Furthermore, its numerical implementation is relatively straightforward. With a relatively modest number of grid points, Grotzbach and Schumann have applied this method to simulate turbulent flows in channels and annuli. They also included such effects as heat transfer, buoyancy, and wall roughness. There are two disadvantages with this method. First, by ignoring the flow in the vicinity of the walls, one cannot use this approach as a tool for studying the dynamics of wall turbulence and its interaction with the outer layers. Second, the degree of universality of the near-wall boundary conditions and their effects on the outer solution is not yet fully explored. If such boundary conditions can indeed be constructed with a reasonable degree of generality, then this approach can be of considerable practical value for the numerical simulation of high Reynolds number wall-bounded turbulent flows.

Recently Moin and Kim (1981) simulated fully developed turbulent channel flow at Reynolds number 13,800 based on the centerline velocity and channel half-width. These calculations were carried out on the ILLIAC IV computer with up to 64 × 64 × 128 grid points. An eddy-viscosity model was used to relate the subgrid scale stresses to the strain rate of the large-scale velocity field. The computational domain included the solid boundaries where no-slip boundary conditions were used. Some of the calculations reported in Moin and Kim (1981) have required over 50 hours of ILLIAC IV time to complete. Therefore, it goes without saying that generally, these types of calculations are not intended for direct utilization as a design tool. However, the amount of information that can be generated from one such calculation is enormous. In addition to instantaneous three-dimensional velocity and pressure field, the computer program output for time-averaged quantities consists of: profiles of the mean velocity, turbulent intensities, turbulent shear stress, skewness and flatness factors; all the terms appearing in the governing equations for the resolvable turbulent intensities and Reynolds stress including pressure-strain correlations; rms vorticity and pressure fluctuations, one-dimensional energy spectra and two-point correlation functions. The overall agreement of the computed turbulence statistics with experimental data was good. The computed flow field was also used to study the detailed time-dependent structure of the flow field. There is excellent qualitative agreement between the computed time-dependent flow structure and those observed in the laboratory. These results are reported in Moin and Kim (1981) and a selection of them were presented during this Conference.

To date, all the numerical simulations of wall-bounded turbulent flows have dealt with flows that are homogeneous in two space dimensions (the direction of inhomogeneity is normal to the wall(s)). There are two main mathematical and computational advantages associated with the calculation of this limited but important class of flows. In the direction in which the flow is homogeneous, one can use periodic

boundary conditions. This choice alleviates the need to prescribe the difficult and largely unknown inflow and outflow boundary conditions. Secondly, in order to obtain sufficient statistical samples of various turbulent quantities one can average these quantities over the dimension(s) in which the flow is homogeneous. The spatial averaging reduces the required averaging time and thus provides for substantial savings in computer cost. It is, therefore, important to emphasize that even for the same number of grid points, the flows with only one homogeneous direction (all the flows that are two-dimensional in the mean) require much more computer time than those with two. However, with relatively minor modifications of the existing computer programs a great deal can be learned about some of the complex turbulent flows. For example, one can introduce uniform blowing through one wall of the channel and uniform suction through the other. Effects of system rotation and imposed periodic oscillations can be included by adding a term to the governing equations being solved.

As mentioned earlier, it is expensive to perform large-eddy simulation of wall-bounded turbulent shear flows. However, it is also true that considerable information about the flow can be generated by one such calculation. Indeed, LES is a powerful research tool in that it provides access to the resolvable portion of velocity and pressure field at thousands of spatial locations, thus establishing a supplementary data base to experimental measurements. It can also furnish considerable guidance to phenomenological turbulence modeling especially with modeling of quantities such as pressure-strain correlations and dissipation tensor that are not easily measured. Moreover, in certain practical problems, such as noise generation and flow over compliant boundaries, LES is perhaps the only predictive computational tool available.

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DISCUSSION

SESSION IX

Launder--Backward-Facing Step (Case 0421)

Results of computed reattachment length for Computors A-E are 5.5 ± 0.3. These differences are too small to draw any significant conclusion with regard to variations in the computors' choice of numerical method. The results of Computor F, however, significantly disagree with those of the other computors. Computor F has expressed concern over the calculated pressure distribution obtained, and considers his results not definitive for the purposes of this comparison.

As a conclusion, therefore, because of the agreement of Computors A-E, turbulence modeling can proceed without significant concern for sensitivity of the results on the choice of numerical method.

It should be stressed, however, that there exists a significant difference between computed reattachment lengths (5.5 ± 0.3) and those obtained experimentally (7.5 ± 0.5).

Launder--Curved Mixing Layer (Case 0331)

In contrast to the previous flow, numerical differences do provide sufficient deviation so that one cannot distinguish between effects due to the turbulence model and those due to numerical schemes. In other words, the influence of numerical methods on the computed results was found to be significant in this strongly curved flow.

Rogallo--Direct Simulation of Homogeneous Turbulence

The results of this simulation should be very important to turbulence modelers. For example, it was found that:

- Dissipation terms are much more anisotropic than expected for the shear case.
- Pressure-strain terms produce anisotropy in the pure rotation case.
- The results strongly suggest that length scales and turbulence energy grow exponentially.

The discussors strongly recommend that these results be considered as a valuable addition to the data bank.

Moin and Kim--Large-Eddy Simulation Study of Channel Flow--New Results

The calculations were carried out for moderately high Reynolds numbers and include the effects of solid boundaries. The results can be used to guide the development of turbulence models. In particular, in the near-wall region, the calculations indicate a net transfer of energy from the vertical component of the turbulent kinetic energy, to the horizontal component. Therefore, modelers should consider inclusion of

this effect in their models for the pressure-strain correlations. Moreover, the calculations have demonstrated that they can be effectively used to study the detailed structure of the flow field. The computations can be further used to relate this structure to statistical correlations.

The discussors advocate that these results be used to supplement the laboratory data.*

General

The NASA-Ames Research Center was commended for providing significant support over the past several years for the numerical simulation of turbulent flows. Efforts should be made to continue these important studies by further investments in manpower and hardware.

Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

U. Mehta asked what the group meant by having the numerics "under control." E. Reshotko replied that since calculations A through E of the backstep flow gave results for the reattachment length that lay in a band distinctly separate from the possible range of experimental data, the numerics could be separated from the effects of the turbulence model. P. Roache pointed out that four of the five calculations used essentially similar numerics and that what surprised him, therefore, was the range of answers that they gave.

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*[Ed.: It needs to be strongly emphasized that the presenters of both the complete simulations and the LES studies are very clear on what these methods will and will not do. The presenters see the methods as important research tools that have and will produce important information about turbulence from study of simple cases; some of this information cannot be measured with currently available instrumentation. The presenters also see both methods are extremely demanding of computer storage space and run time; hence, they will not adapt for use as engineering tools at least for some time to come. This denial of potential for direct use as an engineering design tool contradicts implications (which appear at a number of other points in this volume) by Computors who have not actually worked with direct simulation of LES methods.]

Chairman: J. P. Johnston

Technical Recorders:

R. Luxton
P. Eibeck

Technical Reporters

W. J. McCroskey
Case 0441

J. K. Eaton
Cases 0421, 0422, 0423. 0424, and 0431

INCOMPRESSIBLE SEPARATED FLOWS

Case 0441 -- External Flow

Cases 0421, 0431 -- Internal Flows

Predictive Cases 0422 (P2), 0423 (P3), 0424 (P4) -- Internal Flows

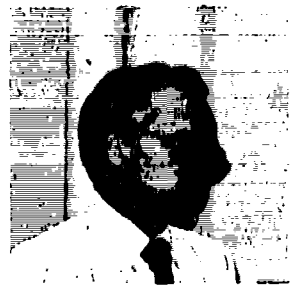
PICTORIAL SUMMARY

Flow 0440. Data Evaluator: A. Wadcock. "Two-Dimensional Stalled Airfoil."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0441 D. Coles A. Wadcock	 $0.62 \leq x/c \leq 2.00$ $\Delta x/c = 0.0111$	Mean surface pressure	125	125	125	125	-	125	Intermittency	-	1.5×10^6 (based on airfoil chord)	Free stream turb. level < 1.5%	Aspect ratio = 2. Probe interference present for attached boundary layer data—separated zone and wake is interference-free.

Method Used—Session X
(External Flow)

Method	Case 0441
BKEZ	●
ML	●
SG	●
BIKX	●
EE	●
Plate No. Vol. III	114-116



INCOMPRESSIBLE SEPARATED FLOW--EXTERNAL FLOW
Two-Dimensional Stalled Airfoil — Flow Case 0441

Technical Report

by

W. J. McCroskey*

INTRODUCTION

This case concerns the low-speed flow over an airfoil at high incidence. The primary data were obtained by the so-called flying hot wire technique (Coles et al., 1978) on an NACA 4412 airfoil model mounted between plane parallel side plates in the GALCIT 3.1 m diameter subsonic wind tunnel (Coles and Wadcock, 1979; Wadcock, 1979). The chord and span of the model were 0.90 m and 1.99 m, respectively.

The test conditions, corresponding to maximum lift on the airfoil, were the following:

$$\alpha = 13.9°$$

$$M = 0.07$$

$$Re = 1.5 \times 10^6$$

$$C_L = 1.67$$

$$C_D = 0.072$$

For these conditions, the boundary layer on the upper surface separated between $x/c = 0.75$ and 0.80. Measurements with the flying hot wire x-array probe started at $x/c = 0.62$ and extended one chord length downstream of the trailing edge. In addition, the static pressure distribution was measured on the upper and lower surfaces of the airfoil.

GENERAL FEATURES OF THE FLOW

Figure 1 shows the measured pressure distribution and, for reference, the predictions of inviscid theory. The comparison of these two results illustrates the relative importance of viscous and inviscid effects for this flow.

Figure 2 compares the measurements with the prediction of a simple viscous correction method for attached boundary layers, due to Pinkerton (1936). The comparison of these two results shows the relative importance of separation in modifying the pressure distribution near the trailing edge.

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*NASA Ames Research Center, Moffett Field, CA 94035.

Figures 3 and 4 show the measured velocity and Reynolds-stress profiles at four stations: (1) ahead of separation, (2) within the separated zone on the airfoil, (3) in the near wake, and (4) approximately one chord length downstream of the tailing edge. According to the evaluator (Waddock, 1981),

> "The main conclusion to be drawn from the Reynolds-stress data is that the separation process is relatively regular up to the trailing edge of the airfoil. The real challenge is to understand the merging process for the two shear layers just downstream of the trailing edge and the subsequent rapid relaxation toward the final state of a conventional wake far downstream."

CALCULATION METHODS

Five computors submitted results for this case. As indicated below, the techniques that were employed varied from a simple vortex method which ignored all of the structure of the turbulence, to a time-averaged Navier-Stokes code with a two-equation k-ε model.

- ML 46 - integral boundary layer for attached flow; semi-empirical vortex model for the wake.
- EE 27 - integral boundary layer (energy integral) zonal model on the airfoil; calculations not extended into the wake.
- SG 42 - integral boundary layer (entrainment with shear-stress lag) zonal model; α_{max} limited to 13.6°.
- BIKX 41 - Boussinesq one-equation model with damped ϵ_T in the Reynolds-averaged Navier-Stokes equations.
- BKEZ 23 - Boussinesq two-equation k-ε model with law-of-the-wall in the Reynolds-averaged Navier-Stokes equations.

RESULTS AND DISCUSSION

The computed pressure distributions are compared with the experimental results in Fig. 5. In this global comparison, some computors achieved good agreement in the leading-edge region but not near the trailing edge, whereas the ones who predicted the correct trailing-edge levels failed to get the right leading-edge suction peak.

Figure 6 shows the trailing-edge region in more detail. With the exception of method BKEZ 23, none of the calculations show the correct pressure difference between the upper and lower surfaces of the airfoil.

The mean-velocity profiles are shown in Fig. 7. No results were provided by computor ML 46, nor by computor EE 27 downstream of the trailing edge. With the exception of BKEZ 23, the results shown are in reasonable agreement with the measurements for the first three stations. However, all results show excessive velocity defects at $x/c = 1.95$, and the wake position according to BKEZ 23 is incorrect.

Figure 8 compares the three sets of computed Reynolds-stress profiles with the measurements. A discrepancy at $x/c = 1.17$ is evident, but otherwise the agreement is satisfactory. It may be mentioned that whereas the Reynolds stresses according to BKEZ 23 at the important stations $x/c = 0.91$ and 1.17 agreed fairly well with experiment, the calculations of $\overline{u^2}$ (Fig. 9) and $\overline{v^2}$ (not shown) did not.

CONCLUDING REMARKS

The precision of maximum lift on an airfoil has long been considered a major challenge in applied fluid dynamics. Therefore, it is gratifying to note that five sets of computed results, employing a wide range of different methodologies, agree fairly well with each other and with the experiment. In this connection, it is interesting to note that in discussions following the presentation of the results, none of the computors felt that turbulence modeling was a limiting factor for this flow.

Nevertheless, it seems evident that all of the methods, and the experiment for that matter, could and should be improved upon. Whether due to problems with the inviscid part of the solution, the viscous-inviscid coupling, spatial resolution with the grid employed, or something else, no computor correctly predicted the measured pressure distribution in both the leading- and trailing-edge regions. Furthermore, as alluded to in advance by the evaluator (Wadcock, 1981), each of the three computors that attempted to calculate the flow in the viscous wake encountered difficulties in the near wake. Of course, the other two provided no information there.

The important issue of drag prediction was addressed only by computor Group 42, and his value of C_D was about half the experimental value. In addition to its practical significance, C_D is probably a useful and sensitive criterion for assessing the prediction of the interaction of the viscous region with the outer flow.

In future comparisons of this type, it would be worthwhile to compute the flow at angles of attack somewhat above and below the angle for maximum lift. Experimental information for these conditions would also be desirable.

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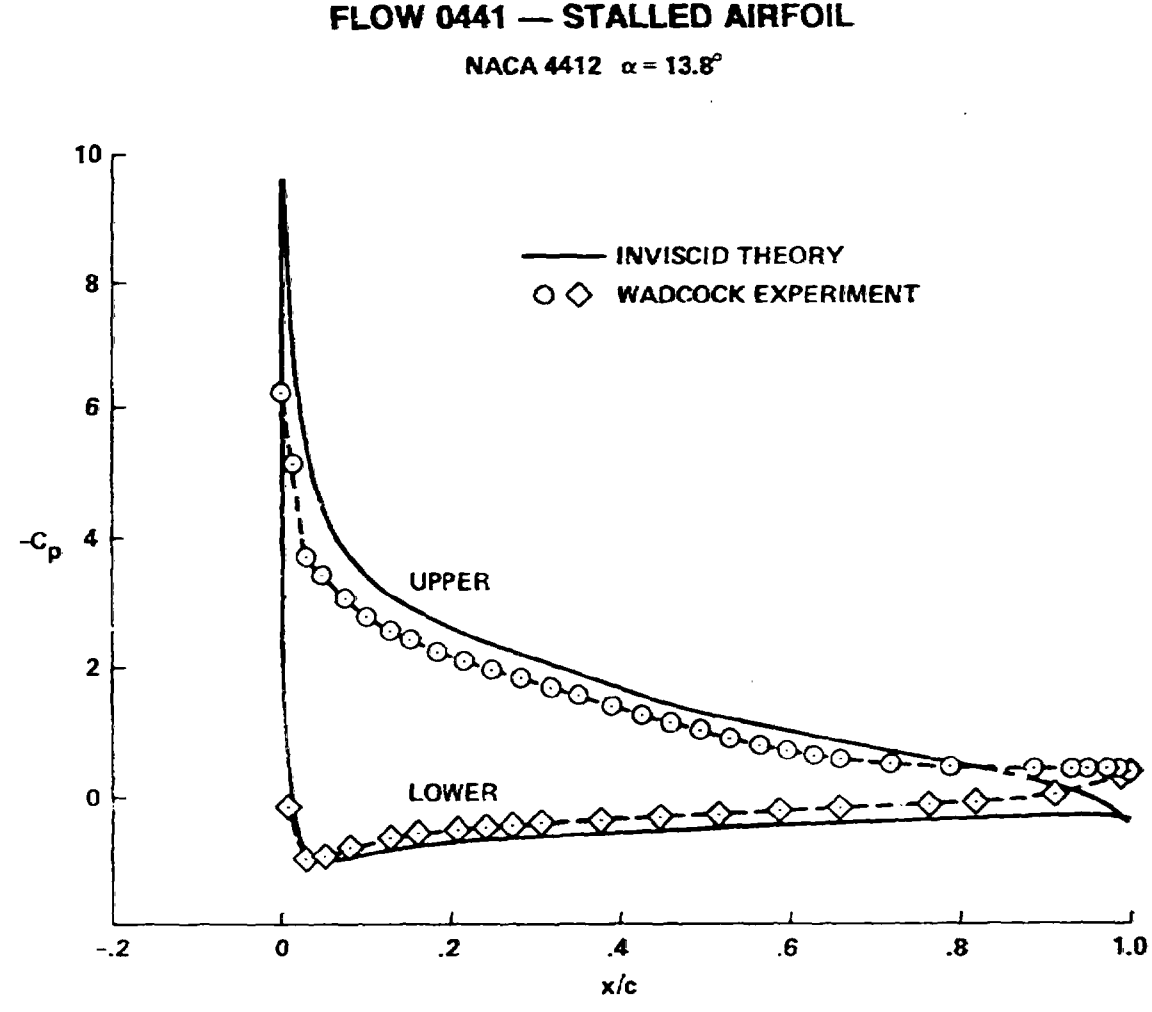

FLOW 0441 — STALLED AIRFOIL
NACA 4412 α = 13.8°
INVISCID THEORY
WADCOCK EXPERIMENT
UPPER
LOWER
$-C_p$
10
8
6
4
2
0
-.2
0
.2
.4
.6
.8
1.0
x/c

Figure 1.

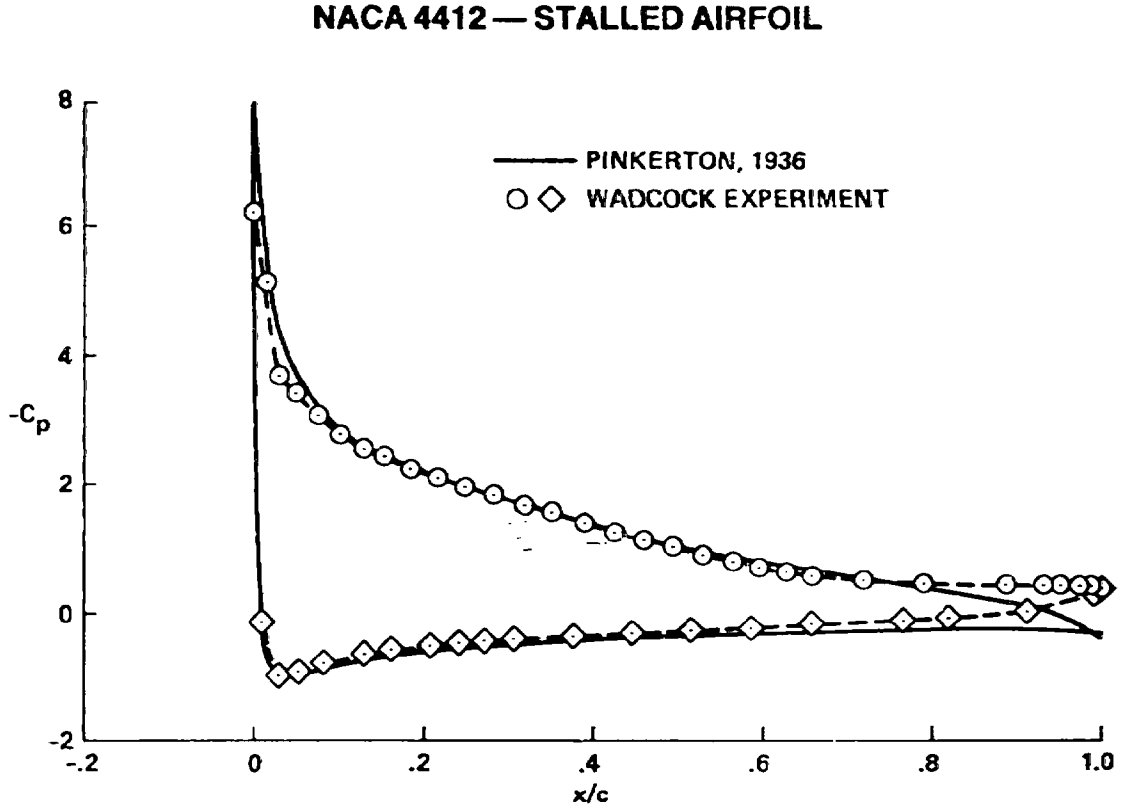

NACA 4412 — STALLED AIRFOIL
PINKERTON, 1936
WADCOCK EXPERIMENT
$-C_p$
8
6
4
2
0
-2
-.2
0
.2
.4
.6
.8
1.0
x/c

Figure 2.

FLOW 0441 — STALLED AIRFOIL

MEAN VELOCITY PROFILES

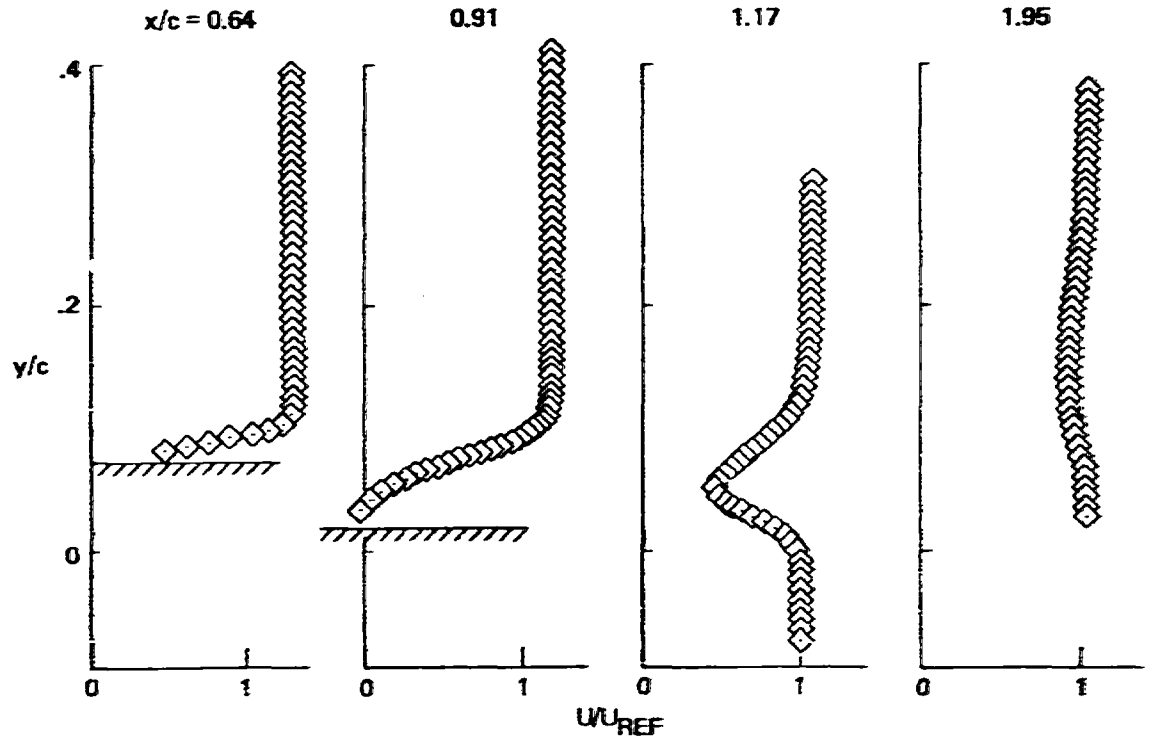


Figure 3.

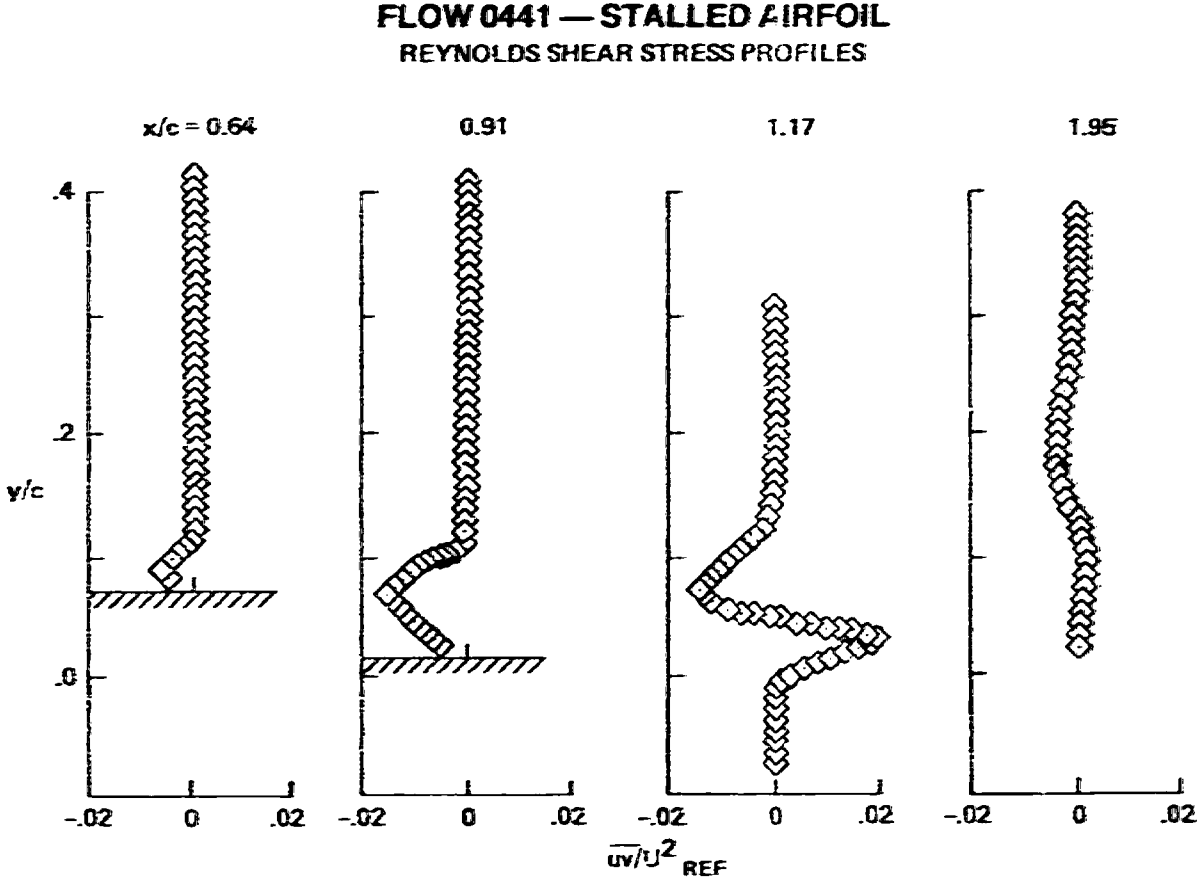

FLOW 0441 — STALLED AIRFOIL
REYNOLDS SHEAR STRESS PROFILES
x/c = 0.64
0.91
1.17
1.95
.4
.2
.0
y/c
-.02
0
.02
$\overline{uv}/U^2_{REF}$

Figure 4.

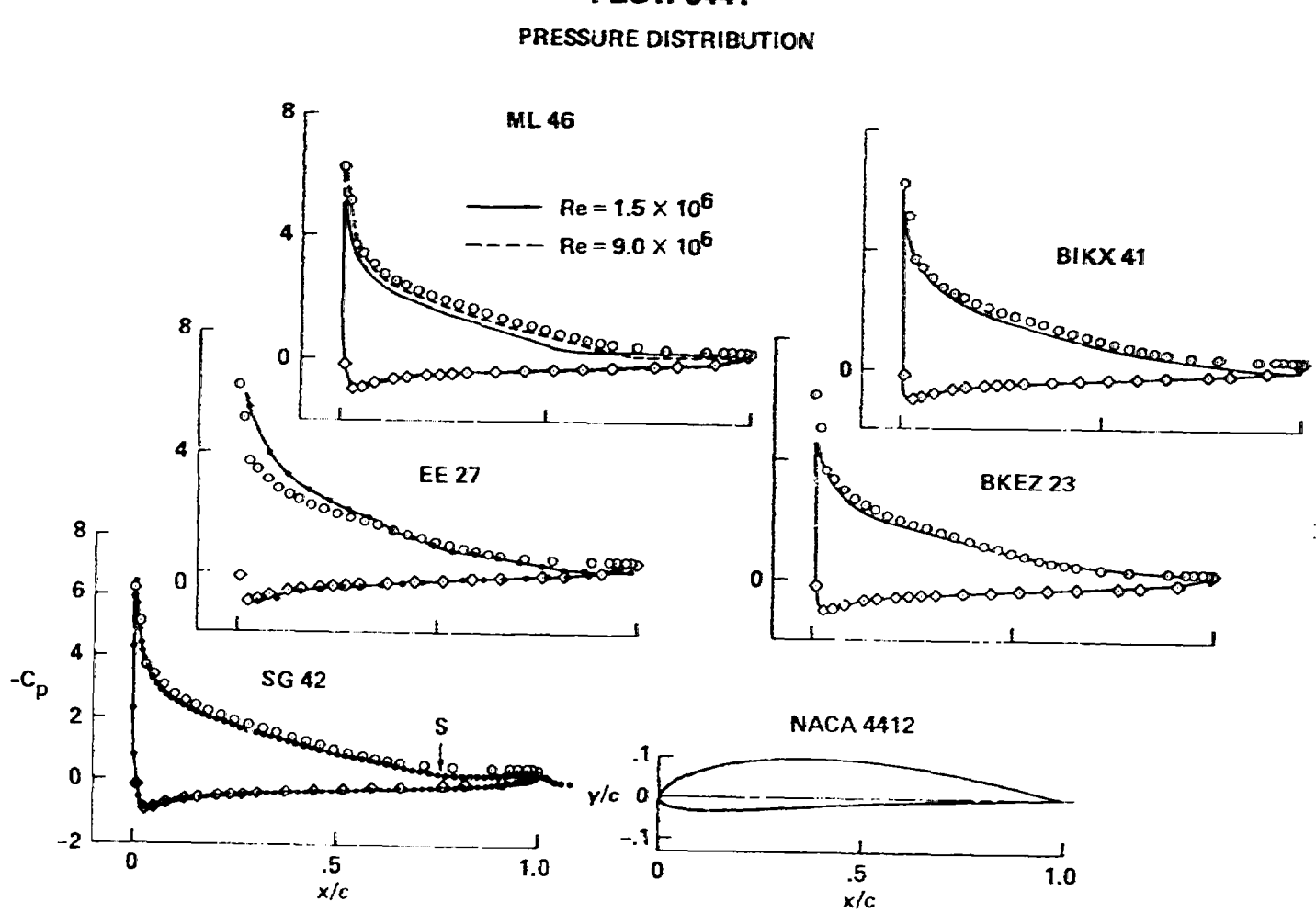

FLOW 0441
PRESSURE DISTRIBUTION
ML 46
Re = 1.5 X 10^6
Re = 9.0 X 10^6
BIKX 41
EE 27
BKEZ 23
SG 42
S
NACA 4412
-Cp
x/c
y/c

Figure 5.

STALLED AIRFOIL

TRAILING EDGE PRESSURE DISTRIBUTION

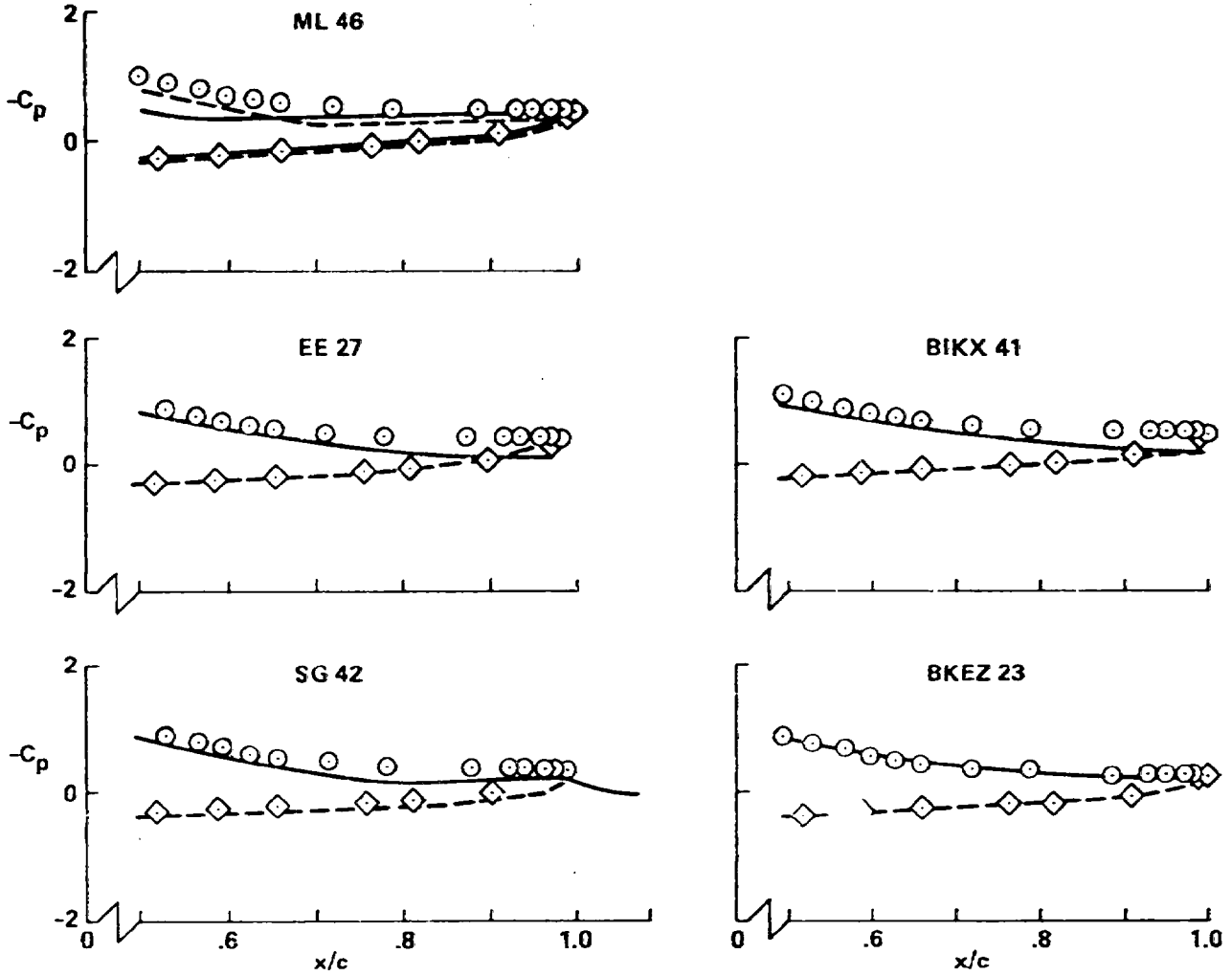


Figure 6.

MEAN VELOCITY PROFILES

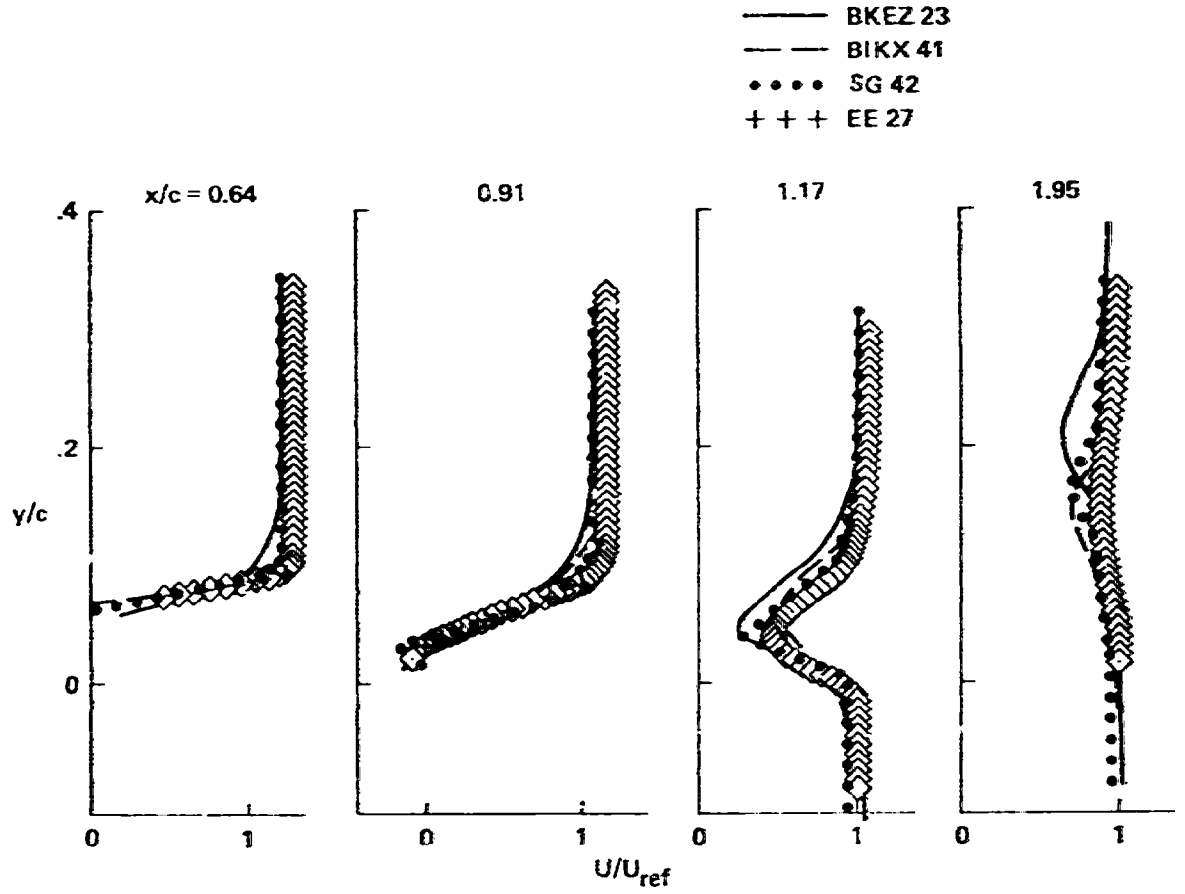


Figure 7.

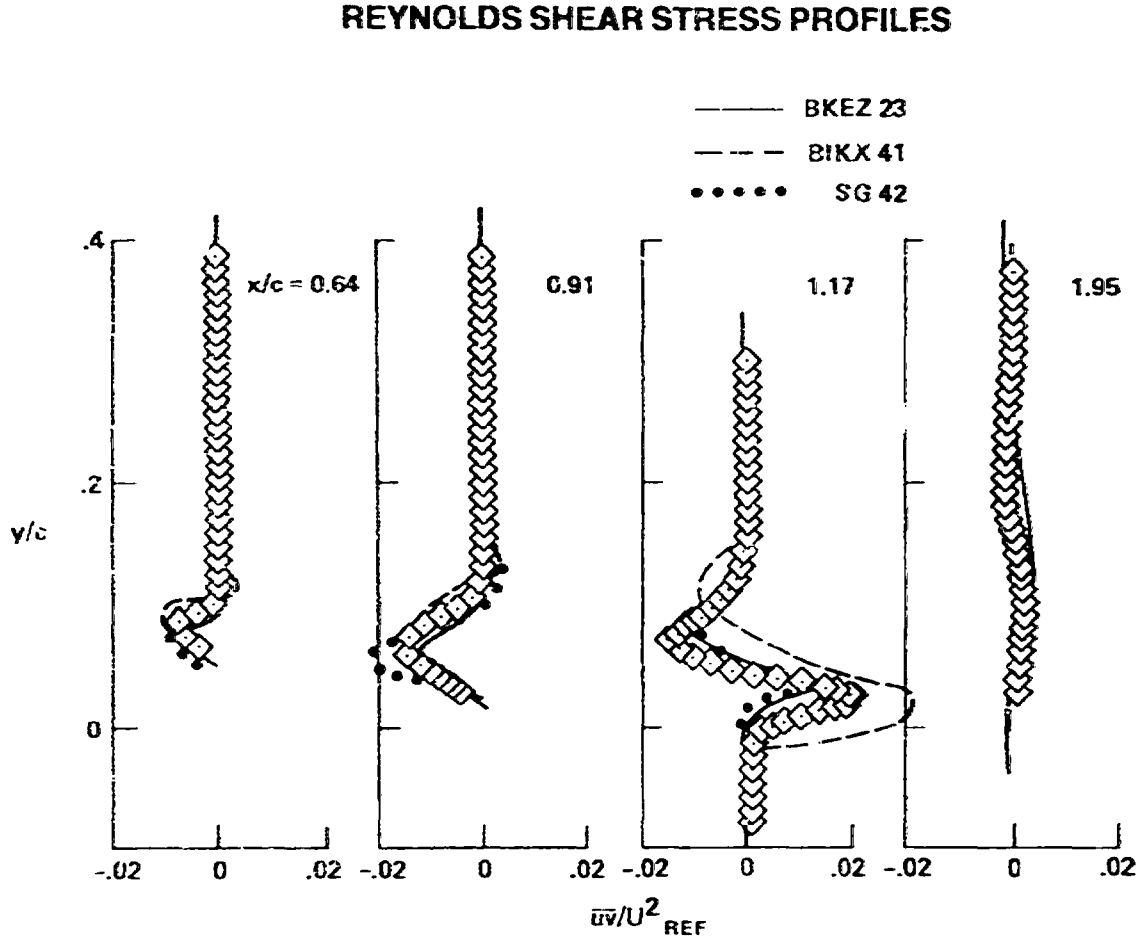

REYNOLDS SHEAR STRESS PROFILES
BKEZ 23
BIKX 41
SG 42
x/c = 0.64
0.91
1.17
1.95
.4
.2
0
y/c
-.02
0
.02
$\overline{uv}/U^2_{REF}$

Figure 8.

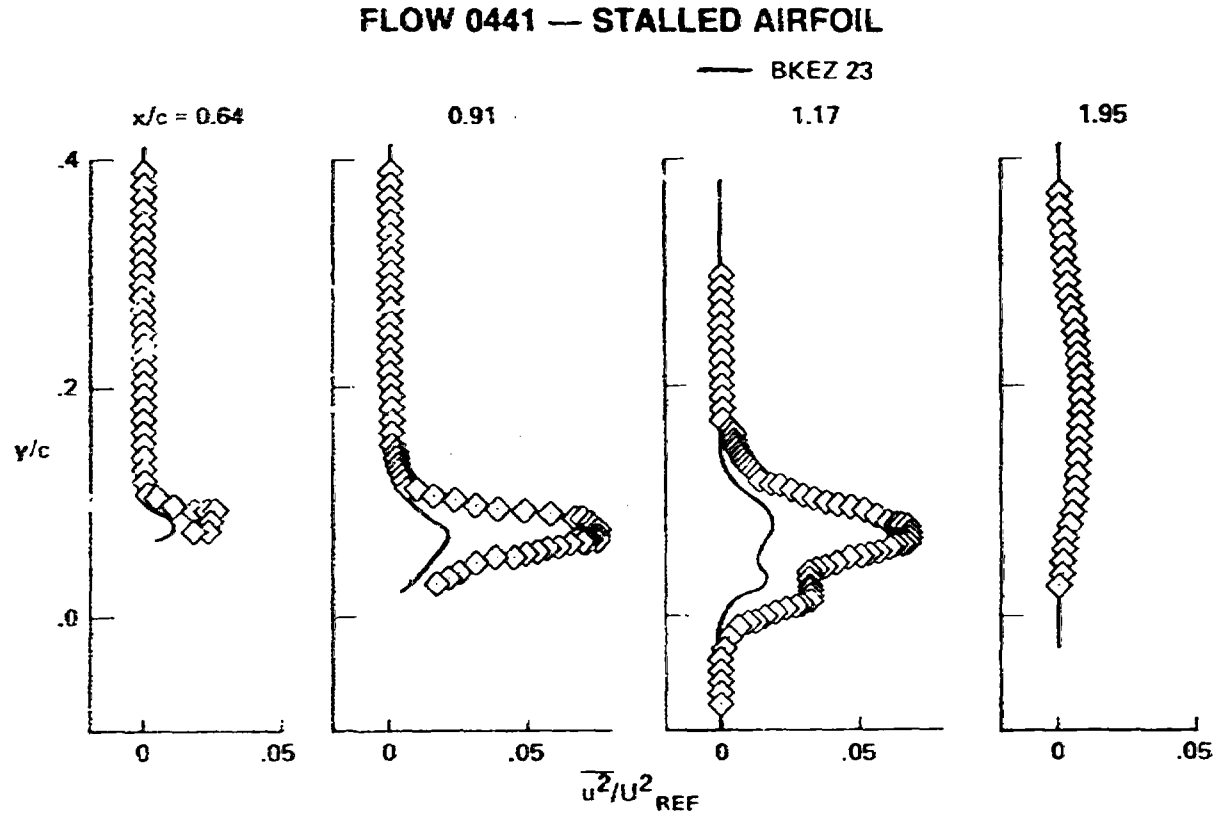
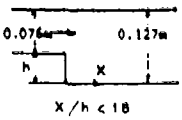

FLOW 0441 — STALLED AIRFOIL
BKEZ 23
x/c = 0.64
0.91
1.17
1.95
.4
.2
.0
y/c
0
.05
$\overline{u^2}/U^2_{REF}$

Figure 9.

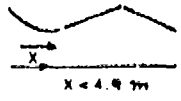
PICTORIAL SUMMARY

Flow 0420. Data Evaluators: J. Eaton and J. Johnston. "Backward-Facing Step Flow."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0421 J. Kim S. Kline J. Johnston		Wall pressure	13	-	6	6	-	6	Intermittancy	Reattachment length about 7h	4.5×10^4 (based on h)	δ/h = 0.3	Aspect ratio 16. Data inadequate in zones of high turbulence and oscillating flow. Laser data needed. Reattached layer well documented. Pressure on upper and lower walls available.

PICTORIAL SUMMARY

Flow 0430. Data Evaluator: R. Simpson. "Diffuser Flow (Separated)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	Initial Condition	Other Notes
Case 0431 R. Simpson T. Chew B. Shivaprasad		< 0 upstream of throat	21[a]	-	21[a]	11[a]	6[a]	11[a]		Preston tube	1.07×10^6 per m	Turbulence intensity 0.1%	Flatness, skewness, and fraction of time flow moves downstream is available. Two-dimensionality was checked. Position of an inviscid streamline near the upper wall is given.
		> 0 downstream	11[†]	11[†]	11[†]	11[†]	4[†]	11[†]	Spectra of u	Surface hot-wire anemometer			

[a] Hot-wire measurement.

[†] Laser Doppler measurement.

Editors' Comment

Cases 0422 (P2), 0423 (P3), and 0424 (P4) are predictive cases which are not described in the Honami-Cantwell pictorial summary here or in Vol. I. The conditions and geometries of these cases are given in context below.

Methods Used--Session X
(Internal Flow)

Method	Case 0421	0422 (P2)	0423 (P3)	0424 (P4)	0431
RSLZ	•	•			
BKEY	•				
AKEZ	•			•	
BKEZ	•	•	•	•	
BOLX	•				
BILX	•			•	•
BIKX	•			•	
BKWY	•				
BOPX	•				•
D5QZ	•				•
EE	•				•
HN					•
Plate No. Vol. III	101-108	180-183	184-186	- †	109-113

†No plates were prepared for Case 0424 (P4) (see Footnote #).

INCOMPRESSIBLE SEPARATED FLOWS--INTERNAL FLOWS

Backward-Facing Step

Cases 0421, 0431; Predictive Cases 0422 (P2), 0423 (P3), 0424 (P4)‡

Technical Report

by

J. K. Eaton*

Four test cases were classified as separated internal flows. All four were nominally two-dimensional flows in enclosed passages which contained large regions of reversed flows. It was expected that these cases would require elliptic codes for adequate calculation of the flow.

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‡Groups 17 and 30 submitted preliminary results for Case 0424 (P4) but these were not presented to the Conference, since the experimental data for comparison purposes were not available. (The Conference policy was not to present any results where data for comparison are not available.)

The four cases were a separated diffuser flow (Case 0431), a backward-facing-step flow (Case 0421), and two modified backward-facing-step flows (Cases 0422 and 0423). The latter two cases were "predictive" test cases, in the sense that data were not revealed to anyone but the present author prior to the 1981 Conference, thus computations were blind or truly "predictive."

Case 0431

Case 0431 is a separated diffuser flow. The data were taken by Simpson, Chew, and Shivaprasad at Southern Methodist University, primarily using Laser-Doppler anemometry. The flow is a turbulent boundary layer which separates under an imposed adverse-pressure gradient (see Fig. 1 and pictorial summary above). The flow remains separated until the end of the tunnel, a fact that caused some questions to be raised about the usefulness of the data. However, most computors indicated that their calculation results were independent of the details of the boundary condition specification at the exit plane.

The case was computed by six groups using eight different methods, including three integral methods and five differential methods. The skin-friction variation was reasonably well predicted by all of the methods (see Fig. 2).* Two of the integral methods (Groups 27 and 45) gave excellent agreement with the skin-friction data. Group 30 used both an inverse boundary-layer code and a viscous-inviscid interaction scheme with the same turbulence model. The viscous-inviscid interaction scheme did appreciably better in computing the skin friction.

The separation location was reported for four of the computations (see Table 1). The calculated separation location was upstream of the measured separation point in all but one case.

Table 1
Separation Location

Group	Method	Separation Location
Data	———	3.4
14	D5Q%	3.28
28	BOPX Parabolic	3.2
28	BOPX Elliptic	3.2
45	HN	3.47

The γ_p data† indicate that separation ($\overline{C_f} = 0$) occurs at approximately $x = 3.4$ m. The measured mean-velocity profiles also indicate that separation occurs between

*The calculations of Group 28 were left off Fig. 2 because their normalization was different from that of the other computations.

†γ_p is the function of the time the flow is moving forward near the wall. At separation $\gamma_p = 0.5$.

x = 3.4 and 3.5 m. The integral method specifically adapted to this type of flow seems to do the best job of predicting the separation location.

The mean-velocity profiles upstream of separation are well predicted by all Groups except Group 28. It seems that almost all methods are capable of calculating an attached boundary layer accurately. Within the separated-flow region, two of the integral methods (EE 27, HN 45) continue to calculate the profiles accurately (see Fig. 3). However, the profile family of Group 28 apparently cannot represent reversed flow. The differential methods do a slightly poorer job in the separated-flow region.

Both Groups 28 and 30 use eddy-viscosity-type models, yet get drastically different results. It is doubtful that the difference can be attributed to numerics, since Groups 28 used both elliptic and parabolic codes and obtained similar results. On the other hand, the results of Group 14, who used a five-equation differential model, are quite similar to the results of Group 30. Both are in reasonably good agreement with the data. The significant difference between Group 30 and Group 28 may be in the wall treatment. Group 28's calculation is very poor in the near-wall region. The reverse-flow region appears to be much too thin.

Reynolds shear stresses were computed by only three of the groups (the differential methods). The five-equation model (D5QX 14) gave excellent results upstream of separation, while the two eddy-viscosity models gave reasonable results (see Fig. 4). Downstream of separation, the five-equation model (D5QX 14) continued to do a good job, while the two eddy-viscosity methods (BOPX 28, BILX 30) failed to predict the correct shear-stress levels. The shear stress grew far too rapidly in Group 28's computation. The shear-stress computations of Groups 14 and 30 were quite similar. In both cases the peak shear stress was significantly underpredicted. Despite the fact that the shear stress is underpredicted, the growth rate of the shear layer is overpredicted by Groups 14 and 30. The mean-velocity profiles show that by x/H = 3.97 the computed free-shear layer is significantly thicker than the data indicate.

Case 0421

Case 0421 is a flow through a two-dimensional, single-sided sudden expansion (see Fig. 5). The data were collected by Kim, Kline, and Johnston of Stanford University. The flow was computed by eleven different groups using a total of 15 different methods. Nine of the methods used a Boussinesq (eddy-viscosity) model for the Reynolds stresses. Of these, seven methods were variations of the k-ε model of turbulence. The other two used a mixing-length model and a k-ℓ model. Three of the methods used algebraic stress models, one used a differential model (D5QZ) and one used a full Reynolds-stress transport model. Finally, there was a single computation of the flow using an integral method.

The single most important parameter to compute in a backward-facing-step flow is the reattachment length. Figure 6 shows the computed reattachment length for each of the 15 methods. The Boussinesq methods with k-ε turbulence modeling all underpredict the reattachment length by about 20%. This is also true of one of the algebraic stress models and the full Reynolds-stress closure. Possible reasons for the consistent underprediction of the reattachment length will be discussed later. The reattachment length was overpredicted by the five-equation model (D5QZ 14) and the one integral method (EE 27). The latter method obtains quite poor agreement with all of the measured parameters, so it would not be expected to calculate the reattachment length correctly.

One of the Boussinesq models which uses a mixing-length formulation (BILX 30) predicted the reattachment length quite well. The "standard" algebraic stress model employed by Group 17C obtained results similar to the Boussinesq models. However, the algebraic stress model AKEZ 17E employing a modified ε equation in which $C_\mu \frac{k^2}{\varepsilon} (\partial U_i/\partial x_j)^2$ replaces $\frac{P_{kk}}{2}$ does an excellent job of calculating the reattachment length. The computors of Group 17 felt that the modified ε equation correctly accounted for the stabilizing effects of shear-layer curvature, resulting in an accurate prediction of reattachment length.

A band of results is shown for Group 44, because three different reattachment lengths were reported. The longest reported reattachment length is close to the measured value. However, other computed parameters--especially the static pressure--are in very poor agreement with the data. In the discussion of numerical checks, it was suggested that some numerical difficulty must have arisen in this calculation. The results were not in agreement with other computations made using an identical turbulence model. Therefore, the good agreement with the reattachment-length data must be regarded as coincidental.

The uncertainty in the reattachment-length measurement was quoted by the data evaluators (the present author and J. P. Johnston) to be ± one step height. Many of the computations of the reattachment length fall near the lower bound of the uncertainty band. However, the static pressure data shown in Fig. 7 confirm that the computed reattachment length is significantly shorter than the actual value.*

The static pressure distribution is generally quite well predicted by almost all of the methods (see Fig. 7). The overall pressure recovery is usually quite well predicted, and the pressure rise between the center of the separation region and a point far downstream is within 10% of the measured value for most of the methods. The worst prediction of pressure recovery was done by the integral method (27) and one of

*[Ed.: See important remarks on this point in discussion, p. 907.]

the Boussinesq models (BKEZ 44). It is not apparent why the results of method 44 differed so markedly from those of other similar methods. The pressure distribution calculated by the differential model (D5QZ 14) is very irregular, perhaps due to too few grid points in this calculation.

Several plots of shear-stress profiles in the reattached flow were produced by each computor. Unfortunately, these plots are not very useful in most cases, because the computed reattachment length is so short. Therefore, the computed layer has had much longer to recover after reattachment than the experimental layer. A reasonable comparison cannot be made under these circumstances. However, a plot of the peak shear stress as a function of distance from reattachment is very revealing. Such a plot is shown for three representative cases in Fig. 8. A "data" curve which was estimated from other workers' measurements* in the separated flow and the actual data points in the reattached flow is also shown. The calculated shear-stress levels were much too high in all but one case (AKEZ 17E). The shear-stress levels calculated by Group 17C and 17E are shown in Fig. 9.

The severe overprediction of the shear stress in the separated shear layer may be the cause of the underprediction of the reattachment length by most computors. Method AKEZ 17E did by far the best job of predicting the shear-stress levels and also accurately calculating the reattachment length. However, methods BOPX/BOLX of Group 30 also accurately calculated the reattachment length while obtaining very large values for the shear stress.

Many computors blamed the poor agreement between measured and computed shear stress on the lack of adequate modeling for curvature effects in the free-shear layer. None of the Boussinesq techniques had curvature-sensitive terms in their turbulence model. On the other hand, method AKEZ 17E uses a modified ε equation which is sensitive to stabilizing curvature. This suggests that modeling of curvature terms is extremely important. However, curvature effects should be quite weak until very near reattachment. The ratio of shear-layer thickness to radius of curvature is quite small upstream of the reattachment zone. In addition, the computed shear-stress levels are much larger than those that occur in a plane (uncurved) mixing layer. In the present author's opinion, more detailed experimental data are needed before we can conclusively say which methods correctly model the effects of stabilizing curvature.

Almost all of the methods had difficulty in predicting the flow in the reattachment region and its downstream recovery. One would expect modeling to be exceptionally difficult in this region, since the flow is changing from a free to attached

*See Eaton, J. K., and Johnston, J. P., "An Evaluation of Data for the Backward-Facing Step Flow," Proc. 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows: Comparison of Computation and Experiment, Vol. I, pp. 275-278.

shear layer. The large-scale structure is grossly distorted and is not characteristic of either a boundary layer or a free-shear layer. An interesting feature of most of the computations is that the shear stress begins to decay well upstream of reattachment, a phenomenon that has also been observed experimentally. An extremely high rate of decay of the shear stress is calculated by most of the methods. Therefore, the shear-stress levels in the recovery region downstream of reattachment are roughly correct, despite the fact that the levels were too high in the separated flow region. Several of the methods (BKEY 33, BKEZ 17C, BKEZ 17F, BOLX 30, and D5QZ 14) show good agreement with the measured shear stress downstream of reattachment.

The computations of mean profiles in the region downstream of reattachment are less satisfactory. The ε profiles are poorly predicted by all but one method. It should be noted that the uncertainty of the data is relatively small in this region, so disagreements indicate an inaccurate computation. For most of the methods it is difficult to compare the computed and experimental velocity profiles because of differences in the reattachment length.* However, in almost all cases it is apparent that the computed mean-velocity profile is not recovering as rapidly as the experimental profile. At $x/h = 13.33$, the computed velocity defect in the inner region is much larger than the actual defect. One would have expected the opposite trend, since the reattachment length was underpredicted.

Two of the methods, BOPX/BOLX 30 and AKEZ 17E, gave good results for the reattachment length, so their mean profiles could be checked in detail. The computed profiles are in reasonably good agreement in the recirculating flow (see Fig. 10). The results get progressively worse in the recovery region following the common trend of too large a velocity defect in the inner region. In the case of Group 30, the deviation is small and the results are still quite good at $x/H = 13.33$. Group 30 uses an algebraically described length scale in the recovery region which is especially adapted to the backward-facing-step flow. Most of the other methods use the k-ε model of turbulence. It would be instructive to compare the length scale produced by the k-ε model to the prescribed length scale used by Group 30. In addition, the calculation of at least one group which used the k-ε model should be compared in more detail to the Kim, Kline, and Johnston data. This could be done by a computor who has access to all of the necessary computational output.

Case 0422 (P2)

Case 0422 (P2) is a predictive case on flow through a modified sudden expansion. The flow geometry is shown in Fig. 11. The opposite-wall angle is variable, allowing the experimenters to vary the applied pressure gradient. The data were provided by H.

*In retrospect, we should have specified velocity profiles at fixed distances from the computed reattachment point.

L. Seegmiller and D. Driver of NASA-Ames Research Center, and were independently checked in part by S. Pronchick at Stanford.

Six computations were performed by three groups, including five computations which employed the k-ε model of turbulence with either the Boussinesq or the algebraic closure scheme. The sixth method (21) used a Reynolds-stress transport model. Group 17F was the only group to supply a complete set of results. Therefore, results from some of the groups are missing on the compiled figures.

The reattachment length was measured and computed as a function of opposite-wall angle (see Fig. 12). All of the methods underpredict the reattachment length. At small angles, the algebraic stress model with a modified ε equation (AKEZ 17E) was the most successful. This was also the case for the normal backward-facing-step flow (Case 0421). The agreement with the data is not as good as it was in Case 0421, but it is within about 10% of the correct answer. It is encouraging to see that method AKEZ 17E correctly predicted the trend for small angles. A more detailed comparison of the computation with data is required to understand why the computation deviates from the experimental trend at larger angles. The reattachment length computations were the only information available for method AKEZ 17E. Method BKEZ 17H seriously underpredicted the reattachment length at small angles, but then correctly predicted the reattachment length at large angles. The trend shown was badly in error.

Detailed profiles of mean velocity and shear stress were provided for a wall angle of 6°. Figure 13 shows that the shear-stress profile in the separated shear layer is well predicted by both the Reynolds-stress model and one of the k-ε Boussinesq models. Computed shear-stress profiles were not available for the other methods. At reattachment, the Reynolds-stress model still gave excellent agreement with the measured shear-stress profile. However, the eddy-viscosity method, BKEZ 17H, overpredicted the shear stress substantially (see Fig. 14). This was the same trend observed in Case 0421. Downstream of reattachment, the shear stress computed by the Reynolds-stress model decayed much too rapidly. By $x/H = 16$, the computed peak shear stress was only one-half of the experimental value. The Boussinesq method did an excellent job of predicting the shear stress in the recovery region, as had been the case for Flow Case 0421.

Computed mean-velocity profiles were provided at the streamwise location of the computed reattachment point for methods BKEZ 07, BKEZ 17H, and RSLZ 21. These profiles are compared with data in Fig. 15. The agreement is reasonably good for all three methods. The computed mean-velocity profiles do not recover from the effects of reattachment as rapidly as the experimental data. Figure 16 shows the mean-velocity profiles at $x/H = 16$. The predictions of method BKEZ 07 are in excellent agreement with the data. However, it should be noted that the predicted reattachment length was approximately three step heights too short for this method. If the recovery region

were correctly modeled, the computed profiles would be more completely recovered by x/H = 16.

In summary, for small wall angles, the computer programs gave similar results in the predictive (0422) and postdictive (0421) cases. That is, parameters which are accurately calculated for Case 0421 are also accurately calculated for Case 0422. This is an encouraging result which indicates that the programs may possibly be used to extrapolate from an available data base. However, for angles larger than about 6°, the computed results began to deviate from the data in a more unpredictable manner. It is clear that the current models are far from achieving universality. Even a wall angle of 10° is only a relatively small geometrical perturbation from the standard backward-facing step.

Case 0423 (P3)

Case 0423 (P3) is another modification of the single-sided, sudden-expansion flow. In this case the entire downstream duct is turned at an angle to the inlet duct (see Fig. 17). The data were provided by R. V. Westphal and J. P. Johnston of Stanford University. Only two computations were done for Case P3, probably because the geometry was more difficult to handle than the other backstep variations. Both groups used eddy viscosity formulations with the k-ε model of turbulence. Figure 18 shows the computed variation in reattachment length. Although method 17 has predicted the trend of the data, both methods drastically underpredict the reattachment length.

The computations obtained relatively good results for the static pressure distribution. Figure 19 shows the static pressure profiles for an angle of 10°. The agreement shown here is typical of all of the computed pressure distributions. Again, the differences between computation and experiment were similar to differences observed in the ordinary backward-facing step. The computed base pressure is somewhat too high, but the pressure recovery from step face to far downstream is approximately correct. One may suspect numerical error in the region of the corner as the cause of the incorrect base-pressure calculation. However, while the two methods used nearly identical turbulence models, they used different numerics. Apparently the model causes problems in the region of the corner.

As with Cases 0421 and 0422, mean-velocity profiles are difficult to compare because of the difference in computed and measured reattachment lengths. In this case a computed mean-velocity profile was requested at the computed reattachment location. Group 17 produced this output, and the agreement with data was excellent (Fig. 20).

In general, although only two computer groups attempted this case, the results were encouraging. Group 17 correctly predicted the trend of increasing reattachment length with increasing angle. In addition, deviations from measured parameters were similar to deviations which were observed in the postdictive backward-facing-step case (0421).

CASE 0431 SEPARATED DIFFUSER FLOW

EE 27 HH 45 HH 22C	INTEGRAL METHODS
BOPX 28	PRESCRIBED EDDY VISCOSITY
BILX 30	1 EQN MODEL
B5QX 14	FIVE EQN MODEL

Figure 1. Sketch of flow and summary of computations.

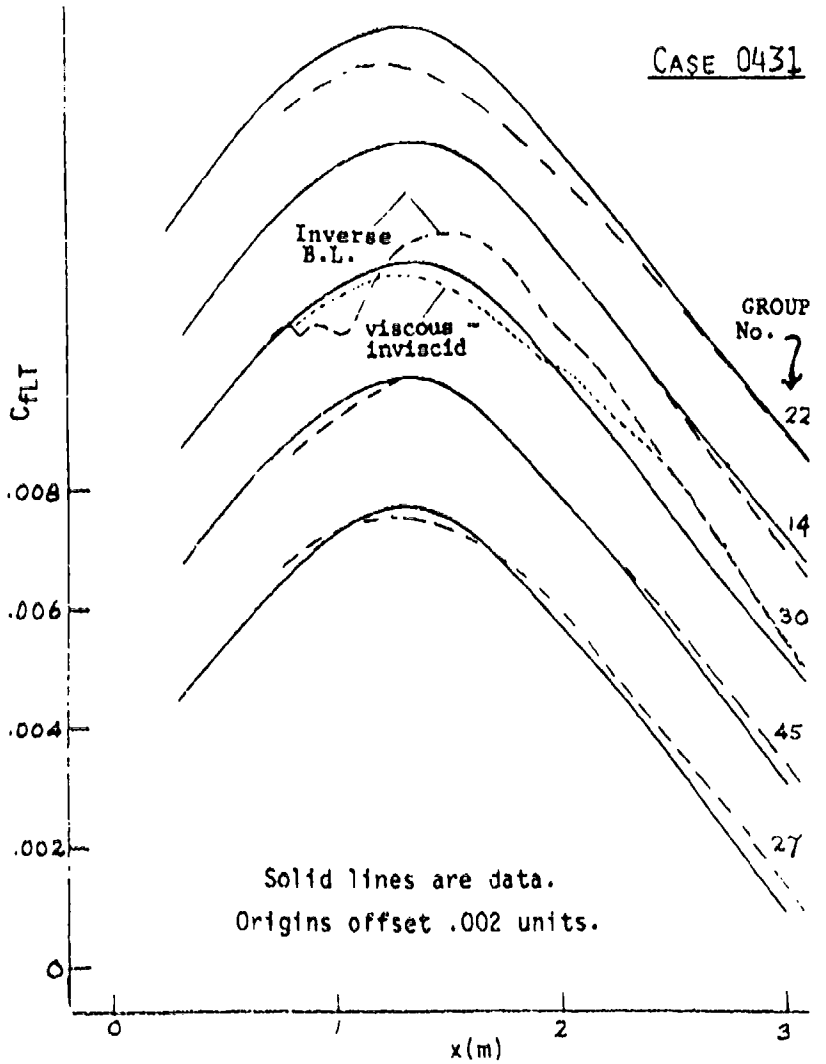


Figure 2. Skin-friction compurations.

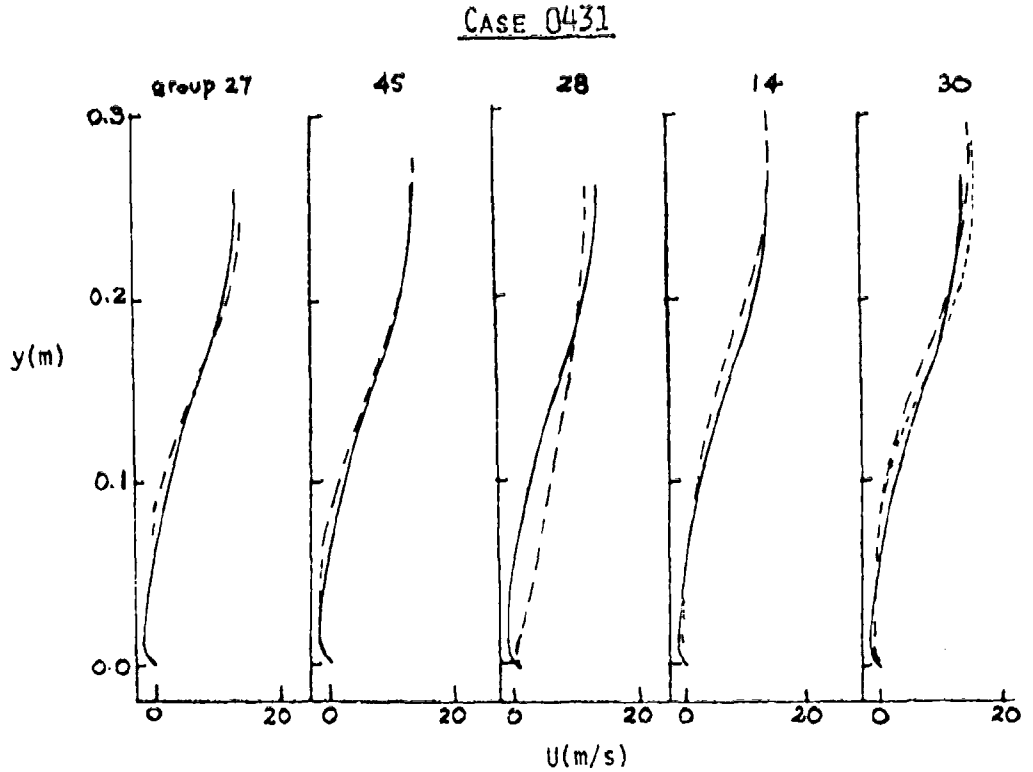


Figure 3. Mean velocity profiles at x = 3.97 m. Solid lines are data.

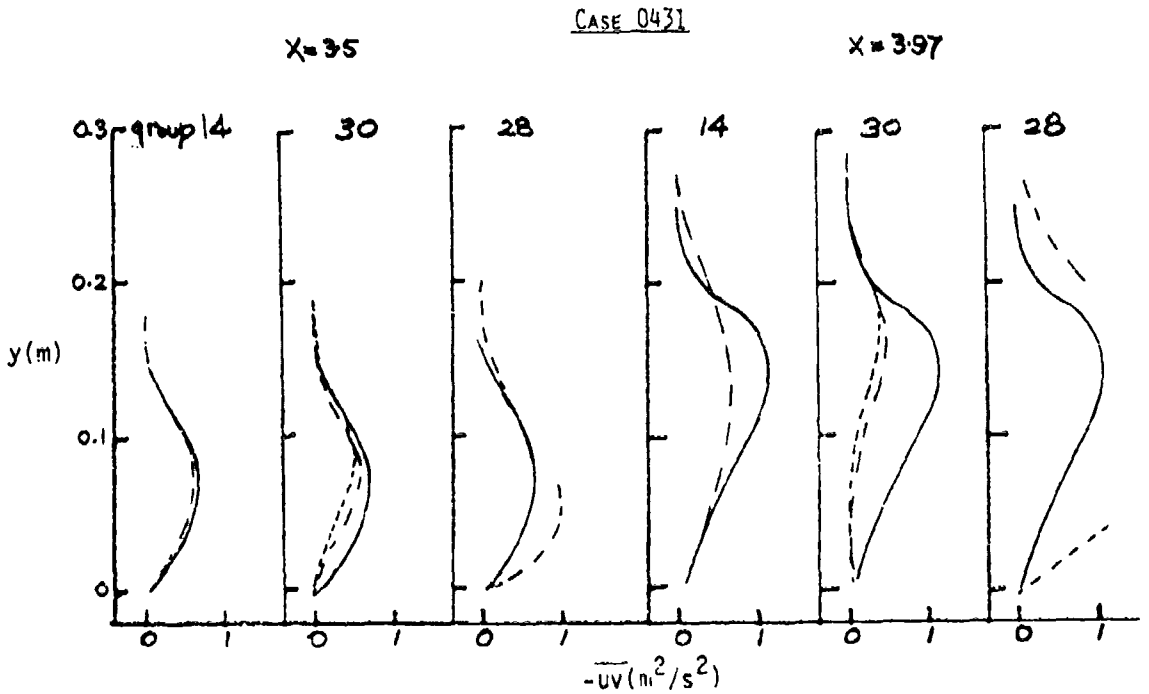


Figure 4. Shear stress data at two locations. Solid lines are data.

CASE 0421

DATA TAKERS: KIM, KLINE, JOHNSTON

COMPUTATIONS

11	GROUPS	7	BKE
15	METHODS	2	AKE
		2	OTHER BOUSSINESQ
		1	OTHER ALGEBRAIC
		2	HIGHER ORDER
		1	INTEGRAL METHOD

Figure 5. Sketch of flow field and summary of computations for Case 0421.

CASE 0421

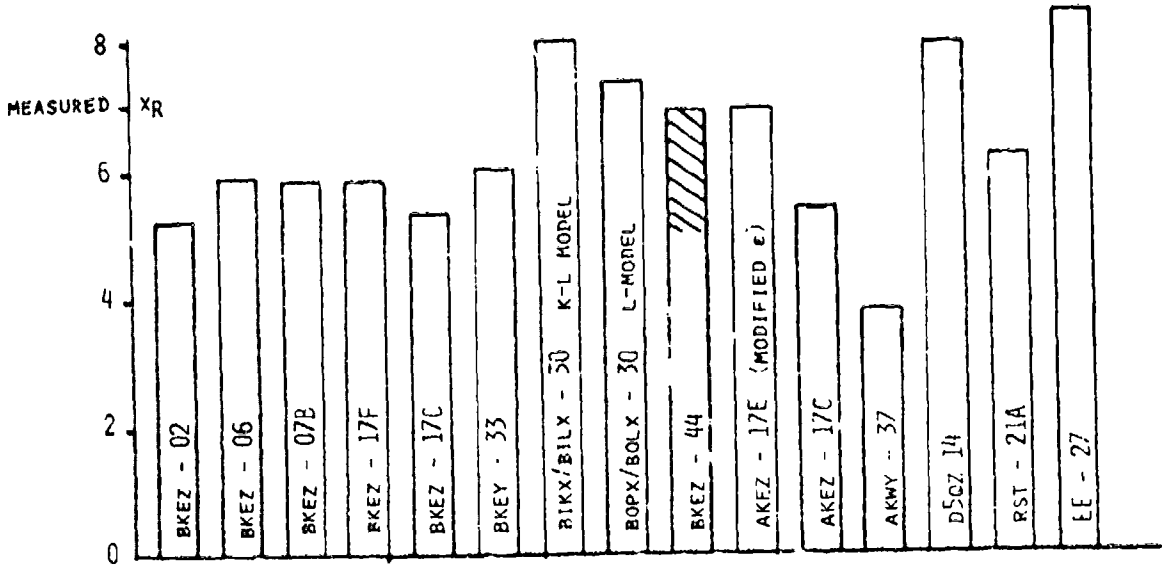


Figure 6. Computations of reattachment lengths.

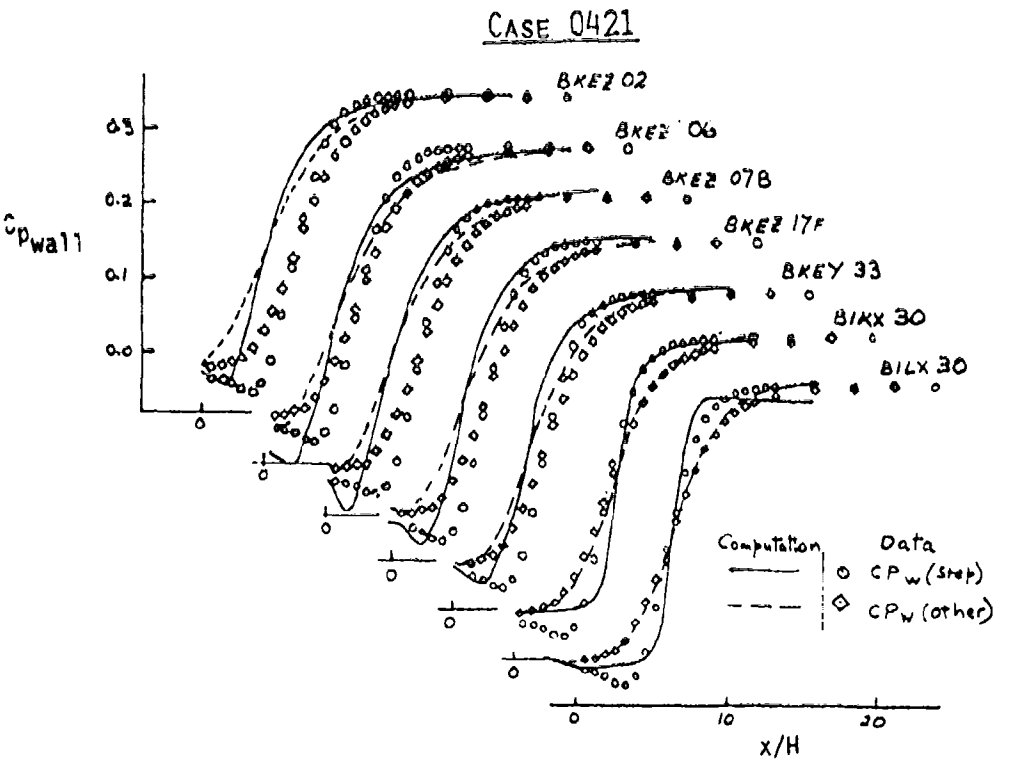


Figure 7A. Computed static pressure distributions.

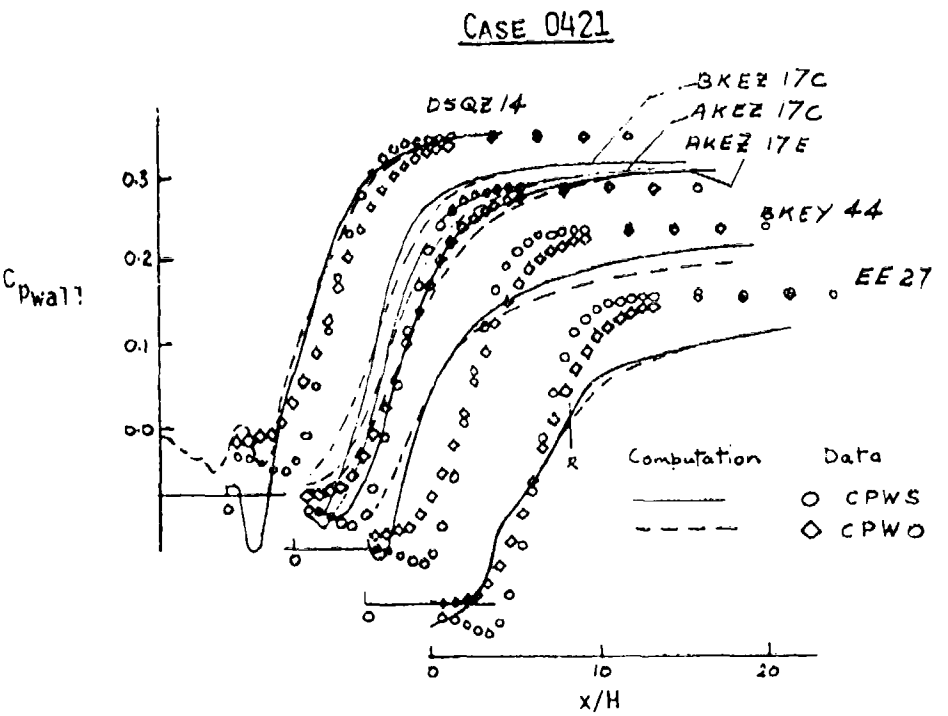


Figure 7B. Computed static pressure distributions.

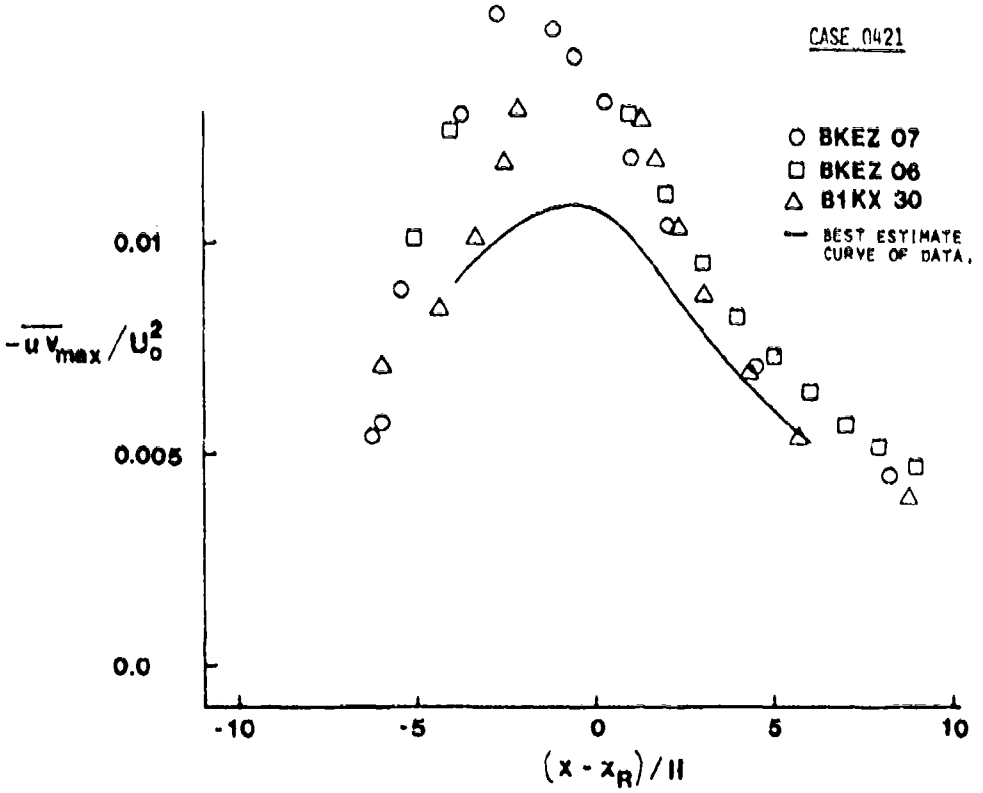


Figure 8. Typical computed shear-stress levels.

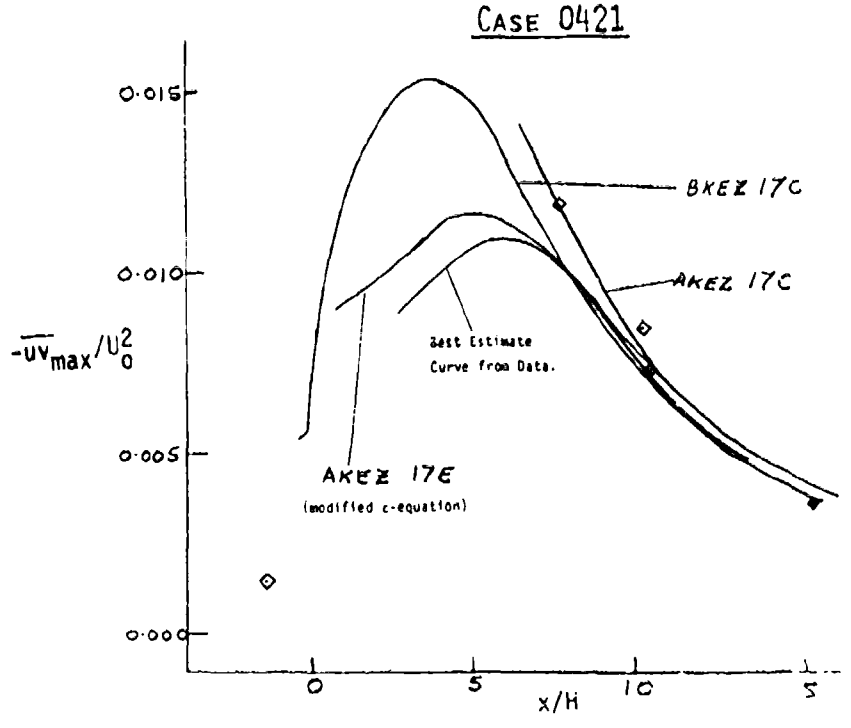


Figure 9. Shear-stress levels for method AKEZ 17.

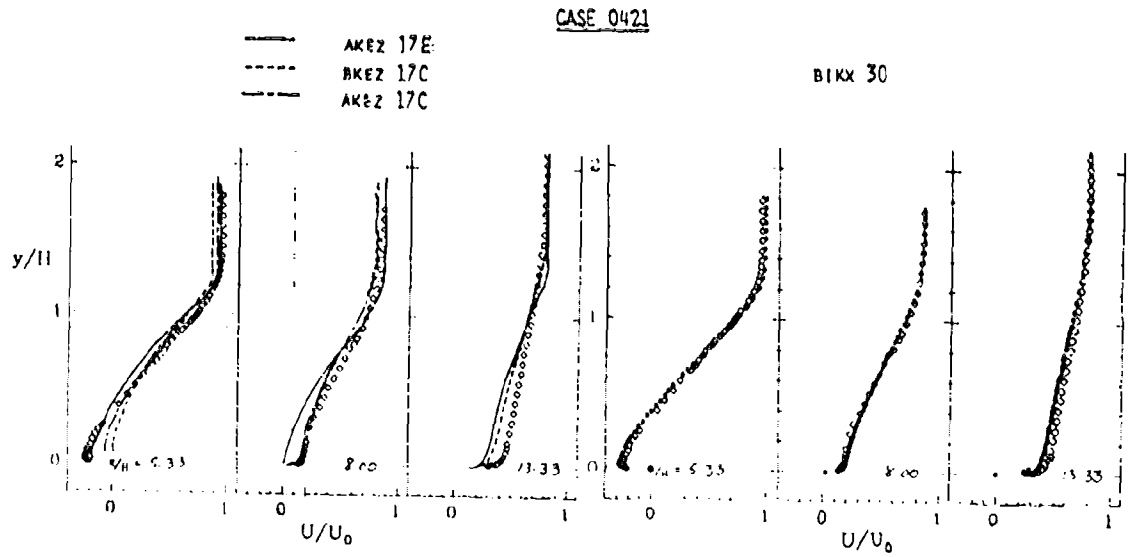
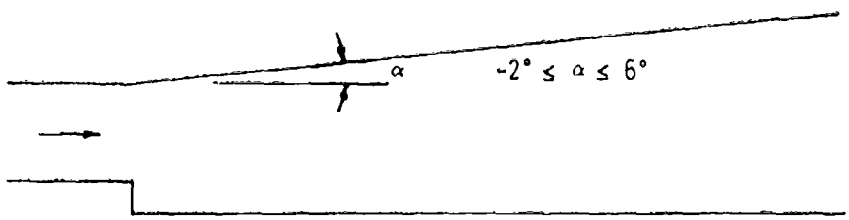


Figure 10. Mean-velocity computations.

PREDICTIVE CASE P2

DATA TAKERS: H. L. SEEGMILLER AND D. DRIVER
NASA-AMES



DATA SUPPLIED:	COMPUTATIONS:
STATIC PRESSURE	GROUP
MEAN VELOCITY	17
SHEAR STRESS	07
SKIN FRICTION	21

Figure 11. Sketch of flow geometry and summary of computations for Case P2.

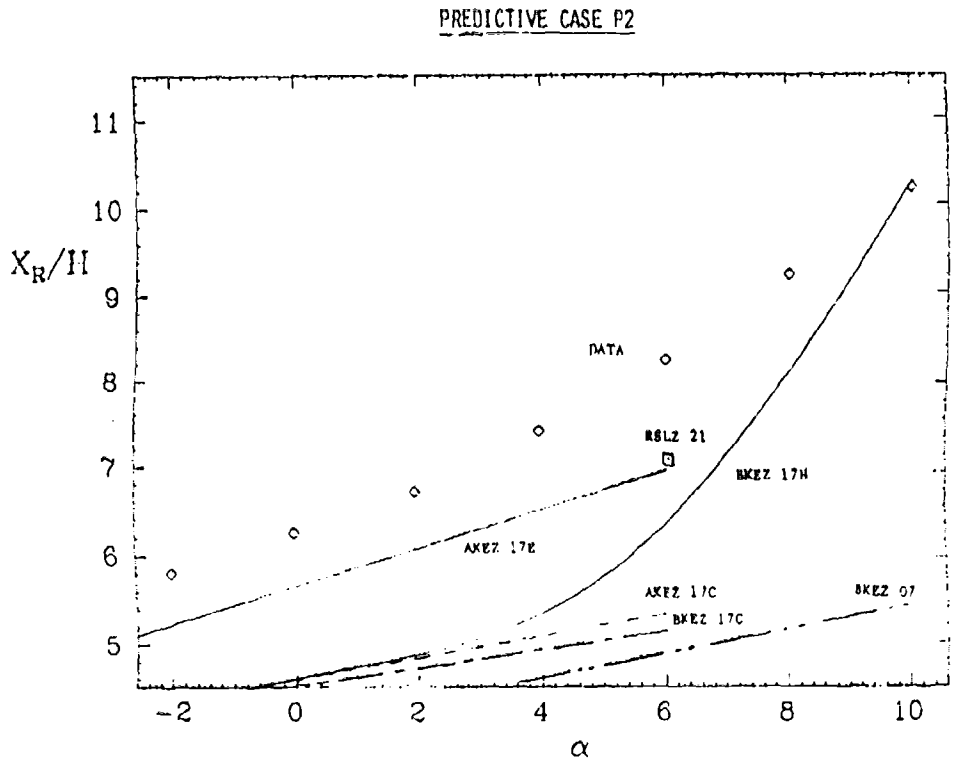


Figure 12. Reattachment length computations for Case P2.

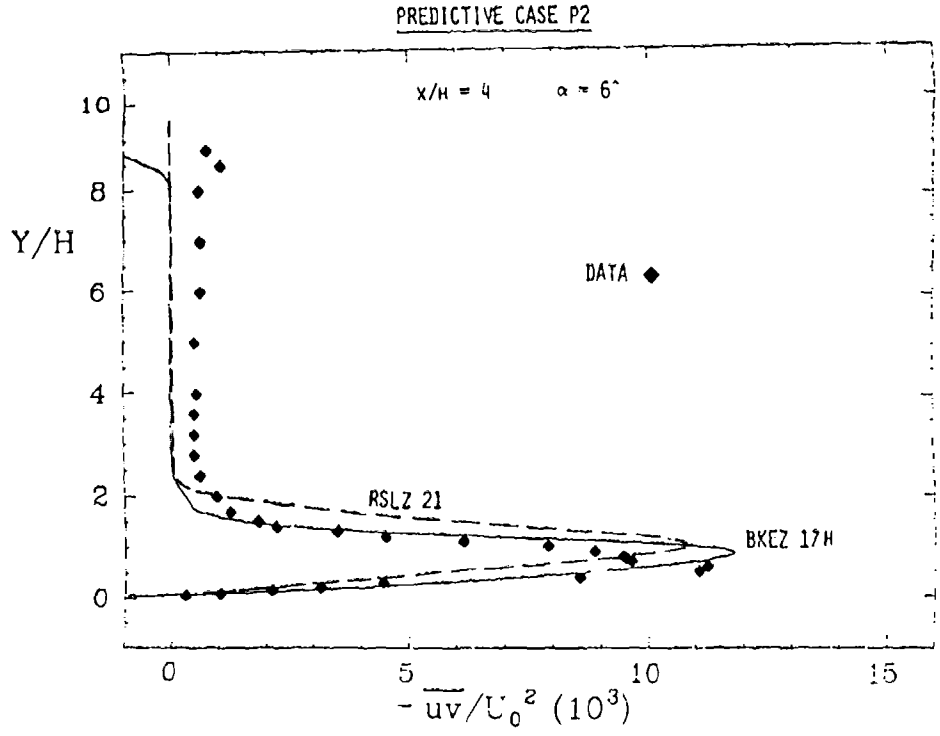


Figure 13 Shear-stress computations in the separated shear layer.

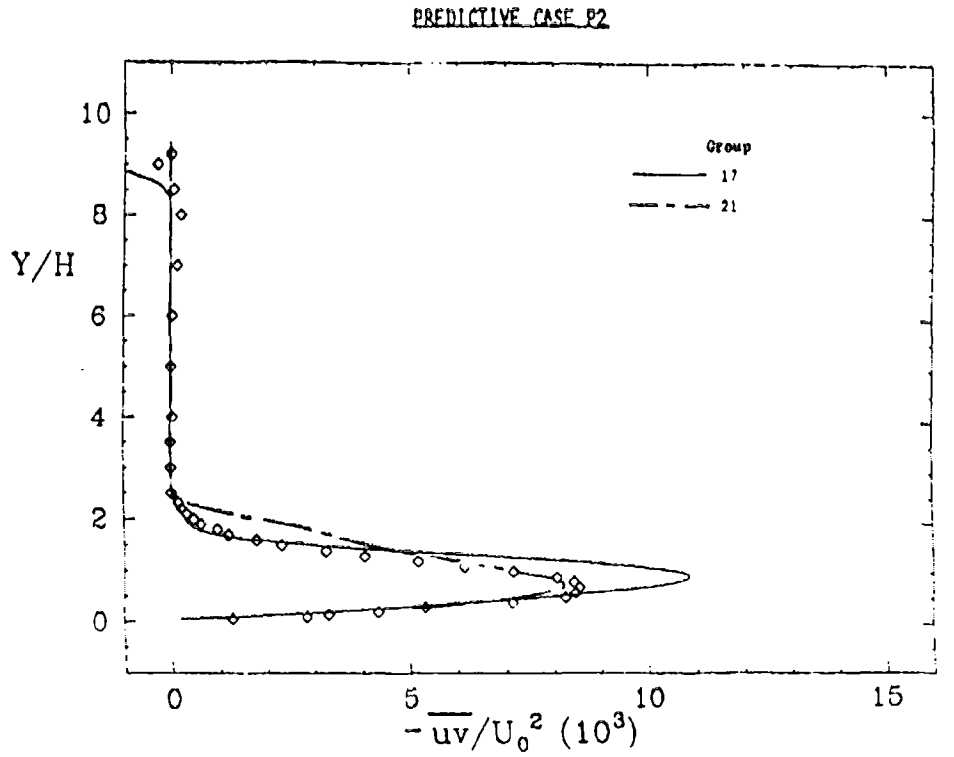


Figure 14. Shear-stress profiles in the separated flow region (x/H = 4) (Case P2).

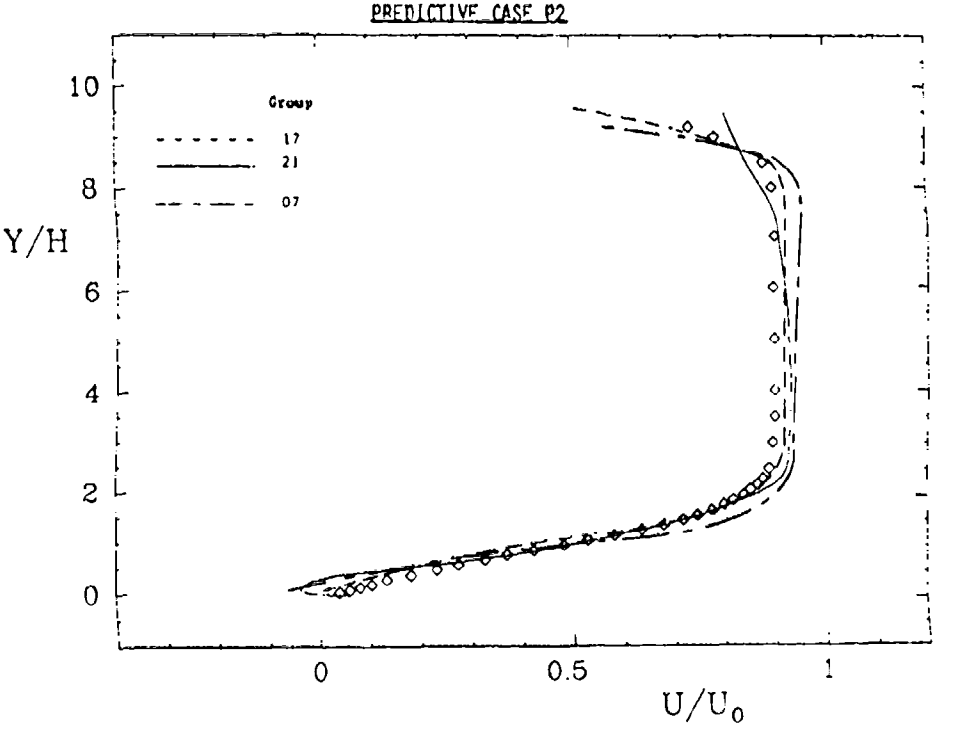


Figure 15. Mean-velocity profiles at reattachment. Diamond symbols are data (Case P2).

PREDICTIVE CASE P2

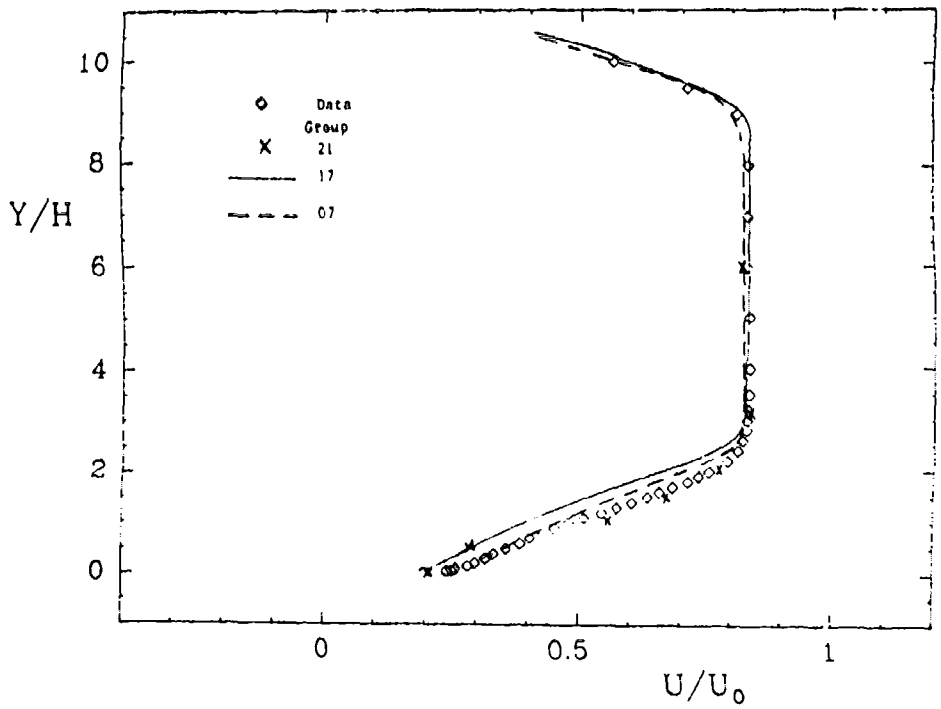
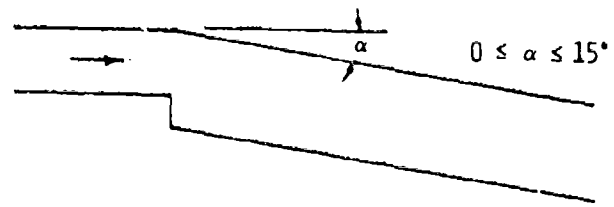


Figure 16. Mean-velocity profile in the recovery region (Case P2).

PREDICTIVE CASE P3

DATA TAKERS: R. V. WESTPHAL AND J. P. JOHNSTON
STANFORD UNIVERSITY



DATA SUPPLIED:

STATIC PRESSURE
MEAN VELOCITY
REATTACHMENT LENGTH
SKIN FRICTION

COMPUTATIONS:

17 3KEZ
07 8KEZ

Figure 17. Sketch of flow geometry and summary of computations for Case P3.

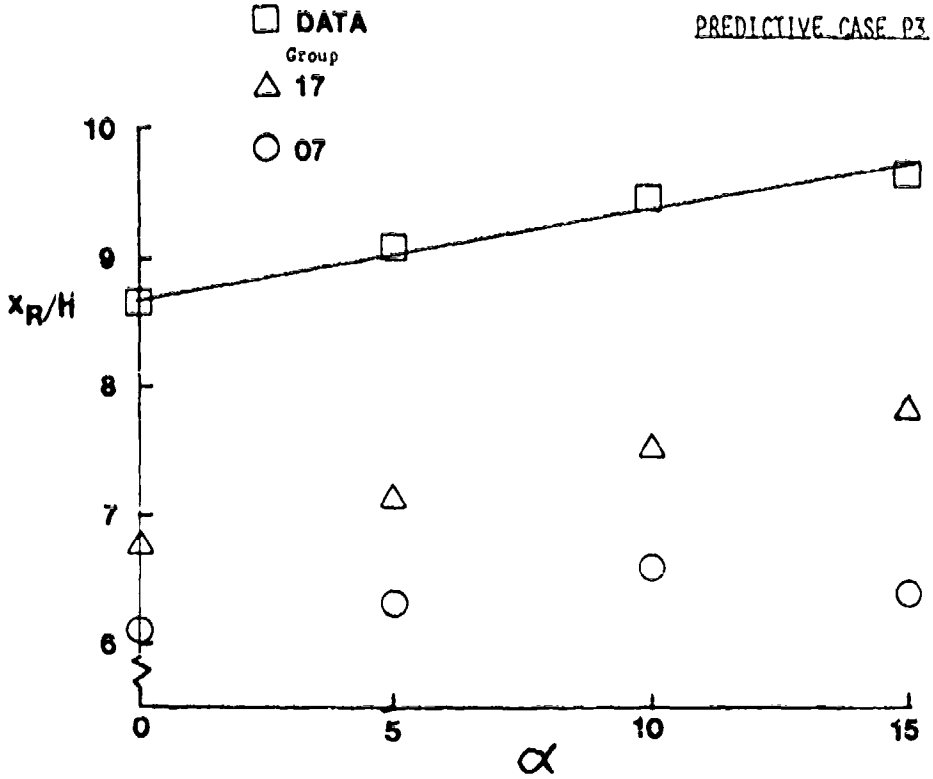


Figure 18. Reattachment length computations for Case P3.

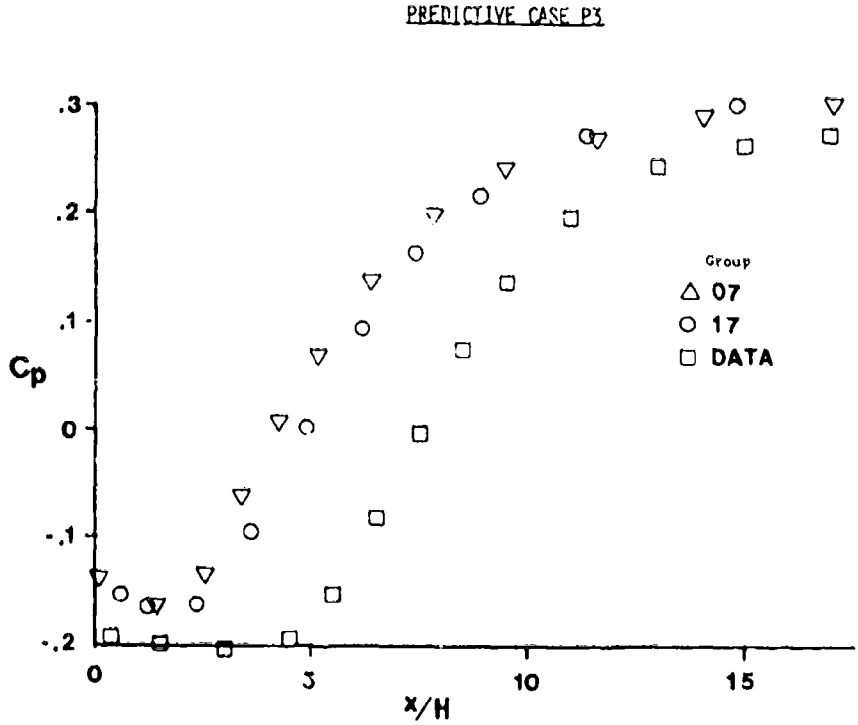


Figure 19. Static-pressure computations for Case P3 ($\alpha = 10°$).

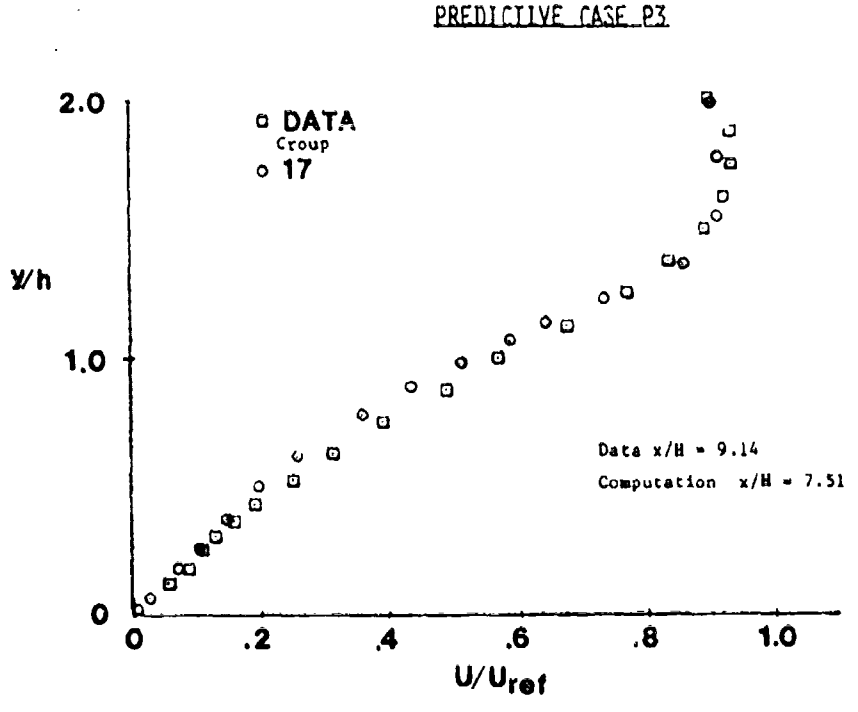


Figure 20. Mean-velocity profile at reattachment for Case P3 (α = 10°).

DISCUSSION

SESSION X

EXTERNAL FLOWS

Two-Dimensional Stalled Airfoil, Case 0441

The group reached a consensus, or near consensus, on the following topics:

Unsteadiness

The group considered whether the flow about the stalled airfoil was subject to self-induced unsteadiness outside of the viscous regions. It was decided there was no evidence to indicate this and that the velocity-fluctuation measurements indicated the contrary. Therefore, there was unanimous agreement that the flow about this airfoil could be considered steady.

Boundary Conditions and Tunnel-Wall Effects

Several computors expressed reservations about the effects of side walls and the vortex generators on these walls, top and bottom walls, and the upstream and downstream tunnel geometry. Some computors included the tunnel walls as outer boundary conditions, and some assumed free-air conditions. H. Moses (Group 27) tried extending the distance to the upper and lower boundaries and got different pressure results. While relatively secondary in importance, it was agreed that the boundary-condition effects could not be ignored; they may have quantitative effects on the results. It appears likely that this experiment will be repeated in different laboratories.

Computational Grids

Navier-Stokes computors (Groups 23 and 41) felt that their grids were not fine enough and/or that truncation errors existed. Group 27 felt the same, and is engaged on further grid refinements. Groups 42 and 46 felt they had an adequate number of grid points.

Importance of Turbulence Modeling

Details of the turbulence downstream of the separation point were not particularly important for calculating the pressure distribution for this airfoil. Extremely simple models of the separated and wake flows seem to give reasonable results. However, turbulence modeling would be important when computing the detailed structure of the wake and is also a factor when determining the separation point. It is interesting to note that for this flow turbulence modeling was not viewed as a limiting factor by any of the computors (see item below).

Computors' Views of Difficulties with This Flow

(a) Group 27 (Integral Method)
- concern about the use of integral methods to model separated flow regions
- lack of wake calculation.

(b) Group 42 (Integral Method)
- numerical problem of coupling inviscid and viscous equations when the separated region gets large.

(c) Group 46 (Vortex Wake Model)
- would like to improve wake model (for example by using an entrainment model).

(d) Group 23 (Time-averaged Navier-Stokes)
- inadequate number of grid points
- outer boundary conditions
- asymptotic behavior of far wake.

INTERNAL FLOWS

Case 0431 (Separated Diffuser Flow, R. L. Simpson)

1. The question of how the downstream boundary condition was applied by the five computors was raised. Groups 14, 45, and 30 were represented, and each said essentially that the downstream boundary condition was no real problem for their computations. It is the impression of the discussors that none of the computors used the actual specified downstream boundary conditions. Either they did not need them, or they concocted a zero-gradient condition appropriate to their own particular method. It was a surprise to discover the the three computors present at the evening discussion were satisfied both with their computations and with the specification of downstream boundary conditions. Murphy, Group 28, had earlier expressed a similar sentiment.*

2. Although the shear stress computed by Mellor's method (Group 14) appeared low in the separated flow region, it was pointed out that agreement was actually quite good everywhere, except near the end of the computational domain. The reason for this was that they specified the downstream boundary condition such that the streamwise derivative of the shear stress was zero, a constraint which influenced the computations at the last station. There was no explanation given for the

*See comment supplied by J. D. Murphy following discussion.

poor agreement on shear stress by Pletcher's method (Group 30) in contrast to his excellent profile predictions.

3. All the computors present were asked if there was any basic reason why the k-ε computer models were not tested against the Simpson diffuser flow. The computor groups present at the evening session, Groups 14, 35 and 30, all commented that time and funding prohibited the calculation of this particular flow in time for the meeting. It also appeared from the discussion that this flow was given a relatively low priority by most computors. This may have been because, unlike the backstep flow, the Organizing Committee did not identify the stalled diffuser as a central case. The Evaluation Committee should note the fact that this practically important flow was not tested against any of the k-ε models.

Case 0421 (Backstep Flow, Parallel Walls--J.K. Eaton & J.P. Johnston)

1. The uncertainty in the experimental value for the location of the long-term mean reattachment point was discussed, and it was generally agreed among those present that the original specification of uncertainty (± one step height) was too conservative. It was tentatively conceded that a lower value of uncertainty of ± 0.3H would be a better estimate on the basis of current knowledge. Moses pointed out that both experimental and predicted pressure distributions supported the view that the reattachment point should lie further downstream than indicated by those methods which underestimated the reattachment length.

2. The Mellor Group (14) reported that their method had computed a fluctuating location of the reattachment point and in the ensuing discussion it was pointed out that a fairly large fraction of the Reynolds stresses probably arose from this unsteady motion of the reattachment point. It is noted that an overestimate of Reynolds shear stress in the separated shear layer should result in an underestimate of the reattachment length. This was not always found.

3. There was some disagreement about the uncertainty applicable to the experimental shear-stress values given in the data. Castro felt that, based on his own measurements, the shear-stress levels in the flow are probably underestimated. Eaton, referring to the available laser-anemometer data for similar flows, felt that the data are largely correct and that the uncertainty band does not embrace the maximum shear stresses predicted in the separated region.

4. It was concluded that curvature effects could be quite strong close to reattachment and that the inclusion in the dissipation equation of a term that was sensitive to curvature would result in better calculation of the reattachment length. None of the eddy-viscosity formulations using the k-ε technique used any curvature terms, and only the algebraic stress models have terms in the dissipation

equation which can react to curvature. The algebraic stress model which used the production as the source term in the dissipation equation did not do well in computing the reattachment length. However, the method which broke this nexus did very well. Pletcher noted that his method did not have any terms which were curvature-sensitive, yet it correctly predicted the reattachment length. Launder cautioned that, although the curvature-sensitive terms allowed his method to predict the correct shear stress in the predictive Case 0421, it did not yield the correct shear stress in Case P2, nor did it do well in the prediction of case P3. It was felt that the major region where the shear stress was over-predicted is near the reattachment point, and this is the region where curvature would be expected to have the largest effect. Castro pointed out that if, in fact, curvature effects were substantial, the shear stress in the free-shear layer should be substantially lower than in a plane mixing layer instead of being higher as observed. Launder indicated that he felt that this was due to competing effects of curvature reducing the shear stress and turbulence feeding back through the recirculating flow and increasing the shear stress. Eaton disagreed with this evaluation, believing that turbulence fed back into the recirculation zone did not significantly change the shear stress in the free-shear layer. Although there was not complete agreement, there was quite general agreement that curvature terms may indeed be important, and will need to be included in some of the simpler models which aim to compute a free-shear layer overlaying a recirculating zone terminating with a reattachment point.

5. There was disagreement about the importance of establishing a fine mesh in the region of the separation corner. Castro and Pollard felt that a finer mesh than those generally used was required and that grid independence had not been proven in this case. Leschziner, on the other hand, felt that a reasonably coarse grid could be used in the region of the separating corner without adversely affecting the results.

In the end there was general agreement that this case (0421, the backward-facing step) was less sensitive to numerical problems than other sharp-corner-separated flows and that the numerical uncertainty estimate given by Launder (roughly 0.3 step heights on the reatachment length) was reasonable. This uncertainty estimate was carried over from the discussion of the first morning session and agreed with the revised estimates of the experimenters. Launder reported that the discussion of numerical problems associated with the flows in this session came to general agreement that predicted reattachment distance was $5.6 \pm 0.3h$ step heights. One must conclude, in view of item 1 of this discussion that there was no substantial overlap between the predicted and the experimental data,

allowing for uncertainties, for this particular flow, with the exception of the results of B1KX-30 and AKEZ-17E.

6. Discussion highlighted the fact that the Launder model, which achieved good agreement with the reattachment length, did poorly in its prediction of the downstream recovery of the mean-velocity profiles. A large number of the methods did very well when predicting at least the peak values of the shear stress downstream of reattachment. However, Rodi pointed out that many of the methods will still do poorly on the mean profiles, despite the fact that they have accurately predicted the shear stress. It was noted that Pletcher's method did predict the downstream development of the profiles very accurately. The discussors were unable to examine the recovery downstream for any of the other methods due to the large discrepancies in the reattachment length between measurements and computation. Zaton pointed out that an adequate comparison requires renormalization of the x-coordinate about the computed reattachment length and that this would be a useful exercise to conduct on the computations that have already been performed. Computors who were willing were asked to supply shear-stress results which they have renormalized and mean profiles at the appropriate renormalized streamwise location.

Predictive Test Cases 0422 (P2), and 0423 (P3)

1. Bradshaw asked why so few computors had satisfied the request to undertake calculations of 0421 and the predictive cases, and it was then pointed out that this request had later been withdrawn. Some confusion seemed to exist on this matter.*

2. In the discussion concerning the reasons why predictive test cases P2 and P3 were not done by very many groups, it appeared that the major stumbling block to the application of the current methods to these cases was the requirement that they must use a non-orthogonal grid in order to avoid the problems of numerical diffusion, where flow lines were essentially not parallel to grid lines and/or the complications of recasting or reformulating the problem in a grid that followed the flow lines. In this context, it seemned strange that many computors chose to tackle the sloping-wall case as opposed to the variable-step-height case, P4, which did not require non-orthogonal grids and did not involve any sloping walls.

3. The poor agreement of the overall pressure recovery for Case P3 was noted. Leschziner indicated that this was probably not a numerical problem, since two methods, using similar models but drastically different numerics, yielded

*[Ed.: Confusion, unfortunately, still exists. The request to do predictive cases, if possible, was not to our knowledge ever withdrawn.]

essentially the same pressure recovery, even though this was significantly different from the data. It was suggested that this was probably a problem of the modeling not correctly representing the flow. The possibility of using universal constants in the methods used to compute these flows is not conclusively demonstrated by the results of the predictive test cases P2 and P3. In the predictive cases, the results of computations were considered to be less satisfactory than those for the back-step case, Case 0421, using the same coefficients in the models. However, universal agreement on this point was not attempted.

Overall conclusions of the discussions of Case 0421 and predictive cases, P2, and P3, were as follows: (i) no one method seems capable, in any one of these cases, of predicting all of the specified conditions; (ii) no method could predict accurately any one of the specified conditions in all of the flows. These cases represented a severe but realistic test of the methods, and it is unfortunate that they were not all prescribed as mandatory case flows.*

Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

G. Lilley questioned whether it could be inferred that, since a reasonable agreement with pressures on the airfoil could be achieved with fairly crude methods, it was also possible to obtain C_L with these methods. J. McCroskey replied that C_L was not specified as an output.

G. Lilley further asked whether the methods would still give good agreement as the angle of incidence was increased beyond separation. J. McCroskey said that this was not the case; in fact, some of the methods were at the limit of their capability to converge, while others thought they could go to higher angles. Melnik stated there was the feeling that some of the codes could predict lift up to $C_{L_{max}}$, and that the method of F. Dvorak could go well beyond this because it uses an empirical correlation to input the length of separation. P. Roache replied that the flow prediction is not sensitive to the separation length; R. Melnik disagreed, saying it was critical in establishing the lift in Dvorak's method.

J. Kim pointed out that the backstep flow contains a large separation region, with different scales and structure than the cases for which the k-ε method was tuned. He wondered whether its shortcomings in this case might be an indication of a failure of the universality of the model. W. Chow felt that the failure was not due to the

*[Ed.: Some methods used in this Conference only do compressible cases. However, we agree that an increase in mandatory flows in some sense would have been desirable, and should be considered in future meetings of this type.]

model in his case, but to the limitations on grid spacing due to his computer space. P. Roache felt the reattachment length to be an ambiguous parameter to use in the backstep flow, since it can be signifiantly altered by changes in numerics. J. Eaton stated that X_R was probably sensitive to numerics, and should therefore not be the sole basis for comparisons. However, it is a critical prediction which must be made. H. Nagib felt that X_R is sensitive to many experimental parameters as well, and that the ability to predict its measured value in the experiments may have been overemphasized. It was generally agreed that variables in addition to the reattachment length must be used for comparisons, but that the calculation of X_R is necessary, since for proper comparisons of other variables, it is necessary to plot them relative to the streamwise distance from reattachment. E. Reshotko pointed out that in supersonic flow the location of reattachment is crucial, since the flow angle influences the base pressure.

COMMENT ON SESSION X

by J. D. Murphy, Group 28

Rather than being satisfied with the treatment of downstream boundary conditions for Case 0431, we simply made the best of a bad situation, since the experimental data at the outflow boundary was neither sufficiently extensive, nor sufficiently dense.

With regard to the quality of the computation, we are content that the solution we have presented is reasonably free of numerical error and hence represents a "good" solution to the time-averaged Navier-Stokes equations incorporating an unmodified Cebeci-Smith model and subject to the specified boundary conditions.

SESSION XI

Chairman: B. J. Cantwell

Technical Recorders:

O. McMillan
M. Lee

Technical Reporter

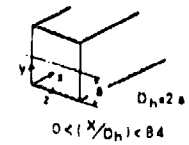
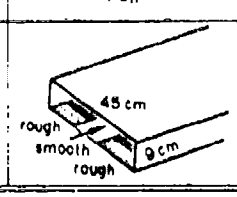
J. B. Jones

INCOMPRESSIBLE DUCT FLOWS

Cases 0111, 0112, 0511, 0512
Predictive Case 0113 (P1)

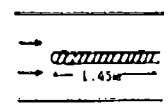
PICTORAL SUMMARY

Flow 0110. Data Evaluator: P. Gessner. "Corner Flow (Secondary Flow of the Second Kind)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Turbulence Profiles Others	C_f	Re	Initial Condition	Other Notes
Case 0111 J. Po E. Lund P. Gessner		34 stations on walls A and B	17	5	-	-	-	3	3 profiles of K	1	$2.5_5 \times 10^5$ (based on bulk velocity and D_h)	symmetric inlet flow	C_p vs x data for laminar flow (Reavers, Goldstein) included with data file. Goldstein's laminar velocity data also included. Nominally uniform, low turbulence level flow at inlet ($x/D_h = 0$).
Case 0112 J. Hinze			1		1	1	1	1		1	$1.5_5 \times 10^5$ (based on hydraulic diam and U_{max})	One wall rough with smooth center-strip	All measurements were made at $x/D_h = 126$. Data include viscous dissipation.

PICTORIAL SUMMARY

Flow 0510. Data Evaluator: R. Dean. "Turbulent Secondary Flows of the First Kind."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Turbulence Profiles Others	C_f	Re	Initial Condition	Other Notes
Case 0511 I. Shabaka	 wing span: 0.127 m wing thickness: 0.051 m	Measured	3	Secondary flow vectors at 3 stations	3	3	3	3	Intermittency and triple products, flatness factor 3 stations	7	$1.1_5 \times 10^5$ (based on wing thickness)	BL thickness at the leading edge was about 0.025 m	Wing-body junction. Calculations should be started downstream of leading edge.
Case 0512 J. Humphrey	 4.0 x 4.0 cm square cross section	Not recorded in bend	U_θ 7	U_r 2	$\overline{u_\theta^2}$ 7	$\overline{u_r^2}$ 2	-	$\overline{u_\theta u_r}$ 2	-	-	4×10^4 (based on duct height)	Nearly fully developed	Curved rectangular duct. Calculations should be started 8.2 duct dia. upstream of bend using Melling's (1975) data.

Editors' Note

Case 0113 is a predictive case, and does not have a pictorial summary. For a picture of the flow geometry, see p. 288, Vol. I.

Methods Used--Session XI

Method	Flow Case				
	0111	0112	0113 (P1)	0511	0512
AKEZ	●	●	●		
AOPX	●			●	●
BKEZ				●	●
BIKX					●
BOPX					●
BKEY					●
BKEX				●	
Plate No. Vol. III	1-7	8	179	121-125	126-132

INCOMPRESSIBLE DUCT FLOWS

Cases 0111, 0112, 0511, 0512 and Predictive Case 0113 (P1)

Technical Report

by

J. B. Jones*

Summary of Computations for Case 0111--Square Duct Entrance Flow†

Three computor groups using methods AKEZ 07A, AKEZ 23, AOPX 22, respectively, submitted results for the developing incompressible flow in a square duct. The specifications called for computations of both streamwise development and distributions across sections normal to the flow of various variables.

Computor Groups 07 and 23 calculated the axial variation of pressure coefficient (C_p) very accurately; the results of Computor Group 22 present too small a value of the dp/dx magnitude with C_p being approximately 20% low at $x/D_h = 84$. Plots of U/U_b vs x/D_h (see Figs. 1 and 2 below) typically show for all computors improved agreement as the distance (y) from the wall increases. The plots of Group 07A show that the fully developed condition is reached too early. A disturbing result of both Groups 22 and 23 is that close to the wall the dU/dx slope is not approaching zero, and in some cases is even increasing, at $x/D_h = 84$.

*Virginia Polytechnic Institute, Blacksburg, VA 24061.

†See also Plates 1 through 7, Volume III.

As appears to be true in all cases involving secondary flow of the second kind, mean cross-flow components are not predicted accurately (see Fig. 3). More significantly, for all flow cases it appears that inaccuracy in the magnitude of cross-flows does not correlate with inaccurate isotach (line of constant velocity) distributions across a section. That is, calculations of mean cross-flow velocities do not serve as a reliable test of isotach distribution calculations.

In the remaining typical plots shown for this case, experimental results are shown by solid lines, contrary to the general convention. The development of isotach patterns along the duct is most accurately matched by the results of Group 22, and these results are shown in Fig. 4. Figure 5 also shows the isotach distribution at $x/D_h = 84$ for all three computors, and the best agreement is seen for Group 22. The results of Group 07A show none of the isotach distortion which results from secondary flow. The Reynolds-stress-distribution results of Group 22 agreed best with the measurements, and these results are shown in Fig. 6. (Note that this plot of a single component of Reynolds stress should not exhibit symmetry about the corner bisector.) The very crowded superposed Fig. 7 showing the distribution of one Reynolds-stress component at $x/D_h = 84$ is readable if one notices that at $z/a = 1.0$, the agreement among all computed results and the measurements makes it possible to identify the Reynolds stress value of each line.

For clarity, Fig. 8 shows lines of constant-turbulence kinetic energy for only two values. For these two values the results of Group 23 are good, but for two intermediate values (40×10^{-4} and 50×10^{-4}) they are poor. A summary of the subjective evaluations of the three sets of computed results in comparison with measurements is given by the following table which indicates that no one of the sets of computations is superior in all respects to the others. G, F, and P stand for good, fair, and poor, with some shading being indicated by the plus and minus signs.

		CONTOURS			
	U vs. x	U	$\overline{uv}$	K	τ_w vs. y
AKEZ 07A	F	P	F	G-	G
AOPX 22	F	G	F+	P	F
AKEX 23	F	F	F-	P	F

Group 22 submitted the results of calculations of the optional laminar flow case (see Plate 187, Vol. III), and in all respects the agreement with measured results is superb.

Summary of Computations for Case 0112--Fully Developed Flow in Rectangular Duct With Peripherally Non-Uniform Roughness*

Computor Group 07A alone submitted results for this flow. Specifications called for results only at $x/D_h = 126$, where the flow would be essentially fully developed.

The isotach distribution shown in Fig. 9 must be regarded as excellent. Notice that the maximum velocity does not occur on the duct centerline. The mean-velocity streamwise component distribution at the duct centerline shown in Fig. 10 is also excellent in view of the complexity of this flow. As in other cases of secondary flow of the second kind, the distribution of a cross-flow component of the mean velocity does not agree well with measurements, as shown by Fig. 11. (This plot is dimensional, so it is worth noting that the mean axial velocity was 15 m/s.) The calculated distributions of the three components of turbulence intensity shown on Fig. 12 are also quite good in view of the complexity of this flow.

Summary of Computations for Case 0113 (P1)--Asymmetric Flow in a Square Duct†

Computor Group 07A alone submitted results for this flow case. The results are generally quite good.

Not shown here is the comparison of calculated and measured pressure coefficient, C_p. The calculated value is approximately 10% above the measured value along the duct.

Figure 13 shows good agreement for the distribution of the axial component of mean velocity. Figure 14 shows remarkably good agreement for the distribution of one of the normal components of the mean velocity.

Figure 15 shows distributions of two of the normal Reynolds stresses and the total turbulence kinetic energy. Both the total kinetic energy and the component normal Reynolds stresses agree very well with experimental values.

The two Reynolds shear stresses shown in Fig. 16 agree reasonably well with measured results. Finally, the wall shear-stress distribution around the channel at the specified axial station is shown in Fig. 17 to be quite good.

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*See also Plate 8, Volume III.

†See also Plate 179, Volume III.

Summary of Computations for Case 0511--Wing-Body Junction*

Two computor groups using methods AOPX 22, BKEZ 07, respectively, submitted results for this flow, but Group 22 submitted two sets, one labeled $C_2 = 0.30$ and the other labeled $C_2 = 0.25$.

The variation of C_f on the body with distance z from the wing is shown in Fig. 18. The results of Group 07 are closer to the data than either of the results obtained by Group 22, but the Group 07 results do show a non-zero slope of C_f vs z at the highest z value shown. C_f on the wing is shown in Fig. 19 to be calculated also more accurately by Group 07; however, it should be noted that the data at $x = 0.6858$ m at y values lower than 0.01 do suggest the shape of the curve shown by Group 22 for the case of $C_2 = 0.30$.

Distributions of the Reynolds stress $\overline{uv}$ at $x = 0.644$ m and $x = 1.254$ m are shown in Fig. 20. (No results for Group 07 are shown at $z = 0.02337$ m because they were not presented by the computor. Group 07 presented results at $z = 0.01003$ and 0.01837.) The effect of the C_2 value on the Group 22 results is significant. It should be noted that the value of $C_2 = 0.25$ is a special case, since this value implies that $(\overline{v^2} - \overline{w^2}) = 0$ and $\overline{vw} = 0$.

The distributions of the mean velocity components U, V, and W are shown in Figs. 21, 22, and 23, respectively at $x = 1.254$ m only. Differences among the results of different computors are less at this location than at the upstream location of $x = 0.644$ m. In general, the results of Group 07 are in fairly good agreement with measurements. The sensitivity of the Group 22 results to the value selected for C_2 is seen to be strong.

Summary of Computations for Case 0512--Square Duct with a 90° Bend†

Six computor groups using methods AOPX 22, BOPX 01, BIKX 41, BKEY 33, BKEZ 07, BKEZ 17G submitted results for the flow through a 90° bend in a duct of square cross-section. Computor Group 22 submitted two sets of results: one with the radial pressure gradient considered and the other without. A review of results showed that in all cases the results with the radial pressure gradient considered were superior to or at least equal to those based on no consideration of the radial pressure gradient; therefore, the results of the calculation based on no radial pressure gradient are not considered further here. Also, Group 17 had submitted results designated as BKEZ 17G and 17I; these are shown on Plates 126 through 132.

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*See also Plates 121 through 125, Volume III.

†See also Plates 126 through 132, Volume III.

Not all computors presented all of the specified results or results for the specified values of parameters. Consequently, for this case, some interpolation among the results of a given computor group had to be made to facilitate comparisons of results.

Results were called for at the beginning of the bend, $\theta = 0$, only three hydraulic diameters downstream from the section at which initial conditions were specified, and at $\theta = 90°$ as well as at two intermediate sections. Comparable results are shown here only for the stations at $\theta = 0$ and $\theta = 90°$.

Figure 24 shows good general agreement on the streamwise mean-velocity component at the inlet to the bend but significant disagreement at the outlet. It is difficult to select a "best" set of results.

The earlier specifications regarding the radial velocity component contained a decimal point error which caused some confusion among computors and caused some not to submit results on this quantity. Qualitatively, all methods except that of Group 22 give reasonable results as shown in Fig. 25.

Figure 26 shows that calculations of the Reynolds-stress distribution at 90° lead only to chaos. Among all the cases involving secondary flow of the second kind, Case 0512 has produced the least satisfactory calculated results.

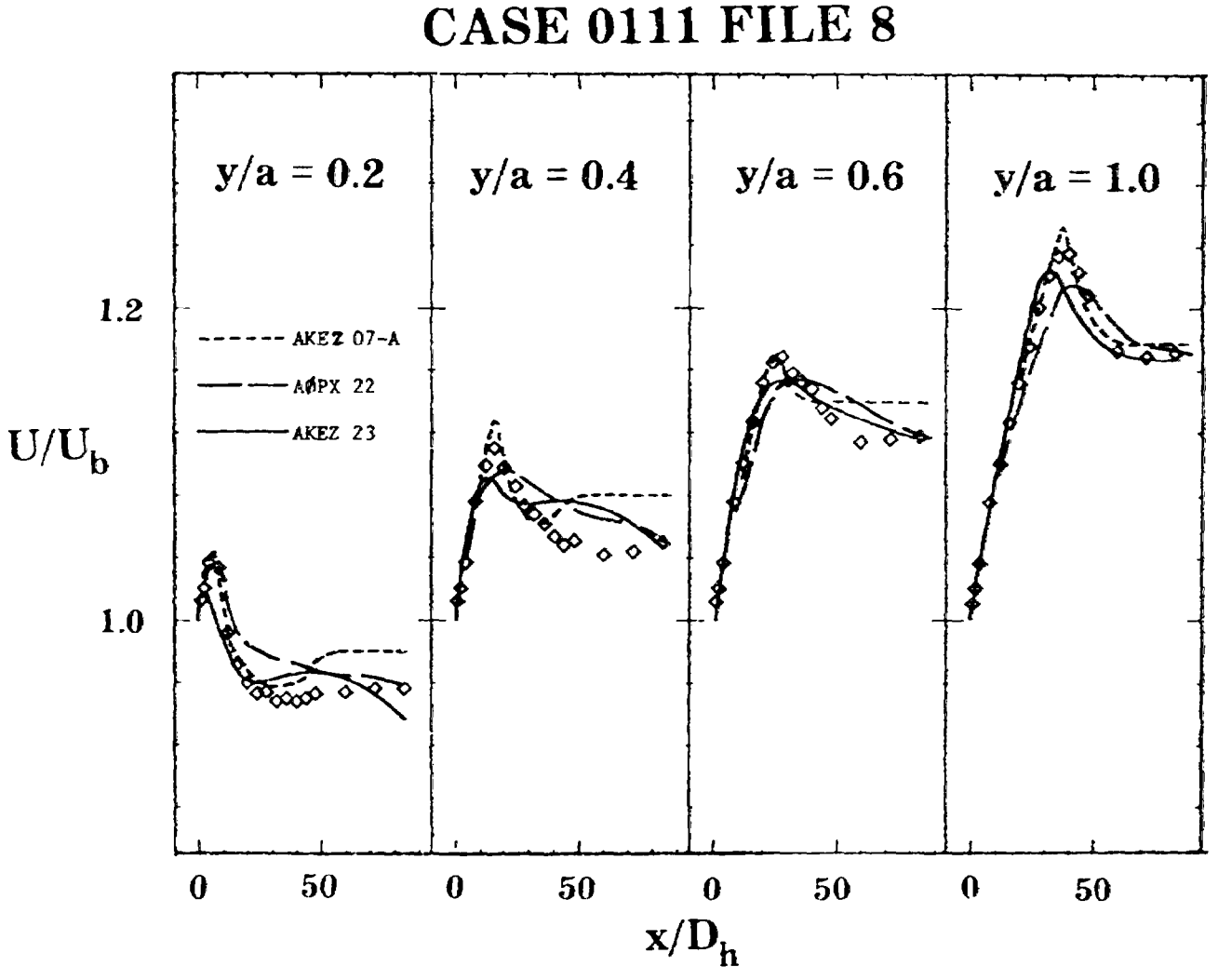

CASE 0111 FILE 8
y/a = 0.2
y/a = 0.4
y/a = 0.6
y/a = 1.0
1.2
1.0
U/U_b
AKEZ 07-A
AØPX 22
AKEZ 23
0
50
0
50
0
50
0
50
x/D_h

Figure 1.

CASE 0111 FILE 9

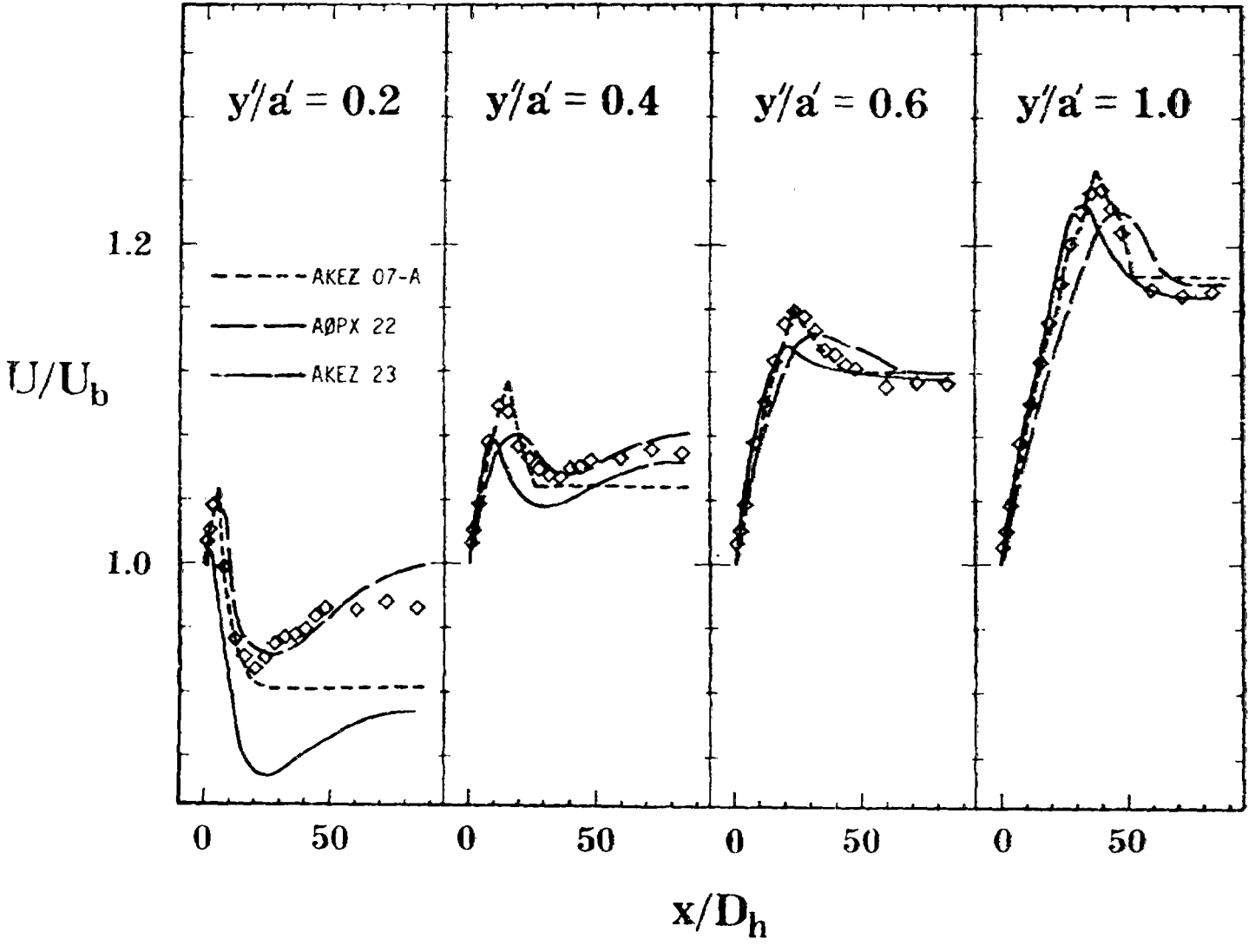


Figure 2.

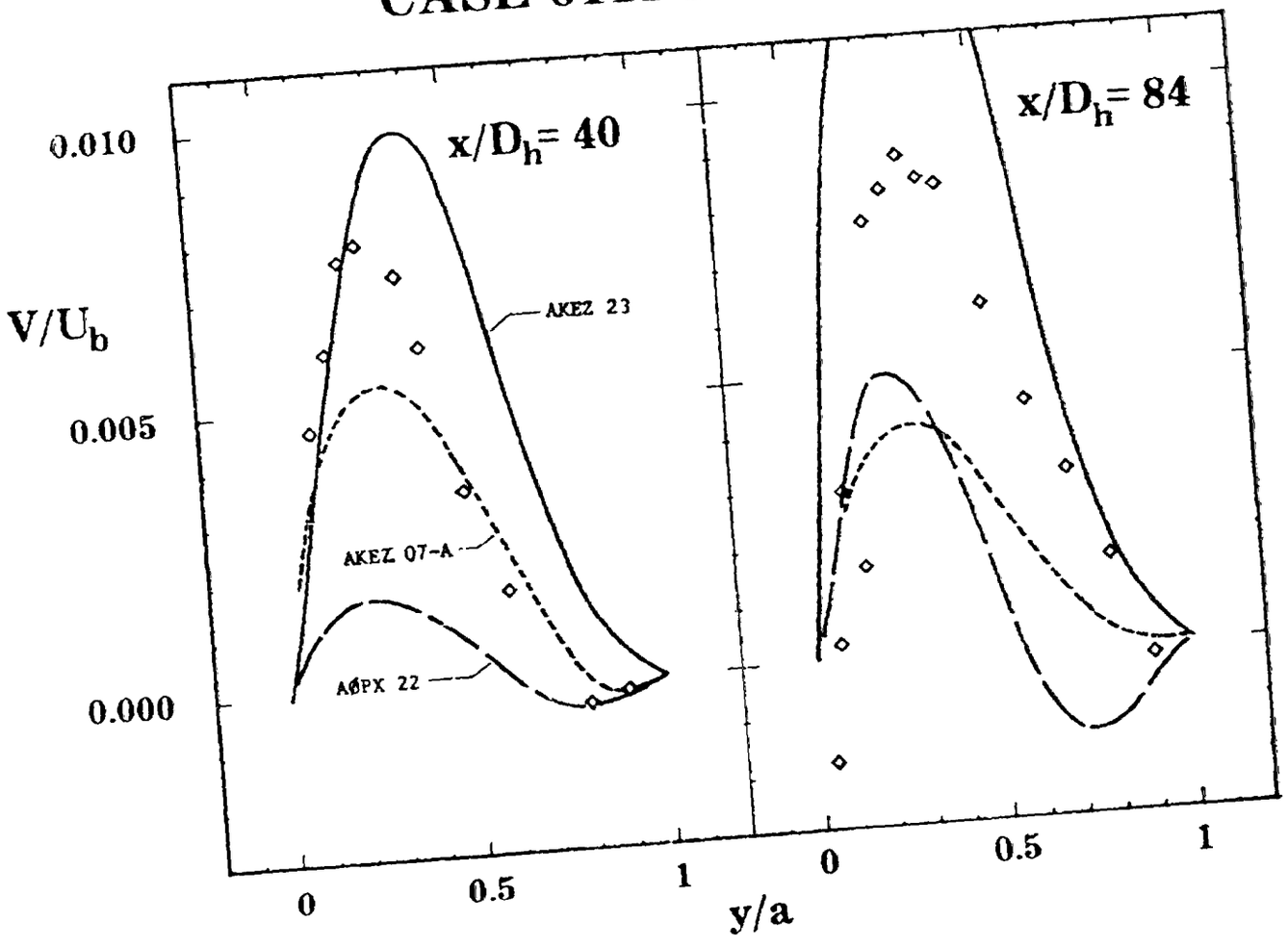

CASE 0111 FILES 10, 12
x/D_h= 40
x/D_h= 84
0.010
0.005
0.000
V/U_b
AKEZ 23
AKEZ 07-A
AØPX 22
0
0.5
1
0
0.5
1
y/a

Figure 3.

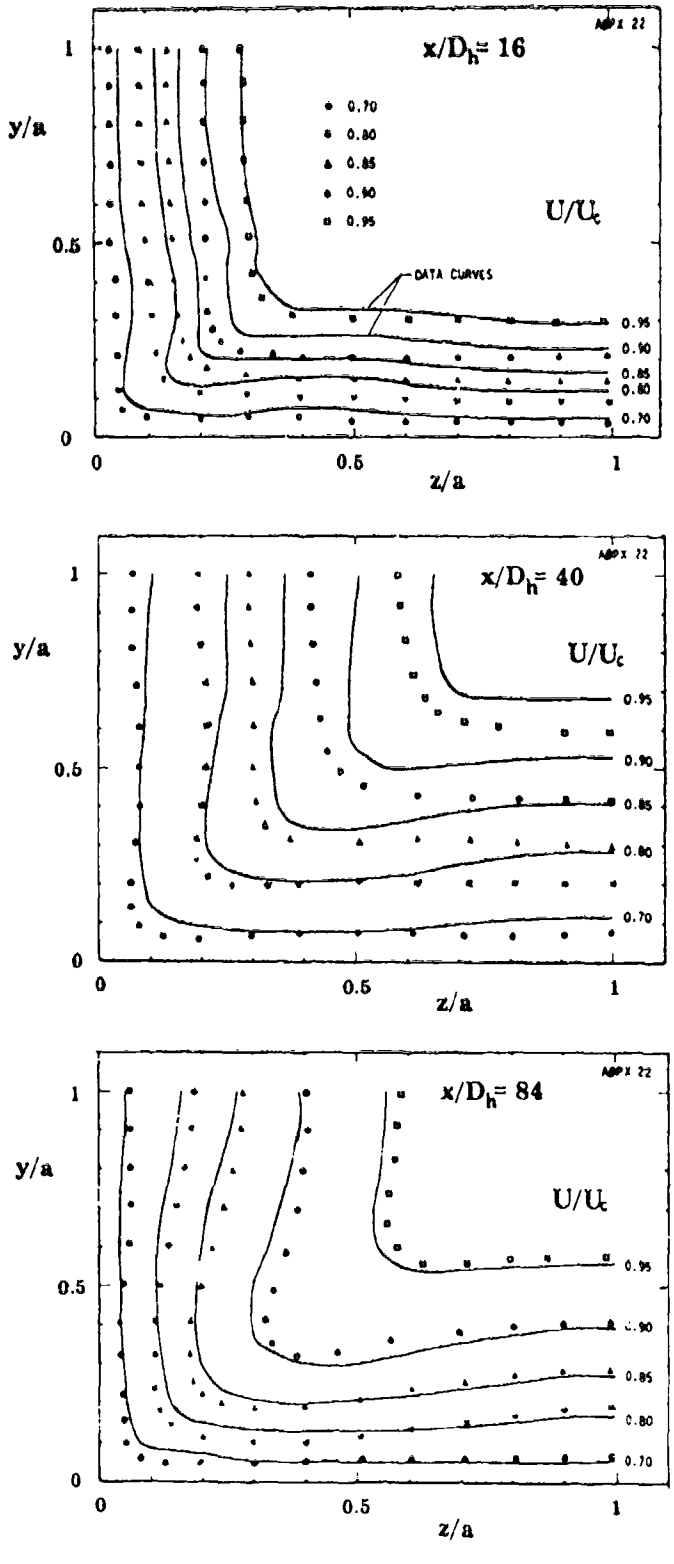


Figure 4.

CASE 0111 FILE 18

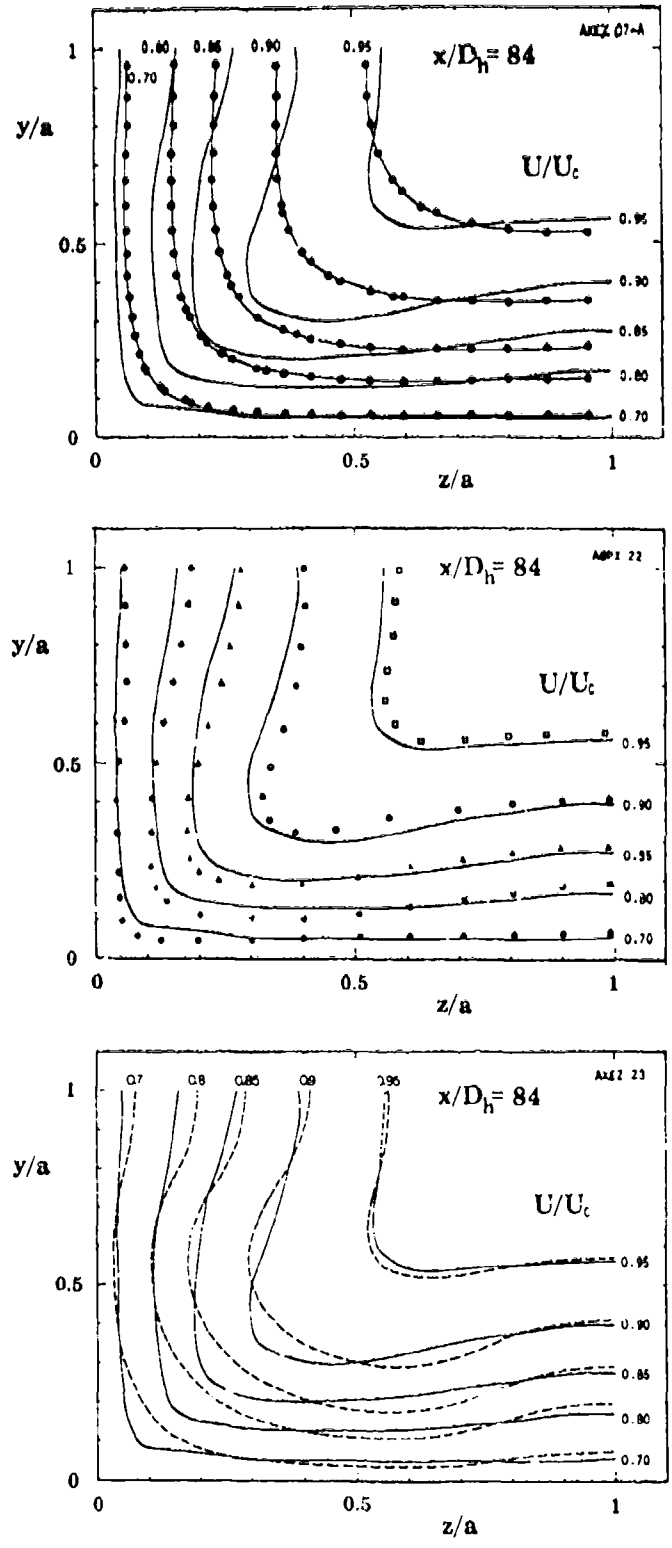


Figure 5.

CASE 0111 FILE 19

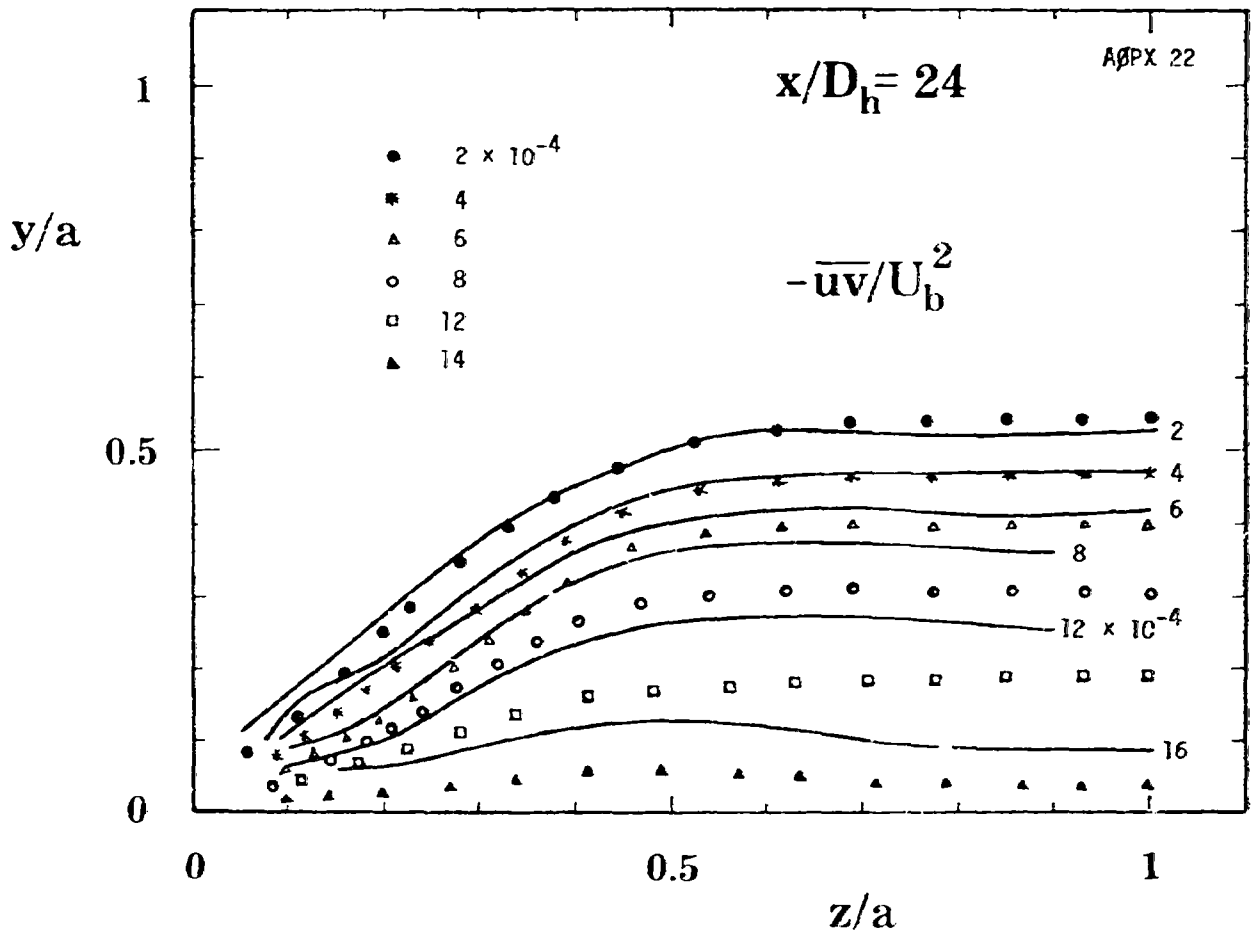


Figure 6.

CASE 0111 FILES 26,27

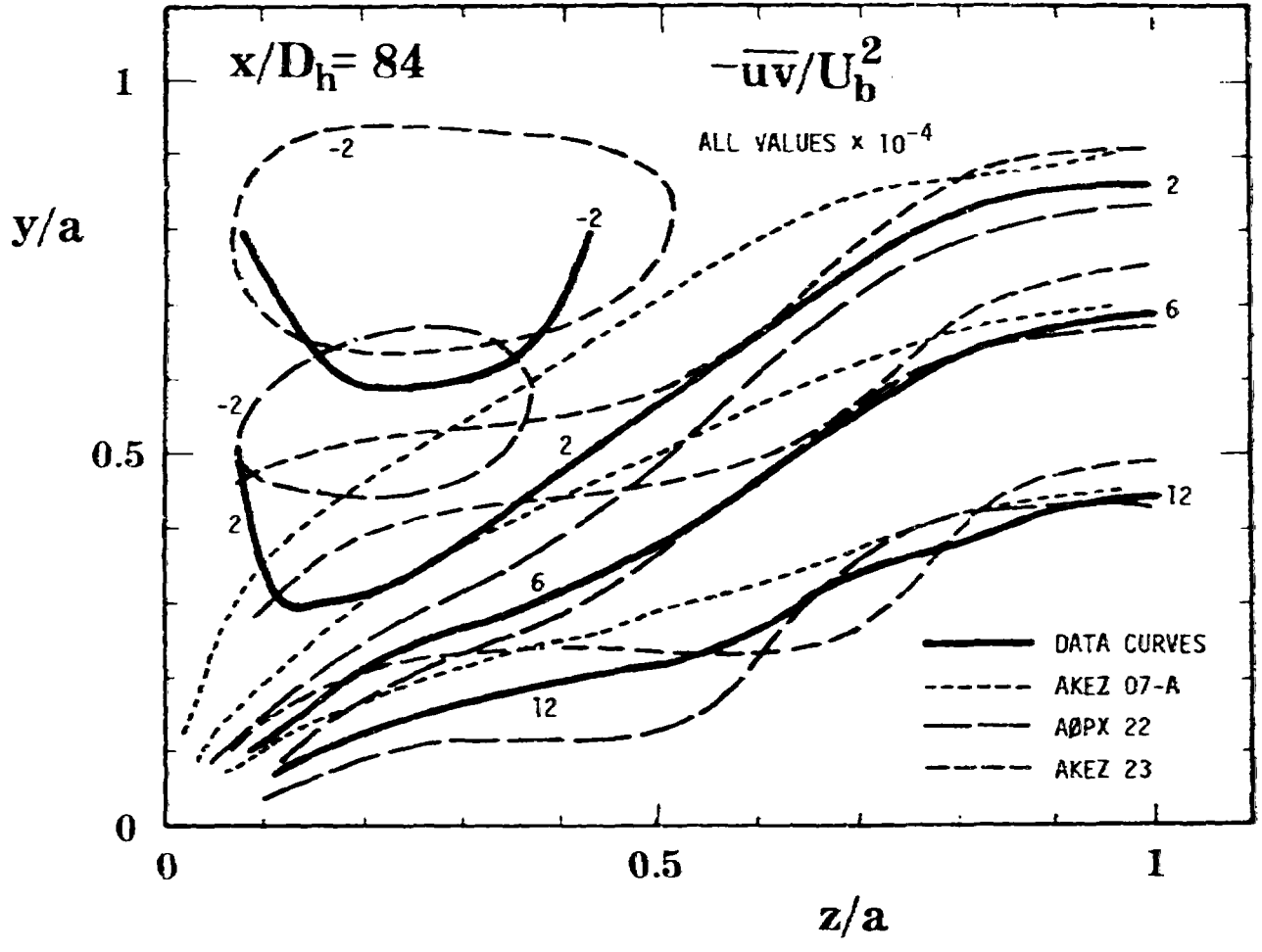


Figure 7.

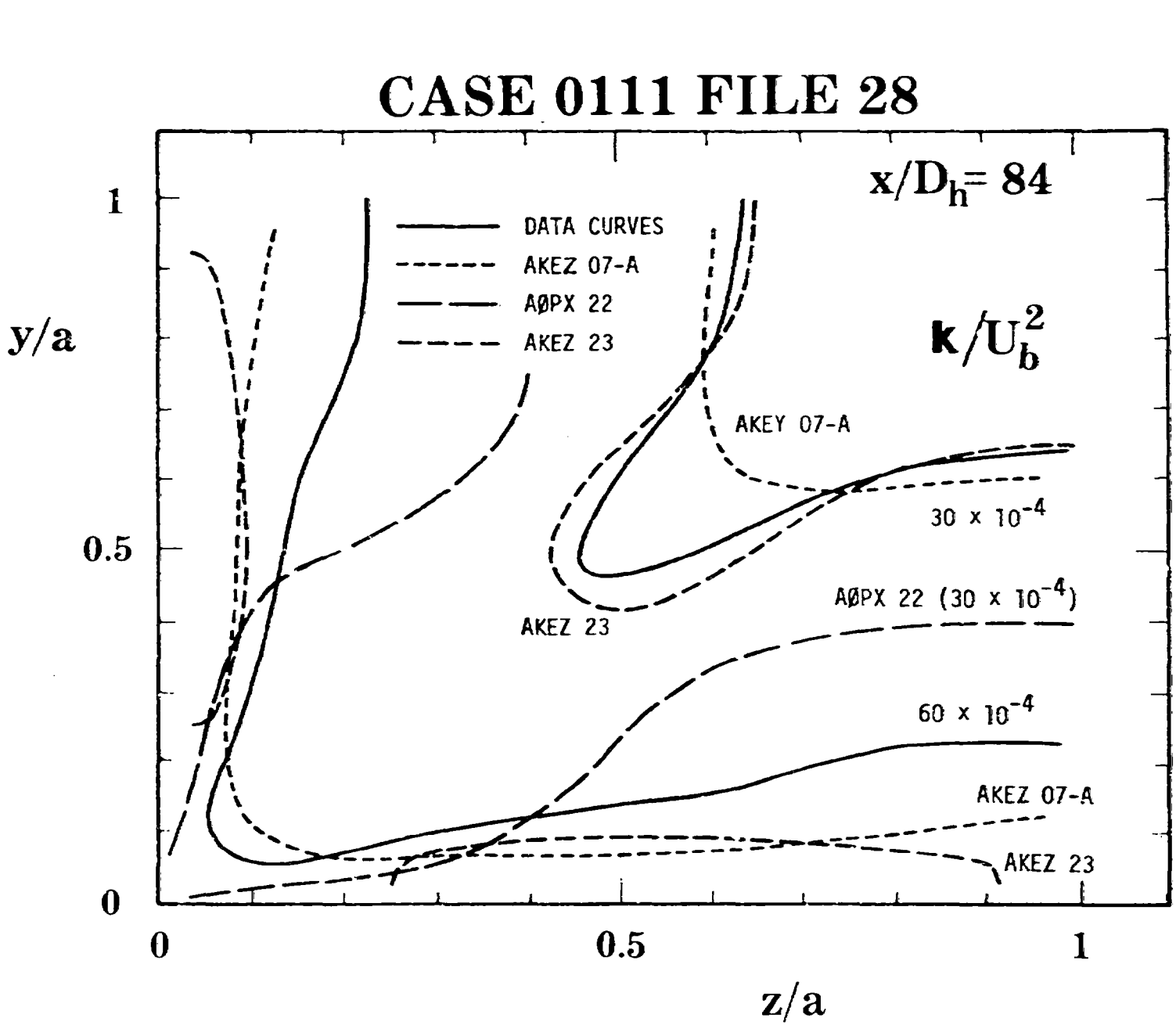

CASE 0111 FILE 28
$x/D_h = 84$
DATA CURVES
AKEZ 07-A
AØPX 22
AKEZ 23
k/U_b^2
AKEY 07-A
30×10^{-4}
AKEZ 23
AØPX 22 (30×10^{-4})
60×10^{-4}
AKEZ 07-A
AKEZ 23
y/a
z/a
0
0.5
1

Figure 8.

CASE 0112 FILE 2

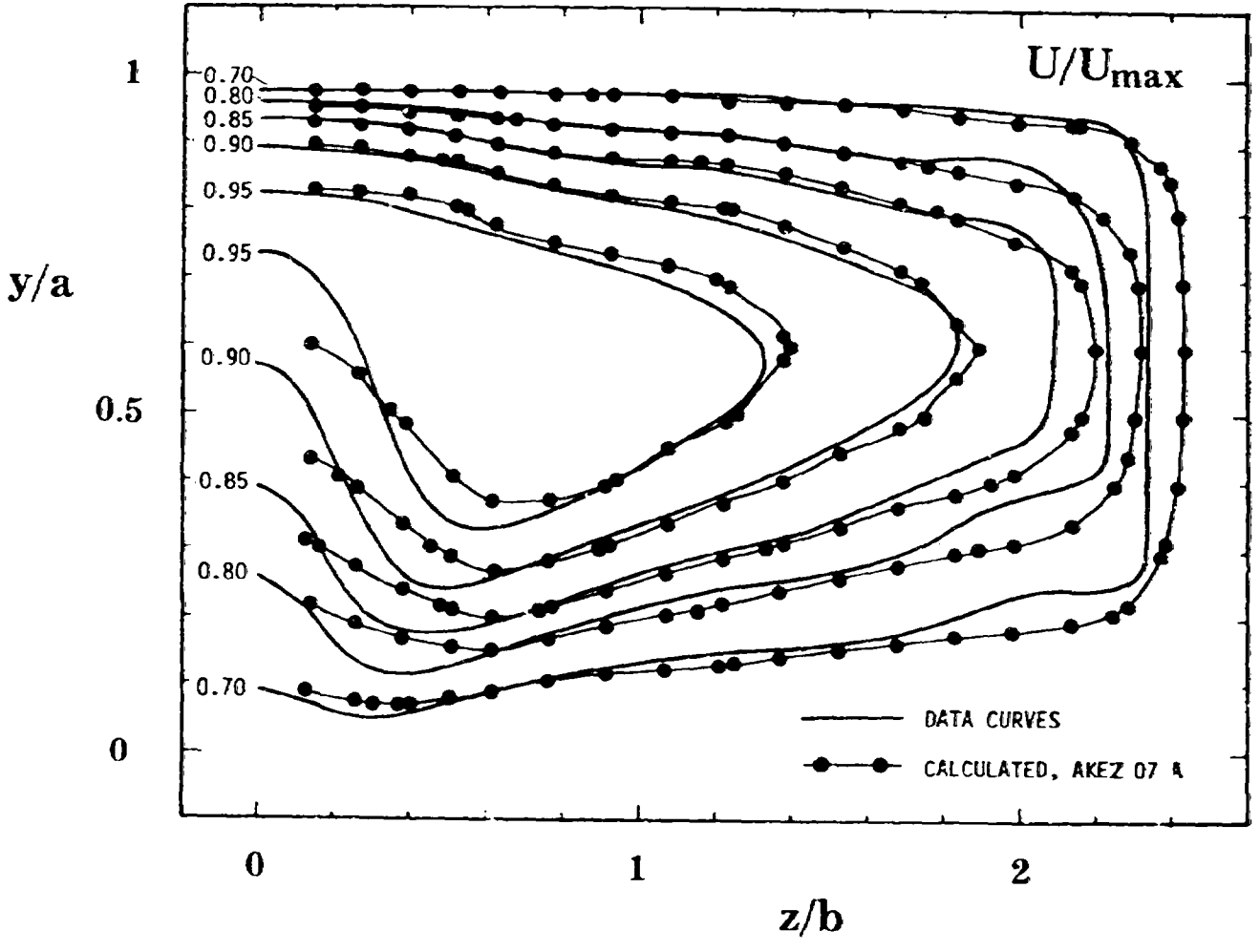


Figure 9.

CASE 0112 FILE 4

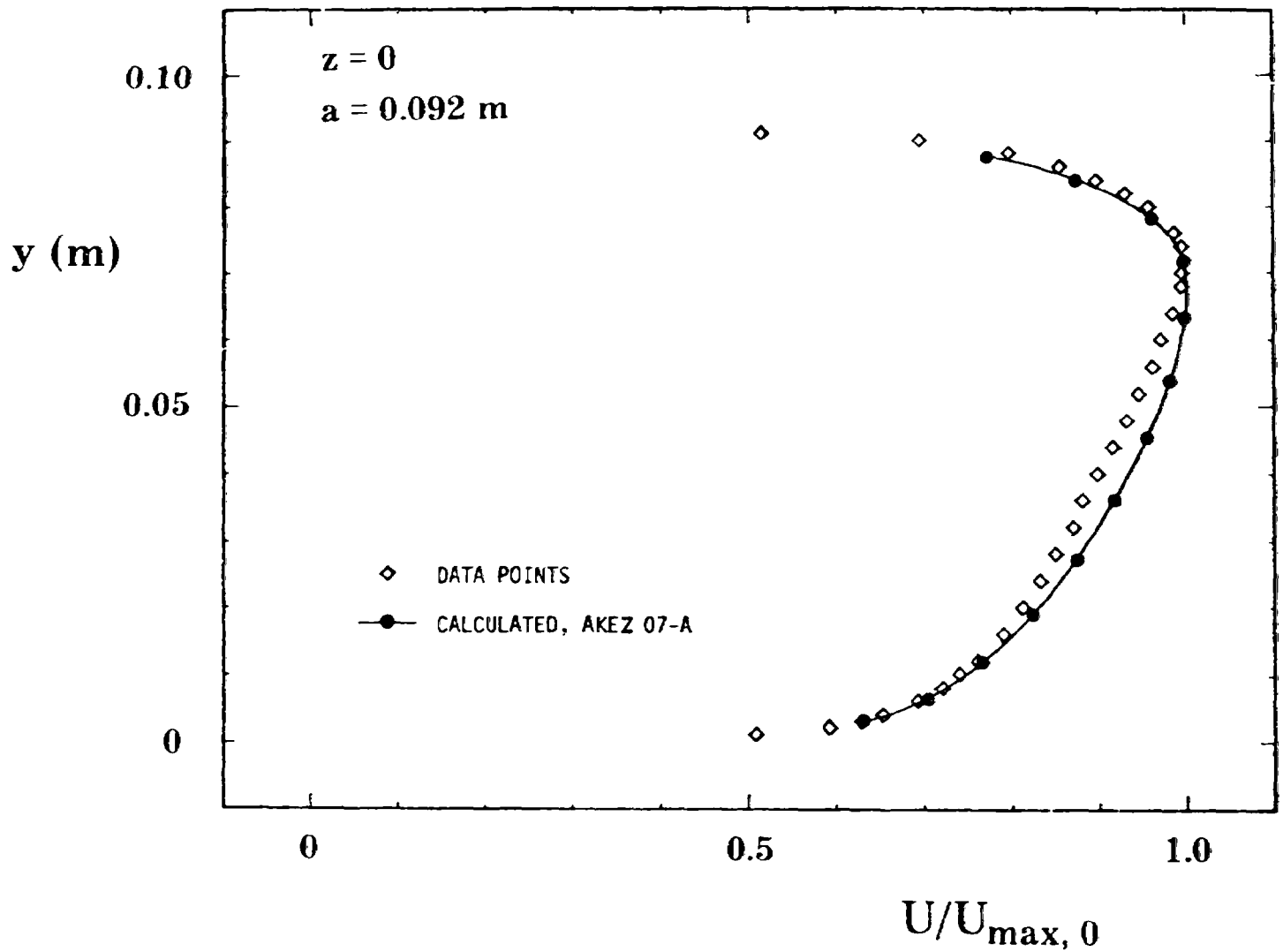


Figure 10.

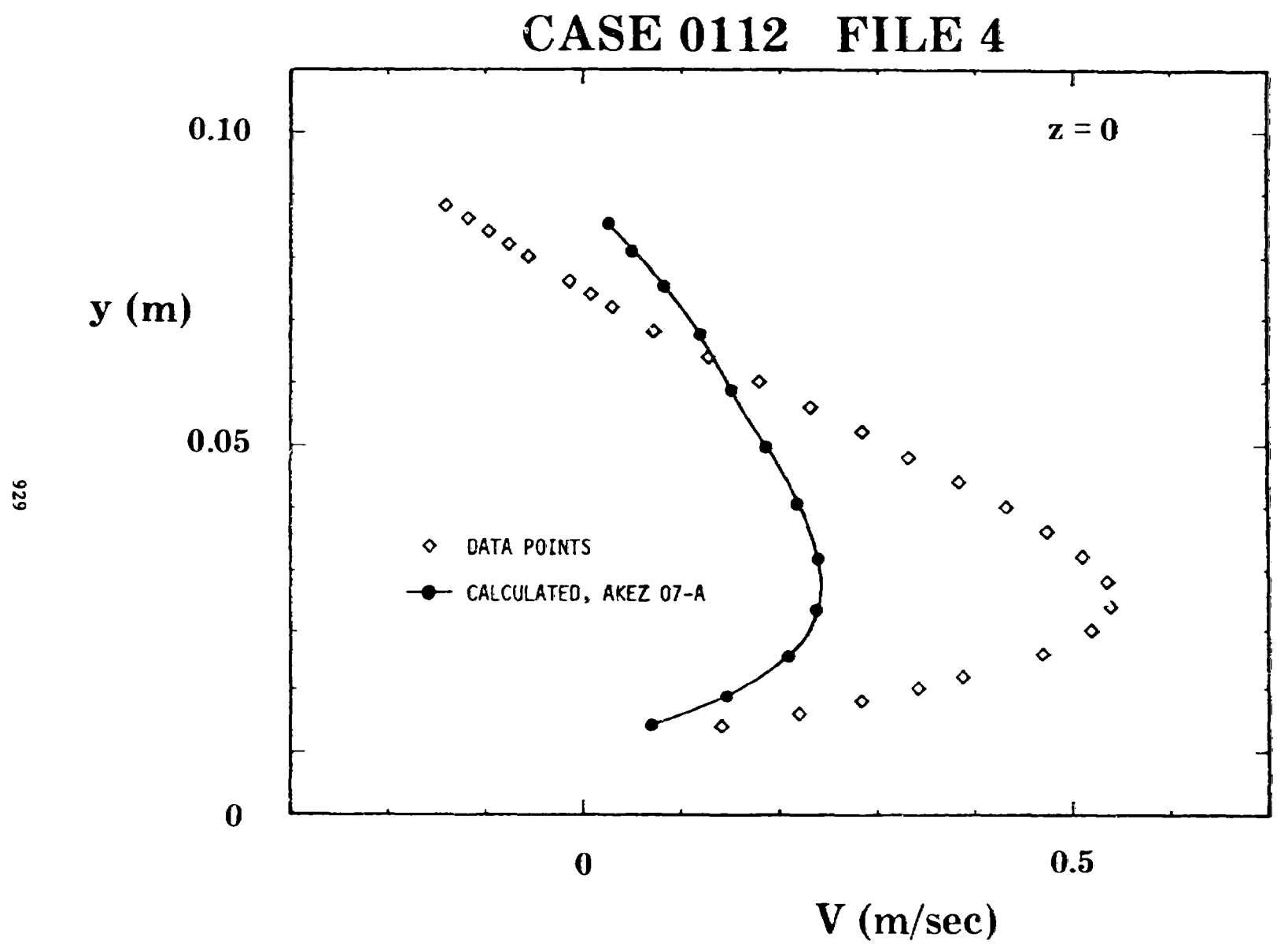

CASE 0112 FILE 4
z = 0
0.10
0.05
0
y (m)
0
0.5
V (m/sec)
DATA POINTS
CALCULATED, AKEZ 07-A

Figure 11.

CASE 0112 FILE 4

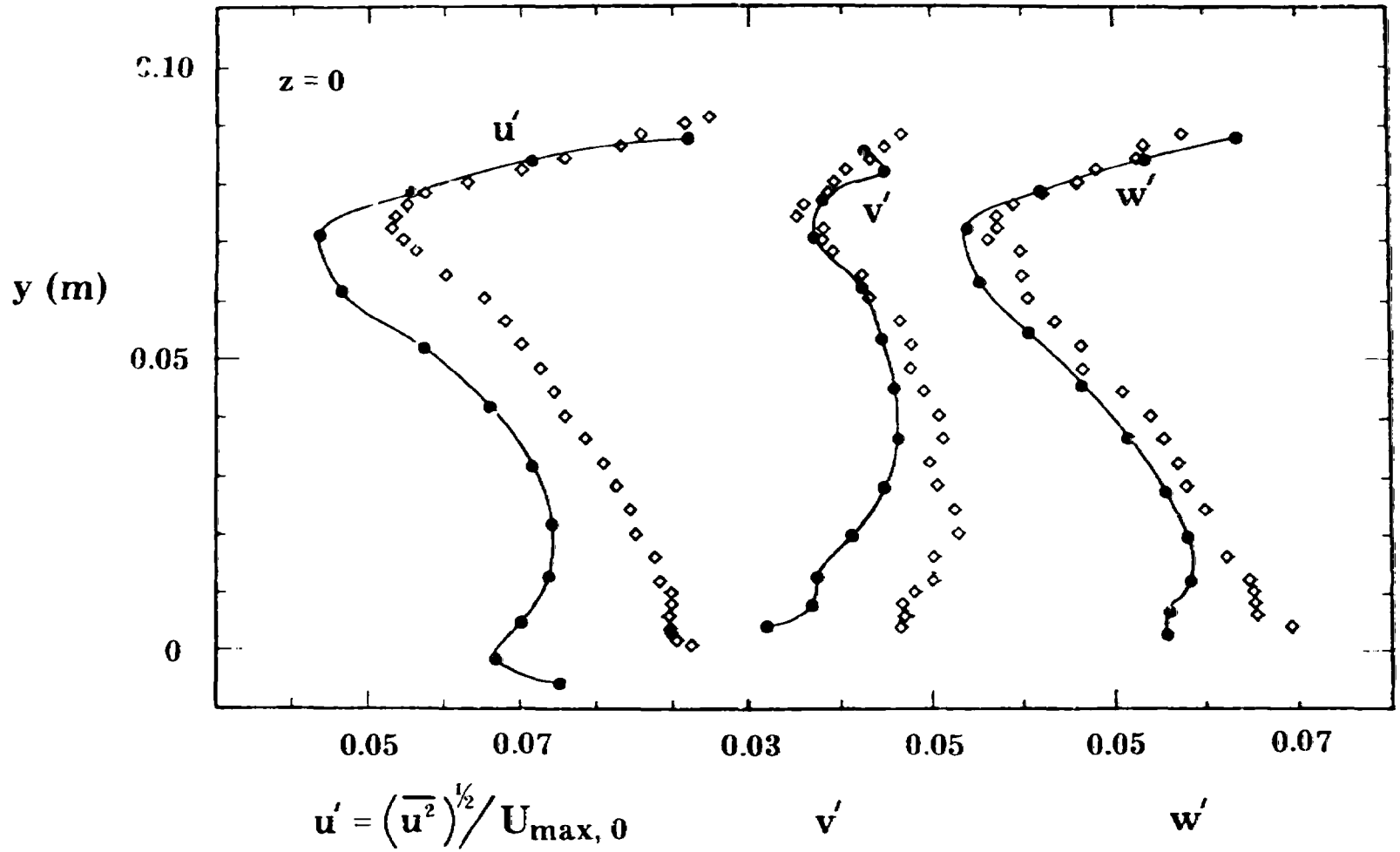


Figure 12.

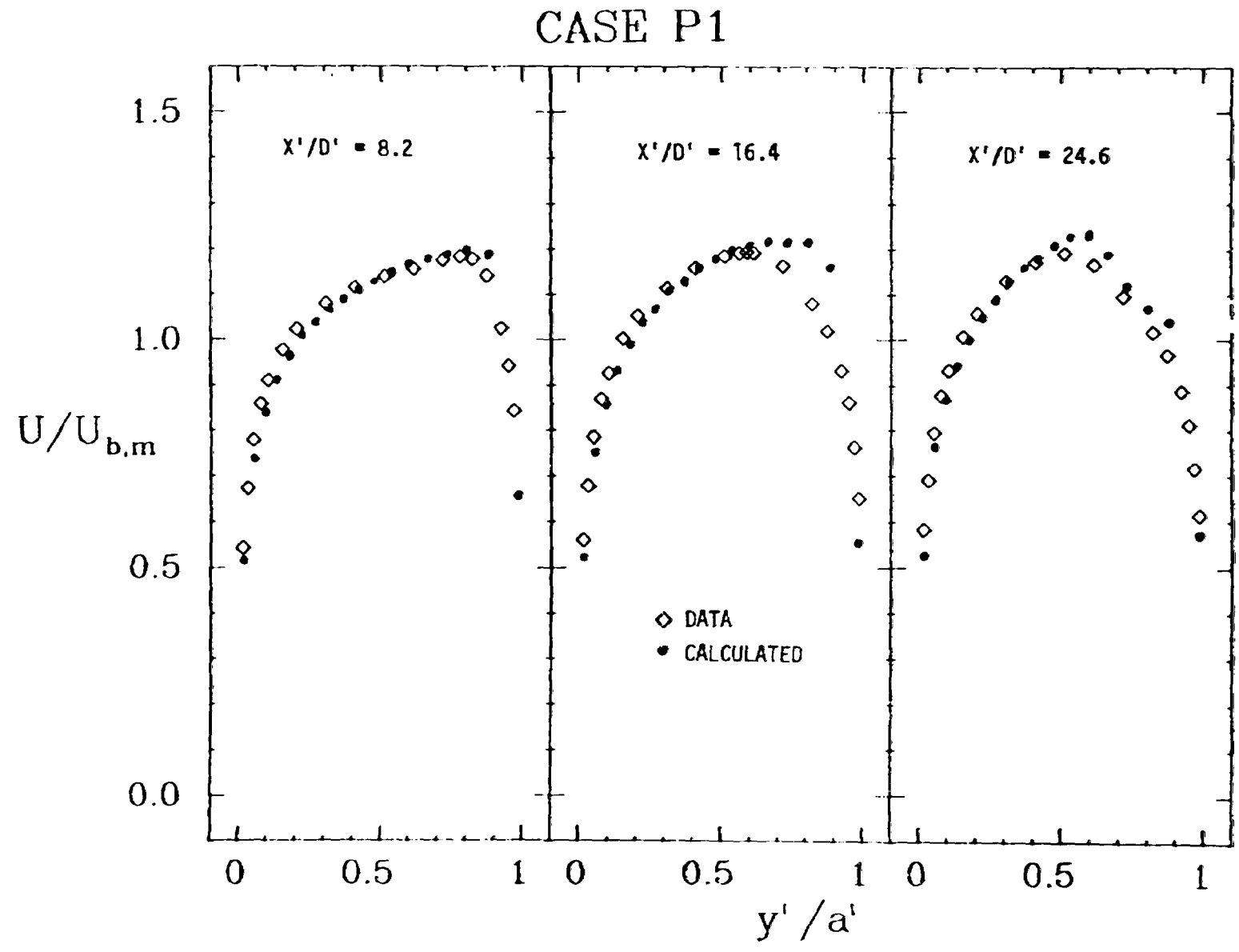

CASE P1
X'/D' = 8.2
X'/D' = 16.4
X'/D' = 24.6
1.5
1.0
0.5
0.0
U/U b,m
◇ DATA
• CALCULATED
0
0.5
1
y' /a'

Figure 13.

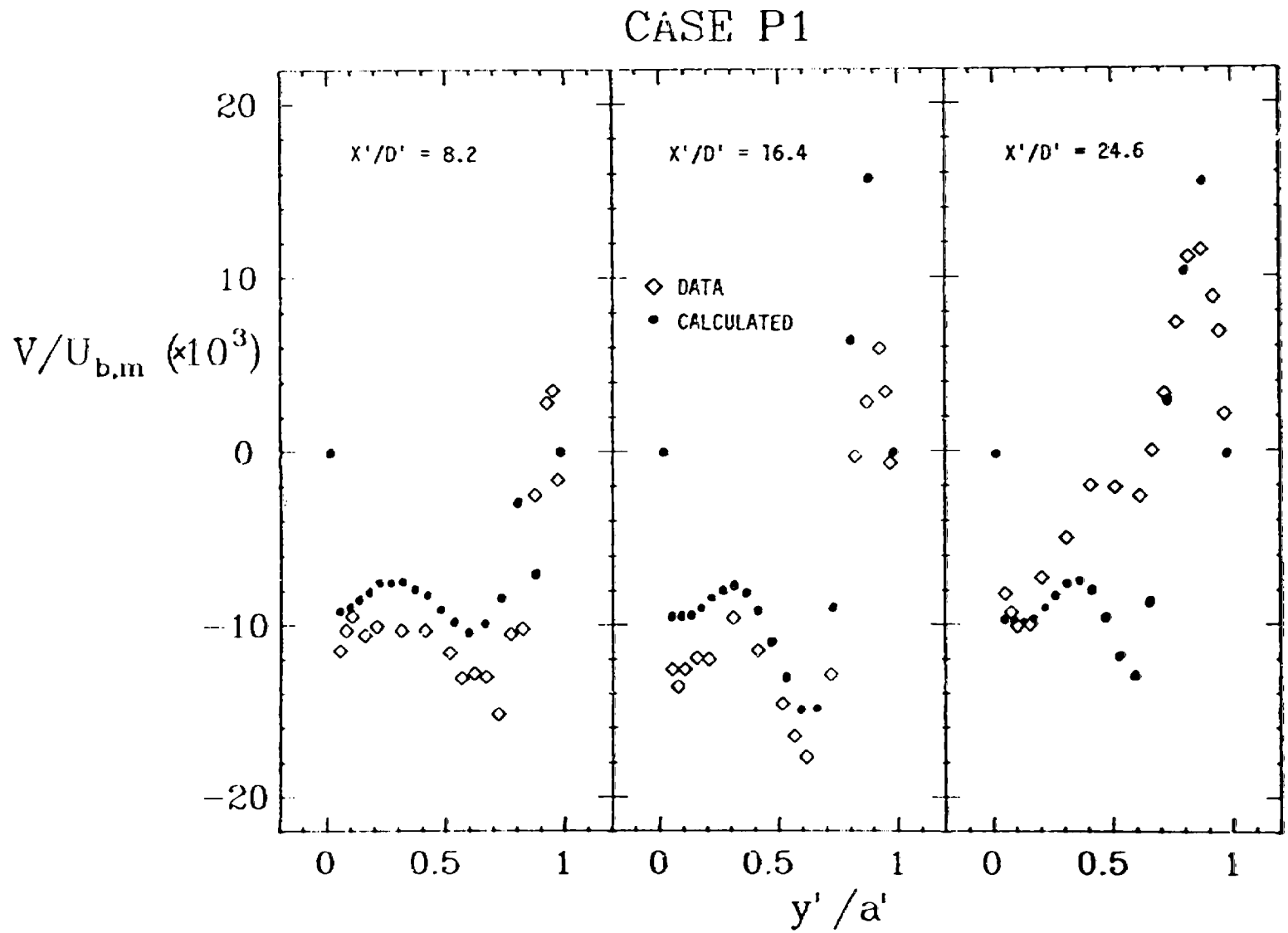

CASE P1
X'/D' = 8.2
X'/D' = 16.4
X'/D' = 24.6
DATA
CALCULATED
V/U_b,m (×10^3)
20
10
0
-10
-20
0
0.5
1
y'/a'

Figure 14.

CASE P1

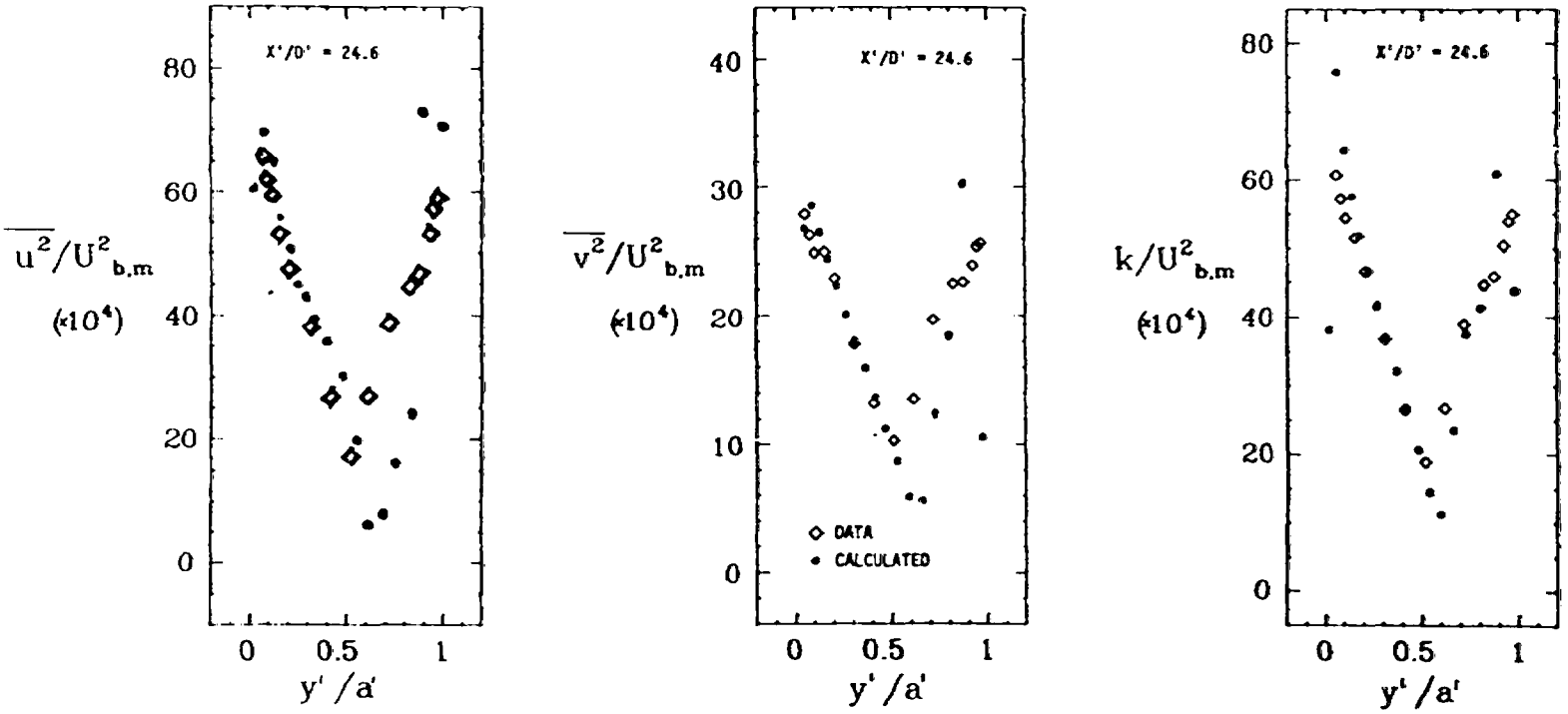
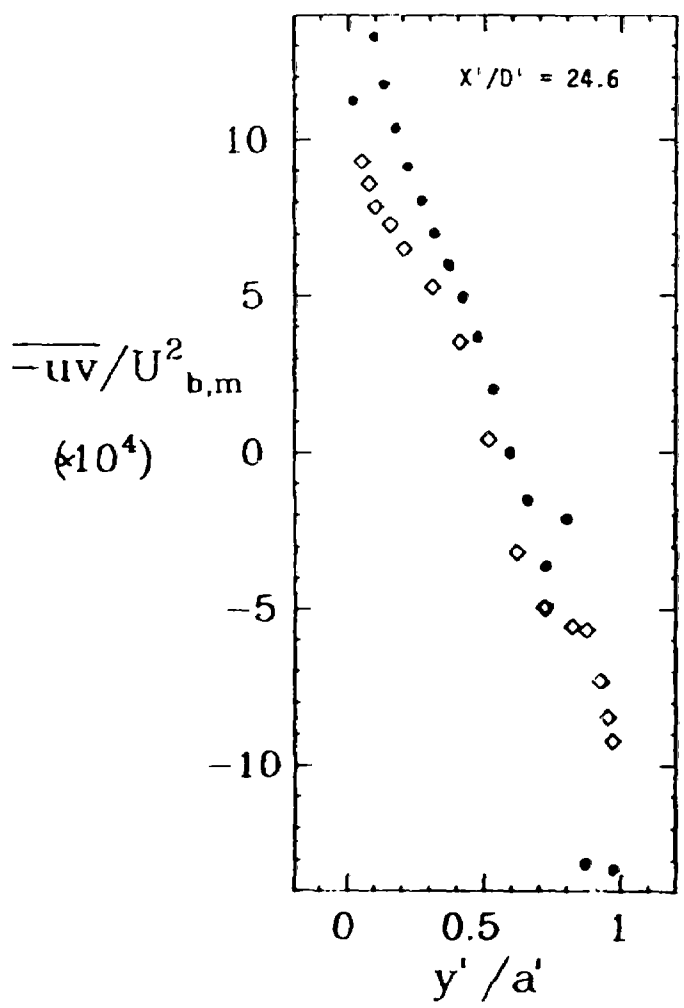


Figure 15.

CASE P1



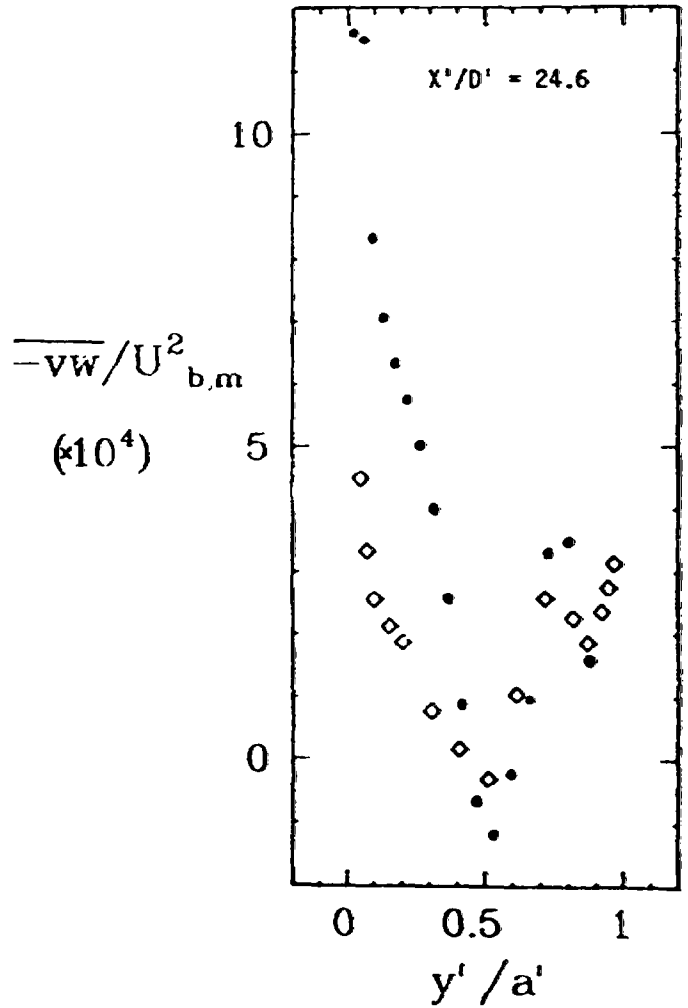


Figure 16

CASE P1

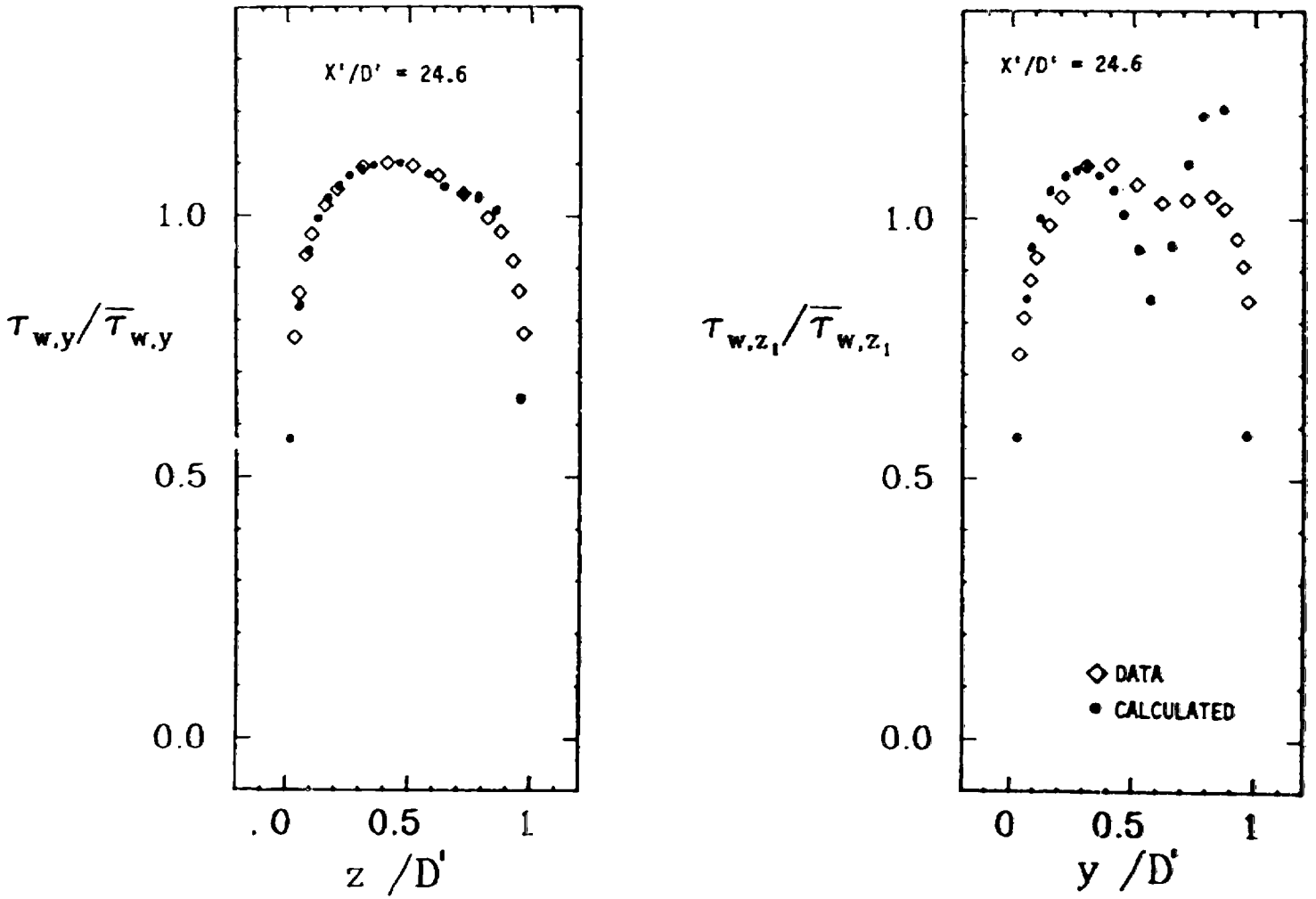


Figure 17.

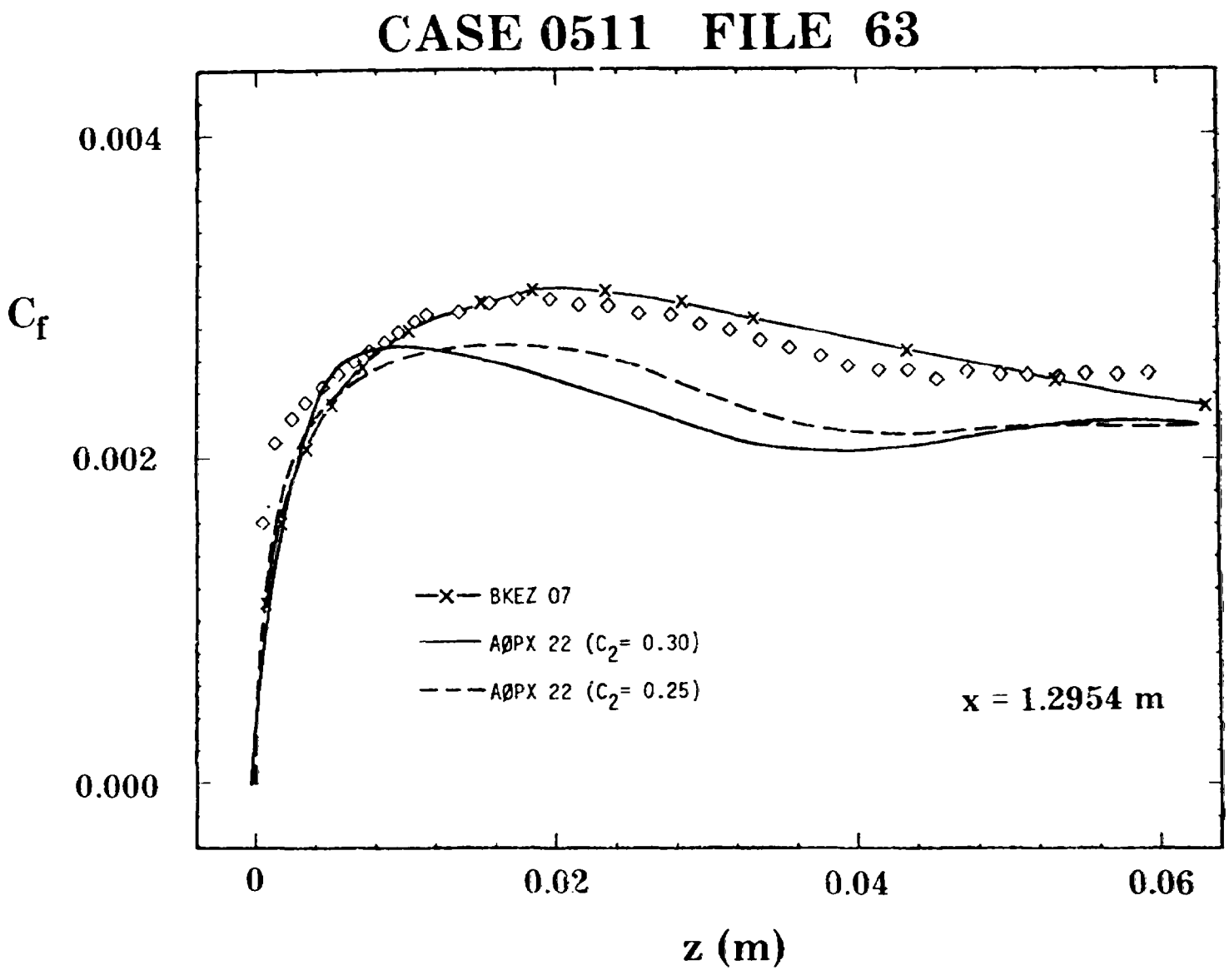

CASE 0511 FILE 63
0.004
0.002
0.000
C_f
0
0.02
0.04
0.06
z (m)
—x— BKEZ 07
—— AØPX 22 (C_2= 0.30)
– – – AØPX 22 (C_2= 0.25)
x = 1.2954 m

Figure 18.

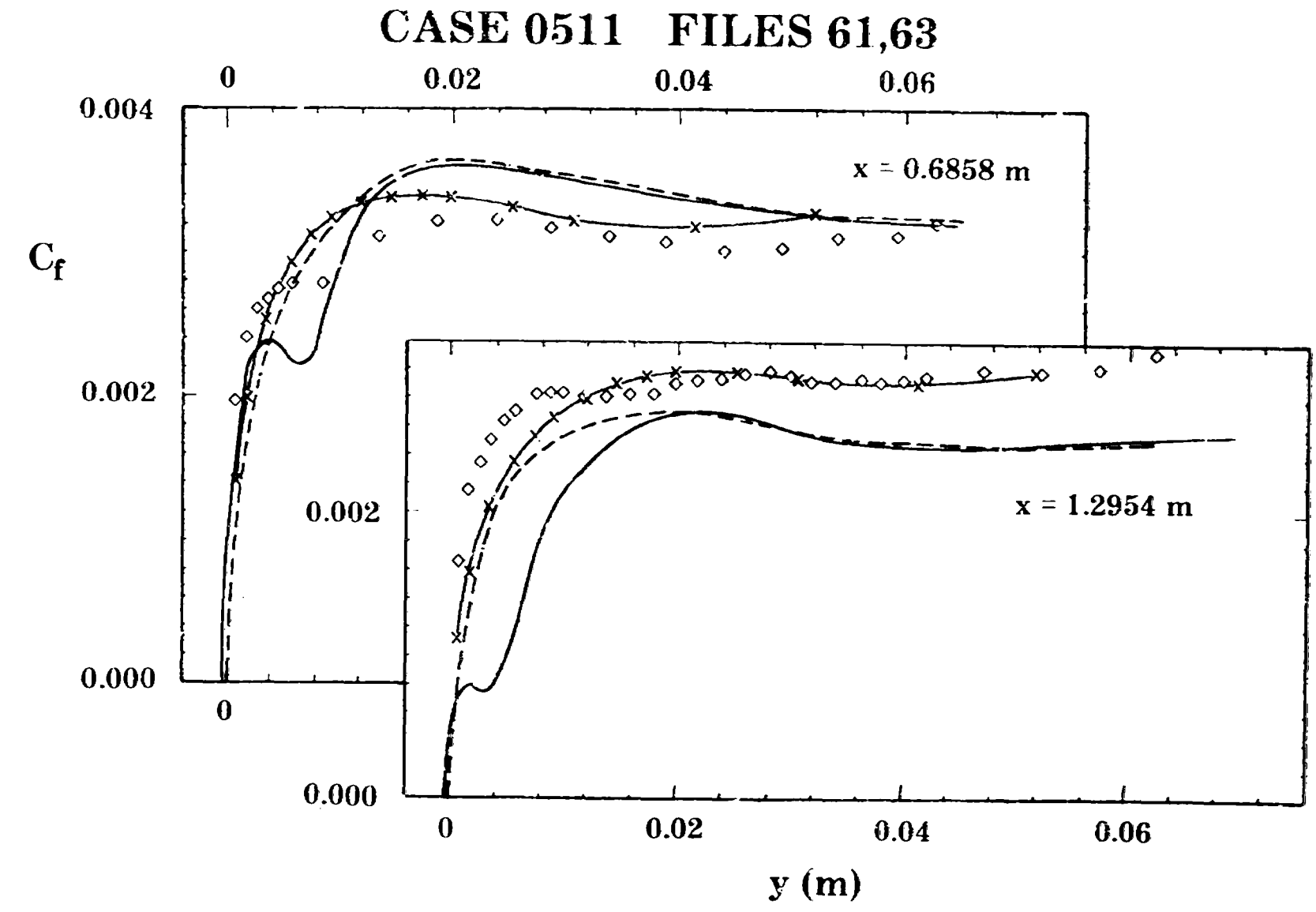

CASE 0511 FILES 61,63
0
0.02
0.04
0.06
0.004
0.002
0.000
C_f
x = 0.6858 m
x = 1.2954 m
y (m)

Figure 19.

CASE 0511
FILES 90, 94, 98, 106, 114, 118, 122, 130

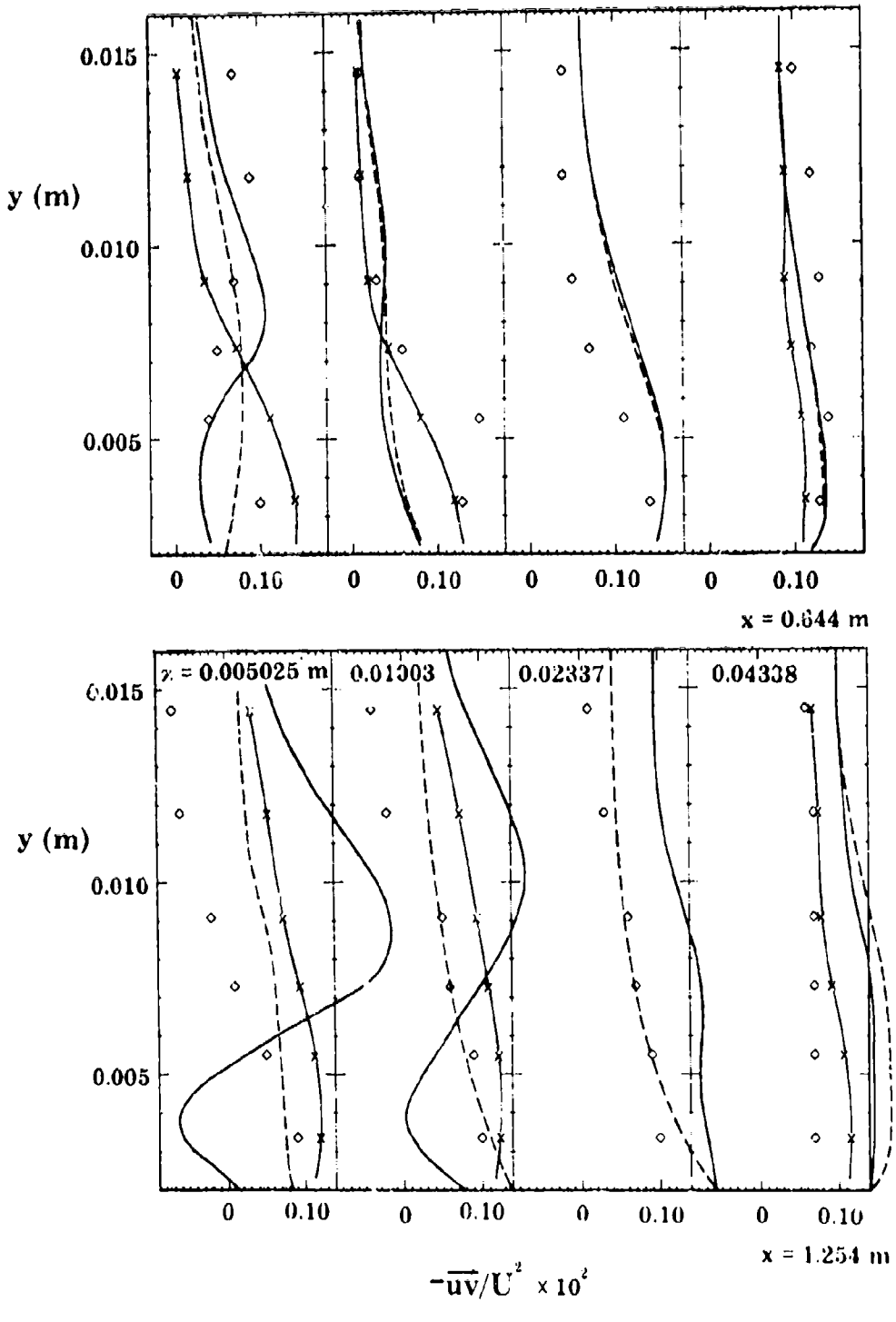


Figure 20.

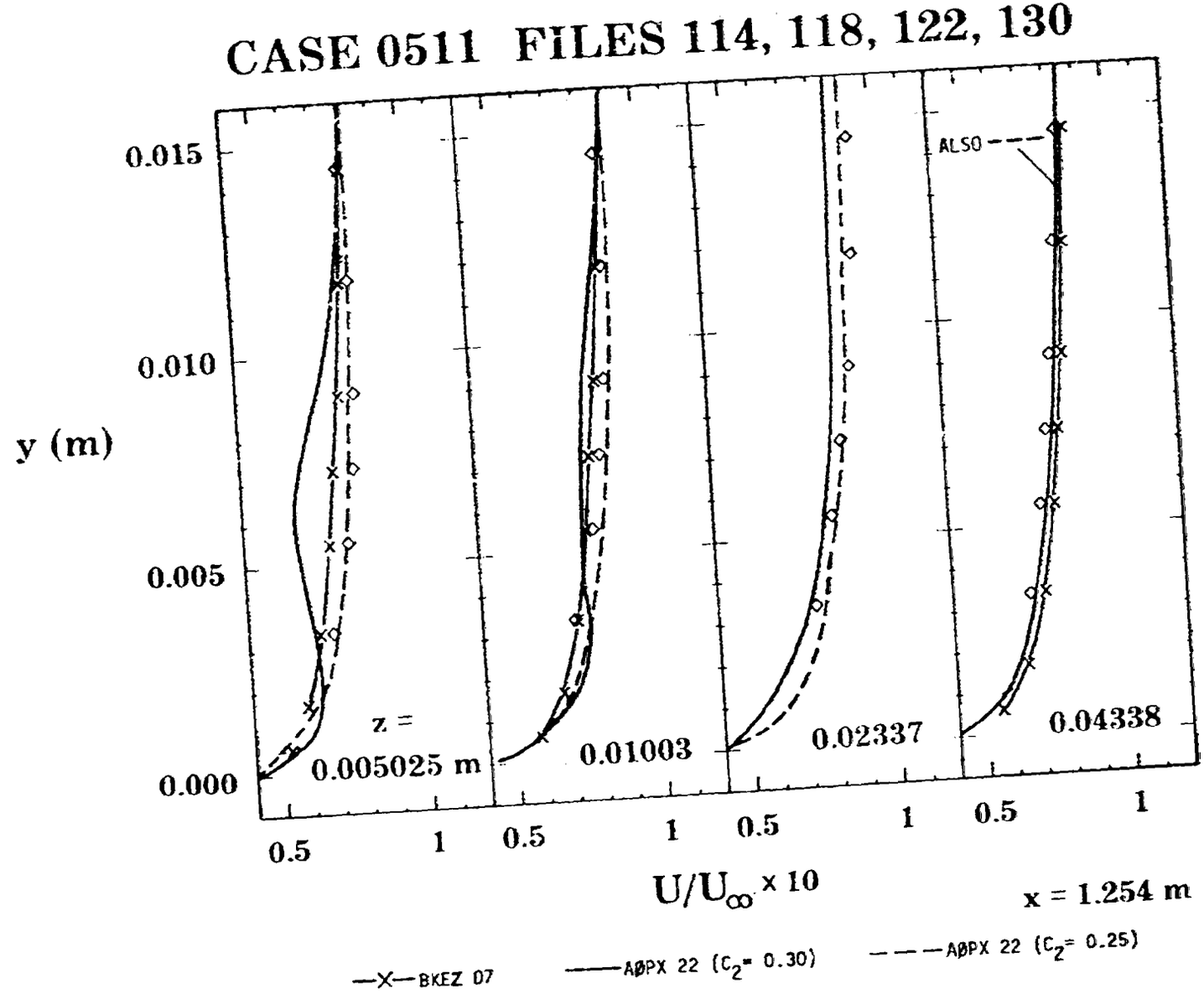

CASE 0511 FILES 114, 118, 122, 130
0.015
0.010
0.005
0.000
y (m)
ALSO
z =
0.005025 m
0.01003
0.02337
0.04338
0.5
1
0.5
1
0.5
1
0.5
1
U/U∞ x 10
x = 1.254 m
—X— BKEZ 07
—— AØPX 22 (C2= 0.30)
— — — AØPX 22 (C2= 0.25)

Figure 21.

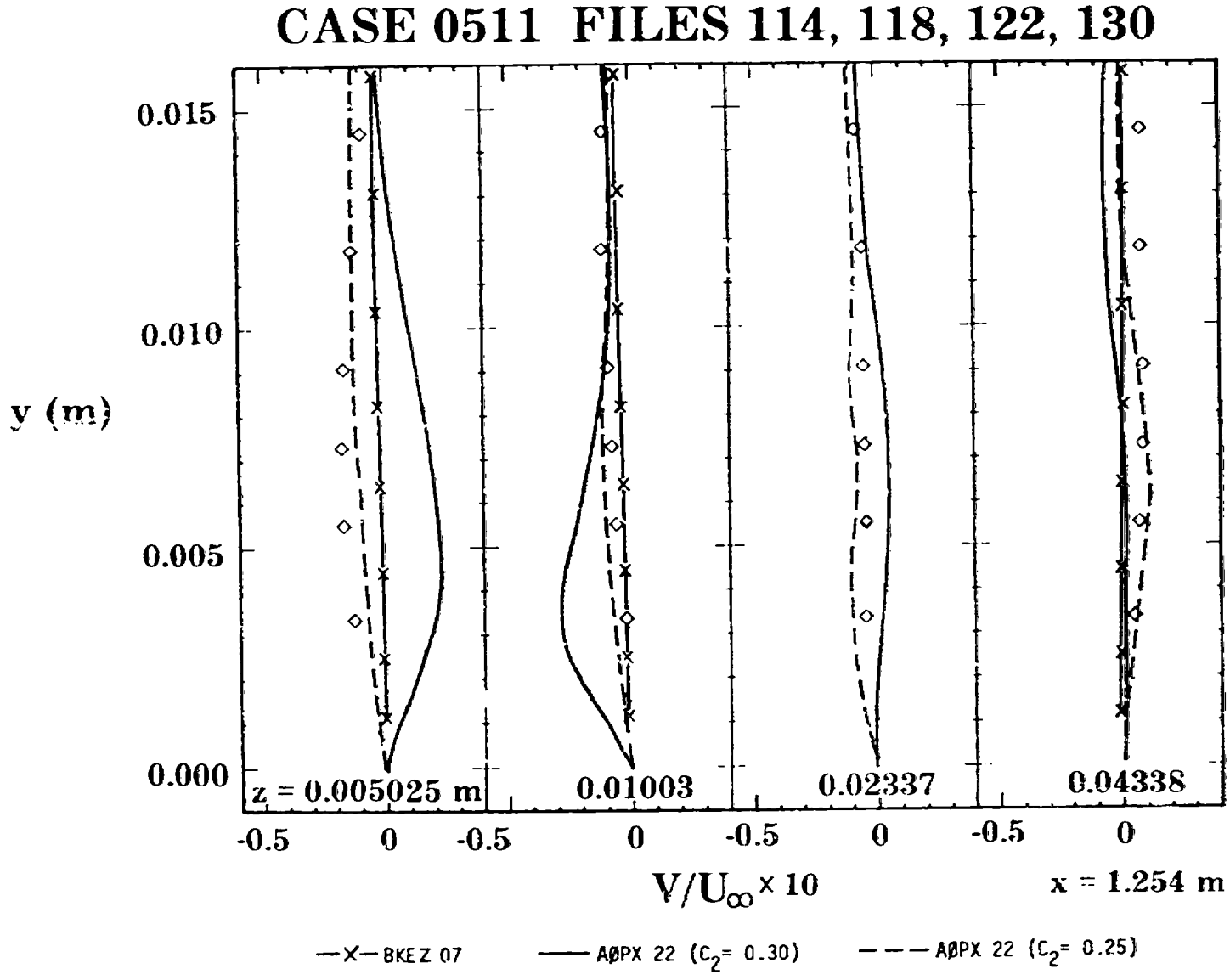

CASE 0511 FILES 114, 118, 122, 130
y (m)
0.015
0.010
0.005
0.000
z = 0.005025 m
0.01003
0.02337
0.04338
-0.5
0
V/U∞ × 10
x = 1.254 m
—X— BKEZ 07
—— AØPX 22 (C2= 0.30)
– – – AØPX 22 (C2= 0.25)

Figure 22.

CASE 0511 FILES 114, 118, 122, 130

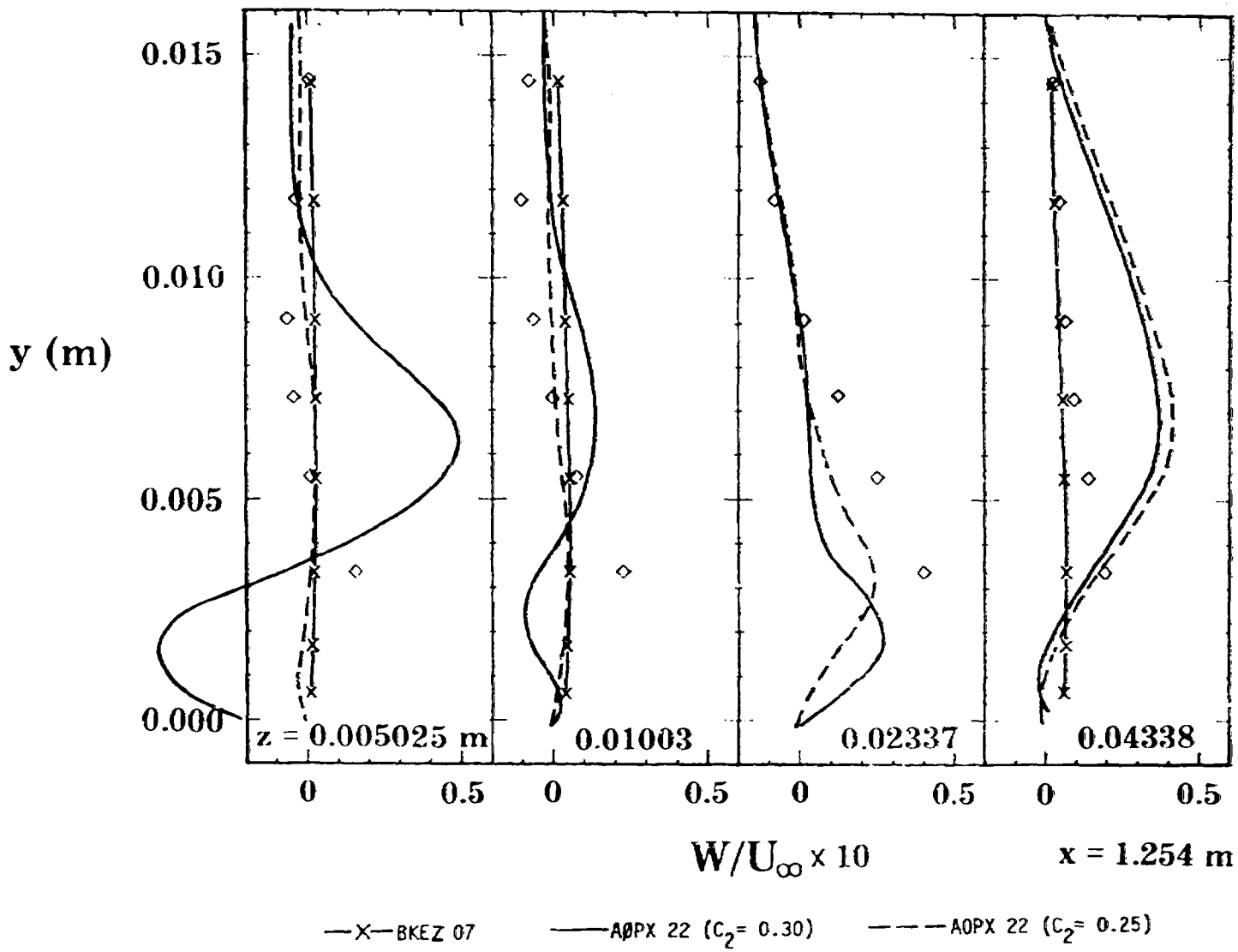


Figure 23.

CASE 0512 FILES 8,17

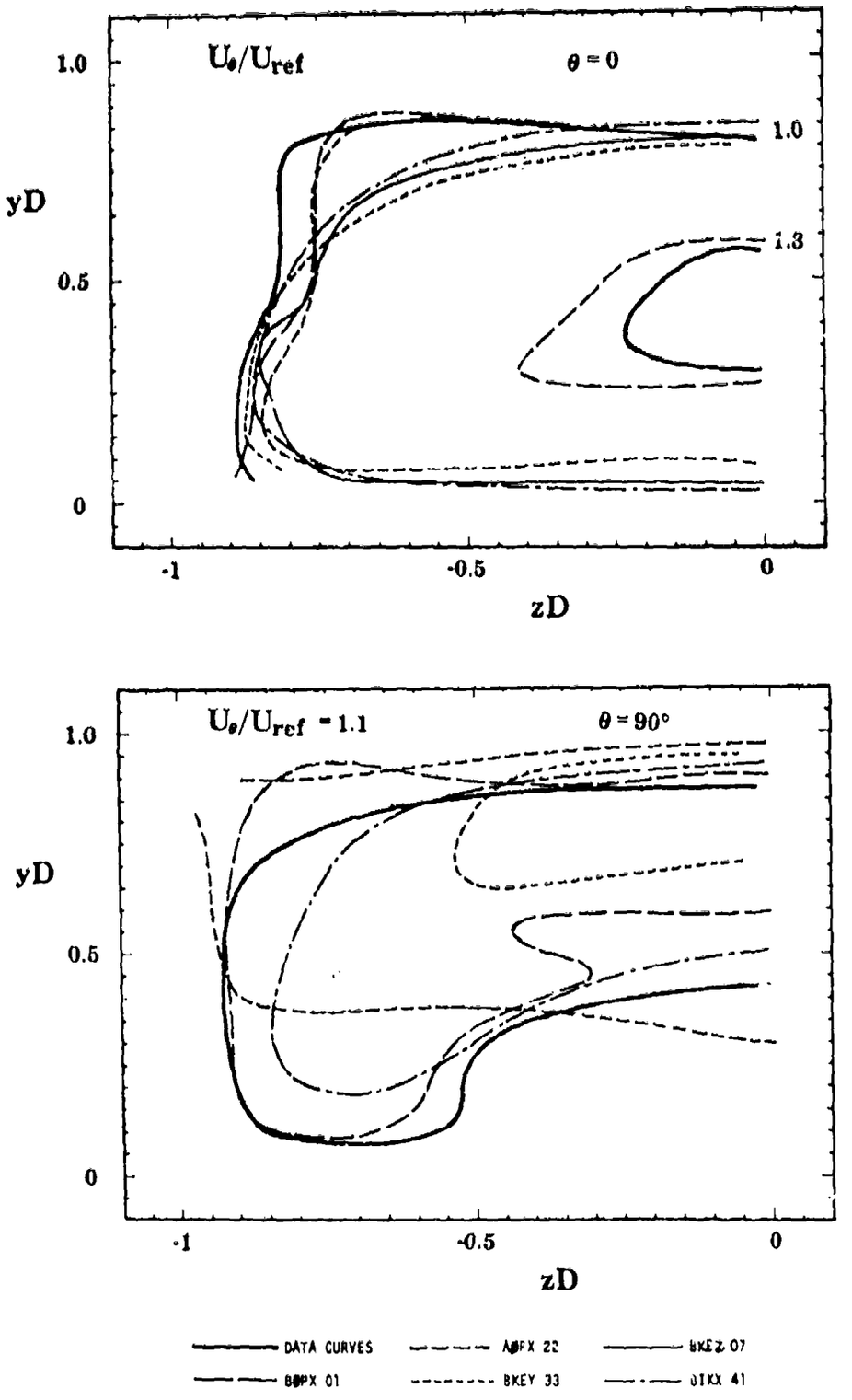


Figure 24.

CASE 0512 FILES 10, 19

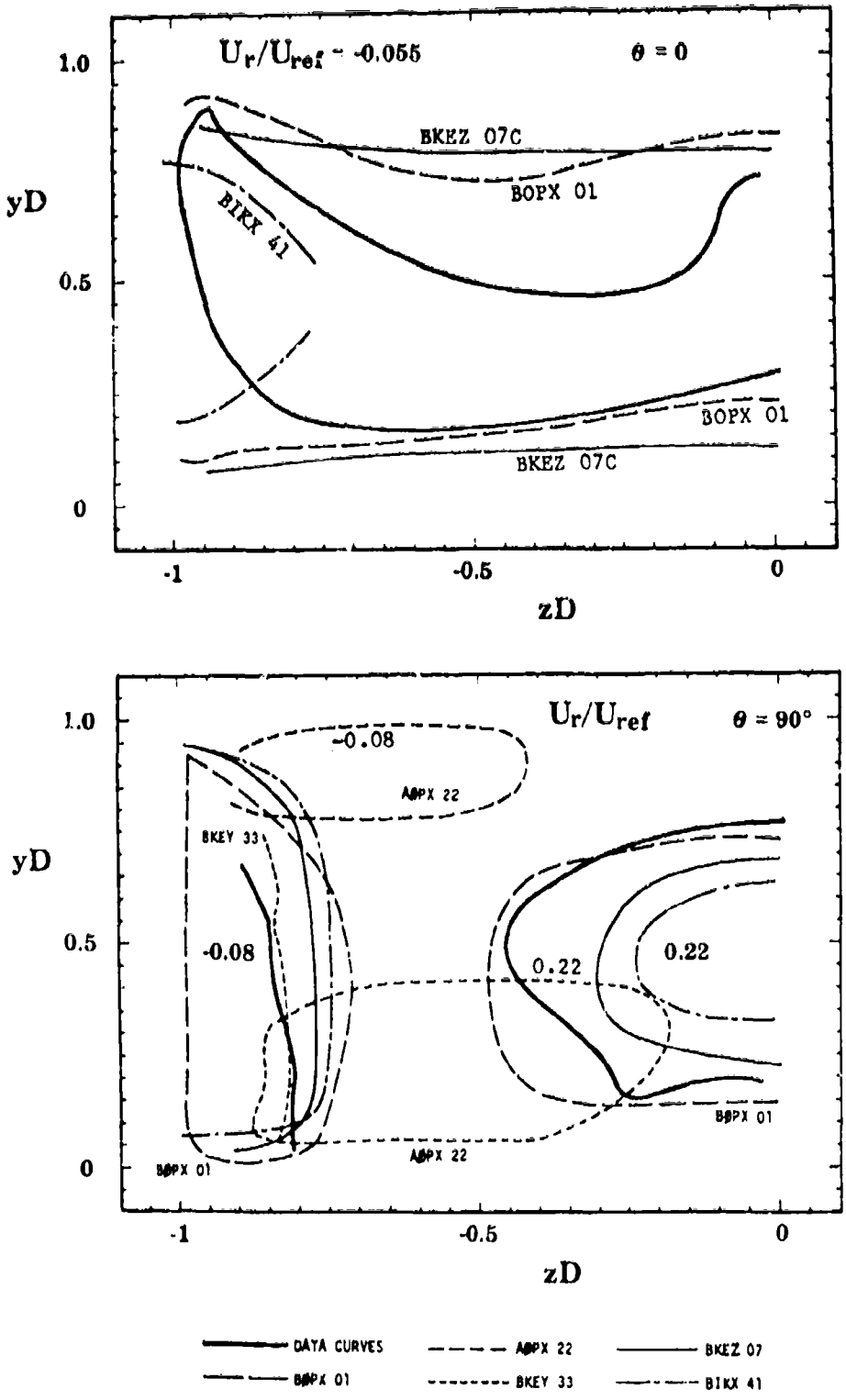


Figure 25.

CASE 0512 FILES 12, 21

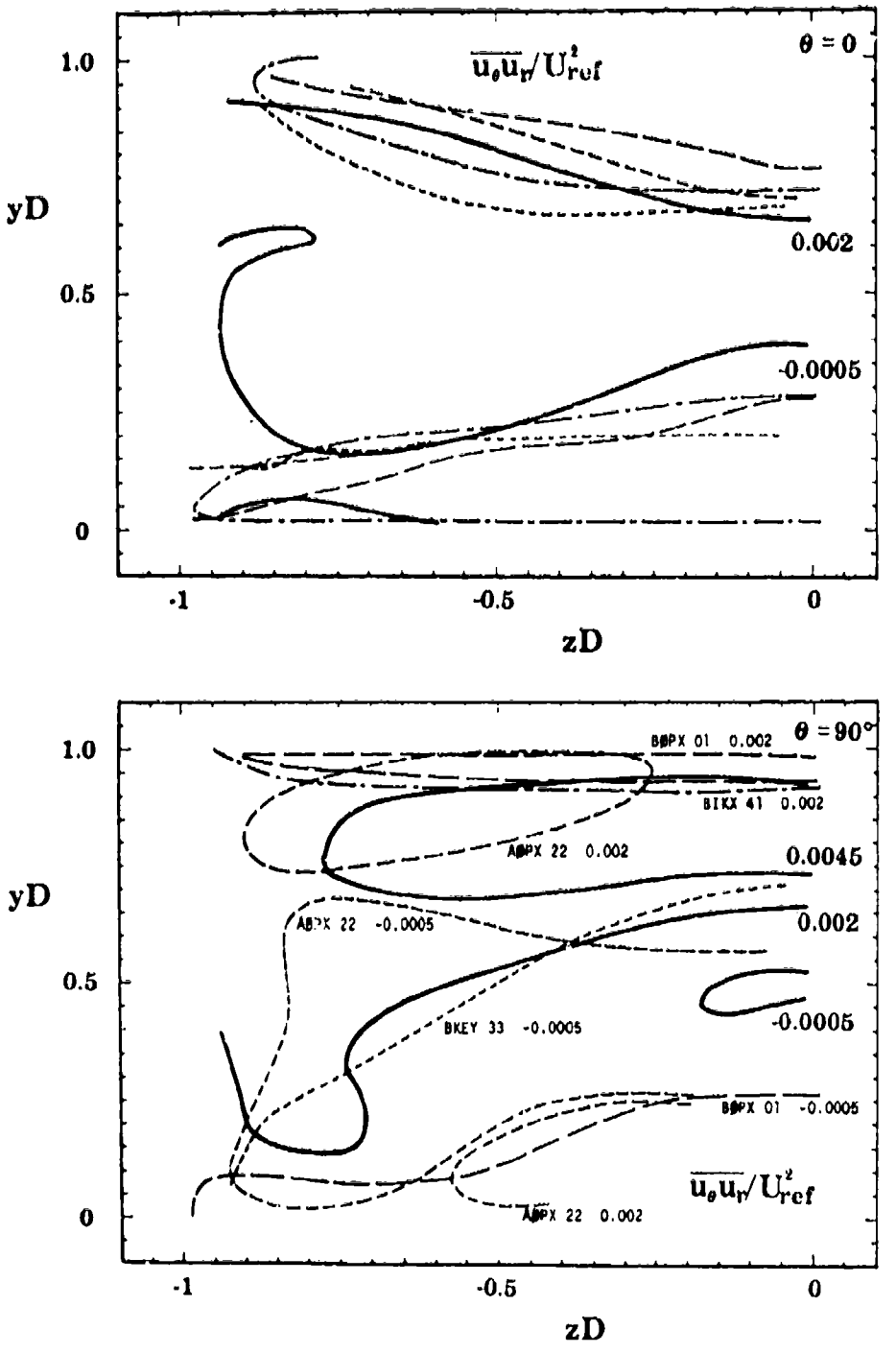


Figure 26.

DISCUSSION

SESSION XI

Case 0113 (P1)

To the question, "Why did the Rodi method (Group 07) work well for Case 0113 (P1) in contrast to the predictions for Case 0111?" W. Rodi replied that the prediction is good because the influence of secondary flow is small, and that it is basically a wall-dominated phenomenon.

Case 0511

Comments by Rodi (Group 07): The decision was made not to use the algebraic stress model, because the secondary motion should be pressure-driven, and, therefore, a refined-stress model is not necessary.

Comments by M. Arnal (Group 22): The decision to use $C_2 = 0.25$ resulted in no turbulence-driven secondary flow in the calculation. The calculation, however, shows a decaying secondary flow because of its presence in the initial condition.

Case 0512

1. The use of Melling's data to initiate the computation: Melling's thesis presented two sets of results for the secondary motion at $x/D_h = 42$. The first set is uncorrected data. The second set is corrected to satisfy the secondary flow continuity equation assuming $\partial U/\partial x = 0$. In Humphrey's calculation, use of the uncorrected initial-condition data led to an extra pair of secondary vortices which disappear when θ equals 45°. These were not observed in the other computations. F. Gessner pointed out that the assumption, $\partial U/\partial x = 0$, may not be valid for Melling's data at $x/D_h = 42$. Humphrey attributed these effects to the symmetry properties of the initial data rather than the magnitude of the secondary velocities. (J. Moore imposed a symmetry assumption on the initial data.)

2. An issue was raised regarding the treatment of the flow near the wall. The issue was highlighted by the fact that in Case 0512 a maximum in the secondary velocity occurs very close to the wall. J. Cousteix used a modified van Driest damping factor for the length scale. Moore used the logarithmic mean of the laminar and turbulent viscosities at the first grid point ($y^+ \sim 20$) to calculate the wall shear stress.

3. An issue arose about the grid independence of the calculated results. W. Rodi felt that none of his results were grid-independent and that the primary flow was subject to numerical diffusion. Typical mesh sizes over the half duct were:

   Moore — 10
   Rodi — 20
   Cousteix — 30

Moore and Arnal felt that their results were grid-independent, although no mesh refinements were tried.

4. The method of Cousteix used a parabolic calculation of the viscous equations with the pressure distribution along the duct centerline derived from inviscid theory with no viscous-inviscid interaction and the same turbulence model was used in the curved- and straight-duct cases by Cousteix' group. F. Gessner made the observation that the boundary-layer assumptions used to develop this model are appropriate in the straight-duct case but may not be in the curved-duct case.

5. Humphrey made the observation that the numerical scheme used for the convection term in Case 0512 is very important because the velocities are skewed with respect to the grid. The resulting cross-stream numerical diffusion smears the streamwise momentum.

Case 0111

1. Rodi stated that neglect of the cross-stream gradients of the secondary motion in the production term in the algebraic Reynolds-stress equation leads to calculated secondary velocities which are much too high when the constant $C_2 = 0.4$ is used as suggested in the LRR work. However, by including these terms agreement is better, although the secondary velocities are somewhat low. Gessner said that an appropriate choice of C_2 in the rapid part of the pressure-strain term together with neglect of the cross-stream gradients can give good results.

   The model used by Cousteix made use of the data of Mojola and Young to tune the values of C_2 (0.3) for stress-driven secondary flow.

2. Gessner and Cousteix include an empirically based streamwise dependence on their mixing-length distribution.

3. Jones asked why the Cousteix calculation for U did not achieve fully developed states near the wall. He suggested it might be due to wall treatment.

Comments Presented in Session XIII

The following comments were presented in Session XIII based on sessions and the written summaries of discussions posted during the meeting. They are included here to provide a unified, continuous presentation of materials:

W. Rodi stated that for the curved-duct calculation, the reference to the model "blowing up" referred only to one particular pressure-strain model, in which they used the unchanged model with the coefficients specified in the LRR paper. When secondary velocity gradients were included, the problem disappeared. He stated that J. Cousteix had also found this. J. Humphrey said that the second pair of vortices only appeared with a higher-order scheme, which no other computor had used.

SESSION XII

Chairman: D. Coles

Technical Recorders:

F. Pierce
R. Moser

Technical Reporter

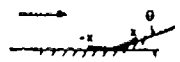
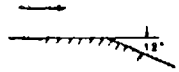
J. Marvin

SUPERSONIC FLOWS — II

Cases 8631, 8632, 8641, 8651, 8661, 8663, 8671
— Extra Strains

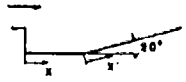
PICTORIAL SUMMARY

Flow 8630. Data Evaluators: M. Rubesin and C. Horstman. "Compressible Flow over Deflected Surfaces."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8631 G. Settles T. Fitzpatrick S. Bogdonoff		> 0	9	-	-	-	-	-	M and p/p_∞	Preston tube	8×10^3 to 7.6×10^6 (based on δ_o)	2.85	θ = 8, 16, 20 and 24°. Wall pressure also available.
Case 8632 J. Dussauge J. Gaviglio		< 0	23	-	2	-	-	-	$\overline{Tu}$	Law of wall	5×10^3 (based on δ_o)	1.76	Prandtl Meyer expansion. Wall pressure also available.

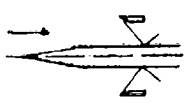
PICTORIAL SUMMARY

Flow 8640. Data Evaluators: M. Rubesin and C. Horstman. "Compressible Flow over Compression Corner with Reattaching Planar Shear Layer."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8641 G. Settles B. Baca D. Williams S. Bogdonoff		> 0	11	-	-	-	-	-	M, p/p_∞ 11	Preston probe	2.2×10^5 (based on δ_o)	2.8	Combines three problems: Free-shear layer development, reattachment, and downstream development.

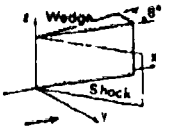
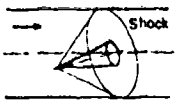
PICTORIAL SUMMARY

Flow 8650. Data Evaluators: M. Rubesin and C. Horstman. "Axisymmetric Shock Impingement (Supersonic)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8651 M. Kussoy C. Horstman		various	10	-	-	4	-	-	$\overline{uv}$ 4	Floating element	3.6×10^5 (based on δ_o)	7.2	Axisymmetric. Two cases - one attached and one separated. Heat transfer measurements included.

PICTORIAL SUMMARY

Flow 8660. Data Evaluators: M. Rubesin and C. Horstman. "Three-Dimensional Shock Impingement (Supersonic)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8661 D. Peake			5	5*	-	-	-	-	Wall pressure	Preston probe	1.5×10^5 (based on δ_0)	2	3D planar shock interaction. *V known in terms of U and flow angle.
Case 8663 M. Rungoy J. Viegas C. Horstman			5	Flow angle at various stations around cone	5	5	5	5	$\overline{uw}$ 5	Buried wire	1.0×10^6 (based on δ_0)	2.17	3D curved shock interaction. Turbulence measurements limited to windward and leeward planes.

PICTORIAL SUMMARY

Flow 8670. Data Evaluators: D. Peake (D. J. Cockrell). "Pointed Axisymmetric Bodies at Angle of Attack (Supersonic)."

Case / Data Taker	Test Rig Geometry	dp/dx or C_p	Number of Stations Measured: Mean Velocity U	Mean Velocity V or W	Turbulence Profiles $\overline{u^2}$	$\overline{v^2}$	$\overline{w^2}$	$\overline{uv}$	Others	C_f	Re	M_∞	Other Notes
Case 8671 W. Rainbird			1	-	-	-	-	-	Surface pressure	Preston tubes and oil flow visualization	2.5 to 3.4 $\times 10^7$ (based on cone length)	1.8	Free-stream turbulence ~ 0.2%. Data at two angles of attack. Measurements at one axial station x/L = 0.85.

Methods Used--Session XII

Method	Flow Case 8631	8632	8641	8651	8661	8663	8671
BIKX			●				
BOPX	●				●	●	●
BKWX	●	●	●	●		●	
SG	●						
Plate No. Vol. III	169-170	171	172	173	174	175	176,177

SUPERSONIC FLOWS — II

Cases 8631, 8632, 8641, 8651, 8661, 8663, 8671

Technical Report

by

J. Marvin*

The set above shows the flows reported during this session. They represent idealized flows encountered in high-speed aerodynamic applications. Most of them are complicated by the presence of shock waves and/or separation. They provide stringent tests of the numerical method and of the turbulence model. The complexity of these flows makes it extremely difficult to separate differences between computation and experiment due to either numerical procedure or turbulence modeling. With the exception of Le Balleur (Group 42), all the computors solved the Reynolds-averaged Navier-Stokes (RANS) equations. Flow Cases 8661 and 8671 were computed with the parabolized form of the RANS equations which neglects derivatives of the stresses in the marching direction. Eddy-viscosity models were used predominantly.

The number of computors attempting any single flow was small, and not all the flows were computed with the same turbulence model. Therefore it is difficult to distinguish advantages of any method or turbulence model. The pressure distributions were usually adequately predicted by all the computations, no matter what method or model was used, but the details in the viscous near-wall regions of the flow were not. This probably reflects the fact that the pressure fields are strongly influenced by the inviscid portions of the flow field. However, a bright aspect of all the computations is that they reproduce, albeit qualitatively, the essential experimentally observed features of these flow field.

Case 8631 (Plates 169-170, Volume III)

Three computors attempted this flow. Two of them use the RANS equations. The other used an interactive scheme based on entrainment and an additional lag equation based on the turbulent kinetic energy. They all do well at predicting the overall pressure distributions, except that the RANS-equations solutions fail to give a plateau over the separated bubble for the 20° corner angle suggesting that the effective viscosity is too large. The viscous-inviscid interactive scheme of Group 42 does better in this regard, probably because of its additional lag equation. However, apparently this method did not predict separation as the skin friction never appears to be negative. The two-equation model (k-ω) of Group 36 seems to perform better overall when the skin-friction and profile data are compared. It also apparently

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predicts the Reynolds-number effects on the locations of separation and reattachment, although the extent of separated flow is underpredicted.

Case 8632 (Plate 171, Volume III)

The expansion corner flow was intended to test the ability of the turbulence model to reflect the decrease in turbulence due to a "sudden expansion." Only Group 36 attempted to calculate this flow. Surface pressure and profiles of velocity are in substantial agreement with the data. This probably should be expected if the numerical procedures are correct because the major regions of compression are outside the sudden expansion region ($0 < x < 0.005$m) and in the high-speed portions of the viscous layer where inviscid effects are dominant. The comparison of velocity fluctuations within the sudden expansion indicates the model fails to predict the experimentally measured decrease.

Case 8641 (Plate 172, Volume III)

This flow tests the ability to model both the free-shear layer and its development subsequent to reattachment. Two groups computed the flow. Both groups show qualitative agreement with the data. Presuming that the computational grids in the two computations were comparable, it appears as though the two-equation k-ω model of Group 36 did better in predicting the initial shear-layer development, e.g., see the profile comparisons for various x. On the other hand, in the developing region downstream of reattachment ($x \geq 0.068$m) neither of the solutions compares very well with the velocity-profile data. This may be due in part to the fact that the experimental reattachment location is not predicted.*

Case 8651 (Plate 173, Volume III)

This hypersonic shock boundary-layer interaction flow was computed by only one group. The computations fail to reproduce most of the details of the flow in the region of the interaction. Evidently the computed interaction zone is much smaller than that found experimentally.

Case 8661 (Plate 174, Volume III)

This swept shock flow was computed by two groups, each using essentially the same algebraic turbulence model. One group solved the RANS equations and the other used a parabolized RANS form. Both reproduce the pressure rise obtained experimentally. However, the solutions using the parabolized RANS code gave poorer representation of the skin-friction and velocity-profile development. However, before conclusions can be drawn regarding these differences it must be determined whether the computational

*[Ed.: Reattached layers have structures quite different from normal turbulent boundary layers; see data in Case 0421, Vol. I, and comments by J. Eaton in this volume, starting on p. 888.]

grids were comparable and whether neglecting the boundary-layer development along the wedge generator, as was done by the group using the parabolized RANS code, influenced their results.

Case 8663 (Plate 175, Volume III)

This complex three-dimensional shock interaction was computed by one group using two different turbulence models. While the pressures along the wall were represented by the computations using either model, the details within the viscous interaction region show significant discrepancies.

Case 8671 (Plates 176-177, Volume III)

Group 31 using the parabolized RANS code with an algebraic turbulence model computed this complex flow involving cross-flow separation. Surface pressures and surface flow angles ($\omega_s{}^\circ$) are predicted quite well, except near the primary separation location ($\phi^\circ \approx 130$). The surface pitot pressure, directly related to the skin friction, shows rather poor agreement as do the turning angles away from the surface. The latter differences result partly from the prediction of the incorrect separation line.

DISCUSSION

SESSION XII

The modelers present agreed during the discussion that supersonic flows tend to be relatively easy to compute, because often the associated shear flows do not determine the overall flow but rather respond to it. A good appreciation of the total flow usually follows from the geometry, with viscous effects showing up mainly in details.

In all of the computations considered here, the wall boundary condition was invariably treated by integration of model equations, rather than by use of a wall function modified for compressible flow. In the multi-equation models, therefore, the first grid point was usually placed below $y^+ = 1$, for the sake of higher precision in determining the surface friction.

One of the flows (Case 8631; 20° compression corner at $M = 3$, with modest separation) was computed using an algebraic model (43/BOPX) and also using a two-equation model (36/BKWX). The algebraic model showed considerable sensitivity of computed reattachment distance to choice of (constant) eddy viscosity in the outer part of the boundary layer. For a plausible choice, reattachment tended to occur too far downstream. The two-equation model is quite successful in representing the velocity field, but not the surface friction. In neither case did a finer mesh improve matters. An unpublished study of the sensitivity of this flow to grid spacing (Horstman) showed noticeable effects on surface friction but not on the position of the separation and reattachment points. The computors present agreed that for this flow (and probably also for Case 8663), the discrepancies observed between computations with different models arise in the models and not in the numerics. The third computation for this flow (42/SG), using an integral method with entrainment equation, showed a strong sensitivity of the computed results to the shape factor for the initial boundary layer. The experimenters who supplied the data noted that the corresponding experimental test showed no particular sensitivity to this parameter.

The performance of different models can also be compared for another flow (Case 8663; three-dimensional incident shock at $M = 2.2$). The same computor group tested an algebraic model (36A/BOPX) and a two-equation model (36/BKWX). The simpler method gave more satisfactory results, e.g., for separation and reattachment along the generator $\phi = 0$, in contrast to the situation in the case of flow 8631.

This finding stimulated some discussion about the differences between boundary-layer behavior in a flow with a shock wave generated at the wall (8631) and in a flow with an externally generated shock wave impinging on a wall (8663). It is possible in the second case that much finer grid spacing may be needed in the outer part of the boundary layer in the impingement region. It was also suggested (Bogdonoff) that it

would be valuable to have experimental data for these two types of flow (with small regions of separation) with the same Mach Numbers, Reynolds Number, overall pressure rise, and initial boundary-layer thickness.

The flow with skewed impinging shock (Case 8661; wedge normal to working surface at M = 2) was computed by two groups. One used a Boussinesq model (36A/BOPX); the other used a Boussinesq model said to be essentially derivable from the first one by parabolization; i.e., by neglecting x-derivatives in diffusion terms (31/BOPX). Possible minor differences in the models and in initial conditions prevent assigning the discrepancies in results solely to the absence of upstream diffusion in one model, and it would be useful to have a computation to clarify this point.

For the hypersonic flow (Case 8651; wing shock generator concentric with cylindrical body at M = 6.8), the one available computation (36/BKWX) gives results which suggest that, among other problems, there is not nearly enough upstream diffusion in the model. The author of a recent catalog (Fernholz)* commented that this flow must be everywhere quite far from local equilibrium, since neither van Driest scaling for the velocity profiles, nor the Crocco energy integral is of any help in organizing the experimental data.

The one computation (31/BOPX) submitted for the cone at angle of attack (Case 8671; M = 1.7) was remarkably accurate. The reason may be found in the point already made, that the inviscid flow is supersonic, conical, and shock-free at the surface. Although the flow in the boundary layer is strongly three-dimensional, apparently no great difficulties were encountered through the use of an isotropic eddy viscosity in the (parabolized) algebraic model.

A discussion of the supersonic cavity flow (Case 8641; step with reattachment on ramp at M = 2.9) was quite productive. Several computors agreed that there seems to be so far no model (with fixed constants) which can represent accurately both a free-shear layer and a subsequent reattaching and relaxing boundary layer. The difficulty is presumably related to the problem of choosing a length scale, at any rate for models which require such a scale. At the end, there was agreement at least to contemplate the need for and uses of a local or adaptive model, in which the specifications are adjusted to suit the kind of flow being computed. Use of such a model was not encouraged by the rules laid down for the present conference.† It was pointed out by an experimenter (Bogdonoff) that such a proposition also implies that experiments should attempt to include (say) measurements of local Taylor microscale, perhaps derived from the auto-correlations of the streamwise velocity component.

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*AGARDograph 263 (1981).

†[Ed.: We are not aware of any rules, procedures, or comments that would discourage local or adaptive methods. See Sessions IV and XV.]

SESSION XIII

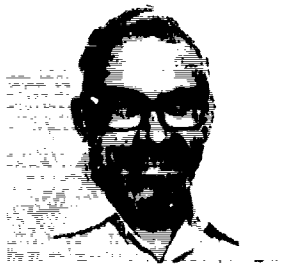
Chairman: E. Reshotko

Technical Recorders:

I. Castro
S. Pronchick

Editor's comment: Session XIII consisted of further comments on materials presented in earlier sessions, including the summary of discussions posted during the meeting. In order to provide a unified, continuous discussion on various points, these comments are shown at the end of the discussion in the sessions to which they apply.

SESSION XIV



Chairman: E. Reshotko

Technical Recorders:

T. Simon
G. Allen

AD-HOC COMMITTEE REPORTS

A USER'S VIEWPOINT ON COMPUTATIONAL FLUID DYNAMICS -- G. Sovran

PREDICTION OF TURBULENT FLOWS--A BOEING VIEW -- E. Tjonneland

AD-HOC COMMITTEE REPORTS

Report of Ad-Hoc Committee of Computors

"What We Have Learned, Where Do We Go?" (B. E. Launder, Chairman)

1. Representatives from thirteen computor groups met on Sept. 17, 1981, to discuss the above questions and a further computor supplied his views in writing. The viewpoints that emerged were subsequently conveyed to the Evaluation Committee.
2. First, it should be said, the 1981 meeting has been a very valuable one for it has helped focus attention on difficult flows. It will stimulate activity and give directions for further work at both fundamental and applied levels.
3. Perhaps a superficial overall impression is that (in the words of the written contributor): "Things are in worse shape than we thought before we came." Here we have seen "unpublishable" results as well as successes, and this open-ness will certainly assist the task of advancing competency in the calculation of complex turbulent flows.
4. The Committee sees a continuing need for work at all the levels of modeling represented at the Conference.
5. Integral methods have emerged as providing powerful, yet economical, approaches provided their limitations are recognized and respected. It is a level at which one is modeling turbulent flows rather than turbulence. A prerequisite is that one has enough experimentally accumulated knowledge to make a reasonable approximation of the evolution of the velocity profile. These approaches seem well adapted to handling interacting flows over airfoils, and we foresee their extensive continued development and exploitation in time-dependent and certain three-dimensional boundary layers.
6. There is no consensus on how reliable integral methods can be made for separated flows. In weak separations, their usefulness has been demonstrated; in strongly separated zones, it is less clear. In any event, further efforts in development will certainly be made.
7. Corner-flow interactions were seen as a type of phenomenon not suited to integral methods as were, likewise, certain supersonic flows, where severe heating, normal pressure gradients and other factors produced such diverse effects on the velocity profile that these could not adequately be modeled by an integral approach.
8. A certain degree of improvement in integral methods was seen as possible by increasing the number of parameters on which the velocity profile depended. (At present - apart from a weak influence of Reynolds number - one-parameter families are habitually used.) It was recognized, however, that one should not go far in this direction or one would have an integral method that was as costly as and less adaptable than a finite-difference treatment.

9. At the differential level, the Conference results have underlined that mixing-length models should not be used in separated compressible flows, for the length to reattachment seems to be consistently overpredicted. Indeed, in this type of flow, as in others, there are clear benefits from proceeding to a two-equation level, where the length-scale evolution can be prescribed.

Nevertheless we see a continuing role for mixing-length and one-equation Boussinesq models for attached boundary layers. The fact that the dissipation length scale $k^{3/2}/\varepsilon$ is, to such a good degree of approximation, a linear function of wall distance, is an experimental fact of life that we ignore at our cost. Thus, it may often be desirable to employ a prescribed-length-scale model in the wall vicinity, even when a differential-length-scale scheme is used for free flows and the outer regions of a flow. Moreover, the use of such a simpler treatment makes it easier numerically to resolve the viscosity-affected near-wall region and may in some cases allow a more accurate representation of the physics.

10. Much has been written elsewhere in the literature about the benefits of employing a second transport equation for predicting turbulent flows. While reaffirming this viewpoint, we here wish to underline the weaknesses of the current "second-variable" transport equations. First, none of the equations manages, in conditions removed from local equilibrium, to preserve as universal a variation of near-wall length scale as experiments show to be needed. In an adverse pressure gradient the ε equation produces too large a value of ℓ near the wall while the ω equation generates excessively large length scales in the outer region; this results, in both instances, in a serious overestimate of skin friction. There appear to be other weaknesses with the production term in the second-scale-variable equation associated with curvature, dilatation, and rapid streamwise straining. A number of variants of the ε equation have been introduced at this meeting, which have shown encouraging trends. These need further testing and evaluation before they can be unequivocally recommended to the turbulence community. What we all feel is that the current status of the second-variable equation does not by any means represent the limits of development. The forms used by most workers are, one might say, naively simple. That these equations should nevertheless produce <u>roughly</u> the right length-scale variation over such a range of conditions is remarkable.

11. Here we want to note that in some cases there are insufficiently precise data available to allow the modeler to distinguish whether weaknesses in his computation are attributable to errors in the pressure-strain model (or to C_μ if a Boussinesq model is employed), or to the form of the second-variable equation. The supersonic mixing layer perhaps provides the prime example.

12. Concerning the connection between stress and strain, we believe the meeting shows clearly the superiority of using the "algebraic" stress-strain relation rather than the simpler Boussinesq formula. In simple strains the former reduces to the latter but in more complex strains, the algebraic treatment seems superior.

13. Nevertheless, there have been a few cases (the axisymmetric diffuser with high-core turbulence or the homogeneous contracting flows), where the algebraic stress approximation for stress transport has produced anomolous results. We see here, over the next few years, a changeover towards differential stress treatments. For two dimensional parabolic flows the extra computation (compared with and "algebraic" treatment) associated with such a model is unimportant, while in elliptic flows, some computors feel--though not all--that a differential-stress scheme (such as that used by Donaldson's and Mellor's groups) may actually facilitate convergence.

14. Turning finally to the stress-transport equations themselves, the most crucial element therein is undoubtedly the pressure-strain approximation and, we believe, the mean-strain or "rapid" part of that process. The flows considered at the meeting show clearly that current models, which incidentally have now been in active use for nearly a decade, need to be refined considerably before current closures can cope confidently with the complexities of three-dimensional strain fields. The difficulties and resultant uncertainties in modeling the wall-reflection part of the pressure-strain process also need to be mentioned.

15. There have been a number of recent proposals for more general schemes. It must be said, however, that writing down general tensorial forms is relatively easy. Transforming them to a useful model is inevitably a lengthy and painstaking business. This is an area where international cooperation could effectively assist the advancement of the state of the art.

16. There are other processes in the stress-transport equations whose modeling is uncertain but these are certainly of lesser importance than the pressure-strain model discussed above.

17. For the future we see substantial benefits from greater interchange among groups using different closure levels. There needs to be an information cascade between higher-order models to simpler ones to help refine and optimize the physics at the simpler level. For example, results from large-eddy and full-simulation, and rapid-distortion theory can, if the test flows are properly chosen, greatly assist the improvement of stress-transport closures.

18. One must bear in mind, however, that as one reduces the closure level, the potential width of applicability will be narrowed. This inherent limitation needs to be openly acknowledged, and greater legitimacy given to models which describe accurately only a limited class of flows or only certain zones of a more complex flow.

19. The Computors' Committee also conveyed a number of other observations to the Evaluation Committee:

(a) It emphasized that numerics were not seen as a problem for the two-dimensional parabolic flows. These flows reflected the success or otherwise of the turbulence model.

(b) It requested the Evaluation Committee to take the time needed to absorb fully the results of the meeting before making its final report. The amount of information to be assessed was truly enormous and it was doubted that this could be fully absorbed—let alone digested—during the conference period.

(c) It expressed its hope that a follow-up meeting to the present one would be held about two years hence in which computors would be invited to submit predictions for a proportion of the test cases assembled for this meeting.

(d) It was the Committee's view that, if the spirit of openness and cooperativeness that has prevailed at this meeting could be continued, we should make good progress in our efforts to reduce complex turbulent flows to computable phenomena.

J. Le Balleur
T. Coakley
J. Cousteix
K. Hanjalić
B. E. Launder (Chairman)
M. A. Leschziner
H. L. Moses
P. Orlandi
R. Pletcher
A. Pollard
W. Rodi
M. Savill
G. Scheuerer
R. Sullivan
J. Viegas
D. C. Wilcox

Discussion

P. Bradshaw: Are all the remarks having to do with the ε-equation or length scale in the k-ε equation also applicable to a full-stress transport model?

B. Launder: We believe so, yes.

B. Newman: Did the group of computors think it was necessary to check the transport equation at the primitive level and to urge the experimenters to measure these terms individually?

B. Launder: So far as length scales are concerned, none of the exact equations contain correlations that are measureable, and in the case of the axisymmetric jet, modelers cannot determine what is attributable to pressure-strain errors and how much to length-scale errors. Individual Reynolds stresses are not known to sufficient accuracy.

P. Orlandi: Did you report about zonal modeling?

B. Launder: This slipped through the cracks, but was implied in my cascade model. One should not feel guilty in adjusting coefficients to treat different regions of flow.

S. Kline: This cascade of information is an idea I agree with, but I have further comments on zonal modeling, and I will talk about it later.*

*See comments in Session IV and Opinion, Comments, and Closure in Session XV.

Report of Ad-Hoc Committee on

Starting/Matching Conditions (from Session V, P. Bradshaw Chairman)

The main reason for large transients in comparison with conference data appears to be that computors who are accustomed to doing real-life predictions (as distinct from postdictions for well-documented test cases) use highly simplified starting conditions, e.g., the turbulent energy profile may be taken from "flat-plate" data, or computed from the modeled TKE equation with convective terms neglected (giving an ordinary D.E. in y for k). All computors who testified to the committee used an empirical length-scale distribution (e.g., mixing length = min(0.41 y, 0.085 δ)) because no test case provides length scale or dissipation data: a possible alternative is to assume dissipation = production = $-\overline{uv}(\partial U/\partial y)$, $\varepsilon \sim (k^2/\overline{uv})(\partial U/\partial y)$; these are nearly equivalent. All computors used the initial mean U-profile supplied in the test case. Two computors assumed mean V = 0 or V = V at wall. This is illegal if U and $\overline{uv}$ are given, because in that case the continuity of momentum equations combine to give an ODE for V(y), so the V = 0 assumption may lead to transients.

Several computors were accustomed to minimizing the effects of starting transients by running a separate computation until θ (say) reached the value specified at the initial station of the test case. This could be done either with "flat-plate" boundary conditions on a backward extrapolation of test-case boundary conditions. (It was not done in the Session V test cases.)

One computor started computations in regions of transpiration with V at the wall equal to zero, rising to the test-case value in a few x-steps. This inevitably leads to large transients.

All computors agreed that length scale or dissipation rate recovers rapidly from poorly chosen initial values so that the absence of length scale for test data is not a severe problem in starting computations (one does of course want length-scale data to check length-scale modeling!).

The committee also discussed "Data needs for computational fluid dynamics." The above-mentioned "tribal customs" for starting calculations were not fully appreciated outside the computor community, but it was accepted that simplified starting procedures were inevitable for industrial-type predictions. It was further agreed that no significant update could be offered to the 1980 Meeting position paper on Data Needs.*

Transients induced by inaccuracy/incompatibility in test data were not regarded as a major source of the results for Session V, but are, of course, important in principle.

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*See Vol. I, p. 23, "Experimental Data Needs for Computational Fluid Dynamics--A Position Paper."

No user of "integral" methods testified to the committee. It was pointed out that one integral method needed to take H different from the experimental value to get good results downstream.

Comments by Chairman: Before condemning the "flat-plate start," let us recall that non-fixed transition in attached boundary layers virtually always occurs in pressure gradients that are small by turbulent-flow standards. Also, note that the apparently excessive concentration on boundary layers in the above discussion is not short-sighted--real-life free-shear layers start from boundary layers. Shear-layer computations which have a strong upstream influence at the inlet boundary (i.e., ellipticity) are unrealistic!

Discussion

H. Nagib: Starting transients are indicative of the incompatibility of the model with the initial conditions in the beginning of the flow.

P. Bradshaw: There were cases of models tripping over their own boot laces at the beginning, but this is not the main reason for the transients.

Following a brief introduction by J. Humphrey on "Numerical Tests," the following discussion took place:

Discussion

T. Han: In the case of secondary flows, where streamlines are not parallel to grid lines, higher-order numerical schemes should be used in the streamwise direction also.

J. Humphrey: Yes, but when there is no reverse flow or strong vorticity in a pressure field, one can use upwind differencing and a semi-elliptic approach, allowing grid refinement in the streamwise direction.

M. Leschziner: But you said that streamwise diffusion was important.

J. Humphrey: If you could determine how streamwise diffusion was affecting your results, you may choose to use upwind differencing, which is easier.

P. Roache: I agree with your results but I question your grid-size study; you go only from 10 to 13?

J. Humphrey: Comparing the two cases, you can see the large difference. Have I not refined enough?

It was generally agreed that this was insufficient grid refinement to reach clear conclusions.

A USER'S VIEWPOINT ON COMPUTATIONAL FLUID DYNAMICS

By

Gino Sovran

USES OF COMPUTATIONAL FLUID DYNAMICS (CFD)

The uses of Computational Fluid Dynamics (CFD) can be divided into two separate categories having different basic objectives. The first is as a technique for generating scientific understanding of the mechanisms involved in, and the behavior of, fluid flows of interest. It can be a valuable complement to the conventional research approaches of mathematical analysis and judicious experimentation. This type of usage occurs primarily in academia and government laboratories, and is an activity of many of the developers of computational codes.

In the second category, CFD is used as a tool for designing the hardware of engineering devices or systems. This occurs primarily in industry, where it complements empirical correlations and parametric testing. In this context, it can contribute to design through: the evaluation of candidate system configurations; the development of final hardware geometry; the performance evaluation of the final configuration; a determination of the sensitivity of performance to off-design values of imposed boundary conditions; the identification of potential problem areas.

In addition to the difference in objectives, these categories also differ in the importance that is placed on the cost-benefit ratio of computations. In an industrial design context, the definition of an acceptable ratio must be consistent with the profit-seeking objectives of the particular company involved.

This paper represents one person's viewpoint on the usage of CFD in the industrial environment. To put it in proper perspective, the particular environment of his experience includes internal flows in ducted systesm and turbomachines, the fluid flows and motions inside the manifolds and cylinders of reciprocating engines, and the flow fields of road vehicles. It can be characterized as mechanical engineering, rather than aeronautical. It is one in which CFD per se has not yet been used extensively, but other forms of computation have.

THE INDUSTRIAL ENVIRONMENT

The objective of CFD in industry is to provide "useful" input for consideration in the making of design decisions. The definition of "useful" is left intentionally vague, but covers a spectrum from only qualitative output (concepts and ideas) to quantitative results (hard numbers). The value of the former is not to be underestimated, particularly for complex flows.

An important characteristic of the industrial environment is that there can be severe consequences for being wrong--in extreme cases, even financial failure. The decision-makers (both the designers themselves and their technical managers) are very

much aware of this. They are therefore cautious about abandoning the tried and true in favor of the new and, to them, unproven. They have to weigh CFD against the other alternatives for providing desired input to design decisions--on the basis of cost, time, and trustworthiness. The number of cases where CFD is the only means of providing input are small. The expectation of decision makers is to be guided by CFD. Their fear is that they will be misled.

The industrial CFD computor who provides input to design decisions must share responsibility for the decisions that are made. Therefore, he also has to be very concerned about the trustworthiness (or, conversely, the uncertainty) of his results.

Implicit in the preceding comments is the possible involvement of four different types of personnel in the usage of CFD for industrial design. These are: code developer, code user, designer, technical manager. In small organizations some of these may be one and the same person.

Because of the highly specialized skills required for code development and the general research-type interests of those who possess them, code developers are very frequently not those who use CFD to provide design inputs. These are provided by code users who are more design-oriented, but whose capabilities in CFD are not as advanced as those of the developers. This creates interface problems. First of all, the codes have to be restructured to make them user oriented. If they are to have widespread use in design, they must be usable in an effective manner by others less skilled in CFD than the code developers. The effort required to transform a complex computer code into a practical design tool can be comparable to that of developing the basic code itself. There is then the problem of adequately documenting the code for the user. A black-box type of comprehension not only limits the user's effectiveness in making computations but also precludes his suggesting any improvements based on his experiences.

Although he may be design-oriented, the code user does not usually fully understand the needs of, and the constraints on, the hardware design itself. This is more appropriately the purview of a designer, with his own set of specialized skills and interests. Effective design requires good communication between code user and designer regarding their individual needs and capabilities. If designers are to be persuaded of the utility of CFD, they must be actively and cooperatively involved in its usage. The difficulty of gaining their confidence should not be underestimated.

The fourth participant in the overall design process is the technical manager. While he bears the greatest responsibility for the quality of the product that evolves, he usually does not have a working knowledge of, or even appreciation for, CFD. Consequently, a major problem is building up in him sufficient confidence in the trustworthiness of CFD inputs that he is willing to base important decisions on them. This usually requires favorable past experiences in utilizing CFD.

Although predictive capability is the goal of all code developers, even postdictive codes can have value. They can be used to increase the utility of test-derived inputs by: providing detailed data that were not measured; identifying controlling physical mechanisms and generating understanding; permitting interpolations and limited extrapolations of system geometry.

DESIGN FACTORS

The use of CFD in industrial design is characterized by a number of factors. The following categorization contains those which the writer considers to be the most significant. They are listed in the order in which they enter a typical design process, not in the order of their relative importance.

(1) System Geometry and Flow Pattern

The real systems and flows of industrial interest are often of very complicated geometry and flow pattern--more complicated than any used for developing and validating CFD codes. The flows are commonly turbulent, three-dimensional, complex--and sometimes even time-dependent. In contrast to the single-component single-phase flows that are the subject of this conference, many industrial flows are multi-component, multi-phase, and even chemically reacting. They frequently contain more than one flow module, and sometimes significant interaction between them. They are often outside the bounds of past experience.

Under any circumstances, the complex geometry of real systems can be very difficult to describe accurately for the computer. Geometrical description is further complicated if the available computer storage is limited. Inadequate definition and resolution of geometry can be very serious problems for CFD, because in many systems small changes are known to influence fluid dynamic behavior substantially.

Good computations require construction of a suitable computational mesh. For systems of complex geometry, this can be a very challenging task. Factors affecting mesh selection are the resolution as well as the accuracy required of computed results. Available computer capacity and computational time are other factors.

In some cases the general flow pattern is known a priori and the objective of CFD is to quantify performance. In others, the flow pattern may be the objective of the computation; e.g., the fluid motions inside the cylinder of a reciprocating engine. In this case even an approximate picture of the flow pattern can be useful.

(2) Boundary Conditions

The boundary conditions, especially at open boundaries, of the device or system being designed are frequently not accurately known. This is particularly true for computations of only part of a total system--which are sometimes required because computer-capacity limitations preclude treatment of the whole system. The effectiveness of such computations is dependent on the accuracy with which the approach-flow

can be specified, as well as the local outflow or discharge conditions (especially for elliptic flows). Inadequate specification of these boundary conditions inherently translates to uncertainty to computed results.

(3) Parametric Studies

Because an inverse design capability does not often exist, or exists only to a limited degree, parametric studies usually have to be relied on so that design configurations can be developed by iterative optimization. The nature of such studies is to examine many candidate configurations and make design decisions from the alternative possibilities. As flow complexity increases the number of possible parametric variations also increases. While these studies are most often done experimentally, an effective CFD capability can be used to reduce the size of the test matrix if the individual compuations can be made at acceptable cost and in reasonable time. This usually requires codes that have been optimized for particular classes of flows.

(4) Trends vs Absolute Accuracy

By its nature, engineering is a matter of best trade-offs between competing factors. Everything being equal, a CFD user wants the results of his computations to be as quantitatively accurate as possible. However, the complexity of industrial flows does not often permit high accuracy. But one should not get overly concerned about absolute accuracy. A code is still useful for design purposes if valid trends as a function of system geometry and/or boundary conditions can be generated. An ability to predict that configuration A is better than configuration B will still permit the pursuit of better designs. However, the computed trends must be trustworthy.

(5) Flow-Specific vs General Codes

Code developers and researchers tend to overemphasize code generality. In an industrial environment primary priority must always be given to those flows which are of importance to the particular company involved. Computational capability for other flows is only of secondary importance. As a general rule it is best to use the code of least complexity that will give the desired results.*

The more general codes are more complex and tend to be of lower computational efficiency for a specific flow than one restricted to and tailored for that flow. There is also time-consuming decision making required between available options to reduce the capability of general codes to that actually needed for specific flows. Among these are: the number of governing equations to use, and in which form; which turbulence model; what wall functions? Complexity and sophistication are not necessarily synonymous.

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*[Ed.: See also report of Evaluation Committee, p. 979, for further comments on sophistication and accuracy of various levels of modeling.]

Because of the iterative optimization required in design, best accuracy and best cost effectiveness can be achieved with a spectrum of codes covering a spectrum of flows. One wants to avoid the F-wrench approach, i.e. having a code that can be applied to anything but fits nothing. Sometimes quick and dirty answers are adequate, particularly for the early sorting in design. The more powerful methods are most appropriate for defining the critical detailed geometry of the final configuration.

What is most desirable is a well-organized arsenal of weapons (codes) covering a range of caliber (capability). For each weapon one should know the physics contained, its limitations, the range of best applicability, the speed of computation, the accuracy, and the type of output. Using a hunting analogy, it is not appropriate to hunt rabbits with an elephant gun. It is best to match the weapon to the size of the game.

(6) Computer System and I/O Hardware

Regardless of the effectiveness of any state-of-the-art code for complex turbulent flows, potential industrial users have to ask some computer-related questions when considering its usage. Is the code so computer-hardware dependent that it is inoperative on the company's computer? Are the speed and storage capacity of the company computer sufficient to make computations feasible? If not, are other systems available on which computations can be made conveniently, at reasonable cost, and more or less on demand? What is the local graphics capability and is its hardware compatible with the code?

(7) Cost

Because of the profit-seeking nature of industrial organizations, cost is a very important factor. It has a strong influence on the extent to which any new technique can be incorporated into the design process, particularly if extensive parametric studies are involved and the technique has to be applied repeatedly. A necessary condition for cost being acceptable is that it does not exceed the available funds in the design kitty. The sufficiency condition is that the cost-benefit ratio can be justified relative to other means of providing the design input. The benefit side of this ratio has to account for whatever uncertainty there is in computed results. If CFD is the only means for generating desired information, then cost may just play a secondary role.

In some industries, such as the aeronautical one, testing can be extremely expensive and so even high CFD costs can be competititve as a means of providing design inputs. As a matter of fact CFD can often be the substantially cheaper alternative. In other industries, where testing cost is not extreme and where complexity of both the system geometry and flow pattern are high, it may be difficult to justify CFD.

Because of cost, excessively complex codes may not be usable in design even though they are accurate. Industry is usually willing to sacrifice some accuracy for lower cost.

(8) Deadlines

Time constraints are a part of the design process. In the case of CFD the total time required to produce comprehensible output must be considered--problem set-up, geometry definition, mesh generation, computation, graphical output. In many cases--not all cases as some advocates would like to believe--it is possible to get results more quickly than by testing. However, complex codes for complex flows can take a long time to generate output--sometimes too long. As an example, a CFD code that takes two days to predict tomorrow's weather is not of much use for weather forecasting.* This is not to say that such a code would not have value in a research ... for generating general understanding of weather patterns.

COMPUTOR INFLUENCE

The effectiveness of most tools depends on the technique of the worker who uses them, and this is also true for computational codes. The antithesis of technique is captured by the term "cookbook," and few of the modern codes can be handled in a cookbook fashion. With a hammer and a chisel most people can only make gravel from a piece of stone. With these same tools one man created the Pieta. The inherent and implicit influence of the computor on CFD results presents a problem for this conference--which is trying to evaluate the capabilities of various types of codes, not the skills of the computors who use them.

The computor influence depends on the skill, experience, and knowledge of the computor--what he knows about the code, and what he knows about the flow. Some specific ways in which code output can be influenced are through the selections made regarding: the location of computational boundaries and the specification of the fluid dynamic conditions at them; the computational mesh; the selection and definition of modular zones for zonal models; turbulence model; wall functions at solid boundaries; time step in time-dependent flows; iteration procedure and path; degree of "convergence" represented by results. In general, the more complex the code the more technique involved in its use, and the greater the computor influence.

CONSTRAINTS

No existing code used for design purposes contains the complete Navier-Stokes equations. Consequently, results need to be carefully interpreted by the user before

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*This succinct and beautifully direct statement is not original with the writer, but he does not know its origin.

they are offered as input to design decisions. This needs to be done in the light of how well the fluid physics captured in the code match those actually involved in the flow of interest. The more the physics that is not accounted for, the more interpretation is required and the greater the uncertainty of the results.

The usefulness of a code also depends on how user-oriented it is. The set-up of new problems and the selections made between available options should not be difficult. The code should be robust so that it will operate and produce answers even in the hands of inexpert users. It should be insensitive to iteration procedure and path. A blow-up can be very confidence-shattering to a designer-user. To him, any blow-up is a major problem, even though caused by something recognized as only a minor difficulty and readily correctable by the code developer. It is the designer's perception of code reliability that controls the extent to which he is willing to utilize CFD inputs, not the code developer's.

TYPES OF OUTPUT

The types of output data useful for design purposes (rather than for code verification or development) that come out of CFD depend on the nature of the computational technique employed (e.g. integral vs differential) and the specifics of the particular code involved. In general there are the following categories:

a) Overall-performance indicators of the fluid system under design--such as lift, drag, pressure-recovery coefficient. These establish the engineering worth of the system.

b) Detailed mean-flow data--such as velocity profiles and pressure distributions. They can provide a basis for understanding the flow so that it can be more effectively "managed" to produce desired levels of overall performance. They can be particularly important when more than one performance parameter must reach specified levels, e.g. lift and drag; heat transfer and friction factor.

c) Detailed turbulence intensity and scale. This can be useful input for designs involving combustion, aerodynamic noise, structural vibration, and mixing processes.

d) Coherent structures. This can be additional useful input for the same design problems as turbulence data.

How many of these types of data need to be generated? Only as many as are needed for, or can be used in, the design problem at hand.

DATA MANAGEMENT AND DISPLAY

The large quantities of numerical output that can be generated need to be organized into comprehensible information if they are to be effectively used--and preferably in visual form. This can involve several levels: one that facilitates the interpretation and understanding of results by the computor himself; another that

permits effective communication of results to the designer; and a third to make the output understandable to decision-making technical managers. The desirability of different presentations for different audiences (i.e., different strokes for different folks) is particularly the case for complex flows, and especially if they are three-dimensional.

The value of effective graphical display is demonstrated by the fact that some industrial concerns have a complete computer system dedicated to data handling and display.

CONCLUDING REMARKS

From the engineering point of view, the ultimate use of CFD is for industrial design. However, full utilization in this context must be consistent with the characteristics of the industrial environment--which follow, in one way or another, from the profit-making requirement of industry. The nature of this environment and the factors typical of the design process need to be well understood by developers of computational codes if CFD is to play the important role in engineering design that its proponents feel it can.

ACKNOWLEDGMENT

In composing a general paper such as this it is difficult to be explicitly aware of all the sources that one has used—influenced as one must be by his own work experiences and philosophy, general discussions with others over the years, and writings on the general subject area. To any, who feel they recognize concepts and/or phrasings they believe to be their own, I extend my apologies, and my thanks.

Discussion

P. Roache: It is ironic that the important subject of grid generation has not been mentioned until now. There will be an AFOSR-sponsored conference on adaptive grid-generation techniques in Spring 1982 at Nashville.

PREDICTION OF TURBULENT FLOWS--A BOEING VIEW*

By

E. Tjonneland†

ABSTRACT

The primary role of computational fluid dynamics (CFD), including the prediction of complex turbulent flows, is to assist in the design of aircraft or components configured to maximize the aircraft performance by control of the flow through or about the aircraft. The goal of CFD technology development is thus an improved ability to predict the flows characteristic of modern aircraft. The advantages of using CFD for design are reduced aircraft-development time, cost and risk, removal of the existing data base as a design constraint, and the ability to produce superior products for a competitive dynamic market. Our overall approach to the development of this flow-analysis technology is based on the R&D process--an early identification of key problem areas and a coordinated effort to resolve these to achieve specific goals. In the context of this, the 1980-81 Stanford conferences on complex turbulent flows are viewed as the most significant events in fluid mechanics in this decade. Key results from these meetings are the selection of trustworthy data sets for a wide range of turbulent flows, the creation of a data library (containing the data cases on magnetic tape), and an evaluation of the available turbulence models and computational procedures through comparisons between computer results and data for the test flows.

We are today on the threshold of predicting the complex three-dimensional flows of real aircraft. The challenge of the next decade will be to use the results of the 1980-81 Stanford conferences to this end. Toward this goal we advocate a definition of key problems which must be resolved to achieve this goal and a coordinated attack on these problems. These problems, as we see them, are as follows: (1) The infinite number of flows which must be considered if we continue to classify flows on the basis of geometry. (2) An inadequate understanding of how well current models work to predict both basic and complex turbulent-flow phenomena. Some understanding of this should also result from the present meeting. (3) Inadequate procedures for numerical error assessment. We must be able to distinguish between numerical errors and modeling errors if we are to achieve improved modeling. If we can coordinate our efforts to resolve these problems, tremendous progress can be made in the next decade in the prediction of complex three-dimensional flows.

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*The author gratefully acknowledges the helpful suggestions by and discussions with my coworkers Dr. S. F. Birch, Mr. C. K. Forester, and Dr. G. C. Paynter who contributed to the preparation of this article.

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Introduction

The goals of a typical corporation are to maximize profit and ensure survival of the corporation. One strategy for accomplishing this is to market a technically superior product at a competitive price. A technical advantage has tremendous leverage in the marketplace. The aerospace market is very competitive, both nationally and internationally. At stake in this competition are our national security and economic welfare. The high stakes and heavy competition in the aerospace marketplace require a constant high sustained effort to develop improved products and to satisfy rapidly changing market requirements. The industry attempts to compete in this marketplace through application of the Research and Development (R&D) process.

The R&D process was defined by the Wright Brothers (Ritchie, 1978). It is ubiquitous through the industry and has remained essentially unchanged since the period of the Wright Brothers. In a typical R&D cycle to develop a new aircraft, reasonable overall performance objectives, based on past experience, as embodied by preliminary design tools, are defined and translated into specific performance objectives for the components. For example, for a given vehicle the wing, control system, propulsion system, inlet, nozzle, installation, etc., each would have specific performance objectives defined from the preliminary design process. A detailed plan is prepared which will lead to achievement of the desired component and overall objectives. Work on the various components of the aircraft is closely coordinated to ensure component compatibility and to avoid duplication of effort in the integration process. Potential problem areas are defined as early as possible and research to resolve these problems is coordinated within the constraints of the overall program.

Aircraft design is by component buildup as shown in Fig. 1. In a typial design process, once a design concept has been defined for a given component or assembly, parametric model-scale tests are run to define performance for a range of configurations. On the basis of this performance information, configurations are selected for full-scale validation and optimization. Full-scale tests are run to confirm that the results of the model-scale parametric tests scaled. On the basis of the full-scale test results, a final design is chosen for flight test or for fabrication. The potential role of CFD in this is to replace parametric model-scale testing with parametric analysis as the basis for configuration selection. The rising cost of wind tunnel testing means fewer configurations can be evaluated in a given design process using a test-based approach. This implies that the risk associated with achieving a successful design is increased. The increasing availability of powerful computers and rapid progress in algorithms for solution of the partial differential equations of fluid mechanics make CFD an increasingly attractive alternative to parametric model-scale testing in the design process. Use of parametric analysis rather than parametric tests for design offers the advantages of reduced development cost, time, and risk. It

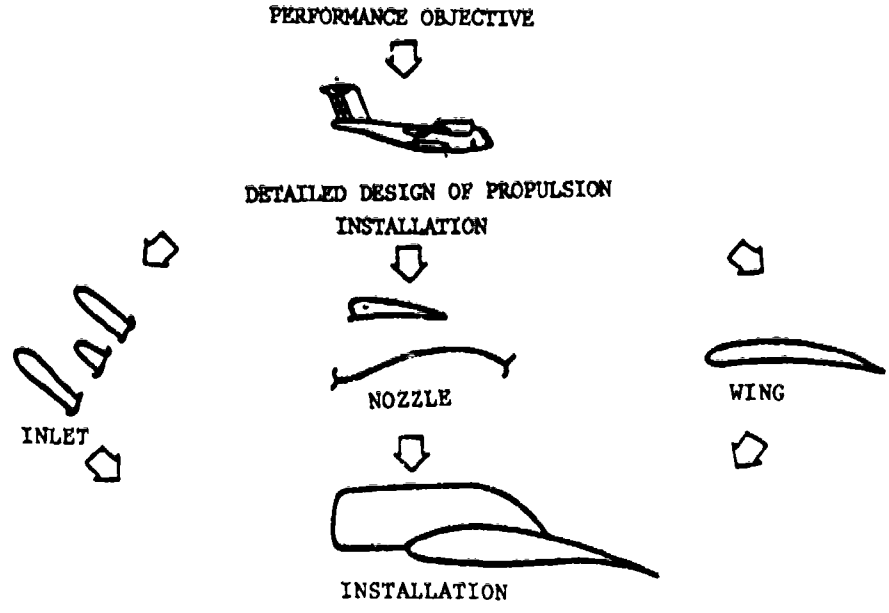

Figure 1. Design by component buildup.

also removes as a design constraint the existing test data base. At present, the design process is a mix of test- and analysis-based approaches. The available analyses are used to reduce the size of the configuration test matrix. The analysis is also used to scale and interpret the test results. As prediction methods improve in accuracy, availability, cost, usability, etc., the design process will use more analysis and less testing.

Because of the potential market advantages, the aerospace industry is hard at work developing a flow-analysis technology which will predict the flow through or about aircraft. As one might expect, we at Boeing are applying a strategy similar to the R&D process to the development of this new flow-analysis technology. Our current lack of understanding and inability to model complex two-dimensional and three-dimensional turbulent flows have been identified as critical problems. In the context of our work to develop an improved analysis capability, the results of the 1980-81 Stanford conference should lead to an improved understanding and an improved ability to model the complex flows of real aircraft.

Flow Analysis Technology Development

The overall goal with our CFD technology is an improved ability to predict the flows characteristic of modern aircraft in a design environment. These flows and geometries are complex, three-dimensional, and viscous. The design environment implies hundreds of configurations may have to be considered. This design environment also implies the use of commercially available computers. The time and effort required to set up, run and understand a flow with a given analysis procedure or procedures is a significant part of the cost of a design process.

Our overall flow-analysis strategy is to use zonal modeling, which is to couple multiple analyses in various flow regimes to take advantage of simplifications that are available through the local nature of the flow. For example, a three-dimensional potential flow analysis coupled to a three-dimensional, parabolized, time-averaged Navier-Stokes procedure might be used for a nozzle installation with a close coupling between the aerodynamic flow over the wing and the flow from the turbo fan engine as illustrated in Fig. 2. A zonal modeling strategy is used because the use of a single analysis for the complex flows of a real aircraft would imply solution of the three-dimensional Navier-Stokes equations of large flow regions. This is impractical on current and projected commercially available computers because of the large storage required, high cost, and the difficulty in setting up the solution. Solution of the full Navier-Stokes equations is generally unnecessary because the flow in most regions of interest is adequately described with simpler inviscid and boundary-layer or thin-shear-layer methods. Zonal modeling is also very compatible with aircraft design by component buildup. It is very desirable to be able to use analyses of varying levels of accuracy depending on the stage of the design process.

A "broad front" strategy is applied to the development of the necessary flow-analysis technology. This strategy is characterized by an early identification of key problem areas and a coordinated effort on the individual analysis components to achieve specific goals. This strategy avoids duplication of work on the various analysis components and ensures the compatibility of the various analysis elements (numerics, turbulence modeling, mesh generation, geometry description, etc.) used in the analysis of a given flow. The advantages of a "broad front" strategy are that the time and resources required to develop a strong flow-analysis capability are minimized. Typical

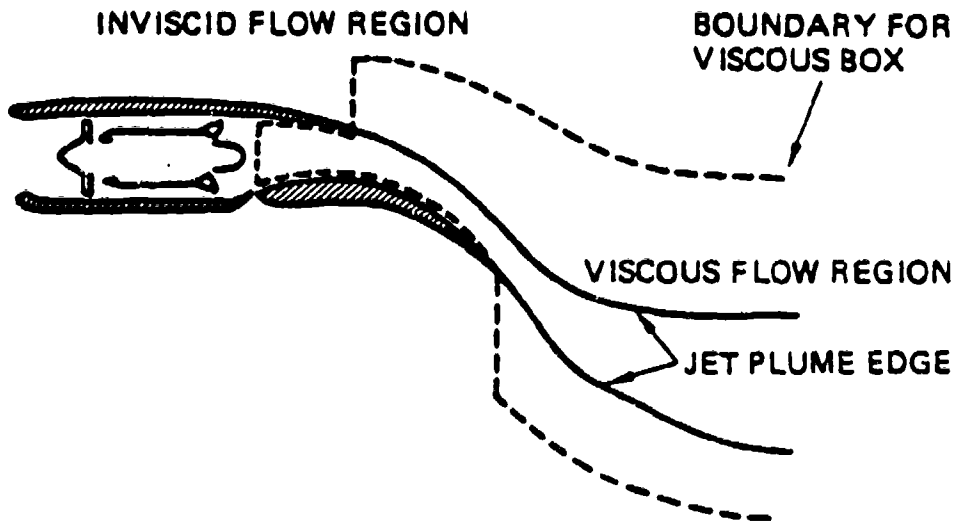

Figure 2. Zonal flow analysis of a powered lift installation.

problem elements for a given analysis development are (1) numerics, (2) mesh generation, (3) turbulence modeling, (4) modeling and validation experiments, and (5) data handling and display. The Boeing plan for flow-technology development for the design of propulsion components and installations was described by Welliver (1977).

Turbulence Modeling

From our viewpoint, the current status with regard to turbulence modeling is that we can predict many two-dimensional and a few three-dimensional flows of interest. We generally use models based on two-dimensional data to predict these three-dimensional flows. The argument for this is that turbulence phenomena are essentially three-dimensional, even when the mean flow is two-dimensional. Therefore, models developed for two-dimensional flows should work for at least some three-dimensional flows of interest. As noted recently by Hall (1981), the available turbulence models seem to work reasonably well where the turbulent structure changes slowly. Examples of these flows are the boundary layers, mixing layers, wakes, and jets in mild adverse or favorable pressure gradients. The available models do not seem to work where rapid changes in turbulence structure occur. Examples of these flows are regions of separation, reattachment, and shock boundary-layer or shear-layer interaction.*

Our overall objective in the turbulence modeling area is once again an improved ability to predict the two-dimensional and three-dimensional complex turbulent flows of real aircraft. We are interested in flows from low subsonic to high supersonic Mach numbers and Reynolds numbers ranging from transition to flight values. Desirable turbulence-modeling features are compatibility with the available numerical procedures, simplicity to minimize the computing cost, and a wide range of application.

The 1980-81 Stanford meetings, viewed in the context of the industrial CFD technology development, are the most significant events in fluid mechanics in this decade. The accomplishments of these conferences are the selection of trustworthy data sets for a wide range of complex flows, the creation of a data library with these flows available on magnetic tape, and an evaluation of the available turbulence models and computational methods through comparisons between computed results and the experimental data for these flows.

Challenge of the Future

We are now on the threshold of predicting the complex three-dimensional flows of real aircraft, and the results of the present Stanford meetings will undoubtedly play an important role in future develoments. The first step toward this goal, however, is

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*Mean-flow properties through such interactions are predicted reasonably well with current methods. The relaxation of the turbulent layer downstream of the interaction is not well understood. [Ed.: The mean flow properties are not well predicted downstream of separation and reattachment--see comments, pp. 890 and 951.]

a definition of key problem areas. These problems, as we see them are: (1) There are an infinite number of flows to be considered if we continue to classify flows primarily on the basis of geometry. (2) There is an inadequate understanding of how well current models work for both simple and complex turbulent flows. An improved understanding should result from the present meetings. (3) Our present procedures for numerical error assessment are inadequate. We must be able to distinguish between numerical errors if we are to achieve improved modeling.

The major challenge, in our view, is the development of an approach to turbulence research that will allow for a meaningful study of the very complex flows encountered in industrial applications—essentially combinations of the complex flows considered at this conference. It is obvious that we must first understand individual flow phenomena before we can hope to predict various combinations of these phenomena, but it is far from obvious that a simple extrapolation of procedures we now use to study these simpler flows will lead to a real ability to understand and to ultimately calculate the very complex flows encountered in most practical applications.

As the complexity of the flow increases the difficulty and expense of experimental studies also increases, so that the number of flows that can be studied, with a more or less fixed budget, decreases. But since the total possible variations increases with complexity, the fraction of the flows that can be studied experimentally, within a given general classification, decreases dramatically. Because of this we must expect that most flows encountered in practical applications will differ in some potentially important way from any previously studied flow. Faced with this situation, how does one select a calculation method for a given flow, and having performed the calculation, how does one estimate the reliability of the result? This problem is really more basic than simply developing an improved method for classifying turbulent flows. Its solution requires, at minimum, the development of a broad-based approach to turbulence research, that includes both model development and experimental design. The work by Bradshaw (1975) and his associates, over the past decade, has probably been the most successful general attack on this problem to date, but an enormous quantity of work still remains to be done before we can claim to have any general ability to calculate the behavior of turbulent flows. We believe that progress in the calculation of complex viscous flows over the next decade will depend largely on our response to this challenge.

Most of the flows encountered in industrial applications are complex and three-dimensional, and we are going to continue to apply the available modeling to these complex flows. Almost no detailed data exist. Surely we should document at least a few flows of current interest as "benchmarks" against which current models can be evaluated. Flows which might be considered are those associated with (1) multi-element airfoils, (2) multi-element airfoils with control surfaces, (3) multi-stream

engine exhaust/nacelle flow interaction, (4) the near wake of single- and multi-element airfoils, and (5) vortex generators under shear flows.

Numerical error assessment is at present a tedious and difficult task. Data on the residual error as a function of iteration cycle and truncation error as a function of mesh density, distortion, and location are gathered by repetitive solutions for various meshes. Computer codes are commonly "validated" for a given flow by comparing computed results with experimental data. Examples of flow calculations with an inadequate mesh in which poor agreement between computed and measured flow properties is attributed to inadequate turbulence modeling are all too familiar. We must develop automatic error monitors and mesh-adjustment procedures so that we can sort out modeling errors from grid-related errors. Until we do this, we will continue to misinterpret how well our present turbulence models work, especially for complex flows where mesh selection is difficult. Multigrid methods appear to be particularly well suited to resolving these problems (Forester, 1981). Since multigrid methods use a range of mesh densities, truncation-error information is readily available as the solution develops. This error information can be used for mesh adjustment and solution acceleration.

Concluding Remarks

The use of CFD for aircraft design is new. This use has occurred only over the last 15 years by industry. This is the beginning. The competitive aerospace market requires improvement and increasing utilization of our present CFD technology. We can expect radical change in aircraft design over the next decade because of the use of this technology. We must apply a well-defined and coordinated attack toward specific objectives to achieve improved turbulence modeling. Improved turbulence modeling is an essential element in the prediction of the complex three-dimensional flows of real aircraft. The ability to predict these flows, over the next decade, in a design environment, will have an important impact on our national security and prosperity.

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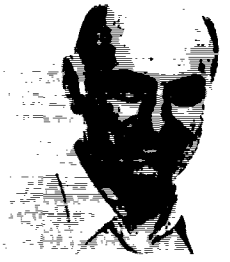
SESSION XV



Chairman: G. Sovran

Technical Recorders:

W. Feiereisen
E. Adams

EVALUATION COMMITTEE REPORT — H. W. Emmons

DISCUSSION OF EVALUATION COMMITTEE REPORT

EDITORS' INTRODUCTORY REMARKS TO UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD

UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD--
A Personal OPINION — S. J. Kline

COMMENTS ON UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD
— P. Bradshaw
— A. M. Savill
— P. J. Roache
— G. S. Settles
— A.R.A.P. Computor Group

CLOSURE ON ZONAL MODELING--THE ROAD AHEAD — S. J. Kline

EVALUATION COMMITTEE REPORT

This conference, which is the second part of the complex turbulent flows meeting has presented turbulence modelers with a selection of the best experimental data on a wide range of turbulent flows--66 cases in all. Thirty-five different computer groups used 67 different methods.* This resulted in 1266 curves comparing calculation results with experimental data.

This tremendous computational effort supplies a great deal of detailed information which will take some years to digest--and additional calculations--to fully understand.

The Evaluation Panel with nine man-weeks of total effort has been able to do no more than discern certain major features of the status of our 1981 computational capabilities. We invite all present and future computors to study the results of this conference comparisons as an aid to the important areas to be further developed.

A comparison of the results of the 1968 and 1981 turbulence conferences shows the considerable increase in capability. The most remarkable increase in capability is in the range of turbulent flows that can be calculated with some degree of success.

In this conference significant progress was shown in the calculation of separated flows, boundary-layer shock-wave interaction, calculation of the various turbulent velocity components u', v', w', the decay of turbulence toward laminar flow, and transonic flows. Both elliptic and parabolic problems can now be done with fair success. In 1968 there were only a few programs based upon field equations with a one-equation model. There were no two-equation models. We now have a much wider range of models with a wide range of applications.

The increase of capacity in available calculating machines over the past decade has been remarkable and, as is expected, the turbulence calculators have expanded their desires and efforts correspondingly. Thus, some of the problems calculated for this 1981 Conference could not have been attempted in 1968, even if the physical models had been formulated at that time.

Having noted all these important advances, we must not suppose that there is nothing more to do. In fact there are no flows and no methods which are wholly satisfactory. Indeed one of the test cases used in the 1968 Conference, the attached boundary-layer flow, has been also used in this Conference and the computed results are shown to be in about the same agreement with data as was shown in 1968.

In attempting to evaluate our present standing we recognize two areas of critical importance:

- The mathematical model of the basic physics of turbulence.
- The numerical technique chosen for its solution.

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*[Ed.: See also remarks in editors' conclusions regarding number of computer groups.]

The mathematical models which use directly the Navier-Stokes equations are so demanding of computer memory and computer time on the largest computers that only special scientific studies of turbulence can be treated in this way, and then only at low Reynolds numbers. Higher Reynolds number can be achieved but only when some suitable approximation to the flow equations is introduced such as the "Large Eddy Simulation Method," or perhaps at some date in the future when computer capabilities have increased by some orders of magnitude. These are very important studies but are not of direct use in this conference.

The models considered are those represented by the taxonomy developed for this conference. The main groups are: integral models, one-equation models including Boussinesq models, algebraic stress models and multiple-equation models for the Reynolds Stresses.

The numerics affect the results of the current models both in the algorithm chosen and the number and distribution of nodal points and other computational details.

The result of any given computation depends intimately on both the mathematical model of turbulence and the numerical technique chosen. It is impossible in many of the flows computed to separate the physical from the numerical limitations of the present work. In a few cases--but only few--grid refinement or other tests for solution accuracy were used and even some of these still showed significant changes in the numerical results.

The Evaluation Committee therefore had to judge the current status of the computing of turbulence flows as a complex of model plus numerical status.

With this "global" view in mind we constructed a matrix of computations given as Fig. 1. Figure 1 shows only the number of cases computed in each area. We in fact tried a simple quantitative evaluation of every flow calculation in its agreement with the specified data. This, an at best approximate assessment, is not included here because the cases computed were too sparse to give statistical significance to the results. Futhermore, it confirmed the impression gained during the week of presentations, that every method had its strong and weak points. No method had any significant universality. Likewise no method proved to be universally bad.

This table, together with our examination of individual flows and individual methods, have permitted us to draw some tentative conclusions (C-1 to C-7).

C-1. The most important conclusion is that all methods are well worthy of further study and refinement.

C-2. The weakest point of present one-point closure models is the ε-equation. The computed results of many flows can be brought into good agreement with the data by tweeking the ε-equation constants (as ε itself). A better equation should make

Number of Methods / Number of separate flow cases tackled		Strain	Case Number	Classification of Method for Calculating Turbulent Flow					
					One-Point Closure				Two-Point Closure
				Integral Methods	Prescribed Eddy Viscosity or ℓ	Boussinesq (B)	Algebraic (A)	Reynolds Stress (RS)	2ED
Incompressible	Free Shear	Simple	0311, 0381, 0471	• •	• •	8 9	2 2	2 2	•
		Extra	0331, 0382	• •	• •	5 5	5 5	2 2	•
	Wall B.L.	Simple	0141, 0211, 0261 0612	9 17	4 10	11 26	6 8	2 4	•
		Extra	0142, 0143, 0132, 0232, 0233, 0241, 0242, 0244, 0263	3 6	3 5	5 18	4 16	2 5	•
Compressible	Transonic	Simple (attached)	8601, 8621	5 5	3 3	2 2	1 1	• •	•
		Extra (attached + separated)	8623, 8611, 8612, 8691	1 2	• •	1 1	2 2	• •	•
	Supersonic	Simple (attached)	8101, 8201, 8403, 8411	2 5	3 4	2 4	1 3	1 4	•
		Extra (a) (attached)	8501, 8301	• •	3 5	• •	1 1	3 3	•
	Supersonic	Extra (b) (attached + separated)	8631, 8632, 8651, 8661, 8663, 8671 8641	1 1	3 5	2 6	• •	• •	•
Incompressible	External	Extra (stalled airfoil)	0441	3 3	• •	2 2	1 1	• •	•
	B.L.	Extra (relaminar)	0281	• •	• •	2 2	• •	• •	•
	Internal	Extra (attached)	0111, 0112, P1(0113) 0511, 0512	• •	2 4	5 6	2 2	• •	•
	Internal	Extra (separated)	0421, 0431, P2(0422), P3(0423)	3 4	3 4	11 14	3 3	3 4	•
Incompressible	Homogeneous	Various	0371, 0372, 0373, 0374, 0375, 0376	• •	• •	• •	2 18	8 72	2 22

Figure 1. Matrix of computations (an approximate guide).

these changes in value an automatic part of the calculation. A corollary effect of the ε-equation defects is the too large length scale in adverse pressure gradients and near separation.

C-3. The use of the algebraic stress models would be expected to be better than a scalar Boussinesq method. In fact, in calculating certain flows, as the turbulence-induced secondary flows in a corner, the algebraic methods give a fair answer while Boussinesq methods give none. However, an evaluation of the flows presented in this conference show no significant difference on the average. The Evaluation Committee agrees that a significant difference should result as the algebraic methods are futher improved.

C-4. The methods which have included an integration to the wall have been somewhat better than those assuming a "law of the wall." Clearly a more general "law of the wall" could fix this discrepancy. However, we believe that the number of influences on the wall profile is so large and the computational capacity is so large that, except in various special cases, an integration to the wall is preferable.

C-5. The fact that none of the present methods are influenced by rotation of the turbulent flow is an indication that present models are deficient in this respect. This question needs further study.

C-6. Present methods cause supersonic mixing layers to spread too fast. The repair of this defect should help guide further model improvement.

C-7. In turbulence models the simpler the treatment, the narrower the range of application. Thus we have a progression from the universally applicable (but impractical) full Navier-Stokes model to the narrowly applicable integral models. The progression is Full Navier-Stokes > Large Eddy Simulation > Reynolds Stress Models > Kinetic Energy and Decay Models > Mixing Length Models > Integral Models.

Model development and computer capability is slowly working up this list. However, this list is very significant in two additional respects: (1) The computation time in general decreases from full Navier-Stokes to integral models, so that for practical work an integral model is preferable if it is applicable. (2) Each of the easier models are based upon a greater amount of empirical knowledge. There is an important possibility of improving the simpler models by solving a series of flows by a more advanced method in order to determine appropriate empirical information for the simpler models. The more advanced model may in some cases be a better source of guidance than experiment, because the advanced model can supply information about quantities difficult or impossible to measure.

C-8. The fact that different numerics including algorithms and grid- and time-spacing yield different results was well brought out in the presentations, but was not adequately considered by the computors. We realize that halving the grid spacing is often computationally prohibitive on presently available machines. However, some testing of numerics is not prohibitive, especially for parabolic methods, such as use of higher-order accurate numerical methods for two-dimensional flows.

Although the results of the rough "quantitative" assessment could not be expected to produce statistically significant information on the relative successes of the computations in matching data, due to the small samples involved, nevertheless certain conclusions and trends (T-1 to T-5) were found and are reported here.

T-1. Before this meeting many workers in turbulent fluid mechanics were probably of the opinion that Homogeneous Turbulent Flows could be calculated to a suitable accuracy if one or another of the more sophisticated turbulent-flow models were used. What came out of these calculations was that the Reynolds-stress models (RS) were only slightly worse than the two-point closure schemes (eddy damped quasi-normal hypothesis), but the results of the one-point closure scheme using an algebraic model were significantly worse. Of course, the task set computors was that of finding the time decay of the three (normal) Reynolds-stress components as well as the Reynolds shear stresses, and hence it was not surprising that one-point closure schemes using algebraic stress models were less than satisfactory. However, it was interesting to note the differences that came out of these computations and careful study of these results will no doubt bring forth some important information for turbulent-flow modelers, since they base many of their models on the results from homogeneous flows. The lack of success in these flows gives a measure of how far we are away from generating a universal turbulent flow model.

T-2. In the case of the flat-plate boundary layer in compressible flow with insulated and variable wall temperature, the results of all computations using a variety of methods showed a spread of results roughly equal to that of the experimental data. This spread of about ± 10% was probably higher than we had expected, and is of importance when we consider the corresponding accuracy which we can expect for calculations on more complex turbulent flows involving compressible flows.

T-3. The general accuracy of results involving separated flows was significantly worse than for corresponding attached flows. For a flow involving separation, the (RS) methods did no better than the less sophisticated approaches, and, in a restricted sense, the integral methods gave the best accuracy.

T-4. Similar conclusions to (3) were found for free-shear layers, where again the (RS) methods did no better than other methods, such as the algebraic (A) or Boussinesq (B) methods.

T-5. For attached boundary layers the (RS) methods showed some advantages over other methods, although again in a restricted sense the integral methods performed satisfactorily. For flows involving secondary flows the one-point closure methods gave useful results, but the methods in their present form are clearly in need of considerable refinement.

Although it is not possible to determine from the present results the relative merits of the various existing models, it is clear that this enormous computing effort has been well worthwhile in clarifying many fine details and in providing many specific tasks which, when added to the present results, will make the next ten years as profitable as the last ten.

Some of the suggestions (S-1 to S-7) which occur to the Evaluation Panel are listed for whatever they may be worth to the next ten years of computing effort:

S-1. One of the more important steps, which will make model comparisons more meaningful, will result from a more detailed concern for the numerical problems leading to accuracy of solutions. Only in this way is it possible to distinguish the precision of the physical modeling from the inherent numerical errors related to algorithm and grid. The effects of adjustment of a single model may be judged by use of poor numerics; different models cannot be safely compared. This is a very serious question deserving much greater consideration in the future.

S-2. The Conference has shown that many complex turbulent flows, incorporating both internal and externa flows, can be computed to satisfatory engineering accuracy, although improvements and extensions are desirable and necessary. The design of codes for inexpensive engineering use, can often justify less than perfect numerics. For this purpose codes should be approached as a package--a turbulence model with constants adjusted to give adequate results for a class of flows together with a narrow class of numerical methods and grid sizes.

S-3. It appears to the Evaluation Committee that more work is needed on the homogeneous flows by use of the Reynolds stress model. Success here could then cascade down through the simpler models to show the way to their improvement.

S-4. The models need special attention to the following:

a. the ε-equation
b. the pressure-strain correlations
c. the effect of adverse pressure gradients
d. the effect of rotation

e. compressible flow mixing layers

f. detailed integration to the wall in some simple but adequate way.

S-5. The items under (4) above are open research questions. The answer may come from an ingenious analysis, trial and error, or simply a good guess. On the other hand, the best approach may in some cases be a carefully performed experiment. Every known technique should be tried as a guide to better future procedures.

S-6. It is clear that there are great difficulties involved in truly three-dimensional computations. However, the limited success of the three-dimensional computations presented to this conference is an encouragement for further efforts in this direction.

S-7. It may be desirable for a small computor group to examine the totality of results of the present calculations to locate those holes, where a few additional calculations would permit more definitive method comparisons, to select a few specific cases to be computed by everyone, and to hold a limited conference in a few years, which would give time for new computer codes and flow models to be tried and tested, and computed output to be available for comparison with the 1980 Data Bank. Such a conference could be held as a specific session at some national meeting.

If this is done, the Organizing Committee should require a detailed statement of not only the physical model used, but also the numerical method used, including plots indicating the exact number and location of the grid points. Also required should be an exact statement as to how and where the boundary conditions are satisfied.

The Evaluation Panel is aware of the fact that we are not able to do what we all wish were possible, namely to say this is better than that, so that work in the future be more narrowly directed. Nevertheless, certain flows such as the airfoil at transonic speeds and the airfoil at low speed at a large angle of attack in particular have been shown to be capable of flow prediction—features which were not thought remotely possible at the 1968 Conference. At the 1968 Conference, it was also felt that there was likely to be little interest in the future for integral methods once the two-equation and higher-order models had been further developed and improved. We see at this Conference that for the calculation of certain global features of turbulent shear flows, integral methods continue to perform adequately and for many engineering purposes are sufficient and preferable.

We have been most impressed by the great advances made since the 1968 Conference. We feel most encouraged for the future by the tremendous effort already expended and the spirit of cooperation, both betwen us and the computors, and the natural respect

and coooperation between the computors themselves. We believe this conference provided an important push into the future in spite of few clear-cut evaluative decisions.

This report represents the consensus of the full Evaluation Committee

Members:

H. W. Emmons, Chairman

D. R. Chapman
P. G. Hill
G. M. Lilley
M. Lubert
M. V. Morkovin
W. C. Reynolds
P. Roache
J. Steger

DISCUSSION

EVALUATION COMMITTEE REPORT

J. McCroskey: What does the Evaluation Committee think about the time scale for the next conference, the number of flows to be selected, and how to select those flows?

H. W. Emmons: The committee has only briefly discussed this. My personal opinion is that there should be about six cases, chosen by the computors themselve, to bring out particular issues. A small group of computors should be organized to select flows from the existing data base.

P. Joubert: I believe the experimenters should pick the cases.

S. Bogdonoff: I am worried about the data base upon which we are building the computational structure. Experimental boundary conditions are as difficult as computational ones. Most of the flows have been measured by only one laboratory. Also, I don't think six test cases are enough. We must have flows that have been independently confirmed by several laboratories, and which test a range of the parameters instead of just a single value.

H. W. Emmons: I believe there are six reliable data sets, although further experimental work is always welcome. I was thinking of just six cases for the next conference in two years.

S. Bogdonoff: Are the experiments we are using really the experiments we need? Some of the quantities required by computors are not always measured.

H. W. Emmons: Experimenters would do well to look at the proceedings of this conference to learn what kind of data is required by computors.

S. J. Kline: We need both experimenters and computors on the Organizing Committee for the next conference. That conference should include both simple flows and ones that push the state of the computational art.

B. Lakshminarayana: Another comment in regard to the next conference. If it takes place in only two years we shall see the same results we have now. Let the

proceedings of this conference come out, allow time for the results to crystallize, and then maybe in five years hold the next conference.

H. W. Emmons: Two years was suggested by the computors themselves, in the spirit that there be only a small number of flows and that all computors would do something new.

M. V. Morkovin: I think two years from now is a good time for a new conference. There's still much "gold in them thar hills." Computors have learned to ask their own questions; everyone has learned here. We have a lot to deal with already without any new test cases.

S. J. Kline: I agree with Bogdonoff that some other experiments should be considered. The Evaluation Committee asked people to regard some of the present test cases as central, but this was largely disregarded. We should have experiments which test a range of parameters, done by at least two laboratories, and with full uncertainty analysis. We were not expecting a clear-cut result from this Conference like the 1968 boundary-layer conference, but at least we now have a picture of the state of the art as of this week. I agree with Morkovin; we should show the computors where to go next. Do the computors see goals?

R. Melnik: We thought we would find holes in the present data base and we did. These need to be filled with future test cases and we need recommendations from this conference in order to be able to get the funding to do the experiments.

P. Joubert: I like the idea of setting up some standard experimental test cases and having them performed in different laboratories.

B. Cantwell: Six cases are probably too few to cover the 18 flow modules that Kline has outlined. I don't think you'll find six cases to do all that.

H. W. Emmons: The six cases are for two years from now.

B. Cantwell: Would you care to recommend the six cases?

H. W. Emmons: Not off the cuff.

P. Roache: I would recommend an AIAA meeting organized around one good test case which everyone would do. A variable expansion channel or a variable backstep would be a good idea. You would ask for everything normalized on the reattachment length.

B. Cantwell: That experiment is a variation of P4 for which the data never materialized.

D. Chapman: I agree with Emmons. The next conference should be restricted to a lesser number of flows which have been independently checked by several laboratories.

M. V. Morkovin: I want to see experiments performed at more than one laboratory.

P. Orlandi: Most closures used in the conference were one-point methods; there was only one two-point method. I think two-point methods will spread. We will need

turbulence spectra for the initial conditions. The homogeneous flow results depend on initial conditions.

S. J. Kline: We could set up a committee to deal with Orlandi's question on initial conditions and with Joubert's and Bogdonoff's suggestions. We need both experimenters and computors on this committee.

G. Sovran: The questica really being addressed here is whether the quality of the present data base warrants additional computational effort.

I'd like to address a question to the Evaluation Committee. What kind of information would you like to have had that you did not have?

H. W. Emmons: A few test cases which everyone had to do would have allowed us to make a better comparison of the methods. Also, more detail on the grids and numerics would have been helpful.

M. Lubert: We need to have information on how the residual errors were computed. Also, future Evaluation Committees should receive the computational results before the conference--we need more time! Finally, we need a combination of computors and experimenters to design the experiments to be used as test cases.

E. Reshotko: Are there any points that the Evaluation Committee feels can be addressed by further comparisons of the computations of this conference?

H. W. Emmons: We should have had more time for our evaluation, but we didn't. We don't have specific general recommendations for further evaluations, but we should ask for suggestions on what could be done in the next few weeks.

S. J. Kline: I'd also like to know if anyone has recommendations to the Evaluation Committee for work in the next few weeks or months?

G. Sovran: I have a question for the computors. Are there other elements of an evaluation that you would like to have seen?

J. Murphy: I want to suggest that we not rank methods in a single queue. Some consideration needs to be given to what method is best for a given flow. For example, you cannot compare an integral method to a full Navier-Stokes simulation. In industry, efficiency, cost, and relative accuracy are important.

H. W. Emmons: Within its range of applicability an integral method can be compared with a time-averaged Navier-Stokes solution.

R. Melnik: There are some situations where integral methods are more accurate because you cannot fit enough points in computer memory for a differential calculation of the same accuracy.

N. Mansour: Algebraic turbulence models should be better than Boussinesq models, but this was not proven by the results of this conference.

B. Launder: We need more time, a year was not enough for the computations. We know how to use the data tape now, so two more years should be enough.

The computor groups feel they can distinguish between the results of algebraic and Boussinesq schemes. We would like to discuss with the Evaluation Committee what leads them to conclude otherwise.

W. C. Reynolds: The conclusion is that overall there is not clear evidence that algebraic is better than Boussinesq. In some cases Yes, in some No.

W. Rodi: Our group took the attitude that for some flows algebraic turbulence models are needed, and we used them; in others they are not, and we didn't.

K. Hanjalić: Computors need more upstream history from the experimenters for initial conditions.

G. Sovran: How much of the computational success demonstrated in this conference was the result of computor skill and ingenuity in postdicting known experimental results? Should we have had more predictive cases?

H. W. Emmons: I don't think significant improvement would be gained from secrecy. Each method is tuned to a flow situation. No method is universal.

E. Reshotko: We do have an answer here. Predictive cases were handled no better and no worse than other flows.

J. Eaton: Predictive experimenters went to an awful lot of work for this conference. Was this necessary? The agreement was not as good in the predictive cases.

I. Castro: I do not believe separated-flow experiments are as accurate as boundary-layer experiments. It is dangerous to do predictive cases in these flows.

S. J. Kline: Case P2 (0422) was done in two laboratories; this was done on purpose. I do not see much difference between the computational results for the predictive and postdictive cases.

Experiments should be performed in at least two laboratories, and the uncertainty checked.

H. Nagib: When you suggested that simple flows were computed well in this conference, for example attached boundary layers, were you including recovery from curvature?

H. W. Emmons: Clearly NOT; even in boundary-layer computations, improvement is needed.

B. Lakshminarayana: Does the Evaluation Committee have any recommendations on turbulence modeling?

H. W. Emmons: We have no specific recommendations on this. The Evaluation Committee has no key to Mother Nature that you don't have.

P. Hill: How much progress has been made on flat-plate boundary layers approaching separation? I was pleased with the ability to calculate beyond separation but we now need to go back and make sure we can compute up to separation with confidence.

D. Chapman: It is unimportant to see if method A or B is better. It is more important to see how each can be improved.

EDITORS' INTRODUCTORY REMARKS TO UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD

Discussion of the question of universal versus zonal modeling occurred at three different times as part of the 1981 meeting. An extended oral discussion occurred as part of the evening meeting following Session IV. S. J. Kline added a postscript to that discussion during Session XVI. This postscript was then rewritten and extended by S. J. Kline as an OPINION shortly after the meeting. The OPINION was sent to all attendees of the 1981 meeting with the statement that written comments on it from attendees would be published. Five such written comments were received: from P. Bradshaw, A. M. Savill, P. J. Roache, G. S. Settles, and C. duP. Donaldson et al. To preserve the record and also make sense of these continuing discussions in the Proceedings, the editors have placed them all in two places.* The postscript to Session IV and the oral discussions concerning it have been placed at the end of the discussions to Session IV in this volume. The extended Opinion, the written discussions on it, and a closure by S. J. Kline follow these introductory remarks in order to place the major evaluative conclusions in one section. The postscript to Session IV and its discussion do overlap the presentations in Session XV to some degree, but also contain several quite different comments and viewpoints.

*With the exception of a short but important comment by J. Lumley which is an integral part of his Reporter's remarks; see p. 767.

UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD

A Personal OPINION

By S. J. Kline

At the beginning of this meeting, I indicated that a (perhaps the) central question to be considered at the current time was whether a universal closure model for turbulent flows has been or can be created, or, alternatively, whether it would be necessary to do what can be called "zonal modeling" in order to obtain results of engineering accuracy for practical flows in the near and intermediate future. This same question was addressed in a group of active researchers and government monitors at the NASA Langley Research Center in May 1980, and resulted in a sharp and relatively even division of opinions. At that time I abstained from the discussion since I felt it would have been inappropriate for me, as Chairman of the Organizing Committee for this Conference, to take a public position beforehand. This topic was also covered at some length in response to the excellent questions prepared by Phil Klebanoff for the follow-up dinner session from Session IV on Monday night of the present meeting. The question of universality of closure models is a crucial one because it influences the central strategy of how one models turbulent flows.

For all these reasons, I have been paying attention to the question as the computed results for this meeting accumulated, and during the discussions and presentations of the meeting. These added experiences have considerably clarified my own thinking. It therefore seems appropriate to expand my earlier remarks (see Session IV) on the topic of universal versus zonal modeling from several viewpoints, and then draw some personal conclusions.

The discussion begins with remarks on the general nature of models in physical science since that underlies the philosophy we employ. The arguments for a universal approach are then given. Next are sections discussing: (i) the functional nature of the Reynolds stresses needed for closure; (ii) the physics of turbulent flows; (iii) experiences with modeling prior to 1968, between 1968 and 1981, and finally the experiences of this Conference. The conclusions follow from these discussions.

ON PHYSICAL MODELS

When I was in high school, our science teachers told us that we were learning the universal laws of nature. In college I found that several central things my high school teachers had presented (such as the planetary picture of the atom and Newton's laws of motion) had already been overtaken by scientific revolutions* in some cases limiting the domain of applicability to something far less than "universal" and in other cases completely replacing the principle or concept with different improved

*In the sense of T. Kuhn.

models. However, it was not for another dozen years after my undergraduate work that I came to the full realization that what my high school and university teachers had told me were the laws of nature were not that at all. They were rather, models made up by human minds to describe nature. Some of these models are of astounding elegance, breadth and accuracy; they are high pinnacles in the achievements of the human intellect. Nevertheless, they all are models with more or less breadth, but always with some limits of range of domain, and also with some residual uncertainties. At bottom, each of these principles, models or concepts is a truth assertion made by humans about nature.

Any single truth assertion of this sort is derived from and is intended to be true for a class of paradigmatic systems, and no more. There is a limit, in some cases very broad, and in other cases very narrow to the relevant class of systems. It is easy to prove this is so. One need only consider any truth assertion about nature whatsoever. We all know any such assertion can be made false, trivially, merely by changing the definition of the system. Once we clearly recognize that we are discussing human-made truth assertions about nature, and that all such assertions are inextricably tied to classes of systems our vision of the nature of these assertions is considerably sharpened. We see that the question is not whether a model is totally universal; none are adequate for all systems.* The relevant question becomes, "what is the domain over which this particular model gives adequately accurate predictions about nature?" That is precisely the question that confronts us in turbulence modeling.

THE ADVANTAGES OF A UNIVERSAL MODEL

The foregoing discussion makes clear that there is nothing "good" or "bad" about more or less universality of modeling. The important questions are pragmatic ones, "What works? What represents nature accurately?"

From this pragmatic view, it is clear that a universal closure model for turbulence would have advantages. It would allow turbulent flows to be modeled once and for all. It might be constructed more easily and quickly than a variety of models each fitted to special circumstances. Most important, it would provide us with assurance that we could do true PREdictive computations rather than merely POSTdictive computations with which we have long been familiar.

A universal model would also appeal to our sense of scientific fitness and elegance. But here we must be careful; the test is not elegance or seeming fitness, nor is it some subliminal desire to emulate Newton or Einstein. The only proper test is the pragmatic one, and our design needs demand we adhere tightly to the pragmatic

*I can provide, on request, a recent more detailed study concerning the universality of the principles of physical science and their relation to more complex systems that defines the operational modifier "adequate" in this statement.

test. There may be other aspects of universality that I have missed. I would appreciate comments particularly from those who have or do take the position that we should focus on the search for a universal closure model. As an editor, I agreed to include in the proceedings any substantive comments on this discussion that reached me from attendants at the 1981 meeting prior to 30 November 1981 (see comments below).

THE FUNCTIONAL NATURE OF THE REYNOLDS STRESSES

What is the functional nature of a complete turbulence closure model? When we time-average the Navier-Stokes equations, we lose information, and that information is inherent in the Reynolds stresses. The Reynolds stresses, for incompressible flow, are a symmetric second-order tensor that is in general a multipoint function of four variables. Such a tensor is a complex quantity mathematically. We need to hold that complexity in mind for a moment in this discussion. We also need to hold in mind that for compressible flows we must deal with a number of different variables each of which has this complexity even if we use the simpler forms of the equations given by Favre averaging.

A geometric analogy may help our thinking. Suppose we imagine the terrain of a rough, glaciated mountain chain such as the Sierra Nevada of California or the Alps. Consider the nature of a model that might describe the topography: the peaks, valleys, spires, crevices, boulders, cirques, moraines and other features of such a mountain chain. Would we expect a simple algebraic equation with a few adjustable constants to describe such complex terrain? I think not. Nor would we expect a simple ordinary differential equation with one or a few adjustable constants to do the job.

SOME REMARKS ON THE PHYSICS OF TURBULENCE

How complex is turbulence as a phenomenon? Is it a relatively homogeneous terrain, or is it like the Sierra Nevada, composed of a very complex topography? Investigations over the past century answer this question quite thoroughly. The Introduction to Volume I of these Proceedings contains a list of twenty-two quantities that each can significantly affect the nature of turbulence. These include not only pressure gradient, but various forms of wall curvature, body forces, additives, and many other effects. Nor are we sure this list of 22 effects is yet complete. Contrary to earlier ideas, turbulence is not a single or even a simple set of states; it is a very complex and variable set of states that often react in unanticipated ways to a great variety of circumstances.

What we have learned about the structure of turbulence, mostly in the past 25 years, tells us the same thing. We can be quite sure that shear-flow turbulence is neither totally random not totally coherent; the available data effectively deny any such possibility. Shear-flow turbulence is rather quasi-coherent, or if you prefer, quasi-random, and such phenomena are inherently complex. Even more to the point for

this discussion, we know that the quasi-coherent parts of the turbulent flows, what we call the large- or medium-eddy structures, are not the same in different kinds of turbulent flows. The structures observed in the near-wall region of attached boundary layers, that are so beautifully illustrated in the paper of P. Moin and J. Kim in this conference using Large-Eddy Simulation, are quite different from those observed in free-shear layers by such observers as Browand, Brown and Roshko, Bradshaw and many others. Work in the Stanford HTTM group has recently shown that the structure in the region near detachment of a turbulent layer from a faired surface is distinctly different from that in the layer well before detachment--a point I will return to below. Nor does this exhaust the list of identifiable coherent structures characterizing particular flow zones.

HISTORY OF CLOSURE MODELS IN BRIEF

What is our experience with universality in turbulence modeling? Before 1968, there was general disbelief that adequate models for turbulent boundary layers existed. The 1968 AFOSR-IFP-Stanford Conference showed that a number of adequate models for attached incompressible turbulent boundary layers did exist, provided: (i) they were not too close to detachment; and (ii) they were not reattached layers (Tillmann Ledge flow). Similar results were found in the NASA 1969 and 1972 Conferences for attached compressible layers and for the far zone of free shear layers. In the context of this discussion it is important to note two things: (i) each of these three earlier conferences dealt with a class of flows with a single kind of flow structure; (ii) the modeling failed or was far less successful when we considered behavior beyond the edges of each class. Difficulties of type (ii) were encountered near detachment, for reattached layers, and for the near zone of free-shear layers in these earlier conferences.

What are the results in the 1980-81 AFOSR-HTTM-Stanford Conference. Several major results are relevant to the question of universality of modeling:

(i) No single model presented is accurate over the entire range of cases in this conference.

(ii) There is no correlation between sophistication (i.e., level) of model and accuracy of results over the full range of usable models (see Evaluation Commitee report).

(iii) Several times in discussions, individual computors reported success on some class followed by degradation of results when attempts were made to extend range of domain using a single method.* Much more evidence on this same point is evident in the fa[illegible]

*Method is used here in the sense of this Conference as an invariant procedure with fixed constants.

groups explicitly shift method when moving from one class of flows to another.

(iv) If one looks over the total results presented by Computors in this meeting one finds for nearly all the flows tried one or more methods providing quite accurate results. However, the accurate method(s) vary from one case to the next. Moreover, the most accurate methods on several relatively difficult cases are integral procedures; examples include: the transonic airfoil cases; the curved wall flows; and the planar diffuser of Simpson and his co-workers operating in transitory stall. This does not imply that integral procedures are more accurate or powerful; there are other cases where integral procedures give no output, and also cases where higher-order methods have given better results.

The lesson to be drawn from the preceding paragraph seems clear. The evidence is strong that, at 1981, there is a definite trade-off between accuracy of output and the range of domain attempted. That was also true in 1968, 1969 and 1972, but it was not so obvious or important because we studied classes of flows each with a single type of structure. An example of this trade-off is seen in the diffuser flows. The most accurate method for the diffuser in transitory stall fails badly when used on the Pozzorini high-core turbulence diffuser flow since the method does not model the turbulence-turbulence interaction between the core and the boundary layer. This method does work particularly well in the detaching flow or transitory stall because, in large part, it incorporates a specific, different modeling of the detachment zone as contrasted with the fully attached layer. It is not a Prandtl-like two-zone method, but rather a three-zone method employing distinct models for the potential flow, the attached layer and the detachment zone.

Similarly integral methods specifically tailored to the calculation of transonic airflows perform distinctly better in prediction of displacement thickness at the trailing edge than "more general" differential methods applied to this problem. The differences in accuracy are of design significance as emphasized by Melnik.

The message seems to be clear. When we know enough about the physics, the structure of a given flow zone, (and often this is far from a vast amount of knowledge), and if we systematically build this knowledge into our modeling, we obtain reasonably good accuracy. This seems true both for direct modeling of terms in model equations and for indirect modeling as in use of correlations in integral procedures. When we try to extend models even at the level of algebraic-stress or Reynolds-stress modeling to a very broad class of flows, at best we loose accuracy, and we may get quite inadequate results for specific purposes. It is not clear in the transonic airfoil cases

if the poorer results of differential procedures arise from less accurate modeling or from loss of sufficient grid refinement with models this complex owing to computer size/cost limits. In the diffuser in transitory stall the situation is clear; it is improved modeling, more physical input, that makes the difference.

What does this say about the appealing idea of building universal models starting with the homogeneous cases to set constants and then systematically enlarging the range of domain without altering the constants set from the simpler flows? The evidence given above is not encouraging for this approach. We also recognize the fact that homogeneous flows have quite different structure features from shear flows. The idea that homogeneous flows extend by any kind of regular "expansion procedure" to shear flows is very questionable. The simplest case of isotropic homogeneous turbulence in fact contains no production and therefore does not provide a suitable model for flows containing production, as noted by Kline.* Moreover, the boundary conditions on homogeneous flows cannot, in general, simulate those for shear flows, and the boundary constraints are critical in determining structure.

What are we to do then? If we try to build separate models for various flow cases or applications, we face a hopeless task. There are, after all, a manifold of infinities of scientific flow cases, and a higher-order manifold of infinities of geometries of commercial importance. The task would be endless. Fortunately, we do not need to model cases separately. In most flows of interest there are only a limited number of identifiable structural flow zones. By structural flow zones, I intend to denote a zone, a part or all of a flow field, that has roughly the same kind of flow structure. For the present I purposely leave the word "roughly" undefined. A tentative second cut at a list of such structural zones is given in Appendix I below; it contains twenty items. Several comments are crucial.

The list is not finished. It needs study and trials to see what works, to find what items might be consolidated, to see what needs to be added. We know a good bit about how to model many of these structural flow zones already. One need only look through the methods of this and the 1968, 1969, and 1972 Conferences. Certain items on the list delineate problems needing research (reattachment zones, shock/boundary-layer interactions, recirculating zones for example). We also know a good bit about how to patch and/or match flow zones. In some cases, we need only a reasonable sliding of model constants, in others well-developed techniques of asymptotic matching can be used. The computer has no particular difficulty in keeping track of where various zones lie throughout a computation; we know where we will need to patch or match. The various cases of complex strains and turbulent-turbulent interactions can be fitted into zonal modeling through treatment as subroutines in appropriate cases.

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*IUTAM meeting Kyoto 1967 published as supplement to J. Phys. Fluids.

The idea of zonal modeling tied to structural flow zones is not new. It is in fact central to the famous 1904 paper of Prandtl, to the analysis of isotropic flow and shocks, and to many of the methods presented in this Conference. What has perhaps not been sufficiently emphasized is the importance of the tie between successful models and the physics--that is the structure features of particular flow zones. The important point is the following: It will almost certainly accelerate progress if we iterate turbulence models with experiments on structure, not in general, but rather for particular structural flow zones one by one.

Let us return for a moment to the functional viewpoint and the analogy of the mountain range. We know one accurate model for turbulence in Newtonian fluids; it is the three equations: (i) continuity; (ii) Navier-Stokes; (iii) the viscous-energy equation--each in unaveraged form. As Peter Bradshaw has put it, God gave us one good model. Why should there be another model that is vastly simpler? We would not expect to find such a simple model for the topography of the Sierra Nevada, and nothing in the evidence cited above suggests turbulence is much simpler nor encourages the view that a simple, universal, turbulence model that is not-too-slow and also adequately accurate can be found.

Given the review above, I have little faith that we will find a single, reasonably fast, accurate turbulence closure model. The evidence suggests, to me at least, that the profitable road for engineering calculations in the near and intermediate future is systematic exploitation of zonal models tied directly to structure features of the flow. When we have done that in special cases, we have often succeeded. What remains now is to extend the approach to more general cases and to perfect the very important details.

The argument is sometimes made, as noted above, that zonal modeling will be more work and will, therefore, proceed more slowly than universal modeling. The argument seems plausible, but will certainly be untrue if no single, simple, adequate closure model exists. Nothing is slower than a search for the non-existent. Thus, I do not consider the conclusion I reach as discouraging. It seems to me quite the contrary. The argument suggests, apparently for the first time, what seems to be a feasible route toward achievement of what has eluded scientific research for more than a century. Let me put this differently. When we look for a single, not-too-slow closure model of engineering accuracy, we see failure not only in 1981 but many times before. In 1981 we do not even have a universal model for the known cases of homogeneous flows. When we look for not-too-slow methods of engineering accuracy for given structural flow zones where we know enough of the physics, we see successes not only in 1981 but many times before. In 1981 we see successes of this sort for many more types of cases than in 1968; significant progress has occurred. In 1981 some cases still are not well handled. For the most part, these are cases where we lack knowledge

about the physics for one or more flow zones in the field and have tried to use models created for other types of flow zones. Examples include cases 0421, 0422 (P2), 0423 (P3), and 0411.

Before ending this discussion, I need to be clear on one more point. Nothing in this discussion is intended to suggest cessation of effort on higher-order or broader models for complex turbulent flows. The higher-order approaches inform the lower order as the Evaluation Committee emphasizes. Computer power will continue to increase. In a decade much more effective use of Reynolds-stress models should be possible. We do, however, need to recognize more clearly that the zonal approach holds more promise than we have thought, and that design needs impel us toward development of the simpler forms of such zonal models in parallel with further development of higher-order models.

APPENDIX I

KNOWN STRUCTURAL FLOW ZONES OMITTING HYPERSONICS AND WAVE PHENOMENA*

1. Inviscid flow
2. Laminar boundary layer
3. Transition
4. Homogeneous turbulent flows
5. Attached turbulent boundary layers, two-dimensional
6. Attached turbulent boundary layers, three-diemnsional
7. Reattaching/detaching zones
8. Mixing layer
9. Axisymmetric wake
10. Plane jet
11. Axisymmetric wake
12. Plane wake
13. Recirculation zone (fully stalled zone)
14. Secondary flow, type 1
15. Secondary flow, type 2
16. Mach No. effects
17. Shock/boundary-layer interaction, two-dimensional
18. Shock/boundary-layer interaction, three-dimensional
19. Trailing edge interactions
20. Large-scale vortical motion

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*The list should include Wall Jets as noted by A. M. Savill (see below). Thanks are also due to Gino Sovran for some improvements to this list over an earlier version.

COMMENTS ON "UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD"

From P. Bradshaw*

I like the Zonal Modeling paper very much. I think the following points need covering as well:

(i) One must concede that replacing constants by zone-dependent functions may make extrapolation hazardous. Models tuned for special zones may be real death traps.

(ii) Zone-dependence as such is only half way to the goal of parameter-dependence, and interpolation between zonal models is not necessarily easy: consider how one would change from a mixing-layer model to a boundary-layer model near reattachment (behind a step, say). Where possible I would want to use "wall effect functions" with (length scale)/y as a parameter, rather than a separated/attached switch with smoothing.

(iii) Modelers will need strong encouragement to test out their models over as many zones as possible and to link up with others who are modeling "adjacent" zones.

Finally, mountainous country is particularly liable to contain mineshafts and electric power pylons!

From A. M. Savill†

At the recent Stanford Conference on Complex Turbulent Flows, Steve Kline put forward the suggestion that, as far as practical engineering calculations are concerned, it would be better to concentrate on developing an approach based on zonal models rather than continue the search for a universal closure model for turbulent flows. This was very much a personal opinion which he put forward to stimulate discussion and comment, but he has subsequently expanded and written up his remarks as the foregoing paper entitled "Universal or Zonal Modeling--The Road Ahead." In this he points out that the results of the 1981 Conference show that when models are specifically tailored to the calculation of a particular type or class of flow it is possible to obtain good accuracy, but when one tries to extend even the higher-order closure models to cover a broad range of flows, the results are often quite inadequate. There appears to be a trade-off between accuracy of output and range of application. Such a conclusion is supported by the earlier 1968, 1969, and 1972

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Conferences, although not brought out so clearly because they concentrated on individual flow classes.

Obviously this is not very encouraging for those who are attempting to build universal models of turbulence and Kline suggests the alternative, at least for engineering calculations in the immediate future, is to consider practical flows as made up of identifiable flow zones, model these individually and then patch together the solutions over the whole flow field. This is not a new idea, but has tended to provoke sharp divisions of opinion, and this was certainly the case at Stanford judging by the immediate response and subsequent private discussions. As one of the attending computors I have several comments which I should like to put forward here.

The term "zone" is used by Kline to define a region of flow typified by a particular structure which is specific to that region and distinct from the structure found in any other zone. He has put forward a preliminary list of 20 or so such structural zones (see his Appendix I), but points out that this is tentative and probably incomplete. Certainly one immediately notices the omission of the axisymmetric and plane-wall jets, so it is likely the final list would be somewhat longer. The prospect of having to develop individual models for such a wide range of flows is rather daunting, but in practice this may not be necessary. Already there is strong evidence for a degree of structural similarity between some of the flow zones. For example, from work carried out over the last two decades, at the Cavendish Laboratory under the direction of Dr. A. A. Townsend, it has become apparent that at least three types of turbulent eddy are common to all the plane-shear flows.

These are: inclined roller eddies (which are perhaps surprisingly well modeled by Rapid Distortion Theory); transverse* entraining motions (which are responsible for the entrainment of external, irrotational fluids); and isotropic small scales. Between them these eddies contain most of the turbulence energy and so make the major contributions to all the turbulence parameters of interest. They represent a sort of underlying universal structure. However, there are differences between the various shear flows because the scales of the eddies and their relative intensities change from flow to flow. Thus the wake contains intense, large anisotropic swirling motions (Lasmos) which have a scale comparable with the wake half-width, and are responsible for the large-scale overturning which occurs during periods of active entrainment. On the other hand, the entrainment in the boundary layer occurs as the result of small-scale nibbling associated with the tops of hairpin vortices (which in section appear as transverse motions with a width an order of magnitude less than the boundary-layer

*The term "transverse" is used here to imply that the principal circulation of the eddies concerned is in the vertical plane although their spanwise extent may be only two to three times their scale in the stream direction.

thickness), and so is wholly quiescent. In the sense that Kline has used the term, these flows do have different "structure," but in fact it seems the inherent eddy structure is the same in each case.

This obviously simplifies the modeling, because it means one does not have to start completely from scratch with each individual case; once one of these flows has been adequately modeled it should not be difficult to extend the model to cover all four satisfactorily even allowing for the special problems associated with the different boundaries. Furthermore, Bradshaw's group at Imperial College has shown that it is also possible to consider some flows as combinations of these. A developing channel flow may be thought of as two merging layers time-sharing at the centerline. It has also been suggested that a wall-jet might be modeled as a jet sitting above a boundary layer which might explain the omission of this case from the tentative list of zones, except that the jet may as easily be modeled as two merging shear layers.

There is now some evidence to suggest that the same sort of structural similarity exists in the axisymmetric flows as well. If this proves to be the case it will lead to further simplifcations. Of course, this still leaves a large number of zones that would presumably require special individual treatment, and one problem that particularly concerned attendants of the Stanford Conference was how the models would fare on fully three-dimensional boundary layers. It will be interesting to see the outcome of the April 1982 Eurovisc Conference in Berlin, but already some computors are confident that the more advanced models will be able to handle such flows. The feeling is that, because the turbulence structure one models in the axisymmetric and two-dimensional flows is inherently three-dimensional, the extension to fully three-dimensional flows will not be as great a problem as some anticipate.

Taking all these points into consideration, the idea of zonal modeling starts to look a more feasible proposition. However, there are still at least two potential stumbling blocks when it comes to putting the procedure into practice. The first of these is the problem of zone identification. It is not immediately obvious how this would be achieved unless one can make a very good guess at the mean-flow geometry. The possibilities for error in an iterative scheme, up-dating the zone identification on the basis of the mean velocity output, seem very large. And yet it is vital that any calculation scheme should be able to recognize changes in environment, and automatically switch from one zonal model to another. In a sense, this is a problem common to all approaches to turbulence modeling. We have already seen examples of "universal" models which switch parameters on and off in different flow regions. The question is, do they recognize when to do this? In some cases the answer is clearly no; the basic model may be unchanged, but the individual parameters have to be externally tuned for each flow case. However, other models do appear to have this facility. Launder's group has produced very good results for two backstep flows with

an algebraic k-ε model in which a tensor invariant curvature transformation leads to an increase in dissipation when the streamlines start to curve. This is obviously an acceptable development, and helps to show that there is not after all such a great difference between the zonal and universal approaches to turbulence modeling. Indeed if a universal model is self-adapting to changes in environment, it is essentially a zonal model.

Unfortunately some confusion has arisen because the term "model" has been used in two senses; first to denote a particular system of modeled equations at a particular level of closure, and secondly to describe the way a given scheme is applied to different flows. Some people would question whether a model can be universal if its parameters are changed; does it not then become a different model? The point is that a true zonal approach might use completely different turbulence models for different flow regimes, whereas a truly universal approach would attempt to model the whole of every flow with just one set of parameters. Naturally these are very extreme lines. As we have seen, the former is unlikely to be necessary, while the latter is unlikely to be possible [emphasis added by editors]. In practice the goal of the universal modelers must surely be to develop a model which adapts itself to all the different flows with the least amount of what we might call internal tuning. This will certainly have to be based on at least an algebraic, or more likely, a full-Reynolds-stress closure scheme.* It would represent the ultimate, or perhaps one could say the universal zonal model; one in which the same turbulence model is used in each zone. The only difference, then, is that the zonal modeler is prepared to forego this elegance in favor of a jury-rigged scheme in which various simpler models are used, where possible, with the aim of providing a serviceable procedure for more immediate application to engineering calculations. If it were possible to build up a working zonal scheme very quickly, then there might be little point in pursuing the search for universal models (at least so far as industry is concerned; presumably there would still be academic interest). But this is certainly not the case. Even if we can reduce Kline's twenty individual zones to about ten pretty distinct cases, there are still the possible twenty-two distortions to consider. Via either path we are a long way from any scheme capable of predicting the complex flows of engineering systems.

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*In fact, much might be achieved with a simpler model. For instance, I have previously suggested (a) that present three-dimensional k-ε schemes might be improved by using their mean-velocity and shear-stress output to divide a complex flow up into regions dominated by different types of distorted shear flows, (b) using a highly structural model developed at the Cavendish to calculate representative values for the stress intensities, and (c) using these in place of isotropic values for the normal stresses, while modifying the shear stresses. This process of "Region Identification" would result in a crude form of a universal zonal model.

It is obvious that all routes towards this goal must be pursued, and that these two approaches are in many ways complementary.

Assuming that satisfactory zone identification can be achieved, there is still the problem of patching/matching flow zones to be overcome. If the same turbulence model is in use in adjacent zones, this would probably only require some reasonable sliding of model constants, and if this is not the case it should be possible to make use of the well-developed techniques of asymptotic matching as Kline suggests, although there may be greater difficulty if two very dissimilar turbulence models are involved. However, even if the numerical problems can be surmounted, these procedures might not be very realistic physically. We know that, within a single zone, combinations of strains may have odd effects on eddies resulting in unexpected changes in the turbulence parameters. One strain may so orientate a particular group of eddies that they are then preferentially amplified by a second strain. Alternatively one set of eddies may be selectively suppressed while another type gain energy and so rise to prominence in the flow. Some new structures such as longitudinal eddies may persist for times long compared with typical turbulence time-scales. The turbulence structure always depends on the strain history and so memory effects are important. Eddies passing out of one zone into another region of the flow would thus carry a "knowledge" of their earlier environment, and might influence the dynamics in the second zone sufficiently to invalidate the assumption that each region can be modeled in isolation.

In some cases there will be a clear-cut changeover from one type of flow to another. One example is a flow recently studied by Townsend where a boundary layer separates from the roof of a wind tunnel and a curved mixing layer is initiated. Here the turbulence production is so rapid immediately downstream of the origin of the free-shear layer that its structure completely overwhelms any old boundary-layer turbulence, even though the sense of the imposed curvature is stabilizing. However, we know from forcing studies that the condition of nozzle boundary layers can affect the spreading rates of jets; not by changing the form of the jet-eddy structure, but its growth. Here there is a lingering influence of the flow upstream.

Another case where some interaction could be expected is the hypothetical situation of a plane jet impinging at an angle onto a convex wall. Here what would otherwise be a highly stabilized laminar or perhaps transitional boundary layer would be greatly influenced by the "free-stream" jet turbulence and almost certainly fully turbulent. The degree of interaction would depend upon the intensity and at least a typical scale of this turbulence, and probably the actual form of the turbulent eddies in both regions. In general then it seems likely that successful patching will require further experimental work on such interactions between shear layers particularly regarding the changes in the structure of the turbulent eddies.

All of the above discussion assumes that future models will take more account of the highly structural picture of turbulent flows that has emerged in recent years. This seems vital if we are to continue to make progress and both experimenters and modelers can help the process along. The modelers could provide feedback to the experimenters by explaining where and why they have had to make modifications to reproduce experimental data. At the same time experimenters should aim to provide information on the types, scales, and intensities of eddies within each flow and the manner in which they respond to the various extra strain rates so that the modeling can be placed on a more secure physical foundation.

From P. J. Roache*

I am in complete agreement with the opinion on zonal modeling expressed by Professor Kline.

Perhaps researchers simply have been tricked by the limitations of language. In speaking of "turbulence," there is implicit an overly simple binary classification scheme; flows are either "laminar" or "turbulent." A better terminology would be something like "laminar" and "non-laminar." With a retrospective view of the progress over the last half-century, it seems absurd--if not arrogant--to think that we can lump together all of these "non-laminar" flows and solve them with anything but the original non-averaged Navier-Stokes equations.

As an analogy, consider gray versus non-gray. It is true that we can divide our visual perceptions into "gray" and "color," but having done so does not give us any information on the color spectrum. "Color" is more complex than that, and non-laminar flows are too complex to all be dumped in the same category of "turbulent."

Any attempt to invent more universal models may meet with some limited success, but the approach is overly and unnecessarily limiting. However, the zonal modeling approach suggested by Kline is much more demanding on turbulence modelers, since they would have to develop a facility and understanding, as well as separate computer code modules, for the different zones of turbulent flow which they intend to model. This is a very large obstacle to be overcome, and it is probable that several cycles of new researchers (graduate students), i.e., 8 to 10 years, will pass before the approach becomes common, but ultimately it seems certain to become the engineering approach of choice.

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From G. S. Settles*

Congratulations on your "Universal or Zonal Modeling—The Road Ahead," which makes eminent sense to me. I am making it required reading for my graduate students.

Your initial comments on universality remind me of an elegant paper by John A. Wheeler: "Beyond the Black Hole" (in Some Strangeness in the Proportion, A Centennial Symposium to Celebrate the Achievements of Albert Einstein, ed. H. Woolf, Addison-Wesley, 1980). You might especially enjoy reading the section of Wheeler's paper entitled "Up the Staircase of Law and Law Transcended to Mutability."

But more to the point, your comments on zonal modeling based on a physical understanding of flow structures strike a particularly respondent chord with me. Here at Princeton we've been saying this for some time, but we also ask where will the physical understanding come from? Who will undertake (and is qualified to undertake) the necessary, detailed, well-defined, parametric experiments to reveal the physics? Who will define what these experiments should be? Who will recognize the importance of the results and build models from them? And who will fund this process?

My shelves are filled with reports on fluid mechanics experiments—most of them useless for this purpose. The Stanford Conference distilled a mere handful of these to be put forward as standards of a sort, but those of us who are experimentalists recognized serious inadequacies even in these experiments, both in scope and in quality. (As a simple example of the latter, consider anybody's skin-friction measurements in a pressure gradient, including my own: no one knows how to do it accurately.)

Further, those few experiments which are "true classics beyond reproach" seem to have sprung from a fortunate confluence of people and circumstances, not from any concerted or directed program in the fluid mechanics community to produce such experimental results. I would like to see the point made strongly that "the road ahead" desperately needs a strong, directed effort in experimentation to go hand-in-hand with modeling and computation.

Anyhow, congratulations once again on a successful conference which I learned from and thoroughly enjoyed.

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From C. duP. Donaldson, W. S. Lewellen, B. Quinn, R. D. Sullivan, R. I. Sykes, and A. K. Varma*

Elsewhere in these proceedings, Professor Kline argues persuasively for a zonal modeling approach to complex turbulent flows. This approach certainly has some merit, but we believe it can also easily be oversold. A zonal approach works best when it is possible to show from a more universal, fundamental approach under what conditions the appropriate zonal approximations are valid. Without the guidance from the more universal model, the zonal model can only be used if, at least, a rough estimate of the solution is available from experiment or other means. Too much emphasis on zonal models will prove to be costly because of the increased experimental burden.

It may be argued that with the increased computing power that will be available before long, predictive calculations will be carried out directly from the Navier-Stokes equations and there will be no need for Reynolds-stress closure with or without tensor-scale equations. However, as the computing power grows, some of it will be devoted to extending the complexity or the number of flows that can be handled instead of to increasing the accuracy or respectability of the methods. It is our belief that Reynolds-stress closure will be very useful in the foreseeable future and attempts to extend its scope should be encouraged.

*These remarks were submitted as a part of the Computors' Summary by the A.R.A.P. Group (Aeronautical Research Association of Princeton, Inc., McLean, VA 22102). They have been moved, in toto and without modification, to this section by the Editors to provide a unified discussion.

CLOSURE ON UNIVERSAL OR ZONAL MODELING--THE ROAD AHEAD

S. J. Kline

Professor Bradshaw's remarks are, as usual, concise, very pertinent scientifically, and in addition humorous--a rare combination. His points are well taken, and after further consideration, I want to expand on one of them: the question of merging from one zone to another.

My first remarks on merging are, on reconsideration, too optimistic. We know some things about this already. For example, the observations of Kim (1978) on a reattached layer following a backward-facing step, shows that some of the large eddies generated by the free-shear layer pass into the outer portion of the downstream boundary layer. These eddies affect the structure and properties of the boundary layer for more than 50 boundary-layer thicknesses. Hence, the boundary layer "recovers" toward equilibrium only very slowly. The standard correlations for correlating equilibrium boundary layers fail badly in this recovery (readjustment) region. Similar very slow readjustments were observed by Gillis (1980) and by Simon (1980) for flow over a flat wall downstream from a convex wall. Similar slow readjustments to equilibrium are being observed in current work on a flat plate downstream from a concave wall by Jeans (1982) and by Simonich (1982). In these slowly readjusting zones, all the standard turbulence models we have tried perform badly in predicting results. New models will be needed.

In other instances, the transition is much easier as found by Bardina et al. (1981) in the transition from attached to detached layers. This appears typically where the "new" structure is much stronger than the old one and thus quickly overwhelms the older structure. This same point is made by A. M. Savill (see above). In the detachment case, a simple sliding of constants in standard models was sufficient. The merging of potential flow and boundary-layer zones has been standard practice for 75 years and causes little, if any, difficulty.

Some approaches to classification and construction of these additional models for readjusting zones can already be seen. While it is too early to discuss them in detail, I do need to mention they exist in order not to leave the impression that they represent an insuperable problem. Bradshaw's suggestions in his comments above seem eminently applicable, and even simpler approximations are discernible. In point of fact, I suspect that we are now doing extremely badly in predicting these readjustment regions not because they are so difficult, but rather because when one uses either a universal model or the model for another simpler zone, readjustments are simply ignored. What we do know about the physics of such readjustment zones is not yet incorporated into our models.

Nor do I want to leave the impression that zonal models restrict one to problems describable by parabolic equations. In the Stanford University HTTM group, we have

already done some cases where strong elliptic effects are present using zonal approaches. These are more difficult, as we must expect, but they are not impossible.

These results do not settle the questions of how to merge zonal models; they do suggest that this problem may not be intractable, but will need considerable specific attention. They do also reinforce the writer's prejudice that at least a modicum of knowledge concerning structure is an essential ingredient for satisfactory modeling of turbulent flows.

Regarding Dr. Savill's comments: they are thoughtful and provocative. The results he describes on structure have the insight we have come to expect from Townsend's group, and will bear watching by the research community. I cannot entirely agree on some points, however.

In the first paragraph Savill remarks, "when one tries to extend even the higher-order closure models to cover a broad range of flows, the results are often quite inadequate." This clause leaves the impression that the results are sometimes adequate; unfortunately, that does not seem to be the case; they are uniformly inadequate thus far for truly broad classes of flows, and we need to face that fact squarely.

Nor can I agree with Dr. Savill's comment about locating zones in the computer. In every case where we have used zonal modeling thus far, the location of zones has not been a problem. We have managed to assign clear tests, recognizable by the computer, that advise when to switch from one model (or constant) to another. Even if this were not true, there is nothing to prevent inspection of output and modification of zone boundaries should that be necessary. I thus do not see the difficulty that seems to be bothering Dr. Savill.

Dr. Savill's point that a "completely" zonal model is probably unnecessary and a completely universal closure model probably impossible is, I believe, very important, and the remark has been underlined by the editors. I agree that intermediate formulations that are universal, when possible, and zonally adaptive, when necessary, are the most likely to be both economical and effective. It seems obvious that one would want to adjust the value of the parameters in a given model for various flow zones and, only if that were unsuccessful, attempt to use fundamentally different models for various flow zones in computation of a single flow field. Further comment on this point appears below in reply to the comments by Donaldson et al.

The various remarks on flow structure and on the similar nature of underlying structures for differing flow situations by Dr. Savill are extremely interesting and in accord with the writer's prejudice for understanding flow structures as a basis for modeling turbulent flows. They are, however, too many and too detailed for complete discussion herein, and are therefore left for later independent consideration and discussions.

I am also in full accord with Dr. Savill that more cross-pollination of ideas between experimenters and computors is vital to maximizing progress in turbulent flows and have so stated at several other points in these Proceedings.

Regarding Dr. Roache's comments, the writer is grateful not only for his support in facing what seem now to clearly be the facts, but also for his articulate and graphic analogies stating the case.

I appreciate Prof. Settles' comments. I agree that most of the experiments proposed for Vol. I are less than "true classics beyond reproach." I would, however, add that given the position papers and discussions of Vol. I, and allowing for the fact that no experiment is "beyond reproach," the purposeful creation of such "true classics" should not be beyond our capabilities.

The reader may also want to reread the remarks of J. Lumley near the end of his Reporter's Comments in this volume regarding universality. These remarks were written after reading the OPINION above by S. Kline, and represent the view of a particularly able and experienced modeler and experimentalist who has long held to the "universal" modeling approach.

Donaldson et al. make three major remarks on zonal modeling versus universal modeling. All three invite further discussion. Donaldson et al. suggest that zonal modeling "... has some merit, but we believe it can also easily be oversold." I have great admiration for the ability of Donaldson and his co-workers. However, on this issue I think the shoe is decidedly on the other foot. Universal modeling has been pretty much the only recognized game in town. The leaders of the theoretical and computational research on turbulence have almost all been "universalists." Nearly the only explicit proponents of "zonalism" have been some experimentalists and some individuals in leading companies, such as Boeing, where the engineers were face-to-face with the need to make design decisions (sometimes with billion-dollar implications) and therefore had an urgent need to guarantee accurate results. If the arguments I make above for a zonal approach are correct, it should then not be surprising that these engineers, under those conditions, chose to use models that were known to be explicitly tailored to specific classes of flow situations in order to guarantee accurate results. This same fact (namely that universalism has been favored and to that extent "oversold") can be seen from the results of a question I asked nearly a decade ago, as a member of a panel of leading experts, who were asked to discuss the question of appropriate roads in future turbulence research. I asked as a question something like the following: "Is it possible that we may have to use various closure models for various flow types?" I noted, as part of the question, that the computer can do this, and that while it would perhaps be inelegant, it might also be eminently practical. The reaction of my four colleagues on the panel was primarily outrage and indignation that I would pose such a question. "Universalism" was clearly the accepted approach.

In sum, I do not think we are in danger of overselling "zonalism"; rather we have been and therefore probably still are in danger of continuing to oversell "universalism." If, as Savill notes, a completely universal model is probably impossible and a completely zonal model is undesirable, then the appropriate question is, "where is a suitable balance between these poles?" It must lie, as John Lumley suggests, with as much "universalism" as we can build while retaining adequate accuracy, and it must be based on information about the flow physics. Just where this point lies we do not yet know. That is a major task of the next round of research. However I do not believe we will answer the question appropriately unless we approach it with an open mind, that is to say, without bias against zonal approaches, which has been very strong in many quarters historically.

In view of the remarks above by Bradshaw and Savill on merging from one zone to another and of Lumley and Savill on "as much universalism as possible," a short expansion on the kinds of zonalism that occur may be helpful at this point. There are basically two levels of approximation for zonal models. In one approximation, different turbulence models are used for different classes of flows but held constant within any one flow field calculation. In a second level of approximation, various models, or at least various constants within a given model, are used to approximate various zones within the computation of a single complex flow field. It is worth noting that the first level is routinely used nearly everywhere, and to that extent seems acceptable. We expect to see computations of boundary layers and potential flows separately or at least different grid spacings used in different zones. We also expect to use different approximations for compressible and incompressible cases for reasons of computational efficiency if for no other. And these are only the most obvious cases. Even more to the point is a distinction in the semantics that had to be delineated in order to create a usable questionnaire for describing methods in this Conference. It was found that many computor groups had a single program, but that a number of methods (in the sense of invariant procedures) were callable within this one "program." Since the word program was used for both purposes, it is clear that much confusion had existed on what approximations were actually used. Moreover, the switching from one method to another within a single program is by definition a form of zonalism; it is merely hidden from the outside observer, and buried in the "program." In fact, this is the practice of the Computor Group led by Donaldson et al. When one looks at the descriptions of what that group has done, as described for this Conference, it is clear that they are using a number of "methods," and to this extent are already using some zonalism. It is true that they enforce certain important constraints (such as invariance) on the mathematics, and this tends to improve their models and moves toward universalism. Nevertheless, the complete set of computations do involve several methods, not one.

It seems strange to me that so many people will accept zonalism at the first level but reject very strongly zonalism at the second level of approximation. After all, if one is to do anything less than a wholly universal model, it then is not a question of whether different approximations are to be used for different situations, but only what is the most expeditious and accurate way to proceed. Both Lumley and Savill have recapitulated this point. They both comment that we do need to seek as much universality as we can in our models in order to increase the range of predictive capacity and to limit effort both experimentally and computationally, and with this I certainly agree. However, it would not seem at all profitable to seek universality for its own sake given what we now know, and this applies to the second level of approximation just as much as to the first level. In the last analysis our tests ought to be pragmatic ones rather than appeals to elegance or a sense of scientific fitness, as I have already stated in the OPINION above.

A second point made by Donaldson et al. is the statement: "A zonal approach works best when it is possible to show from a more universal, fundamental approach under what conditions the appropriate zonal approximations are valid." I do not believe it is presently possible to do this from theory or computation for most assumptions about turbulence models, since we lack complete solutions nearly everywhere and since the potential for asymptotic perturbations in the solutions are generally present in viscous flows. It is not theoretical validation that I had in mind in suggesting the construction of zonal models, nor is it such validation that zonal models have employed in the part. Zonal models have relied for the most part, as Donaldson and co-workers say, on "... rough estimates of the solution from experiment ...." One needs to note that such a rough estimate can be very rough. For example, Prandtl used only the experimental observation that the boundary-layer width is small compared to its length in the flow direction for attached layers at high Reynolds number. In Prandtl's hands, this critical fragment of experimental data is enough to derive boundary-layer theory and thus to set off work on a whole field of analysis (asymptotic perturbation theory) and also provide the modern basis for most analysis and computation in viscous, high-Reynolds-number flows. Thus, I agree that we need to base zonal models on observations of the physics, and this fact is the primary basis for arguing in the Editors' Concluding Remarks in this volume, that the day of the need for experiments in turbulent flows has not passed.

On a third point, I agree entirely with Donaldson and his colleagues that direct solutions of the complete, unaveraged Navier-Stokes equations will not wholly replace Reynolds-stress closures in the foreseeable future. For more than a decade we have had many statements suggesting this replacement will occur; it is overdue, in my opinion, that some one says clearly that they are a disservice to the current user community, as a whole. We cannot do such solutions now. We are a long way from them. Even

if computers get big enough, so that such computations can be done, there is as yet no assurance that the cost of the very large main-frame computer will become so low that the typical industrial user will have effective access for day-to-day design purposes. It is altogether possible that the asymptotic state, even say at the year 2000 A.D., will be one in which important scientific questions and very pressing military or large-dollar industrial applications will be carried out using the full, unaveraged Navier-Stokes equations, but that ordinary day-to-day work will still be done in most companies with faster, simpler methods. This is even more likely when one remembers that scientific solutions tend to be "once and for always," or at most need a few repetitions, but a single design often requires tens or hundreds of repetitions of computations. I am not making a prediction here; I am, however, saying the far future is, as always, very uncertain and therefore hard to predict. It must follow that for engineering design we are obliged to continue along the lines that are possible and that we know can guarantee adequate accuracy. To date the only line of this sort is appropriate zonal models, and it is likely that this will remain so for a considerable period of time.

Closing Comments

The remarks of several workers on the topic of zonal versus universal modeling contain the implication that zonal modeling necessarily implies the use of less physics than universal closure modeling. A bit of reflection will, I believe, show that this implication puts the shoe on the wrong foot. The logic follows.

We agree, in general, that a central purpose of closure models is to replace as much as we can of the information that is lost in the process of time-averaging the governing equations. There is, moreover, general agreement that the governing differential equations for viscous motions are correct at least for Newtonian fluids (see, for example, the report of the committee on this matter in the Proceedings of the 1978 AFOSR/Lehigh Workshop on Coherent Structure in Turbulent Boundary Layers, p. 476). We must note, for the logic, that the underlined word "correct" above implies not only that the governing equations contain the physics, but also that they DO NOT CONTAIN MORE (that is, they do not contain irrelevant information). Nor is there any evidence, known to me, that the governing equations do contain irrelevant information.

If we accept this fact, that the governing differential equations contain all the physics and no more, then it follows that any single universal model that is simpler than the governing equations must represent less physics, that is, some physics will not be represented by the simpler model. This is precisely the genesis of the problems in universal modeling. If we adjust the model so it fits for a few flow classes, it will not fit in at least some other classes because the model does not represent all the physics. But suppose we allow adjustment of the closure model from class to

class, that is, we adopt a zonal view. Now we have the potential to fit the model to the physics class by class; we gain the possibility of using more appropriate information on the physics for any given type of flow zone.

The central purpose of zonal modeling is precisely to enable us to incorporate more physics into the total set of models while still keeping computation size within the limits of available computers. If we could get enough physics into a universal model, we would not need to go to zonal representation.

This point is so central to the entire discussion about the advantages/disadvantages of zonal versus universal modeling, that I want to reiterate it in different words. First, I have long been known as an advocate of understanding the flow structure and incorporating that understanding in our models. I have not changed that position; I remain very firmly attached to the idea that flow structure is important and should be incorporated in our models to the greatest degree possible. Second, we now know that the flow structures controlling the dominating turbulent shear stresses vary significantly from one structural flow zone to another. If our closure models cannot incorporate sufficient physics for all the various structural flow zones, then we need to create simple models that incorporate more of the physics zone by zone. The intent of zonal modeling, as I see it, is to incorporate more physical information, not less. Given this remark, it is appropriate to ask, "What do we learn about the probability of finding an adequately simple, universal model when we look at the total results of this Conference?"

The overall picture which emerges as a result of the enforced considerable further study of those Proceedings during editing suggests the answer, and this answer forms my final comment. When one reads the comments of any one Reporter, and studies the set of flows to which he refers, it seems that some difficulties occur, but they tend to seem less than insuperable for a universal model. John Lumley's comments referred to above are a good example. When one reads the summaries by all the Reporters and the discussions, and examines in detail the total output of the computations and the remarks on that output by the Evaluation Committee, a quite different impression emerges. That different impression focuses strongly the multitude of difficulties standing in the way of achievement of any simple, single, universal closure model. Hence this different impression very strongly reinforces the remarks in the OPINION to which this Closure refers. Let us recapitulate some of these points.

Bushnell remarks on considerable difficulties in compressible cases. (Note particularly the comments on p. 814 regarding Fig. 23 on p. 836.) Models of separating diffusers using standard shear-layer methods universally fail. The rate of growth of the mixing layer as a function of Mach number seems still to be badly predicted by existing models. Lumley in discussing homogeneous flows notes the failure of existing models to predict all the various cases and remarks, "... again these methods have

probably been optimized for a different type of strain"; and elsewhere, "... again it is clear that at least a full-Reynolds-stress model with rapid terms seems to be necessary ...." And the two quotations refer to just homogeneous flows. Lumley also remarks, "... it probably does not make much sense at the present time, from the practical point of view, to calibrate models intended for shear flows based on data from homogeneous flows." And yet this is a keystone of many current "universal" models. Marvin, Eaton, and Simpson, each discussing a different class of flows, all present results illustrating the difficulty of modeling and the poor results obtained in readjusting regions where the flow passes from one flow zone to another (see also remarks above in this Closure). Melnik in discussing the very important case of transonic airfoils, on which he is particularly expert, remarks that no standard models of the universal sort predicted these cases sufficiently accurately for engineering purposes, but that some specially "tailored" integral methods that are in fact also simpler to use in design work are quite successful. The Report of the Ad-Hoc Committee of Computors states simpler methods need to be given more legitimacy, and emphasizes that more attention needs to be given to the range of validity of particular models (see particularly item 18, p. 959). Wilcox and others report that what had been accurate models are significantly degraded when an attempt was made to broaden the domain of applicability. Jones reports that secondary flows of the second kind are not in general predicted well by the existing "universal" models. Eaton reports that the only model for the backstep that obtains good results in mean-velocity profiles for the reattached flows is specially "tailored" for that flow situation.

Nor does the litany of the preceding paragraph exhaust the list of difficulties and deficiencies of "universal" models. It does, however, seem sufficient to make the point that study of the complete results of these Proceedings do not suggest much hope for any simple, single universal closure model. The reader may well want to examine this matter further for him or herself by study of this Volume and of the detailed results provided in Volume III, since the question lies at the center of how to create successful strategies for turbulence modeling.

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SESSION XVI

Chairman: S. Bogdonoff

Technical Recorders:

S. Birch
B. Afshari
G. Lilley

GENERAL DISCUSSION

GENERAL DISCUSSION

S. Bogdonoff (Chairman) opened the Discussion by inviting comments on the Evaluation Committee's Report and its recommendations.

P. Bradshaw: I wish to comment on the remark made by the Evaluation Committee that "most of the turbulent-flow models are performing roughly the same". It is recognized that the simpler models, i.e., integral and mixing length, etc., have been extensively refined over a period of many years. It appears, therefore, that the Reynolds-stress methods, which have as yet not been subjected to the same kind of development, probably retain more potential for further development and refinement.

S. Bogdonoff: Can you suggest what is likely to be needed in this further development of these models and what is the likely outcome?

P. Bradshaw: In the development of turbulent-flow models, use should be made of the exact Reynolds-stress transport equation. Any turbulence model must be recognizable as a simplification of these equations. More thought needs to be given, also, to the "zonal modeling" concept.

E. Reshotko: Following the 1980 meeting the Organizing Committee requested that computors distinguish between numerical errors and inaccuracies in flow models by showing an independence of grid size in their codes by halving the grid size. Would computor groups inform the Conference if they now have better checks to separate the effects of errors in numerics and flow models on the accuracy of their codes?

S. Bogdonoff: This has already been partly discussed at other sessions, but it is an important matter deserving more discussion.

S. Birch: Professor Bogdonoff suggested that experimentalists faced problems with initial conditions, boundary conditions and instrumentation errors that were similar or analogous to the problems encountered by computors. It occurred to me that experimentalists in general do a much better job in dealing with these problems than computors. For example, experimentalists have become very skilled in designing experiments to minimize experimental problems while retaining the essential physics of the phenomenon being studied.

P. Roache: A suitable test case for isolating and studying errors in the numerics is that of the Bradshaw-Castro curved shear layer (Flow Case 0331). A flow-model code could be tested using a body-fitting adaptive grid. He understood this scheme was being attempted by Grosman (Imperial College). This flow is a good choice since it appears to pose no serious problems in a coordinate system aligned with the flow.

B. Launder: I agree with this recommendation.

J. Murphy: I disagree. The problem of error separation is problem dependent. A code that tests satisfactorily for one flow case cannot be used for another flow without a separate check being made on errors.

I. Castro: I fully agree with Murphy. However, as pointed out by Humphrey (Session XIV), certain specially designed laminar flows can be employed to investigate errors which will arise in related turbulent-flow calculations.

P. Bradshaw: The Organizing Committee had earlier suggested the swirling laminar jet as a special test case for this purpose.

E. Reshotko: There were additional cases similar to that mentioned by Bradshaw, but all had been rejected as test cases by the 1980 Conference on the grounds that laminar flows cannot provide a strong enough test of the numerics of turbulent flows.* An alternative method is to use a systematic sensitivity analysis analogous to that used in experimental work, as discussed by Moffat in the paper on Uncertainty Analysis at the 1980 Conference.†

M. Morkovin: In a paper presented to the AIAA meeting at Snowmass (1980) Dwyer advocated an inexpensive way of introducing numerical checks for the prediction of complex turbulent flows.

J. Ferziger: In personal communications with Dwyer it has been established that Dwyer's sensitivity analysis is not cheap; it would cost as much as a complete calculation. Neverthless, the method is of value. In applications to turbulent boundary-layer calculations, using a k-ε model, it has been shown that the output is very sensitive to the ε-equation, especially in the wall region. The Organizing Committee should ask Dwyer for a brief summary of his work and include it with the Proceedings of this Conference.

N. Mansour: I agree with Ferziger's statement on Dwyer's method. It should be noted, however, that Dwyer's method requires the solution of additional equations and this is inconvenient.

S. Kline: The tests proposed by Dwyer were considered but were dropped, because it was not clear that they provided the correct basis for error assessment in all cases. Would Tjonneland comment on the method used by Boeing for this purpose?

E. Tjonneland: The Boeing method is a built-in method to change the grid size and

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*[Ed.: The point is that numerical errors depend on the precise form of the solution equations as used in the computer. The equations used in the computer solution matrix for laminar flows are never the same as those for the "equivalent" turbulent flows. Any result obtained from the laminar equations is, therefore, essentially irrelevant. This point was made by P. Saffman in the 1980 meeting and accepted by the Organizing Committee and the bulk of attendants at that meeting (see Vol. I, p. 218 and pp. 599-603).

†See "Contributions to the Theory of Uncertainty Analysis for Single-Sample Experiments" by R. J. Moffat, Vol. I, p. 40.

improve the accuracy as the calculation proceeds. It results in a calculation having a desired (specified) level of accuracy. The method does not include a sensitivity check.

S. Kline: Could you clarify what precisely you minimize?

E. Tjonneland: The Boeing method is a check on the convergence rate. Further information can be obtained from Forrester.

P. Roache: The sensitivity analysis of numerical methods presents an active research field for many mathematicians. The current position is that such analysis is not even simple for strongly parabolic equations and one-dimensional flow problems. The analysis is much more complex in multi-dimensional systems. There are some recent papers in the mathematical journals (S.I.A.M.) giving the present state-of-the-art.

S. Bogdonoff: I would like to change the direction of this discussion and proceed to the question of three-dimensional mean flows. Why were these not considered at the Conference, especially when we remember that turbulent flow structure is three-dimensional and is flow dependent? Does the Conference believe that an analysis of two-dimensional mean flows presents a good starting point for the understanding of complex turbulent three-dimensional mean flows? Are there any experiments, particularly in three-dimensional complex turbulent supersonic flows, including boundary layers on surfaces at large angles of attack, which show similar characteristics to the two-dimensional flows examined at this Conference?

P. Bradshaw: Even mildly three-dimensional boundary-layer flows show features not found in two-dimensional flows. There is the experiment by van den Berg on a swept wing which shows that the flow needs to be described in terms of an anisotropic eddy viscosity. The results of the three-dimensional test cases will be presented at the 1982 IUTAM meeting in Berlin.

S. Bogdonoff: It would appear essential that the turbulent-flow models being proposed should be checked to ensure that they provide the correct trends in passing from two-dimensional to three-dimensional flows.

M. Morkovin: What is the experience of the European community in working on three-dimensional flows?

E. van den Berg: Most of the European work has been related to experiments on three-dimensional boundary-layer flows. Little work on three-dimensional turbulent-flow modelling has been attempted.

P. Klebanoff: I suggest that the computation of complex three-dimensional laminar flows can shed some light on the related problem of three-dimensional turbulent flow. An example is the effect of three-dimensional roughness elements in a laminar layer.

V. C. Patel: Work on these lines has been done in Japan.

M. Morkovin: Sedney at the Aberdeen Proving Ground, working on data provided by Tani, failed to compute these kinds of flow. He concluded that the standard boundary-layer assumptions are not sufficient for three-dimensional flows.

D. Chapman: At the present time three-dimensional calculations are limited by computer storage. At present it is unpractical to conduct three-dimensional interactive calculations. In the future we can expect many more three-dimensional calculations.

S. Bogdonoff: I was surprised how well Horstman's calculations, using a fairly crude grid, reproduced the main features of the flow studied by Settles.

D. Chapman: I was referring to more complicated three-dimensional flows, including cases where there is a strong interaction between the external flow and the boundary layer. It should be noted that early turbulence models also looked good, but as more data became available we found these models performed poorly.

D. Wilcox: My earlier work in this field showed good output for three-dimensional flows calculations, but it was shown later they were right for the wrong reasons.

W. Hah: We have shown that eddy-viscosity models do not give satisfactory results for three-dimensional boundary layers.

B. Lakshminarayana: There are some three-dimensional codes but they need an input from well-documented three-dimensional experiments such as those performed at Imperial College. The experiments on rotation are a good data set.

B. van den Berg: In Europe there are some on-going experiments on three-dimensional flows.

S. Kline: We now have some of the three-dimensional flow data on tape in the Data Library. These flows will be considered in the IUTAM meeting in Berlin - 1982.

W. Reynolds: The 1968 Conference concluded that it was appropriate to use P.D.E.'s in models for the prediction of the turbulent kinetic energy in turbulent flows. It seems to me that the 1981 Conference has shown that the length scale can be generated by a P.D.E. as well as can be guessed by an experienced mixing-length estimator. Therefore, in truly predictive situations, I would, henceforth, favor length predictions over length estimates based on the past experience of the estimator.

S. Bogdonoff: We have had a wide-ranging discussion in which we have highlighted a number of important conclusions from the 1981 meeting. In closing the meeting I propose a vote of thanks to Professor Kline for the tremendous work he has put into this meeting and its success is largely a result of his own dedicated efforts. Certainly a follow-up meeting is desirable, and we look forward to its early announcement.

S. J. Kline

and

The Presentation Clock

(A gift of the Organizing Committee

1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows)

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EDITORS' CONCLUDING REMARKS

The 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows has been a cooperative learning experience involving a major fraction of the research community. The Editors have been privileged to play several roles in the Conference. These concluding remarks by the Editors may be helpful in providing an overall summary of certain points, particularly those that have become clearer during the editing process. The remarks are supplementary to and do not replace the Report of the Evaluation Committee, the Ad-Hoc Committee of Computors (p. 957), or the other summary elements of the Conference. Readers interested in the overall evaluation of the Computations in 1981 should see these other elements. The Readers' Guide in this volume may be helpful in locating various elements within these Proceedings which are most useful for a given purpose.

A. GOALS

The Conference had three major goals as noted in the Introduction of Vol. I:

1. To reach consensus in the research community on trustworthy data sets that can be used as input for modeling of turbulence in complex flows and as the basis for standard "trials" for checking output of computations.
2. The creation of a "data library" on magnetic tape. This library will hold the data selected as trustworthy in standard normalized form. The data will be computer-readable and widely accessible at a moderate fee.
3. Comparison of the output of current methods of computation for turbulent flows as applies to a set of "basic test cases" covering a broad range of flows.

It is believed that the work of the Conference has essentially met all three goals.

The 1980 meeting on data has identified approximately seventy cases that are useful for input to models and for checking output of computations. These cases have been put into standard form including detailed specifications of test cases for checking output and are presented in detail in Vol. I. These cases have also been entered onto magnetic tape in standard machine-readable form. In addition to the role played in the present Conference, these carefully checked data should also be useful in providing inputs to future specific models for related classes of flows (see comment C below). Either the volumes or the tape can be ordered from:

Complex Turbulent Flows
Department of Mechanical Engineering
Stanford University, CA, USA 94305

The outputs from some 35 Computor Groups have been compared with the standard test cases and discussed in the 1981 Meeting. The full results of these outputs are compiled in Vol. III together with short summaries of the experiences of individual Computor Groups.

B. OTHER ELEMENTS--TAXONOMIES, DATA NEEDS

A number of other elements have also been prepared and presented in these Proceedings in order to provide, in so far as possible, a complete picture of the state of the art in 1980-81 in an understandable, organized format. The complete listing, including certain transitional and explanatory elements prepared by the Editors can be seen in the Table of Contents of the three volumes of these Proceedings. Particularly noteworthy are two sets of comments:

(i) the taxonomies of turbulence models and of numerical methods, and the summary of methods given in Sessions I and II of this Volume. These materials provide for the first time a relatively clear view of what is available and how future methods may fit into the existing frameworks. New students of turbulence may find these materials particularly valuable;

(ii) the position papers, the discussions, and the reports from ad-hoc committees in Vol. I constitute a far more complete discussion of the data needs, the instrument limitations, and the methods for the control of experiments in complex turbulent flows than have previously been available. These comments should provide significant information for experimentalists concerned with the creation of data for use in creating models or checking output for complex turbulent flows.

A point concerning data needs that was discussed in the 1980 meeting, but has become much more strongly emphasized by the experiences of the Computor Groups reported at the 1981 meeting and in Vol. III is the following. In many flow situations, particularly where boundary layers approach or reach detachment, the results of computations are strongly dependent on initial conditions including not only the mean flow but also the time-averaged fluctuating quantities (e.g., Reynolds stresses and entrainment rates) together with their gradients over the entire inlet section. More data, specifically varying such quantities over the inlet section, and more careful measurements of fluctuations at upstream boundaries are definitely needed in all cases intended for construction of models or checking output.

C. DOES A UNIVERSAL CLOSURE MODEL EXIST? — THE BASES OF ASSESSMENT

In the Introduction to the Conference (see Vol. I), S. J. Kline pointed to the question of whether a universal, single, simple turbulence closure model does in fact exist, and noted that this is a central question in directing research efforts. The

work of the Conference has greatly clarified this question, and has thereby made clearer the appropriate direction for future researches in computation of complex turbulent flows. Considerable comment on this question appears in the discussion to Session IV of the 1981 meeting, and an extensive separate document entitled "Universal or Zonal Modeling--The Road Ahead, a Personal OPINION" by S. J. Kline with comments from attendants and a Closure appears in the general conclusions in this Volume (see Session XV).

As stated in that OPINION and its closure, given currently available results, the likelihood of finding a single, simple universal closure model is extremely small. The OPINION strongly suggests that successful computation of complex turbulent flows will need to follow zonal methods when even moderate accuracy is mandatory. One further comment on this point is not obvious on the surface, but emerges from studying the complete results including comments by all the Reporters, the session discussions, the Evaluation Committee Report and the complete results in Vol. III. If one looks at any given group of flows, some difficulties for creating a universal closure model are apparent, but they may not seem insuperable. However, when one looks at the entire range of cases contained in this volume, and one also recalls that these cases contain no VERY complex flows, a different, stronger picture emerges. The true difficulties of finding a single, simple, universal turbulence closure model become more apparent, and the likelihood of finding such a model is seen to be extremely small.

The preceding paragraph is not in any way intended to deny the important utility of computation in complex turbulent flows. Computation has become an important "third force" in turbulent flows, and will, no doubt, remain in this role. Moreover, as explained in the OPINION in Session XV, by clarifying the road along which successes have been achieved and seem most likely to be achieved in the future (that is, via appropriate zonal modeling tailored to classes of flows), we can expect that more rapid, sound progress can be realized than by attempting to achieve the very unlikely (or perhaps impossible) goal of a single, simple, universal closure model.

This conclusion indicates that a systematic, broad comparison of results has been imperative as a basis for forming a sound opinion about the current state of the art and appropriate paths for future research in complex turbulent flows. The study of a few cases does NOT suffice. The same result was true in 1968. Only after systematic study of a broad range of cases in 1968, 1969, and 1972 did a clear, proper picture of the state of the art in computation of shear layers emerge; prior to that time the common wisdom was in significant error.

D. NEXT PROBLEMS--FUTURE MEETING(S)

The 1980-81 Conference has clarified the state of the art in the Data and in Computations of Complex Turbulent Flows. In so doing, it has focused more clearly

some remaining questions for the next phase of research. The two central questions for the immediate future appear to be:

1. Given that zonal methods, tailored to certain classes of flows, are the road along which accurate engineering predictions (or postdictions) can be constructed, what methods are best suited to particular classes of flows and industrial applications?
2. How can the possible errors arising from numerical procedures be clearly separated from errors owing to turbulence models, and what procedures, if any, can be built into codes to guarantee a prescribed level of numerical accuracy?

These two questions will be major topics of a follow-on meeting tentatively scheduled for 1984 in Southampton, England. Reader's interested in this meeting can obtain information from:

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E. COMPARISON OF 1968 AND 1981 EXPERIENCES--IMPLICATIONS FOR INDUSTRIAL USERS

The experiences with computations and Computor Groups in the 1980-81 Conference has been decidedly different than that in the 1968 AFOSR-IFP-Stanford Conference on Computation of Turbulent Boundary Layers. These differences have implications of importance to potential industrial users.

In the 1968 Conference some 28 methods by 26 Computor Groups were identified. All 28 methods were represented. All Computor Groups did all 16 mandatory cases; many did the 16 optional cases as well. In 1968 no Computor Group dropped out (one individual who had a temporary, serious medical problem had his method programmed and presented by the Organizing Committee in the 1968 Proceedings). Nor did any known Computor Group decline to submit results to the 1968 meeting. Thus the 1968 results are complete in coverage of the field. These results indicated a basic success in computation of incompressible turbulent boundary layers.

The situation in the 1981 meeting was quite different. Some 55 Computer Groups indicated they would submit computations. Only 35 ultimately did submit results. Over the nine months available, no Computor Group did all or nearly all the cases presented (even where several Groups worked together co-operatively). Moreover, for many Computor Groups there was significant reduction in the number of cases covered between the initial estimates and the final computations presented. Ultimately, less than two-thirds of the cases promised were submitted. The results presented in 1981 include very few attempts to do any of the four predictive cases (in which experimental data were taken concurrently under the auspices of the Organizing Committee and

were therefore not known beforehand by the Computors). This occurred even though written requests by the Organizing Committee asked that these cases be done whenever possible. Finally, several well-known Computor Groups declined to submit any output to the 1980-81 Conference on the grounds that the situation was too complex, particularly for elliptic fields, and therefore that nothing would be learned. These complexities were summarized by H. McDonald in a presentation at the 1980 meeting which appears in Vol. I.

These differences between the 1968 and 1981 meetings have implications that are important to potential users of computations of complex turbulent flows. The computation of individual shear layers (as in 1968 and two subsequent meetings coordinated by NASA in 1969 and 1972) is not extraordinarily difficult and does not consume extremely large amounts of computer time. For many situations existing methods are quite adequate for computation of shear-layer cases to adequate engineering accuracy. The computation of complete Complex Turbulent Flow Fields presents a quite different picture. Despite certain recent, very optimistic, published accounts, computations of complete complex turbulent flow fields remains a difficult problem in 1982. The difficulties are well attested by several facts: (i) the very significant differences between intended and actual computations presented to the Conference, (ii) the lack of any single method that seems adequate for the full range of cases presented for study by the 1980 meeting; (iii) the lack of any VERY complex turbulent flow that is adequately documented by data—although many such flows are important in industrial applications; (iv) the disappointing failure of increased complexity of turbulence modeling to provide increased accuracy of output in results;* (v) the seemingly general failure of existing methods to extrapolate over a wide variety of flow cases and thus to assure adequate predictive capacity. For further details, the reader should see the Report of the Evaluation Committee and the OPINION on Zonal Modeling in Session XV of this Volume.

These facts, taken together suggest some caution on the part of potential industrial users. The computation of complex turbulent flows has advanced enormously in the past decade and a half, but it is not yet a mature art. The range of flows that can be usefully attacked has increased enormously in the past decade, and some method that is reasonably accurate can be found for most, if not all, complex turbulent flow fields. However, certifiable methods for very complex flow fields, as for example behind ship hulls with propellers or the motions in the vicinity of buildings in a city, do not yet exist. Nor is any one method usable for all the flows that can be attacked for the reasons summarized in C above. Users will therefore need to exercise

*An excellent discussion of some underlying reasons for this failure and the uses of various levels of approximation is given by J. Cousteix, p. 650 (herein).

some caution in the selection of methods, and should recognize that the task of computation of complete complex turbulent flow fields is neither inexpensive nor fast in 1982, and will probably remain in this condition for some years despite the fact that the field is still developing rapidly and will very probably continue to do so.

F. COMPUTATION AND EXPERIMENT

Nothing presented at the 1980 or the 1981 meeting of this Conference suggests that the era of needful experimental study of the flow physics of turbulent flows is past. In fact, the contrary conclusion seems fully justified. Successful methods of computation of complex turbulent flows invariably still build in some data and thereby adjust the values of some parameters (or functions) to provide accurate output for a given class of flows. The need for zonal modeling re-emphasizes the need to provide further physical information concerning some important "structural flow zones" as for example, detaching zones, reattaching zones, recirculating zones in separated flows, and three-dimensional shock/boundary-layer interactions to mention only a few. More details can be found in the discussions of Vol. I. A number of reporters, as for example John Lumley, specifically call for further study of the physics as needful input to improved models of turbulence. Other researchers, as for example Gary Settles (see p. 1005), note that the number of experiments currently under way seems low compared to the amount of effort being placed on computational methods in turbulent flow. These are both opinions the Editors share. Since we are trying to represent by statistical averages motions that are inherently and often grossly unsteady, it is usually quite difficult to build physical information from observations into the working mathematical models. Moreover, most existing data were not taken with the idea of building computer models in mind. Hence we expect the needs for data and for improved incorporation of data into mathematical models to remain an issue requiring continuing attention for some time. The comments in the discussions of Vol. I of these Proceedings provide considerable information for Researchers on the type of data needed and the necessary precautions and cross-checks required to insure data that are useful as input for modeling and for checking the output of computations in turbulent flows, as noted in item B above.

G. ORGANIZATIONAL PRODEDURES

The nature of the 1980-81 AFOSR-HTTM-Stanford meeting on Complex Turbulent Flows was of a distinctly different character than typical scientific meetings and hence required devising and utilizing several new organizational procedures for selecting data, for summarizing results and for closing discussions in useful form. These procedures may be useful for some other meetings and are accordingly described in the Introduction to Vol. I.

H. FURTHER USE OF THE PROCEEDINGS

The Editors believe that the preceding remarks are appropriate as a supplement to the Conclusions in this Volume. Given the complexity and extent of the field at this time, it is not to be expected that these remarks exhaust all possibilities. It is rather to be expected that further conclusions will be realized as many workers have time to study the materials of these Proceedings.

Stanford, CA
July 1982

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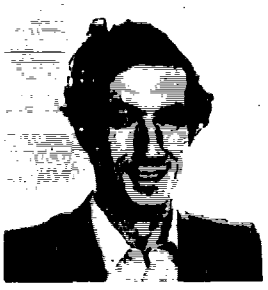

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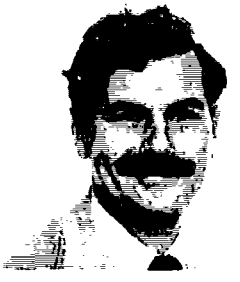

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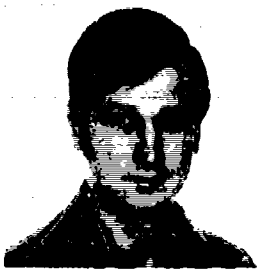

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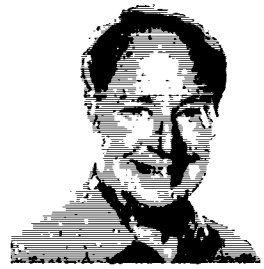

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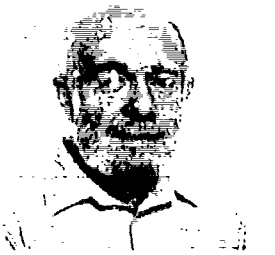

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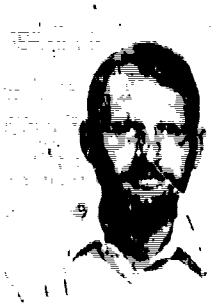

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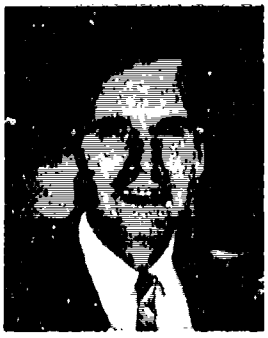

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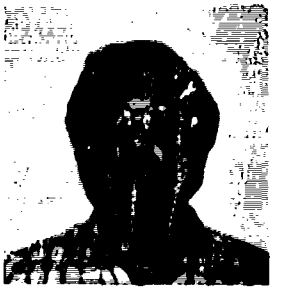

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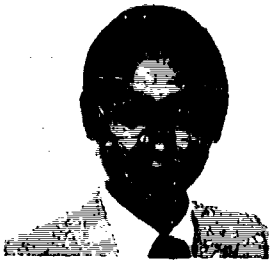

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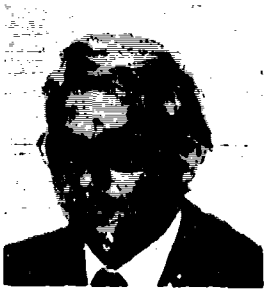

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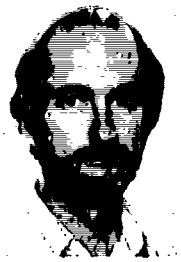

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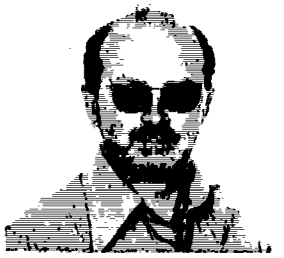

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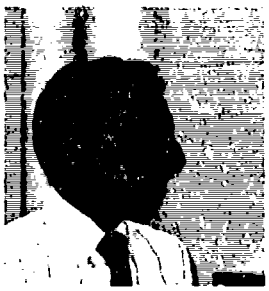

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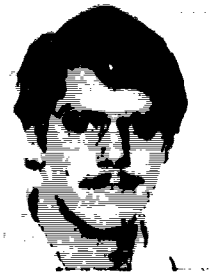

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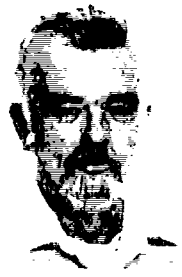

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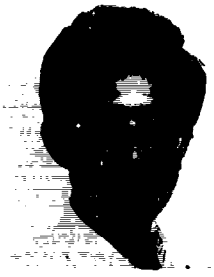

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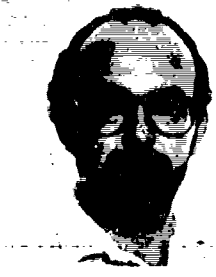

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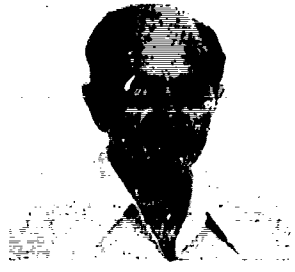

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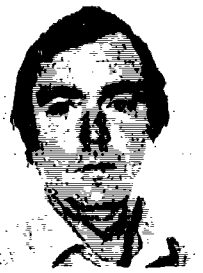

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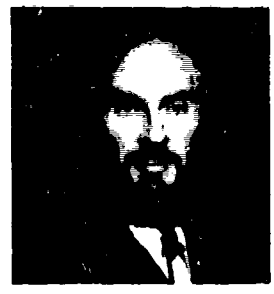

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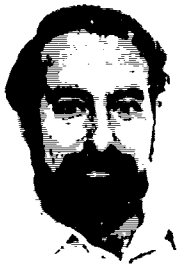

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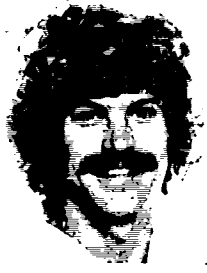

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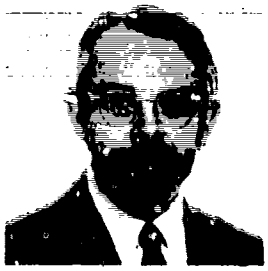

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