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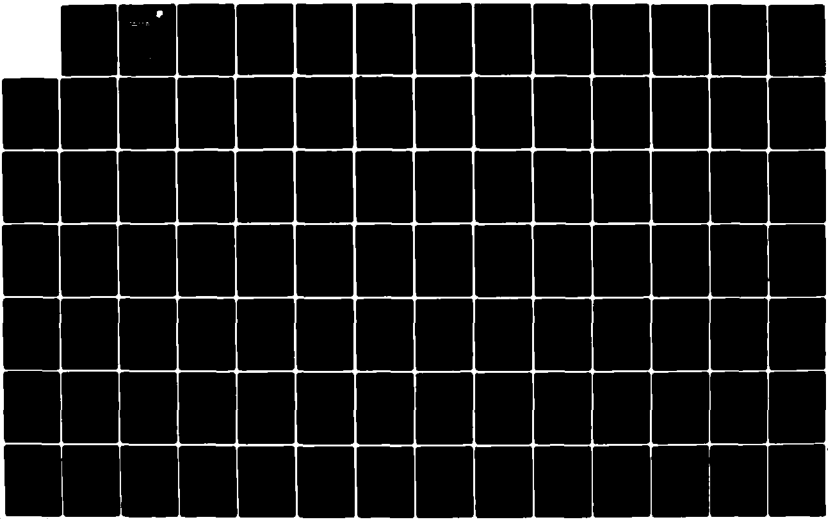

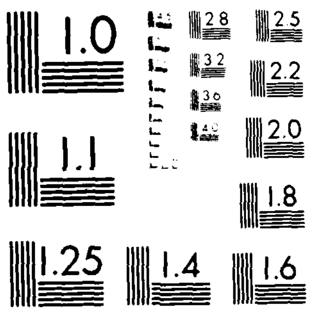

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Phase Report
April 1983

EMC MODELNG AND ANALYSIS - A PROBABILISTIC APPROACH

Syracuse University

Abner Ephreth and Donald D. Weiner

This effort was funded totally by the Laboratory Directors' Fund

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report provides a new and fundamental basis for EMC analysis, i.e. a probabilistic approach. The advances in high speed, high density integrated circuit (IC) technology provides the impetus for investigating new concepts in electromagnetic compatibility/electromagnetic interference (EMC/EMI). Performance criteria, acceptable performance, EMI performance curve and performance threshold are concepts related to susceptiblity level in a probabilistic manner. In addition, the interactions at

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) different levels (e.g. system, subsystem, equipment, component) are also discussed. Because large portions of systems are being replaced by complex ICs and because the electromagnetic environment and equipment susceptibility are in reality, random in nature, a probabilistic approach enables one to develop a statistical macromodel. In such an approach, detailed circuit models and functions are replaced by statistical models where probability density functions are used to evaluate probabilities and statistical averages associated with various responses at various operational levels.

CONTENTS

1. INTRODUCTION

Modern electronic systems must frequently operate in hostile electromagnetic environments dominated by unwanted signals. The unwanted signals are referred to as interference whether or not they cause unacceptable system performance. Undesirable electromagnetic interactions can occur between different 1) systems, 2) subsystems within a system, 3) equipments within a subsystem, and 4) components within an equipment. Electromagnetic compatibility (EMC) exists when, even in the presence of the unwanted signals, the equipments, subsystems, and systems perform together in an acceptable manner. In other words, if unintentional electromagnetic radiations and/or responses do not sufficiently degrade system performance, EMC is said to apply. On the other hand, electromagnetic interference (EMI) results when acceptable performance is prevented because of the interfering signals. The term, interference, is seen to have a dual meaning because it refers to both the unwanted signals and their effect.

In order to determine acceptable performance, a performance criterion is needed which provides a standard, rule, test, or measure for judging quality of operation. Depending upon the specific application and the type of system, subsystem, or equipment involved, different criteria are appropriate. Several examples are cited below. In speech communication systems, where intelligibility is the important factor, two widely used criteria are articulation score and articulation index. In television and graphics display systems, resolution is often chosen as the performance criterion. Bit error rate and probability of error are commonly used criteria in digital communication systems. Performance criteria used in analog data systems are mean square error, peak error, and average absolute error. With respect to radar systems, the probability of detection for a specified false alarm probability is a frequently used criterion. In every case, once a performance criterion has been chosen, acceptable performance can be specified by selecting an allowable range of values for the performance criterion. This range is usually based upon mission requirements.

In the evaluation of system performance, the desired and interfering signals are usually characterized in terms of convenient parameters such as average power, peak amplitude, energy, and peak power. The EMI performance curve is a plot of the selected performance criterion versus some function of the desired and interfering signal parameters. For example, in Figure 1

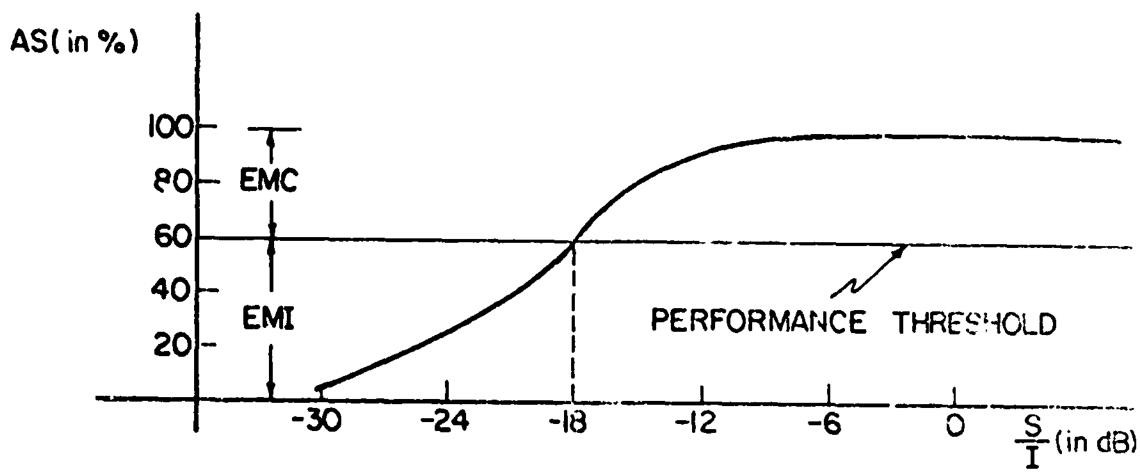

Figure 1 EMI performance curve for a speech communication system.

is shown a typical EMI performance curve for a speech communication system. For this system the performance criterion selected is the articulation score, AS, which gives the percentage of words correctly interpreted in a listener panel test. The average desired signal power, S, and the average interfering power, I, are the desired and interfering signal parameters suitable for this plot. The EMI performance curve consists of a plot of the articulation score versus the signal-to-interference ratio, S/I. Use of the signal-to-interference ratio implies that performance is independent of the absolute levels of S and I. Rather, it is their ratio which determines the amount of intelligibility. As would be expected, the articulation score is seen to be an increasing function of the signal-to-interference ratio until, for large enough values of S relative to I, essentially perfect intelligibility is achieved.

Given the EMI performance curve, it is possible to specify the range of operation over which acceptable performance occurs. The performance threshold is that value of the performance criterion which demarcates the regions of acceptable and unacceptable performance. These regions, of course, are determined in accordance with mission requirements. By way of example, the performance threshold in Figure 1 has been set at an articulation score equal to 60%. This implies, for the application of interest, that acceptable performance results when the articulation score is greater than or equal to 60% while scores less than 60% yield unacceptable performance. To put it another way, EMC exists provided AS $\geq$ 60% while EMI prevails when AS < 60%.

Given the EMI performance curve and numerical values for the performance threshold and the desired signal parameter, it is possible to specify the level of interference above which EMI exists. For example, with respect to Figure 1, S/I equals -18 dB at the performance threshold. Therefore, when

$S = -10$ dBm, interference with average power greater than 8 dBm produces EMI. On the other hand, with $S = -10$ dBm, EMC exists when $I \leq 8$ dBm. By definition, the <u>susceptibility level</u>, L, is the level of the interfering signal that results, for a specified desired signal, in operation at the performance threshold. Although the numerical value of L depends upon the desired signal, the susceptibility level is primarily a characteristic of the system, subsystem, or equipment involved. Returning to the example of Figure 1, assume $S = -10$ dBm. The corresponding susceptibility level is given by $L = 8$ dBm. However, an improved design of the speech communication system of Figure 1 could result in the susceptibility level being increased to a value greater than 8 dBm when $S = -10$ dBm.

It is important to recognize that EMI performance curves are a function of the type of interference encountered. For example, given an FM communications receiver, separate EMI performance curves apply to the different cases of AM, FM, and pulsed radar interference. Although a complete set of EMI performance curves does not exist, many curves have been generated for several types of communications receivers under a wide variety of interfering signals [illegible].

Having introduced the concepts of EMI performance curve, performance threshold, and susceptibility level, attention is now focused on EMC modeling and analysis. Historically, approaches to this problem proceeded from a deterministic point of view in which the signals, coupling paths, and equipment characteristics are assumed to be known. Uncertainties were accounted for by utilizing "worst-case" models. However, many modern electromagnetic environments and electronic equipments have become so complex they no longer can be modeled either realistically or efficiently using a deterministic approach. This is particularly true of equipments containing microelectronics. In particular, digital circuits are currently being designed with the latest integrated circuit technologies where hundreds and even thousands of active devices are fabricated on tiny silicon chips. The resulting circuits are so complicated they cannot be analyzed, or even simulated on a computer, in order to determine their EMI performance.

In recent years macro models have been introduced for simulating complicated circuits. Ideally, the behavior of the macro model closely approximates that of the original circuit even though the macro model employs significantly fewer active devices. Limited success has been achieved thus far. For example, EMI in an operational amplifier containing 25 transistors has been successfully predicted by a macro model employing only 2 transistors. In addition, macro models are currently being developed for predicting EMI in simple digital logic circuits.

However, the electromagnetic environment and equipment susceptibility are, in reality, random in nature. By utilizing a probabilistic approach, statistical macro models can be developed which simplify the problem of characterizing complicated signals and circuits. In such an approach, detailed circuit models are replaced by statistical models where probability density functions are used to evaluate probabilities and statistical averages associated with the response. Following the lead of Dr. Capraro of Rome Air Development Center [2], this report proposes and analyzes a probabilistic EMC model which is useful when the deterministic approach to EMC modeling and analysis is inappropriate. Additional material relevant to this work can be found in the paper by Bossart, Shekleton, and Lessard [3].

2. THE PROBABILISTIC MODEL

For simplicity, consider the EMC problem illustrated in Figure 2 where both a desired signal, s(t), and an interfering signal, i(t), couple

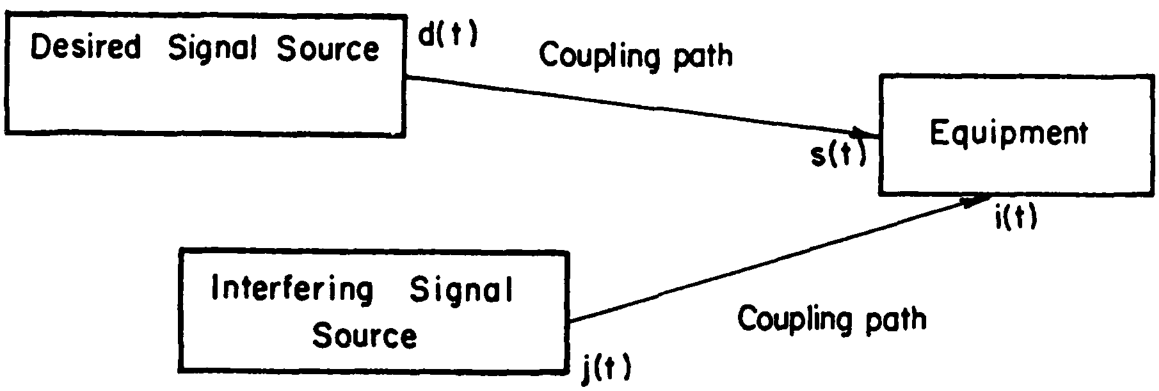

Figure 2 A simple EMC problem.

into an equipment. If the desired signal is to convey information, it must be unpredictable. In fact, according to information theory, the information content of a signal increases with its uncertainty. Unpredictability in the desired signal is also introduced by the coupling path. Typically, the received signal, s(t), must be distinguished from the transmitted signal, d(t). For example, unpredictable signal distortion may arise when the attenuation and phase characteristics of the coupling path are unknown. Also, the transmitted signal may undergo fading, whereby the received signal amplitude is found to fluctuate in a random fashion. In addition, additive noise may be introduced in the coupling path so that the received signal actually consists of the sum of signal plus unknown noise. In the probabilistic approach, uncertainty in the desired signal is incorporated by treating s(t) as a sample function from a random process.

The interfering signal may enter the equipment directly through the input port. It may also penetrate the equipment indirectly by coupling on to cables such as power lines or by propagating through apertures in the case. As with s(t), the interfering signal i(t) is likely to be unpredictable. Some of the uncertainty may be due to the transmitted signal, j(t), also being an information bearing signal. Additional uncertaint may be introduced by the coupling path. Finally, the mechanism by which i(t) enters the equipment may not be completely known. This is especially true when the interfering signal couples onto wires or cables and/or through apertures. As a result, i(t) is also treated in the probabilistic EMC model as a sample function from a random process.

The final uncertainty arising in the EMC problem is associated with the equipment itself. It is not possible to manufacture "identical" resistors, diodes, transistors, etc. having precisely the same parameter values. Parasitic elements, whose values cannot be controlled, may play an important role in an equipment's behavior. The positioning of wires and cables may differ from one "identical" equipment to another. Also, equipments are apt to age differently. Therefore, it is highly unlikely that the EMI performance curves of "identical" equipments will be identical. This is illustrated in Figure 3 where the EMI performance curves for many "identical" speech communication

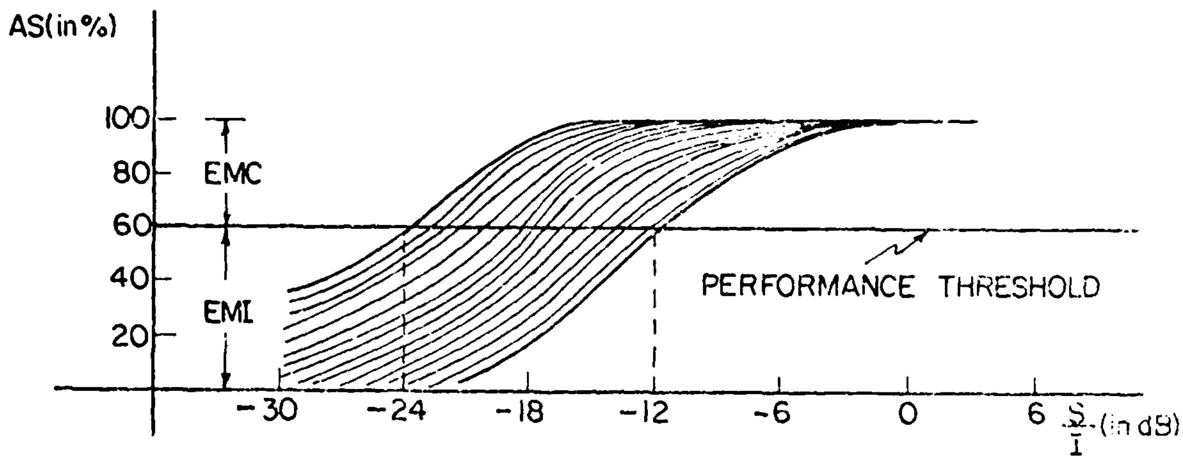

Figure 3 EMI performance curves for many "identical" speech communication systems subjected to the same desired and interfering signals.

systems are shown. As in Figure 1, the selected performance criterion is the articulation score which is plotted versus the signal-to-interference ratio, S/I. In generating Figure 3, it is assumed that each equipment is exposed to the same electromagnetic environment (i.e., each equipment experiences the same desired and interfering signals). Observe, in Figure 3, that at the performance threshold the signal-to-interference ratio varies from -24 dB to

-12 dB, depending upon the equipment under test. Consequently, for a given desired signal, there is a 12 dB variation in the susceptibility level, L. For example, when S = -10 dBm, L ranges from +14 dBm to +2 dBm. In the probabilistic approach, the uncertainty in the susceptibility level is incorporated by treating L as a random variable.

In summary, the probabilistic model proposed for the EMC problem of Figure 2 is obtained by treating

1) the desired signal, s(t), as a sample function from a random process,

2) the interfering signal, i(t), as a sample function from a random process,

3) the equipment as a sample from an ensemble of randomly distributed "identical" equipments whose EMI performance curves may be characterized in terms of the random susceptibility level, L. For a specified desired signal, L indicates the minimum level of the interfering signal above which EMI occurs.

The remainder of this report is devoted to use of the probabilistic model in determining EMC and EMI.

3. THE GIVEN SIGNAL AND INTERFERENCE EMI AND EMC PROBABILITY FUNCTIONS

For simplicity in developing the proposed probabilistic EMC model, discussion is again centered around the simple EMC problem depicted in Figure 2. For case of discussion, it is assumed that knowledge of only the desired and interfering signal average powers are required to determine EMI performance. Of course, depending upon the particular problem at hand, knowledge of other signal parameters such as peak amplitude, energy, or peak power may be needed.

Assume s(t), the desired signal at the equipment, is known. Let uncertainty in the equipment's EMI performance curve be characterized by the random susceptibility level, L. (In this report random variables are denoted by upper case letters while values assumed by the random variables are represented by lower case letters.) Let the average powers of the desired and interfering signals be denoted by the random variables S and I, respectively. Given S = s, the random variable L equals the average power of the interfering signal required to cause the equipment to operate at its performance threshold.

The random behavior of L is governed by its conditional cumulative distribution function (CDF) which is denoted by $F_{L|S}(\ell|s)$. By definition, $F_{L|S}(\ell|s)$ is the probability, given S = s, that the random variable L is less than or equal to the value ℓ. Symbolically, this is written as

$$F_{L|S}(\ell|s) = P[L \leq \ell|s]. \qquad (1)$$

In general, it is difficult to theoretically determine the conditional CDF of L. However, it can be obtained empirically through laboratory measurements.

For this purpose, it is necessary to have a large number, N, of "identical" equipments all exposed to the same desired and interfering signals. Let $N_{EMI}(\ell)$

denote the number of equipments experiencing EMI when the average power of the interfering signal equals ℓ. The experiment proceeds by 1) holding the desired signal average power at the constant value S = s, 2) gradually increasing the average power of the interfering signal, and 3) recording the value of $N_{EMI}(\ell)$. A typical result of the experiment is shown in Figure 4.

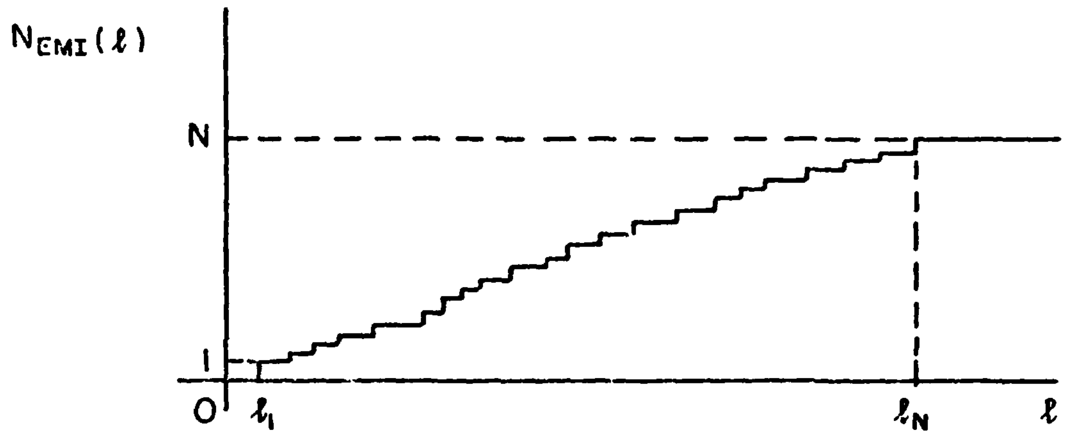

Figure 4 Experimental result in measurement of conditional CDF of L.

Note that $N_{EMI}(0) = 0$ because $\ell = 0$ implies the absence of an interfering signal. The value of the interference average power at which the first equipment from the group of "identical" equipments reaches its performance threshold is given by $\ell = \ell_1$. As the average power of the interfering signal is increased, more and more of the equipments experience EMI. Finally, the Nth and last equipment from the group of "identical" equipments reaches its performance threshold when the interference average power equals ℓ_N. Of course, an equipment's susceptibility level must be less than ℓ when it suffers EMI with the interference average power equal to some value ℓ. It follows that $N_{EMI}(\ell)/N$ is an estimate of $P[L < \ell|s]$. Dividing the ordinate in Figure 4 by N and smoothing the resulting curve yields an approximation for the conditional CDF of L, as shown in Figure 5.

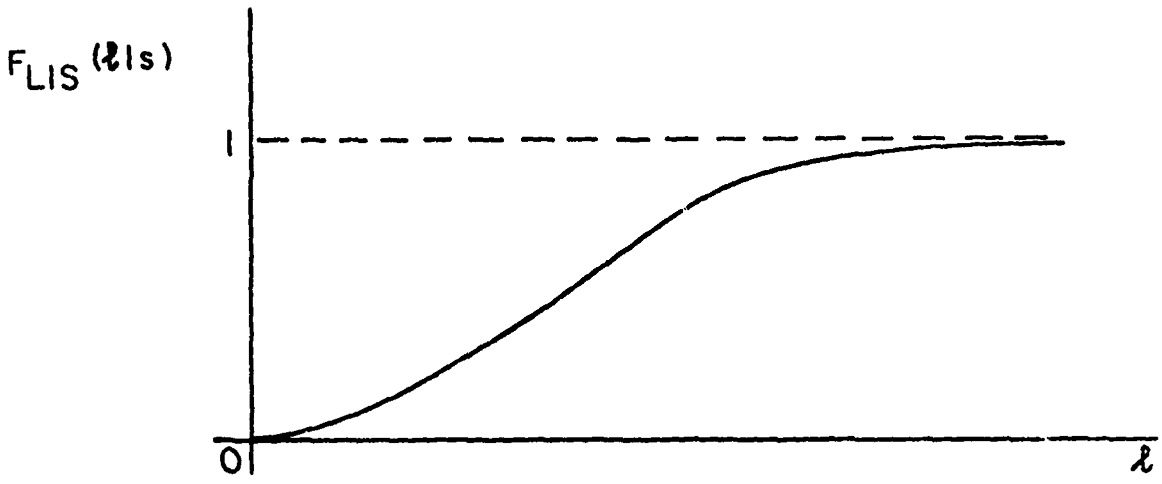

Figure 5 Approximation to conditional CDF of L.

The curve in Figure 5 is conditioned on the presence of a desired signal s(t) whose average power is given by S = s. In general, as different sample functions from the ensemble of s(t) excite the equipment, the value of S changes and a family of curves for the CDF of L is generated. A typical family is shown in Figure 6 where, as would be expected, the susceptibility level is seen to increase with increasing values of the desired signal average power.

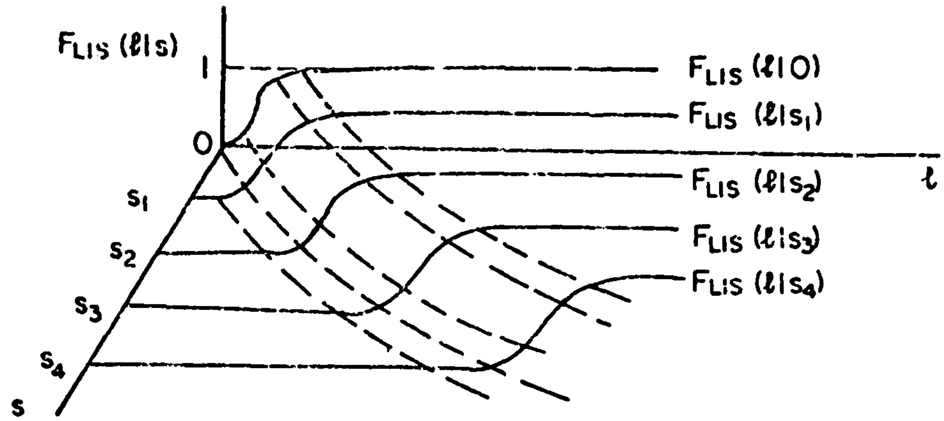

Figure 6 Family of conditional CDF's of L.

The conditional CDF of L, $F_{L|S}(\ell|s)$, can be given a second interpretation. Assume the average powers of both the desired and interfering signals are known. In particular, let S = s and I = i. EMI results when the susceptibility level is less than the average power of the interfering signal. Note that

$$F_{L|S}(i|s) = P[L \leq i|s] \tag{2}$$

is the probability that the susceptibility level is less than or equal to i, given S = s. Hence, $F_{L|S}(i|s)$ equals the conditional probability of EMI under the circumstance that S = s and I = i. Therefore, $F_{L|S}(i|s)$ is referred to as the given signal and interference EMI probability function.

On the other hand, EMC exists when the susceptibility level is greater than the average power of the interfering signal. The given signal and interference EMC probability function is defined to be

$$C_{I|S}(i|s) = P[\ L > i|s] = 1 - F_{L|S}(i|s). \tag{3}$$

By definition, $C_{I|S}(i|s)$ is the probability that the susceptibility level is greater than i, given S = s. Consequently, $C_{I|S}(i|s)$ equals the conditional probability of EMC under the circumstances that S = s and I = i. Since $C_{I|S}(i|s)$ is the complement of a CDF, it is a nonincreasing function whose value ranges from unity to zero with increasing i. A typical plot of the given signal and interference EMC probability function is shown in Figure 7.

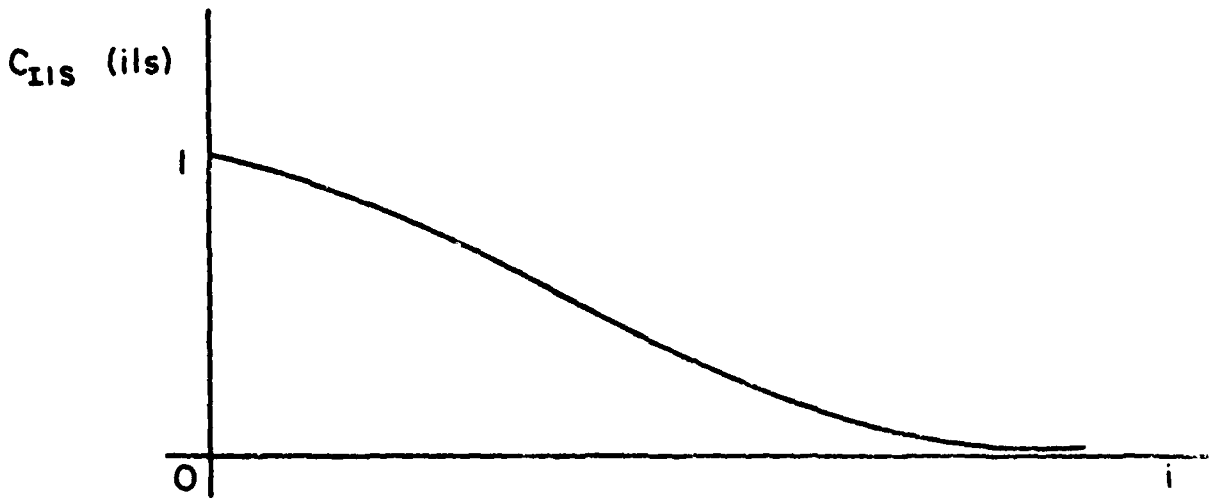

Figure 7 Typical plot of conditional EMC probability function.

4. DERIVATION OF EQUIPMENT EMC IN A RANDOM ELECTROMAGNETIC ENVIRONMENT

The given signal and interference EMI and EMC probability functions are applicable only if the average powers of the desired and interfering signals are both known. When the equipment operates in a random electromagnetic environment, S and/or I may not be known. Useful results are then obtained by performing ensemble averages over the unknown random quantities.

Assume the probability density functions (PDF's) of S and I are denoted by $f_S(s)$ and $f_I(i)$, respectively. The conditional PDF of the susceptibility level, L, is obtained by differentiating the conditional CDF of L with respect to ℓ. In particular,

$$f_{L|S}(\ell|s) = \frac{dF_{L|S}(\ell|s)}{d\ell} \quad . \tag{4}$$

In general, the conditional PDF of L varies from one value of s to another. A typical family of curves for $f_{L|S}(\ell|s)$ is shown in Figure 8. It should be

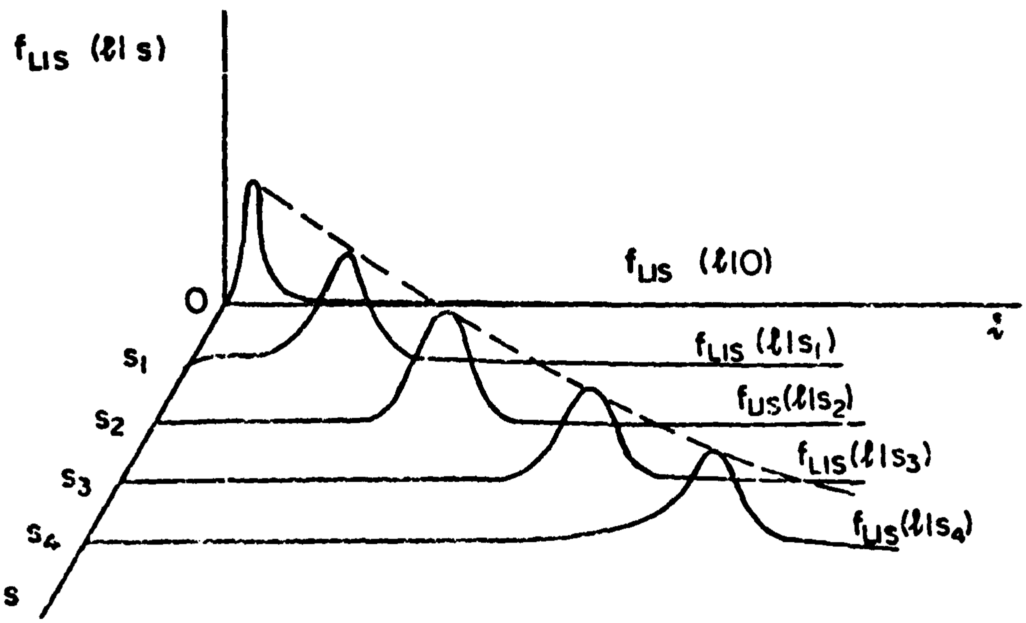

Figure 8 Family of conditional PDF's of L.

noted that L and S are likely to be statistically dependent random variables even when S and I are statistically independent.

The first case considered is the situation in which $S = s$ is known but I is unknown. Averaging $C_{I|S}(i|s)$ over all possible values of I, there results

$$C_S(s) = \int_{-\infty}^{\infty} C_{I|S}(i|s)\ f_I(i)di. \tag{5}$$

Since

$$C_{I|S}(i|s) = P[L > i|s] = \int_i^{\infty} f_{L|S}(\ell|s)d\ell, \tag{6}$$

it follows that $C_S(s)$ can be expressed as

$$C_S(s) = \int_{-\infty}^{\infty} \int_i^{\infty} f_{L|S}(\ell|s)\ f_I(i)d\ell\ di. \tag{7}$$

Given $S = s$, $C_S(s)$ is the expected probability of EMC obtained by averaging over all possible values of the unknown interfering signal average power. Consequently, $C_S(s)$ is referred to as the <u>given signal EMC probability function</u>. As implied by the notation, $C_S(s)$ is a function of the particular value of the known desired signal average power.

A second case arises when S is unknown but it is know that $I = i$. Averaging $C_{I|S}(i|s)$ over all possible values of S, we obtain

$$C_I(i) = \int_{-\infty}^{\infty} C_{I|S}(i|s)\ f_S(s)ds. \tag{8}$$

Substitution of (6) into (8) yields

$$C_I(i) = \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L|S}(\ell|s)\ f_S(s)d\ell\ ds$$

$$= \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L,S}(\ell,s)d\ell\ ds \qquad (9)$$

where $f_{L,S}(\ell,s)$ is the joint PDF of the random variables L and S. $C_I(i)$ is referred to as the given interference EMC probability function. Given I = i, $C_I(i)$ is the expected probability of EMC obtained by averaging over all possible values of the unknown desired signal average power. Analogous to $C_S(s)$, $C_I(i)$ is a function of the particular value of the known interfering signal average power.

Finally, consider the case in which both S and I are unknown. Averaging $C_{I|S}(i|s)$ over all possible values of S and I, there results

$$C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{I|S}(i|s)\ f_{S,I}(s,i)ds\ di \qquad (10)$$

where $f_{S,I}(s,i)$ is the joint PDF of the random variables S and I. Typically, S and I are statistically independent. Then

$$f_{S,I}(s,i) = f_S(s)\ f_I(i). \qquad (11)$$

Use of (6) and (11) in (10) yields

$$C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{I|S}(i|s)\ f_S(s)\ f_I(i) ds\ di$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L|S}(\ell|s)\ f_S(s)\ f_I(i) d\ell\ ds\ di \qquad (12)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L,S}(\ell,s)\ f_I(i)\ d\ell\ ds\ di.$$

C is referred to as the EMC probability. It is the expected probability of EMC obtained by averaging over all possible values of the unknown desired and interfering signal average powers. In analogy to the concept of reliability, C is also referred to as the system compatibility.

By definition, the probability of EMI is the complement of the probability of EMC. It follows that

$1 - C_S(s)$ = the given signal EMI probability function,

$1 - C_I(i)$ = the given interference EMI probability function,

and

$1 - C$ = the expected EMI probability.

To elaborate, given S = s, $[1 - C_S(s)]$ is the expected probability of EMI obtained by averaging over all possible values of the unknown interfering signal average power. Similarly, given I = i, $[1 - C_I(i)]$ is the expected probability of EMI obtained by averaging over all possible values of the unknown desired signal average power. Finally, $(1 - C)$ is the expected probability of EMI obtained by averaging over all possible values of the unknown desired and interfering signal average powers.

5. THE EMI RATE FUNCTION

The EMI rate function, for a specified desired signal, is denoted by $\lambda_s(i)$. By definition, given $S = s$, $\lambda_s(i)di$ equals the probability an equipment will experience interference for $i < I \leq i + di$ provided compatibility exists with $I \leq i$. To be more specific, let A_1 and A_2 denote the conditional events

A_1: equipment experiences EMI with $i < I \leq i + di$, given $S = s$.

A_2: equipment does not experinece EMI with $I \leq i$, given $S = s$.

In terms of the events A_1 and A_2,

$$\lambda_s(i)di = P(A_1|A_2) = \frac{P(A_1A_2)}{P(A_2)} . \tag{13}$$

Since, given $S = s$, the joint event A_1A_2 is equivalent to the equipment's susceptibility level being in the interval $(i, i + di)$, it follows that

$$P(A_1A_2) = P[i < L \leq i + di|S = s] = f_{L|S}(i|s)di. \tag{14}$$

In addition, given $S = s$, the event A_2 is equivalent to the equipment's susceptibility level being greater than i. Therefore,

$$P(A_2) = G_{L|S}(i|s) = 1 - F_{L|S}(i|s). \tag{15}$$

Subsitution of (14) and (15) into (13) yields

$$\lambda_s(i)di = \frac{f_{L|S}(i|s)di}{1 - F_{L|S}(i|s)} . \tag{16}$$

Dividing through by di, we obtain

$$\lambda_s(i) = \frac{f_{L|S}(i|s)}{1 - F_{L|S}(i|s)} = \frac{\frac{dF_{L|S}(i|s)}{di}}{1 - F_{L|S}(i|s)} . \tag{17}$$

Making the change of variables $i = \ell$ in (17) and rearranging terms, the differential equation for $F_{L|S}(\ell|s)$ becomes

$$\frac{dF_{L|S}(\ell|s)}{d\ell} + \lambda_s(\ell)\, F_{L|S}(\ell|s) = \lambda_s(\ell). \tag{18}$$

Because no interference results when the interfering signal average power is zero, the associated initial condition is

$$F_{L|S}(0|s) = 0. \tag{19}$$

The resulting solution of the differential equation for $F_{L|S}(\ell|s)$ is given by

$$F_{L|S}(\ell|s) = 1 - e^{-\int_0^{\ell} \lambda_s(u)du} . \tag{20}$$

The complement of $F_{L|S}(\ell|s)$ is $C_{I|S}(\ell|s)$. Hence,

$$C_{I|S}(i|s) = 1 - F_{L|S}(i|s) = e^{-\int_0^{i} \lambda_s(u)du} . \tag{21}$$

Observe that knowledge of the EMI rate function $\lambda_s(i)$ is sufficient to completely determine the given signal and interference EMI and EMC probability functions.

The conditional PDF of L, $f_{L|S}(\ell|s)$, is also completely determined by the EMI rate function. Differentiation of (20) with respect to ℓ yields

$$f_{L|S}(\ell|s) = \frac{dF_{L|S}(\ell|s)}{d\ell} = \lambda_s(\ell)\, e^{-\int_0^{\ell} \lambda_s(u)du} \quad . \tag{22}$$

This result can also be derived by direct substitution of (21) into (17). Similarly, substitution of (21) into (5), (8) and (10) results in the expressions

$$C_S(s) = \int_{-\infty}^{\infty} e^{-\int_0^{i} \lambda_s(u)du} f_I(i)di, \tag{23}$$

$$C_I(i) = \int_{-\infty}^{\infty} e^{-\int_0^{i} \lambda_s(u)du} f_S(s)ds, \tag{24}$$

and

$$C = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\int_0^{i} \lambda_s(u)du} f_{S,I}(s,i)ds\ di. \tag{25}$$

Assuming statistical independence between S and I, it is readily seen from (21)-(25) that knowledge of $\lambda_s(i)$, $f_S(s)$, and $f_I(i)$ is sufficient to determine $f_{L|S}(\ell|s)$, $C_{I|S}(i|s)$, $C_S(s)$, $C_I(i)$, and C. Note that $\lambda_s(i)$ characterizes the equipment while $f_S(s)$ and $f_I(i)$ characterize the desired and interfering signals, respectively.

The concepts introduced above are now illustrated by two examples.

Example 1: Assume S and I are statistically independent uniformly distributed random variables with PDF's

$$f_S(s) = \begin{cases} \frac{1}{s_o} , & 0 \leq s \leq s_o \\ 0 , & \text{otherwise} \end{cases} \tag{26}$$

and

$$f_I(i) = \begin{cases} \frac{1}{i_o} , & 0 \leq i \leq i_o \\ 0 , & \text{otherwise.} \end{cases} \tag{27}$$

Also, assume the EMI rate function is given by

$$\lambda_s(i) = \lambda_o(1 - \frac{s}{s_o}). \tag{28}$$

Observe that the EMI rate function is a constant with respect to i but decreases linearly with respect to s. It follows that

$$\int_o^{\ell} \lambda_s(u)du = \lambda_o \ell(1 - \frac{s}{s_o}). \tag{29}$$

Substitution of (29) into (20) and (22) yields

$$F_{L|S}(\ell|s) = 1 - e^{-\lambda_o \ell(1 - \frac{s}{s_o})} \tag{30}$$

and

$$f_{L|S}(\ell|s) = \lambda_o(1 - \frac{s}{s_o})e^{-\lambda_o \ell(1 - \frac{s}{s_o})} . \tag{31}$$

The susceptibility level, L, of the equipment is seen to obey an exponential distribution. Observe that ℓ varies from 0 to ∞ even though the range over which $f_I(i)$ is nonzero extends from 0 to i_o. With reference to [illegible] and [illegible], the given signal and interference EMC probability function is expressed as

$$C_{I|S}(i|s) = e^{-\lambda_o i\left(1 - \frac{s}{s_o}\right)} \quad . \qquad \text{[illegible]}$$

A sketch of $C_{I|S}(i|s)$ is shown in Figure 9a. In the absence of interference,

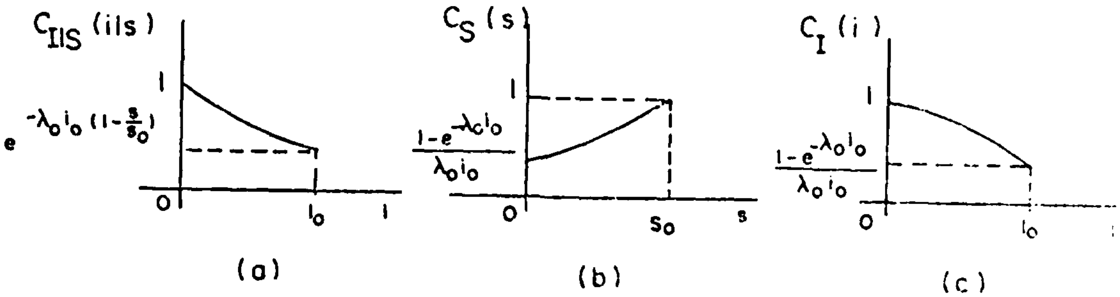

Figure 9 Sketches of a) $C_{I|S}(i|s)$, b) $C_S(s)$, and c) $C_I(i)$.

the probability of EMC should equal unity. Note that $C_{I|S}$[illegible] = 1, as anticipated. Also, reflecting the fact that the EMI rate function equals zero when $s = s_o$, $C_{I|S}(i|s_o)$ equals unity, independent of i. Given $s \neq s_o$, the prob[illegible] of EMC decreases for increasing values of the interference average power. [illegible] given signal EMC probability function is obtained by use of (27) and [illegible] in (23). Specifically,

$$C_S(s) = \int_0^{i_o} e^{-\lambda_o i(1 - \frac{s}{s_o})} \frac{1}{i_o}\, di$$

$$= \frac{1}{\lambda_o i_o (1 - \frac{s}{s_o})} [1 - e^{-\lambda_o i_o (1 - \frac{s}{s_o})}] \,. \tag{33}$$

$C_S(s)$ is sketched in Figure 9b. Because the EMI rate function decreases linearly with respect to s, $C_S(s)$ is seen to be an increasing function of s. When $s = s_o$, $C_S(s_o) = 1$, as was the case with $C_{I|S}(i|s)$. The given interference EMC probability function is obtained by use of (26) and (29) in (24). In particular,

$$C_I(i) = \int_0^{s_o} e^{-\lambda_o i(1 - \frac{s}{s_o})} \frac{1}{s_o}\, ds$$

$$= \frac{1}{\lambda_o i} [1 - e^{-\lambda_o i}]. \tag{34}$$

A sketch of $C_I(i)$ appears in Figure 9c. As would be expected, $C_I(i)$ is a decreasing function of i with $C_I(0) = 1$. By coincidence,

$$C_S(0) = C_I(i_o) = \frac{1 - e^{-\lambda_o i_o}}{\lambda_o i_o} \,. \tag{35}$$

Consequently, the EMC probability averaged over all possible values of I, given

$S = 0$, is identical to the EMC probability averaged over all possible values of S, given $I = i_o$. Observe that $S = 0$ and $I = i_o$ represent worst case situations with respect to the desired and interfering signal average powers, respectively. Finally, the EMC probability is obtained by use of (26), (27), (11), and (29) in (25). This yields

$$C = \int_o^{i_o}\int_o^{s_o} e^{-\lambda_o i(1 - \frac{s}{s_o})} \frac{1}{s_o} \frac{1}{i_o} \, ds \, di$$

$$= \frac{1}{i_o} \int_o^{i_o} \frac{1 - e^{-\lambda_o i}}{\lambda_o i} \, di = \frac{1}{i_o} \int_o^{i_o} C_I(i) di \tag{36}$$

$$= \frac{1}{s_o} \int_o^{s_o} \frac{1 - e^{-\lambda_o i_o (1 - \frac{s}{s_o})}}{\lambda_o i_o (1 - \frac{s}{s_o})} \, ds = \frac{1}{s_o} \int_o^{s_o} C_S(s) ds.$$

Unfortunately, a closed form expression is not possible for C. However, a value for C can be obtained using numerical integration. In fact, C is nothing more than $1/i_o$ times the area under $C_I(i)$ or, equivalently, $1/s_o$ times the area under $C_S(s)$.

Example 2: As in example 1, assume S and I are statistically independent uniformly distributed random variables with PDF's specified by (26) and (27). Now, however, let the EMI rate function increase linearly with respect to i and, as in example 1, decrease linearly with respect to s. Specifically, let

$$\lambda_s(i) = \lambda_o i(1 - \frac{s}{s_o}). \tag{37}$$

Observe that

$$\int_o^{\ell} \lambda_s(u) du = \frac{\lambda_o}{2} (1 - \frac{s}{s_o}) \ell^2. \tag{38}$$

Consequently, the CDF and PDF of L are given by

$$F_{L|S}(\ell|s) = 1 - e^{-\frac{\lambda_o}{2}(1 - \frac{s}{s_o})\ell^2} \tag{39}$$

and

$$f_{L|S}(\ell|s) = \lambda_o \ell (1 - \frac{s}{s_o}) e^{-\frac{\lambda_o}{2}(1 - \frac{s}{s_o})\ell^2} . \tag{40}$$

These are recognized as the CDF and PDF, respectively, of the Rayleigh distribution. Following the procedure of example 1, it can be shown that

$$C_{I|S}(i|s) = e^{-\frac{\lambda_o}{2}(1 - \frac{s}{s_o})i^2} , \tag{41}$$

$$C_S(s) = \frac{\sqrt{2\pi}}{i_o\sqrt{\lambda_o(1 - \frac{s}{s_o})}} [\mathrm{erf}\ (i_o\sqrt{\lambda_o(1 - \frac{s}{s_o})}\ ) - \frac{1}{2}\ ] \tag{42}$$

where

$$\mathrm{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\ dt, \tag{43}$$

and

$$C_I(i) = \frac{2}{\lambda_o i^2} [1 - e^{-\frac{\lambda_o}{2} i^2}]. \tag{44}$$

Once again, a closed form expression for C does not exist. As in example 1, the probability of EMC equals unity in the absence of interference and/or when $S = s_o$ corresponding to a zero EMI rate function. It follows that

$$C_{I|S}(0|s) = C_{I|S}(i|s_o) = C_S(s_o) = C_I(0) = 1, \qquad ([illegible])$$

as can be verified by examining (41) - (44).

Typically, the conditional CDF of the susceptibility level, $F_{L|S}(\ell|s)$, can be determined only by experiment. $f_{L|S}(\ell|s)$, the conditional PDF of L, is then found by numerical differentiation. However, to facilitate analysis, it is convenient to approximate the resulting PDF by a well known distribution. In many cases, analytical expressions can then be derived for $C_{I|S}(i|s)$ and $C_S(i)$. The given signal and interference EMC probability function and the EMI rate function are tabulated in Appendix A for some commonly used PDF's.

6. THE COMPATIBILITY MARGIN

As mentioned earlier, compatibility exists provided the average power of the interfering signal is less than the susceptibility level of the equipment involved. In some cases, the probability of EMC is more readily evaluated by introducing the random variable

$$Y = L - I. \tag{46}$$

In terms of Y, compatibility exists provided $Y \geq 0$. Y also gives the amount of average power by which the interfering signal can be increased before EMI occurs. For this reason, Y is referred to as the compatibility margin. In this section, expressions are derived for $C_S(s)$, $C_I(i)$, and C in terms of the compatibility margin.

The given signal and interference EMC probability function is defined in (3) to be

$$\begin{aligned} C_{I|S}(i|s) &= P[L > i|s] = 1 - F_{L|S}(i|s) \\ &= \int_i^\infty f_{L|S}(\ell|s)d\ell. \end{aligned} \tag{47}$$

$C_{I|S}(i|s)$ has the following interpretation. Let an equipment be selected at random from an ensemble of "identical" equipments. Apply to the equipment a known desired signal with average power $S = s$ and a known interfering signal with average power $I = i$. $C_{I|S}(i|s)$ equals the probability of EMC. Alternatively, assume the desired and interfering signals with average powers $S = s$ and $I = i$, respectively, are applied to the entire ensemble of "identical" equipments. $C_{I|S}(i|s)$ then equals the fraction of equipments that are likely to experience compatibility.

The given signal EMC probability function is defined in (5) to be

$$
\begin{aligned}
C_S(s) &= \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_I(i)di \\
&= \int_{-\infty}^{\infty} \left[\int_{i}^{\infty} f_{L|S}(\ell|s)d\ell \right] f_I(i)di \qquad (48) \\
&= \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L|S}(\ell|s)\, f_I(i)d\ell\ di.
\end{aligned}
$$

The corresponding region of integration is the shaded region, R_1, shown in Figure 10a. In R_1 note that $\ell \geq i$. Now introduce into (48) the change of variable

$$y = \ell - i. \qquad (49)$$

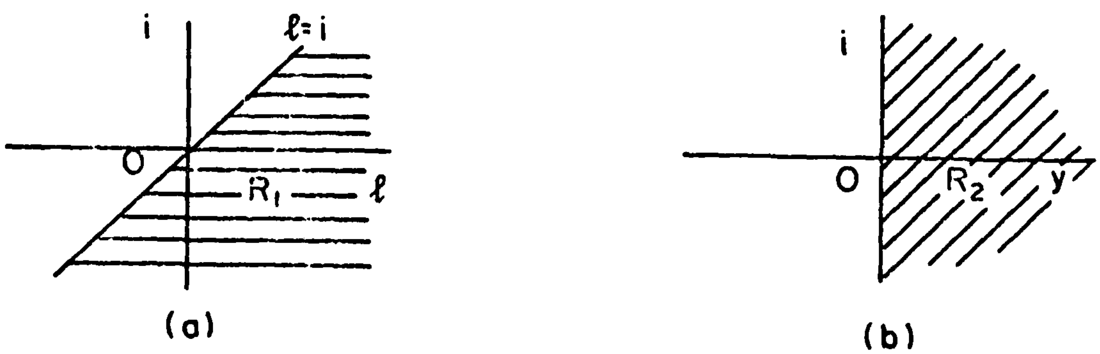

Figure 10 Regions of integration for $C_S(s)$.

The new region of integration, R_2, in the i-y plane is shown in Figure 10b. The boundary $\ell = i$ in Figure 10a corresponds to the boundary $y = 0$ in Figure 10b. Also, the points in R_1 for which $\ell \geq i$ correspond to those in R_2 for which $y \geq 0$. It follows that

$$C_S(s) = \int_0^{\infty} [\int_{-\infty}^{\infty} f_{L|S}(y + i|s)\ f_I(i)di]dy. \tag{50}$$

Assuming statistical independence between the desired and interfering signals, it is possible to write

$$f_{I|S}(i|s) = f_I(i). \tag{51}$$

The expression for $C_S(s)$ may then be written as

$$C_S(s) = \int_0^{\infty} [\int_{-\infty}^{\infty} f_{L|S}(y + i|s)\ f_{I|S}(i|s)di]dy. \tag{52}$$

Now consider the random variable $Y = L - I$. If L and I are statistically independent, it can be shown that the conditional density function of Y is given by [4]

$$f_{Y|S}(y|s) = \int_{-\infty}^{\infty} f_{L|S}(y + i|s)\ f_{I|S}(i|s)di. \tag{53}$$

Finally, substitution of (53) into (52) results in

$$C_S(s) = \int_0^{\infty} f_{Y|S}(y|s)dy. \tag{54}$$

As anticipated, the probability of EMC given S = s, equals the probability that the compatibility margin, Y, is greater than or equal to zero. $C_S(s)$ has the following interpretation. Let an equipment be selected at random from an ensemble of "identical" equipments. Apply to the equipment a known desired signal with average power S = s and an interfering signal with unknown average power. $C_S(s)$ equals the probability of EMC. Alternatively, assume the desired signal with average power S = s and the interfering signal with unknown average power are applied to the entire ensemble of "identical" equipments. $C_S(s)$ then equals, on the average, the fraction of equipments likely to experience compatibility when the experiment is repeated many times with the interfering signal chosen randomly on each repitition of the experiment.

Example 3: Let the conditional PDF of L be Gaussian with mean s and variance σ_L^2. In particular, assume

$$f_{L|S}(\ell|s) = \frac{1}{\sqrt{2\pi}\,\sigma_L} e^{-\frac{(\ell - s)^2}{2\sigma_L^2}} . \tag{55}$$

Also, let

$$f_I(i) = \frac{1}{\sqrt{2\pi}\,\sigma_I} e^{-\frac{(i - m_I)^2}{2\sigma_I^2}} . \tag{56}$$

Hence, the interference average power is also assumed to be Gaussian with mean m_I and variance σ_I^2. Since average powers are nonnegative, s and m_I are assumed to be sufficiently large such that (55) and (56) are negligibly small for negative values of ℓ and i, respectively. Direct use of (48) yields

$$C_S(s) = \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L|S}(\ell|s)\, f_I(i)\, d\ell\, di \tag{57}$$

$$= \frac{1}{2\pi\,\sigma_L\,\sigma_I} \int_{-\infty}^{\infty} \int_{i}^{\infty} e^{-\left[\frac{(\ell - s)^2}{2\sigma_L^2} + \frac{(i - m_I)^2}{2\sigma_I^2}\right]} d\ell\, di . \tag{58}$$

The above expression for $C_S(s)$ is not readily evaluated. However, assuming I to be statistically independent of both S and L, the given signal EMC probability function can be determined from (54) where $Y = L - I$ is the compatibility margin. Since L and I are statistically independent Gaussian random variables, it follows that Y is a Gaussian random variable with mean

$$m_Y = s - m_I \tag{59}$$

and variance

$$\sigma_Y^2 = \sigma_L^2 + \sigma_I^2 . \tag{60}$$

Therefore, the conditional PDF of Y is given by

$$f_{Y|S}(y|s) = \frac{1}{\sqrt{2\pi}\,\sigma_Y} e^{-\frac{(y - m_Y)^2}{2\sigma_Y^2}} . \tag{61}$$

Substitution of (61) into (54) results in

$$C_S(s) = \int_0^\infty f_{Y|S}(y|s)\,dy = \frac{1}{\sqrt{2\pi}\,\sigma_Y} \int_0^\infty e^{-\frac{(y - m_Y)^2}{2\sigma_Y^2}} dy . \tag{62}$$

Introducing the change of variables

$$z = \frac{y - m_Y}{\sigma_Y} , \tag{63}$$

(62) becomes

$$C_S(s) = \int_{-\frac{m_Y}{\sigma_Y}}^{\infty} e^{-\frac{z^2}{2}} dz = \int_{-\infty}^{\frac{m_Y}{\sigma_Y}} e^{-\frac{z^2}{2}} dz = \mathrm{erf}\left(\frac{m_Y}{\sigma_Y}\right) \tag{64}$$

where erf(x) is defined in (43).

The given interference EMC probability function is defined in (8) to be

$$C_I(i) = \int_{-\infty}^{\infty} C_{I|S}(i|s)\; f_S(s)ds$$

$$= \int_{-\infty}^{\infty} [\int_{i}^{\infty} f_{L|S}(\ell|s)d\ell]f_S(s)ds \qquad (65)$$

$$= \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L,S}(\ell,s)d\ell\; ds.$$

The corresponding region of integration is the shaded region, R_3, shown in Figure 11. Interchanging the order of integration,

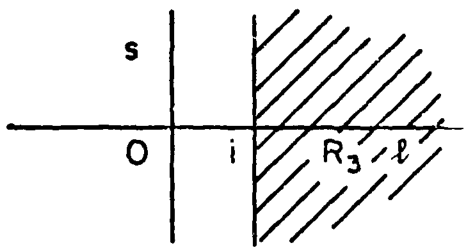

Figure 11 Region of integration for $C_1(i)$.

$C_I(i)$ becomes

$$C_I(i) = \int_{i}^{\infty} [\int_{-\infty}^{\infty} f_{L,S}(\ell,s)ds]d\ell. \qquad (66)$$

However, the marginal PDF of L is related to the joint PDF of I and S according to the relation

$$f_L(\ell) = \int_{-\infty}^{\infty} f_{L,S}(\ell,s)ds. \tag{67}$$

Substitution of (67) into (66) yields

$$C_I(i) = \int_{i}^{\infty} f_L(\ell)d\ell \tag{68}$$

where i is the known average power of the interfering signal. Now consider the random variable

$$Y = L - i \tag{69}$$

where i is recognized to be a known constant. The conditional PDF of Y is related to the marginal PDF of L as follows:

$$f_{Y|I}(y|i)dy = f_L(\ell)d\ell. \tag{70}$$

Hence, the given interference EMC probability function can be expressed as

$$C_I(i) = \int_{0}^{\infty} f_{Y|I}(y|i)dy. \tag{71}$$

Analogous to (54), the probability of EMC, given I = i, equals the probability that the compatibility margin, Y, is greater than or equal to zero. $C_I(i)$ has the following interpretation. Let an equipment be selected at random from an ensemble of "identical" equipments. Apply to the equipment a known interfering signal with average power I = i and a desired signal with unknown average power.

$C_I(i)$ equals the probability of EMC. Alternatively, assume the interfering signal with average power $I = i$ and the desired signal with unknown average power are applied to the entire ensemble of "identical" equipments. $C_I(i)$ then equals, on the average, the fraction of equipments likely to experience compatibility when the experiment is repeated many times with the desired signal [illegible] randomly on each repitition of the experiment.

Finally, assume I to be statistically independent of both S and L. The EMC probability is defined in (12) to be

$$
\begin{aligned}
C &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_S(s)\, f_I(i)\, ds\, di \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Big[\int_{i}^{\infty} f_{L|S}(\ell|s)\, d\ell\Big] f_S(s)\, f_I(i)\, ds\, di \qquad (72) \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{i}^{\infty} f_{L,S}(\ell,s)\, f_I(i)\, d\ell\, ds\, di.
\end{aligned}
$$

Integrating first with respect to s, C is given by

$$
C = \int_{-\infty}^{\infty} \int_{i}^{\infty} \Big[\int_{-\infty}^{\infty} f_{L,S}(\ell,s)\, ds\Big] f_I(i)\, d\ell\, di. \qquad (73)
$$

Substitution of (67) into (73) yields

$$
C = \int_{-\infty}^{\infty} \int_{i}^{\infty} f_L(\ell)\, f_I(i)\, d\ell\, di. \qquad (74)
$$

Observe the simlarity between (74) and the last equation in (48). The only difference is that the conditional PDF of L appears in (48) whereas the marginal

PDF of L appears in (74). Applying the same procedure used to convert (48) into (54), it follows that

$$C = \int_0^\infty f_Y(y)dy \tag{75}$$

where Y = L - I. Once again, the probability of EMC equals the probability that the compatibility margin, Y, is greater than or equal to zero. Note that $C_S(s)$ in (54), $C_I(i)$ in (71), and C in (75) use the PDF's $f_{Y|S}(y|s)$, $f_{Y|I}(y|i)$, and $f_Y(y)$, respectively. C has the following interpretation. Let an equipment be selected at random from an ensemble of "identical" equipments. Apply to the equipment desired and interfering signals whose average powers are unknown. C equals the probability of EMC. Alternatively, assume the desired and interfering signals whose average powers are unknown are applied to the entire ensemble of "identical" equipments. C then equals, on the average, the fraction of equipments likely to experience compatibility when the experiment is repeated many times with the desired and interfering signals chosen randomly on each repetition of the experiment.

Example 4: As in example 3, all random variables are assumed to be Gaussian. In particular, the PDF's of $f_{L|S}(\ell|s)$, $f_S(s)$, and $f_I(i)$ are approximated by

$$f_{L|S}(\ell|s) = \frac{1}{\sqrt{2\pi}\,\sigma_L} e^{-\frac{(\ell - s)^2}{2\sigma_L^2}}, \tag{76}$$

$$f_S(s) = \frac{1}{\sqrt{2\pi}\,\sigma_S} e^{-\frac{(s - m_S)^2}{2\sigma_S^2}}, \tag{77}$$

and

$$f_I(i) = \frac{1}{\sqrt{2\pi}\,\sigma_I}\, e^{-\frac{(i - m_I)^2}{2\sigma_I^2}} . \tag{78}$$

Given (76), (77), and (78), it is desired to evaluate $C_I(i)$ and C using (71) and (75), respectively.

The first step is to determine the marginal PDF of L. With reference to (67),

$$f_L(\ell) = \int_{-\infty}^{\infty} f_{L,S}(\ell,s)\,ds. \tag{79}$$

However,

$$f_{L,S}(\ell,s) = f_{L|S}(\ell|s)\; f_S(s)$$

$$= \frac{1}{2\pi\,\sigma_L\,\sigma_S}\, e^{-\frac{1}{2}\left[\frac{(\ell - s)^2}{\sigma_L^2} + \frac{(s - m_S)^2}{\sigma_S^2}\right]} . \tag{80}$$

By completing the square in the exponent with respect to s, it can be shown that

$$f_{L,S}(\ell,s) = \frac{1}{2\pi\,\sigma_L\,\sigma_S}\, e^{-\frac{1}{2}\left[\frac{\sigma_S^2 + \sigma_L^2}{\sigma_L^2\,\sigma_S^2}\right]\left[s - \frac{\ell\sigma_S^2 + m_S\,\sigma_L^2}{\sigma_S^2 + \sigma_L^2}\right]^2}$$

$$\cdot\, e^{-\frac{1}{2}\,\frac{(\ell - m_S)^2}{\sigma_S^2 + \sigma_L^2}} . \tag{81}$$

Substitution of (81) into (79) results in

$$f_L(\ell) = \frac{e^{-\frac{1}{2}\frac{(\ell - m_S)^2}{\sigma_S^2 + \sigma_L^2}}}{2\pi\ \sigma_L\ \sigma_S} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left[\frac{\sigma_S^2 + \sigma_L^2}{\sigma_L^2\ \sigma_S^2}\right]\left[s - \frac{\ell\sigma_S^2 + m_S\ \sigma_L^2}{\sigma_S^2 + \sigma_L^2}\right]^2} ds$$

$$= \frac{e^{-\frac{1}{2}\frac{(\ell - m_S)^2}{\sigma_S^2 + \sigma_L^2}}}{2\pi\ \sigma_L\ \sigma_S} \sqrt{2\pi}\ \frac{\sigma_L\ \sigma_S}{\sqrt{\sigma_S^2 + \sigma_L^2}} \tag{82}$$

$$= \frac{1}{\sqrt{2\pi}\ \sqrt{\sigma_S^2 + \sigma_L^2}}\ e^{-\frac{1}{2}\frac{(\ell - m_S)^2}{\sigma_S^2 + \sigma_L^2}}\ .$$

Therefore, L is a Gaussian random variable with mean m_S and variance $(\sigma_S^2 + \sigma_L^2)$. Since average powers are nonnegative, m_S and m_I are assumed to be sufficiently large such that $f_S(s)$, $f_I(i)$, $f_{L,S}(\ell,s)$, and $f_L(\ell)$ are negligibly small for negative values of their arguments.

To evaluate $C_I(i)$ by means of (71), it is necessary to determine $f_{Y|I}(y|i)$ where $Y = L - i$ and i is a known constant. Since L is Gaussian, the conditional PDF of Y is also Gaussian with mean $(m_S - i)$ and variance $(\sigma_S^2 + \sigma_L^2)$.

Specifically,

$$f_{Y|I}(y|i) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_S^2 + \sigma_L^2}} e^{-\frac{1}{2}\frac{[y-(m_S-i)]^2}{\sigma_S^2+\sigma_L^2}} . \tag{83}$$

Applying the same procedure used to evaluate (62), it follows that

$$C_I(i) = \int_0^\infty f_{Y|I}(y|i)dy = \mathrm{erf}\left(\frac{m_S - i}{\sqrt{\sigma_S^2 + \sigma_L^2}}\right) \tag{84}$$

where erf(x) is defined in (43).

Finally, to evaluate C by means of (75), it is necessary to determine $f_Y(y)$ where $Y = L - I$. Let I be statistically independent of L. Since L and I are both Gaussian, the marginal PDF of Y is also Gaussian with mean $(m_S - m_I)$ and variance $(\sigma_S^2 + \sigma_L^2 + \sigma_I^2)$. In particular ,

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_S^2 + \sigma_L^2 + \sigma_I^2}} e^{-\frac{1}{2}\frac{[y-(m_S-m_I)]^2}{\sigma_S^2+\sigma_L^2+\sigma_I^2}} . \tag{85}$$

Applying the same procedure used to evaluate (62) and (84), it follows that

$$C = \int_0^\infty f_Y(y)dy = \mathrm{erf}\left(\frac{m_S - m_I}{\sqrt{\sigma_S^2 + \sigma_L^2 + \sigma_I^2}}\right) . \tag{86}$$

It is of interest to compare the expression for $C_S(s)$ in example 3 with those obtained for $C_I(i)$ and C in example 4. In each case, the expressions involve identical functions of the means and variances of $f_{Y|S}(y|s)$, $f_{Y|I}(y|i)$, and $f_Y(y)$, respectively.

It is not always possible to obtain analytical expressions for $C_S(s)$, $C_I(i)$, and C, even when well known PDF's are specified for $f_{L|S}(\ell|s)$, $f_S(s)$, and $f_I(i)$. Numerical integration techniques must then be used to evaluate the integrals in (5), (8), and (12) or, equivalently, in (54), (71), and (75). To illustrate some cases where analytical expressions are possible,

$$\begin{aligned} C_S(s) &= \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_I(i)di \\ &= \int_0^{\infty} f_{Y|S}(y|s)dy \end{aligned} \tag{87}$$

is evaluated in Appendix B for some well known PDF's of $f_{L|S}(\ell|s)$ and $f_I(i)$. Consistent with the property that the susceptibility level of an equipment is likely to increase as the desired signal average power is increased, the conditional PDF's of L are specified in Appendix B such that larger susceptibility levels are more probable with larger values of s. This results in $C_S(s)$ being a nondecreasing function of s.

Knowledge of $f_{Y|S}(y|s)$, $f_{Y|I}(y|i)$, and $f_Y(y)$ can provide additional insight into $C_S(s)$, $C_I(i)$, and C, respectively. When certain symmetries in the underlying PDF's exist, general conclusions can be derived concerning the probability of EMC. For example, assume $f_{L|S}(\ell|s)$ and $f_I(i)$ are both symmetrical functions about the same mean m. Therefore, for all x,

$$f_{L|S}(m + x|s) = f_{L|S}(m - x|s) \tag{88}$$

and

$$f_I(m + x) = f_I(m - x) \ . \tag{89}$$

From (53) the conditional PDF of Y = L - I is given by

$$\begin{aligned} f_{Y|S}(y|s) &= \int_{-\infty}^{\infty} f_{L|S}(y + i|s)\ f_{I|S}(i|s)di \\ &= \int_{-\infty}^{\infty} f_{L|S}(y + i|s)\ f_I(i)di \end{aligned} \tag{90}$$

where $f_{I|S}(i|s)$ has been replaced with $f_I(i)$ by assuming statistical independence between S and I. Observe that

$$f_{Y|S}(-y|s) = \int_{-\infty}^{\infty} f_{L|S}(-y + i|s)\ f_I(i)di. \tag{91}$$

Introduction into (91) of the change of variable

$$i = m + x \tag{92}$$

and utilization of the symmetry properties stated in (88) and (89) results in

$$\begin{aligned} f_{Y|S}(-y|s) &= \int_{-\infty}^{\infty} f_{L|S}(m + x - y|s)\ f_I(m + x)dx \\ &= \int_{-\infty}^{\infty} f_{L|S}(m - x + y|s)\ f_I(m - x)dx. \end{aligned} \tag{93}$$

Finally, with the change of variable

$$w = m - x, \tag{94}$$

(93) becomes

$$\begin{aligned} f_{Y|S}(-y|s) &= \int_{-\infty}^{\infty} f_{L|S}(y + w|s)\ f_I(w)dw \\ &= f_{Y|S}(y|s). \end{aligned} \tag{95}$$

Hence, $f_{Y|S}(y|s)$ is an even function of y. It follows that

$$C_S(s) = \int_0^{\infty} f_{Y|S}(y|s)\ dy = \frac{1}{2} \tag{96}$$

independent of the particular PDF's for $f_{L|S}(\ell|s)$ and $f_I(i)$ as long as the conditions in (88) and (89) are satisfied. Similarly, it can be shown that $C_I(i) = 1/2$ provided $f_L(\ell)$ is a symmetric function about the mean i. Also, $C = 1/2$ when $f_L(\ell)$ and $f_I(i)$ are both symmetrical functions about the same mean m.

7. MULTIPLE PERFORMANCE THRESHOLDS

Thus far, discussion has been restricted to the special case where the EMI performance curve is subdivided into two performance categories by a single performance threshold, as shown in Figure 1. In certain applications it may be desirable to subdivide the EMI performance curve into three or more performance categories. For example, it may be useful to categorize performance as either 1) acceptable, 2) marginal, or 3) unacceptable, as illustrated in Figure 12. The EMC performance threshold is that value of the performance

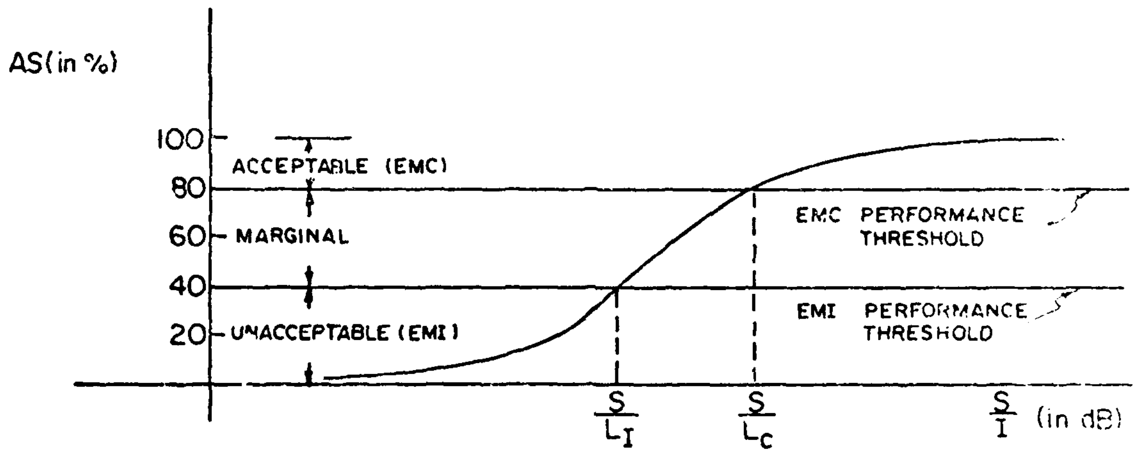

Figure 12 EMI performance curve of a speech communication system subdivided into three performance categories.

criterion which demarcates the regions of acceptable and marginal performance. Similarly, the EMI performance threshold is that value of the performance criterion which demarcates the regions of marginal and unacceptable performance. As in the single threshold case, EMC and EMI are said to exist when the performance is acceptable and unacceptable, respectively. By definition, the EMC susceptibility level, L_C, is the level of the interfering signal that results, for a specified desired signal, in operation at the EMC performance threshold. In addition, the EMI susceptibility level, L_I, is defined to be the level of the interfering signal that results, for a specified desired signal, in operation at the EMI performance threshold.

In the probabilistic approach, the susceptibility levels, L_C and L_I, are treated as random variables whose behaviors are governed by the conditional CDF's

$$F_{L_C|S}(\ell_C|s) = P[L_C \leq \ell_C|s] \tag{97}$$

and

$$F_{L_I|S}(\ell_I|s) = P[L_I \leq \ell_I|s]. \tag{98}$$

The corresponding PDF's are

$$f_{L_C|S}(\ell_C|s) = \frac{dF_{L_C|S}(\ell_C|s)}{d\ell_C} \tag{99}$$

and

$$f_{L_I|S}(\ell_I|s) = \frac{dF_{L_I|S}(\ell_I|s)}{d\ell_I}. \tag{100}$$

In the ensuing discussion, it is assumed that performance is degraded as the average power in the interfering signal is increased. It follows that L_C and L_I obey the inequality

$$L_C < L_I. \tag{101}$$

EMC exists when the susceptibility level, L_C, is greater than the average power of the interfering signal. Given S = s and I = i, the conditional probability of EMC is given by

$$\begin{aligned} C_{I|S}(i|s) &= P[L_C > i,\ L_I > i|s] \\ &= P[L_I > i|(L_C > i),s]\ \ P[L_C > i|s]. \end{aligned} \tag{102}$$

Because of the inequality in (101),

$$P[L_I > i|(L_C > i),\ s] = 1. \tag{103}$$

Therefore, (102) simplifies to

$$\begin{aligned} C_{I|S}(i|s) &= P[L_C > i|s] = 1 - F_{L_C|S}(i|s) \\ &= \int_i^{\infty} f_{L_C|S}(\ell_C|s)d\ell_C. \end{aligned} \tag{104}$$

$C_{I|S}(i|s)$ is known as the <u>given signal and interference EMC probability function</u>.

EMI exists when the susceptibility level, L_I, is less than the average power of the interfering signal. Given S = s and I = i, the conditional probability of EMI is given by

$$\begin{aligned} U_{I|S}(i|s) &= P[L_I \leq i,\ L_C \leq i|s] \\ &= P[L_C \leq i|(L_I \leq i),s]\ \ P[L_I \leq i|s]. \end{aligned} \tag{105}$$

Because of the inequality in (101),

$$P[L_C \le i \mid (L_I \le i), s] = 1. \tag{106}$$

Therefore, (105) simplifies to

$$\begin{aligned} U_{I|S}(i|s) &= P[L_I \le i|s] = F_{L_I|S}(\ell_I|s) \\ &= \int_{-\infty}^{i} f_{L_I|S}(\ell_I|s)\,d\ell_I. \end{aligned} \tag{107}$$

$U_{I|S}(i|s)$ is known as the <u>given signal and interference EMI probability function</u>.

Marginal performance exists when the average power of the interfering signal is greater than or equal to the EMC susceptibility level, L_C, but is less than the EMI susceptibility level, L_I. Given S = s and I = i, the conditional probability of marginal performance is given by

$$M_{I|S}(i|s) = P[L_C \le i,\ L_I > i|s]. \tag{108}$$

However, performance is either acceptable, unacceptable, or marginal. Since these are disjoint events,

$$C_{I|S}(i|s) + U_{I|S}(i|s) + M_{I|S}(i|s) = 1. \tag{109}$$

Solving for $M_{I|S}(i|s)$ and utilizing (104) and (107), it follows that

$$\begin{aligned} M_{I|S}(i|s) &= 1 - C_{I|S}(i|s) - U_{I|S}(i|s) \\ &= 1 - (1 - F_{L_C|S}(i|s)) - F_{L_I|S}(i|s) \\ &= F_{L_C|S}(i|s) - F_{L_I|S}(i|s) \\ &= \int_{-\infty}^{i} [f_{L_C|S}(\ell|s) - f_{L_I|S}(\ell|s)]\,d\ell. \end{aligned} \tag{110}$$

$M_{I|S}(i|s)$ is known as the given signal and interference marginal performance probability function.

The given signal and interference EMC, EMI, and marginal performance probability functions are applicable only if the average powers of the desired and interfering signals are both known. When S = s is known but I is unknown, the appropriate probabilities are obtained by averaging over the unknown average power of the interfering signal. Let $f_I(i)$ denote the PDF of I. Then

$$C_S(s) = \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_I(i)\,di$$

$$= \int_{-\infty}^{\infty}\int_{i}^{\infty} f_{L_C|S}(\ell_C|s)\, f_I(i)\,d\ell_C\, di \qquad (111)$$

= given signal EMC probability function,

$$U_S(s) = \int_{-\infty}^{\infty} U_{I|S}(i|s)\, f_I(i)\,di$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I|S}(\ell_I|s)\, f_I(i)\,d\ell_I\, di \qquad (112)$$

= given signal EMI probability function.

and

$$M_S(s) = \int_{-\infty}^{\infty} M_{I|S}(i|s)\, f_I(i)\,di$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{i} [f_{L_C|S}(\ell|s) - f_{L_I|S}(\ell|s)]\, f_I(i)\,d\ell\, di \qquad (113)$$

= given signal marginal performance probability function.

When $I = i$ is known but S is unknown, the appropriate probabilites are obtained by averaging over the unknown average power of the desired signal. Let $f_S(s)$ denote the PDF of S. Then

$$
\begin{aligned}
C_I(i) &= \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_S(s)ds \\
&= \int_{-\infty}^{\infty}\int_{i}^{\infty} f_{L_C|S}(\ell_C|s)\, f_S(s)d\ell_C\, ds \\
&= \int_{-\infty}^{\infty}\int_{i}^{\infty} f_{L_C,S}(\ell_C,s)d\ell_C\, ds
\end{aligned}
\qquad \text{([illegible])}
$$

= given interference EMC probability function,

$$
\begin{aligned}
U_I(i) &= \int_{-\infty}^{\infty} U_{I|S}(i|s)\, f_S(s)ds \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I|S}(\ell_I|s)\, f_S(s)d\ell_I\, ds \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I,S}(\ell_I,s)d\ell_I\, ds
\end{aligned}
\qquad \text{([illegible])}
$$

= given interference EMI probability function,

and

$$
\begin{aligned}
M_I(i) &= \int_{-\infty}^{\infty} M_{I|S}(i|s)\, f_S(s)ds \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{i} [f_{L_C|S}(\ell|s) - f_{L_I|S}(\ell|s)]f_S(s)d\ell\, ds \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_C,S}(\ell_C,s)d\ell_C\, ds - \int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I,S}(\ell_I,s)d\ell_I\, ds
\end{aligned}
\qquad \text{([illegible])}
$$

= given interference marginal performance probability function.

Finally, when both S and I are unknown, the appropriate probabilities are obtained by averaging over both unknown average powers. Assuming statistical independence between S and I, there results

$$
\begin{aligned}
C &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_S(s)\, f_I(i)\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{i}^{\infty} f_{L_C|S}(\ell_C|s)\, f_S(s)\, f_I(i)\,d\ell_C\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{i}^{\infty} f_{L_C,S}(\ell_C,s)\, f_I(i)\,d\ell_C\,ds\,di \\
&= \underline{\text{EMC probability}},
\end{aligned}
\tag{117}
$$

$$
\begin{aligned}
U &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{I|S}(i|s)\, f_S(s)\, f_I(i)\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I|S}(\ell_I|s)\, f_S(s)\, f_I(i)\,d\ell_I\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I,S}(\ell_I,s)\, f_I(i)\,d\ell_I\,ds\,di \\
&= \underline{\text{EMI probability}},
\end{aligned}
\tag{118}
$$

and

$$
\begin{aligned}
M &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{I|S}(i|s)\, f_S(s)\, f_I(i)\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{i} [f_{L_C|S}(\ell|s) - f_{L_I|S}(\ell|s)]\, f_S(s)\, f_I(i)\,d\ell\,ds\,di \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_C,S}(\ell_C,s)\, f_I(i)\,d\ell_C\,ds\,di - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{i} f_{L_I,S}(\ell_I,s)\, f_I(i)\,d\ell_I\,ds\,di \\
&= \underline{\text{marginal performance probability}}.
\end{aligned}
\tag{119}
$$

Comparison of (111)-(119) with the corresponding expressions previously developed for the single performance threshold case reveals the two sets of expressions to be quite similar. It can be shown that this similarity extends to the general case for which (n - 1) performance thresholds are used to subdivide system performance into n categories.

8. SAMPLE SIZE CONSIDERATIONS IN THE EXPERIMENTAL DETERMINATION OF $F_{L|S}(\ell|s)$

Consider, once again, an EMI performance curve with a single performance threshold such as appears in Figure 3. In the probabilistic approach, the EMI performance curve for an equipment is characterized in terms of the random susceptibility level, L. The random behavior of L is governed by its conditional CDF, $F_{L|S}(\ell|s)$, which is defined in (1). An experimental procedure for determining $F_{L|S}(\ell|s)$ through laboratory measurements was described in Section 3. It was pointed out, for this purpose, that it is necessary to have a large number, N, of "identical" equipments all exposed to the same desired and interfering signals. In this section, a method proposed by Kolmogorov and Smirnov [5-7] is presented for determining a suitably large value of N.

Denote the empirically obtained conditional CDF of L by $E^N_{L|S}(\ell|s)$. According to Kolmogorov and Smirnov, as long as $F_{L|S}(\ell|s)$ is a continuous CDF, then

$$P[E^N_{L|S}(\ell|s) - \delta < F_{L|S}(\ell|s) < E^N_{L|S}(\ell|s) + \delta] = \gamma \tag{120}$$

provided

$$N \geq \left(\frac{a}{\delta}\right)^2 \tag{121}$$

where $\delta \leq 0.2$ and a is determined from Table 1 [8].

TABLE 1

PARAMETERS USED IN KOLMOGOROV-SMIRNOV TEST

γ	0.80	0.85	0.90	0.95	0.99
a	1.07	1.14	1.22	1.36	1.63

Equation (120) means that, with probability γ, the inequality $|F_{L|S}(\ell|s) - E^N_{L|S}(\ell|s)| < \delta$ is satisfied for all ℓ in the interval $(-\infty, +\infty)$. In other words, whatever the CDF $F_{L|S}(\ell|s)$, so long as it is continuous, the random domain between the boundaries $E^N_{L|S}(\ell|s) - \delta$ and $E^N_{L|S}(\ell|s) + \delta$ contains $F_{L|S}(\ell|s)$ completely, with probability γ. Note that the domain quoted depends on the empirical distribution $E^N_{L|S}(\ell|s)$ obtained using the particular sample of N equipments. Should the experiment be repeated using a different sample of N equipments, $E^N_{L|S}(\ell|s)$ is likely to change causing the domain of width 2δ to be centered about a different function.

In the statistical literature, the interval $[E^N_{L|S}(\ell|s) - \delta, E^N_{L|S}(\ell|s) + \delta]$ is called the <u>confidence interval</u> for estimating $F_{L|S}(\ell|s)$. The boundaries $E^N_{L|S}(\ell|s) - \delta$ and $E^N_{L|S}(\ell|s) + \delta$ are called <u>confidence limits</u> for $F_{L|S}(\ell|s)$. The probability γ is called the <u>confidence coefficient</u>. Multiplying γ by 100%, 100γ% is called the <u>confidence level</u>. Thus, (120) specifies a confidence domain for an unknown cumulative distribution function.

<u>Example 5</u>: Let $\delta = 0.15$ and $\gamma = 0.90$. What should be the sample size, N, of the experiment?

From Table 1, a = 1.22. Substituting into (121), there results

$$N \geq \left(\frac{1.22}{0.15}\right)^2 = 66.15. \tag{122}$$

Hence, 67 equipments are needed if, with probability 0.90, $F_{L|S}(\ell|s)$ is to be completely contained within the confidence interval of width 0.3 centered on the empirical CDF.

<u>Example 6</u>: Let $\delta = 0.1$ and $\gamma = 0.975$. What should be the sample size, N, of the experiment?

Note that $\gamma = 0.975$ does not appear in Table 1. Consequently, it is necessary to interpolate between the tabulated values. Using linear interpolation,

$$\frac{0.975 - 0.95}{a - 1.36} = \frac{0.99 - 0.95}{1.63 - 1.36} . \tag{123}$$

Solution of (123) for a yields

$$a = 1.36 + \frac{(0.025)(0.27)}{(0.04)} = 1.53 . \tag{124}$$

Using (121), the inequality on N becomes

$$N \geq \left(\frac{1.53}{0.1}\right)^2 = 234. \tag{125}$$

Therefore, 234 equipments are needed if, with probability 0.975, $F_{L|S}(\cdot|s)$ is to be completely contained within the confidence interval of width 0.2 centered on the empirical CDF.

9. APPROXIMATION OF $F_{L|S}(\ell|s)$ BY A WELL KNOWN DISTRIBUTION

When $F_{L|S}(\ell|s)$ is determined in the laboratory, it is convenient, for analytical reasons, to approximate the experimentally obtained CDF by a well known distribution. In general, several different distributions may provide acceptable approximations to the experimental data. This section presents some statistical techniques for rejecting candidate distributions which are not supported by the data.

The procedure consists of two steps. First, two measures which contain information relative to the shape of a distribution, the coefficients of skewness and kurtosis, are used to make preliminary selections of candidate distributions. The candidates are then tested to see whether any should be rejected on the basis of a significant statistical deviation between the experimental data and the distribution being tested. The approximation to $F_{L|S}(\ell|s)$ is chosen from those distributions which passed the test. However, it is not possible to state whether any one distribution which passed the test is better than any other.

The coefficients of skewness and kurtosis are related to the kth central moment of a random variable where k = 1,2,3,· · ·. Let the mean of the random variable L be denoted by m_L. The k^{th} central moment of L is defined to be

$$\mu_k = E[(L - m_L)^k] \qquad (126)$$

where E[ ] denotes the statistical operation of expected value. By definition,

$$\alpha_3 = \frac{\mu_3}{\mu_2^{3/2}} = \text{coefficient of skewness} \qquad (127)$$

and

$$\alpha_4 = \frac{\mu_4}{\mu_2^{\,2}} = \underline{\text{coefficient of kurtosis}}. \qquad (12c)$$

The coefficient of skewness is a measure of the asymmetry of the PDF. The coefficient of kurtosis is a measure of the "peakedness" or "flatness" of a PDF in the central part of the distribution.

When experimentally determining $F_{L|S}(\ell|s)$, it is assumed that each of N "identical" equipments is exposed to the same desired and interfering signals. As explained in Section 3, the experiment proceeds by 1) holding the desired signal average power at the constant value S = s, 2) gradually increasing the average power of the interfering signal, and 3) recording the value of $N_{EMI}(\ell)$ where $N_{EMI}(\ell)$ denotes the number of equipments experiencing EMI with the average power of the interfering signal equal to ℓ. A typical result of the experiment is shown in Figure 4. Let ℓ_i denote the value of the interference average power at which the i^{th} equipment reaches its performance threshold. An unbiased estimator of the mean m_L is given by

$$\overline{m}_L = \frac{1}{N}\sum_{i=1}^{N} \ell_i. \qquad (13a)$$

Similarly, unbiased estimators of μ_2, μ_3, and μ_4 are

$$\overline{\mu}_2 = \frac{1}{N-1}\sum_{i=1}^{N} (\ell_i - \overline{m}_L)^2, \qquad (13b)$$

$$\overline{\mu}_3 = \frac{N}{(N-1)(N-2)}\sum_{i=1}^{N} (\ell_i - \overline{m}_L)^3, \qquad (13c)$$

and

$$\bar{\mu}_4 = \frac{N^2}{(N-1)(N^2-3N+3)} \sum_{i=1}^{N} (\ell_i - \bar{m}_L)^4 - \frac{6N-9}{N^2-3N+3} (\bar{\mu}_2)^2 . \tag{132}$$

Using (130)-(132) in (127) and (128), estimates of the coefficients of skewness and kurtosis become

$$\bar{\alpha}_3 = \frac{\bar{\mu}_3}{(\bar{\mu}_2)^{3/2}} \tag{133}$$

and

$$\bar{\alpha}_4 = \frac{\bar{\mu}_4}{(\bar{\mu}_2)^2} . \tag{134}$$

Preliminary selections of some well known distributions for approximating $F_{L|S}(\ell|s)$ are made by comparing the estimates, $\bar{\alpha}_3$ and $\bar{\alpha}_4$, with the known values, α_3 and α_4, of the various distributions under consideration. Those distributions for which a reasonable match exists become candidates for further consideration. In general, the PDF's of the well known distributions contain two or more parameters for which numerical values must be determined in order for the distribution to be completely specified. These parameters are assumed to be chosen such that the mean and variance of the distribution equal, respectively, the sample mean, $\bar{m}_L$, and the sample variance, $(\bar{\sigma}_L)^2 = \bar{\mu}_2$. Therefore, with respect to the mean and variance, there is no loss in generality by considering the standardized random variable

$$X = \frac{L - m_L}{\sigma_L} \tag{135}$$

where the variance of L is denoted by $\sigma_L^2 = \mu_2$. The standardized random variable, X, has zero mean and unit variance. It can be shown that the coefficients of skewness and kurtosis are invariant under standardization of the random variable. In other words, the distributions for L and X, respectively, have the same numerical values of α_3 and α_4. For the purpose of comparing the shapes of the various distributions, it is convenient to work with the standardized random variables. Analytical expressions for PDF's whose random variables have been standardized to zero mean and unit variance are tabulated in Appendix C along with their coefficients of skewness and kurtosis.

Having selected a group of possible distributions for approximating $F_{L|S}(\ell|s)$ on the basis of comparing $\bar{\alpha}_3$ and $\bar{\alpha}_4$ with α_3 and α_4, respectively, it is then desirable to perform a "goodness of fit" test on each candidate distribution. This enables a distribution to be rejected when a significant statistical deviation exists between the experimental data and the distribution being tested. Even though a distribution is not rejected, it cannot be accepted with confidence. In addition, if several distributions are not rejected, the goodness of fit test cannot be used to accept one distribution over another. The best that can be said regarding distributions which pass the test is that they were not rejected. The goodness of fit test prevents acceptance of a distribution which is likely to make a poor approximation to $F_{L|S}(\ell|s)$.

N, the number of "identical" equipments employed in the experiment, determines which goodness of fit test to use. For small values of N (i.e., $N \leq 30$), the Kolmogorov-Smirnov test is suggested. On the other hand, the χ^2 test is preferred when N is large (i.e., $N > 30$).

Case 1 - Small N ($N \leq 30$): Once again, it is convenient to work with standardized distributions having zero mean and unit variance. With respect to the data, the sample mean, $\bar{m}_L$, is given by (129) while the sample variance, $(\bar{\sigma}_L)^2 = \bar{\mu}_2$, is given by (130). The experimental data is standardized according to the relation

$$x_i = \frac{\ell_i - \bar{m}_L}{\bar{\sigma}_L} \quad ; \quad i = 1, 2, \ldots, N. \tag{136}$$

Denote the CDF of the standardized distribution to be tested by $F_X(x)$. Analytical expressions for various standardized CDF's are tabulated in Appendix D.

The Kolmogorov-Smirnov test is presented here without proof. It proceeds as follows:

1) Standardize the experimental data by utilizing (136).

2) Obtain ananalytical expression $F_X(x)$ for the distribution under test, (Appendix D may be of some value in this regard.)

3) For each experimental data point, evaluate $F_X(x_i)$.

4) Construct an experimental standardized CDF by using the relation

$$P(X_E \leq x_i) = \frac{N_{EMI}(\ell_{i_+})}{N} \tag{137}$$

where $N_{EMI}(\ell_{i_+})$ denotes the number of equipments experiencing EMI with the average power of the interfering signal equal to a value just slightly larger than ℓ_i. The random variable X_E denotes the experimental data. The experimental CDF has the shape of a staircase function such as shown in Figure 4.

5) Find the maximum distance given by

$$D_{max} = \max_i \left| F_X(x_i) - \frac{N_{EMI}(\ell_{i_+})}{N} \right|. \tag{138}$$

6) Assign a numerical value to the conditional probability which is defined to be

$$\alpha = P\,(\text{rejecting distribution} \mid \text{distribution is a suitable approximation}). \qquad ([illegible])$$

This probability is known as the <u>significance level</u> of the test.

7) Refer to the Table in Appendix E. Find the tabulated value corresponding to the applicable sample size, N, and significance level, α. Denote the tabulated value by D_N^α.

8) Reject the distribution if $D_{max} > D_N^\alpha$.

<u>Example 7</u>: Assume 10 "identical" equipments are used in the experiment for determining $F_{L|S}(\ell|s)$. The results of the experiment are summarized in Table A. The sample mean is given by

$$\overline{m}_L = \frac{1}{10} \sum_{i=1}^{10} \ell_i = 1011.1 \text{ mw.} \qquad ([illegible])$$

The sample standard deviation is given by

$$\overline{\sigma}_L = \left[\frac{1}{9} \sum_{i=1}^{10} (\ell_i - \overline{m}_L)^2\right]^{1/2} = 68.48 \text{ mw.} \qquad ([illegible])$$

Therefore, the standardized data are obtained using

$$x_i = \frac{\ell_i - 1011.1}{68.48}\ . \qquad ([illegible])$$

x_i is tabulated in Table A along with x_i^3 and x_i^4 which are needed in computing the coefficients of skewness and kurtosis. Since x_i has zero mean and unit variance, it

TABLE A

[illegible]	[illegible]	[illegible]	x_i^3	x_i^4	$F_X(x_i)$	$P(X_E \le x_i)$	$\lvert F_X(x_i)-P(X_E \le x_i)\rvert$
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3287	.4	.0713
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.6466	.6	.0466
[illegible]	[illegible]	[illegible]	[illegible]	.3570	.7724	.8	.0276
[illegible]	[illegible]	[illegible]	[illegible]	5.144	.0723	.1	.0277
[illegible]	997	[illegible]	-.0087	.0018	.4125	.5	.0875
6	1098	[illegible]	[illegible]	2.593	.8979	1.0	.1021
7	975	[illegible]	-.1719	.0956	.290	.3	.010
8	1059	[illegible]	.2415	.2387	.7488	.7	.0488
9	909	-1.491	[illegible]	4.942	.0744	.2	.1256
10	1083	1.050	1.158	1.216	.8498	.9	.0502

follows that

$$\bar{\alpha}_3 = \frac{(10)}{(9)(8)} \sum_{i=1}^{10} x_i^3 = -0.406 \qquad (143)$$

and

$$\bar{\alpha}_4 = \frac{(100)}{(9)(73)} \sum_{i=1}^{10} x_i^4 - \frac{(51)}{(73)} = 1.532. \qquad (144)$$

In Appendix C, the Weibull distribution is the only distribution for which negative values of α_3 appear. Likely candidates are those special cases for which β = 4, 5, 6, 7, and 8.

Let us test the Weibull distribution for which β = 4. From Appendix D, it is seen that

$$F_X(x) = [1 - e^{-\frac{(x + 3.564)^4}{239.14}}] \, u(x + 3.564). \qquad (145)$$

Values of $F_X(x_i)$, $P(X_E \leq x_i)$, and $|F_X(x_i) - P(X_E \leq x_i)|$ are also tabulated in Table A. Observe that

$$D_{max} = 0.1256. \qquad (146)$$

Assume the significance level is set at $\alpha = .05$. Referring to Appendix E, it is found that

$$D_{10}^{.05} = 0.409. \qquad (147)$$

Since $D_{max} < D_{10}^{.05}$, the distribution is not rejected.

Case 2-Large N ($N > 30$): For large values of N, the χ^2 test is preferable. It is presented here without proof. The χ^2 test proceeds as follows:

1) Standardize the experimental data by utilizing (136)

2) For the distribution under test, divide the range of X into 10 cells such that it is equally probable X will fall into each cell. In the statistical literature, each cell is referred to as a decile. The decile boundaries for some standardized CDF's are tabulated in Appendix F.

3) Count the number of standardized data, x_i, contained within each cell. Denote the number of data in the k^{th} cell by n_k; $k = 1, 2, \ldots, 10$.

4) Compute the quantity P_k defined by

$$P_k = \frac{n_k}{N} \; ; \; k = 1, 2, \ldots, 10. \qquad (148)$$

5) Evaluate the statistic

$$\chi^2 = N \cdot \sum_{k=1}^{10} \frac{(P_k - 0.1)^2}{0.1} . \tag{149}$$

6) Let M denote the number of parameters in $F_X(x)$, the CDF under test, whose numerical values must be determined in order for the distribution to be completely specified. The <u>degrees</u> <u>of</u> <u>freedom</u> are given by $P = 10 - 1 - M = 9 - M$.

7) Assign a numerical value to the significance level, α, defined by (139).

8) Refer to the Table in Appendix G. Find the tabulated value corresponding to the applicable degrees of freedom, P, and significance level, α. Denote the tabulated value by $(\chi^2)_P^\alpha$.

9) Reject the distribution if $\chi^2 > (\chi^2)_P^\alpha$.

The above discussion is applicable when $N > 100$. Should $30 < N \leq 100$, step 2 should be modified such that the range of X is subdivided into 5 equally probable cells. The cell boundaries can be obtained from Appendix F by selecting the 5 intervals (x_0, x_2), (x_2, x_4), (x_4, x_6), (x_6, x_8), and (x_8, x_{10}). The statistic equivalent to that in step 5 is now given by

$$\chi^2 = N \cdot \sum_{k=1}^{5} \frac{(P_k - 0.2)^2}{0.2} . \tag{150}$$

Finally, the degrees of freedom specified in step 6 are now given by $P = 5 - 1 - M = 4 - M$. All other steps remain the same.

Independent of whether 10 or 5 cells are employed, best results are obtained when the number of data points in each cell exceeds three.

Example 8: Assume 20 "identical" equipments are used in the experiment for determining $F_{L|S}(\ell|s)$. For this value of N, the Kolmogorov-Smirnov test should be used. However, the χ^2 test will be applied. The small value of N has been specified in order to make the example more tractable. The results of the experiment are summarized in Table B. The sample mean is given by

$$\bar{m}_L = \frac{1}{20} \sum_{i=1}^{20} \ell_i = 1007.35 \text{ mw.} \tag{151}$$

The sample standard deviation is given by

$$\bar{\sigma}_L = [\frac{1}{19} \sum_{i=1}^{20} (\ell_i - \bar{m}_L)^2]^{1/2} = 49.51 \text{ mw.} \tag{152}$$

Therefore, the standardized data are obtained by using

$$x_i = \frac{\ell_i - 1007.35}{49.51} . \tag{153}$$

x_i is tabulated in Table B along with x_i^3 and x_i^4 which are needed in computing the coefficients of skewness and kurtosis. Since x_i has zero mean and unit variance, it

TABLE B

i	ℓ_i (mw)	x_i	x_i^3	x_i^4
1	966	-.835	-.582	.486
2	976	-.633	-.254	.161
3	1023	.316	.032	.010
4	1038	.619	.237	.147
5	1064	1.144	1.497	1.713
6	1036	.579	.194	.112
7	1035	.558	.174	.097
8	932	-1.522	-3.526	5.366
9	910	-1.966	-7.599	14.939
10	1035	.558	.174	.097
11	978	-.593	-.209	.124
12	950	-1.158	-1.553	1.798
13	1013	.114	.002	.000
14	964	-.876	-.672	.589
15	1091	1.690	4.827	8.157
16	1058	1.023	1.071	1.095
17	1045	.760	.439	.334
18	1043	.720	.373	.269
19	1036	.579	.194	.112
20	954	-1.078	-1.253	1.350

follows that

$$\bar{\alpha}_3 = \frac{(20)}{(19)(18)} \sum_{i=1}^{20} x_i^3 = -.377 \tag{154}$$

and

$$\bar{\alpha}_4 = \frac{(400)}{(19)(343)} \sum_{i=1}^{20} x_i^4 - \frac{(111)}{343} = 1.945. \tag{155}$$

With reference to Appendix C, it would be logical to select the Weibull distribution. However, let us test the triangular distribution. Because of the small value of N, let the range of X be subdivided into 5 cells. Referring to Appendix F, the 5 cells are given by (-2.45,-.900),(-.900,-.259),(-.259,.259), (.259,.900), and (.900,2.45). The standardized data in each cell, n_k for each cell, P_k, and $(P_k - 0.2)^2$ are tabulated in Table C. The χ^2 statistic becomes,

TABLE C

Cell Number	1	2	3	4	5
Cell intervals	(-2.45,-.900)	(-.900,-.259)	(-.259,.259)	(.259,.900)	(.900, 2.45)
standardized data in each cell	8,9,12,20	1,2,11,14	13	3,4,6,7 10,17,18,19	5,15,16
n_k	4	4	1	8	3
P_k	0.2	0.2	.05	0.4	0.15
$(P_k - 0.2)^2$	0	0	.0225	.04	.0025

according to (150),

$$\chi^2 = 20 \sum_{k=1}^{5} \frac{(P_k - 0.2)^2}{0.2} = 6.53 \tag{156}$$

Since the triangular distribution contains two parameters, the degrees of freedom are given by

$$P = 4 - M = 2. \tag{157}$$

Assume the significance level is set at $\alpha = 0.05$. Referring to Appendix C, it is found that

$$(\chi^2)_2^{0.05} = 5.99. \tag{158}$$

Since $\chi^2 > (\chi^2)_2^{0.05}$, the distribution is rejected.

10. CONCLUSION AND SUGGESTIONS FOR FUTURE WORK

An inherent feature of many EMC problems is the randomness associated with the desired signals, interfering signals, and equipments involved. In this work a probabilistic approach is proposed which introduces the following concepts:

1) $F_{L|S}(i|s)$, the given signal and interference EMI probability function,

2) $C_{I|S}(i|s)$, the given signal and interference EMC probability function,

3) $C_S(s)$, the given signal EMC probability function,

4) $C_I(i)$, the given interference EMC probability function,

5) C, the EMC probability (also referred to as the compatibility),

6) $\lambda_s(i)$, the EMI rate function

7) Y, the compatibility margin.

Several examples illustrating the interrelationships between these concepts are presented.

For problems where a deterministic approach is inappropriate, the probabilistic approach can provide an improved EMC model. In addition, by utilizing statistical parameters, statistical macromodels can be developed which simplify the characterization of complex signals and equipments. This approach appears to be especially well suited for the EMI modeling of [illegible] circuits. Because of their tremendous complexity, it is neither desirable nor possible to carry out an exact circuit modeling and analysis in order to determine EMC.

Central to the probabilistic approach is the experimental determination of $F_{L|S}(\ell|s)$. A method for determining a suitably large value of N, the number of equipments to be used in the experiment, is presented. For analytical reasons, it is desirable to approximate the experimentally determined $F_{L|S}(?|s)$ by a well known distribution. Some statistical techniques for rejecting candidate distributions not supported by the experimental data are discussed.

It is recommended that randomness associated with microcircuit susceptibility be investigated. To what extent is the susceptibility level, L, random ? If the susceptibility level is random, can its PDF be determined? Assuming the PDF is known, can the probability of EMC and/or EMI be successfully determined? To develop answers to these questions, it is suggested that the random susceptibility of the 7400 TTL NAND gate, a simple integrated circuit, be studied experimentally. If the study is concluded successfully, more complicated microcircuits should then be investigated.

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APPENDIX A

$C_{I|S}(i|s)$ AND $\lambda_s(i)$ FOR SOME COMMON PDF's

In this appendix, given $f_{L|S}(\ell|s)$, $C_{I|S}(i|s)$ and $\lambda_s(i)$ are derived using the relations

$$C_{I|S}(i|s) = \int_i^\infty f_{L|S}(\ell|s)\,d\ell$$

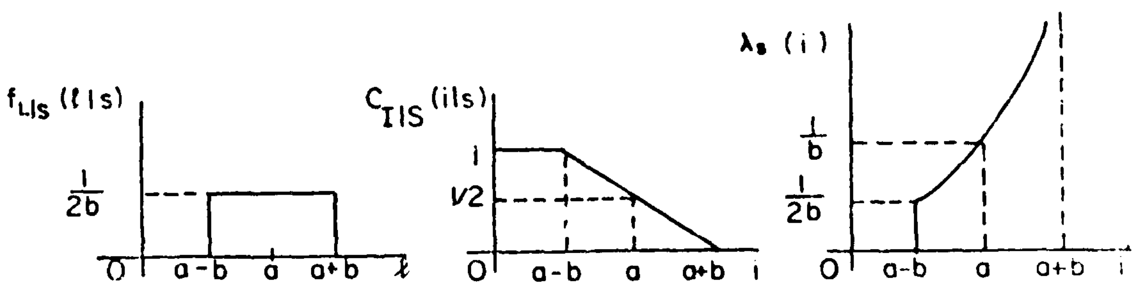
$$\lambda_s(i) = \frac{f_{L|S}(i|s)}{C_{I|S}(i|s)} \,.$$

1. Uniform Distribution

$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{1}{2b} & , \quad a-b \le \ell \le a+b \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$C_{I|S}(i|s) = \begin{cases} 1 & , \quad i \le a-b \\ \dfrac{1}{2b}\,[a+b-i] & , \quad a-b \le i \le a+b \\ 0 & , \quad a+b \le i \end{cases}$$

$$\lambda_s(i) = \begin{cases} \dfrac{1}{a+b-i} & , \quad a-b \le i \le a+b \\ 0 & , \quad \text{otherwise} \end{cases}$$

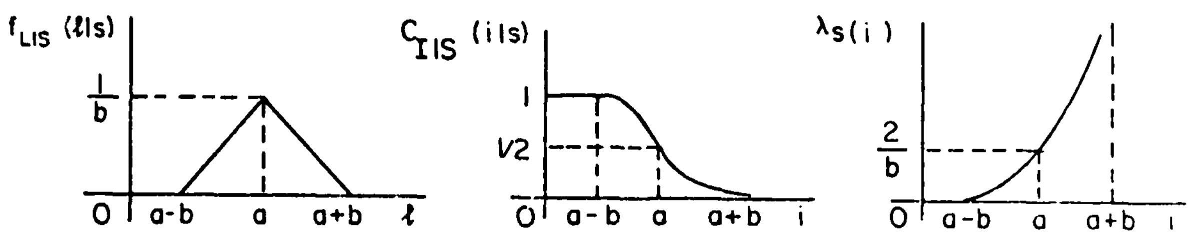


2. Triangular Distribution

$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{\ell}{b^2} - \dfrac{a-b}{b^2} & , \quad a-b \le \ell \le a \\[2ex] \dfrac{-\ell}{b^2} + \dfrac{a+b}{b^2} & , \quad a \le \ell \le a+b \\[2ex] 0 & , \quad \text{otherwise} \end{cases}$$

$$C_{I|S}(i|s) = \begin{cases} 1 & , \quad i \le a-b \\[1ex] 1 - \dfrac{1}{2}\left[\dfrac{i-(a-b)}{b}\right]^2 & , \quad a-b \le i \le a \\[2ex] \dfrac{1}{2}\left[\dfrac{a+b-i}{b}\right]^2 & , \quad a \le i \le a+b \\[2ex] 0 & , \quad a+b \le i \end{cases}$$

$$\lambda_s(i) = \begin{cases} 2\dfrac{i-(a-b)}{2b^2 - [i-(a-b)]^2} & , \quad a-b \le i \le a \\[3ex] \dfrac{2}{a+b-i} & , \quad a \le i \le a+b \\[2ex] 0 & , \quad \text{otherwise} \end{cases}$$

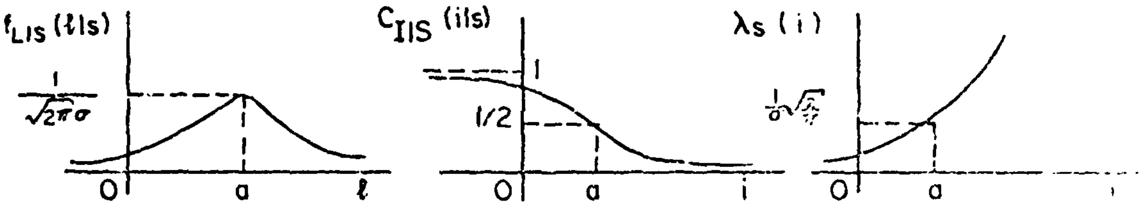


3. Gaussian Distribution

$$f_{L|S}(\ell|s) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(\ell - a)^2}{2\sigma^2}} \equiv G(\ell, a, \sigma)$$

$$C_{I|S}(i|s) = \mathrm{erfc}\left(\frac{i - a}{\sigma}\right), \quad \mathrm{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}}\, dt$$

$$\lambda_s(i) = \frac{G(i, a, \sigma)}{\mathrm{erfc}\left(\frac{i - a}{\sigma}\right)}$$



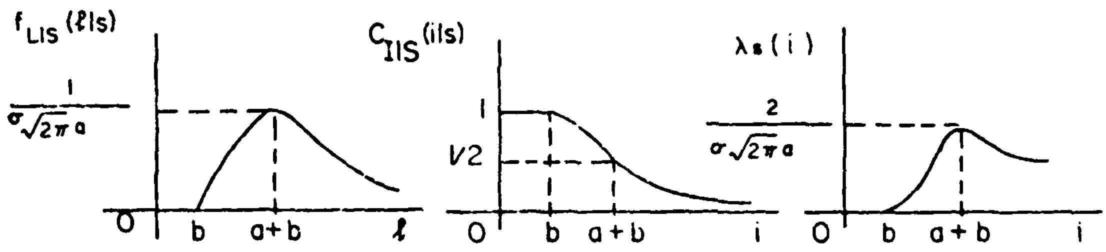
4. Log-Normal Distribution

$$f_{L|S}(\ell|s) = \frac{1}{\sqrt{2\pi}\,\sigma(\ell - b)}\, e^{-\frac{[\ln(\ell - b) - \ln a]^2}{2\sigma^2}}\, u(\ell - b)$$

$$= \frac{G[\ln(\ell - b),\ \ln a,\ \sigma]}{\ell - b}\, u(\ell - b),\ b > 0$$

$$C_{I|S}(i|s) = \begin{cases} 1 & ,\ i \le b \\ \operatorname{erfc}\left(\frac{1}{\sigma} \ln \frac{i - b}{a}\right) & ,\ b < i \end{cases} \qquad u(\ell - b) = \begin{cases} 1, & \ell \ge b \\ 0, & \ell < b \end{cases}$$

$$\lambda_s(i) = \begin{cases} 0 & ,\ i < b \\ \dfrac{G[\ln(i - b),\ \ln a,\ \sigma]}{(i - b)\ \operatorname{erfc}\left(\frac{1}{\sigma} \ln \frac{i - b}{a}\right)} & ,\ i \ge b \end{cases}$$

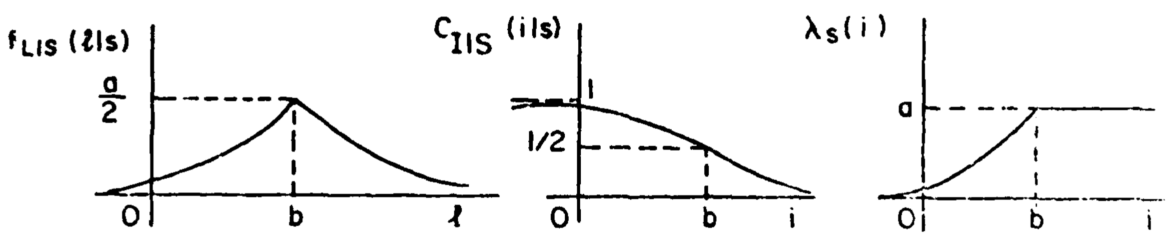


5. Laplace Distribution

$$f_{L|S}(\ell|s) = \frac{a}{2}\, e^{-a|\ell - b|}$$

$$C_{I|S}(i|s) = \begin{cases} 1 - \frac{1}{2}\, e^{-a(b - i)} & , \quad i \le b \\ \frac{1}{2}\, e^{-a(i - b)} & , \quad b < i \end{cases}$$

$$\lambda_s(i) = \begin{cases} \dfrac{a}{2e^{a(b - i)} - 1} & , \quad i \le b \\ a & , \quad b < i \end{cases}$$

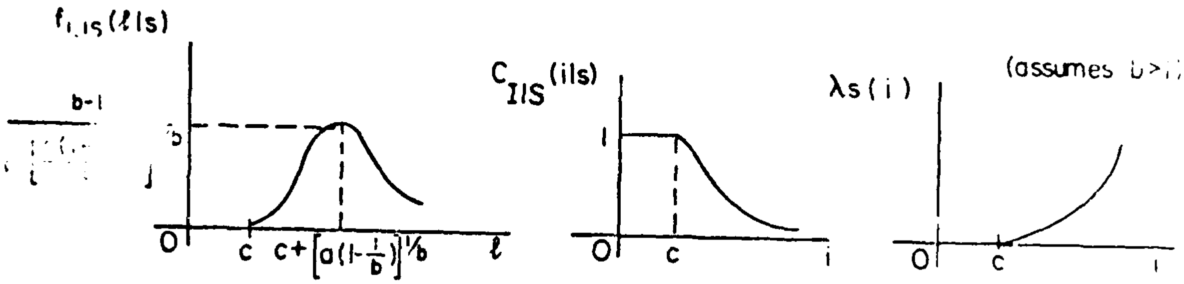


6. Weibull Distribution

$$f_{L|S}(\ell|s) = \left(\frac{b}{a}\right)(\ell - c)^{b-1}\, e^{-\frac{(\ell - c)^b}{a}}\, u(\ell - c) \quad ; \quad a,\ b,\ c \quad 0$$

$$C_{I|S}(i|s) = \begin{cases} 1 & , \ i < c \\ e^{-\frac{(i - c)^b}{a}} & , \ c < i \end{cases}$$

$$\lambda_s(i) = \left(\frac{b}{a}\right)(i - c)^{b-1}\, u(i - c)$$



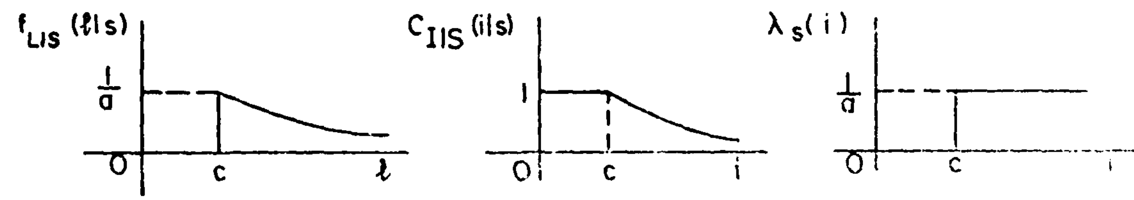
Special Cases of the Weibull Distribution

6.1 b = 1 results in the exponential distribution

$$f_{L|S}(\ell|s) = \frac{1}{a}\, e^{-\frac{(\ell - c)}{a}}\, u(\ell - c)$$

$$C_{I|S}(i|s) = \begin{cases} 1 , & i < c \\ e^{-\frac{(i - c)}{a}} , & i \geq c \end{cases}$$

$$\lambda_s(i) = \frac{1}{a}\, u(i - c)$$

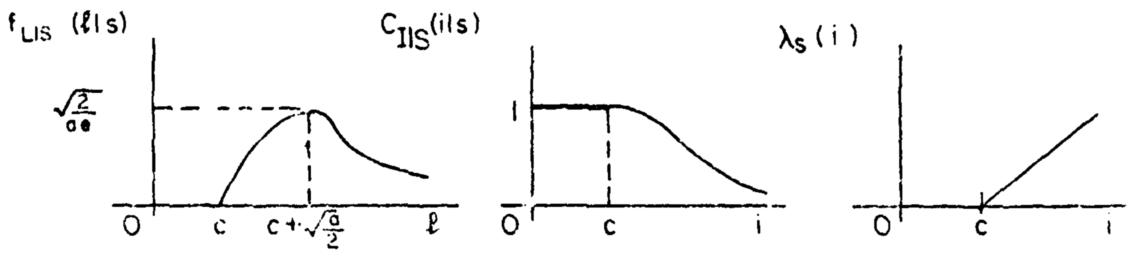


b = 2 results in the Rayleigh Distribution

$$f_{L|S}(\ell|s) = \frac{2}{a}(\ell - c)\, e^{-\frac{(\ell - c)^2}{a}}\, u(\ell - c)$$

$$C_{I|S}(i|s) = \begin{cases} 1, & i < c \\ e^{-\frac{(i - c)^2}{a}}, & i > c \end{cases}$$

$$\lambda_s(i) = \frac{2}{a}(i - c)\, u(i - c)$$

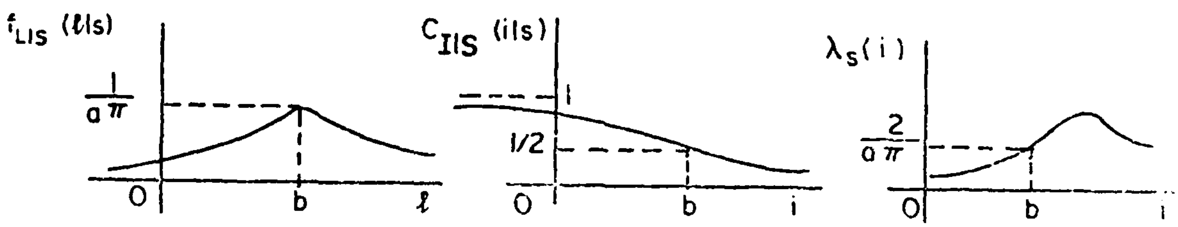


7. Cauchy Distribution

$$f_{L|S}(\ell|s) = \frac{\frac{a}{\pi}}{a^2 + (\ell - b)^2}$$

$$C_{I|S}(i|s) = \frac{1}{2} - \frac{1}{\pi}\tan^{-1}\left(\frac{i - b}{a}\right)$$

$$\lambda_s(i) = \frac{a}{[a^2 + (i - b)^2][\frac{\pi}{2} - \tan^{-1}(\frac{i - b}{a})]}$$



8. Gamma Distribution

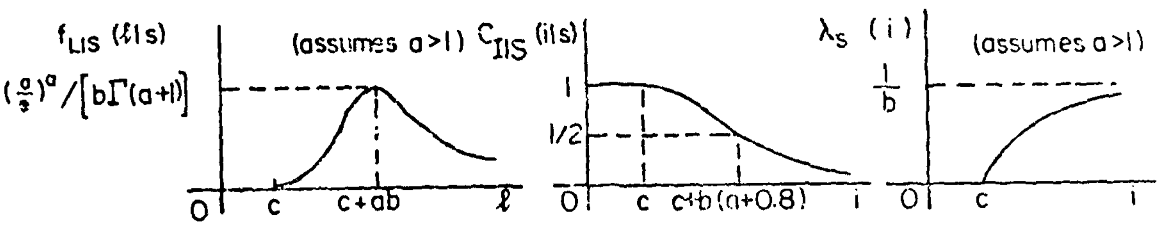
$$f_{L|S}(\ell|s) = \frac{(\ell - c)^a}{\Gamma(a + 1)\, b^{a + 1}}\, e^{-\frac{\ell - c}{b}} \quad u(\ell - c),\ a > -1,\ b > 0$$

$$C_{I|S}(i|s) = \begin{cases} 1 - \dfrac{\Gamma_{\frac{i - c}{b}}(a + 1)}{\Gamma(a + 1)} & , \quad c < i \\ 1 & , \quad i < c \end{cases}$$

$$\lambda_s(i) = \frac{(i - c)^a\, e^{-\frac{i - c}{b}}}{b^{a + 1}\left[\Gamma(a + 1) - \Gamma_{\frac{i - c}{b}}(a + 1)\right]}$$

$$\Gamma(a) = \int_0^\infty t^{a - 1} e^{-t}\, dt \qquad \text{(Gamma function)}$$

$$\Gamma_b(a) = \int_0^\infty t^{a - 1} e^{-t}\, dt \qquad \text{(Incomplete Gamma function)}$$



9. Beta Distribution

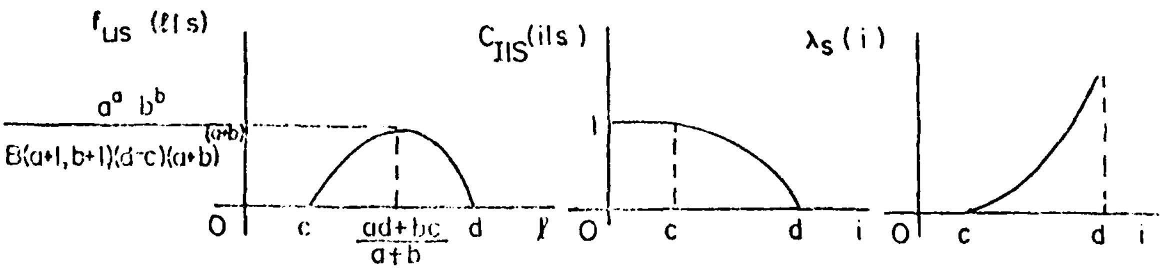
$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{(\ell - c)^a (d - \ell)^b}{B(a+1,\, b+1)\,(d-c)^{a+b+1}} & ;\ c \le \ell \le d,\ d-c=2k > 0,\ a,b > -1 \\ 0 & ,\ \text{otherwise} \end{cases}$$

$$C_{I|S}(i|s) = \begin{cases} 1 & ,\ i < c \\ 1 - \dfrac{B_{\frac{i-c}{2k}}(a+1,\, b+1)}{B(a+1,\, b+1)} & ,\ c < i < d \\ 0 & ,\ d < i \end{cases}$$

$$\lambda_S(i) = \begin{cases} \dfrac{(i-c)^a\,(d-i)^b}{(2k)^{a+b+1}\,\{B(a+1,\, b+1) - B_{\frac{i-c}{2k}}(a+1,\, b+1)\}} & ,\ c \le i < d \\ 0 & ,\ \text{otherwise} \end{cases}$$

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \qquad \text{(Beta function)}$$

$$B_C(a,b) = \int_0^C t^{a-1}(1-t)^{b-1}\,dt,\ 0 \le C < 1 \qquad \text{(Incomplete Beta function)}$$



Special Case of the Beta Distribution

9.1 a = b = 0 results in the uniform distribution

$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{1}{2k} \;, & c \leq \ell < d \\ 0 \;, & \text{otherwise} \end{cases}$$

APPENDIX B

EXAMPLES OF ANALYTICAL EXPRESSIONS FOR $C_S(s)$

Given $f_{L|S}(\ell|s)$ and $f_I(i)$, the given signal EMC probability function may be evaluated using

$$C_S(s) = \int_{-\infty}^{\infty} C_{I|S}(i|s)\, f_I(i)\,di$$

$$= \int_0^{\infty} f_{Y|S}(y|s)\,dy$$

where $C_{I|S}(i|s)$ and $f_{Y|S}(y|s)$ are determined from (3) and (53), respectively.

1. $f_{L|S}(\ell|s)$ Uniform and $f_I(i)$ Uniform

$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{1}{2a} , & s - a \le \ell \le s + a \\ 0 , & \text{otherwise.} \end{cases}$$

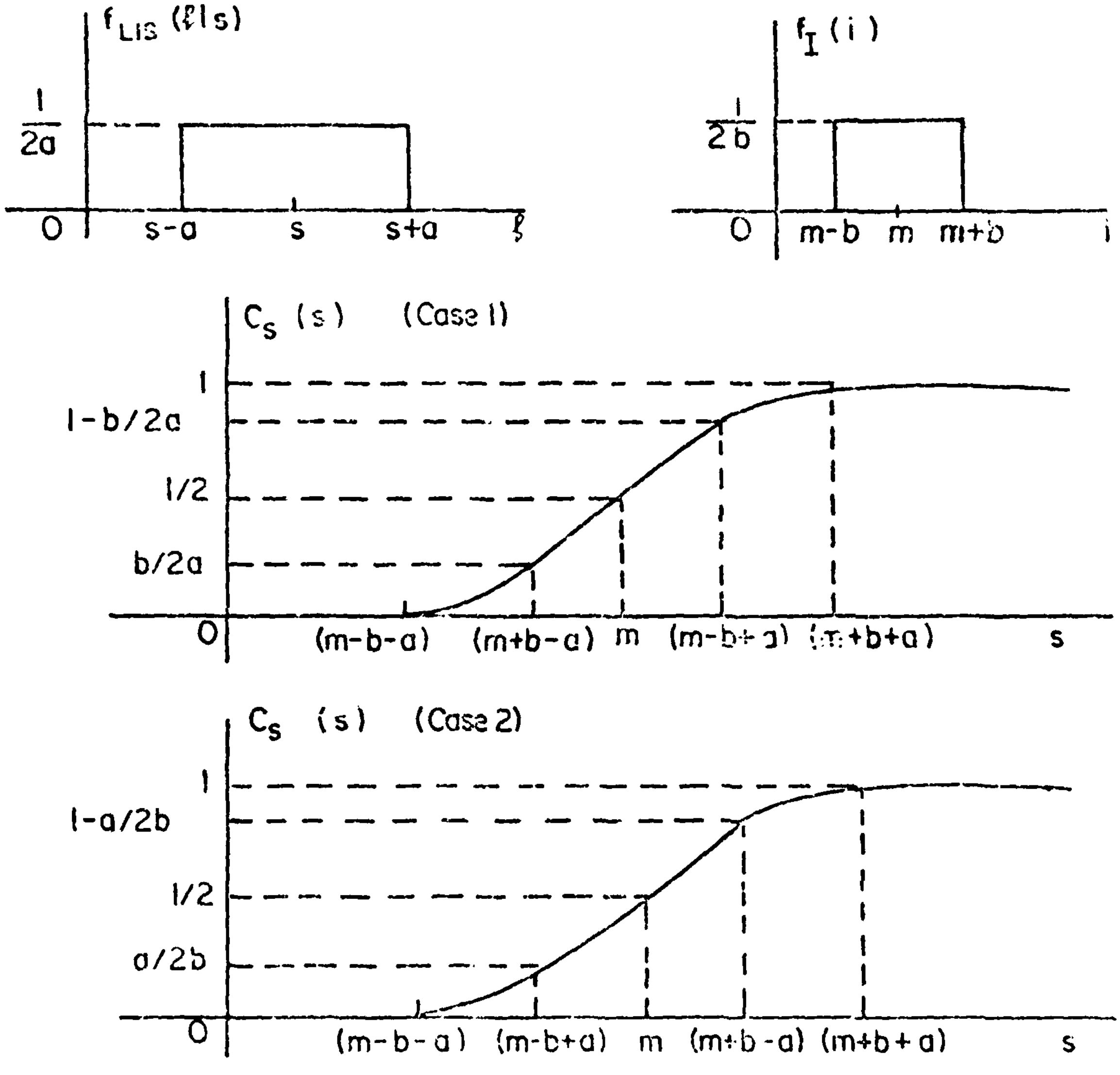
$$f_I(i) = \begin{cases} \dfrac{1}{2b} , & m - b \le i \le m + b \\ 0 , & \text{otherwise.} \end{cases}$$

Case 1: $b < a$

$$
C_S(s) = \begin{cases}
0 & , \quad s \leq m - b - a \\[2ex]
\dfrac{(s + a + b - m)^2}{8ab} & , \quad m - b - a \leq s \leq m + b - a \\[2ex]
\dfrac{s + a - m}{2a} & , \quad m + b - a \leq s \leq m - b + a \\[2ex]
1 - \dfrac{[s - (m + b + a)]^2}{8ab} & , \quad m - b + a \leq s \leq m + b + a \\[2ex]
1 & , \quad m + b + a \leq s
\end{cases}
$$

Case 2: $a < b$

$$
C_S(s) = \begin{cases}
0 & , \quad s \leq m - b - a \\[2ex]
\dfrac{(s + a + b - m)^2}{8ab} & , \quad m - b - a \leq s \leq m - b + a \\[2ex]
\dfrac{s + b - m}{2b} & , \quad m - b + a \leq s \leq m + b - a \\[2ex]
1 - \dfrac{[s - (m + b + a)]^2}{8ab} & , \quad m + b - a \leq s \leq m + b + a \\[2ex]
1 & , \quad m + b + a \leq s
\end{cases}
$$



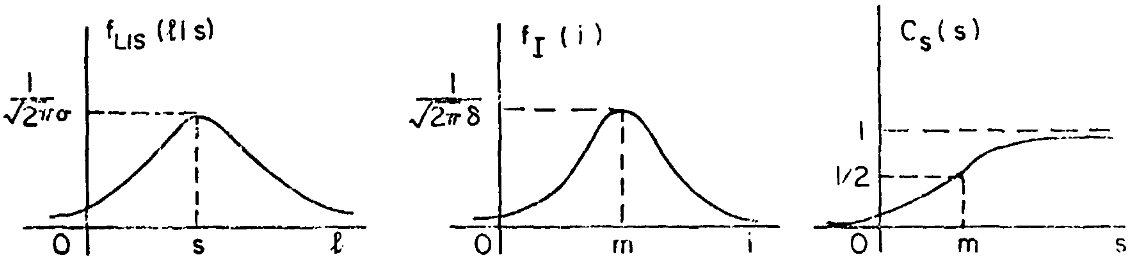
$f_{L|S}(\ell|s)$
$\frac{1}{2a}$
0
s-a
s
s+a
ℓ
$f_I(i)$
$\frac{1}{2b}$
0
m-b
m
m+b
i
$C_s(s)$ (Case 1)
1
1-b/2a
1/2
b/2a
0
(m-b-a)
(m+b-a)
m
(m-b+a)
(m+b+a)
s
$C_s(s)$ (Case 2)
1
1-a/2b
1/2
a/2b
0
(m-b-a)
(m-b+a)
m
(m+b-a)
(m+b+a)
s

2. $f_{L|S}(\ell|s)$ Gaussian and $f_I(i)$ Gaussian

$$f_{L|S}(\ell|s) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(\ell - s)^2}{2\sigma^2}}$$

$$f_I(i) = \frac{1}{\sqrt{2\pi}\,\delta}\, e^{-\frac{(i - m)^2}{2\delta^2}}$$

$$C_S(s) = \operatorname{erf}\left(\frac{s - m}{\sqrt{\sigma^2 + \delta^2}}\right)$$



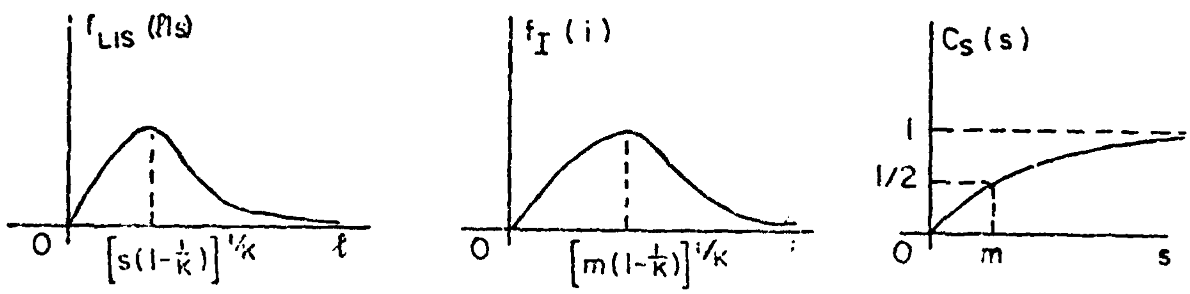
3. $f_{L|S}(\ell|s)$ Weibull and $f_I(i)$ Weibull (identical shape parameter, K)

$$f_{L|S}(\ell|s) = \frac{K}{s}\,\ell^{K-1}\,e^{-\frac{\ell^K}{s}}\,u(\ell)$$

$$, \quad u(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$f_I(i) = \frac{K}{m}\,i^{K-1}\,e^{-\frac{i^K}{m}}\,u(i)$$

$$C_S(s) = \frac{s}{s+m}$$ (Note that $C_S(s)$ is independent of K)



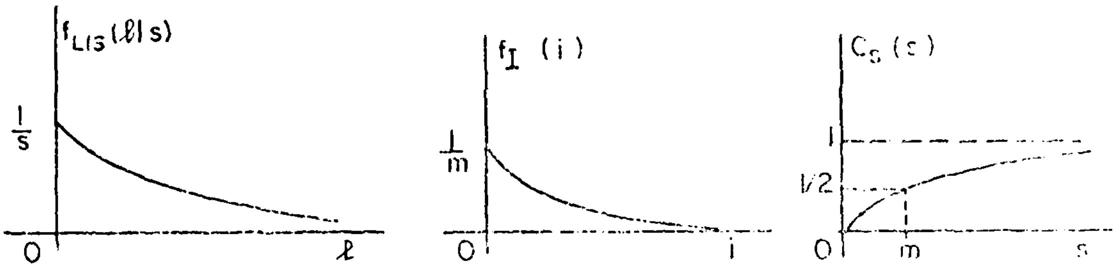
Special Cases of the Weibull Distribution

3.1 K = 1 results in the exponential distribution

$$f_{L|S}(\ell|s) = \frac{1}{s} e^{-\frac{\ell}{s}} u(\ell)$$

$$f_I(i) = \frac{1}{m} e^{-\frac{i}{m}} u(i)$$

$$C_S(s) = \frac{s}{s+m}$$



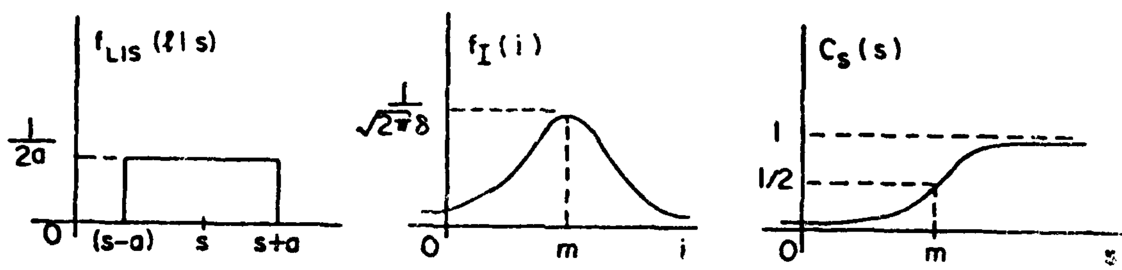
4. $f_{L|S}(\ell|s)$ Uniform and $f_I(i)$ Gaussian

$$f_{L|S}(\ell|s) = \begin{cases} \dfrac{1}{2a} , & s - a \le \ell \le s + a \\ 0 , & \text{otherwise} \end{cases}$$

$$f_I(i) = \frac{1}{\sqrt{2\pi}\,\delta} \; e^{-\frac{(i-m)^2}{2\delta^2}}$$

$$C_S(s) = \mathrm{erf}\left(\frac{s-a-m}{\delta}\right) + \frac{s+a-m}{2a}\left[\mathrm{erf}\left(\frac{s+a-m}{\delta}\right) - \mathrm{erf}\left(\frac{s-a-m}{\delta}\right)\right]$$

$$- \frac{\delta}{a\sqrt{2\pi}} \; e^{-\frac{(s-m)^2 + a^2}{2\delta^2}} \; \sinh\left[\frac{a(s-m)}{\delta^2}\right]$$

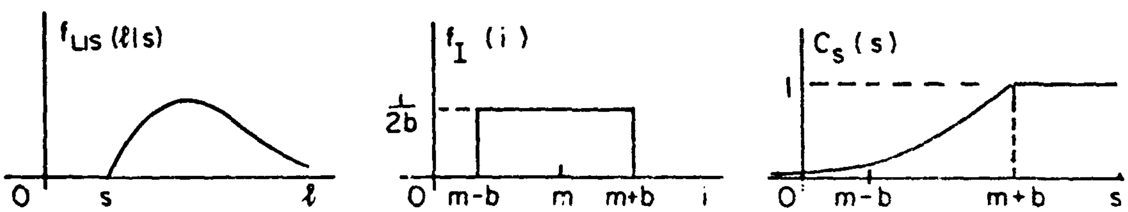


5. $f_{L|S}(\ell|s)$ Weibull and $f_I(i)$ Uniform

$$f_{L|S}(\ell|s) = \frac{K}{a}(\ell - s)^{K-1} e^{-\frac{(\ell - s)^K}{a}} u(\ell - s)$$

$$f_I(i) = \begin{cases} \frac{1}{2b}, & m - b \le i \le m + b \\ 0, & \text{otherwise} \end{cases}$$

$$C_S(s) = \begin{cases} \frac{a^{1/K}}{2b} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(nK+1)} \left[\left(\frac{m+b-s}{a^{1/K}}\right)^{nK+1} - \left(\frac{m-b-s}{a^{1/K}}\right)^{nK+1} \right], & s \le m - b \\ \frac{s+b-m}{2b} + \frac{a^{1/K}}{2b} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(nK+1)} \left(\frac{m+b-s}{a^{1/K}}\right)^{nK+1}, & m - b \le s \le m + b \\ 1, & m + b \le s \end{cases}$$

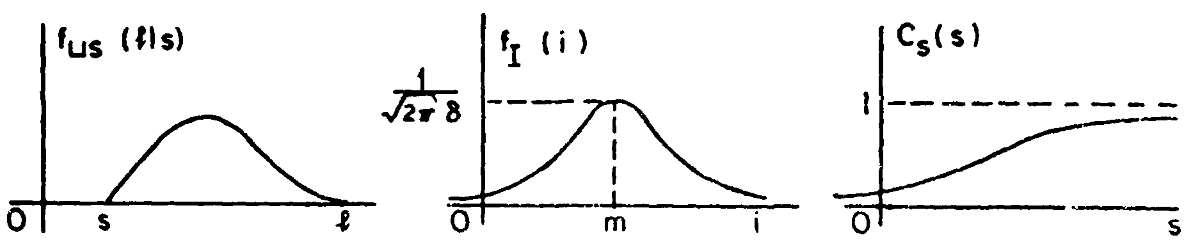


6. $f_{L|S}(\ell|s)$ Weibull and $f_I(i)$ Gaussian

$$f_{L|S}(\ell|s) = \frac{K}{a}(\ell - s)^{K-1}\, e^{-\frac{(\ell - s)^K}{a}}\, u(\ell - s)$$

$$f_I(i) = \frac{1}{\sqrt{2\pi}\,\delta}\, e^{-\frac{(i - m)^2}{2\delta^2}}$$

$$C_S(s) = \mathrm{erf}\left(\frac{s - m}{\delta}\right) + \frac{1}{\sqrt{2\pi}\,\delta}\, e^{-\frac{(s - m)^2}{2\delta^2}} \int_0^\infty e^{-\left(\frac{x^K}{a} + \frac{x^2}{2\delta^2} + \frac{s - m}{\delta^2}x\right)}\, dx$$



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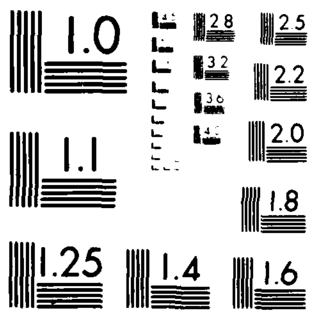

MICROCOPY RESOLUTION TEST CHART

APPENDIX C

ANALYTICAL EXPRESSIONS FOR STANDARDIZED PDF's AND THEIR COEFFICIENTS OF SKEWNESS AND KURTOSIS

PDF	ANALYTICAL EXPRESSION	α_3	α_4
Uniform	$f_X(x) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} \leq x < \sqrt{3} \\ 0, & \sqrt{3} < \lvert x \rvert \end{cases}$	0	1.8
Triangular	$f_X(x) = \begin{cases} \frac{x}{6} + \frac{1}{\sqrt{6}}, & -\sqrt{6} \leq x < 0 \\ -\frac{x}{6} + \frac{1}{\sqrt{6}}, & 0 \leq x < \sqrt{6} \\ 0, & \sqrt{6} < \lvert x \rvert \end{cases}$	0	2.4
Gaussian	$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$	0	3
Log Normal	$f_X(x) = \frac{1}{C\sqrt{2\pi}(x-b)} e^{-\frac{[\ln(x-b)-\ln a]^2}{2C^2}} u(x-b)$ $a = [e^{C^2}(e^{C^2}-1)]^{-1/2}$ $b = -(e^{C^2}-1)^{-1/2}$	$(e^{C^2}-1)(e^{C^2}+2)$	$(e^{4C^2}+2e^{3C^2}+3e^{2C^2}-3)$
Special Cases of Log Normal	C = 1/4	0.778	4.096
	C = 1/2	1.750	8.898
	C = 3/4	3.263	26.54
	C = 1	6.185	113.9
	C = 2	414.36	9.22×10^6

PDF	ANALYTICAL EXPRESSION	α_3	α_4
Laplace	$f_X(s) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}\,\lvert x \rvert}$	0	6
Weibull	$f_X(x) = \frac{\beta}{\alpha} (x - \gamma)^{\beta - 1} e^{-\frac{(x-\gamma)^\beta}{\alpha}} u(x - \gamma)$		
	$\alpha = [\Gamma(1 + \frac{2}{\beta}) - \Gamma^2(1 + \frac{1}{\beta})]^{-\frac{\beta}{2}}$		
	$\gamma = -\alpha^{\frac{1}{\beta}} \Gamma(1 + \frac{1}{\beta})$		
Special Cases of Weibull	$\beta = 0.5$	6.62	87.72
	$\beta = 1$	2	9
	$\beta = 2$	0.63	3.24
	$\beta = 3$	0.16	2.73
	$\beta = 4$	-0.09	2.76
	$\beta = 5$	-0.26	2.91
	$\beta = 6$	-0.39	3.09
	$\beta = 7$	-0.48	3.28
	$\beta = 8$	-0.56	3.48
	$\beta = 9$	-0.63	3.69
	$\beta = 10$	-0.68	3.92

PDF	ANALYTICAL EXPRESSION	α_3	α_4
	$f_X(x) = \frac{(x-c)^a}{\Gamma(a+1)b^{a+1}} e^{-\frac{x-c}{b}} u(x-c)$		
Gamma	$b = (a+1)^{-\frac{1}{2}}$	$\frac{2}{\sqrt{a+1}}$	$\frac{3(a+3)}{a+1}$
	$c = -(a+1)^{1/2}$		
Special Cases of Gamma	a = 1	1.41	6
	a = 2	1.16	5
	a = 3	1	4.5
	a = 4	0.89	4.2
	a = 5	0.82	4
	a = 6	0.76	3.86
	a = 7	0.71	3.75
	a = 8	0.67	3.67
	a = 9	0.63	3.6
	a = 10	0.60	3.55

APPENDIX D

ANALYTICAL EXPRESSIONS FOR STANDARDIZED CDF's

CDF	ANALYTICAL EXPRESSION
Uniform	$F_X(x) = \begin{cases} 0 , & x < -\sqrt{3} \\ \frac{x}{2\sqrt{3}} + \frac{1}{2} , & -\sqrt{3} \leq x \leq \sqrt{3} \\ 1 , & \sqrt{3} < x \end{cases}$
Triangular	$F_X(x) = \begin{cases} 0 , & x \leq -\sqrt{6} \\ \frac{x^2}{12} + \frac{x}{\sqrt{6}} + \frac{1}{2} , & -\sqrt{6} \leq x \leq 0 \\ -\frac{x^2}{12} + \frac{x}{\sqrt{6}} + \frac{1}{2} , & 0 \leq x \leq \sqrt{6} \\ 1 , & \sqrt{6} \leq x \end{cases}$
Gaussian	$F_X(x) = \text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$
Laplace	$F_X(x) = \begin{cases} \frac{1}{2} e^{\sqrt{2}x} , & x \leq 0 \\ 1 - \frac{1}{2} e^{-\sqrt{2}x} , & 0 \leq x \end{cases}$

CDF	ANALTYICAL EXPRESSION
Log Normal	$F_X(x) = \mathrm{erf}[\frac{1}{C} \ln (\frac{x - b}{a})]u(x - b)$ $a = [e^{C^2} (e^{C^2} - 1)]^{-1/2}$ $b = -[e^{C^2} - 1]^{-1/2}$
Special Cases of Log Normal	$C = 1/4;\ F_X(x) = \mathrm{erf}[4 \ln (\frac{x + 3.938}{3.817})]u(x + 3.938)$ $C = 1/2;\ F_X(x) = \mathrm{erf}[2 \ln (\frac{x + 1.876}{1.656})]u(x + 1.876)$ $C = 3/4;\ F_X(x) = \mathrm{erf}[\frac{4}{3} \ln (\frac{x + 1.151}{0.869})]u(x + 1.151)$ $C = 1;\ F_X(x) = \mathrm{erf}[\ln (\frac{x + 0.763}{0.463})]u(x + 0.763)$ $C = 2;\ F_X(x) = \mathrm{erf}[\frac{1}{2} \ln (\frac{x + 0.137}{0.0185})]u(x + 0.137)$
Gamma	$F_X(x) = \frac{\Gamma_{(\sqrt{a+1}\,x + a + 1)}(a + 1)}{\Gamma(a + 1)} u(x + \sqrt{a + 1})$

CDF	ANALYTICAL EXPRESSION
	$a = 0;\ F_X(x) = \frac{\Gamma_{x+1}(1)}{\Gamma(1)}\, u(x+1) = [1 - e^{-(x+1)}]u(x+1)$
	$a = 1;\ F_X(x) = \frac{\Gamma_{\sqrt{2}\,x+2}(2)}{\Gamma(2)}\, u(x+\sqrt{2})$
Special Cases of Gamma	$a = 2;\ F_X(x) = \frac{\Gamma_{\sqrt{3}\,x+3}(3)}{\Gamma(3)}\, u(x+\sqrt{3})$
	$a = 3;\ F_X(x) = \frac{\Gamma_{2x+4}(4)}{\Gamma(4)}\, u(x+2)$
	$a = 4;\ F_X(x) = \frac{\Gamma_{\sqrt{5}\,x+5}(5)}{\Gamma(5)}\, u(x+\sqrt{5})$
Weibull	$F_X(x) = [1 - e^{-\frac{(x-\gamma)^\beta}{\alpha}}]u(x-\gamma)$ $\alpha = [\Gamma(1+\frac{2}{\beta}) - \Gamma^2(1+\frac{1}{\beta})]^{-\beta/2}$ $\gamma = -\alpha^{1/\beta}\,\Gamma(1+\frac{1}{\beta})$

CDF	ANALYTICAL EXPRESSION
	$\beta = 1/2;\ F_X(x) = [1 - e^{-\frac{(x + 0.447)^{1/2}}{0.473}}]u(x + 0.447)$
	$\beta = 1\ ;\ F_X(x) = [1 - e^{-(x + 1)}]u(x + 1)$
Special Cases of Weibull	$\beta = 2\ ;\ F_X(x) = [1 - e^{-\frac{(x + 1.913)^{2}}{4.66}}]u(x + 1.913)$
	$\beta = 3\ ;\ F_X(x) = [1 - e^{-\frac{(x + 2.759)^{3}}{29.445}}]u(x + 2.759)$
	$\beta = 4\ ;\ F_X(x) = [1 - e^{-\frac{(x + 3.564)^{4}}{239.14}}]u(x + 3.564)$

APPENDIX E

TABLE FOR KOLMOGOROV-SMIRNOV TEST [9]

Sample size (N)	Significance level (α) .20	.15	.10	.05	.01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.734
5	.446	.474	.510	.563	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.409	.486
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.391
17	.250	.266	.286	.318	.380
18	.244	.259	.278	.309	.270
19	.237	.252	.272	.301	.361
20	.231	.246	.264	.294	.352
25	.21	.22	.24	.264	.32
30	.19	.20	.22	.242	.29

APPENDIX F

DECILE BOUNDARIES FOR SOME STANDARDIZED CDF's

DISTRIBUTION		x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Uniform		-1.732	-1.386	-1.039	-.693	-.346	0	.346	.693	1.039	1.386	1.732
Triangular		-2.45	-1.354	-.900	-.552	-.259	0	.259	.552	.900	1.354	2.45
Gaussian		$-\infty$	-1.282	-.842	-.524	-.253	0	.253	.524	.842	1.282	∞
Log Normal	c = 1/4	-3.938	-1.168	-.846	-.590	-.355	-.121	.128	.413	.773	1.321	∞
	c = 1/2	-1.876	-1.004	-.789	-.602	-.417	-.220	.003	.276	.646	1.267	∞
	c = 3/4	-1.151	-.819	-.689	-.564	-.432	-.282	-.101	.136	.483	1.121	∞
	c = 1	-.763	-.634	-.564	-.489	-.404	-.300	-.167	.019	.311	.905	∞
	c = 2	-.137	-.135	-.133	-.130	-.125	-.118	-.106	-.084	-.037	.103	∞
Laplace		$-\infty$	-1.138	-.648	-.361	-.158	0	.158	.361	.648	1.138	∞
Weibull	β = 1/2	-.447	-.445	-.436	-.418	-.389	-.340	-.259	-.123	.132	.739	∞
	β = 1	-1.00	-.895	-.777	-.643	-.489	-.307	-.084	.204	.609	1.303	∞
	β = 2	-1.913	-1.213	-.894	-.623	-.370	-.116	.153	.456	.825	1.363	∞
	β = 3	-2.759	-1.302	-.887	-.570	-.292	-.029	.237	.523	.855	1.314	∞
	β = 4	-3.564	-1.330	-.867	-.530	-.2[illegible]5	.018	.277	.548	.858	1.273	∞

DISTRIBUTION		x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Gamma	a = 0	-1.000	-.894	-.776	-.646	-.488	-.306	-.082	.204	.611	1.302	∞
	a = 1/2	-1.225	-.987	-.815	-.644	-.461	-.259	-.022	.272	.670	1.330	∞
	a = 1	-1.414	-1.040	-.832	-.638	-.441	-.227	.016	.313	.705	1.341	∞
	a = 2	-1.732	-1.098	-.846	-.628	-.412	-.188	.060	.356	.741	1.343	∞
	a = 2.5	-1.871	-1.115	-.850	-.622	-.402	-.176	.076	.371	.748	1.338	∞
	a = 3	-2.000	-1.130	-.852	-.618	-.393	-.164	.087	.381	.759	1.342	∞
	a = 4	-2.236	-1.149	-.854	-.610	-.381	-.148	.107	.399	.769	1.341	∞

APPENDIX G

TABLE FOR χ^2 TEST [9]

Degrees of Freedom (P)	Significance level (α) .995	.99	.975	.95	.90	.80	.70	.50	.30	.20	.10	.05	.025	.01	.005
1	.000	.000	.001	.004	.016	.064	.148	.455	1.07	1.64	2.71	3.84	5.02	6.63	7.88
2	.010	.020	.051	.103	.211	.446	.713	1.39	2.41	3.22	4.61	5.99	7.38	9.21	10.6
3	.072	.115	.216	.352	.584	1.00	1.42	2.37	3.66	4.64	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.07	3.83	5.35	7.23	8.56	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	4.59	5.53	7.34	9.52	11.0	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.38	6.39	8.34	10.[illegible]			16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.18	7.27	9.34	11.8	13.4	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	6.99	8.15	10.3	12.9	14.6	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	7.81	9.03	11.3	14.0	15.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	8.63	9.93	12.3	15.1	17.0	19.8	22.4	24.7	27.7	29.8

Degrees of Freedom (P)	Significance level (α)														
	.995	.99	.975	.95	.90	.80	.70	.50	.30	.20	.10	.05	.025	.01	.005
14	4.07	4.66	5.63	6.57	7.79	9.47	10.8	13.3	16.2	18.2	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	10.3	11.7	14.3	17.3	19.3	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.2	12.6	15.3	18.4	20.5	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.0	13.5	16.3	19.5	21.6	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	12.9	14.4	17.3	20.6	22.8	26.0	28.9	31.5	34.8	37.2
19	6.83	7.63	8.91	10.1	11.7	13.7	15.4	18.3	21.7	23.9	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	14.6	16.3	19.3	22.8	25.0	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	15.4	17.2	20.3	23.9	26.2	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	16.3	18.1	21.3	24.9	27.3	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	17.2	19.0	22.3	26.0	28.4	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	18.1	19.9	23.3	27.1	29.6	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	18.9	20.9	24.3	28.2	30.7	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	19.8	21.8	25.3	29.2	31.8	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	20.7	22.7	26.3	30.3	32.9	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	21.6	23.6	27.3	31.4	34.0	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	22.5	24.6	28.3	32.5	35.1	39.1	42.6	45.7	49.6	52.3

Degrees of Freedom (P)	Significance level (α)														
	.995	.99	.975	.95	.90	.80	.70	.50	.30	.20	.10	.05	.025	.01	.005
30	13.8	15.0	16.8	18.5	20.6	23.4	25.5	29.3	33.5	32.6	40.3	43.8	47.0	50.9	53.7
40	20.7	22.1	24.4	26.5	29.0	32.3	34.9	39.3	44.2	47.3	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.3	34.8	37.7	41.4	44.3	49.3	54.7	58.2	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	50.6	53.8	59.3	65.2	69.0	74.4	79.1	83.4	88.4	92.0

MISSION
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