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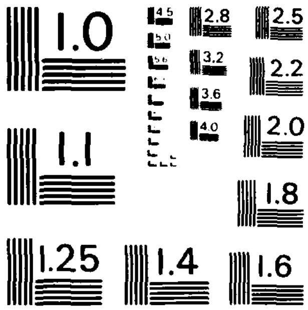
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During this period, research continued in the area of numerical solution of the incompressible Navier-Stokes equations. In particular, the following topics have been addressed -- (1) The question of achieving stable discretizations of the incompressibility constraint, (2) The problem of obtaining accurate solutions in the limit of large Reynolds numbers, (3) Devising efficient numerical solution algorithms for solving the nonlinear algebraic systems of equations arising from the discretization step. A necessary condition for convergence of the discrete approximation was obtained. A major result achieved has been (CONT

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ITEM #20, CONT.: to show that many often used low order element pairs are, in fact, unstable in the sense of this criterion. In addition, new, simple low order element pairs were introduced and proved to be stable. Concerning the second topic noted above, the investigator has taken an approach based on the idea that h (the discretization parameter) needs to be small only in certain locations, namely in boundary layers. Finally, solving the algebraic systems which arise from the discretization remains a major difficulty. Two new approaches have been developed, one dependent on time marching to the steady state limit, and the other based on an adaptation of a method used in structural mechanics to the fluids case. Four new scientific papers have been generated in the report period, to be published in the refereed literature.

AFOSR-TR- 83-1092

Interim Technical Report

Period ending 5/31/83, Grant AFOSR-82-0213

Grantee: Professor R. A. Nicolaides

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1. Introduction

Three basic aspects of the numerical solution of incompressible Navier-Stokes equations have been considered, namely

1) The question of achieving stable discretizations of the incompressibility constraint.
2) The problem of obtaining accurate solutions in the limit $Re \to \infty$.
3) Devising of efficient numerical solution algorithms for solving the nonlinear algebraic systems of equations arising from the discretization step.

The incompressible Navier-Stokes equations are

$$\frac{\partial u}{\partial t} - \nu\Delta \underline{u} + (\nabla \underline{u})\underline{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\underline{f}$$
$$\text{div}\ \underline{u} = 0 \qquad (1.1)$$

with, typically, $\underline{u} = 0$ on fixed boundaries of the flow field and, possibly, conditions on $\underline{u}$ specified at infinity (external flows). In (1.1), $\underline{u}$ represents the velocity vector, p the pressure, ρ the constant density, ν the kinematic viscosity, and $\underline{f}$ the body force per unit mass. For simplicity, we consider henceforth

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the case of an internal flow, in which the flow domain Ω is bounded.

A standard weak form for (1.1) in the stationary case is as follows: find $\underline{u} \in \vec{H}_0^1(\Omega)$ such that

$$\nu \int_\Omega \nabla \underline{u} : \nabla \underline{v} - \frac{1}{2} \int_\Omega \underline{u} \cdot \nabla \underline{u} \cdot \underline{v} - \underline{u} \cdot \nabla \underline{v} \cdot u$$

$$- \int_\Omega p \operatorname{div} \underline{v} = \int_\Omega \underline{f} \cdot \underline{v} \qquad \forall\ \underline{v} \in \vec{H}_0^1(\Omega) \tag{1.2}$$

$$\int_\Omega q \operatorname{div} \underline{u} = 0 \qquad \forall\ q \in L_0^2(\Omega)$$

where $\vec{H}_0^1(\Omega)$ is the first order Sobolev space of vector functions zero on the boundary $\partial\Omega$ and $L_0^2(\Omega)$ is the class of square integrable functions with mean zero over Ω. The density has been normalized to $\rho = 1$ in (1.2).

Numerical approximations to $(\underline{u},p)$ are generated by the following scheme: pick finite dimensional subspaces $V^h \subset \vec{H}_0^1(\Omega)$ and $S^h \subset L_0^2(\Omega)$ and seek $(u^h,p^h) \in V^h \times S^h$ such that

$$\nu \int_\Omega \nabla \underline{u}^h : \nabla \underline{v}^h - \frac{1}{2} \int_\Omega \underline{u}^h \cdot \nabla \underline{u}^h \cdot \underline{v}^h - \underline{u}^h \cdot \nabla \underline{v}^h \cdot \underline{u}^h$$

$$- \int_\Omega p^h \operatorname{div} \underline{v}^h = \int_\Omega f \underline{v}^h \qquad \forall\ \underline{v}^h \in \vec{H}_0^1(\Omega) \tag{1.3}$$

$$\int_\Omega q^h \operatorname{div} \underline{u}^h = 0 \qquad \forall\ q^h \in L_0^2(\Omega)$$

In terms of this formulation, our work on 1) above deals with stable choices of the spaces V^h and S^h as summarized in section 2 below. Work on 2) concerns the case $\nu \to 0$ in (1.2)-(1.3) and is summarized in section 3. Finally, the topic

3) concerns the solution of the nonlinear systems of algebriac equations arising from (1.3) and is discussed in the fourth section.

2. Work relating to discretization

This work is a continuation of that undertaken during the previous year's grant AFOSR-80-0091. Mathematically, the following condition is necessary for convergence of the discrete approximations $(\underline{u}^h, p^h)$ as $h \to 0$:

$$\sup_{\substack{\underline{v}^h \in V^h \\ |v^h|=1}} \int_\Omega q^h \operatorname{div} \underline{v}^h \geq \gamma \|q^h\| \qquad \forall\, q^h \in S^h \tag{2.1}$$

where $\gamma > 0$ is independent of h. Previously, we gave a local (elementwise) test which applied to all elements except the simplest pairs. Applying this test reveals whether a given element pair (V^h, S^h) is stable or not. Unfortunately, the simplest pairs of elements are of considerable interest in practical computing, so it is necessary to somehow prove or disprove that (2.1) is valid for them. A major result achieved has been to show that many often used low order element pairs are, in fact, unstable in the sense of (2.1). In addition, new, simple low order element pairs which we proved to be stable have been introduced. Moreover, a "postprocessing" operation has been given which can often be applied to the unstable numerical results which restabilize them. This work was carried out with Ph.D. student J. M. Boland. Mr. (now Dr.) Boland has been supported by AFOSR during his studies and graduated Ph.D. in January 1983 from Carnegie-Mellon University. He is presently Assistant Professor of Mathematics in the University of Pittsburgh. I am confident he will become an outstanding applied mathematician.

The work referred to in this section is the subject of references [1] and [2] listed below.

3. The second topic, the inviscid limit $\nu \to 0$, has lead to the following conclusions. If h is kept fixed and solutions computed for a sequence of values of $\nu \to 0$ then, as is well known, the numerical solutions develop oscillations of nonphysical character of ever increasing amplitude. Essentially, this is caused by the fact that no mechanism exists in the numerical scheme for switching off the no-slip boundary condition, so that for $\nu = 0$ we have an overdetermined system of equations. The oscillations are a manifestation of this potential overdetermination. On the other hand, if ν is kept fixed, and $h \to 0$, then mathematical theorems are available to quarantee (subject to conditions of the type 2.1) convergence of numerical solutions to the true solution, (u,p). The problem with letting $h \to 0$ is the increasingly large size of equation systems that must be solved, if ν is at all small. The approach taken by the author is based on the idea that h needs to be small only in certain locations, namely in boundary layers, and that, crucially, the thickness of such layers is always $0(\nu^{1/2})$, rather than $0(\nu)$ as would be the case for normal (as opposed to shear) layers. Normal layers do not

seem to be an issue in incompressible flows, except, possibly as a consequence of an artificially imposed outflow condition, or some similar situation. The $0(\nu^{1/2})$ thickness means that $h = 0(\nu^{1/2})$ too for stable solutions free of oscillations, and this condition is not, in practice, too severe. This latter approach is the only one known to the author which actually solves the posed problem, rather than some perturbed problem, perturbed by adding artificial viscosity of amount far larger than the physical viscosity.

Work on this topic continues, and will be reported in [3].

4. Solving the algebriac systems which arise from (1.3) remains a major difficulty, from the point of view of the amount of computational labor/cost. Two new approaches have been developed, one dependent on time marching to the steady state limit, and the other based on an adaptation of a method used in structural mechanics to the fluids case. To describe the first method, consider the iteration

$$\begin{aligned} \frac{1}{\delta t}(u^{n+1}-u^{n}) + N(u^{n}) + BP^{n+1} &= F \qquad n = 0,1,\ldots \\ B^{T}u^{n+1} &= g \qquad u^{o} \text{ arbitrary.} \end{aligned} \tag{4.1}$$

In (4.1), the first term is a discretization of $\frac{\partial u}{\partial t}$, the second denotes the viscous and convection terms evaluated at the previous time level, and B represents a discrete gradient operator. The fact that the discrete divergence operator is the transposed gradient operator is a property of the continuous problem inherited by the finite element formulation.

To solve (4.1), multiply the first equation by B^T and use the second at n and $n + 1$ to get the equation for p^{n+1},

$$B^T B p^{n+1} = B^T F - B^T N(u^k). \tag{4.2}$$

In (4.2) the variable of interest is Bp^{n+1}, rather than p^{n+1} itself, and a special technique has been devised for obtaining this vector, without first finding p^{n+1}. Once this equation is solved, u^{n+1} can be computed directly from (4.1). The procedure may now be repeated to get p^{n+2} and so on. This algorithm has worked well in practice, except that too many time steps are needed to reach the steady state. This situation is common to most marching algorithms.

The second, more direct approach which has shown considerable promise is based on the following idea. The region Ω is partitioned into subdomains Ω_i, in a way which respects element boundaries. Each subdomain will contain numerous elements. In each subdomain, the discrete Navier-Stokes equation is formed by the standard procedure. The subdomain equations will interact only across their common boundaries, and the equations for a given subdomain can be transformed in such a way as to express the interior variables for the subdomain in terms of its (unknown) boundary variables. Linking the equations for the boundary variables gives a smaller subsystem for solution. Once this subsystem is solved, the subdomain boundary values are known and so the subdomain interior values may be computed. This procedure, which has been used in other fields for many years, was not used to solve the Navier-Stokes equations because the subdomain equations are singular (due to the arbitrariness in the pressure field) and so the subdomain equations cannot be solved. A way of handling this problem has been devised and is reported in [4]. The method is highly parallelizable, because the subdomain equations can be processed simultaneously.

Papers prepared during grant period

1. Counterexample to uniform stability of bilinear-constant, velocity-pressure finite elements (with J. M. Boland) [to appear, Numerische Mathematik].
2. Stable and semistable low order finite elements for viscous flows (with J. M. Boland) [to appear, SIAM J. Num. Analysis].
3. Solution technique for discrete incompressible Navier-Stokes equations [in preparation].
4. Gaussian elimination with noninvertible pivots (with M. D. Gunsburger) [to appear, J. Linear Algebra].

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