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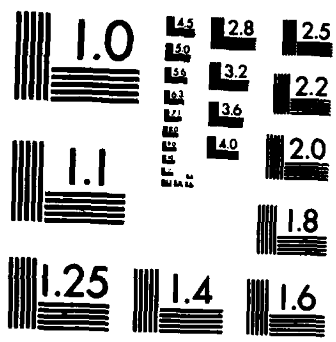

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COMMENTS ON MEASURES OF PERFORMANCE OF SELECTED SIGNAL DETECTORS

D. E. Marsh

14 July 1983

NOSC

NAVAL OCEAN SYSTEMS CENTER
San Diego, California 92152

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

This document discusses various measures of signal processor performance, their interrelationships, and the relationship between analytically and emperically derived performance models. It also presents the derivation of performance equations for a broadband coherent signal processor structured as a two channel device so as to allow study of correlated noise situations as well as of general correlation detectors beyond the constraints of matched filter or replica correlation schemes.

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RELATIONSHIPS AMONG VARIOUS MEASURES OF PERFORMANCE OF ENERGY DETECTORS

The purpose of this section is to discuss various measures of performance, including detection threshold (DT), recognition differential (RD), and minimum detectable signal (MDS). These measures have a common ground in that they represent the input ratio of signal power to noise power spectral density required to achieve a given probability of detection (PD), when signal is present, or probability of false alarm (PFA), when signal is absent. The relationship between the latter terms is often displayed graphically for various parameterizations of input signal-to-noise power ratio, processing bandwidth (W), and integration time (T) in the form of "receiver operating characteristic" (ROC) curves. Such curves depend on the processor's output statistics, whose distribution functions are usually hypothetical.

In this section an analytical model is first derived for the probability of detection of a narrowband processor. This model is then reconciled with an empirically derived model of performance.

DERIVATION OF ANALYTICAL MODEL

Consider first the statistics of a processor's output as represented by figure 1. The abscissa is the output voltage x. The probability P(x) that noise alone or signal plus noise will be observed to have the value x is given by the incremental area under the oridinate. The noise statistics have a mean μ_N and a standard deviation σ_N; the signal-plus-noise statistics have a mean μ_{S+N} and a standard deviation σ_{S+N}. The term k represents the detector threshold; that is, if a value of x greater than k is observed, a signal will be declared present. This threshold is set $d_1\,\sigma_N$ above the noise mean to achieve a desired probability of false alarm. The probability of detection will depend on the distance between threshold k and the signal-plus-noise mean. This distance is equal to $d_2\,\sigma_{S+N}$. The terms d_1 and d_2 are components of the detection index d.

When the probability function P(x) is assumed to be Gaussian, the detection index is defined as

$$d = \frac{(\mu_{S+N} - \mu_N)^2}{\sigma_N^2} \quad . \tag{1}$$

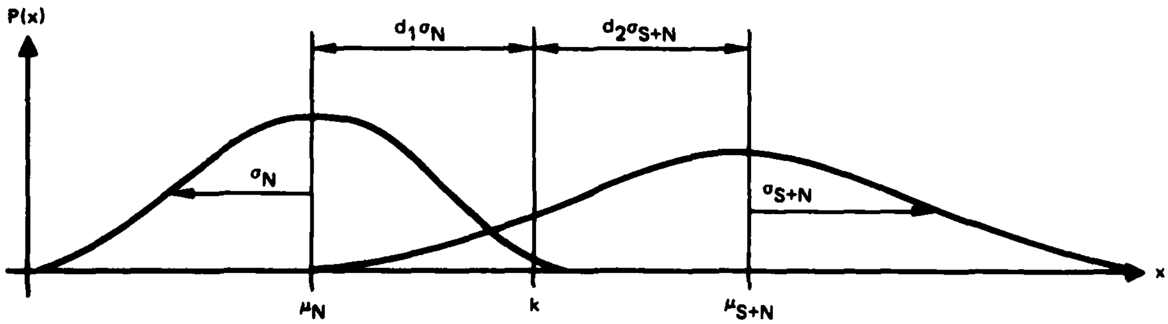

Figure 1. General representation of statistics of processor performance.

This is the parameter used by Peterson, Birdsall, and Fox,[1] whose ROC curves are presented in figure 2. In this case the probability of detection is defined by

$$PD = \left(2\pi\, \sigma_{S+N}^2\right)^{-1/2} \int_k^{\infty} \exp\left[-\frac{1}{2}\, \frac{(x - \mu_{S+N})^2}{\sigma_{S+N}^2}\right] dx. \tag{2}$$

The threshold is set at a multiple of the standard deviation of the noise above the noise mean:

$$k = \mu_N + d_1\, \sigma_N. \tag{3}$$

By a change of variable the probability of detection becomes

$$PD = (2\pi)^{-1/2} \int_{(k-\mu_{S+N})/\sigma_{S+N}}^{\infty} \exp\,(-y^2/2)\; dy. \tag{4}$$

Expressing the lower limit of equation (4) in terms of equation (3) yields

$$\frac{k - \mu_{S+N}}{\sigma_{S+N}} = \frac{\mu_N + d_1\, \sigma_N - \mu_{S+N}}{\sigma_{S+N}}$$

$$= -\left(\frac{\mu_{S+N} - \mu_N}{\sigma_{S+N}}\right) + d_1\, \frac{\sigma_N}{\sigma_{S+N}}\, , \tag{5}$$

which permits equation (4) to be rewritten as

$$PD = (2\pi)^{-1/2} \int_{-\infty}^{C} \exp\,(-x^2/2)\; dx, \tag{6}$$

where

$$C = \left(\frac{\mu_{S+N} - \mu_N}{\sigma_N}\right) \frac{\sigma_N}{\sigma_{S+N}} - d_1\, \frac{\sigma_N}{\sigma_{S+N}}\, . \tag{7}$$

All terms in the above equations except d_1 are fixed by the nature of the processor and its input. The choice of d_1 determines probability of detection and probability of false alarm.

[1] W.W. Peterson, T.G. Birdsall, and W.C. Fox, "The Theory of Signal Detectability," Trans IRE, vol PGIT-4, 1954.

For square-law processors the output statistics have the following values:

$$\mu_{S+N} = S + N, \quad \sigma_{S+N} = (S + N)/(TW)^{1/2},$$

$$\mu_N = N, \quad \sigma_N = N/(TW)^{1/2}.$$

Applying these values to equation (1) yields

$$d = TW(S/N)^2. \tag{8}$$

Solving for the ratio of input signal power to noise power spectral density yields

$$S/N_o = (dW)^{1/2}/T^{1/2}, \quad N = N_oW. \tag{9}$$

This ratio expressed in decibels is called the detection threshold, which for a square law processor is given by

$$DT = 5 \log d + 5 \log W - 5 \log T. \tag{10}$$

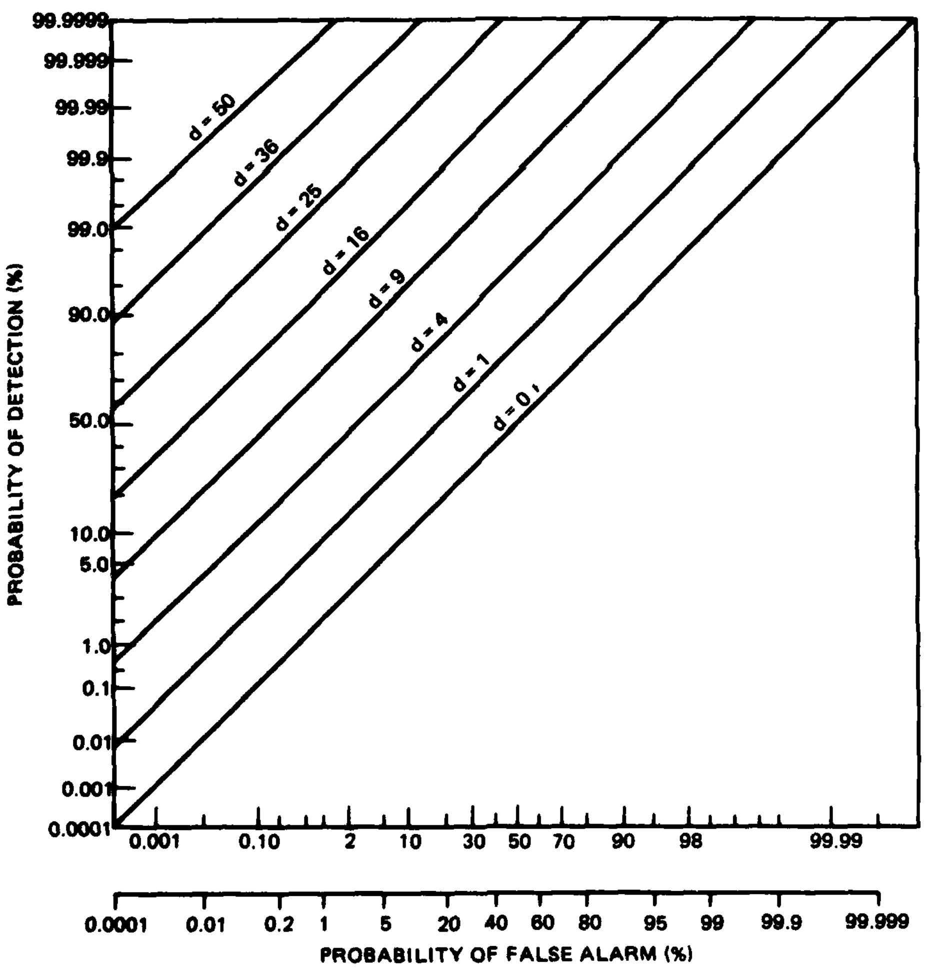

Figure 2. Receiver operating characteristic (ROC) curves for matched filter and autocorrelator.

By equation (7) it can be observed that if one chooses d_1 to be equal to the square-root of the detection index d, then C is zero, and the probability detection is 0.5. The detection threshold for this probability is sometimes referred to as the recognition differential.

If the probability P(x) is assumed to have a chi-squared distribution, the nomographs of Pryor[2] rather than the ROC curves of Peterson et al [1] are applicable. In this case performance is parameterized by a threshold detector ratio defined as the square root of equation (1), which expressed in terms of figure 1 is:

$$d_1 = \frac{\mu_{S+N} - \mu_N}{\sigma_N} . \tag{11}$$

As before, for the square-law processor, the output statistics have the following values:

$$\mu_N = N, \quad \mu_{S+N} = S + N, \quad \sigma_N = N/(TW)^{1/2} .$$

Equation (11) thus becomes

$$d_1 = (S/N)\,(TW)^{1/2} . \tag{12}$$

Solving for the ratio of input signal power to noise power spectral density yields

$$S/N_o = d_1(W/T)^{1/2} . \tag{13}$$

The minimum detectable signal defined by Pryor is this ratio expressed in decibels:

$$MDS = 10 \log d_1 + 5 \log W - 5 \log T. \tag{14}$$

Implicit in Pryor's[2] derivation of equation (14) is processor operation at a probability detection of 0.5. Pryor explicitly states that, when $d_1 = 1$, this expression yields the basic MDS and that PD = 0.5 and PFA = 0.16. This is the same result one obtains under the Gaussian hypothesis. Note that, as previously observed, if $d^{1/2} = d_1$ in equation (7), PD = 0.5. If in addition $d_1 = 1$, then the area under P(x) beyond 1 σ_N is 0.158. For these particular conditions, recognition differential (RD), detection threshold (DT), and minimum detectable signal (MDS) are the same. For other conditions detection thresholds derived from tabulations of Gaussian cumulative density functions differ slightly from minimum detectable signal levels based on chi-squared statistics. Results obtained with the more readily accessible Gaussian tables are usually conceded to converge to those from the possibly more accurate chi-squared tables when the latter have at least 20 to 30 degrees of freedom.

Table 1 shows values of detection threshold and minimum detectable signal for several probabilities of detection and false alarm for a square-law processor with 36 degrees of freedom, ie, TW = 18. It can readily be seen that at performance levels of interest the difference in the required input ratio of signal power to noise power spectral density predicted by the two statistical models is negligible. For convenience the Gaussian model will, thus, be used in the remainder of this analysis.

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[2]Naval Ordnance Laboratory, NOL-TR-71-92, Calculation of the Minimum Detectable Signal for Practical Spectrum Analyzers, by C.N. Pryor, 2 August 1971.

Table 1. Detection threshold (DT) and minimum detectable signal (MDS) as a function of probability of detection (PD) and probability of false alarm (PFA).

PD	PFA	DT (dB)	MDS (dB)
0.5	0.16	-23.3	-23.3
0.5	10^{-4}	-17.6	-16.6
0.5	10^{-6}	-16.5	-15.1
0.9	10^{-6}	-13.9	-13.5

RECONCILIATION WITH EMPIRICAL MODEL

The foregoing analytic measures of performance must now be reconciled with empirical measures. Equation (15) shown below is a widely used empirically derived model

$$PD = (2\pi)^{-1/2} \int_{-\infty}^{SE/\sigma_{SE}} \exp(-y^2/2)\, dy, \quad (15)$$

where signal excess SE (in dB) is normally distributed with zero mean and standard deviation σ_{SE} of 7.8 dB. The performance indicated by equation (15) is at an unspecified but low probability of false alarm.

The analytic expression of equation (6) takes into account only the statistical behavior signals of constant power masked by noise of constant power, whereas the empirical model of equation (15) includes the composite effect of waveform statistics and power fluctuations, as well as the effect of the human operator on the detection process. At the same time, however, the analytical and empirical expressions are of the same form. Thus, if one equates the upper limit of the integrals, one obtains

$$C = SE/\sigma_{SE}, \quad (16)$$

where C is as defined in equation (7). If one now notes from figure 1 that

$$\mu_{S+N} - \mu_N = d_1\, \sigma_N + d_2\, \sigma_{S+N}, \quad (17)$$

and if one then substitutes equations (16) and (17) into equation (7), one obtains

$$d_2 = SE/\sigma_{SE}. \quad (18)$$

Furthermore, one may note that the ratio of signal power-to-noise power spectral density at the processor input is given by

$$S/N_o = SE + DT = SL - PL + AG - N_o. \quad (19)$$

Where SL is the source level of the signal, PL is propagation loss, and AG is array gain in accord with usual sonar equation definitions.

Using equation (18) one may rewrite this expression as

$$S/N_o = (d_2 \sigma_{SE} + DT) . \tag{20}$$

Power fluctuations are thus shown to require a straightforward increase in input signal-to-noise power ratio with respect to the detection threshold predicted solely by the analytical model. The amount of increase depends on the empirical constant σ_{SE} (7.8 dB) and the factor d_2 used to establish the processor's probability of detection in the absence of fluctuations.

DERIVATION OF PERFORMANCE EQUATIONS FOR A BROADBAND COHERENT RECEIVER

The general problem is one of binary hypothesis testing, where

$$H_0 \quad \begin{cases} r_1(t) = n_1(t) \\ r_2(t) = n_2(t) \end{cases}$$

$$H_1 \quad \begin{cases} r_1(t) = s_1(t) + n_1(t) \\ r_2(t) = s_2(t) + n_2(t) \end{cases}$$

We choose to use as test statistic the finite-time cross-correlation between the two received signals given by

$$\chi = \int_0^T r_1(t)\, r_2(t)\, dt \tag{21}$$

We recognize that, in general, this is not an optimal test statistic for deciding which hypothesis is true. The statistics for χ will not, in general, be Gaussian even though all signals and noises can be Gaussian. Thus, the applicability of the performance index d, particularly as an output signal-to-noise ratio, must be viewed cautiously. The general formulations and signal representations are taken from Urkowitz[3], who addressed the question of energy detection of one signal. The following extensions were derived in collaboration with Dr. E.L. Titlebaum, of the University of Rochester.

Assume that the received signals are deterministic and have been spectrally limited by a low-pass filter whose bandwidth is W Hz. The noise signals are assumed to be statistically independent white Gaussian processes. This may require inclusion of a prewhitening filter in each channel. The noise signals are assumed to have zero means with *two-sided* power density spectra

$$S_{n_1}(\omega) = N_1 \text{ watts/Hz}$$

[3]H. Urkowitz, Energy Detection of Unknown Deterministic Signals, Proc. IEEE, vol 55, pp 523–531, April 1967.

and

$$S_{n_2}(\omega) = N_2 \text{ watts/Hz}$$

over the band of W Hz.

Following Urkowitz[3], we represent each noise voltage with 2TW samples and form

$$n_i(t) = \sum_{k=1}^{2TW} a_i(k) \operatorname{sinc}(2Wt - k), \qquad i = 1, 2 \tag{22}$$

where

$$a_i(k) = n_i(k/2W) .$$

For each i, the set of noise samples $\{a_i(k)\}$ is assumed to be zero-mean, uncorrelated Gaussian random variables with identical variance

$$\operatorname{var}[a_i(k)] = \sigma_{n_i}^2 = 2N_iW . \tag{23}$$

We also have that

$$\int_0^T n_i^2(t)\, dt = \frac{1}{2W} \sum_{k=1}^{2TW} a_i^2(k) . \tag{24}$$

The signal processes which are assumed to be constant with respect to any ensemble average, eg,

$$E[s_i(t)] = s_i(t) .$$

are similarly sampled every 1/2W second yielding the expansion

$$s_i(t) = \sum_{k=1}^{2TW} S_i(k) \operatorname{sinc}(2Wt - k), \qquad i = 1, 2, \tag{25}$$

where

$$S_i(k) = s_i(k/2W) ,$$

and has moments

$$E[S_i(k)\, S_i(\ell)] = \sigma_i^2\, \delta_{k\ell} , \tag{26}$$

$$E[S_1(k)\, S_2(\ell)] = \rho\, \sigma_1\, \sigma_2\, \delta_{k\ell} , \tag{27}$$

where

$$\delta_{k\ell} = \begin{cases} 1, k = \ell \\ 0, k \neq \ell \end{cases}$$

We also have that

$$\int_0^T s_i^2(t)\,dt = \frac{1}{2W} \sum_{k=1}^{2TW} S_i^2(k) \triangleq E_i \tag{28}$$

and

$$\int_0^T s_1(t)\, s_2(t)\,dt = \frac{1}{2W} \sum_{k=1}^{2TW} S_1(k)\, S_2(k) \triangleq \rho \sqrt{E_1 E_2}. \tag{29}$$

Here ρ is interpreted as the projection of one (energy normalized) signal on the other. We will express the final answer for d in terms of the average power for each signal using the relation

$$P_i = E_i/T . \tag{30}$$

Now we define d as

$$d \triangleq \frac{\{E[\chi|H_1] - E[\chi|H_0]\}^2}{\text{var}\,[\chi|H_1]}, \tag{31}$$

Finally the test statistic, χ, may be expressed as

$$\chi = \frac{1}{2W} \sum_{k=1}^{2TW} r_1\,(k/2W)\, r_2\,(k/2W)\,, \tag{32}$$

where under H_0

$$r_i(k/2W) = a_i(k), \quad i = 1, 2 \tag{33}$$

and under H_1

$$r_i(k/2W) = S_i(k) + a_i(k), \quad i = 1, 2 \tag{34}$$

and we proceed to calculate the quantities in this expression.

1. $E[\chi | H_0]$

Since both signals and noises may be correlated but still independent of each other the first quantity becomes

$$E[\chi|H_0] = \frac{1}{2W} \sum_{k=1}^{2TW} E[a_1(k)\, a_2(\ell)] \tag{35}$$

$$E[a_1(k)\, a_2(\ell)] = \rho_N\, \sigma_{n_1}\, \sigma_{n_2}\, \delta_{k\ell}\,. \tag{36}$$

Using equation (36) in (35) we have that

$$E[\chi|H_0] = T\, \rho_N\, \sigma_{n_1}\, \sigma_{n_2}\,. \tag{37}$$

2. $E[\chi | H_1]$

For the second term we have

$$E[\chi|H_1] = \frac{1}{2W} \sum_{k=1}^{2TW} E\left\{[S_1(k) + a_1(k)]\ [S_2(k) + a_2(k)]\right\} \tag{38}$$

$$= \frac{1}{2W} \sum_{k=1}^{2TW} S_1(k)\, S_2(k) + T\, \rho_N\, \sigma_{n_1}\, \sigma_{n_2}\,,$$

and using equation (29) we obtain

$$E[\chi|H_1] = \rho \sqrt{E_1 E_2} + T\, \rho_N\, \sigma_{n_1}\, \sigma_{n_2}\,, \tag{39}$$

3. $E[\chi^2 | H_1]$

For the term $E[\chi^2 | H_1]$ we have

$$E[\chi^2|H_1] = \frac{1}{4W^2} \sum_{k,\ell=1}^{2TW} E\left\{[S_1(k) + a_1(k)]\ [S_2(k) + a_2(k)]\right.$$

$$\left.\cdot\ [S_1(\ell) + a_1(\ell)]\ [S_2(\ell) + a_2(\ell)]\right\}. \tag{40}$$

There are 16 terms in the summand which can be characterized as follows:

1. $S_1(k)\, S_2(k)\, S_1(\ell)\, S_2(\ell)$
2. $S_1(k)\, S_2(k)\, S_1(\ell)\, a_2(\ell)$
3. $S_1(k)\, S_2(k)\, a_1(\ell)\, S_2(\ell)$
4. $S_1(k)\, S_2(k)\, a_1(\ell)\, a_2(\ell)$
5. $S_1(k)\, a_2(k)\, S_1(\ell)\, S_2(\ell)$
6. $S_1(k)\, a_2(k)\, S_1(\ell)\, a_2(\ell)$
7. $S_1(k)\, a_2(k)\, a_1(\ell)\, S_2(\ell)$
8. $S_1(k)\, a_2(k)\, a_1(\ell)\, a_2(\ell)$
9. $a_1(k)\, S_2(k)\, S_1(\ell)\, S_2(\ell)$
10. $a_1(k)\, S_2(k)\, S_1(\ell)\, a_2(\ell)$
11. $a_1(k)\, S_2(k)\, a_1(\ell)\, S_2(\ell)$
12. $a_1(k)\, S_2(k)\, a_1(\ell)\, a_2(\ell)$
13. $a_1(k)\, a_2(k)\, S_1(\ell)\, S_2(\ell)$
14. $a_1(k)\, a_2(k)\, S_1(\ell)\, a_2(\ell)$
15. $a_1(k)\, a_2(k)\, a_1(\ell)\, S_2(\ell)$
16. $a_1(k)\, a_2(k)\, a_1(\ell)\, a_2(\ell)$

Taking expectations of each we have 16 terms, eight of which are nonzero. We will simply catalogue the nonzero results here:

$$\text{Term } 1 = \rho^2 E_1 E_2 .$$

$$\text{Term } 4 = T\, \rho_N\, \sigma_{n_1}\, \sigma_{n_2} \cdot \rho \sqrt{E_1 E_2} .$$

$$\text{Term } 6 = \frac{1}{2W} E_1\, \sigma_{n_2}^2 .$$

$$\text{Term } 7 = \frac{1}{2W} \rho_N\, \sigma_{n_1}\, \sigma_{n_2} \cdot \rho \sqrt{E_1 E_2} .$$

Term 10 = term 7 .

$$\text{Term } 11 = \frac{1}{2W} E_2\, \sigma_{n_1}^2 .$$

Term 13 = term 4 .

$$\text{Term } 16 = E[\chi^2 | H_0] .$$

Summing all these results yields that

$$E[\chi^2 | H_1] = \rho^2 E_1 E_2 + 2 \left[T\, \rho_N\, \sigma_{n_1}\, \sigma_{n_2}\, \rho \sqrt{E_1 E_2} \right] + \frac{1}{2W} \left[\sigma_{n_1}^2 E_2 + \sigma_{n_2}^2 E_1 \right] + \frac{1}{W} \left[\rho_N\, \sigma_{n_1}\, \sigma_{n_2}\, \rho \sqrt{E_1 E_2} \right] + \frac{T\, \sigma_{n_1}^2\, \sigma_{n_2}^2}{2W} \left[1 + (1 + 2TW)\, \rho_N^2 \right] . \tag{41}$$

4. *Var [χ|H₁]*

We calculate this by subtracting from equation (41) the square of (39) which after eliminating common terms yields that

$$\mathrm{var}\,[\chi|H_1] = \frac{1}{2W}\left\{\sigma_{n_1}^2 E_2 + \sigma_{n_2}^2 E_1 + 2\rho\sqrt{E_1E_2}\,\rho_N\,\sigma_{n_1}\sigma_{n_2} + T\,\sigma_{n_1}^2\sigma_{n_2}^2\left[1+\rho_N^2\right]\right\}. \tag{42}$$

Now we may calculate d to be

$$d = \frac{2W\rho^2 E_1E_2}{\sigma_{n_1}^2 E_2 + \sigma_{n_2}^2 E_1 + 2\rho\sqrt{E_1E_2}\,\rho_N\,\sigma_{n_1}\sigma_{n_2} + T\,\sigma_{n_1}^2\sigma_{n_2}^2\left(1+\rho_N^2\right)} \tag{43}$$

Now using equation (30), so that we have everything expressed in terms of average powers, results in

$$d = 2TW\,\frac{\rho^2}{\left(\frac{\sigma_{n_2}^2}{P_2}\right) + \left(\frac{\sigma_{n_1}^2}{P_1}\right) + 2\rho\rho_N\left(\frac{\sigma_{n_1}^2\sigma_{n_2}^2}{P_1P_2}\right)^{1/2} + \left(\frac{\sigma_{n_1}^2\sigma_{n_2}^2}{P_1P_2}\right)\left(1+\rho_N^2\right)} \tag{44}$$

We now apply equation (45) in several special cases.

Case 1

For the case of equal signal-to-noise ratios $\left(\sigma_{n_1}^2/P_1\right) = \left(\sigma_{n_2}^2/P_2\right) = \left(\sigma_n^2/P\right)$, we obtain

$$d = 2TW\,\frac{\rho^2}{2\left(\frac{\sigma_n^2}{P}\right) + 2\rho\rho_N\left(\frac{\sigma_n^2}{P}\right) + \left(\frac{\sigma_n^2}{P}\right)^2\left(1+\rho_N^2\right)}$$

Case 1a

For the case of uncorrelated noise, $\rho_N = 0$, then we have

$$d = 2TW\rho^2\,\frac{\left(\frac{P}{\sigma_n^2}\right)^2}{1 + 2\left(\frac{P}{\sigma_n^2}\right)}$$

Case 2

For the case of energy detection of one channel with low signal-to-noise ratio, where $\rho = \rho_N = 1$, $P_1 = P_2 = P$, $\sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2$, and $(\sigma_n^2/P \gg 1)$, then we obtain

$$d \cong TW \left(\frac{P}{\sigma_n^2}\right)^2 .$$

This is the usual result for a simple square law detector.

Case 3

For the case of one high signal-to-noise ratio and one low, $\sigma_{n_1}^2/P_1 \gg 1$, $\sigma_{n_2}^2/P_2 \ll 1$, we obtain

$$d \cong 2TW \rho^2 \left(\frac{P_1}{\sigma_{n_1}^2}\right) .$$

which corresponds to the result for a matched filter detector.

The results are comparable to those found in Urick[4], except that he did not consider any cases for which the signal components in each channel were, in any way, different. Thus, to compare results, ρ^2 must be set to unity.

[4]R.L. Urick, Principles of Underwater Sound for Engineers, New York, McGraw-Hill, chap 12, 1967.

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