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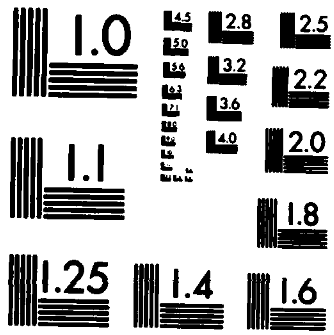
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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

THE FORECASTING OF FUTURE INVENTORY AND THE OPTIMIZATION OF TRAINING REQUIREMENTS WITHIN THE AIRBORNE COMMUNITY

by

Donald Bruce Chung

December 1983

Thesis Co-advisors: G. T. Howard
P. R. Milch

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The Forecasting Of Future Inventory And The Optimization Of Training Requirements Within The Airborne Community

by

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ABSTRACT

In an era of modernization, new weapons systems generate new manpower requirements for the airborne community within the United States Army. The problem of forecasting yearly requirements and inventories has become increasingly complex. This thesis formulates a methodology which applies the Markov Theory to manpower planning in order to forecast yearly inventories. It also discusses the strategy of dynamic programming in determining the optimal numbers of soldiers with certain skill levels and job types whc should enter into each type of special training. Further, this methodology is applied to the Career Management Fields of 11 and 13 in forecasting inventories for fiscal years 1985 and 1986 and in determining the optimal numbers of soldiers to enter into each type cf special training within the airborne community.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

In the late 1970's, in face of the surmounting Soviet Threat, the United States Army began to modernize its force by developing hardware commensurate with the present-day levels of technology. New fighting platforms and systems were developed and procured. Concurrently, new manpower requirements for specialized training were developed. Key questions of whom to train and how to train them had to be answered in order to maximize the total effectiveness of both machine and organization.

The problem of determining the maximum number of personnel who can enter into special training from the different types of jobs and classes of military manpower has become increasingly complex as the United States Army continues its force modernization toward the end of the decade of the eighties. It also has become increasingly critical to forecast the yearly requirements for personnel to be qualified by special training because of the fiscal restrictions established by Congress pertaining to such training. The task is further complicated when those requirements are desired by type of job and grade level.

Currently, force modernization has its greatest impact on the airborne community which encompasses personnel of all types of special training involving military parachuting. The preceding manpower problems are further complicated in this community by school capacity and budget constraints. The remainder of this chapter provides pertinent background information about the airborne community whose personnel analysis is the subject of this thesis and defines the problems peculiar to the airborne community. It also develops the critical questions pertaining to the number of personnel who can undergo each type of special training from among the different types of jobs and grade levels.

Chapter 2 discusses the formulation of the model. It highlights the theoretical and mathematical concepts pertinant to the model formulation, and formulates the design of the model which will forecast inventory end-strength requirements for each type of special training and projects those requirements by job type and grade level into future years by utilizing Markov Chain theory. Additionally, it examines the application of dynamic programming as a viable optimization strategy in determining the special training requirements in the airborne community. Aggregation of the forecasting and optimization phases of the overall model and a description of their interface will be discussed in the remainder of the chapter.

Chapter 3 discusses the execution of the model. Sensitivity analysis of the data is applied to the budget constraint, attrition factors, promotion and recruitment rates.

Chapter 4 discusses the potential of the model as a decision making tool and as a manpower planning model.

A. THE AIRBORNE COMMUNITY

1. Military Occupation Specialty (MOS)

Each soldier in the United States Army is awarded a military occupation specialty (MOS) upon the completion of basic training (BT) and advanced individual training (AIT). Both types of training are designed to provide the soldier with the basic skills of his specific job. Each MOS refers to a specific type of job and job skill. (eg. cook (94B), mechanic (63B), mortarman (11C), infantryman (11B)). There are 363 MOS's for which a soldier may be trained.

2. Skill Qualification Identifier (SQI)

After attaining the basic job skills and being awarded a MCS, a soldier can undergo special training which will award a skill qualification identifier (SQI) upon successful completion of that training. There are twenty-nine SQI's within the United States Army. This skill identifier indicates that the individual is qualified to perform some specific skill different from his basic job type. For example, an infantryman (11B) who successfully completes airborne training is a qualified military parachutist and is awarded the SQI of 'P'. This soldier's complete job type and job skills would then be 11B-P.

3. Grade and Skill Levels

A soldier in the United States Army may be promoted through various grade levels. Initially, a soldier enters the service at grade level one. In order to be promoted to the next grade level, an individual must undergo a specific selection process. This selection process is based upon his past performance, leadership qualities, and time in grade and time in service criteria. This selection process for promotion is similar at each grade level. For example, A sergeant (E-5) is promoted to the grade level of staff sergeant (E-6) once he has 12 months time in grade and 36 months time in service and he is selected by a centralized selection board. The nine enlisted grade levels as correlated with rank are listed below.

GRADE	RANK
E-1	Private
E-2	Private
E-3	Private First Class
E-4	Specialist
E-5	Sergeant
E-6	Staff Sergeant

E-7	Sergeant First Class
E-8	Master Sergeant
E-9	Sergeant Major

Additionally, the soldier may move to the next skill level (SL) which is partially based upon demonstrated technical and tactical competency. These skill levels are closely aligned to the grade levels. Movement to the next skill level is based upon grade promotion. Maintaining the grade level is partially a function of demonstrating competency in the corresponding skill level. For example, An infantry sergeant, E-5, (SL 2) who is promoted to staff sergeant, E-6, (SL 3) must demonstrate technical competency at that skill and grade level. The relationship between skill and grade levels is listed below.

GRADE	SKILL LEVEL
E-1 to E-4	1
E-5	2
E-6	3
E-7	4
E-8 to E-9	5

4. Career Management Field (CMF)

Each soldier in the United States Army has a career pattern that he can follow. This pattern is a network of jobs as specified by MOS and SL. Examples of this career progression are diagrammed in Figures 1.1 and 1.2 and represent the career progressions for CMF 11 and CMF 13, respectively. There are twenty-eight CMF's in the United States Army. Only CMF 11 and CMF 13 will be considered in this thesis.

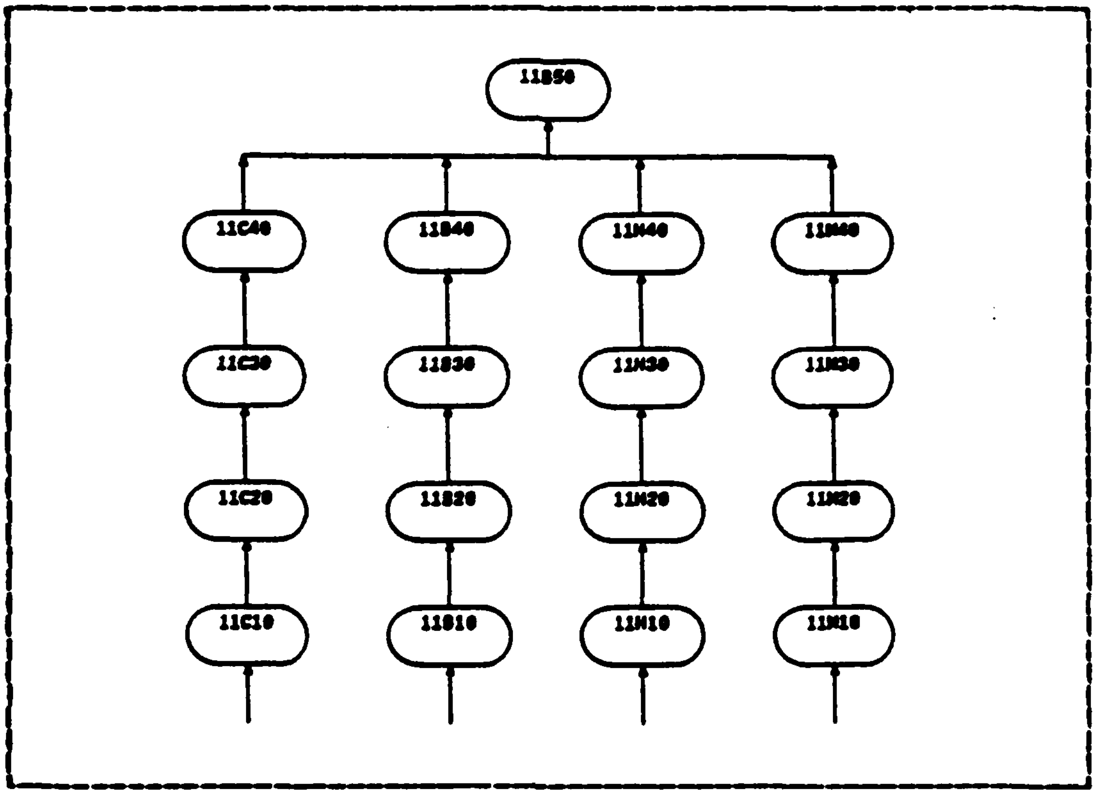


Figure 1.1 Career Progression Pattern for CMF 11.

5. <u>Special Training Within the Airborne Community</u>

The airborne community consists of four types of special training:

A) Airborne--referring to the 82nd ABN Division, 502 ABN Brigade, XVIII ABN Corps, U.S.Central Command (USCENCOM), and other miscellaneous units (A total of 20,000 soldiers)

B) Ranger--referring to 2 battalions. (1500 soldiers)

C) Pathfinder--referring to a small unit of no more than 40-50 soldiers.

D) Special Forces--referring to the 5th, the 7th, and the 10th Special Forces Groups, and the JFK

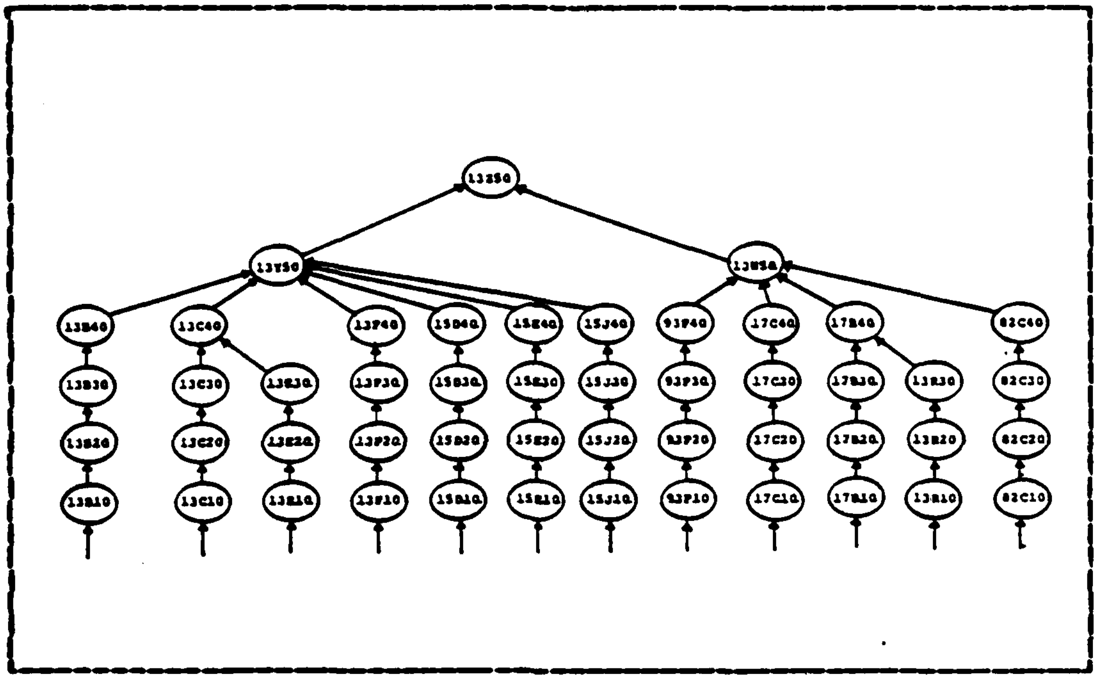


Figure 1.2 Career Progression Pattern for CMF 13.

Center and miscellaneous units. (A total of 6000 soldiers)

Airborne training awards the skill qualification for basic military parachuting. Each graduate of this training is awarded the SQI of 'P'. A soldier is paid an additional $85 while he is in training. Once a soldier completes the training, he is paid $85 per month while he remains as an active military parachutist. Active military parachuting or "on jump status" is defined as having conducted at least one jump in a ninety day period.

Recruitment for airborne training largely consists of soldiers from among MOS 11B and the SL's 1 and 2. Recruitment exists from among soldiers of all MOS's and SL's.

Ranger training awards the skill qualification for small unit operations and requires soldiers to be parachutist qualified prior the the conduct of training. Each graduate is awarded the SQI of 'V'. A soldier is paid an additional $85 per month while he is in training.

Recruitment for ranger training mainly occurs from among MOS 11B and SL's 2 and 3. However, recruitment is not conducted from soldiers of SL 1.

Special Forces training awards the skill qualification for proficiency in conducting covert operations and requires soldiers to be parachutist qualified prior to the conduct of training. Each graduate is awarded the SQI of 'S'. A soldier is paid an additional $85 per month while he is in training.

Recruitment for Special Forces training largely consists of soldiers from among MOS 11B and SL's 3 and 4. Recruitment occurs from among all categories of MOS and SL.

6. Duty Position

In the United States Army, each soldier holds a duty position which is a job category specified by his MOS, SL, and SQI. In this thesis, reference to duty position will denote a particular specification of MOS, SL, and SQI, in that order. The SL designation used is a two-digit code with the first digit being the skill level and the second digit being a zero. For example, a soldier with a skill level of one will have a SL of 10. If his MOS is 11B-infantryman and his SQI is P-parachutist, then that soldier's duty position is 11B10P.

7. Authorizations Within the Airborne Community

The force vacancies created by attrition and the personnel movements within the airborne community reflect the total shortages between the authorized inventory level and the on-hand inventory level. All shortages can be calculated by MOS, skill level, and SQI.

Shortages = Authorized Inventory - Current Inventory

These levels of authorization by year are given by the Personnel Structure and Composition System (PERSACS) document and will be assumed to be known and non-negotiable. Levels of authorization do not necessarily equal the strengths authorized by the Table of Organization and Equipment (TOE) documents. The PERSACS generates the authorizations of each duty position specified by MOS, SQI and grade level and reflects levels annually determined based upon current military levels, missions and budget constraints. For example, the 82d Airborne Division has a rapid deployment mission and must be constantly manned well above the TOE strength levels. The PERSACS normally generates authorizations for that unit which are either at or above TOE strength levels.

Changes in the force structure may change the authorized levels and will be reflected in the PERSACS. For this study, it will be assumed that changes to the force structure will be reflected in each year's authorization levels. If the authorization level is 80% of the TOE authorized strength level, then all shortages in each category will be filled to the community's authorization level (80% of the strength level specified by the TOE). The assumption will be made that the manpower pool factor is also incorporated into the desired stock levels, as explained below.

8. <u>Manpower Pool Factor</u>

In order to support the assignment policy of rotation in and out of the airborne community, the manpower pool factor was created. This simply requires that for a soldier to have the opportunity to professionally develop, there must exist another soldier who is qualified to fill the vacancy created upon the former's departure. For example, an Infantry sergeant, E-5, now serving in the 82nd Airborne Division, must have a qualified replacement serving in a position outside the airborne community.

9. <u>Attrition Within the Airborne Community</u>

A shortage within the airborne community can result from personnel who leave the community by either conducting an expiration of term of service (ETS) move, or by retirement. ETS movement can occur by:

1) Voluntary departure once an individual's obligation is met.

2) Involuntary departure as a result of administrative or punitive discharge. (e.g. an individual is discharged for the good of the service under provisions of Army Regulation 600-200, Chapter 10, or he is dishonorably discharged under courts-martial.)

Shortages may also result when personnel conduct a permanent change of station (PCS) out of the airborne community. This attrition is a voluntary reassignment out of the airborne community and a soldier automatically revokes (terminates) his military parachutist qualification. For example, a soldier in the 82nd Airborne Division may no longer want to be a military parachutist and requests reassignment to the 2nd Armored Division, a unit outside of the airborne community. Prior to his assignment, he must voluntarily withdraw the military parachutist qualification and the SQI of 'P' from his official military record.

10. Personnel Movements Within the Airborne Community

Shortages may also result from promotions and demotions within each segment of the community. For example, a sergeant, E-5, when promoted to staff sergeant, E-6, creates a vacancy for an E-5 in his segment of the community.

Transfers within the community result from individuals who conduct a permanent change of station (PCS) from one segment of the community to another segment of the community. For example, A sergeant, E-5, from the 2d Ranger Battalion is reassigned to the 82nd Airborne Division. When this type of transfer occurs, a shortage in the losing segment's community results, while a shortage in the gaining segment of the community is filled. Note that these reassignments by PCS are both voluntary and involuntary, and are unforseen at the beginning of the year. It will be assumed all intra-community PCS assignments are negligible. The present policy of assignment is that those soldiers qualified in any SQI of the airborne community will rotate in and out of the community in subsequent assignments. This is to insure the professional development of the soldier and is in accordance with the "whole man" concept.

11. Inventory Levels Within the Airborne Community

On-hand inventory levels are recorded by the United States Army Military Personnel Center (MILPERCEN) as ending inventory levels of the fiscal year (FY). These inventory levels are recorded by MOS, skill level, and SQI. Ending inventory levels for a year will be assumed to be the same as the beginning inventory levels for the following year. For example, ending inventory level for the FY 1978 will be the same as the beginning inventory level for FY 1979.

12. Funding of Special Training Within the Airborne Community

The funding for each type of special training in the airborne community is extracted from the congressional budget which is allocated to the U.S. Army for the purpose of paying the hazardous duty pay for those soldiers on "jump status" and for those soldiers undergoing special training peculiar to the airborne community.

13. Frequency of Training and School Capacity

The three types of special training in the airborne community are conducted at different times during the year and vary in class size and length. Airborne training is a three week course beginning every four weeks except during two weeks in December. Fifty classes are cycled throughout the year with each class limited to 400 soldiers. Ranger training is conducted five times per year with each class limited to 200 soldiers. The training period is eight weeks in duration. Special Forces training is conducted twelve times per year with each class limited to 100 soldiers. The training period is twelve weeks in duration.

B. PROBLEM DEFINITION

In the United States Army, the necessity to maintain detailed inventories of qualified personnel within the airborne community has required manpower planners to develop techniques and models to forecast the force by each duty position and to predict the training requirements needed to maintain the prescribed inventory levels for each duty position. These two requirements dictate that MILPERCEN, the proponent for personnel assignments, be able to produce timely predictions of future force levels, training requirements, and the effects of any change within and outside the airborne community.

1. Forecasting Future Stocks

Manpower planning is matching the supply of people with the jobs available [Ref. 1]. This is particularly applicable in the airborne community where the duty position is the composite specification of MOS, skill/grade level, and SQI. Those duty positions which are vacant at the end of the fiscal year are the jobs available for the following fiscal year. In the airborne community, each duty position (e.g. 11B10P--Infantry private, parachutist qualified) can be considered as a specific state into which a soldier can be recruited or promoted and out of which he can be promoted or attrited.

Attrition is the most fundamental of all flows [Ref. 2]. Attrition in the airborne community is both voluntary and involuntary. In this case, attrition may include intra-community and inter-community transfers resulting in PCS and/or ETS movements which can be voluntary or involuntary and have a high degree of variability.

The objective of forecasting is to predict the future inventory levels of the airborne community given total recruitment into the community and current flows within and out of the community.

2. Optimization of Training Requirements

Once the ending inventory levels are predicted for a specific time period, shortages in certain job types can be determined. The budget with which to train new soldiers and maintain the current force levels sets limits on how many soldiers can enter into special SQI training. Additionally, the capacity of the school which conducts each type of special SQI training restricts the number of soldiers who can enter into that respective SQI training. The questions to be answered are:

1) How many soldiers should enter each type of special SQI training?
2) What duty positions should these soldiers fill?
3) How many soldiers will the budget and school capacity allow to enter into special SQI training?

C. OVERVIEW

The remaining chapters of this thesis will formulate a model to forecast future force levels in the airborne community by duty postion and to determine the number of soldiers to be trained by duty postion. Only those duty positions of CMF 11 and CMF 13 will be considered in the discussion, construction, and execution of the model.

II. MODEL FORMULATION

The formulation of the model will consist of the development of two different sub-models which together will forecast future stocks and determine the optimal levels to which training requirements can be filled.

A. FORECASTING MODEL

Markov analysis can be used to predict future movements of personnel and end strength inventories by duty position in the airborne community. For the remainder of this thesis, the fiscal year (FY) will be the time period considered. To implement this analysis, certain assumptions about the airborne community must be met.

An assumption required by a Markov process is that the end strength inventories of each duty position within the airborne community are dependent only on the beginning strength inventories of each duty position and the promotion, attrition, and recruitment flows during the fiscal year.

A second assumption is that each member of the personnel system is subject to only one flow during a single fiscal year. This assumption may be violated in reality as some soldiers may be promoted and reclassified to a new MOS within the same year. However, the frequency of this occurrence is very small and is normally prohibited by existing policies. A soldier who is reclassified into a new MOS is normally withheld from promotion consideration while a soldier who is promoted is restricted from changing his MOS. However, a soldier may be promoted and attrited from the airborne community within the fiscal year. This occurrence is largely limited to SL 1. Promotions in higher SL's incur an additional time-in-service obligation and preclude attrition during the same fiscal year.

A third assumption is that the fractional flows within and out of the system remain constant over the time interval for which the forecast is being made. In reality, these proportions will change yearly. In Chapter 4, the relaxation of this assumption will be discussed in further detail.

1. <u>The Development of the Markov Process</u>

The Markov process is based upon the equation

$$\underline{N}(t) = \underline{N}(t-1)P + \underline{R} \qquad \text{(eqn 2.1)}$$

where P is the transition matrix whose individual elements p_{ij} represent the fractional flow rate with which personnel from a particular duty position i move to another duty position j during the fiscal year. $\underline{N}$ represents the force level vector whose individual components n_i are the numbers of soldiers in duty position i at the beginning of a particular fiscal year [Ref. 3]. $\underline{R}$ represents the recruitment vector whose individual elements r_i are the numbers of soldiers who enter into duty position i at the end of a particular fiscal year.

The P-matrix is a representation of the interrelationships among the MOS's, SL's, and SQI's. If there were only one MOS and one SQI then the transition matrix would be just a representation of the existing promotion policies and attritions from each duty position of the overall force. For example, if the MOS was 11B-Infantryman and the SQI was P-Parachutist, then the duty positions would be 1) 11B10P, 2) 11B20P, 3) 11B30P, 4) 11B40P, 5) 11B50P. The corresponding transition matrix is listed in Figure 2.1 and represents the promotion and staying rates within each skill level of the airborne community consisting of one MOS and one SQI.

SL	1	2	3	4	5
1	X	X			
2		X	X		
3			X	X	
4				X	X
5					X

Note: Only non-zero elements are indicated by an 'x'.

Figure 2.1 Transition Matrix I.

When no interrelationships exist between the SQI's or the MOS's then a series of separate transition matrices for each MOS-SQI combination is generated. The current force level vector N can be partitioned into smaller separate components. Each component is passed through its corresponding transition matrix of the type shown in Figure 2.1. The resulting force levels of the two components are then aggregated to form the predicted force level vector at the following time period. For example, if the MOS's were 11B-infantryman and 13B-cannon crewman, the SQI was P-parachutist for both, and the SL's were 1-5, then the duty positions would be as listed in Table I.

The force level vector $\underline{N}$ would consist of all duty positions as listed in Table I. But, since no interdependence between the two MOS's exist, the force level vector $\underline{N}$ can be decomposed by MOS into two smaller force level

TABLE I

Duty Position

MOS	SL	SQI
11B	10	P
11B	20	P
11B	30	P
11B	40	P
11B	50	P

MOS	SL	SQI
13B	10	P
13B	20	P
13B	30	P
13B	40	P

vectors $\underline{N}_1$ and $\underline{N}_2$ which correspond to MOS 11B and MOS 13B, respectively. These force level vectors can be expressed as:

$$\underline{N}_1 = (11B10P, 11B20P, 11B30P, 11B40P, 11B50P)$$

$$\underline{N}_2 = (13B10P, 13B20P, 13B30P, 13B40P)$$

Given that the current FY is designated by t-1, the force level vector N(t) of the next FY can be determined as $\underline{N}(t) = (\underline{N}_1(t), \underline{N}_2(t))$ where the force lavel vectors $\underline{N}_1(t)$ and $\underline{N}_2(t)$ are computed as:

$$\underline{N}_1(t) = \underline{N}_1(t-1)P1 + \underline{R}$$

$$\underline{N}_2(t) = \underline{N}_2(t-1)P2 + \underline{R}\ .$$

Here, P1 and P2 represent the transition matrices similar to transiticn matrix I.

Interrelationships among several SQI's within cne MOS create composite matrices. If the MOS remains fixed, but the SQI's and grade/skill levels vary, a composite matrix results due tc the interdependence of the two SQI's. In this case, a composite matrix would be generated.

For example, if the MOS was 11B-Infantryman, the SQI's were P-parachutist and V-ranger, and the skill levels were 1-5, then the duty positions for that type of system would be as listed in Table II.

TABLE II

Duty Positions

MOS	SL	SQI	MOS	SL	SQI
11B	10	P	11B	10	V
11B	20	P	11B	20	V
11B	30	P	11B	30	V
11B	40	P	11B	40	V
11B	50	P	11B	50	V

In this example, the SQI's have a fiscal relationship as described in Chapter 1. The composite matrix listed in Figure 2.2 represents the matrix which is appropriate in this case.

Similarly, a composite matrix is created if there is an interdependence between a set of MOS's. If such a relationship exists then the current force level vector cannot be partitioned and a composite matrix results. The actual structure of the matrix is a function of the number of MOS's, SQI's, and grade/skill levels.

For example, if the MOS's were 11B-infantryman and 11C-mortarman, with the SQI's and skill levels the same as in the previous example, then the duty positions listed in Table III exist.

The matrix listed in Figure 2.3 is the composite matrix that is then generated and is used to predict force levels of the overall system. In the airborne community,

DUTY POSITION	i	1	2	3	4	5	6	7	8	9	10
11B10P	1	x	x				x				
11B20P	2		x	x				x			
11B30P	3			x	x				x		
11B40P	4				x	x				x	
11B50P	5					x					x
11B10V	6	x					x	x			
11B20V	7		x					x	x		
11B30V	8			x					x	x	
11B40V	9				x					x	x
11B50V	10					x					x

Note: Only non-zero elements are indicated by an 'x'.

Figure 2.2 Transition Matrix II.

TABLE III

Duty Positions

MOS	SI	SQI
11B	10	P
11B	20	P
11B	30	P
11B	40	P
11B	50	P
11B	1C	V
11B	20	V
11B	30	V
11B	40	V
11B	50	V

MOS	SL	SQI
11C	10	P
11C	20	P
11C	30	P
11C	40	P
11C	10	V
11C	20	V
11C	30	V
11C	40	V

DUTY POSITION	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
11B10P	1	x	x				x				x					x			
11B20P	2		x	x				x				x					x		
11B30P	3			x	x				x				x					x	
11B40P	4				x	x				x				x					x
11B50P	5					x									x				
11C10P	6	x					x	x			x					x			
11C20P	7		x					x	x			x					x		
11C30P	8			x					x	x			x					x	
11C40P	9				x					x				x					x
11B10V	10	x					x				x	x				x			
11B20V	11		x					x				x	x				x		
11B30V	12			x					x				x	x				x	
11B40V	13				x					x				x	x				x
11B50V	14					x									x				
11C10V	15	x					x				x					x	x		
11C20V	16		x					x				x					x	x	
11C30V	17			x					x				x					x	x
11C40V	18				x					x				x					x

Note: Only non-zero elements are shown by an 'x'.

Figure 2.3 Transition Matrix III.

movements among MOS's are negligible. The underlying factor that binds the sub-communities together is the movements from one SQI to another. The transition rates of the separate airborne sub-communities (i.e. parachutist, ranger, special forces) indicate how personnel move within each respective sub-community. The fractions of personnel that move among the sub-communities also contribute to the determination of the force levels of the next fiscal year.

2. Generation of Transition Matrices for CMF 11 and CMF 13

The architecture of a transition matrix for any CMF is generated by imbedding a series of matrices. The process which generates the transition matrix is 1) begin with the SQI matrix, 2) imbed the MOS matrix within the SQI matrix, and 3) imbed the skill levels within the resulting matrix. At each step, relationships among either the SQI's and/or the MOS's determine whether a composite matrix or a series of separate matrices is generated. For CMF 11 and CMF 13, the following MOS's exist among the three SQI's of the airborne community as listed in Tables IV and V, respectively.

TABLE IV

CMF 11

MOS	Parachutist SQI 'P'	Ranger SQI 'V'	Special Forces SQI 'S'
11B	x	x	x
11C	x	x	x
11H	x		

'x' denotes that the MOS is authorized within the specific SQI.

The first step of generating the transition matrix for either CMF is to construct the SQI matrix. All three SQI's are related by the congressional funding as mentioned in the previous chapter. Thus, a composite SQI matrix is generated as illustrated in Figure 2.4. This composite SQI matrix would be the same for both CMF 11 and CMF 13.

TABLE V

CMF 13

MOS	Parachutist SQI 'P'	Ranger SQI 'V'	Special Forces SQI 'S'
13B	x		
13C	x		
13E	x		
13F	x	x	
93F	x		
17C	x		
17B	x		
13R	x		
82C	x		
13Y	x		
13W	x		
13Z	x		

'x' denotes that the MOS is authorized within the specific SQI.

The second step of generating the transition matrix for CMF 11 or CMF 13, is to imbed the MOS's which exist in each SQI of the airborne community within the SQI composite matrix. Each MOS is related by the career progression pattern as described in Chapter 1. The resulting matrices for CMF 11 and CMF 13 are diagrammed in Figures 2.5 and 2.6, respectively.

The final step in generating the transition matrices for both CMF 11 and CMF 13 is to imbed the skill levels within the matrices previously generated in the second step. The transition matrices generated for CMF 11 and CMF 13 are shown in Figures 2.7 and 2.8, respectively.

This basic architecture of the final transition matrices for the corresponding CMF's will be utilized in the next chapter when the execution of the Forecasting Model is discussed.

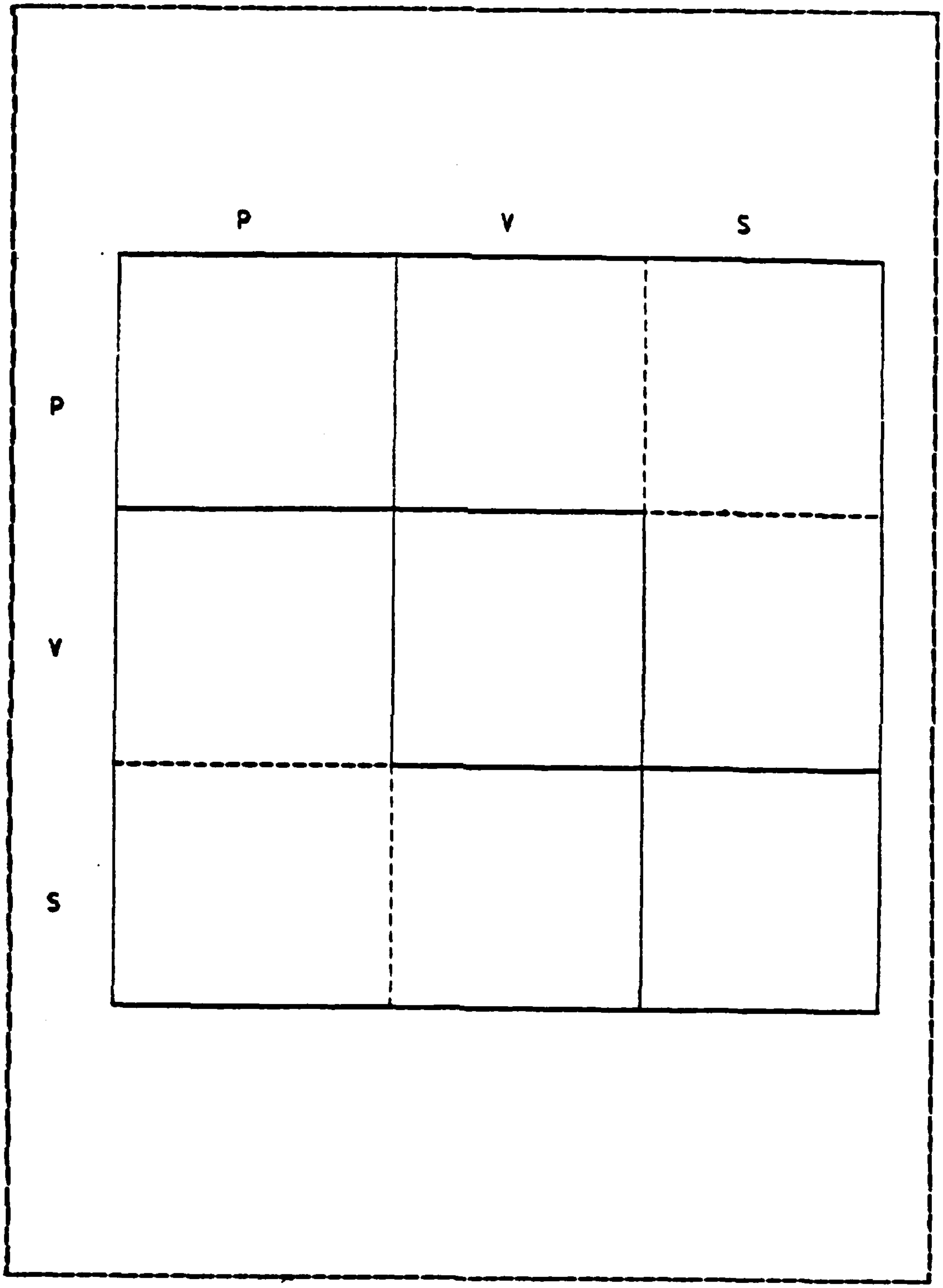


Figure 2.4 SQI Matrix.

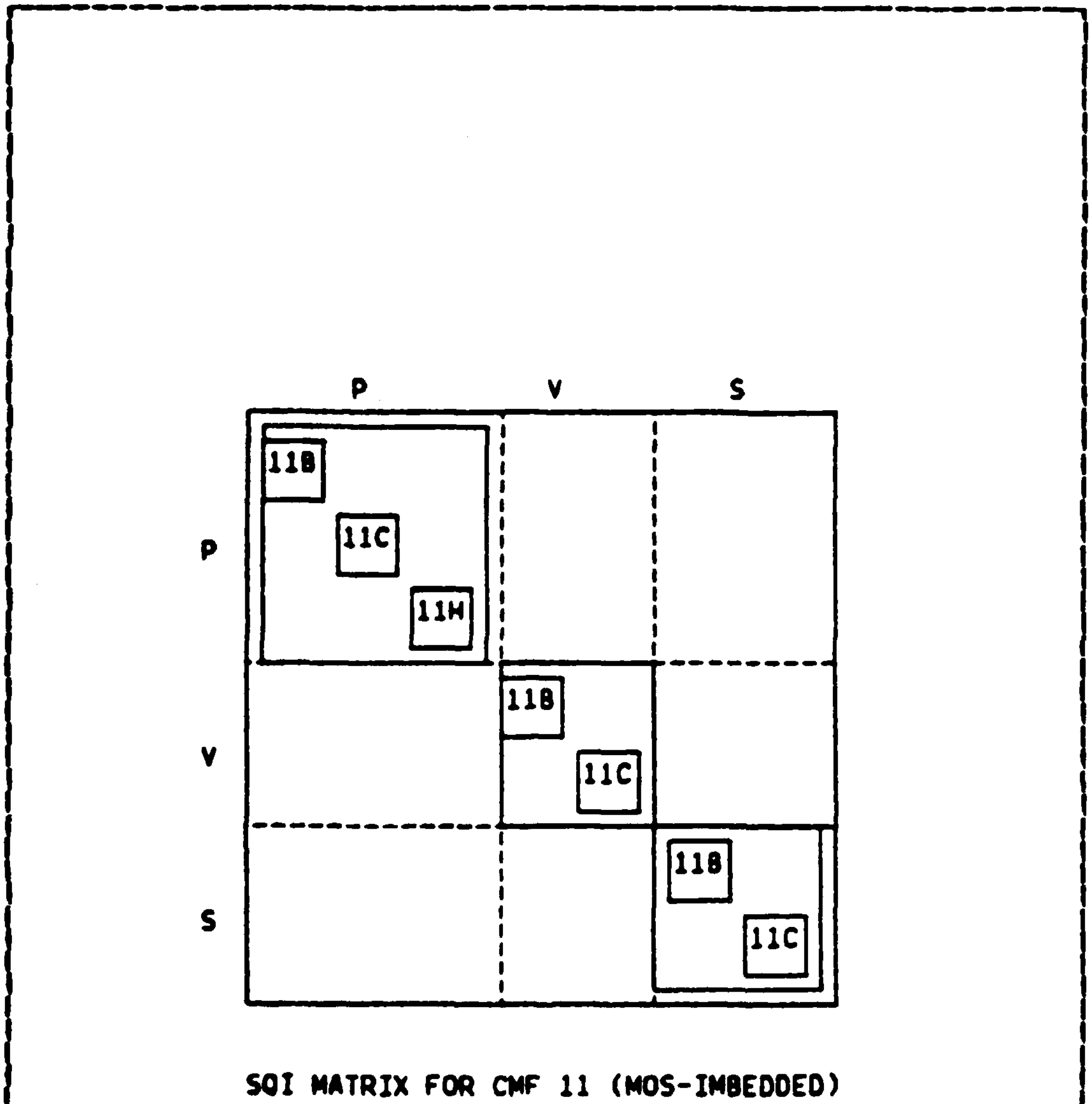


Figure 2.5 MOS Matrix for CMF 11.

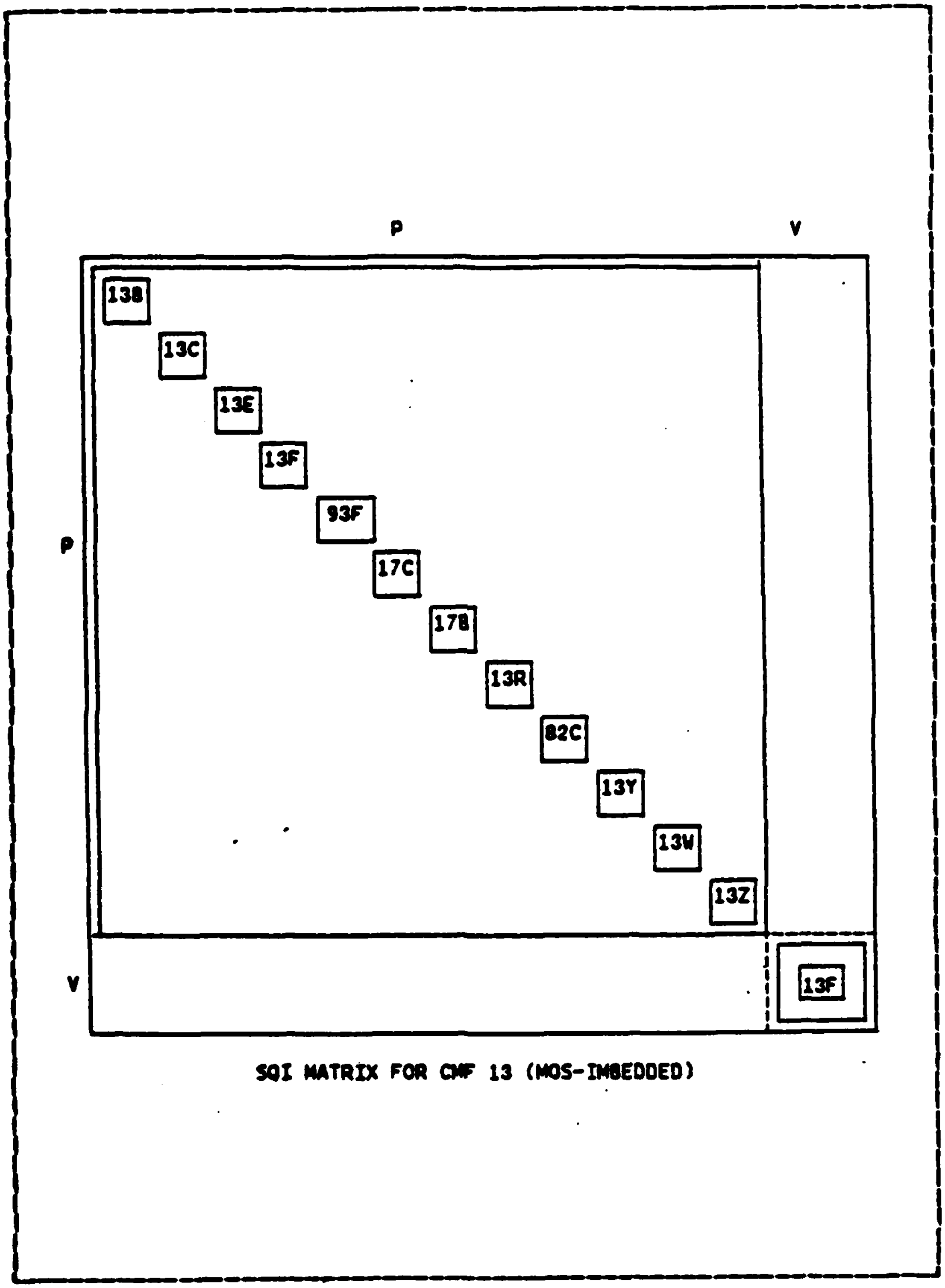


Figure 2.6 MOS Matrix for CMF 13.

DUTY POSITION	q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	w_i
11B10P	1	X	X																														X
11B20P	2		X	X																													X
11B30P	3			X	X																												X
11B40P	4				X	X																											X
11B50P	5					X																											X
11C10P	6						X	X																									X
11C20P	7							X	X																								X
11C30P	8								X	X																							X
11C40P	9					X				X																							X
11H10P	10										X	X																					X
11H20P	11											X	X																				X
11H30P	12												X	X																			X
11H40P	13					X								X																			X
11B10V	14														X	X																	X
11B20V	15															X	X																X
11B30V	16																X	X															X
11B40V	17																	X	X														X
11B50V	18																		X														X
11C10V	19																			X	X												X
11C20V	20																				X	X											X
11C30V	21																					X	X										X
11C40V	22																		X				X										X
11B10S	23																							X	X								X
11B20S	24																								X	X							X
11B30S	25																									X	X						X
11B40S	26																										X	X					X
11B50S	27																											X					X
11C10S	28																												X	X			X
11C20S	29																													X	X		X
11C30S	30																														X	X	X
11C40S	31																											X				X	X

Figure 2.7 Transition Matrix for CMF 11.

DUTY POSITION	q	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	w_i
13B10P	32	X	X																																									X
13B20P	33		X	X																																								X
13B30P	34			X	X																																							X
13B40P	35				X																																X							X
13C10P	36					X	X																																					X
13C20P	37						X	X																																				X
13C30P	38							X	X																																			X
13C40P	39								X																												X							X
13E10P	40									X	X																																	X
13E20P	41										X	X																																X
13E30P	42											X	X																															X
13E40P	43												X																								X							X
13F10P	44													X	X																													X
13F20P	45														X	X																												X
13F30P	46															X	X																											X
13F40P	47																X																				X							X
93F10P	48																	X	X																									X
93F20P	49																		X	X																								X
93F30P	50																			X	X																							X
93F40P	51																				X																	X						X
17C10P	52																					X	X																					X
17C20P	53																						X	X																				X
17C30P	54																							X	X																			X
17C40P	55																								X													X						X
17B10P	56																									X	X																	X
17B20P	57																										X	X																X
17B30P	58																											X	X															X
17B40P	59																												X									X						X
13R10P	60																													X	X													X
13R20P	61																														X	X												X
13R30P	62																												X			X												X
82C10P	63																																X	X										X
82C20P	64																																	X	X									X
82C30P	65																																		X	X								X
82C40P	66																																			X		X						X
13Y50P	67																																				X		X					X
13W50P	68																																					X	X					X
13Z50P	69																																						X					X
13F10V	70																																							X	X			X
13F20V	71																																								X	X		X
13F30V	72																																									X	X	X
13F40V	73																																				X						X	X

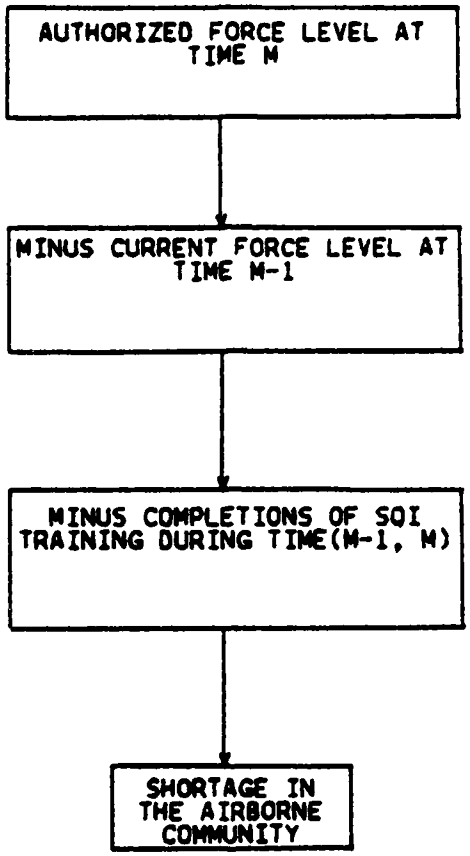
Figure 2.8 Transition Matrix for CMF 13.

B. OPTIMIZATION MODEL

1. The Development of the Objective Function

The first task in the development of an optimization model is to determine its objective or goal. The number of soldiers who should enter training is a function of the shortage at the current fiscal year for each duty position. The number of soldiers who will fill the vacancies in the next fiscal year is a fraction of those soldiers who will complete the required SQI training. If no shortage exists in a particular duty position then no requirement for a trained recruit is generated. However, if a shortage does exist, then that shortage generates the requirement for a qualified person of that particular SQI. The application of the course completion rate pertaining to a particular SQI determines the number needed to enter such SQI training. Thus, if the number of shortages is minimized then the number required to enter each SQI training is directly affected. Ideally, it would be desirable to have no shortage at all in any duty position. This would mean all duty positions would be filled to their authorized levels. But, budgetary constraints do not always allow all duty positions to be filled to their authorized levels. Hence, a goal of the optimization model is to reduce the overall shortage within the airborne community.

The shortage in the airborne community for any time period m is graphically represented in Figure 2.9. Mathematically, this shortage can be expressed as:



This shortage can be further defined as:

Authorized Force Level at time m	-	Current Force Level at time m-1	+	Course Completion Rate	•	Entrants into SQI Training during (time m-1, m) .

Note: The current force level at time m-1 is the attrited force level generated by the Markov process described in the previous section.

Figure 2.9 Shortage in the Airborne Community.

$$S = A - (N + (b \cdot X))$$

where S represents shortage

A represents the authorized inventory level at time period m

N represents the current force level at time period m - 1

b represents the course completion rate

X represents the number of personnel who enter SQI training .

For each job type, this relationship can be expressed as:

$$S_{ijk} = A_{ijk} - (N_{ijk} + b_k \cdot X_{ijk}) \qquad \text{(eqn 2.2)}$$

where i denotes the specific MOS

j denotes the specific grade/skill level

k denotes the specific SQI.

The symbol S_{ijk} represents the shortage for each duty position as specified by MOS i, SL j, and SQI k. We let A_{ijk} and N_{ijk} be the same quantities as in the preceding equation except that each refers to a specified duty position described by the subscripts i, j, and k. The course completion rate is represented as b_k for SQI k and is not dependent on i or j because no distinction is made by MOS and/or SL while a soldier is undergoing SQI k training. The symbol X_{ijk} represents the number of personnel with MOS i and SL j who should enter into SQI k training in order to fill the vacancy in a duty position specified by i, j and k. In accordance with the optimization of training requirements as discussed in Chapter 1, the decision variable chosen for this optimization is X_{ijk}.

As discussed in the previous chapter, the authorized inventory levels are provided by the PERSACS document. The course completion rate of each type of SQI training is provided from empirical data while current force levels are either provided by historical data for the initial time period or by the predicted force levels of the succeeding time periods as generated by the Markov process described in the previous section. The interpretation of $(b_k \bullet X_{ijk})$ is the number of qualified soldiers who will enter into the airborne community. Since the elements of the expression $(A_{ijk} - N_{ijk})$ are derived from known data, this expression can represented as a single term a_{ijk}.

$$a_{ijk} = (A_{ijk} - N_{ijk}) .$$

By substituting the term a_{ijk} into Equation 2.2, the result is:

$$S_{ijk} = a_{ijk} - (b_k \bullet X_{ijk}). \qquad \text{(eqn 2.3)}$$

Thus, since the objective is to minimize the sum of all shortages of personnel for each duty position as specified by MOS i, SL j, and SQI k, the objective function can be mathematically expressed as:

$$\text{Minimize} \sum_{i,j,k} (a_{ijk} - (b_k \bullet X_{ijk})).$$

This objective function assesses the same penalty to each vacancy of each type of duty position. The penalty of the first vacancy is equal to the penalty of the second and third and so on. Although this scheme may be mathematically feasible, it does not realistically capture the dynamics of this problem. In the airborne community, unit readiness is inversely related to the shortage of personnel.

For example, as the shortage of personnel increases the unit effectiveness decreases. If a unit of one hundred men is short ten men then it is considered to be able to continue its primary mission. But, if that same unit is short twenty men, it is considered able to continue its primary mission subject to certain restrictions. When that unit is short forty men then it is considered unable to perform any of its primary missions.

The question of what relationship really exists between shortage and readiness has not really been quantified. However, it is accepted that an inverse relationship does exist between shortage and readiness [Ref. 4]. Also, the assumption will be made that there is no penalty for being over the authorized level in any duty position. The marginal difference between each shortage can be viewed as a penalty and must increase with each successive vacancy. As a result, the desired form of the objective function is graphically shown in Figure 2.10. Thus, the linear relationship is clearly not adequate.

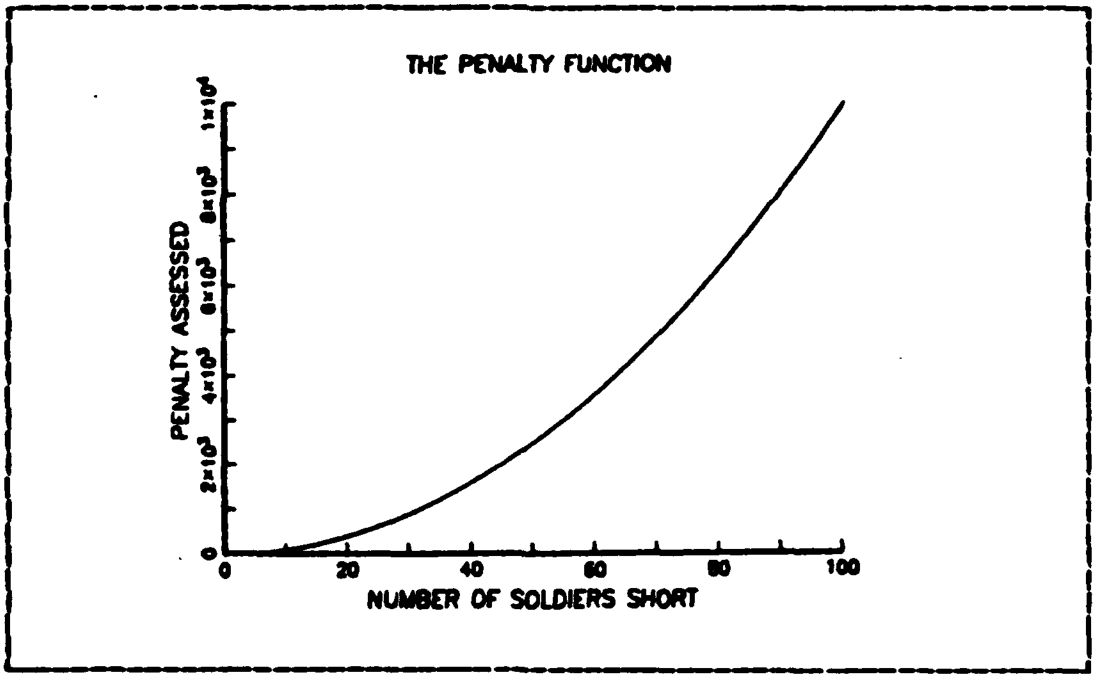


Figure 2.10 Penalty Versus Shortage.

The rate at which the objective function increases for each additional vacancy will be assumed to be quadratic and the objective function will be expressed as

$$\text{Minimize} \sum_i \sum_j \sum_k [a_{ijk} - (b_k \cdot X_{ijk})]^2.$$

The quadratic function satisfies the inverse relationship described earlier in this section. This choice of objective function is arbitrary, but it does have the property of penalizing each additional shortage with increasing severity. Other objective functions could be handled using the same methodology discussed below.

2. The Development of the Constraints

There are two factors that restrict the decision variables X_{ijk}. One is the budget while the second is the school capacity. The total number of soldiers who enter SQI training cannot exceed the capacity L of the respective school during the corresponding time period. Also, the total cost of training all soldiers who enter all SQI training cannot exceed some budget level, B, which is allocated for SQI training. Mathematically, the school capacity constraint can be expressed as:

$$\sum_i \sum_j X_{ijk} \le L_k \qquad k=1,2,\ldots \quad .$$

For example, if five hundred soldiers can be accommodated in ranger training throughout the year by the United States Army Ranger School, then the total number of soldiers of all MOS's and skill/grade levels that can enter into that type of SQI training is limited to 500. This limitation can be the result of living accomodations, student-cadre ratio,or any other factor which sets a physical restriction on the number of students that can be effectively trained. The budget constraint can be mathematically expressed as:

$$\sum_{i}\sum_{j}\sum_{k} t_k \cdot X_{ijk} \leq B .$$

where t_k is the training cost for the kth type of SQI training. The derivation of this cost will be discussed in the following section.

3. The Cost of Training

While the soldier is undergoing training, hazardous duty pay is paid until that training is completed. If a soldier fails to complete the entire course, he is paid a pro-rated sum dependent on the length of training completed. Some soldiers will fail in the early portions of the course while some will fail in the latter portions of the course. If times of failure are assumed to be uniformly distributed over the fiscal year, the average time of failure is the midpoint of the training period, and the cost of that failure is half the cost of training a soldier for the entire fiscal year.

Once a soldier completes the training and is assigned to a unit within the airborne community which is on 'jump status', he is paid hazardous duty pay until he leaves the airborne community. This cost represents the cost to man the force and is dependent on the time when a soldier enters the airborne community. Some soldiers will enter in the beginning while some will enter during the latter portion of the fiscal year. As a result, the distribution of entry times into the airborne community by soldiers just completing SQI training is also assumed to be uniformly distributed over the fiscal year. Therefore, the average time of entry is the midpoint of the fiscal year, and the cost of manning that soldier is half the cost of manning a soldier for the entire fiscal year.

Not all soldiers who enter into SQI training will be assigned to a duty position which is a part of a unit on 'jump status'. Therefore, this manning cost will only apply to those soldiers who fill duty positions which belong to units on 'jump status'. The percentage of shortages of 'jump status' units with respect to the total shortages can be expressed as:

$$PCT_k = \frac{\text{the number of "jump" vacancies in SQI } k}{\text{the total vacancies in SQI } k} \qquad \text{(eqn 2.4)}$$

The total cost of training for any fiscal year is the sum of the hazardous duty pay for soldiers who complete and for those who fail SQI training and the hazardous duty pay for the remaining fiscal year for soldiers who complete training and are subsequently assigned to a "jump" unit. This is graphically represented in Figure 2.11. Mathematically, this cost T_k is expressed as

$$T_k = (c_k \bullet b_k \bullet X_{ijk}) + (c_k/2 \bullet (1-b_k) \bullet X_{ijk}) + (m/2 \bullet b_k \bullet X_{ijk}) \bullet PCT_k$$

where T_k represents the cost to train all soldiers in SQI k during the fiscal year

c_k represents the cost to train one soldier in SQI k

b_k represents the course completion rate of SQI k training for the fiscal year

m represents the cost to man one soldier in the airborne community for a fiscal year

X_{ijk} represents the number who enter training during the fiscal year

PCT_k represents the percentage of 'jump status' vacancies in a fiscal year

By algebraic manipulation, the following expression results.

$$T_k = 1/2\{[(c_k + m \bullet PCT_k)\bullet b_k] + c_k\} \bullet X_{ijk}$$

Since X_{ijk} is the variable equal to the number of soldiers who enter into SQI k training from MOS i and SL j. The cost incurred by each soldier entering into a particular type of SQI training during the fiscal year is

$$t_k = 1/2\{[(c_k + m \bullet PCT_k)\bullet b_k] + c_k\}. \qquad \text{(eqn 2.5)}$$

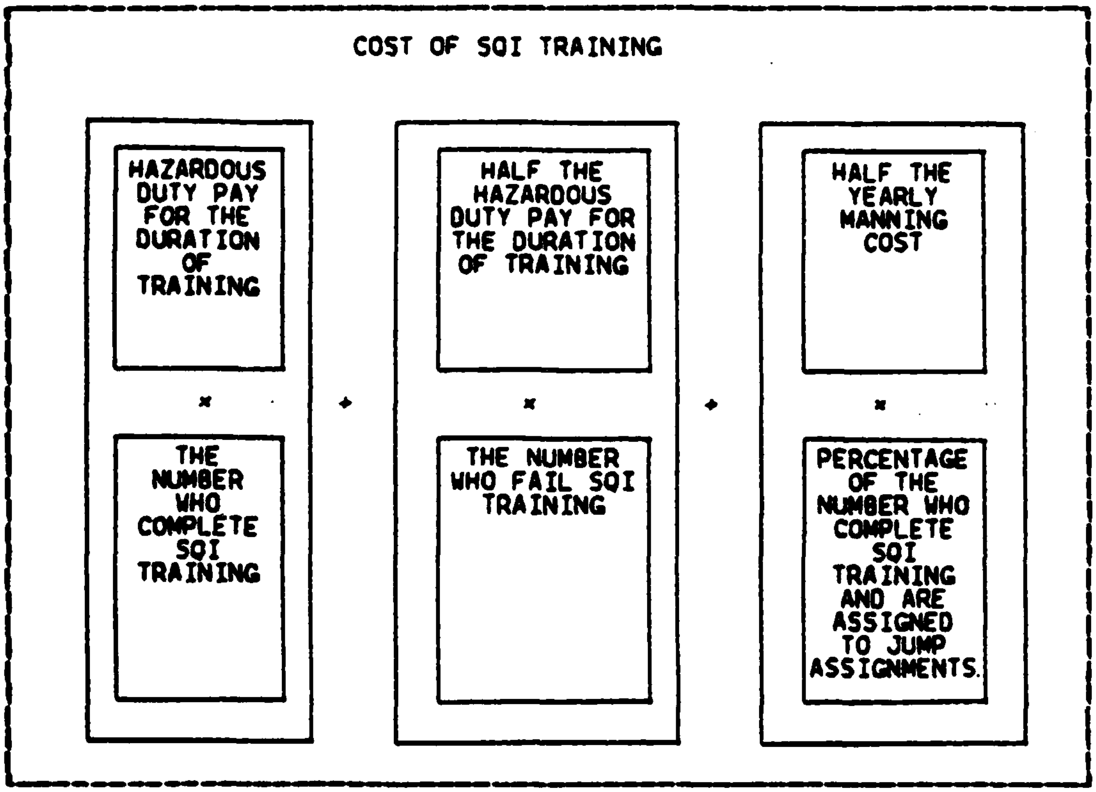


Figure 2.11 Cost of SQI Training.

4. The Optimization Problem

The mathamatical representation of the optimization problem is:

$$\text{Minimize} \quad \sum_i \sum_j \sum_k (a_{ijk} - b_k \cdot X_{ijk})^2 \qquad \text{(objective function)}$$

$$\text{Subject to} \quad \sum_i \sum_j \sum_k t_k \cdot X_{ijk} \le B \qquad \text{(budget constraint)}$$

$$\sum_i \sum_j X_{ijk} \le L_k \qquad \text{(school capacity)}$$

$$X_{ijk} \ge 0 \qquad \text{(non-negativity)}$$

Note that this problem can be viewed as three subproblems related by a budget restriction. This allows the original problem to be re-written in the following format.

$$\begin{array}{l|c|c|c|l}
 & \text{PARACHUTIST} & \text{RANGER} & \text{SPECIAL FORCES} & \\
\text{MIN} & \sum_{i,j} (a_{ij1} - b_1 \cdot X_{ij1})^2 & + \; (a_{ij2} - b_2 \cdot X_{ij2})^2 & + \; (a_{ij3} - b_3 \cdot X_{ij3})^2 & \\
 & \sum_{i,j} t_1 \cdot X_{ij1} & + \; t_2 \cdot X_{ij2} & + \; t_3 \cdot X_{ij3} & \le B \\
 & \sum_{i,j} X_{ij1} & & & \le L_1 \\
 & & \sum_{i,j} X_{ij2} & & \le L_2 \\
 & & & \sum_{i,j} X_{ij3} & \le L_3
\end{array}$$

If the optimal portion of the budget allocated to each subproblem is known then each subproblem can be solved independently. Thus, the critical question is "how should the overall budget be allocated to the three subproblems?"

5. Dynamic Programming

Dynamic Programming is an optimizing strategy [Ref. 5] normally applied to a class of problems which require a sequence of related decisions and is ideally

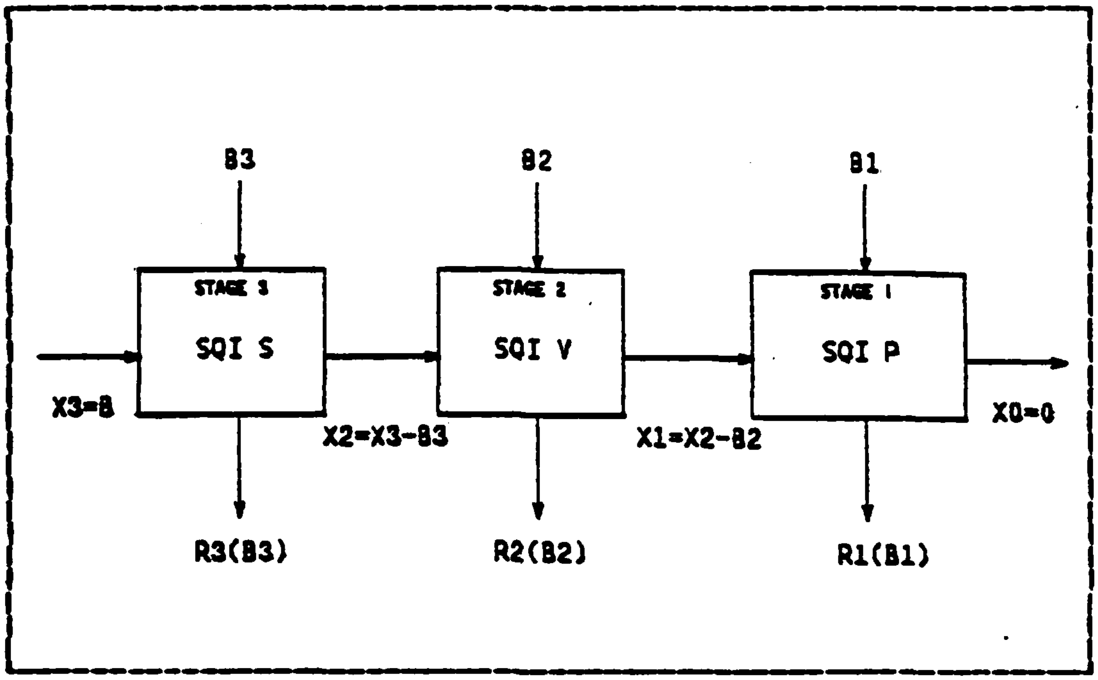


Figure 2.12 Stage Diagram for the Airborne Community.

suited for the question just posed. The problem can be viewed as shown in Figure 2.12 where each stage corresponds to one of the subproblems. For example, stage 1 represents the subproblem pertaining to parachutist training. Entering into a particular stage k is the state variable X_k which corresponds to the remaining budget to be allocated. Also associated with each stage k is a decision variable B_k which represents the amount of the budget allocated to stage k and a stage return function $R_k(B_k)$ which gives the shortage penalty associated with that stage (SQI) as a function of the decision variable B_k. The construction of the stage return function is discussed in detail later in this section.

The original question pertaining to the allocation of the budget among the three subproblems can be stated as:

$$\underset{B_k}{\text{Minimize}} \quad \sum_{k=1}^{3} R_k(B_k)$$

$$\text{Subject to} \quad \sum_{k=1}^{3} B_k \leq B$$

$$\text{and} \quad B_k \geq 0 \qquad \text{for } k = 1,2,3.$$

If $f_k(X_k)$ is defined to be the optimal (minimum) total penalty from the stages 1 through k, then the dynamic programming recursive equations for all stages except the first can be mathematically written as:

$$f_k(X_k) = \text{MIN}\,[R_k(B_k) + f_{k-1}(X_{k-1})], \quad X_{k-1} = X_k - B_k$$

where $R_k(B_k)$ represents the shortage penalty for the kth SQI

$f_{k-1}(X_{k-1})$ represents the remaining minimum penalty associated with the stages 1,2,...k-1 after the decision Bk has been made and a budget of Xk-1 remains.

For this specific problem, when there are three stages, the recursive equations can be expressed as:

$$\text{STAGE 3} \quad f_3(X_3) = \underset{B_3}{\text{MIN}} [R_3(B_3) + f_2(X_2)], \quad X_2 = X_3 - B_3$$

$$\text{STAGE 2} \quad f_2(X_2) = \underset{B_2}{\text{MIN}} [R_2(B_2) + f_1(X_1)], \quad X_1 = X_2 - B_2$$

$$\text{STAGE 1} \quad f_1(X_1) = \underset{B_1}{\text{MIN}} [R_1(B_1)].$$

These recusive equations reflect the principle which was stated by Bellman [Ref. 6] as the principle of optimality.

> "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Specifically, no matter what decision is made on how much is to be allocated to parachutist training, the decisions pertaining to ranger and special forces training must constitute an optimal policy with respect to the remaining budget. Thus, the recursive equation for $f_3(X_3)$ combines the immediate penalty $R_3(B_3)$ with the optimal penalty from the ranger and special forces SQI which is expressed as a function f_2 of the remaining budget $X_3 - B_3$.

The solution of the recursive equations listed above begins with the computation of $f_1(X_1)$ for all values of X_1 which range between zero and the budget restriction B. Once this is completed, the function $f_2(X_2)$ is computed for all values of X_2 between 0 and B. Finally, the function $f_3(X_3)$ is calculated for the given value of budget B.

The preceding discussion assumed that the return functions $R_k(B_k)$ are available; and, if so, then the strategy of dynamic programming can be used to determine the optimal allocation of budget B to the three stages corresponding to the three types of SQI training. The construction of the stage return function will be discussed in the following section.

a. Determination of the Return Function

The stage return function $R_k(B_k)$ represents the penalty associated with SQI k when B_k units of the budget are allocated to kth type of SQI training. The construction of the stage return function $R_k(B_k)$ for one type of SQI training will be discussed.

The optimization problem for the kth type of SQI training can be mathematically expressed as:

$$\text{Minimize} \quad \sum_i \sum_j (a_{ijk} - b_k \bullet X_{ijk})^2 \text{ (objective function)}$$

$$\text{Subject to} \quad \sum_i \sum_j t_k \bullet X_{ijk} \leq B_k \text{ (budget constraint)}$$

$$\sum_i \sum_j X_{ijk} \leq L_k \text{ (school capacity)}$$

$$X_{ijk} \geq 0 \quad \text{(non-negativity)}$$

where Bk represents a portion of the overall budget.

Note that the amount of budget allocated in a particular type of SQI training is a function of the cost of training t_k and the number entering training X_{ijk}. Further, the solution to this problem must yield an optimal value with respect to each value of B_k. This optimal value is the return function value $R_k(B_k)$ described in the previous section.

The objective function can be expressed as:

$$Z = t_k^2 \sum_{i,j} ((a_{ijk}/b_k) - X_{ijk})^2$$

and the constraints can be written as

$$\sum_i \sum_j X_{ijk} \le B_k/t_k$$

$$\sum_i \sum_j X_{ijk} \le L_k$$

$$X_{ijk} \ge 0.$$

Note that only one constraint will be active. If $L_k \ge B/t_k$, then the first constraint is the more restrictive one. On the other hand, if $L_k \le B/t_k$, the second constraint will be the more restrictive one. Because of this unique structure, the problem can be rewritten as

$$\text{Minimize} \quad Z = b_k^2 \sum_{i,j} ((a_{ijk}/b_k) - X_{ijk})^2$$

$$\text{Subject to} \quad \sum_i \sum_j X_{ijk} \le CP_k$$

$$X_{ijk} \ge 0$$

where $CP_k = \text{MIN}\{L_k, B_k/t_k\}$.

The capacity restriction CP_k places an upper bound on the total number of training slots which can be allocated to that SQI. Moreover, a training allocation will be made in the category specified by the subscripts i, j, and k, having the greatest "shortage" (a_{ijk}/b_k).

This problem can now be viewed as a single resource allocation problem. Each additional man is allocated where the marginal decrease in the objective function is greatest since the penalty is most severe for the position of greatest shortage. In order to facilitate this allocation scheme, all shortages within a particular SQI category are ordered with the category of greatest shortage first. Allocations are made until the remaining shortage of the first category is equal to the shortage of the second category. The allocations then continue with alternating allocations being made to these two categories until both shortage levels are equal to the shortage of the third category. The allocation scheme continues in this way until all shortage levels are reduced to zero or until the capacity restriction CP_k become binding.

Once an allocation is made, the value for the objective function of the subproblem is calculated. These values with their corresponding budget values constitute the return function $R_k(B_k)$ for that particular SQI category. Further, these values are optimal for the corresponding budget values. Also, the allocations are the optimal distribution plan for a specified level of the budget.

An example of the construction of the stage return function $R_k(B_k)$ is listed in Tables VI and VII as the distribution plan and the return function values, respectively. In this example, the two duty positions 11B20V and 11B30V have existing shortages of three and two, respectively, and the cost of training a soldier in this particular SQI was ten dollars. The allocation procedure as described above was programmed for a computer. The FORTRAN program is listed in Appendix A.

b. Application of a Special Algorithm

In the preceding problem, the solution process viewed the whole problem as three budgetary-related subproblems in a particular sequence. However, this problem

TABLE VI

Distribution Plan

1st ALLOCATION:

Shortage Initial	Shortage Remaining	Allocation	
3	2	1	11B20V
2.	2.	0.	11B30V

2nd ALLOCATION:

Shortage Initial	Shortage Remaining	Allocation	
3.	1.	2.	11B20V
2.	2.	0.	11B30V

3rd ALLOCATION:

Shortage Initial	Shortage Remaining	Allocation	
3.	1.	2.	11B20V
2.	1.	1.	11B30V

4th ALLOCATION:

Shortage Initial	Shortage Remaining	Allocation	
3.	0.	3.	11B20V
2.	1.	1.	11B30V

5th ALLOCATION:

Shortage Initial	Shortage Remaining	Allocation	
3.	0.	3.	11B20V
2.	0.	2.	11B30V

TABLE VII

Return Function Values

Budget	Z-Value
10	100
20	81
30	64
40	49
50	36

Note: The Z-value is the value of the objective function for a particular SQI.

pertaining to the airborne community has the distinct property that allocating to the category of greatest shortage is equivalent to allocating to the position which contributes the greatest marginal decrease in the objective function. This problem can now be viewed as a "knapsack" problem which asks for optimal allocations of parachutists, rangers, and special forces soldiers to training, given a particular budget. If the specified budget is treated as a knapsack, it can be filled by adding training allocations of designated costs which when added to the knapsack will marginally decrease the overall shortage of the personnel system. The cost for each training allocation is the training cost for each soldier sent into a designated type of SQI training. Each training allocation in the knapsack is a different item with a specific value and cost. Thus, the optimization problem can be restated as:

$$\text{Minimize} \quad S = \sum_{k=1}^{3} v_k(X_k)$$

$$\text{Subject to} \quad \sum_{k=1}^{3} t_k \bullet X_k \leq B$$

$$X_k \geq 0$$

where S represents the shortage function ((A-F)-Bk•N)

v_k represents the return (penalty) function for the kth type of SQI training

t_k represents the training cost for the kth type of SQI training

X_k represents the number of soldiers allocated to the kth type of SQI training.

The return function v_k is a marginally decreasing penalty function where each additional allocation decreases the shortage function S at a marginally decreasing

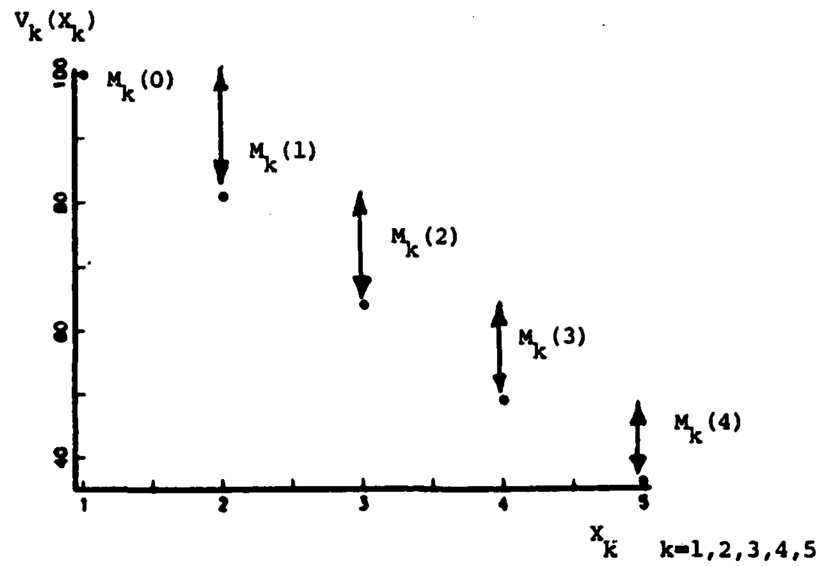


Figure 2.13 The Return (Penalty) Function.

rate. This is graphically represented in Figure 2.13. The return function can further be defined as:

$$v_k(X_k) = \sum_{i=0}^{Xk} M_k(i)$$

where $M_k(i)$ represents the marginal return (decrease) associated with the increase in allocation from X_k=i-1 to X_k=i.

The marginal return $M_k(i)$ is mathematically defined as:

$$M_k(i) = v_k(i_{k+1}) - v_k(i_k)$$

An algorithm which exploits the structure of the knapsack problem is the backward-looking algorithm discussed by Dreyfus and Law [Ref. 7], although that algorithm is presented only in the context of constant marginal returns. In the problem pertaining to the airborne community, the values of v_i are decreasing but the implementation of the algorithm is the same. In this problem, the decision is "into which category do we place the first and each subsequent training allocation?" Once an allocation is made to the ith category, a value v_i is obtained and the remaining available budget is $B-t_i$. The recurrence relation for f(B) can be written as:

$$f(B) = \underset{k=1,2,3}{\text{Minimum}} \{M_k(X_k(B-t_k) + f(B-t_k)\}$$

where f(B) represents the optimal (minimal) total penalty that can be obtained from the three SQI categories when the available budget is B.

$X_k(B)$ represents the optimal value of X_k when the available budget is B.

Therefore, given a specific budget, an optimal scheme of allocations among the three types of SQI training can be computed.

Two conditions make the algorithm easier to implement. First, that the costs and the budget are all integer or can be scaled to be integer. For example, if the budget was $10.40 then it can be scaled to 1040 cents. Second, that the greatest common divisor among the budget and costs is one.

An example of this algorithm is graphically represented. The costs t_i, the marginal returns v_i, and the categories of shortage are listed in Table VIII. A horizontal line is shown in Figure 2.14 which represents the available budget in dollars.

TABLE VIII

Costs and Marginal Returns

TRAINING A		TRAINING B		TRAINING C	
B1	V1	B2	V2	B3	V3
1	-5	1	-18	1	-46
2	-4	2	-17	2	-43
3	-3	3	-11	3	-9
4	-2	4	-8	4	-1
5	-1	5	-7	5	0

Costs of training : tA=1, tB=2, tC=3
Budget: B=10
Shortage: SA=5, SB=5, SC=5, j=3

```
|----|----|----|----|----|----|----|----|----|----|
0    1    2    3    4    5    6    7    8    9    10
```

Figure 2.14 Available Budget.

At any point P, a template of SQI training costs t_k graphically portrays the possible paths by which the budget corresponding to point P could have been reached. At point P (budget = 7), the optimal solution is sought and the graphical representation of the situation is shown in Figure 2.15. Allocation vectors are also shown for budgets 4, 5, and 6, and represent the optimal allocation among the three types of training at the point directly above.

From the point P, an allocation can be determined by looking back to the allocations made at the budget levels 4, 5, and 6. From the budget levels 4, 5, and 6, a training allocation can be made to training types C, B, and A, respectively. The corresponding costs and marginal

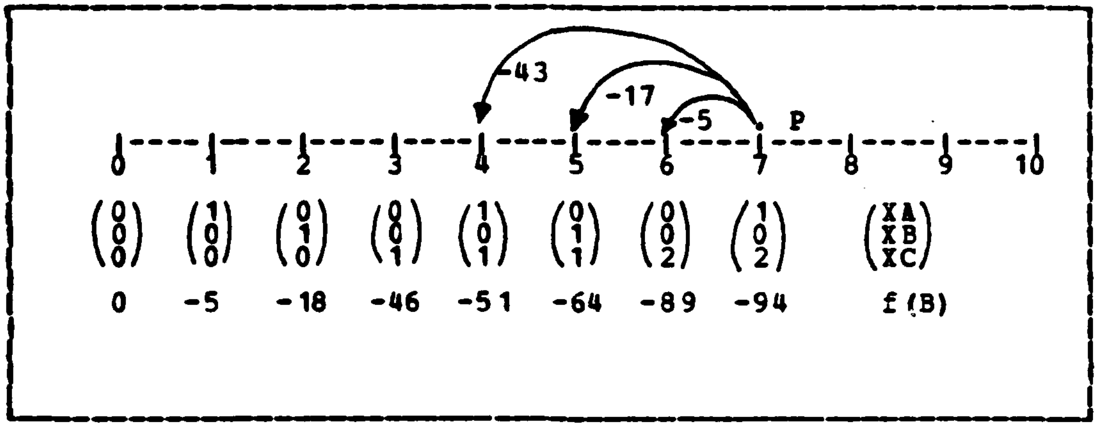


Figure 2.15 Available Budget.

values for the allocations to training types C, B, and A, are \$3, \$2, \$1, and -43, -17, -5, respectively. The optimal solutions at 4,5 and 6, were -51, -64, and -89. The three possible values for point P are:

Marginal Value	+	[f(B-t)]		Possible Value
-43	+	-51	=	-94
-17	+	-64	=	-81
-5	+	-89	=	-94

Given a budget of seven, the optimal allocation vector can be formed by the addition of a training allocation in category C to the allocation vector of budget 4 or the addition of a training allocation in category A to the allocation vector of budget 6. (i.e. The vector (1,0,1) for budget equal to four becomes (1,0,2) for a penalty of -94.)

Once the optimal allocation of the budget is determined, the distribution plan generated as discussed in the previous section and listed in Table VI designates the specific duty positions for each type of training. The application of this algorithm to CNF 11 and CNF 13 will be discussed further in the next chapter.

C. THE AGGREGATE MODEL

The forecasting and optimization models when linked together form the aggregate model which meets the goals established in Chapter 2. There are two bonds that exist between the models. The first connection is an output-input linkage between the forecasting and optimization models. The force vector $\underline{N}$ which is generated by the forecasting model is an integral part of the quadratic objective function within the optimization model. The individual components n_q of the force vector as defined earlier represent the number of soldiers in duty position q at the end of the fiscal year. The subscript q refers to a duty position specified by MOS i, SL j, and SQI k. The correspondence between subscript q of the forecasting model and the subscripts i, j, k, of the optimization model is seen in Figures 2.7 and 2.8 of section A.1. This definition is the same for the force level variable N_{ijk} of the optimization model. The correlation between duty position, MOS, SL, and SQI as discussed in Chapter 1 leads to the following relationship between the two models:

$$n_q = N_{ijk} . \qquad \text{(eqn 2.6)}$$

Hence, the force vector $\underline{N}$ is the output-input link which is generated by the forecast model and subsequently is the input to the optimization model.

The second connection is also of the output-input type. The optimal value of the decision variable X_{ijk} is essential in generating the recruitment vector $\underline{R}$ of the forecast model. The quantity $(B_k \bullet X_{ijk})$ represents the number of soldiers with MOS i and SL j who must complete SQI k training. The individual components r_q of the recruitment vector represent the number of soldiers who must enter into duty position q at the end of the fiscal year. Since the completion of SQI training is a prerequisite for entrance into any airborne subcommunity, r_q is equivalent to the number of soldiers in a particular MOS and SL who complete a specific type of SQI training during the fiscal year. Because of the correlation between duty position, MOS, SL, and SQI, the following relationship results:

$$r_q = b_k \bullet X_{ijk} \qquad \text{(eqn 2.7)}$$

This relationship between $\underline{X}$ and $\underline{R}$ is the second output-input link between the optimization and forecasting model.

The process of the aggregate model is cyclic. A graphical representation of this process is shown in Figure 2.16.

Implementation of the aggregate model begins by obtaining the number of promotions within and attritions out of the airborne community from historical data. Also, the authorizations and current force levels pertaining to the initial fiscal year t must be obtained.

Once the above data has been obtained, the process will continue with the determination of the recruitment vector $\underline{R}$ by the optimization model. The recruitment vector is then used in the forecasting model above to generate the force vector $\underline{N}$ for the next fiscal year t+1. Subsequently, the force vector $\underline{N}$ for fiscal year t+1 is used in the optimization model as part of the objective function to generate the recruitment vector $\underline{R}$ for the next FY t+1. If the forecast pertains to multiple years, then the process is repeated until the multi-year forecast is completed. The execution of the aggregate model and its application to both CMF's 11 and 13 will be discussed in the following chapter.

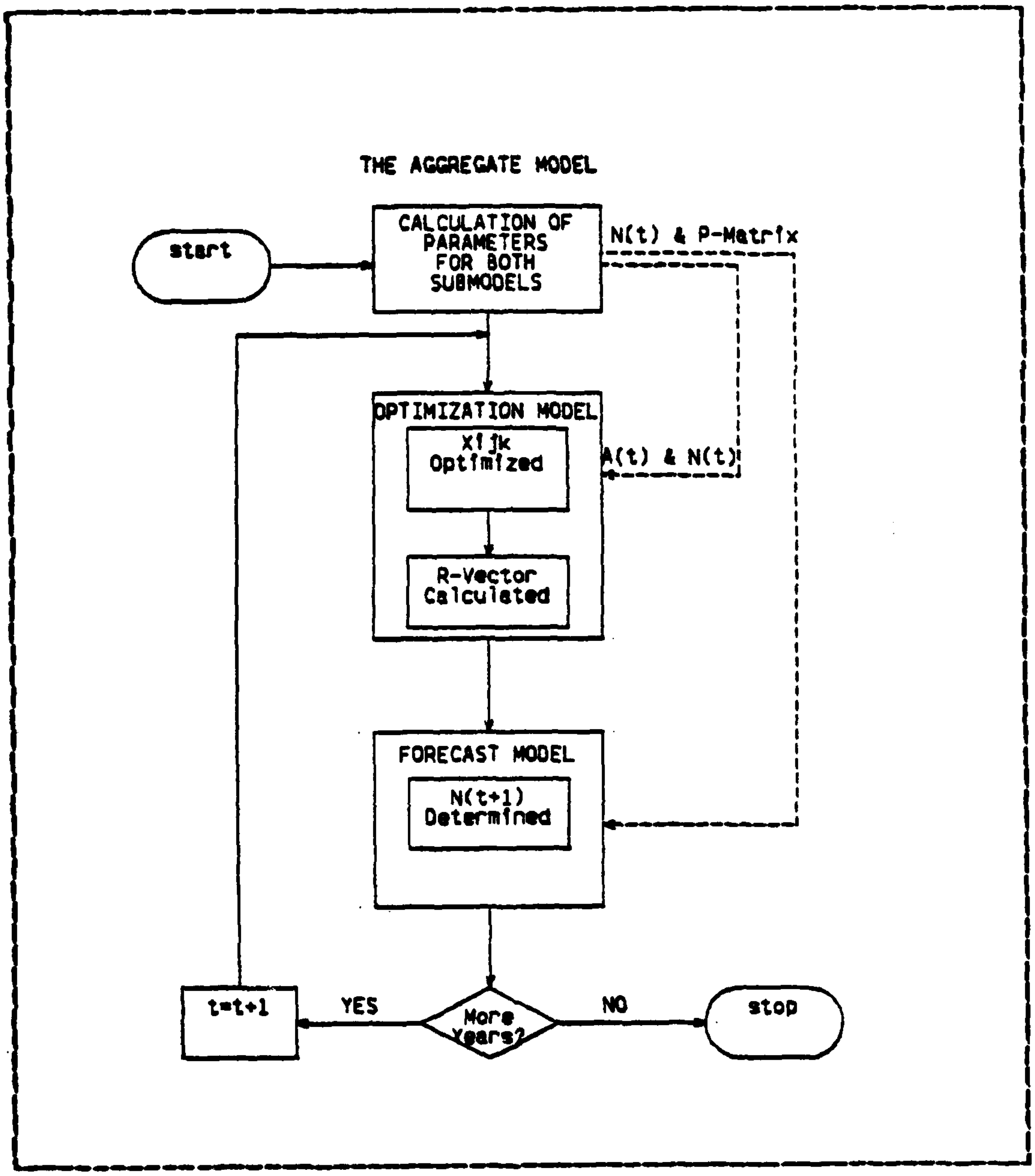


Figure 2.16   Aggregate Model Process.

III. EXECUTION OF THE MODEL

In this chapter, the data required to estimate the parameters for the optimization and forecasting models will be discussed. The parameters of both models will be calculated from empirical data of fiscal year 1983. The results generated by the aggregate model which are applicable to CMF 11 and CMF 13 and pertain to the subsequent fiscal years will be discussed at the end of this chapter.

A. DISCUSSION OF DATA

1. The Optimization Model

There are four parameters which must be calculated before the model can be run. The first two parameters are the course completion rate b_k and the training cost t_k for for each of the three SQI's which affect the operation of the optimization model. The other two parameters are the overall training budget B and the school capacity L_k for each of the three SQI's. They are considered given and non-negotiable. However, to operate the optimization model for only CMF 11 and CMF 13, an estimated percentage of the last two parameters affected by the respective CMF's needs to be determined. If all CMF's were considered in the operation of the optimization model then the budget and the school capacity would be the amounts originally stated.

a. Course Completion Rate

The course completion rates b_k, (k = P,V,S) for the three types of SQI training conducted during FY 83 were calculated by the Army Training Requirements and Resource System (ATTRS). These rates are listed in Table IX.

TABLE IX

Course Completion Rates (FY83)

SQI Training	Course Completion Rate
P-parachutist	.81
V-ranger	.64
S-special forces	.55

b. Cost of Training

The training costs t_k, (k = P,V,S), for the three types of SQI are calculated by using Equation 2.5. The critical step in calculating the training cost tk is to calculate the percentage of soldiers who go to follow-on "jump" assignments upon completion of the SQI training. The empirical data used to calculate this percentage, PCT_k, (k=P,V,S) for the three type of SQI's is listed in Tables X, XI, XII. Using Equation 2.4 in Section B.3 of Chapter 2, the corresponding percentages are listed in Table XII.

Two additional items are needed to calculate the training cost t_k for each type of SQI. The first item is the individual hazardous duty pay prorated to the duration of training. The following hazardous duty pays C_k, (k=P,V,S) for the three type of SQI's are:

C_P = $63.75 (3 weeks @ $85.00 per month)
C_V = $170.00 (8 weeks @ $85.00 per month)
C_S = $255.00 (12 weeks @ $85.00 per month).

The second item needed is the yearly manning cost m which represents the hazardous duty pay given to a soldier on "jump" status during the fiscal year. This cost is $1020. With the course completion rates listed in Table IX, the

TABLE X

SQI P (FY 83)

DUTY POSITION	AUTHORI-ZATION	INVENTORY W/O POOL	INVENTORY W/POOL	"JUMP" SHORTAGE	TOTAL SHORTAGE
11B10P	4306 (8612)	4409	6222	--	2390
11B20P	676 (1352)	965	1444	--	--
11B30P	621 (1242)	621	1065	--	177
11B40P	249 (498)	355	816	--	--
11B50P	137 (274)	177	448	--	--
11C10P	573 (1146)	894	1163	--	--
11C20P	224 (448)	144	214	80	234
11C30P	14 (28)	62	120	--	--
11C40P	47 (94)	43	121	4	--
11H10P	523 (1046)	504	646	19	400
11H20P	87 (174)	176	233	--	--
11H30P	92 (184)	71	147	21	37
11H40P	22 (44)	28	61	--	--
13B10P	529 (1058)	583	672	--	386
13B20P	73 (146)	117	169	--	--
13B30P	63 (126)	68	126	--	--
13B40P	24 (48)	37	85	--	--
13C10P	8 (16)	3	3	5	13
13C20P	6 (12)	0	2	6	10
13C30P	0 (0)	1	3	--	--
13C40P	8 (16)	16	29	--	--
13E10P	56 (112)	80	92	--	20
13E20P	23 (46)	28	39	--	7
13E30P	16 (32)	12	20	4	12
13E40P	3 (6)	0	0	3	6
13F10P	213 (426)	40	40	173	386
13F20P	121 (242)	94	122	27	120
13F30P	44 (88)	45	75	--	13
13F40P	17 (34)	24	47	--	--
93F10P	11 (22)	6	6	5	16
93F20P	4 (8)	6	7	--	1
93F30P	2 (4)	1	5	1	--
93F40P	2 (4)	4	4	--	--
17C10P	35 (70)	35	36	--	34
17C20P	18 (36)	15	18	3	18
17C30P	8 (16)	5	7	3	9
17C40P	6 (12)	0	2	6	10
17B10P	3 (6)	1	1	2	5
17B20P	1 (2)	1	1	3	1
17B30P	1 (2)	2	4	--	--
17B40P	1 (2)	2	4	--	--
13R10P	29 (58)	9	9	20	49
13R20P	7 (14)	2	3	5	11
13R30P	2 (4)	5	7	--	--
82C10P	34 (68)	47	54	--	14
82C20P	17 (34)	9	13	8	21
82C30P	6 (12)	17	25	--	--
82C40P	5 (10)	7	17	--	--
13Y50P	7 (14)	24	43	6	10
13W50P	7 (14)	2	4	5	10
13Z50P	3 (6)	3	5	--	1
TOTAL				400	4411

NOTE: The figures in parenthesis are the authorizations which include the 2.0 MANPOWER POOL FACTOR described in Chapter 1.

TABLE XI

SQI V (FY 83)

DUTY POSITION	AUTHORIZATION	INVENTORY W/O POOL	INVENTORY W/POOL	"JUMP" SHORTAGE	TOTAL SHORTAGE
11B10V	58 (116)	212	248	--	--
11B20V	142 (284)	253	315	--	--
11B30V	220 (440)	200	293	20	147
11B40V	83 (166)	127	207	--	--
11B50V	32 (64)	46	126	--	--
11C10V	0 (0)	15	17	--	--
11C20V	6 (12)	17	19	--	--
11C30V	6 (12)	8	15	--	--
11C40V	0 (0)	5	9	--	--
13F10V	0 (0)	4	4	--	--
13F20V	18 (36)	15	22	3	14
13F30V	8 (16)	5	8	3	8
13F40V	2 (4)	4	9	--	--
TOTAL				26	169

NOTE: The figures in parenthesis are the authorizations which include the 2.0 MANPOWER POOL FACTOR described in Chapter 1.

training costs t_k, (k=P,V,S) can now be calculated by using Equation 2.5 of Section B.3 in Chapter 2 and are listed in Table XIV.

c. Training Budget

The training budget of the airborne community for FY 83 was 2,800 man-years. This is converted into budget dollars by multiplying each man-year by the manning cost. The total budget to fill the shortages for all duty positions is $2,856,000. However, CMF 11 and CMF 13 are only a portion of all the duty positions which encompass the airborne community. Since training allocations will be made

TABLE XII

SQI S (FY 83)

DUTY POSITION	AUTHORIZATION	INVENTORY W/O POOL	INVENTORY W/POOL	"JUMP" SHORTAGE	TOTAL SHORTAGE
11B10S	6 (12)	159	164	--	--
11B20S	7 (14)	92	98	--	--
11B30S	102 (204)	306	337	--	--
11B40S	398 (796)	313	359	85	437
11B50S	483 (966)	343	550	140	416
11C10S	0 (0)	18	18	--	--
11C20S	0 (0)	39	43	--	--
11C30S	14 (28)	69	74	--	--
11C40S	144 (288)	81	92	63	196
TOTAL				288	1049

NOTE: The figures in parenthesis are the authorizations which include the 2.0 MANPOWER POOL FACTOR described in Chapter 1.

to these duty positions authorized in FY 84, the percentage of authorized duty postions which CMF 11 and CMF 13 comprise with respect to the total airborne community will be used to estimate that portion B of the total budget which is allocated for training soldiers from the two CMF's in the three SQI's. The total numbers of authorizations in CMF 11 and CMF 13 are 18,544 and 2,882, respectively, while the total number of authorizations in the airborne community is 51,582. Therefore, the portion of the budget is:

$$B = [(18544 + 2882)/51582] \bullet 2856000$$
$$= (.415) \bullet 2856000 = \$1,185,600.$$

TABLE XIII

Calculation of PCT

$$PCT_P = 400/4411 = .091$$
$$PCT_V = 26/169 = .154$$
$$PCT_S = 288/1049 = .275$$

where k=P-Parachutist, V-Ranger, S-Special Forces.

TABLE XIV

Calculation of Training Costs

$$t_P = 1/2\{([\ 63.75+(1020\bullet.091)\ ]\bullet.81)+63.75\} = \$\ 95.29$$
$$t_V = 1/2\{([\ 170.00+(1020\bullet.154)\ ]\bullet.64)+170.00\} = \$189.67$$
$$t_S = 1/2\{([\ 255.00+(1020\bullet.275)\ ]\bullet.55)+255.00\} = \$274.64$$

d. School Capacity

The same situation exists for the school capacities I_k, (k=P,V,S) for the three type of SQI's. The same percentage of authorizations pertaining to the two CMF's can be used to estimate that portion of each school capacity which will be allocated to train soldiers from CMF 11 and CMF 13. The overall capacities for FY 84 are:

SQI P : 20,000 (50 classes/year @ 400 per class)

SQI V : 1,000 (5 classes/year @ 200 per class)

SQI S : 1,200 (12 classes/year @ 100 per class).

Therefore, the portion of each school capacity used by the optimization model is:

$$I_P = [.415] \bullet 20000 = 8300$$
$$I_V = [.415] \bullet 1000 = 415$$
$$I_S = [.415] \bullet 1200 = 498.$$

2. The Forecasting Model

a. Stock Data

The data required for the execution of the forecasting model is divided into two categories: 1) stock data and 2) flow data. Flow data will be discussed in the next subsection. Stock data is denoted as $n_i(t)$ which refers to the number of soldiers in a specific duty position i at a particular time t. The aggregation of these duty positions n_i, $i=1,2,...k$ is the stock vector $\underline{N}(t)$. This is mathematically expressed for k duty positions as:

$$\underline{N}(t) = (n_1, n_2, ... n_i, ... n_k).$$

Within the scope of this thesis, the total number of duty positions k is 73.

The stock data required for the execution of the forecast model is:

1) the authorized number of soldiers in each duty position
2) the current number of soldiers in each duty position

The two types of stock data are obtained from sources described in the previous chapters.

Authorizations were extracted from the PERSACS document dated 13 October 1983. The authorizations for CMF

11 and CMF 13 by SQI are listed in Tables X, XI, XII, and reflect the authorizations for FY 84. The authorizations for FY 85 and FY 86 are listed by CMF in Tables XXVII and XXVIII.

The current inventory by duty position for FY 83 which pertain to CMF 11 and CMF 13 was provided by MILPERCEN. This data was extracted from the Enlisted Master File (EMF) dated September 1983 and is listed by SQI in Tables X, XI, XII, for CMF 11 and CMF 13. There are two inventory columns listed:

1) the current inventory excluding the manpower pool which was discussed in Chapter 1.
2) the current inventory with the manpower pool included. The first category are only "jump" positions. All duty positions of this type which are listed under the SQI 'P' are all "jump" assignments at Ft. Bragg, North Carolina.

Two empirical distributions which will be used to estimate other data are: 1) the relative frequency of each type of SQI within each MOS-SL combination, and 2) the relative frequency of each type of SL within each MOS. The first set of frequencies is listed in Table XV and represents the proportion of soldiers present in each type of SQI for a particular MOS-SL combination of both CMF 11 and CMF 13. The second set of relative frequencies is listed in Table XVI and represent the proportion of soldiers present in each type of SL for a particular MOS in CMF 11 and CMF 13. These distributions will primarily be used in estimating the attrition rates.

b. Flow data

The second type of data required by the forecasting model is flow data which is denoted as $n_{ij}(t,t+1)$

TABLE XV

Relative Frequency for SQI

MOS-SL	INVENTORY W/POOL	SQI P	SQI V	SQI S
11B10	6634	.938	.037	.025
11B20	1857	.777	.170	.053
11B30	1695	.628	.173	.199
11B40	1382	.590	.150	.260
11B50	1124	.399	.112	.489
11C10	1198	.971	.014	.015
11C20	276	.775	.069	.156
11C30	209	.574	.072	.354
11C40	222	.545	.041	.414
11H10	646	1.0	---	---
11H20	233	1.0	---	---
11H30	147	1.0	---	---
11H40	61	1.0	---	---
13B10	672	1.0	---	---
13B20	169	1.0	---	---
13B30	126	1.0	---	---
13B40	85	1.0	---	---
13C10	3	1.0	---	---
13C20	2	1.0	---	---
13C30	3	1.0	---	---
13C40	29	1.0	---	---
13E10	92	1.0	---	---
13E20	39	1.0	---	---
13E30	20	1.0	---	---
13E40	0	1.0	---	---
13F10	44	.909	.091	---
13F20	144	.847	.153	---
13F30	83	.904	.096	---
13F40	56	.839	.161	---
93F10	6	1.0	---	---
93F20	7	1.0	---	---
93F30	5	1.0	---	---
93F40	4	1.0	---	---
17C10	36	1.0	---	---
17C20	18	1.0	---	---
17C30	7	1.0	---	---
17C40	2	1.0	---	---
17B10	1	1.0	---	---
17B20	1	1.0	---	---
17B30	4	1.0	---	---
17B40	4	1.0	---	---
13R10	9	1.0	---	---
13R20	3	1.0	---	---
13R30	7	1.0	---	---
82C10	54	1.0	---	---
82C20	13	1.0	---	---
82C30	25	1.0	---	---
82C40	17	1.0	---	---
13Y50	43	1.0	---	---
13W50	4	1.0	---	---
13Z50	5	1.0	---	---

TABLE XVI

Relative Frequency for SL

MOS	SL1	SL2	SL3	SL4	SL5
11B	.523	.146	.134	.109	.088
11C	.629	.145	.110	.116	---
11H	.594	.215	.135	.056	---
13B	.639	.160	.120	.081	---
13C	.081	.054	.081	.784	---
13E	.609	.258	.133	.000	---
13F	.135	.440	.254	.171	---
93F	.273	.318	.227	.182	---
17C	.571	.286	.111	.032	---
17B	.100	.100	.400	.400	---
13R	.474	.158	.368	---	---
82C	.496	.119	.229	.156	---
13Y	---	---	---	---	1.0
13W	---	---	---	---	1.0
13Z	---	---	---	---	1.0

and represents the number of soldiers moving between duty positions i and j during the interval t to t+1. For brevity, the notation presented by Bartholomew [Ref. 8] will be used where $n_{ij}(t)$ is equivalent to $n_{ij}(t,t+1)$.

The types of flow data required for the execution of the forecasting model are:

1) the numbers of soldiers moving among all duty positions during the fiscal year.

2) the numbers of soldiers separating from the U.S. Army from all duty positions during the fiscal year.

3) the numbers of soldiers terminating their SQI, and thus, moving out of the airborne community from all duty positions during the fiscal year.

4) the numbers of soldiers who enter the airborne community by completing SQI training during the fiscal year.

The first type of flow data is used in estimating the transition and staying rates. The second and third types of flow data are used in estimating the attrition rates while the fourth type of flow data is generated by the optimization model.

There are many techniques with which to estimate the transition and staying rates p_{ij}. In the forecasting model, the flow data $n_{ij}(t)$ and the stock data $n_i(t)$ will be used. The individual transition rate p_{ij} denotes the fractional flow rate at which soldiers from duty position i move to another duty position j during the fiscal year [Ref. 9]. The estimate of p_{ij} is defined as:

$$p_{ij} = n_{ij}(t)/n_i(t) \qquad i,j = 1,2,\ldots \qquad \text{(eqn 3.1)}$$

If the flow and the stock data pertaining to the estimator p_{ij} are available for several time periods for which the rates do not differ significantly then the estimate of p_{ij} can be mathematically expressed as:

TABLE XVII

Army-wide Promotion Rates (FY 83)

SL 1 to SL 2	.168
SL 2 to SL 3	.128
SL 3 to SL 4	.126
SL 4 to SL 5	.086
SL 5 to 00Z	.072

Note: The category of 00Z is a single category whose paygrade is E-9 and skill level is 5 but organizationally is higher in the formal rank structure. Promotion to this category will attrite the soldier from CMF 11 and CMF 13.

$$p_{ij} = \frac{\sum_T n_{ij}(t)}{\sum_T n_i(t)} \qquad i,j = 1,2,\ldots \quad .$$

where the summations are over all the T values for which both stocks and flows are available.

In this situation, the transition rates in both matrices listed in Figures 2.7 and 2.8 in Section A.1 of Chapter 2 are mostly zeroes except for:

1) promotion rates $p_{i,i+1}$ which lie above the main diagonal and whose values are listed in Table XVII. The flow data was unavailable and the Army-wide rates were used as the estimates of $p_{i,i+1}$.

2) staying rates $p_{i,i}$ which lie along the main diagonal. Although no data was available to estimate these rates, the identity

$$\sum_{j=1}^{k} p_{ij} + w_i = 1 \qquad i,j = 1,2,3\ldots k \qquad \text{(eqn 3.2)}$$

can be applied to compute these rates once the promotion rates $p_{i,i+1}$, the attrition rates w_i and the other promotion rates discussed below are determined.

3) promotion rates which pertain to the promotion of soldiers from SL 4 to SL 5 in MOS's 11C, 11H, 13B, 13C, 13E, 13F, 93F, 17C, 17B and the promotion of soldiers from SL 3 to SL 4 in MOS 13R. These off-diagonal elements are the result of the career progression pattern listed in Figures 1.1 and 1.2 of Section A.4 of Chapter 1 which pertain to CMF 11 and CMF 13, respectively. For example, $p_{22,18}$ in Figure 2.7 is the promotion rate of soldiers in duty position 22(11C40V) to duty position 18 (11B50V). Note that once a soldier with MOS 11C in SL 4 is promoted his MOS becomes 11B as illustrated in Figure 1.1.

The same procedure in estimating the transition rates can be applied to the estimation of the attrition rate w_i. The number of soldiers leaving the airborne community from duty position i during the fiscal year t is denoted as $n_{i,k+1}(t)$ where the category k+1 represents a "holding" category outside the airborne community [Ref. 10]. The estimate of the attrition rate w_i is defined as:

$$w_i = \frac{\text{the number of soldiers attriting from duty position i during the fiscal year (t,t+1)}}{\text{the number of soldiers in duty position i at the beginning of the fiscal year t}}$$

and can be mathematically expressed as:

$$w_i = n_{i,k+1}(t)/n_i(t) \quad i,j = 1,2,..k \,. \quad \text{(eqn 3.3)}$$

In this model, attrition is made up of soldiers who separate from military service and soldiers who "terminate" their SQI thereby leaving the airborne community. Therefore, $n_{i,k+1}(t)$ is equal to the sum of the above two quantities.

The separation data provided by MILPERCEN was extracted from the MOSFILE: Airborne Losses dated 11 October 1983 and are categorized by MOS and SL only and are listed in Table XVIII. In order to obtain the number of soldiers who separated from military service by duty position, the relative frequencies of the SQI's listed in Table XIV are applied to each MOS-SL combination. This results in separations by duty positions as listed in Table XVIII. The underlying assumption in applying this frequency distribution is that all soldiers of a particular MOS-SL combination who separate from the U.S. Army are distributed among the three SQI's in the same proportions as soldiers of that same MOS-SL combination are distributed in the entire airborne community.

The termination data provided by MILPERCEN reflected only the number of soldiers of each MOS assigned to units at Fort Bragg, North Carolina who terminated their SQI during FY 83. In order to estimate the number of soldiers of each duty position who terminated their SQI's during FY 83, three assumptions are made:

1) soldiers of a specific MOS in the entire airborne community terminate their SQI's at the same rate as soldiers of the same MOS who are assigned to Ft. Bragg.
2) soldiers terminating their SQI's in each MOS are distributed among the five SL in the same proportions as soldiers in the entire airborne community.
3) soldiers within each MOS-SL combination who terminate their SQI's are distributed among the three SQI's in the same proportions as soldiers of the same MOS-SL combination within the entire airborne community.

For soldiers located at Ft. Bragg, within each MOS, the termination rate pertaining to CMF 11 and CMF 13 are listed in Table XIX and are used as estimators of the

TABLE XVIII

Separations (FY 83)

MOS-SL	TOTAL SEPARATIONS	SQI P	SQI V	SQI S
11B10	1094	1026	27	41
11B20	269	209	14	46
11B30	26	16	5	5
11B40	4	2	1	1
11B50	2	1	1	0
11C10	122	118	2	2
11C20	23	17	4	2
11C30	2	1	1	0
11C40	0	0	0	0
11H10	157	157	---	---
11H20	37	37	---	---
11H30	4	4	---	---
11H40	0	0	---	---
13B10	115	115	---	---
13B20	20	20	---	---
13B30	10	10	---	---
13B40	0	0	---	---
13C10	0	0	---	---
13C20	0	0	---	---
13C30	0	0	---	---
13C40	0	0	---	---
13E10	10	10	---	---
13E20	5	5	---	---
13E30	1	1	---	---
13E40	0	0	---	---
13F10	52	47	5	---
13F20	24	20	4	---
13F30	1	1	0	---
13F40	1	1	0	---
93F10	5	5	---	---
93F20	1	1	---	---
93F30	0	0	---	---
93F40	0	0	---	---
17C10	14	14	---	---
17C20	2	2	---	---
17C30	1	1	---	---
17C40	0	0	---	---
17B10	4	4	---	---
17B20	0	0	---	---
17B30	0	0	---	---
17B40	0	0	---	---
13R10	0	0	---	---
13R20	0	0	---	---
13R30	0	0	---	---
82C10	12	12	---	---
82C20	3	3	---	---
82C30	1	1	---	---
82C40	0	0	---	---
13Y50	0	0	---	---
13W50	0	0	---	---
13Z50	1	1	---	---

TABLE XIX

Ft. Bragg Termination Rates (FY 83)

MOS	TERMIN-ATIONS	FT. BRAGG INVENTORY	TERMINATION RATE
11B	237	6527	.036
11C	36	1143	.031
11H	44	779	.056
13B	27	805	.034
13C	0	20	.000
13E	4	120	.033
13F	17	231	.074
93F	0	17	.000
17C	5	55	.091
17B	1	6	.167
13R	0	16	.000
82C	9	80	.113
13Y	0	2	.000
13W	0	24	.000
13Z	0	3	.000

termination rates for the entire airborne community. The number of soldiers who terminate within a particular MOS during the fiscal year from the airborne community were calculated using the estimated termination rates and are listed in Table XX.

With the number of soldiers who terminated within each MOS of the airborne community determined, a SL distribution of soldiers from each MOS can be applied based upon the second assumption to estimate the number of soldiers by SL who terminated during the fiscal year. These

TABLE XX

Terminations by MOS (FY 83)

MOS	TERMINATION RATE	INVENTORY W/POOL	TERMIN-ATIONS
11B	.036	12692	457
11C	.031	1905	59
11H	.056	1087	61
13B	.034	1052	36
13C	.000	37	0
13E	.033	151	5
13F	.074	327	24
93F	.000	22	0
17C	.091	63	6
17B	.167	10	2
13R	.000	19	0
82C	.113	109	12
13Y	.000	43	0
13W	.000	4	0
13Z	.000	5	0

values were generated using the relative frequency distribution and are listed in Table XXI.

The third assumption allows the application of the relative frequency distribution for the three SQI's listed in Table XV. As a result, the number of soldiers by duty position who terminated their SQI during the fiscal year are listed in Table XXII.

The sum of the two numbers of separations namely number of attritions listed in Table XVIII and the number of terminations of jump status listed in Table XXII result in

TABLE XXI

Terminations by MOS and SL (FY 83)

MOS	TERMINATIONS	SL1	SL2	SL3	SL4	SL5
11B	457	239	67	61	50	40
11C	59	37	9	6	7	--
11H	61	36	13	8	4	--
13B	36	23	6	4	3	--
13C	0	0	0	0	0	--
13E	5	3	1	1	0	--
13F	24	3	11	6	4	--
93F	0	0	0	0	0	--
17C	6	3	2	1	0	--
17B	2	0	0	1	1	--
13R	0	0	0	0	--	--
82C	12	6	1	3	2	--
13Y	0	--	--	--	--	0
13W	0	--	--	--	--	0
13Z	0	--	--	--	--	0

the number of soldiers in each duty position who attrited from the airborne community during the fiscal year. This is denoted as $n_{i,k+1}(t)$ and is listed in Table XXIII. The estimate of w_i of the attrition rates for CMF 11 and CMF 13 can be calculated according to Equation 3.3 above and are listed in Tables XXIV and XXV, respectively.

TABLE XXII

Terminations by Duty Position (FY 83)

MOS-SL	TOTAL TERMINATIONS	SQI P	SQI V	SQI S
11B10	239	224	6	9
11B20	67	52	4	11
11B30	61	38	12	11
11B40	50	30	13	7
11B50	40	16	20	4
11C10	37	36	1	0
11C20	9	7	1	1
11C30	6	3	2	1
11C40	7	4	3	0
11H10	36	36	---	---
11H20	13	13	---	---
11H30	8	8	---	---
11H40	4	4	---	---
13B10	23	23	---	---
13B20	6	6	---	---
13B30	4	4	---	---
13B40	3	3	---	---
13C10	0	0	---	---
13C20	0	0	---	---
13C30	0	0	---	---
13C40	0	0	---	---
13E10	3	3	---	---
13E20	1	1	---	---
13E30	1	1	---	---
13E40	0	0	---	---
13F10	3	3	0	---
13F20	11	9	2	---
13F30	6	5	1	---
13F40	4	3	1	---
93F10	0	0	---	---
93F20	0	0	---	---
93F30	0	0	---	---
93F40	0	0	---	---
17C10	3	3	---	---
17C20	2	2	---	---
17C30	1	1	---	---
17C40	0	0	---	---
17B10	0	0	---	---
17B20	0	0	---	---
17B30	1	1	---	---
17B40	1	1	---	---
13R10	0	0	---	---
13R20	0	0	---	---
13R30	0	0	---	---
82C10	6	6	---	---
82C20	1	1	---	---
82C30	3	3	---	---
82C40	2	2	---	---
13Y50	0	0	---	---
13W50	0	0	---	---
13Z50	0	0	---	---

TABLE XXXIII

Attrition from the Airborne Community (FY 83)

MOS-SL	TOTAL ATTRITIONS	SQI P	SQI V	SQI S
11B10	1333	1250	33	50
11B20	336	261	18	57
11B30	87	54	17	16
11B40	54	32	14	8
11B50	42	17	21	4
11C10	159	154	3	2
11C20	32	24	5	3
11C30	8	4	3	1
11C40	7	4	3	0
11H10	193	193	---	---
11H20	50	50	---	---
11H30	12	12	---	---
11H40	4	4	---	---
13B10	138	138	---	---
13B20	26	26	---	---
13B30	14	14	---	---
13B40	3	3	---	---
13C10	0	0	---	---
13C20	0	0	---	---
13C30	0	0	---	---
13C40	0	0	---	---
13E10	13	13	---	---
13E20	6	6	---	---
13E30	2	2	---	---
13E40	0	0	---	---
13F10	55	50	5	---
13F20	35	29	6	---
13F30	7	6	1	---
13F40	5	4	1	---
93F10	5	5	---	---
93F20	1	1	---	---
93F30	0	0	---	---
93F40	0	0	---	---
17C10	17	17	---	---
17C20	4	4	---	---
17C30	2	2	---	---
17C40	0	0	---	---
17B10	4	4	---	---
17B20	0	0	---	---
17B30	1	1	---	---
17B40	1	1	---	---
13R10	0	0	---	---
13R20	0	0	---	---
13R30	0	0	---	---
82C10	18	18	---	---
82C20	4	4	---	---
82C30	4	4	---	---
82C40	2	2	---	---
13Y50	0	0	---	---
13W50	0	0	---	---
13Z50	1	1	---	---

TABLE XXIV

Attrition Rates (CMF 11)

DUTY POSITION	ATTRITIONS (FY 83)	INVENTORY (FY 82)	ATTRITION RATES
11B10P	1250	5007	.250
11B20P	261	1078	.242
11B30P	54	738	.073
11B40P	32	550	.058
11B50P	17	295	.058 (.130)
11C10P	154	721	.214
11C20P	24	174	.138
11C30P	4	117	.034
11C40P	4	66	.061
11H10P	193	627	.308
11H20P	50	141	.355
11H30P	12	123	.098
11H40P	4	34	.118
11B10V	33	53	.623
11B20V	18	96	.188
11B30V	17	281	.060
11B40V	14	331	.042
11B50V	21	544	.039 (.111)
11C10V	3	3	1.00
11C20V	5	45	.089
11C30V	3	75	.040
11C40V	3	69	.043
11B10S	50	236	.212
11B20S	57	273	.209
11B30S	16	263	.061
11B40S	8	168	.048
11B50S	4	97	.041 (.111)
11C10S	2	13	.154
11C20S	3	15	.200
11C30S	1	19	.053
11C40S	0	4	.000

NOTE: The ending inventory (FY 82) is the beginning inventory (FY 83) as described in Chapter 1. Also, the figures in parentheses are the attrition rates including those soldiers promoted out of the airborne community.

TABLE XXV

Attrition Rates (CMF 13)

DUTY POSITION	ATTRITIONS (FY 83)	INVENTORY (FY 82)	ATTRITION RATES
13B10P	138	488	.283
13B20P	26	134	.194
13B30P	14	104	.135
13B40P	3	69	.043
13C10P	0	3	.000
13C20P	0	0	.000
13C30P	0	0	.000
13C40P	0	21	.000
13E10P	13	74	.176
13E20P	6	32	.188
13E30P	2	14	.286
13E40P	0	0	.000
13F10P	50	216	.231
13F20P	29	83	.349
13F30P	6	63	.095
13F40P	4	19	.211
93F10P	5	12	.417
93F20P	1	4	.250
93F30P	0	9	.000
93F40P	0	0	.000
17C10P	17	35	.486
17C20P	4	10	.400
17C30P	2	9	.222
17C40P	0	1	.000
17B10P	4	13	.308
17B20P	0	3	.000
17B30P	1	10	.100
17B40P	1	3	.333
13R10P	0	0	.000
13R20P	0	1	.000
13R30P	0	1	.000
82C10P	18	42	.429
82C20P	4	13	.308
82C30P	4	24	.167
82C40P	2	11	.182
13Y50P	0	31	.000 (.072)
13W50P	0	6	.000 (.072)
13Z50P	1	5	.200 (.272)
13F10V	5	14	.357
13F20V	6	20	.300
13F30V	1	4	.250
13F40V	1	6	.167

NOTE: The ending inventory (FY 82) is the beginning inventory (FY 83) as described in Chapter 1. Also, the figures in parentheses are the attrition rates including those soldiers promoted out of the airborne community.

B. EXECUTION OF THE MODEL

1. Optimization for $\underline{R}(83)$

The numbers of soldiers determined to enter into each type of SQI training by duty position are listed under columns 'X1' and 'X2' of Table XXVI for CMF 11 and CMF 13, respectively. These numbers under column 'X1' and 'X2' are the solutions, X_{ijk}, of the optimization model. These solutions were computed using the FORTRAN computer program RECMOD [Ref. 11]. As explained in the previous chapter, the numbers $R_1(84)$ and $R_2(84)$ of soldiers who complete each type of SQI training and enter into the airborne community in CMF 11 and CMF 13, respectively, can be computed by Equation 2.7 in Section C of Chapter 1. As a result, the number of soldiers who enter into the airborne community are also listed in Table XXVI under columns $R_1(84)$ and $R_2(84)$. The recruitment vector $\underline{R}(84)$ which represents the number of soldiers who enter into the airborne community in FY 84 can be expressed as the catenation of the two recruitment vectors $\underline{R}_1(84)$ and $\underline{R}_2(84)$:

$$\underline{R}(84) = (\underline{R}_1(84), \underline{R}_2(84))$$

2. Forecast $\underline{N}(84)$

As described in the previous chapter, two transition matrices were generated for CMF 11 and CMF 13. In Figures 3.1 and 3.2, the transition matrices with the computed transition probabilities p_{ij} for each CMF are shown. By applying Equation 2.1 of Section A.1 of Chapter 2, the predicted force level vectors $\underline{N}_1(84)$ and $\underline{N}_2(84)$ can be determined for CMF 11 and CMF 13, respectively, and are listed in Tables XXVII and XXVIII. Note that the individual elements of both force vectors $\underline{N}_1$ and $\underline{N}_2$ are listed by duty

TABLE XXVI

Training Requirements and Recruitment (1984)

q	DUTY POSITION	X1	R1(84)
1	11B10P	2951	2390
2	11B20P	0	0
3	11B30P	219	177
4	11B40P	0	0
5	11B50P	0	0
6	11C10P	0	0
7	11C20P	289	234
8	11C30P	0	0
9	11C40P	0	0
10	11H10P	494	400
11	11H20P	0	0
12	11H30P	46	37
13	11H40P	0	0
14	11B10V	0	0
15	11B20V	0	0
16	11B30V	0	0
17	11B40V	683	437
18	11B50V	650	416
19	11C10V	0	0
20	11C20V	0	0
21	11C30V	0	0
22	11C40V	306	196
23	11B10S	0	0
24	11B20S	0	0
25	11B30S	267	147
26	11B40S	0	0
27	11B50S	0	0
28	11C10S	0	0
29	11C20S	0	0
30	11C30S	0	0
31	11C40S	0	0

q	DUTY POSITION	X2	R2(84)
32	13B10P	477	386
33	13B20P	0	0
34	13B30P	0	0
35	13B40P	0	0
36	13C10P	16	13
37	13C20P	12	10
38	13C30P	0	0
39	13C40P	0	0
40	13E10P	25	20
41	13E20P	9	7
42	13E30P	15	12
43	13E40P	7	6
44	13F10P	477	386
45	13F20P	148	120
46	13F30P	16	13
47	13F40P	0	0
48	93F10P	20	16
49	93F20P	1	1
50	93F30P	0	0
51	93F40P	0	0
52	17C10P	42	34
53	17C20P	22	18
54	17C30P	11	9
55	17C40P	12	10
56	17B10P	9	7
57	17B20P	1	1
58	17B30P	0	0
59	17B40P	0	0
60	13R10P	60	49
61	13R20P	14	11
62	13R30P	0	0
63	82C10P	17	14
64	82C20P	26	21
65	82C30P	0	0
66	82C40P	0	0
67	13Y50P	0	0
68	13W50P	12	10
69	13Z50P	1	1
70	13F10V	0	0
71	13F20V	22	14
72	13F30V	13	8
73	13F40V	0	0

TABLE XXVII

Recruitment (CMF 11)

q	DUTY POSITION	N1(84)	A1(85)	R1(85)	N1(85)	A1(86)	R1(86)	N1(86)
1	11B10P	6222	8570	2559	6011	8570	2513	6057
2	11B20P	1444	1358	0	1811	1358	0	1970
3	11B30P	1065	1274	59	1215	1274	10	1264
4	11B40P	816	492	0	832	492	0	864
5	11B50P	448	274	0	476	274	0	502
6	11C10P	1163	1146	427	719	1146	275	871
7	11C20P	214	448	0	586	448	0	551
8	11C30P	120	30	0	128	30	0	183
9	11C40P	121	94	0	118	94	0	117
10	11H10P	646	722	0	739	722	335	387
11	11H20P	233	120	0	229	120	0	243
12	11H30P	147	128	0	181	128	0	170
13	11H40P	61	26	0	67	26	0	76
14	11B10V	248	118	66	52	118	41	77
15	11B20V	315	282	25	257	282	72	210
16	11B30V	293	468	42	426	468	47	421
17	11B40V	207	168	0	217	168	0	243
18	11B50V	126	34	0	131	34	0	136
19	11C10V	17	0	0	0	0	0	0
20	11C20V	19	12	0	15	12	0	12
21	11C30V	15	12	0	15	12	0	14
22	11C40V	9	0	0	10	0	0	11
23	11B10S	164	12	0	102	12	0	63
24	11B20S	98	18	0	93	18	0	79
25	11B30S	337	206	0	287	206	0	245
26	11B40S	359	1026	236	790	1050	94	956
27	11B50S	550	1052	109	943	1064	25	1039
28	11C10S	18	28	16	12	28	4	24
29	11C20S	43	366	334	32	342	0	358
30	11C30S	74	118	52	66	118	8	110
31	11C40S	92	282	0	294	282	0	291

NOTE: A1 represents the authorizations for CMF 11 for the fiscal year listed within the parentheses. The subscript q corresponds to the row and column of the transition matrix shown in Figure 3.1.

TABLE XXVIII

Recruitment (CMF 13)

q	DUTY POSITION	N1(84)	A1(85)	R1(85)	N1(85)	A1(86)	R1(86)	N1(86)
32	13B10F	672	110	0	755	110	0	414
33	13B20F	169	14	0	227	14	0	281
34	13B30F	126	18	0	115	18	0	114
35	13B40F	85	16	0	90	16	0	93
36	13C10F	3	12	0	15	12	0	12
37	13C20F	2	8	0	12	8	0	13
38	13C30F	3	0	0	3	0	0	4
39	13C40F	29	10	0	27	10	0	25
40	13E10F	92	10	0	80	10	0	52
41	13E20F	39	4	0	49	4	0	47
42	13E30F	20	8	0	29	8	0	23
43	13E40F	0	0	0	9	0	0	12
44	13F10F	40	94	0	410	94	0	246
45	13F20F	122	48	0	191	48	0	169
46	13F30F	75	22	0	87	22	0	92
47	13F40F	47	4	0	42	4	0	40
48	93F10F	6	10	0	18	10	3	7
49	93F20F	7	4	0	6	4	0	7
50	93F30F	5	2	0	5	2	0	5
51	93F40F	4	2	0	4	2	0	4
52	17C10F	36	70	24	46	70	30	40
53	17C20F	18	36	3	33	36	10	26
54	17C30F	7	16	0	16	16	1	15
55	17C40F	2	12	0	13	12	0	14
56	17B10F	1	6	0	8	6	2	4
57	17B20F	1	2	0	2	2	0	3
58	17B30F	4	2	0	3	2	0	3
59	17B40F	4	2	0	4	2	0	4
60	13R10F	9	58	2	56	58	9	49
61	13R20F	3	14	0	15	14	0	22
62	13R30F	7	4	0	7	4	0	8
63	82C10F	54	38	2	36	38	21	17
64	82C20F	13	22	0	37	22	0	27
65	82C30F	25	6	0	19	6	0	18
66	82C40F	17	4	0	16	4	0	14
67	13Y50F	43	6	0	55	6	0	66
68	13W50F	4	4	0	16	4	0	18
69	13Z50F	5	4	0	5	4	0	4
70	13F10V	4	0	0	0	0	0	0
71	13F20V	22	36	8	28	36	12	24
72	13F30V	8	16	0	17	16	0	16
73	13F40V	9	4	0	8	4	0	9

NOTE: A1 represents the authorizations for CMF 11 for the fiscal year listed within the parentheses. The subscript q corresponds to the row and column of the transition matrix shown in Figure 3.2.

position under column N_i in both tables. The force level vectors were calculated using a revised version of the of the computer program for the Markov Chain Model developed by Bartholomew and Forbes [Ref. 12], available in the APL language on the NPS mainframe (IBM 3033). The force level vector $\underline{N}(84)$ is an input into Equation 2.3 of Section B.1 in Chapter 2, from which elements a_{ijk} are calculated and used in the optimization model in calculating the decision variable X_{ijk}. From Equation 2.7 of Section C of Chapter 2, the recruitment vector $\underline{R}(85)$ is determined.

3. Forecasting for $\underline{N}(85)$ and $\underline{N}(86)$

By using the predicted force level of FY 84 and assuming all costs and completion rates to remain the same throughout the following year, the recruitment vector $\underline{R}(85)$ is determined by the optimization model. The recruitment vectors $\underline{R}_1(85)$ and $\underline{R}_2(85)$ are listed in Tables XXVII and XXVIII, respectively. The predicted force level vectors $\underline{N}_1(85)$ and $\underline{N}_2(85)$ were generated by the forecasting model and their values are listed in Tables XXVII and XXVIII, respectively.

This procedure can be repeated to calculate the recruitment vectors $\underline{R}_1(86)$ and $\underline{R}_2(86)$, and the force level vectors $\underline{N}_1(86)$ and $\underline{N}_2(86)$. The values of these vectors are also listed in Tables XXVII and XXVIII.

C. DISCUSSION OF RESULTS

1. Analysis of the Optimization Parameters

a. Discussion of the Budget

The budget is a critical item in the optimization model as it provides the constraint for which the

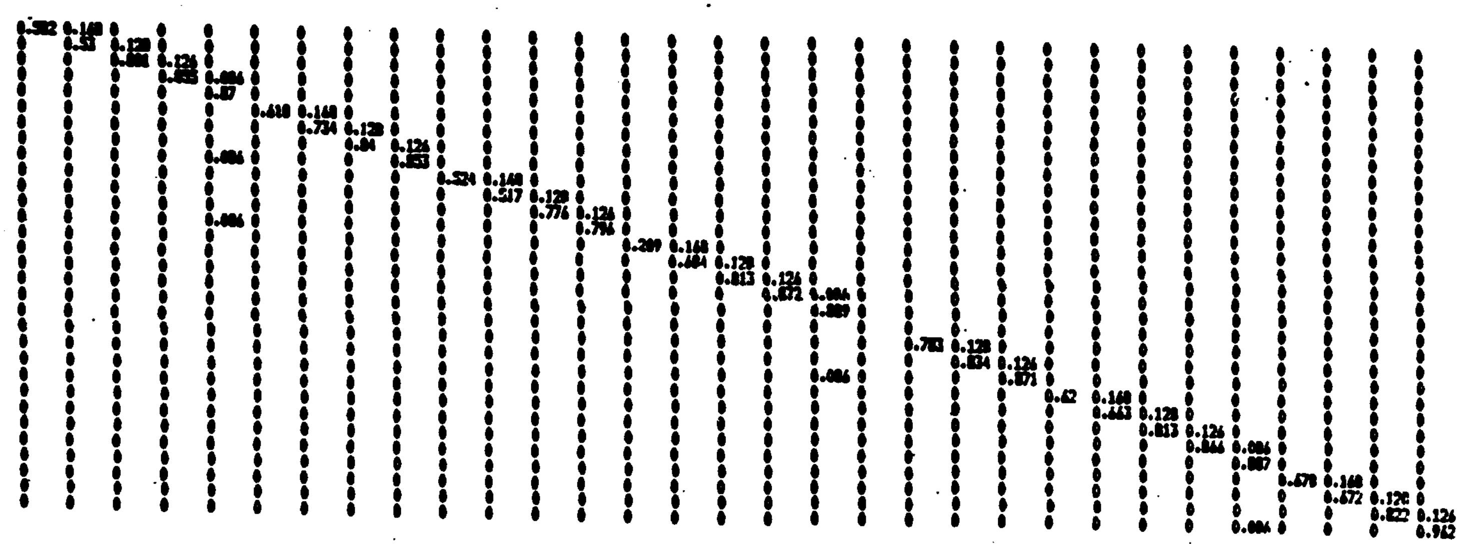


Figure 3.1 Transition Matrix (CHF 11).

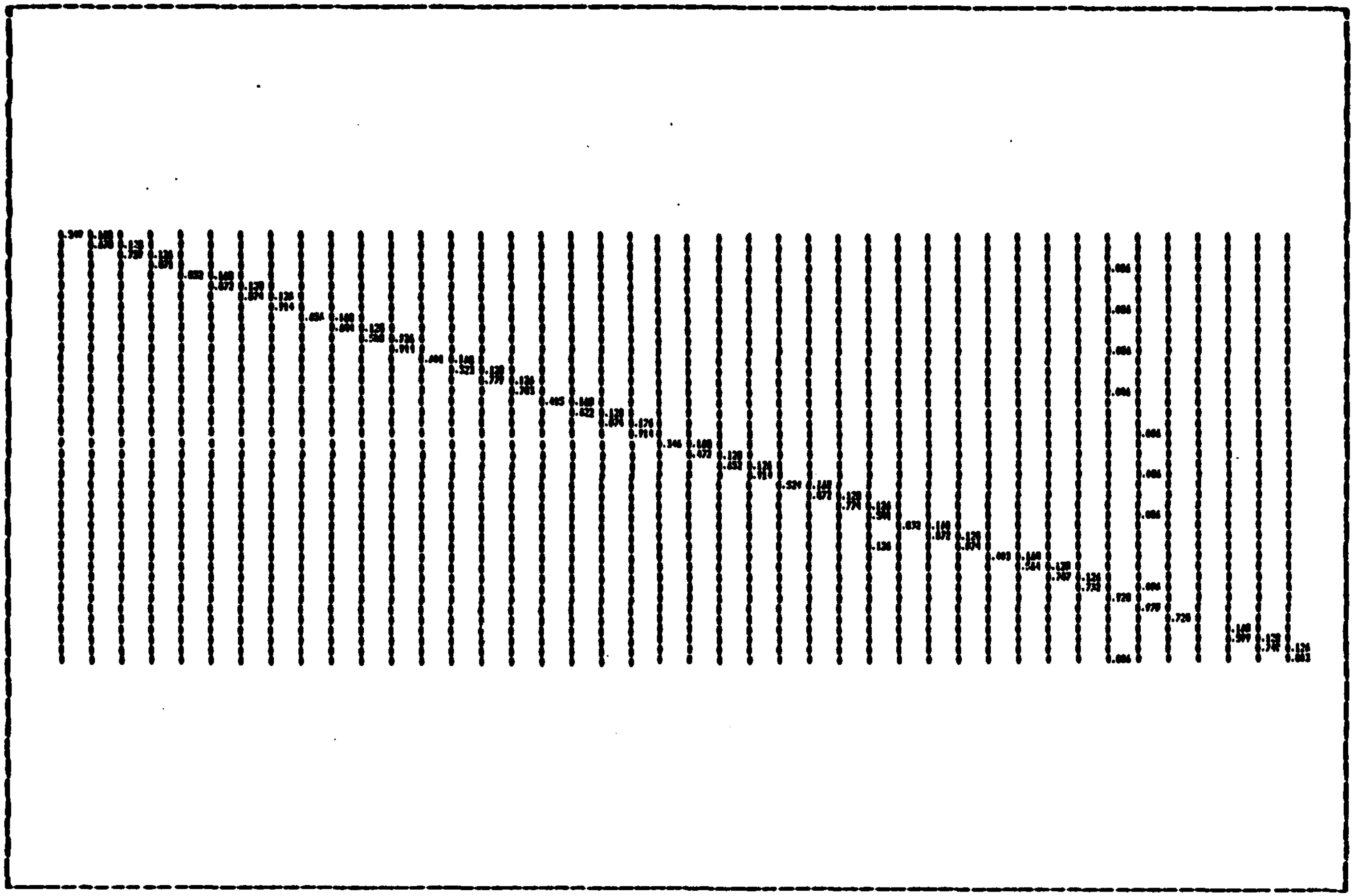


Figure 3.2   Transition Matrix (CMP 13).

penalty function must be minimized. For example, when the budget is less than $224,200, the training allocations are made to parachutist training only. Above that dollar value, the allocations are made between the parachutist and special forces training until the budget of $596,864 is reached. Above that amount allocations are made among all three types of SQI training.

To determine the number of allocations made to each SQI category, the computer program RECMOD FORTRAN [Ref. 13] was used. The efficiency of this algorithm is questionable for large budget values and a heuristic approach can be used to solve the problem. Assuming that the return functions $R_k(B_k)$ are continuous the slope, which is the rate of change per dollar, of each function is defined as:

$$S_k = M_k/t_k \qquad k=1,2,3$$

where S_k represents the slope as the rate of change per dollar of the return function of the kth SQI

M_k represents the marginal decrease in the kth SQI

t_k represents the total cost of training one soldier in the kth SQI.

Lagrange multiplier analysis shows that an optimal answer must occur where $S_1=S_2=S_3$. A search for S can be conducted from an initial value of the slope. A bracketing sequence is applied to find the optimal slope where the budget and training allocations are within an acceptable error.

For example, when a budget B=$450,000 is desired, the optimal allocation vector is (P,V,S)=(3631,0,382) as generated by the computer program. The heuristic approach begins by arbitrarily selecting an initial slope (S = 5.0) and determining the corresponding allocation for each SQI and the associated cost. The cost is then compared to the given budget. If they are equal, the allocation is optimal. If not, a new value of the slope is considered and the process is repeated. The search for the optimal slope concludes with S = -6.44 and the allocation vector is (P,V,S)=(3637,0,380). The budget corresponding to this allocation is $450,015. It is worth noting that the allocation vector is the optimal solution for the corresponding budget $450,015. However, since the true penalty functions are discrete, the slopes are not continuous functions. Since no values of the slope exist between discrete values, a slope value is approximated by the rate of change per dollar of the nearest discrete value. Hence, the method is not guaranteed to be optimal. In Table XXIX, a comparison of the two allocation vectors is shown. The error of the heuristic approach for this problem is 0.5236 percent. This heuristic approach will be used for further analysis of the budget.

TABLE XXIX

Computer vs Heuristic Method

SQI	Computer Results	Heuristic Results	Error (%)
P	3631	3637	0.1652
V	0	0	0.0
S	382	380	0.5236

The budget used for these calculations was $450,000.

In Figure 3.3, the numbers of allocations in each category are plotted against different budget levels.

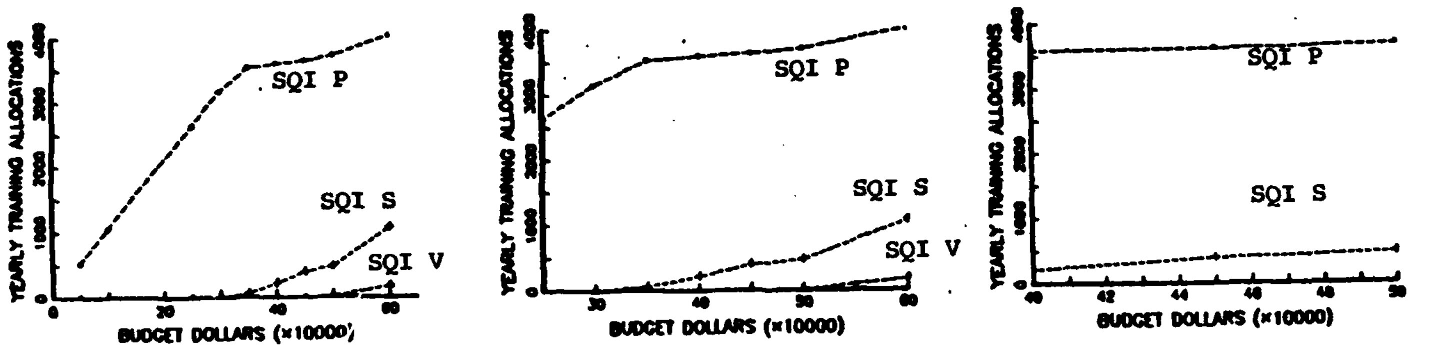


Figure 3.3 Training Allocations Per SQI vs. Budget Levels.

Note that for a budget of $400,000 to $500,000, the number of allocations in SQI 'P' range from 3525 to 3705. It is within this budget range that the number of training allocations remain almost constant in parachutist training. Within this range, training allocations to SQI 'P' vary only 5.106 percent.

Also, the numbers of allocations in SQI 'S' for a budget ranging between $446,600 and $452,310, are 370 to 388. Within this range, the training allocations to SQI 'S' have a variation of 4.9 percent.

Within both budget intervals mentioned above, no training allocations to SQI 'V' are made. The smaller budget interval of $446,600 to $452,310 is where the heuristic method provides an allocation which would be no more than 4.9 percent in error. Further, it is within this range that a change in the budget will not appreciably alter the existing, optimal allocation of training slots.

b. Training Cost

An analysis of the training costs was conducted by using a budget of $450,000 and the current course completion rates listed in Table IX. The training cost of each SQI was varied by ±20% from its FY83 costs listed in Table XIV. The result of varying only the training cost for SQI 'P' is shown as the graph on the left of Figure 3.4, where the resulting change in each SQI allocation can be seen. In Table XXX, the change in the training allocations in any SQI with a corresponding change in the training cost of SQI 'P' is listed. Note that the numbers within the parentheses reflect the percent of change from the FY83 training cost of $95.26. A ten percent error in estimating the cost of parachutist training will not affect the allocations to SQI 'P' significantly. However, it will result in a thirty percent change in the allocations to SQI 'S'. As the cost of

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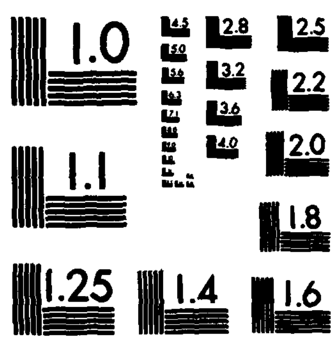

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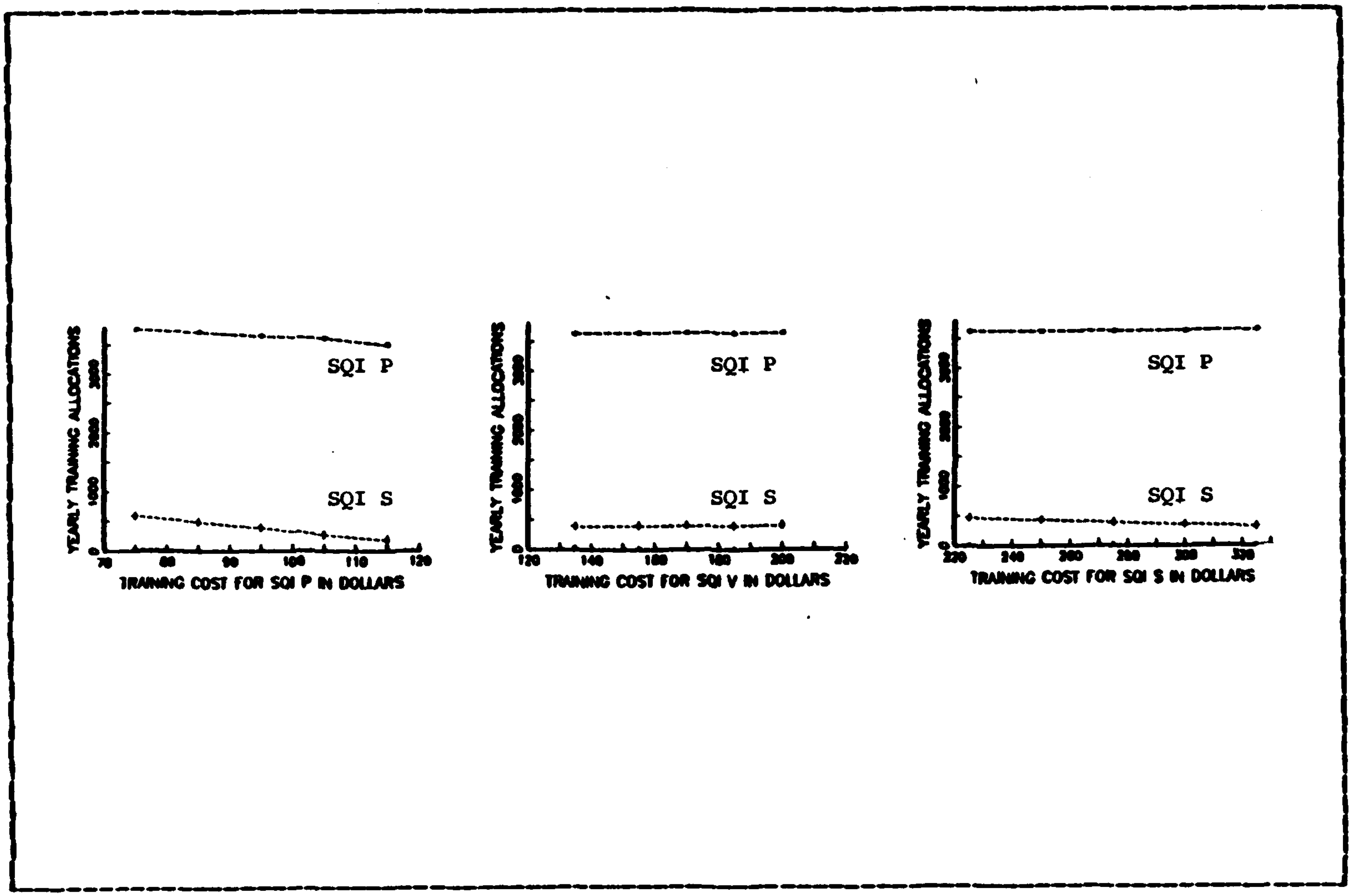

Figure 3.4 Training Allocations Per SQI vs. Training Cost.

TABLE XXX

Change in Training Allocations vs Training Cost of SQI P

Training Cost for SQI P	Change in Training Allocations SQI P	SQI V	SQI S
75	130(3.58)	35(100)	207(54.19)
65	76(2.09)	9(100)	103(26.96)
55.29	-------	-----	---------
105	42(1.16)	0(0)	166(30.37)
115	115(4.35)	0(0)	198(51.83)

training a soldier in SQI 'P' decreases, more of the budget is available for other types of training. As a result, the training allocations made to all three SQI's increase. Conversely, the allocations made to the SQI's decrease as the cost for training a soldier in SQI 'P' increases.

The graph in the middle of Figure 3.4 shows the training allocations made to each type of SQI while varying the cost of ranger training. The following changes in each SQI allocation occurs with a change in the training cost of

TABLE XXXI

Change in Training Allocations vs Training Cost of SQI V

Training Cost for SQI V	Change in Training Allocations SQI P	SQI V	SQI S
150	1(.028)	15(100)	7(1.832)
170	0(0)	0(0)	0(0)
189.67	----	----	----
210	0(0)	0(0)	0(0)
230	0(0)	0(0)	0(0)

SQI 'V' and is listed in Table XXXI. The numbers in parentheses are the percent of change from the FY83 training cost of $189.67. Notice that a ten percent error in estimating the cost of training a soldier in SQI 'V' has no effect on the training allocations.

The graph on the right of Figure 3.4 shows the training allocations made to each type of SQI training while varying only the cost of special forces training. The following changes in the training allocations within each SQI results with the corresponding change in the training

TABLE XXXII

Change in Training Allocations vs Training Cost of SQI S

Training Cost for SQI S	Change in Training Allocations SQI P	SQI V	SQI S
225	5 (.138)	0 (0)	87 (22.78)
250	5 (.138)	0 (0)	42 (11.00)
274.64			
300	10 (.275)	0 (0)	35 (9.162)
115	35 (.964)	0 (0)	69 (18.06)

cost of SQI 'S' and is listed in Table XXXII. The numbers in parentheses are the changes in percent of the training allocations from the FY83 cost of $274.64. A ten percent error in estimating the cost of special forces training does not appreciably affect the allocations to the SQI's of 'P' and 'V'. But, the ten percent error may lead to as much as an eleven percent change in the the training allocations to SQI 'S'.

Overall, changes in the training costs affect one SQI category. The magnitude of its impact on the allocations is dependent on the distribution of shortages by SQI within the airborne community. In this case, the higher the cost of a particular type of training the more effect it has on its own category when any of the costs of training are changed.

c. Course Completion Rate

An analysis of the course completion rates was conducted using a budget of $450,000 and the FY83 costs of training listed in Table XIV. Each course completion rate was varied to observe the change in the training allocations within each SQI. The result of varying the course completion rate for parachutist training is shown as the graph on the left of Figure 3.5 and the changes within each SQI are listed in Table XXXIII. The numbers in parentheses represent the percent of change in the allocations within each SQI from the FY83 course completion rate of 0.81. Underestimating the course completion rate does not alter the training allocations significantly. For a course completion rate within the range of 0.81 to 0.3, the largest percentage error is 7.97%.

The changes in allocations resulting from the variation of the course completion rate for ranger training are listed by SQI in Table XXXIV. The numbers in parentheses are the percent of change in the training allocations for each SQI from the FY83 course completion rate of 0.64. The graph in the middle of Figure 3.5 shows the training allocations as only the course completion rate varies. The course completion rate for ranger training has little effect upon the training allocations of all three SQI's.

The changes in training allocations resulting from the variation of the course completion rate for special

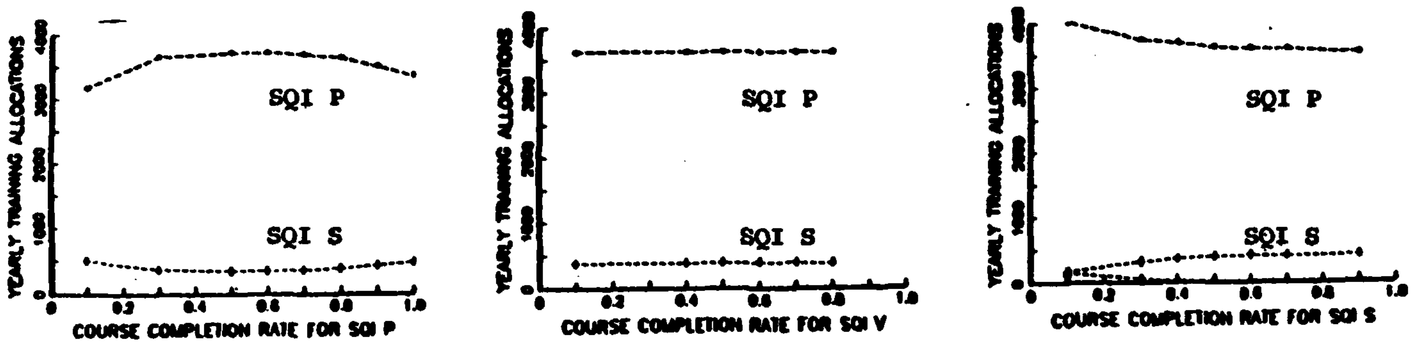

Figure 3.5 Training Allocations Per SQI vs Completion Rates.

TABLE XXXIII

Change in Training Allocations vs Completion Rate of SQI P

Completion Rate for SQI P	Change in Training Allocations SQI P	SQI V	SQI S
0.9	136 (3.75)	0 (0)	47 (12.30)
0.81	--------	----	--------
0.7	55 (1.52)	0 (0)	19 (4.974)
0.6	84 (2.31)	0 (0)	29 (7.592)
0.5	84 (2.31)	0 (0)	29 (7.592)
0.3	29 (.797)	0 (0)	10 (2.618)
0.1	441 (12.1)	0 (0)	140 (36.65)

TABLE XXXIV

Change in Training Allocations vs Completion Rate of SQI V

Completion Rate for SQI V	Change in Training Allocations SQI P	SQI V	SQI S
0.8	3 (.0826)	13 (100)	7 (1.832)
0.7	0 (0)	0 (0)	0 (0)
0.64	0 (0)	0 (0)	0 (0)
0.5	0 (0)	0 (0)	0 (0)
0.4	0 (0)	0 (0)	0 (0)
0.1	0 (0)	0 (0)	0 (0)

forces training are shown as the graph on the right of Figure 3.5 and are listed by SQI in Table XXXV. In this case, overestimating the course completion rate for special forces training has little effect on the training allocations in any SQI while underestimating that rate can significantly alter the training allocations in all three SQI's. This phenomenon occurs because a low course completion rate

TABLE XXXV

Change in Training Allocations vs Completion Rate of SQI S

Completion Rate for SQI S	Change in Training Allocations SQI P	SQI V	SQI S
0.7	26(.7160)	0(0)	9(2.356)
0.6	26(.7160)	0(0)	9(2.356)
0.55	-------	----	-------
0.4	64(1.763)	16(100)	32(8.377)
0.3	114(3.140)	56(100)	74(19.37)
0.1	379(10.44)	136(100)	215(56.28)

allows the school capacity constraint to be the binding one. The low course completion rate requires more soldiers to enter into training. This requirement can be met by the budget but the school is limited by the number of soldiers it can train.

Overall, changes in the course completion rates significantly affect the training allocations but are dependent on the distribution of the shortages within each SQI relative to the overall shortage of the airborne community. In this situation, if the relative frequency of the shortages within the airborne community is small then increases in the course completion rate result in increases in the allocations among all SQI's with little or no effect. If the relative frequency of the shortages is large then any change in the course completion rate results in significant changes of allocations within all SQI's. The course completion rates are not highly sensitive as they can vary, in every instance, by at least 10 percent before an appreciable difference (greater than 4%) in the training allocations occur.

2. Discussion of Forecasting Parameters

The forecasting parameters are the individual elements p_{ij} and w_i of the transition matrix and the attrition vector, respectively. These parameters are highly sensitive and together alter the values of the force level vector $\mathbf{N}(t)$. This section discusses how promotion, attrition, and retention policies are incorporated into the forecast model as changes in the parameters p_{ij} and w_i. The discussion assumes that an initial transition matrix and force level vector have been generated.

a. Promotion Policy

Changes in the promotion policy can apply to a specific MOS and/or SI. Also, a change in the force level vector can be determined as the promotion rate pertaining to a specific duty position is changed. For example, the FY84 promotion policy may be changed to insure that soldiers in duty position 11B40P are promoted at the rate of 0.250 instead of the previous rate of 0.086. The previous attrition rate of 0.058 is left unchanged while and the staying rate of 0.856 is changed to 0.692. The rest of the transition matrix is not altered. The resulting new force level vector $\mathbf{N}_1(85)$ is the same as in Table XXIX except the numbers for duty positions 11B40P and 11B50P are 699 and 609, respectively. This policy change resulted in 133 fewer soldiers in duty position 11B40P and 123 more soldiers in duty position 11B50P. In this manner an appropriate promotion rate may be found that will have the desired increase or decrease in the number of soldiers of a particular duty position. However, as the example demonstrates, accompanying changes in the number of soldiers of another duty position may also result.

If the decision is made to promote a specific SL at a particular rate then the effect on the inventory of the airborne community may be determined by the forecasting model. For example, if there exists a shortage of soldiers in SL 2 and one wants to explore the effect of promoting more soldiers into that skill level, then for every MOS and SQI, the promotion rate to SL 2 in the transition matrix may be changed. While keeping the attrition rate the same, the staying rates are re-calculated using Equation 3.2 in Section A.2. The resulting new force level vector, $\underline{N}(t)$, as generated by the forecasting model will show the effects of the new promotion policy to SL 2.

Similiarly, policies which produce changes in combinations of MOS, SL, and SQI may be incorporated into the forecasting model in the same manner.

b. Attrition Policy

If the attrition rates are changed and the promotion rates are held fixed, a new staying rate must be determined using Equation 3.2 in the same manner as described above. In this way, policy decisions affecting attrition rates may be incorporated into the forecasting model in the same manner as those affecting promotion rates were. For example, if an estimate is made that a new policy will result in a 20 percent attrition rate among soldiers in duty position 11B10P from the airborne community and the promotion rates to SL 2 remain the same as listed in Table XVII, then a new overall attrition rate of 0.632 may be calculated using Equation 3.2 for 11B10P soldiers. When these new rates are set appropriately into the transition matrix, the effects of the new policy may be seen in the resulting new force level vector.

c. Retention Policy

Policy decisions of retention affecting duty positions, MOS's, SL's, or SQI's, may be incorporated into the forecasting model in the same manner as described above. For example, if a new policy is to be evaluated that is thought to have the effect of retaining 80% of 11B10P soldiers, then the staying rate of all soldiers in duty position 11B10P is tc be changed to 0.80. Assuming the promotion rates to remain the same as those listed in Table XVII, the attrition rate of 0.032 may be calculated by Equation 3.2 of Section A.2. These new rates may then be placed in the transition matrix and a new new force level vector may be generated by the forecasting model. The effects of the new retention policy on the future inventory may thus be evaluated.

IV. SUMMARY AND CONCLUSIONS

During this transitional period in which the United States Army continues its force modernization, new weapons systems introduced into the airborne community are translated into new personnel and training requirements. The model formulated and discussed in this thesis can systematically monitor trends in shifting manpower demands. Also, this model is a planning aid for manpower decision-makers in answering "what if" questions and providing timely predicted outcomes to alternate courses of action. It provides the optimal distribution plan for each type of special training upon which assignments can be based subject to budget and school capacity constraints. This facet allows the model to potentially link inventory forecasts with the distribution of the manpower to the force.

A. SUMMARY

This thesis formulates a methodology which forecasts future force levels and determines the number of soldiers to be trained as applied to the CMF's 11 and 13 within the airborne community. A model is formulated which consists of two sub-models. The first sub-model is a forecasting model which applies Markov Theory to manpower planning while the second sub-model is an optimization model which employs the strategy of dynamic programming. The application of theory to both sub-models is discussed during the formulation of the aggregate model. The aggregate model was constructed and applied to CMF 11 and CMF 13. Empirical data of FY83 was used in the execution of the model. Prior to analyzing the output of the model, data preparation is discussed. An analysis of the output is conducted to observe model phenomena.

Since the scope of this thesis applies only to CMF 11 and CMF 13, a portion of the budget and the school capacities is used in the execution of the aggregate model. In reality, neither the budget nor the school capacities are divided among the CMF's. Therefore, conclusions derived from the results of the model are only applicable within the limited scope of this thesis. No global conclusions can be made about the parameters of this model. More study of the model is needed and areas for further research are listed in the final section of this chapter.

B. CONCLUSIONS

The methodology that is used in this thesis to solve the problem stated in Chapter 1 with respect to CMF's 11 and 13 can be applied to the entire airborne community. The forecasting model employs the transition matrix for each CMF and binds them together as described above. Whether the CMF's are 11 and 13, or all the CMF's of the airborne community, the procedure of the forecasting model may remain unchanged. Also, the optimization model considers the allocation of the budget among the three SQI's and considers all MOS's and SL's within each SQI of the airborne community separately in generating the return function mentioned in Chapter 2. The MOS's and SL's are categorized by SQI before the optimization model is applied, regardless of the number of MOS's and SL's.

The aggregate model can be a reliable tool for a manpower decision-maker. The model allows the forecasting of annual inventories of the airborne community and provides discernable information pertaining to training requirements, promotion rates, and attrition rates. The model can be used to evaluate current policies pertaining to promotion, attrition, and retention. Also, it can show how good those policies are in achieving desired force levels or else how counter-productive those same policies are. Further, it can provide timely feedback to the decision-maker about policies which effect the parameters of the model and change the inventories within the airborne community.

The optimization model is robust in that changes in the parameters will not appreciably affect the optimal solution. However, there are instances in which a change in a parameter can significantly alter the solution. When given new budget levels, training costs, and course completion rates, the optimization model can provide timely feedback to a decision-maker in the number of soldiers who should enter and complete each type of special training and who will subsequently enter into the airborne community.

C. RECOMMENDATIONS

1. Application of the Model to the Entire Airborne Community

Two of the assumptions on which the model was formulated are major in the application of this model to all the MOS's, SI's, and SQI's in the airborne community. Both assumptions can be relaxed so that the methodology discussed in this thesis can be applied to any finite number of MOS's, SI's, and SQI's within the airborne community. The two assumptions are that the CMF's are mutually exclusive and that intra-community movements are negligible e.g. a soldier in the parachutist community conducting a PCS move to the ranger community is a rare event.

In regard to the first assumption, CMF's are interdependent as all soldiers, no matter in what CMF they begin, can be promoted to the position of 00Z50. This position can refer to any SQI and only soldiers in SL 5 can move to this position.

To deal with this, a single equation can be stated which links all CMF's together.

$$N_j(t) = N_j(t-1) \bullet (1-w_j) + \sum_{i \neq j} n_i(t-1) \bullet (p_{ij})$$

where j represents the position 00Z50

i represents the duty positions from which promotion to 00Z50 can originate

p_{ij} represents the promotion rate from duty position i to position j.

For example, in CMF 11 and CMF 13, the single equation linking the two CMF's together is:

$$\begin{aligned} N_j(t) = N_j(t-1) \bullet (1-w_j) &+ (n_5(t-1) \bullet p_{5j} \\ &+ n_{18}(t-1) \bullet p_{18j} + n_{27}(t-1) \bullet p_{27j} \\ &+ n_{67}(t-1) \bullet p_{67j} + n_{68}(t-1) \bullet p_{68j} \\ &+ n_{69}(t-1) \bullet p_{69j}) \end{aligned} \quad \text{(eqn 4.1)}$$

For FY85, the projected number of soldiers in the position 00Z50 at the beginning of the year is:

$$\begin{aligned} N_j(85) = N_j(84) \bullet (1-w_j) &+ (448 \bullet 0.072 \\ &+ 126 \bullet 0.072 + 550 \bullet 0.072 + 43 \bullet 0.072 \\ &+ 4 \bullet 0.072 + 5 \bullet 0.072) \\ \cong N_j(84) \bullet (1-w_j) &+ 85. \end{aligned} \quad \text{(eqn 4.2)}$$

Further, the number of soldiers in position 00Z50 at the beginning of the year was 114, i.e. $N_j(84) = 114$. The number who left during FY83 was two while the number of soldiers in that position at the beginning of FY83 was 113, providing a rate of attrition of $w_j = 2/113 = 0.017$. Hence, the number of soldiers $N_j(85)$ in position 00Z50 at the beginning of FY85 is predicted to be 197 by Equation 4.2. The application of an equation similar to Equation 4.2 can tie the forecasts for the entire airborne community together.

The second assumption was that intra-community transfers were negligible. This assumption does not detract from the operation of the model but restricts the model in accounting for those soldiers who move among subcommunities during the fiscal year. This restriction forces all shortages to be filled by newly trained personnel. One alternative is to estimate the intra-community PCS rates from empirical data in the same manner as promotion and attrition rates were estimated. Another alternative is to determine from empirical data all soldiers who are qualified to conduct an intra-PCS move and estimate a percentage of the eligible soldiers who will conduct an intra-PCS move. For example, a ranger-qualified soldier in a duty position within the parachutist subcommunity who is able to conduct a PCS move to the ranger subcommunity without having to undergo ranger training is an "eligible" soldier. If the total number of soldiers who are qualified in both SQI 'P' and SQI 'V' in the parachutist subcommunity is known, then a percentage reflecting the number of soldiers "eligible" to conduct a PCS move may be used as the estimate of the probability that a soldier moves from the parachutist to the ranger subcommunity during the fiscal year. This technique of estimation requires that the number of soldiers qualified in several SQI's in each SQI subcommunity be known at the beginning of the year.

2. <u>Other Areas of Study to Enhance the Model</u>

There are many potential areas which remain to be investigated and can potentially increase the efficiency and effectiveness of this model. The areas of study are:

1) The study of each CMF within the airborne community to determine the relationships that generate the structure for the transition matrix pertaining to each CMF. The determination of each transition matrix allows the relaxation of the assumption pertaining to the independence of each CMF and the application of the methodology of this thesis to the entire airborne community.

2) The estimation of the transition probabilities to include both inter-community and intra-community movements. The estimation of the transition probabilities is critical and may be determined over several time periods. The present unavailability of flow data is the major obstacle in this area.

3) The evaluation of the model as a decision-making aid and its integration and implementation within the United States Army as a manpower planning guide. The real value of this model can be evaluated once it can answer questions pertaining to the entire airborne community.

4) The optimization of training requirements by a generalized network algorithm as an efficient optimization alternative.

5) The efficiency and effectiveness of the heuristic algorithm pertaining to the optimization model.

6) the validation of the model using empirical data to determine its effectiveness in personnel prediction and optimization.

APPENDIX A

THE RETURN (PENALTY) FUNCTION

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C****************************************
C*  THE RETURN (PENALTY) FUNCTION       *
C*          DONALD B. CHUNG              *
C****************************************
C THIS RETURN FUNCTION IS BASED ON THE CHARACTERISTIC OF
C ASSIGNING MEN TO THE DUTY POSITION OF GREATEST SHORTAGE.
C THE AUTHORIZATIONS AND CURRENT INVENTORIES ARE READ IN AND
C THE SHORTAGE OF EACH DUTY POSITION IS CALCULATED.  THESE
C SHORTAGES ARE THEN ORDERED WITH THE LARGEST SHORTAGE
C CATEGORY FIRST.  ASSIGNMENTS ARE THEN MADE IN INCREMENTS
C DESIGNATED BY THE USER.  THE PROGRAM WILL RUN UNTIL THE
C MAN LIMIT OR TRAINING BUDGET IS REACHED.  A RETURN
C FUNCTION IS CREATED ALONG WITH A DISTRIBUTION PLAN FOR
C EACH INCREMENTAL ASSIGNMENT.
C*********** VARIABLE AND CONSTANT DEFINITIONS ************
C   M      THE NUMBER OF DUTY POSITIONS WITHIN A SQI
C   COST   THE TRAINING COST PER MAN FOR TYPE OF SQI TRAINING
C   MEN    THE MAXIMUM NUMBER OF MEN TO BE ALLOCATED TO A SQI
C   INC    THE NUMBER OF MEN TO BE ASSIGNED AT EACH ITERATION
C   CCR    THE COURSE COMPLETION RATE FOR A PARTICULAR SQI
C   A      THE NUMBER AUTHORIZED WITHIN A DUTY POSITION
C   F      CURRENT NUMBER OF SOLDIERS WITHIN A DUTY POSITION
C   DUP    THE NUMBER OF CATEGORIES OF EQUAL SHORTAGE
C   TB     THE TRAINING BUDGET
C   ZI     THE INITIAL VALUE OF THE OBJECTIVE FUNCTION
C   JOB    THE DUTY POSITION BY MOS, SL, SQI
C   Z      THE VALUE OF THE OBJECTIVE FUNCTION EXCLUDING THE
C          INITIAL VALUE
C****************** VARIABLE DECLARATIONS ****************
      INTEGER M,MEN,I,J,Q,L,K,INC,DUP,G,H,V,D,MM
      REAL COST,N(1000),A(1000),S(1000),Z(9000),TB,TEMP,
     *F(1000),ZI,R(1000),EXTRA,T,CCR,ZZ(9000),SS(1000)
      DIMENSION JOB (9000,4),HOLD(9000,4),T(9000)
      LOGICAL NOX
      DATA BLANK/' '/
C*********************************************************
      READ(3,500)M,COST,MEN,INC,CCR
      WRITE(6,660) M
      J = 0
      ZI = 0
      TB = 0
      DO 200  I=1,M
         READ(3,510) A(I),F(I), (JOB(I,K),K=1,4)
         S(I) =  A(I)-F(I)
         WRITE(6,600)S(I), (JOB(I,K),K=1,4)
            IF (S(I).GT.0.)ZI = ZI + (S(I)**2)
200   CONTINUE
      WRITE(6,610)
         WRITE(6,620)TB,J,ZI
      WRITE(6,605) BLANK
250   CONTINUE
         NOX = .TRUE.
         Q = M-1
         DO 285  I=1,Q
            IF(.NOT.(S(I).LT.S(I+1)))GO TO 260
               TEMP = S(I)
               S(I)  = S(I+1)
               S(I+1) = TEMP
               NOX = .FALSE.
```

```
          DO 275 K=1,4
             HOLD(I,K)  = JOB(I,K)
             JOB(I,K)  = JOB(I+1,K)
             JOB(I+1,K)  = HOLD(I,K)
275       CONTINUE
260      CONTINUE
         R(I)  = S(I)
285     CONTINUE
      IF(.NOT.(NCX)) GO TO 250
C     ****** HEADING ******
C     WRITE(4,610)
      DUP = 0
      I = 1
      DO 400 J=INC,MEN,INC
         Z(J) = 0
         TB = FLOAT(J) * COST
         I = J
210      IF(.NOT.(S(I).GT.0..AND.L.NE.0)) GO TO 225
            IF(.NOT.(R(I).GT.0..AND.(R(I) - FLOAT(INC)).GT.
     *      S(I+1)))GO TO 215
               IF(.NOT.(DUP.NE.0)) GO TO 213
                  DC 212 H=1,I
                     R(H) =R(H)-FLOAT(INC)/FLOAT(DUP+1)
212               CONTINUE
               GO TC 214
213            CONTINUE
                  R(I) = R(I) - FLOAT(INC)
214            CONTINUE
               L = 0
            GO TO 222
215         IF(.NOT.(R(I).GT.0..AND.(R(I) - FLOAT(INC)).LE.
     *      S(I+1)))GC TO 220
              EXTRA=FLOAT(INC)-((R(I)-S(I+1))*FLOAT(DUP+1))
              IF(.NCT.(EXTRA.GT.0.))GO TO 2166
                 IF(.NOT.(I.EQ.1))GO TO 216
                     R(I) = S(I+1)
                     V = I+1
                     DO 2150 H=1,V
                        R(H) = R(H) - EXTRA/FLOAT(DUP+2)
2150                 CONTINUE
                     DUP = DUP+1
                     I = I + 1
                 GO TO 218
216              IF(.NOT.(I.NE.1))GO TO 218
                    DO 2160 H=1,I
                       R(H)=S(I+1)
2160                CONTINUE
                    V = I+1
                    IF(.NOT.((R(V)*FLOAT(DUP+2)).LT.
     *              FLOAT(INC)))GO TO 2163
                       D=FLOAT(J-1) + (R(V)*FLOAT(DUP+2))
                       TB = D * COST
                       DO 2162 H=1,V
                          R(H) = 0
2162                   CONTINUE
                    GO TO 2165
2163                CONTINUE
                       DO 2164 H=1,V
                          R(H)=R(H)-EXTRA/FLOAT(DUP+2)
2164                   CONTINUE
                       DUP = DUP + 1
                       I = I + 1
2165                CONTINUE
218              CONTINUE
              GO TO 219
2166          CONTINUE
                 DC 2167 H=1,I
                    R(H) = R(H) - FLOAT(INC)/FLOAT(DUP+1)
2167             CCNTINUE
```

```
219               CCNTINUE
                  L = 0
               GO TO 222
220            IF(.NOT.(H(I).LE.0.)) GO TO 222
                  L = 0
222            CONTINUE
            GO TO 210
225         CONTINUE
            DC 300 G=1,N
               IF(.NOT.(S(G).GT.0.)) GO TO 230
               N(G) = S(G) - R(G)
                  Z(J) = Z(J) + (S(G)-(CCR*N(G)))**2
               WRITE(6,615)S(G),R(G),N(G),(JOB(G,K),K=1,4)
               GO TC 232
230            CONTINUE
                  N(G) = 0
232            CONTINUE
300         CCNTINUE
C        ****** HEADING ******
         WRITE(6,610)
            WRITE(6,620)TB,J,Z(J)
C           WRITE(4,630)TB,J,Z(J)
400      CCNTINUE
         T(1)=Z(1)-ZI
         DC 450 I=1,MEN
            T(I+1)=Z(I+1)-Z(I)
 450      CCNTINUE
         NN = MEN + 1
         DC 475 J=1,NN
            WRITE(7,655)J,T(J)
475      CCNTINUE
         STOP
500      FCRMAT(I10,2X,F6.4,2X,I10,I10,F5.3)
510      FORMAT(F14.6,2X,F14.6,2X,I2,A1,I2,A1)
600      FCRMAT(' ',F14.6,2X,I2,A1,I2,A1)
605      FORMAT(' ',A1)
610      FCRMAT(' ',4X,'BUDGET',3X,'MEN',3X,'Z-VALUE')
615      FCRMAT(' ',F9.0,2X,F9.0,2X,F9.0,2X,I2,A1,I2,A1)
620      FORMAT(' ',F9.0,1X,I5,1X,F14.2/)
630      FORMAT(' ',F9.0,1X,I5,1X,F14.2)
650      FORMAT(' ',F14.6)
655      FCRMAT(' ',10X,I5,1X,F14.2)
660      FCRMAT(' ',I10)
         END
```

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02 — 84