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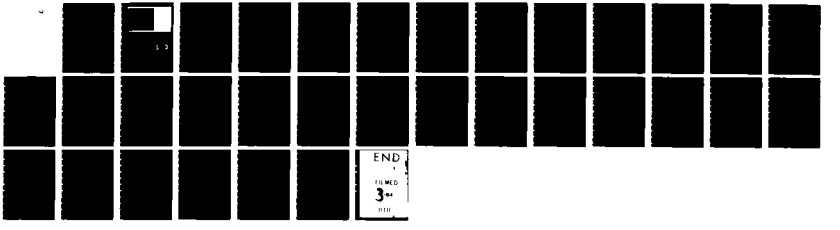

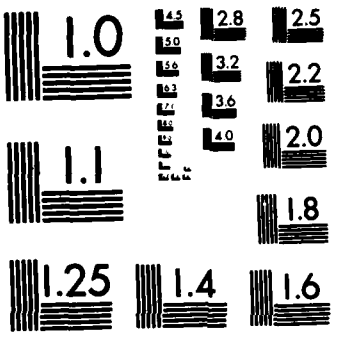

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EDGE WAVES BY BOUNDARY COLLOCATION

L. Reichel and P. Chapman

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

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ABSTRACT

The determination of periodic edge waves along a straight infinite beach can be formulated as a 2-dimensional eigenvalue problem for a generalized wedge. A least squares boundary collocation method for computing the lower discrete eigenvalues is described, with an error estimate for the least eigenvalue. Numerical examples are presented.

AMS(MOS) Subject Classification: 65N25, 65N35

Key Words: Boundary collocation, Edge waves, Eigenvalue problem, Sloping beach

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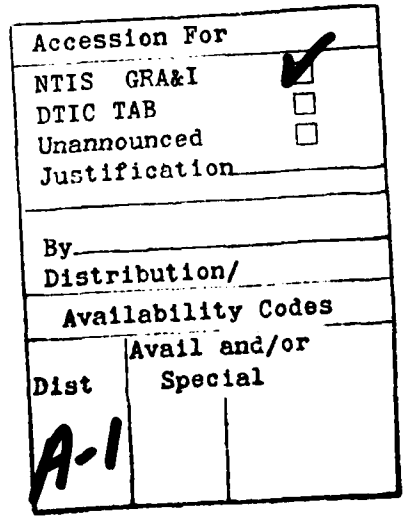
SIGNIFICANCE AND EXPLANATION

Computational eigenvalue problems for unbounded domains often present difficulties in execution. Thus it is of interest to demonstrate the viability of the boundary collocation method for problems on unbounded domains for which few analytic results are known.

For demonstration the Stokes-Ursell edge waves for general bottom profiles are examined. Even though other methods are available for computation of these eigenvalues, the flexibility of the present method might be an advantage in technical problems. It should also be applicable to different problems on semi-infinite domains.



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EDGE WAVES BY BOUNDARY COLLOCATION

L. Reichel and P. Chapman

1. Introduction

The aim of this paper is to describe a computational method for determining traveling edge waves along a straight infinite beach with a cylindrical, not necessarily plane bottom. Assuming the waves are periodic along the shore line (the z-axis), this problem can be reduced to a 2-dimensional eigenvalue problem on a generalized wedge, $\mathcal{D}$, in the x,y-plane, see Ursell [9]. The top boundary of the wedge, representing the (horizontal) free surface of the fluid, is the positive x-axis. The other boundary of the wedge is below the positive x-axis and represents the sea bottom, see Figure 1.1. The y-coordinate of the sea bottom, $y = b(x)$, is assumed to be a smooth (C^2) function of x, with $b(0) = 0$, $b(x) < 0$, $x > 0$ and with a finite, negative $\lim_{x\to\infty} \frac{b(x)}{x}$.

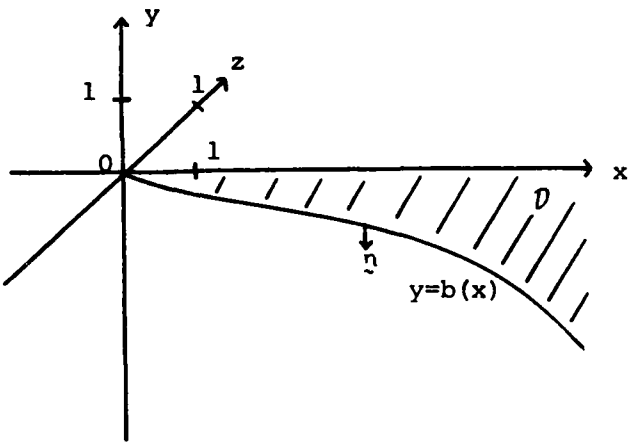

Figure 1.1

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The traveling waves of interest, see Ursell [9], p. 83, are

$$\psi(x,y,z,t) := \cos(kz)\phi(x,y)e^{i\rho t} , \tag{1.1}$$

where ϕ and the real constant ρ satisfy

$$\Delta_{xy}\phi = \phi \text{ on } \mathcal{D}, \tag{1.2}$$

$$\frac{d\phi}{d\underset{\sim}{n}} = \frac{\rho^2}{kg}\phi =: w^2\phi \quad \text{on top: } y=0,\ x>0, \tag{1.3}$$

$$\frac{d\phi}{d\underset{\sim}{n}} = 0 \quad \text{on bottom: } y=b(x),\ x>0, \tag{1.4}$$

$$E := \iint_{\mathcal{D}} \left(|\underset{\sim}{\nabla}\phi|^2 + \phi^2 \right) dS < \infty . \tag{1.5}$$

$\underset{\sim}{n}$ denotes the outward unit normal, g is the gravitational acceleration and k is the same real constant as in (1.1). The dimensionless problem eigenvalue is denoted by w^2. Ursell [9] studied the case of a straight bottom (i.e. $b(x) = cx$) and found a finite, simple, discrete, spectrum with eigenfunctions ϕ of finite energy E (equation 1.5), and a continuous spectrum (of non-finite energy) above a cut off value of w^2.

Ursell [9] further observed that the number of eigenpairs of the problem (1.2-1.5) increases as the interior angle of the wedge decreases. A recent analysis based on shallow water theory of Chapman [1] suggests that this angle is also a controlling parameter when the bottom is not straight.

In the present work the central interests are the following

1. To study the use of a Boundary Collocation Method (BCM) in the computation of eigenpairs.
2. To investigate the role of the wedge angle as a dominant parameter.

The attraction of the BCM lies in its applicability to a variety of domains, its ease of programming, and its conceptual simplicity. The method reduces the computational problem to a 1-dimensional problem. Murashima et al. [4] call the BCM the charge simulation method (CSM) and state "The charge simulation method [...] is often used in some part of Electrical Engineering as an efficient way for solving Laplace's equation. The CSM, is not considered to be straight forward because contour points [= collocation points] and source points [= singular points of the basis functions] must be determined by trial and error." We eliminate the guessing in the CSM by presenting a strategy for selecting collocation points when the set of basis functions is given. We will choose the number of collocation points so that an overdetermined system of equation is obtained. This we solve by a Galerkin method, i.e. the boundary collocation method to be described is a Galerkin-collocation method. For each set of basis functions the determination of eigenpairs is quite rapid, and we carry out the computation for several systematically chosen sets of basis functions. A simply computable error estimate for the lowest eigenvalue reveals which set of basis functions yields best accuracy. This makes it unnecessary for the user to guess good source points. The idea to iteratively choose source points has previously been advocated by Mathon and Johnson [2] for Dirchlet problems. The number of distinct source points we need is, however, much smaller than in the examples in [2]. This is important since the computational effort increases rapidly with the number of distinct source points, see section 2.

This paper is organized as follows: Section 2 contains the description of the BCM, Section 3 contains numerical examples, and in Section 4 we derive an easily computable error estimate for the least eigenvalue. The eigenproblem under consideration has previously been studied by methods of geometrical optics, see Meyer [3] and Shen et al. [8].

2. A Boundary Collocation Method

We first describe the allocation of collocation points for a given set of eigenfunctions. This description is slightly more general than needed for the numerical examples in sections 3, because we want generalizations to other problems to be simple. Presently, we only can motivate the allocation heuristically, but numerical examples show that the allocation is appropriate.

Let $\{(x_j, y_j)\}_{j=1}^{\ell}$ be a point set exterior to $\mathcal{D} \cup \partial\mathcal{D}$. For an arbitrary point (x,y), let (r_j, θ_j) be the polar coordinates for $(x-x_j, y-y_j)$, $j = 1(1)\ell$. We will approximate the eigenfunctions of (1.1)-(1.5) by linear combinations of the functions

$$K_0(r_j),\ K_1(r_j)\cos(\theta_j),\ K_1(r_j)\sin(\theta_j),\ K_2(r_j)\cos(2\theta_j),\ldots$$
$$\ldots, K_p(r_j)\cos(p\theta_j),\ K_p(r_j)\sin(p\theta_j),\quad j=1(1)\ell, \tag{2.1}$$

for some integer $p > 0$. $K_j(r)$ denotes the j^{th} order modified Bessel function of the 2nd kind. Any linear combination of functions (2.1) satisfies (1.2) and (1.5). Conditions (1.3) and (1.4), we impose at collocation points on $\partial\mathcal{D}$.

We turn to the allocation of collocation points. We are led to the allocation strategy by regarding approximation of analytic functions $f(w)$ on regions Ω in the extended complex plane C. The complement of Ω, Ω_c, is assumed to be open, bounded, simply connected, and have a smooth boundary $\partial\Omega$. Let $\{w_j\}_{j=1}^{\ell}$ be a finite set of distinct points in Ω_c, and approximate $f(w)$ by a linear combination of the functions

$$1,\ (w-w_j)^{-1},\ (w-w_j)^{-2},\ldots,\ (w-w_j)^{-\hat{p}},\quad j=1(1)\ell. \tag{2.2}$$

The linear combination is to be determined by interpolation at points on $\partial\Omega$. Regard $w_1, w_2, \ldots, w_\ell$ unit point charges, which generate a potential $-\frac{1}{\ell}\sum_{j=1}^{\ell} \ell n|w-w_j|$. Let $\sigma(w)$ be the charge distribution on $\partial\Omega$ of total charge 1, so that $\sigma(w)$ makes $\partial\Omega$ an equipotential curve in the presence of the point charges at w_j. It is known that $m = \hat{p}\ell+1$ interpolation points (= collocation points) $\{w_k^c\}_{k=1}^m$ on $\partial\Omega$ for determining a linear combination of functions (2.2) should be equidistributed w.r.t. $\sigma(w)$, i.e. w_1^c is arbitrary and w_j^c, $j > 1$, should satisfy

$$\int_{w_k^c}^{w_{k+1}^c} \sigma(w)|dw| = \frac{1}{m}, \quad k=1(1)m-1, \tag{2.3}$$

where integration is done along $\partial\Omega$ in postive direction. For details, see Reichel [6]. $\sigma(w)$ can be determined by solving

$$\left.\begin{aligned} q + \int_{\partial\Omega} \ell n|\tilde{w}-w|\sigma(w)|dw| &= \frac{1}{\ell}\sum_{j=1}^{\ell} \ell n|\tilde{w}-w_j|, \ \tilde{w} \in \partial\Omega \\ \int_{\partial\Omega} \sigma(w)|dw| &= 1, \end{aligned}\right\} \tag{2.4}$$

where q is a constant also to be determined, see [6] or [7]. Let $F(w)$ be a conformal map from Ω_c to the complement of $\mathcal{D} \cup \partial\mathcal{D}$. Let

$$x_j + iy_j := F(w_j), \quad j=1(1)\ell.$$

F can be continued to a bijective map on $\Omega \cup \partial\Omega$, and we let the collocation points (x_j^c, y_j^c), $j=1(1)m$ be defined by

$$x_j^c + iy_j^c = F(w_j^c), \quad j = 1(1)m. \tag{2.5}$$

We motivate this choice by the invariance of the potential under conformal mapping, and by the fact that for some constants $c_p \neq 0$, independent of r_j, θ_j,

$$K_0(r_j) \to c_0 \ell n(r_j) \quad \text{as} \quad r_j \to 0,$$

$$K_p(r_j)e^{-ip\theta_j} \to c_p r_j^{-p_e - ip\theta_j} \quad \text{as} \quad r_j \to 0, \quad p = 1,2,3,\ldots,$$

which shows that the functions (2.1) behave essentially like harmonic functions as $r_j \to 0$, $j=1(1)\ell$. For harmonic functions the same allocation as for analytic functions is suitable.

The application to problem (1.1)-(1.5) permits some simplifications. When allocating the collocation points (x_j^c, y_j^c), we approximate the generalized wedge $\mathcal{D}$ by a wedge D with straight sides and with vertex at the origin. Let the interior angle of D at the origin be $\beta\pi$. Turn D to be symmetric w.r.t. the x-axis and so that D has the negative x-axis in its interior. A conformal mapping F which maps the exterior of the unit circle on the complement of D is explicitly known

$$z = F(w) = \left(\frac{1-w}{1+w}\right)^{2-\beta}. \tag{2.6}$$

The charge density function $\sigma(w(s))$ we seek in the form of a trigonometric polynomial of degree $\hat{n}$. When $\partial\Omega$ is a circle then this approximation of $\sigma(w(s))$ can be determined in $O(\hat{n}\,\ell n(\hat{n}))$ operations, see [5] especially lemma 2 for details. We choose the angle of D so that properly turned, D approximates $\mathcal{D}$ well near the origin. Having determined the collocation points on ∂D, we map them onto $\partial\mathcal{D}$ by some mapping which approximately preserves distance from the vertex.

In the computations we have chosen the number collocation points, m, as 2-3 times n, the number of basis functions (2.1), i.e. $n = (2p + 1)\ell$. The allocation is good in the sense, that doubling the number of collocation points has in no case when checked changed the order of magnitude of the error in the eigenvalues.

We now turn to the computation of eigenvalues and eigenfunctions with a given set of basis functions (2.1). First we allocate m collocation points as described, and make sure one point is placed at the vertex. The vertex point is regarded as 2 points, since we will impose both conditions (1.2) and (1.3) at the vertex. Scale the basis functions (2.1) so that $\max|K_s(r_j)| = 1 \ \forall s,j$, where the maximum is taken over all collocation points. Denote these scaled functions by $f_1(x,y), f_2(x,y),\ldots,f_n(x,y)$, and orthonormalize $\frac{\partial f_j}{\partial n}(x,y)$ with respect to the inner-product

$$(f,g) := \sum_{j=1}^{m} f(x_j,y_j)\ g(x_j,y_j), \tag{2.7}$$

where we sum over all collocation points, and $\frac{\partial}{\partial n}$ denotes the outward normal derivative w.r.t. $\mathcal{D}$. We denote the orthornormalized functions by $\tilde{f}_1(x,y)$, $\tilde{f}_2(x,y),\ldots,\tilde{f}_n(x,y)$. We seek a linear combination

$$\varphi(x,y) = \sum_{k=1}^{n} a_k \tilde{f}_k(x,y) \tag{2.8}$$

and constants w^2 which satisfy (1.2) and (1.3) at the collocation points. This gives rise to an overdetermined generalized eigenvalue problem, which we symbolically write, with $\underline{a} = (a_1, a_2,\ldots,a_n)^T$, as

$$\underbrace{\begin{pmatrix} \frac{d\varphi}{dn} \\ \hline \frac{d\varphi}{dn} \end{pmatrix}}_{n} \underline{a} = w^2 \underbrace{\begin{pmatrix} \varphi \\ \hline 0 \end{pmatrix}}_{n} \underline{a} \quad \begin{matrix} \} \text{ m/2 eq. from top nodes} \\ \} \text{ m/2 eq. from bottom nodes} \end{matrix} \tag{2.9}$$

Multiply (2.9) by the transpose of the orthonormal left hand side matrix. The resulting system we write as

$$\mu^2 \underline{a} = A\underline{a} \ , \quad \mu^2 = 1/w^2 \tag{2.10}$$

(2.10) is an ordinary eigenvalue problem with an $n \times n$ matrix A. We can regard equation (2.10) as Galerlein equations obtained by using the test functions $\frac{\partial \tilde{f}_1}{\partial n}, \frac{\partial \tilde{f}_2}{\partial n}, \ldots, \frac{\partial \tilde{f}_n}{\partial n}$ and inner-product (2.7).

Solving eigenproblem (2.10) yields eigenvalues μ_j^2 and corresponding eigenvectors $\underline{a}_j := (a_{1j}, a_{2j}, \ldots, a_{nj})$. We obtain approximate eigenvalues $\tilde{w}_j^2 := 1/\mu_j^2$, to problem (1.1)-(1.5) and corresponding approximate eigenfunctions

$$\varphi_j(x,y) := \sum_{K=1}^{n} a_{kj} \tilde{f}_K(x,y). \tag{2.11}$$

Enumerate the $\tilde{w}_j^2$ so that $\tilde{w}_1^2 \leq \tilde{w}_2^2 \leq \ldots \leq \tilde{w}_n^2$. In section 4, we derive the following error estimate for $\tilde{w}_1^2$: Let w_1^2 be the true least eigenvalue for (1.1)-(1.5), and let $\partial\mathcal{D}_t$ denote the part of $\partial\mathcal{D}$ which coincides with the x-axis. Let $\partial\mathcal{D}_b := \partial\mathcal{D} - \partial\mathcal{D}_t$. Let s denote arc length. Then

$$|\tilde{w}_1^2 - w_1^2| \leq \int_{\partial\mathcal{D}_t} \frac{d\varphi_1\big(x(s),y(s)\big)}{ds}^2 + \big(\varphi_1(x(s),y(s))\big)^2 |\varepsilon_t(s)| ds + \tag{2.12}$$

$$+ \int_{\partial\mathcal{D}_b} \frac{d\varphi_1\big(x(s),y(s)\big)}{ds}^2 + \big(\varphi_1((x(s),y(s))\big)^2 |\varepsilon_b(s)| ds, \tag{2.13}$$

where

$$\varepsilon_t(t) := \int_0^t \left(\frac{d\varphi_1}{dn}(x(s),y(s)) - \tilde{w}_1^2\varphi_1(x(s),y(s))\right)\left(\frac{d\varphi_1}{ds}(x(s),y(s))\right)^{-1}ds, \qquad (2.14)$$

$$\varepsilon_b(t) := \int_0^t \frac{d\varphi_1}{dn}(x(s),y(s))\left(\frac{d\varphi_1}{ds}(x(s),y(s))\right)^{-1}ds. \qquad (2.15)$$

The parameter t also denotes arc lenth, which is measured from the vertex along $\partial\mathcal{D}_t$ in the integrals (2.12), (2.14), and from the vertex along $\partial\mathcal{D}_b$ in (2.13), (2.15). Especially, $t = 0$ corresponds to the vertex.

The error estimate (2.12)-(2.15), we can use for the <u>determination of a good allocation of source points</u> (x_j, y_j). In all computed examples, we obtained good accuracy with only one source point (x_1,y_1), i.e. in (2.1) we have $\ell = 1$, and we restrict the description to this case. Generalizations to ℓ distinct source points are immediate, but we note that the effort to determine them increases rapidly with ℓ; the determination of ℓ source points yields a 2ℓ-dimensional nonlinear optimization problem.

Since we need not find the best allocation of (x_1,y_1), a good placement suffices, we can use a simple method for its determination. Choose (x_1,y_1) arbitrarily but fairly near the vertex at $\mathcal{D}$. Determine the collocation points, solve the eigenvalue problem (2.10), and evaluate the error estimate (2.12)-(2.15). Repeat for the remaining 4 points in the 5-point-stencil, with points

$$\{(x_1,y_1),\ (x_1+h,y_1),\ (x_1-h,y_1),\ (x_1,y_1+h),\ (x_1,y_1-h)\}. \qquad (2.16)$$

$h > 0$ is selected so that the whole stencil is exterior to $\partial\mathcal{D}$. If the error estimate is smallest at one leg of the stencil, say at (x_1-h,y_1), then move the stencil so that (x_1-h,y_1) becomes its midpoint and compute the

error estimate at stencil points where not known. This way the stencil is moved in the plane until the smallest error estimate is obtained at its center. In this case the stencil is shrunk, i.e. we replace h by $h/2$. Repeat until the stencil size is smaller than some prescribed $\delta > 0$.

3. Numerical Examples

In the examples we denote the computed eigenvalues by $\tilde{w}_k^2$, k=1,2,3,... ordered so that $\tilde{w}_1^2 \leq \tilde{w}_2^2 \leq \tilde{w}_3^2 \leq \ldots$. The number of collocation points, we denote by m, the number of basis functions by n. For wedges with straight sides, we let n := m/2. Increasing the ratio $\frac{m}{n}$ does not change the order of magnitude of the error, which shows that the distribution method for the collocation points is appropriate. For wedges with curved sides we chose $\frac{m}{n} = 3$, since the allocation of nodes for such wedges is somewhat arbitrary; we do not compute the conformal map from the exterior of the unit disk to the exterior of the wedge, but use instead an explicitly known map from the exterior of the unit disk to the exterior of a wedge with straight sides. Again, increasing the ratio $\frac{m}{n}$ does not change the order of magnitude of the error.

Let α denote the interior angle at the vertex of $\mathcal{D}$. All computations were initialized by choosing the source point $(x_1,y_1) = \left(-\sqrt{2}\cos(\frac{\alpha}{2}), \sqrt{2}\sin(\frac{\alpha}{2})\right)$, setting m and n to the smallest values in the tables below and letting h defining the stencil (2.16) be 1/2 for example 3.1 and 1/4 for the other examples. The stencil was moved as described in the end of section 2 until the estimated error was smallest at its center point. This point we below call computed source point, and at this point we computed the eigenvalues for larger values of m and n also.

The discretization replaces the original eigenvalue problem (1.1)-(1.5) by an n-dimensional one that has n eigenvalues, which are approximated by $\tilde{w}_j^2 = \tilde{w}_{j,n}^2$, j = 1(1)n. All eigenvalues of the discrete problem need not correspond to eigenvalues of the continuous problem (1.1)-(1.5), and we need a simple criterion to determine how many of the $\tilde{w}_{j,n}^2$, j = 1(1)n, to accept as approximations of eigenvalues to the continuous problem. Our criterion is

based on the dependence of the $\tilde{w}^2_{j,n}$ on n. For some integer k, depending on the shape of D but independent of n, the $\tilde{w}^2_{j,n}$, $j = 1(1)k$, converge as n increases, as far as this can be judged by numerical computations with increasing but finite n. The convergence of the differences $\tilde{w}^2_{j-1,n} - \tilde{w}^2_{j,n}$ as $n \to \infty$, $j = 2(1)k$, is as fast as the convergence of the $\tilde{w}^2_{j,n}$, $j = 1(1)k$. For $j > k$, the $\tilde{w}^2_{j,n}$ decay with n, but limit values cannot be discerned, only that $\tilde{w}^2_{j,n} > \tilde{w}^2_{k,n}$, $j > k$. The pairwise convergence $\tilde{w}^2_{j+1,n} - \tilde{w}^2_{j,n}$ to 0 as $n \to \infty$, $j > k$ is very pronounced, however. This suggests that the k smallest computed eigenvalues $\tilde{w}^2_{j,n}$, $j = 1(1)k$, approximate eigenvalues of the continuous problem. Numerically we determine k by only regarding differences $\tilde{w}^2_{j-1,n} - \tilde{w}^2_{j,n}$ for increasing values of n. We choose k to be the smallest integer such that

$$\tilde{w}^2_{k-1,n} - \tilde{w}^2_{k,n} + \delta < 0, \quad n \to \infty$$

$$\tilde{w}^2_{j-1,n} - \tilde{w}^2_{j,n} \to 0, \quad n \to \infty, \quad \forall_{j>k},$$

where δ is any constant < 0. We note, in passing, that the 'eigenvalues' $\tilde{w}^2_j$, $j > k$ are more sensitive to changes in computational details, like selection of source point, than the $\tilde{w}^2_j$, $1 \leq j \leq k$. All computations were done on a VAX11/780 in double precision arithmetic, i.e. with 12 significant digits.

Ex. 3.1. We illustrate the numerical method for a domain with known eigenvalues. Let D be a wedge with straight sides and an interior angle of 15 degrees. Then $w^2_\ell = \sin\left(\frac{\pi}{12}(2\ell-1)\right)$, $\ell=1,2,3$, are the eigenvalues found by Ursell [9], and the computations also identified precisely 3 converging eigenvalues by the criterion described above. The error estimate for $\tilde{w}^2_1$ is

defined by (2.12)-(2.15). The table holds for the computed source point $(x_1,y_1) = (-6.351857,\ 2.30591)$.

m	n	$\lvert\tilde{w}_1^2-w_1^2\rvert$	error est. for $\tilde{w}_1^2$	$\lvert\tilde{w}_2^2-w_2^2\rvert$	$\lvert\tilde{w}_3^2-w_3^2\rvert$
22	11	$1.9\cdot10^{-8}$	$8.4\cdot10^{-7}$	$1.0\cdot10^{-4}$	$3.4\cdot10^{-2}$
42	21	$5.2\cdot10^{-10}$	$1.1\cdot10^{-8}$	$2.5\cdot10^{-7}$	$1.8\cdot10^{-3}$

The computations suggest that there are no other eigenvalues than those found by Ursell [9]. ∎

To generate various bottom shapes, a (p_1, p_2, p_3)-family of curves is used and for convenience defined parametrically in the complex plane. For $t \le 0$,

$$\begin{aligned} x(t) + iy(t) &:= \big(x_1(t) + iy_1(t)\big)e^{-ip_1}\cdot p_2 \\ x_1(t) &:= c\cdot u + \frac{5t}{16}(1 - p_3\cdot u) \\ y_1(t) &:= s\cdot v - \frac{5t}{16}(1 + w\cdot v), \end{aligned} \tag{3.1}$$

where $c := 2\cos^2\!\left(\frac{7\pi}{18}\right)$, $s := \sin\!\left(\frac{7\pi}{9}\right)$, $u := \tanh\!\left(-\frac{5t}{8c}\right)$, $v := \tanh\left(1 - \tan\!\left(\frac{5\pi}{18}\right)\cdot\frac{5t}{16s}\right)$, $w := (p_3-1)\tan\!\left(\frac{\pi}{9}\right) - 1$. This curve forms an angle p_1 with the positive x-axis at the origin, $x(0) = y(0) = 0$. The parameter p_2 is a scaling factor introduced to make t approximately equal to arc length for the t-interval under consideration, while p_3 determines the shape of the curve before it asymptotes a straight line (as $t\to-\infty$).

Ex. 3.2. We compute eigenvalues for a region with a lower boundary defined by a curve (3.1) with $p_1 = \frac{\pi}{12}$, $p_2 = 1$, and $p_3 = 4$, see figure

3.1. The inscribed (straight) wedge with angle $\frac{\pi}{12}$ is used when allocating the collocation points.

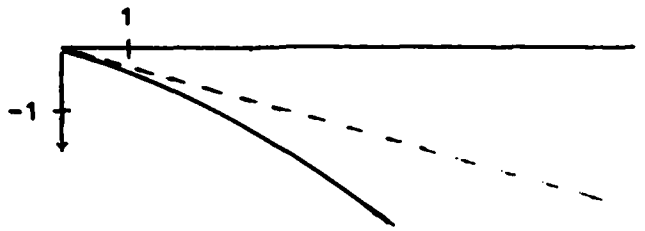

Figure 3.1

k, the number of convergent eigenvalues $\tilde{w}_j^2$ is 2, but we also show $\tilde{w}_{3,n}^2$ as being a typical spurious eigenvalue of the continuous problem. The computed source point is $(x_1,y_1) = (-0.69500, 0.18459)$.

m	n	$\tilde{w}_1^2$	error est. for $\tilde{w}_1^2$	$\tilde{w}_2^2$	$\tilde{w}_2^3$	$\tilde{w}_3^2-\tilde{w}_2^2$
32	11	0.305559	$9.4\cdot10^{-4}$	0.89902	1.67	0.77
62	21	0.305434	$2.0\cdot10^{-4}$	0.86034	1.26	0.40
92	31	0.305430	$3.7\cdot10^{-5}$	0.85668	1.14	0.28

For comparison, we note that the eigenvalues of the inscribed (straight) wedge are those of example 3.1. $\tilde{w}_1^2$ agrees with $\sin(\frac{\pi}{12})$ to 1 significant digit. ■

Ex. 3.3. We compute eigenvalues for the region shown on figure 3.2. The lower boundary is defined by a curve (3.1) with $p_1 = \frac{\pi}{9}$, $p_2 = p_3 = 2$. The (straight) wedge with angle $\frac{\pi}{9}$ indicated is used for allocation of

collocation points.

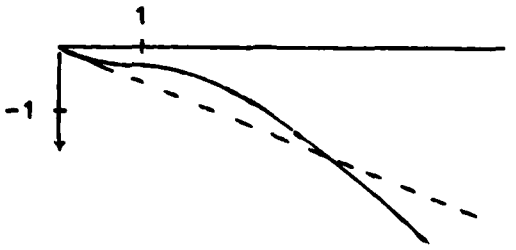

Figure 3.2

The computed source point is $(x_1, y_1) = (-1.74628, -0.10797)$. Two converging eigenvalues $\tilde{w}_1^2$, $\tilde{w}_2^2$ can be identified, but we also show the smallest spurious eigenvalue $\tilde{w}_3^2$.

m	n	$\tilde{w}_1^2$	error est. for $\tilde{w}_1^2$	$\tilde{w}_2^2$	$\tilde{w}_3^2$	$\tilde{w}_3^2 - \tilde{w}_2^2$
32	11	0.29001	$1.3 \cdot 10^{-2}$	0.8194	1.58	0.76
62	21	0.28893	$2.0 \cdot 10^{-3}$	0.7746	1.12	0.34
92	31	0.28888	$2.8 \cdot 10^{-4}$	0.7731	1.06	0.29

The straight wedge on figure 3.2 has eigenvalue $\sin(\frac{\pi}{9}) = 0.342$ and $\sin(\frac{\pi}{3}) = 0.866$, see Ursell [9]. After rounding to 1 correct decimal $\sin(\frac{\pi}{9})$ and $\tilde{w}_1^2$ agree. ∎

4. Error Estimate by Perturbation

The particular problem considered can be embedded in a slightly wider class. Suppose $\mathcal{D}$ is a domain which is a generalization of a wedge that is semi-infinite, with two smooth (C^2) boundaries, a 'top' (t) and a 'bottom' (b), which intersect only at the origin, making there a finite non-zero angle with each other. These boundaries extend to infinity.

The slightly more general eigenvalue problem is: find numbers w^2 and functions $\phi \in C^2$ in $\bar{\mathcal{D}}$ such that

$$\Delta\phi = \phi \quad \text{in} \quad \mathcal{D} \tag{4.1}$$

$$\frac{d\phi}{dn} = 0 \quad \text{on the bottom} \quad \underset{\sim}{x} = \underset{\sim}{x}_b(s), \tag{4.2}$$

$$\frac{d\phi}{dn} = w^2\phi \quad \text{on the top,} \quad \underset{\sim}{x} = \underset{\sim}{x}_t(s), \tag{4.3}$$

where $\underset{\sim}{n}(s)$ is the (continuous) unit outward normal to the boundary, and s is arc length. Also ϕ satisfies a finite energy condition (1.5), at most

$$\phi(\underset{\sim}{x}) \to O\left(|\underset{\sim}{x}|^{-(1+\delta)}\right), \ \delta>0, \quad |\underset{\sim}{x}| \to \infty \ \text{in} \ \bar{\mathcal{D}}. \tag{4.4}$$

For these problems it can be readily shown, by using standard manipulations together with Green's Theorem, that the eigenvalues w^2 are real and the eigenfunctions ϕ_j, ϕ_k belonging to different eigenvalues w_j^2, w_k^2 are orthogonal, in that both the boundary arc length integral

$$\int_t \phi_j\phi_k \, ds = 0, \quad j \neq k, \tag{4.5}$$

and the area integral

$$\iint_D [\phi_j \phi_k + \underset{\sim}{\nabla}\phi_j \cdot \underset{\sim}{\nabla}\phi_k] \, dS = 0, \quad j \neq k \tag{4.6}$$

vanish. Because the relation

$$w_j^2 = \iint_D \frac{[|\underset{\sim}{\nabla}\phi_j|^2 + \phi_j^2] \, dS}{\int_t \phi^2 ds} \tag{4.7}$$

between eigenpairs w_j^2, ϕ_j also holds, it follows that the eigenvalues are all positive, so the notation w_j^2 is justified.

An alternative formulation of the problem for the least eigenvalue now follows. Minimize the functional

$$I_D(\psi) = \iint_D [|\underset{\sim}{\nabla}\psi|^2 + \psi^2] dS \tag{4.8}$$

over all $\psi \in C^2$ in $\bar{D}$, which satisfy the normalizing relation

$$\int_t \psi^2 ds = 1, \tag{4.9}$$

which vanish in $\bar{D}$ sufficiently rapidly that the two integrals (4.8, 4.9) are defined.

The calculus of variations then reveals that a minimizing function ϕ satisfies

$$\left.\begin{aligned} \Delta\phi &= \phi && \text{in } D \\ \frac{d\phi}{dn} &= 0 && \text{on the bottom} \\ \frac{d\phi}{dn} &= \lambda\phi && \text{on the top} \end{aligned}\right\} \tag{4.10}$$

where λ is the Lagrange multiplier. By virtue of the preceeding discussion λ is also the value of $I_{\mathcal{D}}$ at the minimum, and hence the least eigenvalue.

A Perturbation Analysis

The formulation (4.8-4.10) above allows some qualitative statements to be made for problems in which the boundaries of $\mathcal{D}$ undergo perturbations. The aim is to estimate the change in the least eigenvalue for a prescribed perturbation.

Let the top boundary perturbation t' be described by an equation

$$\underset{\sim}{x}(s) = \underset{\sim}{x}_t(s) - \varepsilon_t(s)\underset{\sim}{n}(s) \tag{4.11}$$

$$\varepsilon_t(0) = 0 \tag{4.12}$$

where the boundary perturbation function ε_t is everywhere positive or zero, and together with its derivative, is small

$$\|\varepsilon_t\|, \ \|\frac{d\varepsilon_t}{ds}\| < \varepsilon \ll 1,$$

where

$$\|f\| = \sup_{0<s} |f(s)|.$$

Denote as $\mathcal{D}'$ the region interior to the perturbed top and the original bottom.

Suppose w^2 is the least eigenvalue for the problem (4.1-4.4) on $\mathcal{D}$ with corresponding eigenfunction ϕ, and let Ω^2 be the least eigenvalue for the problem on $\mathcal{D}'$. Since $\mathcal{D}' \subseteq \mathcal{D}$, one would anticipate that $\Omega^2 \leq w^2$; the proof that this is indeed the case follows.

Use as a trial function to estimate Ω^2 the original eigenfunction ϕ, which must first, however, be renormalized. Thus one has the formula

$$N = \int_{t'} \phi^2 ds_{t'} = \int_t \left(\phi - \varepsilon_t \frac{d\phi}{dn}\right)^2 ds_t \left(1 + O(\varepsilon^2)\right)$$

$$= \int_t \left(\phi^2 - 2\varepsilon_t w^2 \phi^2\right) ds_t \left(1 + O(\varepsilon^2)\right) \qquad (4.13)$$

$$= \left(1 - 2w^2 \int_t \varepsilon_t \phi^2 ds\right)\left(1 + O(\varepsilon^2)\right)$$

after using the first term in a local Taylor expansion of ϕ on t to estimate its value on t', the boundary condition (4.4), and after observing that the arc length elements on t' and t satisfy

$$ds_{t'} = ds_t\left(1 + O(\varepsilon^2)\right)$$

by the assumed properties of ε_t.

So the variational result (4.8) yields

$$\Omega^2 \leq \frac{I_{\mathcal{D}'}(\phi)}{N}$$

$$= \frac{\left(I_{\mathcal{D}}(\phi) - I_{\mathcal{D}-\mathcal{D}'}(\phi)\right)}{N} \qquad (4.14)$$

$$= \frac{\left(w^2 - I_{\mathcal{D}-\mathcal{D}'}(\phi)\right)}{N}$$

The quantity $I_{\mathcal{D}-\mathcal{D}'}(\phi)$ can be estimated in the limit $\varepsilon \to 0$

$$I_{\mathcal{D}-\mathcal{D}'}(\phi) = \int_t \left(|\nabla\phi|^2 + \phi^2\right) \varepsilon_t \, ds + O(\varepsilon^2)$$

$$= \int_t \left(\left(\frac{d\phi}{ds}\right)^2 + (1+w^4)\phi^2\right) \varepsilon_t \, ds + O(\varepsilon^2) \qquad (4.15)$$

so that on combining the three formulae (4.13, 4.14, 4.15) it follows that

$$\Omega^2 \leq w^2\left(1-w^{-2}\int_t\left(\frac{d\phi}{ds}\right)^2 \varepsilon_t ds-(w^2+w^{-2})\int_t\phi^2\varepsilon_t ds + O(\varepsilon^2)\right)\left(1-2w^2\int_t\phi^2\varepsilon_t ds\right)^{-1}\left(1+O(\varepsilon^2)\right)$$

$$= w^2\left(1-w^{-2}\int_t\left(\frac{d\phi}{ds}\right)^2 \varepsilon_t ds - (w^{-2}-w^2)\int_t\phi^2\varepsilon_t ds\right)(1+O(\varepsilon^2)). \tag{4.16}$$

Provided $0 < w^2 < 1$ (as is the case in the computed examples) the factor $(w^{-2} - w^2)$ is positive, and because $\varepsilon_t > 0$ by hypothesis, it follows that

$$\Omega^2 \leq w^2. \tag{4.17}$$

For a bottom perturbation which yields a perturbed region $\mathcal{D}' \subseteq \mathcal{D}$ let

$$\underset{\sim}{x}(s) = \underset{\sim}{x}_b(s) - \varepsilon_b(s)\underset{\sim}{n} \tag{4.18}$$

define the perturbed bottom b', where ε_b has properties similar to those of ε_t, that is

$$\|\varepsilon_b\|, \ \left\|\frac{d\varepsilon_b}{ds}\right\| < \varepsilon \qquad \varepsilon_b > 0, \quad \varepsilon_b(0) = 0 \tag{4.19}$$

As a trial function ϕ to estimate the perturbed least eigenvalue Ω^2 of $\mathcal{D}'$ the corresponding eigenfunction ϕ of $\mathcal{D}$ is used. Then the variational formulation requires that

$$\begin{aligned}\Omega^2 \leq I_{\mathcal{D}'}, \ \phi &= I_{\mathcal{D}'}(\phi) \\ &= \left(I_{\mathcal{D}}(\phi) - I_{\mathcal{D}-\mathcal{D}'}(\phi)\right) \\ &= \left(w^2 - I_{\mathcal{D}-\mathcal{D}'}(\phi)\right)\end{aligned} \tag{4.20}$$

Hence the result

$$\Omega^2 \leq w^2 \tag{4.21}$$

again follows.

Thus, in summary, when $\mathcal{D}'$ is obtained from $\mathcal{D}$ by either (or both) boundary perturbation(s) of the type(s) described, (and hence $\mathcal{D}' \subseteq \mathcal{D}$), the least eigenvalues Ω^2, w^2 of $\mathcal{D}'$ and $\mathcal{D}$, respectively, satisfy

$$\Omega^2 \leq w^2. \tag{4.22}$$

The converse is also true, since the boundary perturbation process described is reflexive. That is, if $\mathcal{D}' \supseteq \mathcal{D}$, then

$$\Omega^2 \geq w^2. \tag{4.23}$$

While the results (4.22, 4.23) can be used for a priori estimation using known solutions as comparison, the relationship between the least eigenvalues of $\mathcal{D}$, and $\mathcal{D}'$ (obtained by perturbation) can be refined. First suppose $\mathcal{D}' \supseteq \mathcal{D}$ and the eigenfunction ϕ corresponding to the least eigenvalue w^2 of $\mathcal{D}$ is known. Extend the eigenfunction ϕ to $\mathcal{D}'$ by constructing a preliminary extension Φ_1 which coincides with ϕ in $\mathcal{D}$ and has Φ_1 constant along any given normal $\underset{\sim}{n}'$ to t', b', taking the value on that normal equal to the value of ϕ where the normal intersects t. (Multi-valuedness is excluded by the smoothness assumptions on ε_t, ε_b). Then Φ_1 can be modified to produce a suitable trial function $\Phi \in C^2$ in $\mathcal{D}'$ by introducing large curvature $\left(\frac{d^2\Phi}{dn^2}\right)$ in suitably small transverse neighborhoods of t, b. Such a process can be carried out so that, for any desired m

$$I_{\mathcal{D}}(\Phi) = I_{\mathcal{D}}(\phi)\left(1 + O(\varepsilon^m)\right), \tag{4.25a}$$

and

$$\frac{d\Phi}{ds} = \frac{d\phi}{ds}\left(1 + O(\varepsilon^m)\right), \quad \Phi = \phi\left(1 + O(\varepsilon^m)\right), \quad \text{on t, and b.} \tag{4.25b}$$

Having constructed this Φ, one notes that its normalization is essentially unchanged since

$$\int_{t'} \phi^2 ds'_{t'} = \int_t \phi^2 ds_t \left(1 + O(\varepsilon^2)\right) \tag{4.26}$$

by the properties of the perturbation.

Since $\mathcal{D}' \supseteq \mathcal{D}$ the inequality (4.23) requires that

$$w^2 \leq \Omega^2$$

and the minimum principle (4.8) then demands that

$$w^2 \leq \Omega^2 \leq I_{\mathcal{D}'}(\Phi)$$

or

$$0 \leq \Omega^2 - w^2 \leq I_{\mathcal{D}}(\Phi) + I_{\mathcal{D}'-\mathcal{D}}(\Phi) - w^2. \tag{4.27}$$

But equation (4.25) and the definition of Φ show that

$$I_{\mathcal{D}}(\Phi) - w^2 = O(\varepsilon^m) \tag{4.28}$$

so the basic inequality (4.27) becomes essentially

$$0 \leq \Omega^2 - w^2 \leq I_{\mathcal{D}'-\mathcal{D}}(\Phi). \tag{4.29}$$

In the limit as $\varepsilon \to 0$, the right hand side of the last inequality can be computed by

$$I_{\mathcal{D}'-\mathcal{D}}(\Phi) = \int_t \left(|\nabla\Phi|^2 + \Phi^2\right)|\varepsilon_t| ds_t + \int_b \left(|\nabla\Phi|^2 + \Phi^2\right)|\varepsilon_b| ds_b + O(\varepsilon^2) \tag{4.30}$$

where $\nabla\Phi, \Phi$ are evaluated in the interior of $(\mathcal{D}' - \mathcal{D})$. Here $\frac{d\Phi}{dn}$ vanishes by construction, or $\varepsilon_* = 0$,

$$\begin{aligned} I_{\mathcal{D}'-\mathcal{D}}(\Phi) &= \left\{\int_t + \int_b \left(\left(\frac{d\Phi}{ds}\right)^2 + \Phi^2\right)|\varepsilon_*| ds_*\right\} \left(1 + O(\varepsilon^2)\right) \\ &= \left\{\int_t + \int_b \left(\left(\frac{d\Phi}{ds}\right)^2 + \Phi^2\right)|\varepsilon_*| ds_*\right\} \left(1 + O(\varepsilon^2)\right) \end{aligned} \tag{4.31}$$

where the suffix * denotes t or b as appropriate.

Thus, in the situation described the results (4.27), (4.31) can be summarized as

$$0 \leq \Omega^2 - w^2 \leq I_{\mathcal{D}'-\mathcal{D}}(\phi) \tag{4.32}$$

and $I_{\mathcal{D}'-\mathcal{D}}(\phi)$ can be approximated by the { } factor in equation (4.31).

One can proceed analagously to estimate the relation between the least eigenvalues of $\mathcal{D}$, $\mathcal{D}'$ when $\mathcal{D}' \subseteq \mathcal{D}$ and the eigenpair w^2, ϕ belonging to $\mathcal{D}$ are known. It will follow that a result similar to the above (4.32) holds, namely

$$0 \leq w^2 - \Omega^2 \leq I_{\mathcal{D}-\mathcal{D}'}(\phi) \tag{4.33}$$

after a calculation which is similar to the one used in developing the inequality (4.32), but different in detail.

When $\mathcal{D}'$ is neither wholly interior to $\mathcal{D}$, nor contains it, a two step procedure is carried out to relate the least eigenvalue w^2 of $\mathcal{D}$ to that of $\mathcal{D}'$. Denote as $\mathcal{D}^*$ the region $\mathcal{D} \cup (\mathcal{D}'-\mathcal{D})$, and use as a trial function in both $\mathcal{D}^*$ and $\mathcal{D}'$ the eigenfunction ϕ of $\mathcal{D}$ extended by the rules previously described: that is, where t', b' fall outside $\mathcal{D}$, extend ϕ as a constant along normal lines to these boundaries, taking a value on any such line equal to the value of ϕ where that line strikes t, or b. At boundaries t, b, ϕ is adjusted to satisfy smoothness conditions or boundary conditions as necessary, but in regions whose transverse extent is arbitrarily small in comparison with ε. Noting that $\mathcal{D}^* \supseteq \mathcal{D}$, $\mathcal{D}^* \supseteq \mathcal{D}'$, one deduces the pair of inequalities

$$\begin{aligned} 0 &\leq \Lambda^2 - \Omega^2 \leq I_{\mathcal{D}^*-\mathcal{D}'}(\phi) \\ 0 &\leq \Lambda^2 - w^2 = I_{\mathcal{D}^*-\mathcal{D}}(\phi) \end{aligned} \tag{4.34}$$

from the properties of the trial function, where Λ^2 is the least eigenvalue of $\mathcal{D}^*$. From these inequalities (4.34) it follows that a conservative

estimate is

$$0 \leq |w^2 - \Omega^2| \leq I_{\mathcal{D}^*-\mathcal{D}}(\phi) + I_{\mathcal{D}^*-\mathcal{D}'}(\phi) \tag{4.35}$$

and the last two integrals can be approximated as

$$I_{\mathcal{D}^*-\mathcal{D}}(\phi) + I_{\mathcal{D}^*-\mathcal{D}'}(\phi) \approx \int_t [\left(\frac{d\phi}{ds}\right)^2 + \phi^2 |\epsilon_t|] ds + \int_b [\left(\frac{d\phi}{ds}\right)^2 + \phi^2 |\epsilon_b|] ds \tag{4.36}$$

The inequality (4.35) and the evaluation (4.36) form the basis of the computable error estimate.

An Error Estimate

In estimating the error in the computation of the least eigenvalue, the premise is that the small residuals in

$$\frac{d\phi}{dn} = d_b(s) \tag{4.37}$$

and

$$\frac{d\phi}{dn} - \lambda^2\phi = d_t(s) \tag{4.38}$$

computed on b, t respectively, the boundaries of $\mathcal{D}$ suggest λ^2, ϕ are an eigenpair of the least eigenvalue of a region $\mathcal{D}'$, which is a perturbation of $\mathcal{D}$. (By construction, ϕ is a solution of equation (4.1) in a region including $\mathcal{D}$). Knowledge of the ϕ field, and the location of b, t enable the relative location of b', t' to the established, and hence (using the results of the perturbation analysis) a bound on $|\mu^2 - \lambda^2|$ where μ^2 is the true least eigenvalue of $\mathcal{D}$.

In the case of the bottom boundary, for example, for the given ϕ field it is necessary to determine the nearby curve b'

$$\underset{\sim}{x} = \underset{\sim}{x}_b(s) + \epsilon_b(s)\underset{\sim}{n}(s) \tag{4.39}$$

on which

$$\nabla\underset{\sim}{\phi} \cdot \underset{\sim}{n}' = 0 \tag{4.40}$$

where $\underset{\sim}{n}' \simeq \underset{\sim}{n}$ is the local unit outward normal to b'. When the curvature of b is not large, s and n coordinates may be regarded as Cartesian, the boundary condition becomes approximately

$$\frac{d\phi}{ds}\frac{d\varepsilon_b}{ds} + \frac{d\phi}{dn} \tilde{=} 0 \tag{4.41}$$

on b', or, approximately (equation (4.37))

$$\frac{d\varepsilon_b}{ds} \tilde{=} -d_b \Big/ \left(\frac{d\phi}{ds}\right)\Big|_{\underset{\sim}{x}(s)} . \tag{4.42}$$

Thus the ultimate expression is

$$\varepsilon_b(s) = -\int_0^s d_b \Big/ \frac{d\phi}{ds}\Big|_{\underset{\sim}{x}(s)} ds, \tag{4.43}$$

allowing for the boundary condition $\varepsilon_b(0) = 0$. Similarly, on the top boundary one requires

$$\begin{aligned} 0 &= \nabla\phi \cdot \underset{\sim}{n}' - \mu^2\phi \\ &= \frac{-d\varepsilon_t}{ds}\frac{d\phi}{ds}\left(1 + O(\varepsilon)\right) + \left(\frac{d\phi}{dn} - \lambda^2\phi\right)\left(1+O(\varepsilon)\right). \end{aligned} \tag{4.44}$$

(assuming the calculations are good enough that μ^2 is close to λ^2), so that taking the local approximate values

$$\begin{aligned} \frac{d\phi}{ds}\Big|_{\underset{\sim}{x}+\varepsilon_t\underset{\sim}{n}} &\simeq \frac{d\phi}{ds}\Big|_{\underset{\sim}{x}'} \\ \frac{d\phi}{dn'} - \lambda^2\phi\Big|_{\underset{\sim}{x}+\varepsilon_t\underset{\sim}{n}} &\simeq \frac{d\phi}{dn} - \lambda^2\phi\Big|_{\underset{\sim}{x}} = d_b \end{aligned} \tag{4.45}$$

it follows that

$$\varepsilon_t \simeq \int_0^s d_b \Big/ \frac{d\phi}{ds}\Big|_{\underset{\sim}{x}(s)} ds \tag{4.46}$$

The results (4.43, 4.46) can then be used in the formulae (4.35, 4.36) to estimate the error in the computation. (The formulae (4.43) and (4.46) are constructed in the knowledge that $\left.\frac{d\phi}{ds}\right|_{t,b}$ vanishes only in the limit $|\underset{\sim}{x}| \to \infty$, when ϕ is the eigenfunction corresponding to the least eigenvalue.)

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The determination of periodic edge waves along a straight infinite beach can be formulated as a 2-dimensional eigenvalue problem for a generalized wedge. A least squares boundary collocation method for computing the lower discrete eigenvalues is described, with an error estimate for the least eigenvalue. Numerical examples are presented.

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