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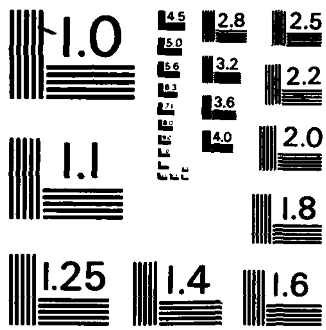
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REPORT SD-TR-83-85

Reliability Analysis of Multi-Components Systems

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Electronics Research Laboratory
Laboratory Operations
The Aerospace Corporation
El Segundo, Calif. 90245

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This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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The reliability of systems consisting of many semiconductor devices is estimated from the results of an accelerated temperature aging test program. This work is an extension of our previous analysis of accelerated aging, in which we assumed that the results from a Student-t analysis can be applied directly to a multi-component amplifier in the classical combinatory manner. In this work we no longer make this assumption but employ a computer simulation to obtain a more efficient estimation of the reliability of an amplifier consisting of many semiconductor devices.

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PREFACE

The author wishes to thank T. H. Davey who first questioned the correctness of the Student-t approximation and K. H. Hurlbut for providing his keen insight into certain aspects of this study. The computer simulations, which are the foundation of this analysis, were performed by A. J. Allard in a cheerful and competent manner.

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I. INTRODUCTION

When a new semiconductor device is introduced, no field failure information exists. Under these conditions, accelerated temperature aging tests are undertaken to obtain initial estimates of the device failure rate under normal operating conditions. The success of such an accelerated aging test depends on the number of devices failed, the method of conducting the accelerated test, and the processing of the failure information. In this study, we are interested in the statistical analysis of an accelerated temperature stress test to estimate the reliability of a multi-element system, such as a power GaAs FET amplifier. In our previous statistical analysis of accelerated aging, we assumed that the cumulative distribution function (cdf), estimated by the Student-t distribution, could be applied directly to estimating the reliability of a multi-component system. In particular, we assumed that the estimated cdf could be used in the classical combinatory analysis.

This procedure is incorrect in that the results of the Student-t analysis are for a single unaged device. The application of the classical combinatory analysis required that the cdf be known exactly, rather than estimated as with the Student-t method.

We have employed the same approach used in developing the Student-t analysis, but extended it to estimating the reliability of a complex multi-component system. This extended Student-t analysis has been accomplished by means of a computer simulation. We have compared the results from the extended Student-t with our previous method, the Student-t approximation. Small differences are observed between these two procedures. The significance of these differences is, of course, a fraction of the intended applications. As a general rule, the Student-t approximation yields a more conservative estimate, because its estimated failure probability is higher than that obtained from the more correct extended Student-t analysis. An exception to this general rule is redundant systems at low failure probabilities.

The extended Student-t method is suggested for use in an indepth reliability analysis of a multi-element system. The Student-t approximation is useful in checking the results of the extended analysis and also for trade-off studies, such as might be performed when an accelerated aging test program is designed.

In the next section, the analysis of accelerated aging test results is reviewed. This material is a condensation of our previous analysis in which we employed the Student-t approximation when dealing with multi-component amplifiers. (See Ref. 1 for a more detailed discussion of accelerated aging and a listing of the appropriate references.)

In Section III, we consider the relatively simple case in which the devices are aged and operated at the same temperature. Although this is not a reasonable procedure in actual practice, it is very useful in introducing our extended Student-t analysis. Section IV describes the application of the extended Student-t analysis to accelerated aging.

Throughout this report we assume that devices exist in only two states: fully operational or failed. Experience indicates that many semiconductor devices do not fail in this catastrophic manner but gradually degrade. The analysis of gradual degradation is treated in a companion report.[2]

II. BACKGROUND

The reliability of a given device is conveniently described by its failure probability density function (pdf), which, by convention, is designated $f(x)$, where x is a random variable. The differential probability that a randomly chosen device will fail between x and $x + \Delta x$ is $f(x)\Delta x$. The cumulative distribution function, $F(t)$, is the probability that a given device will have failed before t and is

$$F(t) = \int_{0}^{t} f(x)dx \tag{1}$$

The survival probability is one minus the cdf when it is assumed, as in the present analysis, that a device can only be in one of two states: fully operational or failed.

In this discussion we need only consider three similar pdf's: normal, standard normal, and log normal distributions. The normal failure pdf with time as the random variable is

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[- \frac{1}{2} \left(\frac{t - \mu}{\sigma}\right)^2 \right] \tag{2}$$

where μ is the mean lifetime and σ the standard deviation — referred to as the scale and shape parameters. The median lifetime is equal to the mean lifetime. The cdf

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[- \frac{1}{2}\left(\frac{t - \mu}{\sigma}\right)^2 dt\right] \tag{3}$$

cannot be expressed in a closed form and must be evaluated by using a tabulation of the standard normal distribution where $\mu = 0$ and $\sigma = 1$. The cdf can be evaluated from the standard normal by

$$F(a) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{(a-\mu)/\sigma} \exp\left(-\frac{1}{2} y^2\right) dy \qquad (4)$$

Probability graph paper (e.g., K & E/46 8003) is useful when working with normal distributions. When the cdf is plotted on this graph paper, a straight line results. Both the median and the mean lifetime occur at cdf = 0.5 (i.e., 50%) and

$$\sigma = \mu - t_{16} \qquad (5)$$

where t_{16} is the time corresponding to $F(t) = 0.16$. The normal pdf is symmetric about μ, and approximately 68% of the failures occur within σ of the median lifetime

Under carefully controlled conditions of elevated temperature and applied bias, the failure pdf of semiconductor devices follows a log normal pdf, expressed as

$$f(t) = \frac{1}{t\sigma(2\pi)^{1/2}} \exp\left\{-\frac{1}{2}\left[\frac{\ln(t/\tau_M)}{\sigma}\right]^2\right\} \qquad (6)$$

In this expression, τ_M is the median lifetime, and σ is the logarithmic standard deviation. The log normal distribution is similar to the normal pdf, except that $t \rightarrow \ln t$. In fact, if we define

$$x = \ln t \qquad (7)$$

we have

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right] \qquad (8)$$

a normal distribution employing not a linear time scale but a logarithmic time scale. By working in logarithmic time, all the properties of the normal distribution apply to the log normal pdf. The time interval $[t, t + \Delta t]$ becomes $[x, x + \Delta x] = [\ln t, \ln t + \Delta t/t]$. The cdf is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{9}$$

which, plotted on probability graph paper using the logarithm of time, results in a straight line. This straight line crosses the 50% probability at $x = \mu$, which is the median of the log lifetime distribution. The median lifetime is

$$\tau_M = \exp \mu \tag{10}$$

In a manner similar to a normal distribution, we have approximately

$$\sigma = \mu - x_{16} = \ln(\tau_M/t \tag{11}$$

where x_{16} and t_{16} correspond to the value x and where 16% of the cumulative failures occur. Approximately 68% of the failures occur between x_{16} and x_{84} (i.e., 2σ, centered around the median lifetime).

Expressions for the real-time (i.e., linear) mean lifetime and variance are available but serve no useful purpose in the present analysis. In fact, we emphasize that knowledge of the failure pdf near the mean lifetime is of little value in analyzing the reliability of complex systems intended for long-life space applications or other similarly complex ground-based systems. Of prime importance is the failure pdf at times much less than the average lifetime. In general, with a system consisting of m components, one desires to know the cdf near the 100/m percentage level. For example, a system consisting of 10^4 identical components in a series would be expected to have a high probability of failure at a time when the average device cdf is 10^{-4}.

If we look at reliability from the viewpoint of cost effectiveness, it is not the failure pdf or cdf that one is interested in, but rather the integral of the cdf over the service life. Consider a system with a specified useful life of t_s. Let the loss in some monetary units for the systems not working in the time interval $[t, t + \Delta t]$ be $R(t)\Delta t$. The expected system loss from failure of the i component is

$$\Delta L_s = \int_o^{t_s} R(t)F(t)dt \tag{12}$$

where F(t) is the cdf for the component. The loss rate, of course, depends on the system-intended employment. If the rate of loss R(t) can be represented as a constant, $\overline{R}$, we have

$$\Delta L_s = \overline{R} \int_o^{t_s} F(t)dt \tag{13}$$

When we have a multi-component system and wish to calculate the failure of the various devices in the system, we employ the results from order statistics. In the simplest case, consider an amplifier consisting of a string of m devices. If any one of these m devices fail, the system fails. We wish to determine the cumulative failure function of the system based on the cumulative failure of the individual devices. In this simple case we proceed as follows. The probability of any device not failing at time t is

$$P(t < \tau_i) = 1 - F_i \tag{14}$$

where F_i is the cumulative failure of the i device. For no failures in a string of m devices, we have

$$P(t < \tau_1, t < \tau_2 \ldots) = \prod_{i=1}^{m} (1 - F_i) \tag{15}$$

or for a string of identical devices

$$P(\text{success}) = (1 - F)^m \tag{16}$$

The cumulative failure probability is therefore

$$F_m = 1 - P(\text{success}) \tag{17}$$

or

$$F_m = 1 - (1 - F)^m$$

For a redundant system we have a collection of m devices, but the system is in the failed state only if all m devices are failed. The probability of an individual device being failed is, of course, F_i and for all devices to be failed, we have

$$P(\tau_1 < t, \tau_2 \text{ etc}) = \prod_{i=1}^{m} P\{\tau_i < t\} \tag{18}$$

$$= \prod_{i=1}^{m} F_i \tag{19}$$

For m devices with identical cumulative failure functions, the cumulative failure function of the redundant collection is

$$F_m = F^m \tag{20}$$

As a particular example, consider a system consisting of two redundant channels in which each channel consists of a string of 10 devices. The system cumulative failure fraction is

$$F_T = \left[1 - \prod_{i=1}^{10} (1 - F_i)\right] \left[1 - \prod_{i=11}^{20} (1 - F_i)\right] \tag{21}$$

or for identical devices

$$F_T = \left[1 - (1 - F)^{10}\right]^2 \tag{22}$$

The cumulative failure function for i failures in a string of m identical devices is

$$F_{Ti} = \sum_{j=1}^{m} \binom{m}{j} F^j (1 - F)^{m-j} \tag{23}$$

which reduces to

$$F_{T1} = 1 - (1 - F)^m \tag{24}$$

and

$$F_{Tm} = F^m \tag{25}$$

for the first and the m failures. The above formulas are useful in calculating the cumulative failure of a parallel combination of devices in which we are allowed to lose a number of devices before the system failure criterion is reached.

Thus far, we have been proceeding as though the failure pdf were known exactly, but in practice, this is not so. In many cases, we do not even know the functional form of the pdf. Only by sampling from the distribution can we estimate the correctness of an assumed functional form, and only when this is established can we estimate the parameters associated with the failure pdf. Knowing the functional form of the failure pdf, the expected cdf is estimated from the sample distribution, not the exact cdf. In the present analysis, it is important to distinguish between the parameters that characterize an exact pdf and the corresponding parameters estimated from sampling. The cumulative failure distribution for a multi-component system is only correct when we know the cumulative failure function for the individual devices exactly. The main purpose of this study is to estimate the system cdf from an estimate of the cdf of individual devices used in the system.

A simple example illustrates the difference between knowing the exact cdf and having only an estimate of the cdf when doing a combinatory analysis. Consider drawing three random numbers from a normal distribution for which we have no prior knowledge of the scale and shape parameters. What is the probability that both the second and third random numbers are greater than the first random number? If we know the exact median value of the normal distribution, the probability of the second or third numbers being greater than the median value is 50%, and the probability of both numbers being greater than the median value is 25%. The probability of either the second or third number being greater than the first (i.e., our estimate of the median value) is still 50%, but the probability of both the second and third being greater than the first number selected is not 25%, but 33-1/3%.

In a single-temperature test, n devices are operated until they all fail. On the basis of the n different failure times experimentally observed, we can estimate the population distribution from this sample distribution. Establishing the validity of an assumed functional form for the failure pdf is outside the scope of this analysis. We assume a log normal pdf and proceed directly to estimating the log normal parameters from the experimentally observed failure times, τ_1, τ_2, $\tau_3 \cdots$

The experimental observed failure times are converted to the corresponding logarithmic x_1, x_2, $x_3 \cdots$, where each of these samples is from a normal distribution whose median value and variance we wish to estimate. Our effective estimation for the scale and the shape parameters are

$$\bar{x} = \sum_{i=1}^{n} x_i \qquad (26)$$

and

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 \qquad (27)$$

where we sum over the n failure times.

Using the above estimates of the scale and shape parameters of a normal pdf gives an inefficient estimate of the cdf. A better estimate of the cdf is obtained from the Student-t analysis, which gives the probability of an unaged device not failing before operating for a time, t,

$$P\left\{\ln t < \bar{x} - t_{n-1}[F(t)]\ s\ \left(1 + \frac{1}{n^2}\right)^{1/2}\right\} = \bar{F}(t) \qquad (28)$$

where t_{n-1} $(\overline{F})$ is the Student-t distribution with n - 1 degrees of freedom corresponding to the confidence level $\overline{F}$, which in our case is the estimated cdf.

Because semiconductor components have excellent reliability, it is very costly to adequately determine the failure pdf under actual operating conditions. This excessive cost is associated with the large sample of devices that must be operated for long test times to predict the failure pdf with a reasonable confidence level. Even if the cost of such a reliability test is not a constraint, the required total test time usually is a constraint. To predict the reliability of an electronic system with a desired system life of t_s, one must determine the failure rate at times comparable to t_s, which, for space applications, can be 10 years or longer. If the failure pdf is exponential, the total test time can be decreased by increasing the sample size since, in this special case, it is the total operating time of the devices that establishes the accuracy of predicting the failure pdf. When the functional form of the failure pdf is unknown, it is dangerous to assume that one can trade sample size against test duration.

Because of this consideration, one often resorts to an accelerated temperature-stress program to estimate the reliability of semiconductor components. The success of such a program is based on the remarkable experimental observations that for a single failure mechanism:

1. The failure pdf at a constant temperature and applied electrical stress is log normal.

2. The logarithmic variance is independent of temperature.

3. The median failure time follows an Arrhenius dependence expressed as

$$\tau_M = \tau_o \exp\left(\frac{\Delta E}{kT}\right) \tag{29}$$

where τ_o and ΔE, the activation energy, depend on the electrical stress but not on temperature.

For an accelerated aging analysis, it is convenient to rewrite the failure pdf as

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x - \ln\tau_o - \Delta EZ}{\sigma}\right)^2\right] \tag{30}$$

where the Arrhenius relation has been reformulated as

$$\mu = \ln\tau_o + \Delta EZ \tag{31}$$

The experimental aging results are represented by n (the number of devices aged to failure) couples (x_i, Z_i) where x_i is the experimental log lifetime observed when the i device is operated at Z_i. In this format, the problem is reduced to simple linear regression and efficient estimators of pdf parameters are

$$\overline{\Delta E} = \frac{\Sigma(x_i - \overline{x})(Z_i - \overline{Z})}{\Sigma(Z_i - \overline{Z})^2} \tag{32}$$

$$\overline{\ln\tau_o} = \overline{x} - \overline{\Delta E}\ \overline{Z} \tag{33}$$

$$s^2 = \frac{1}{n - 2} \sum(x_i - \overline{\ln\tau_o} - \overline{\Delta E}\ \overline{Z_i})^2 \tag{34}$$

where

$$\overline{x} = \frac{1}{n} \sum x_i \tag{35}$$

and

$$\overline{Z} = \frac{1}{n} \sum Z_i \tag{36}$$

The estimated median log lifetime at the operating temperature Z_N is

$$\overline{x_N} = \overline{\ln\tau_o} + \overline{\Delta E}\, Z_N \tag{37}$$

The estimated cdf for an unaged device from the same lot is provided by the Student-t analysis. The probability of an unaged device when operated at Z_N not having failed before an operating t is

$$P\{\ln t < \overline{\ln\tau_o} + \overline{\Delta E}\, Z_N - t_{n-2}[\overline{F}(t)]\, s\, \frac{\sigma_u}{\sigma}\} = \overline{F}(t) \tag{38}$$

where $t_{n-2}[\overline{F}(t)]$ is the Student-t distribution with n - 2 degrees of freedom at a confidence level of $\overline{F}(t)$, our desired estimated cdf. The function

$$(\sigma_u/\sigma)^2 = [1 + \frac{Z_N^2 - 2Z_N\,\overline{Z} + \frac{1}{n}\sum Z_i^2}{\sum(Z_i - \overline{Z})^2}] \tag{39}$$

accounts for the uncertainties in the median lifetime at the operating temperature whereas $t_{n-2}[\overline{F}(t)]$ accounts for the error in the standard deviation.

In our previous analyses, we used the estimated cdf, obtained from application of Eq. (38), in a probability combinational analysis [e.g., Eq. (14) to Eq. (25)] of a multi-component system. In the following sections, an attempt is made to remove this deficiency.

III. PRELIMINARY ANALYSIS OF RELIABILITY OF MULTI-COMPONENT SYSTEM

In this section, we estimate the reliability of a multi-component system operating at a constant temperature from the failure times of n devices that have been aged at the same temperature.

Suppose that n devices have been operated to failure with the results $\overline{\tau_M} = 10^6$ hours and s = 1.0.

Assuming that the lot from which these devices were selected has a uniform log normal distribution, we will estimate the reliability of an amplifier assembled with m additional devices from the same lot. The cdf to be determined is the probability of at least one of the m devices failing after operating for t_o

$$\overline{F}(t_o) = P(\tau' < t_o) \tag{40}$$

where τ' is the lowest lifetime of the m devices used in assembling the amplifier. It is convenient to transform this probability statement as

$$\overline{F}(t_o) = P(x' < \overline{x} + ks) \tag{41}$$

where

$$k = \frac{\ln t_o - \overline{x}}{s} \tag{42}$$

The problem is to estimate the probability of failure at k sample standard deviations below the sample median log lifetime.

Before we undertake the solution to our particular problem, we will consider the less complex case of estimating the cdf of a single device without employing directly the Student-t analysis. In estimating this single device reliability, we consider the relation

$$k = \frac{x_{n+1} - \bar{x}}{s} \tag{43}$$

For any normal pdf, this statistic is independent of the scale and shape parameters of the distribution. One proves this by transforming to a standard normal (e.g., $\mu = 0$ and $\sigma = 1$) employing

$$x_i = \sigma R_i + \mu \tag{44}$$

where R_i is the corresponding random number from a standard normal distribution. Substituting Eq. (44) into Eq. (43) yields

$$k = \frac{\sigma R_{n+1} + \mu - \frac{1}{n} \sum_{i=1}^{n} (\sigma R + \mu)}{\left\{\frac{1}{n-1} \sum_{i=1}^{n} \left[\sigma R_i + \mu - \frac{1}{n} \sum_{i=1}^{n} (\sigma R_i + \mu)\right]^2\right\}^{1/2}} \tag{45}$$

or

$$k = \frac{R_{n+1} - \bar{R}}{\left[\frac{1}{n-1} \sum_{i=1}^{n} (R_i - \bar{R})^2\right]^{1/2}} \tag{46}$$

establishing that the estimated cdf, as a function of k, is independent of the exact scale and shape parameters of the failure pdf.

The function dependence of $\bar{F}(k)$ can easily be obtained by means of a computer simulation. Starting with a standard normal distribution, we selected $n + 1$ random numbers and calculated

$$k = \frac{R_{n+1} - \bar{R}}{s'} \tag{47}$$

where s' is the standard deviation at the first n random numbers. This procedure is repeated for approximately 1000 runs, using $n + 1$ independent random numbers for each of the runs. The various k values are then ranked from lowest to highest to give the cdf as a function of k. Having obtained

$\overline{F}(k)$ for a standard normal, we then use the experimental estimate scale and shape parameters, e.g., $\overline{x}$ and s in the transformation

$$\ln t_o = s\,k + \overline{x} \tag{48}$$

to obtain the cdf as a function of operating time.

Before we present the simulation results for the cdf as a function of k, we consider some initial results to establish a degree of understanding and credibility in our computer simulation. A sample of 10 random numbers is selected from a standard normal 1000 times. In Figure 1 we reproduce the 1000 R_1 random numbers ranked to correspond to a standard normal distribution. Between 1 and 99% reasonable agreement is observed between the ranked R_1 and the theoretical standard normal. We also present, in Figure 1, the ranked $\overline{R}$ (the average from a sample of 10) from the 1000 runs. Throughout the complete range plotted, the results follow the expected normal distribution with $\overline{x} \approx 0$ and $s \approx 1/\sqrt{10}$, as expected. In Figure 2, the order array of the 1000 standard deviations is compared with the expected chi-squared distribution with (10-1) degrees of freedom. In Figure 3, we reproduce the cumulative distribution of the lowest random number in a sample of 10. Close agreement with the expected results is displayed.

In Figure 4, we reproduce the ranked array of 10,000

$$k = \frac{R_{11} - \overline{R}}{s'} \tag{49}$$

computer simulation results. In this presentation, we plotted the first 5000 lowest values of k normally, but reversed the sign for the higher 5000 results to display the symmetry about cdf = 0.5 and k = 0. For reference, we have also plotted the standard normal and the Student-t analysis (for $\overline{x}$ = 0, s = 1, and n = 10). Between 1 and 99% our cdf for k and the Student-t analysis are in excellent agreement, which is to be expected because they both yield the best estimate of the cdf of the n + 1 device.

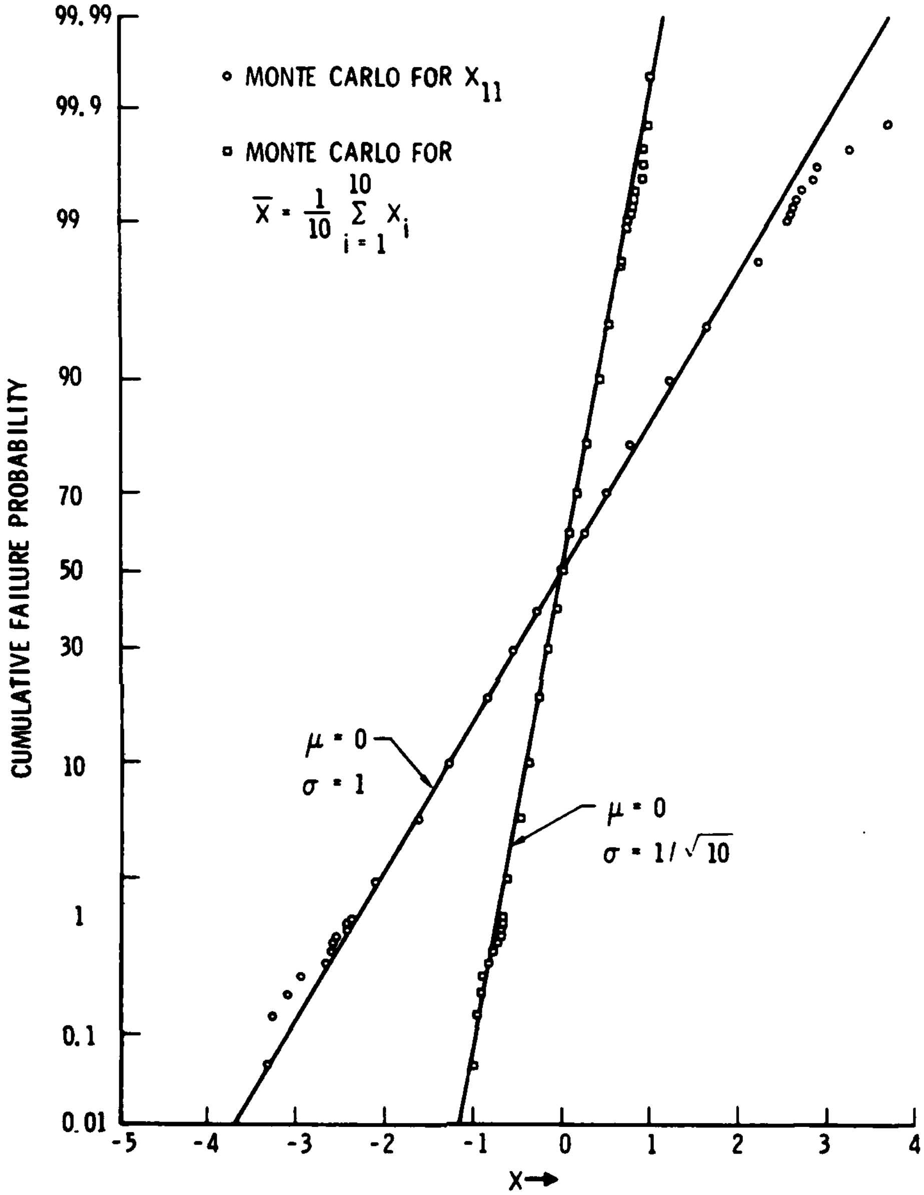

Fig. 1. cdf of 1000 Random Numbers from a Standard Normal and cdf of the Mean of a Sample of 10 from a Standard Normal

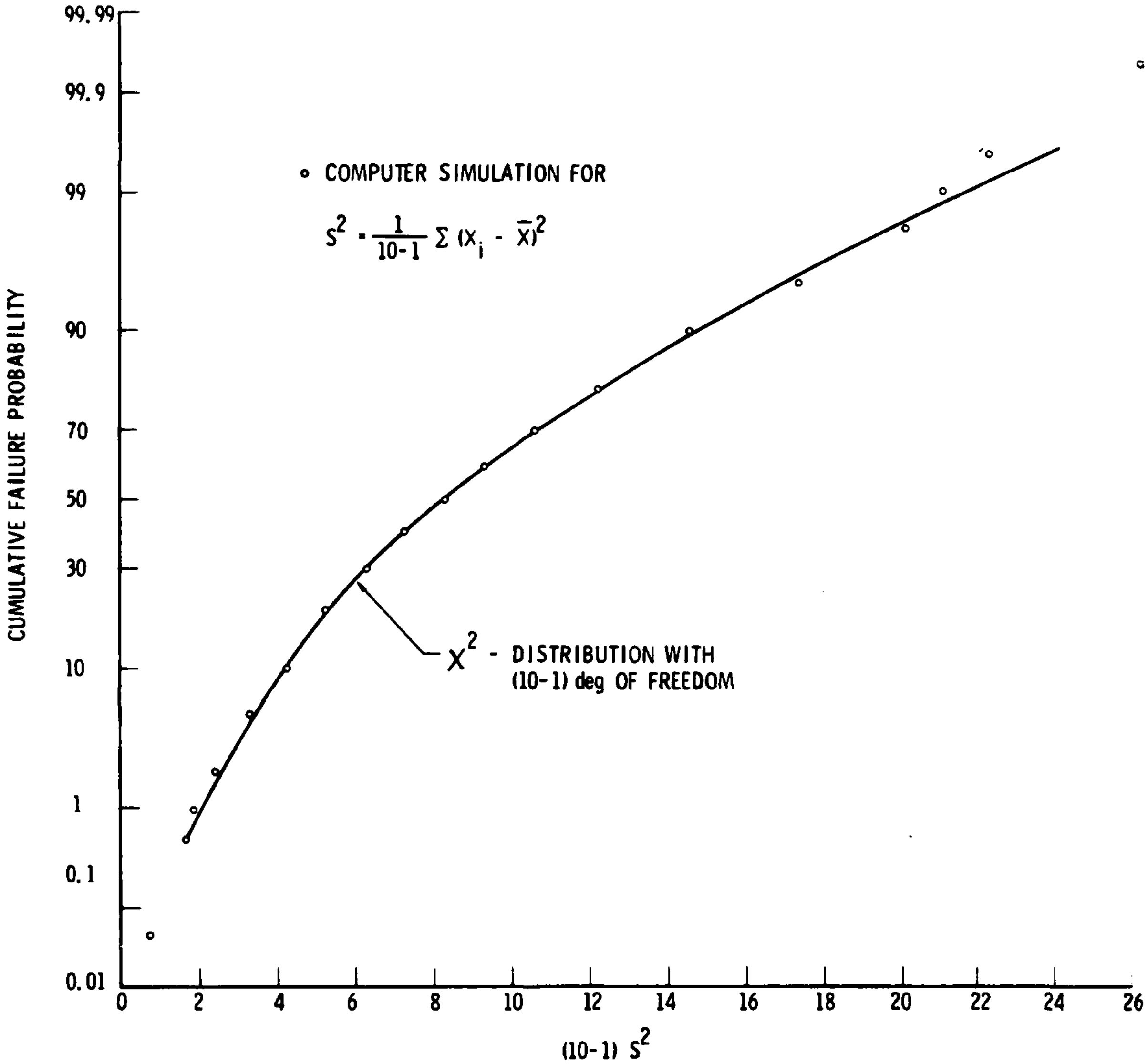

Fig. 2. cdf of the Standard Deviation of a Sample of 10 from a Standard Normal

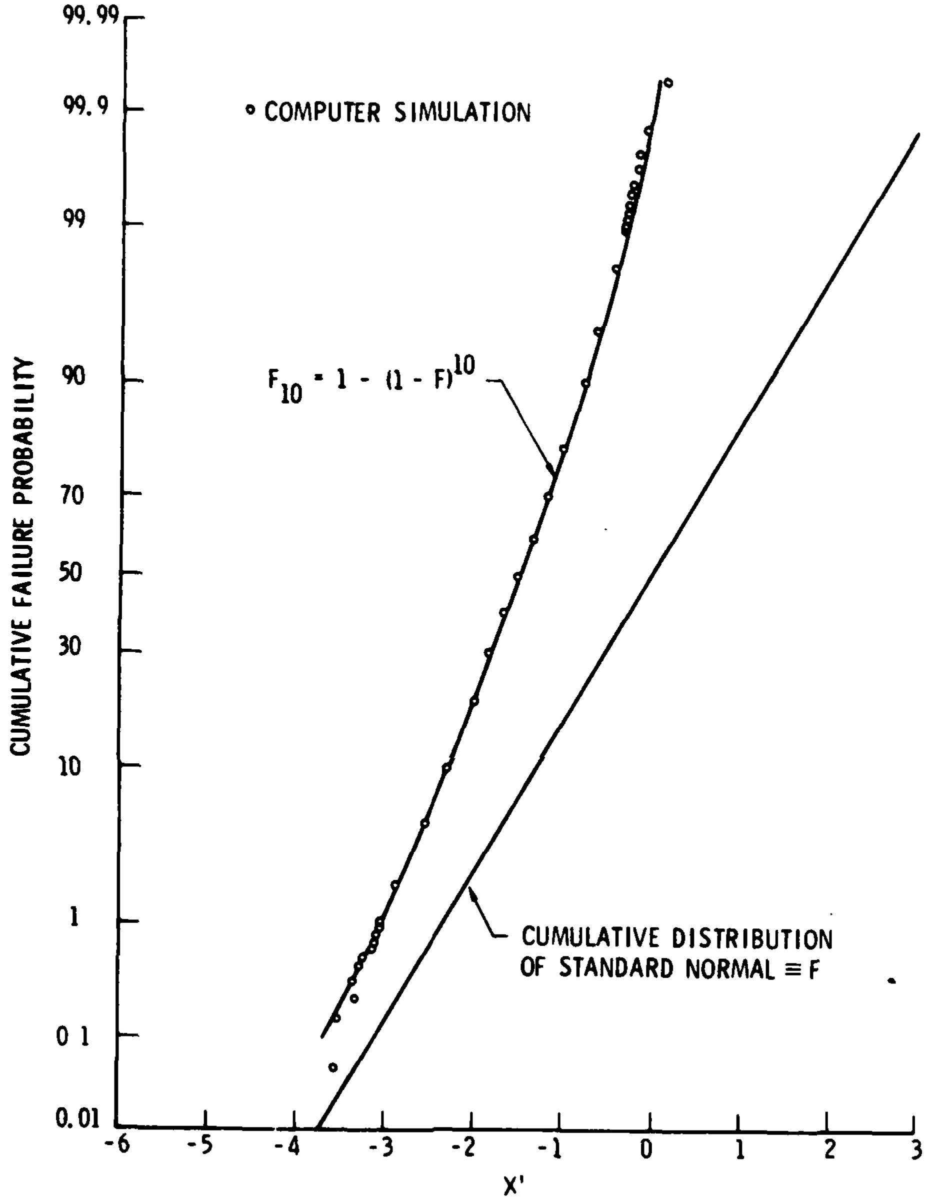

Fig. 3. Distribution from a Computer Simulation of the Lowest of a Sample of 10 Random Numbers from a Standard Normal

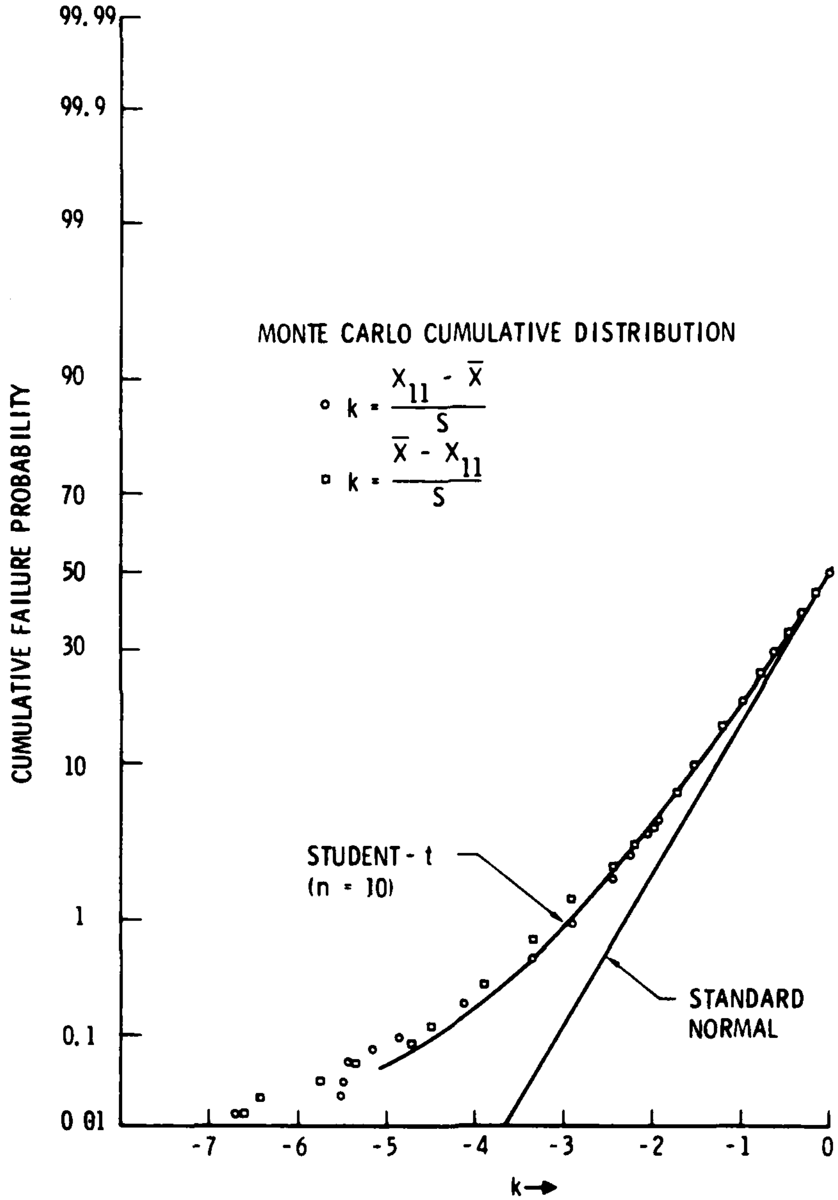

Fig. 4. Comparison between Student-t Analysis and Computer Simulation for a Sample of 10 from a Standard Normal. The computer simulation consisted of 10,000 runs.

We now extend our problem to estimating the reliability of an amplifier consisting of 10 devices, based on the failure results of a sample of devices from the same lot. Following the same logic as before, we consider

$$k = \frac{x' - \overline{x}}{s'} \tag{50}$$

where now x' is the lowest of the m random numbers x_{n+1} to x_{n+m}. This expression is independent of the scale and shape parameters of the failure pdf. Therefore, by performing a computer simulation on k as defined above and ranking the results, we obtain the cdf for the first failure in a string of m devices.

The above simulation has been performed with n and m both equal to 10, and the results displayed in Figure 5. We also have three theoretical curves for reference: the standard normal, the Student-t cdf for a single device, and the cdf for a string of 10 devices using the Student-t approximation. The Monte Carlo results for the reliability of a string of 10 devices closely follow the general shape of the curve derived directly from the Student-t analysis. The small amount of difference that does occur is at failure probabilities above 5%. The Monte Carlo results, which are below the Student-t derived curve, are real and not a result of scatter in the computer simulation.

When the desired $\overline{F}(k)$ is obtained, we transform to our particular case using

$$\begin{aligned} \ln t_o &= s\,k + \overline{x} \\ &= k + \ln 10^7 \end{aligned} \tag{51}$$

These results are plotted in Figure 6 on a linear time scale. This estimated cdf should be used in calculating the system reliability loss, i.e., Eq. (12).

The close agreement displayed in Figure 6 between the computer simulation and the Student-t approximation indicated that, in this case, it is reasonable

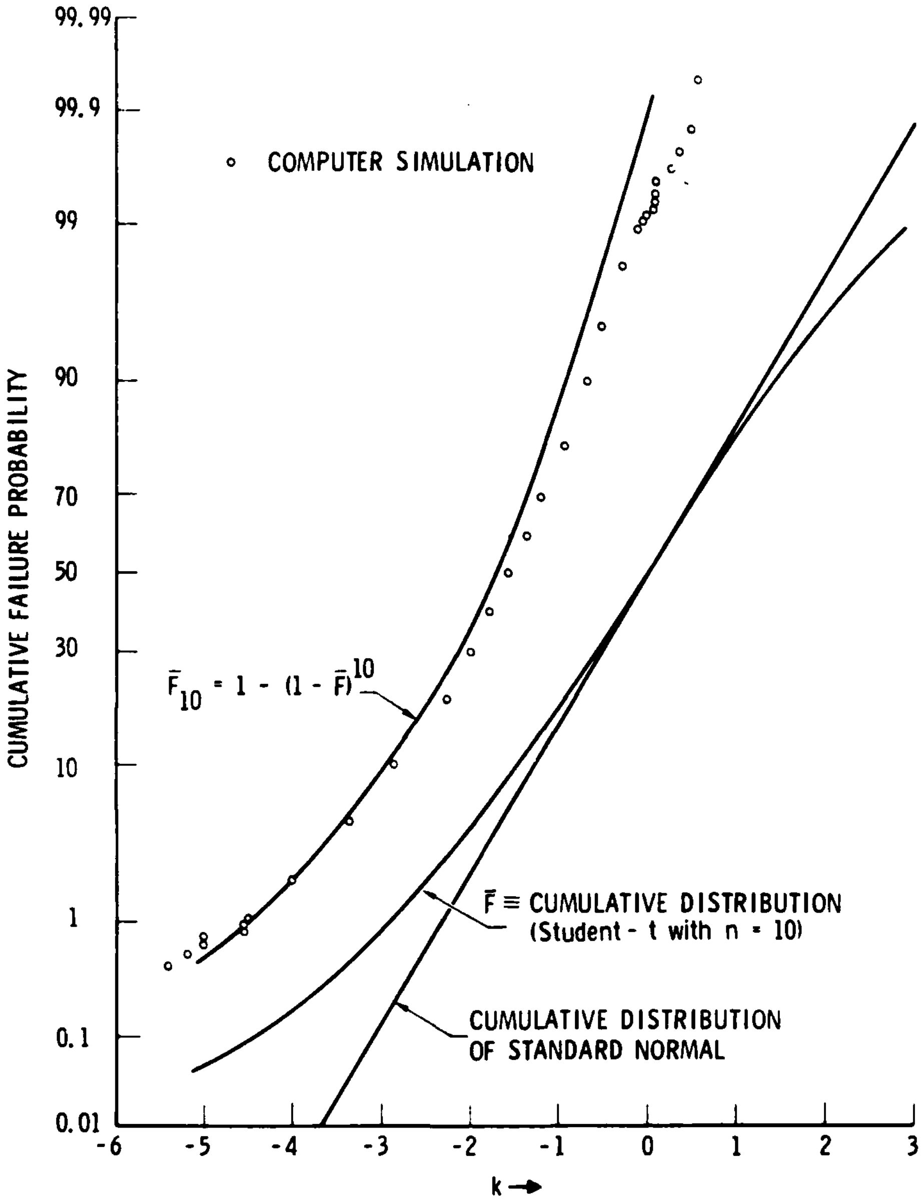

Fig. 5. Comparison between Student-t Approximation and Computer Simulation for a cdf of a String of 10 Devices Estimated from an Aged Sample of 10

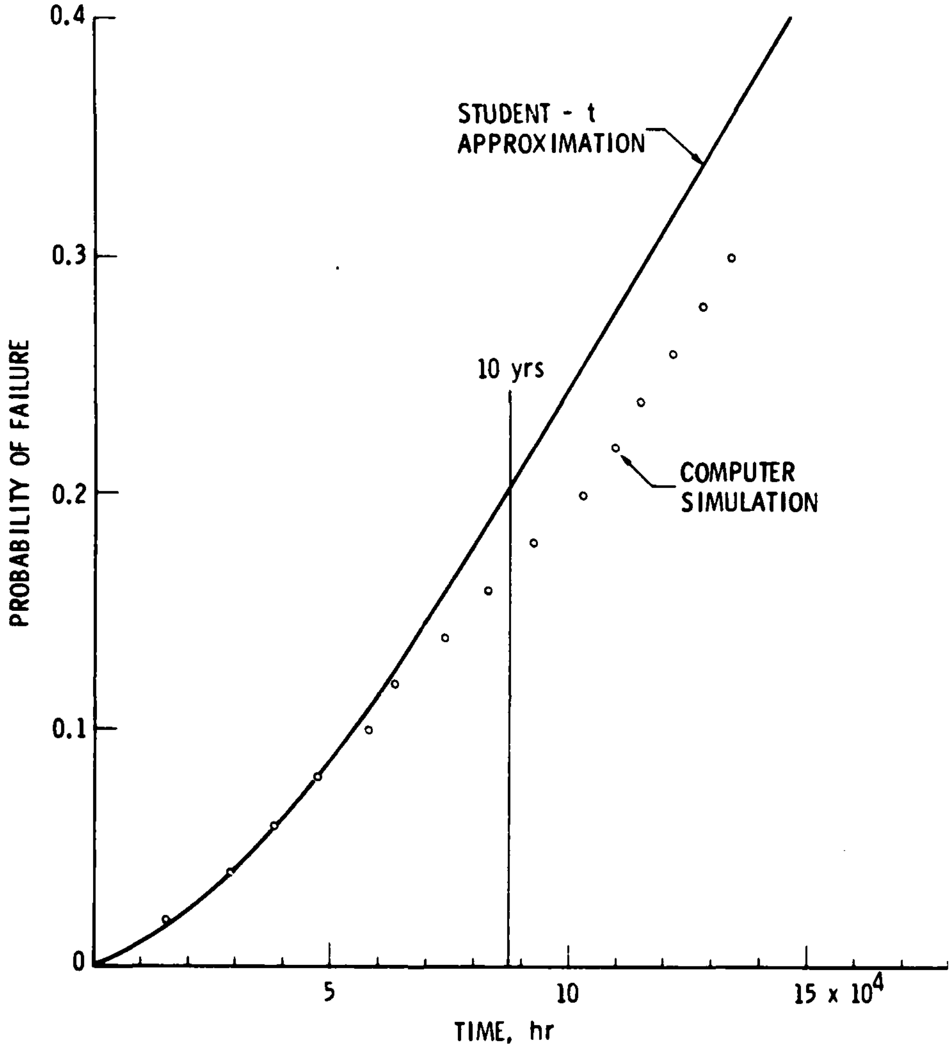

Fig. 6. Probability of Failure for a String of 10 Devices Estimated from an Aged Sample of 10 whose Sample Median Lifetime and Logarithmic Standard Deviation are 10^6 Hr and 1.0

to assume that the cdf obtained from a Student-t analysis can be used directly in estimating the reliability of a multi-component system. The agreement between the exact answer (i.e., computer simulation) and the Student-t approximation depends on the number of devices in the aging test and the system configuration. The agreement between the exact cdf and the Student-t approximation should increase with the number of devices in the aging program and decrease with the number of devices in the system. In Figure 7, we reproduce the results for n = 10/m = 3 and n = 10/m = 30. With only three devices used in the amplifier, the Student-t approximation is nearly exact, whereas with a multi-component system consisting of 30 devices, a significant deviation is observed.

The effect of the aging sample size on the correctness of the Student-t approximation can be seen by comparing Figures 5 and 8. In Figure 8 we present the computer simulation for n = 5 and m = 10. The difference between the Student approximation and the Monte Carlo results is becoming significant for a sample size of five devices, even at low probabilities.

In the preceding examples, we have only considered a simple string of devices and defined the system as failing when any one of the string failed. The computer simulation can easily be extended to more complex arrangements. If our system failure criterion allowed j failures out of m devices, we would perform a computer simulation using

$$k = \frac{x' - \bar{x}}{s'} \tag{52}$$

where x' is the j lowest of m random numbers. For a redundant system containing two identical strings of m devices each, we would use

$$k = \frac{x''' - \bar{x}}{s'} \tag{53}$$

where

$$x''' \text{ highest of } x' \text{ and } x'' \tag{54}$$

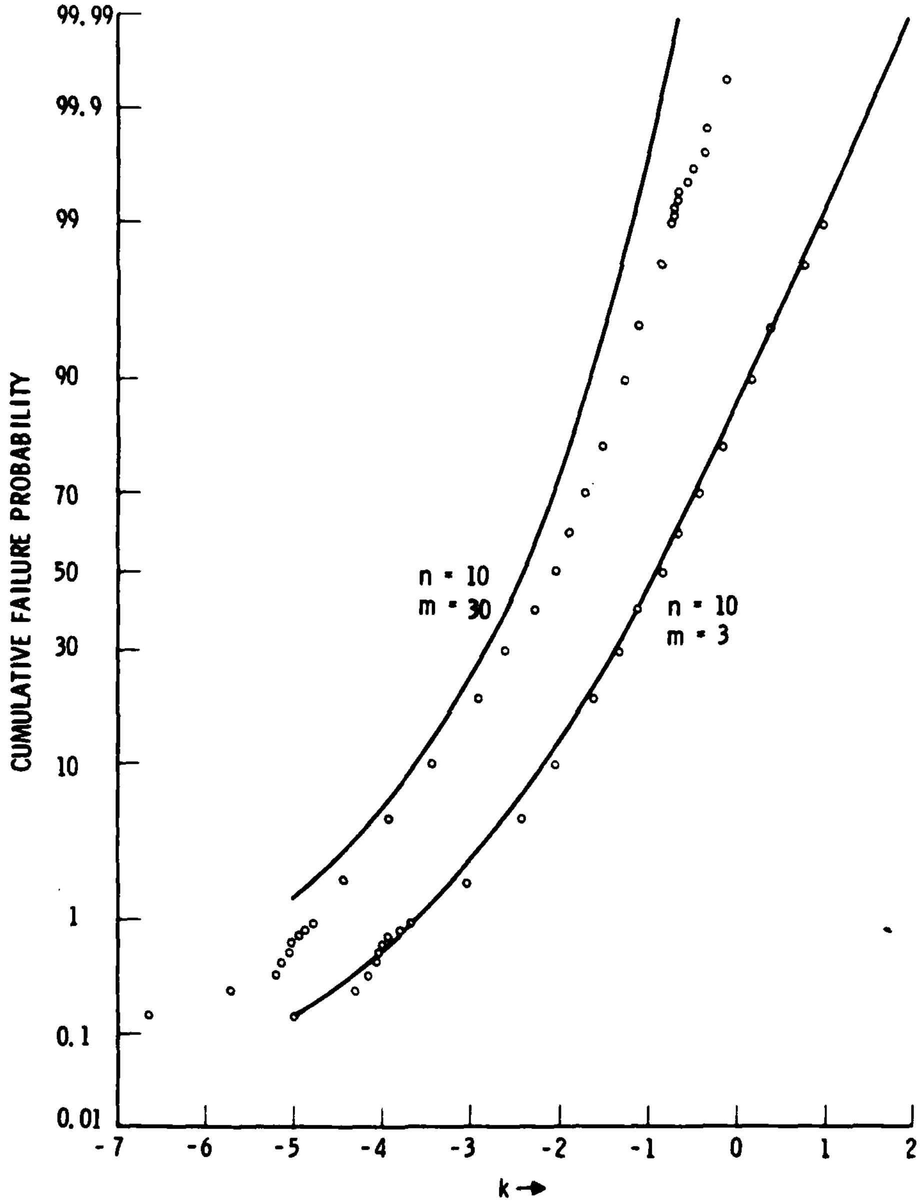

Fig. 7. Same as Fig. 5, Except m = 3 and 30. The solid curves are the Student-t approximation.

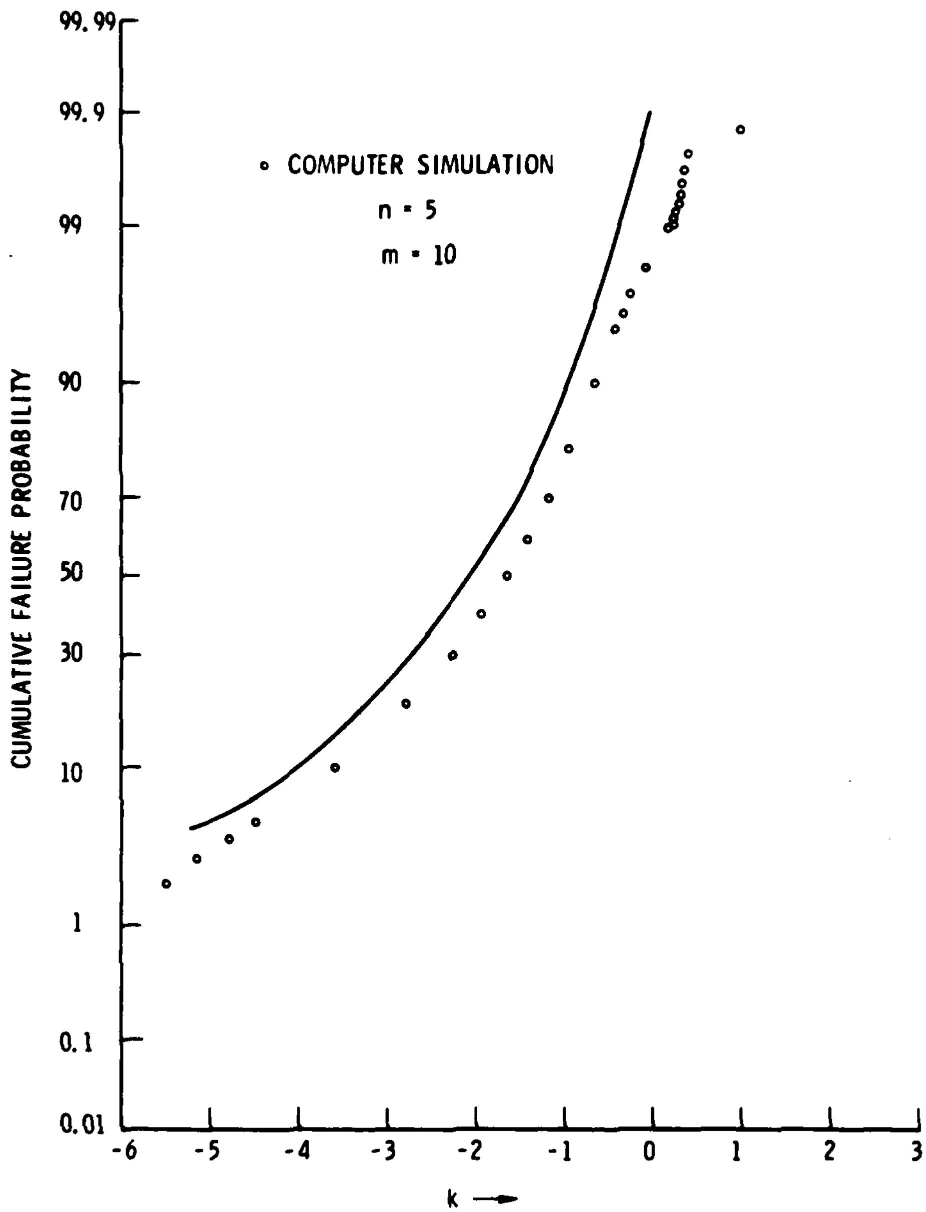

Fig. 8. cdf of a Computer Simulation of the Reliability of a String of 10 Devices Estimated from an Aged Sample Size of 5. The solid curve is the Student-t approximation.

with

$$x' \text{ lowest of } n + 1 \rightarrow n + m \tag{55}$$

and

$$x'' \text{ lowest of } n + m + 1 \rightarrow n + 2m \tag{56}$$

The results for a redundant system are shown in Figure 9. In this case, 10 devices were aged to failure and the results used to estimate the reliability of three different, redundant systems. Each system consisted of two equivalent channels, but with a different number of devices in each channel. The results shown in Figure 9 are for 3, 10, and 30 devices in each channel, or 6, 20, and 60 total devices.

If the redundant channel is not operated while the primary channel is still working, the failure probability in the standby mode is nearly zero. The failure probability for a two-channel standby redundant system is estimated as for an active redundant system except

$$x''' = x' + x'' \tag{57}$$

That is, the failure time of the standby system is the sum of the failure times of each channel.

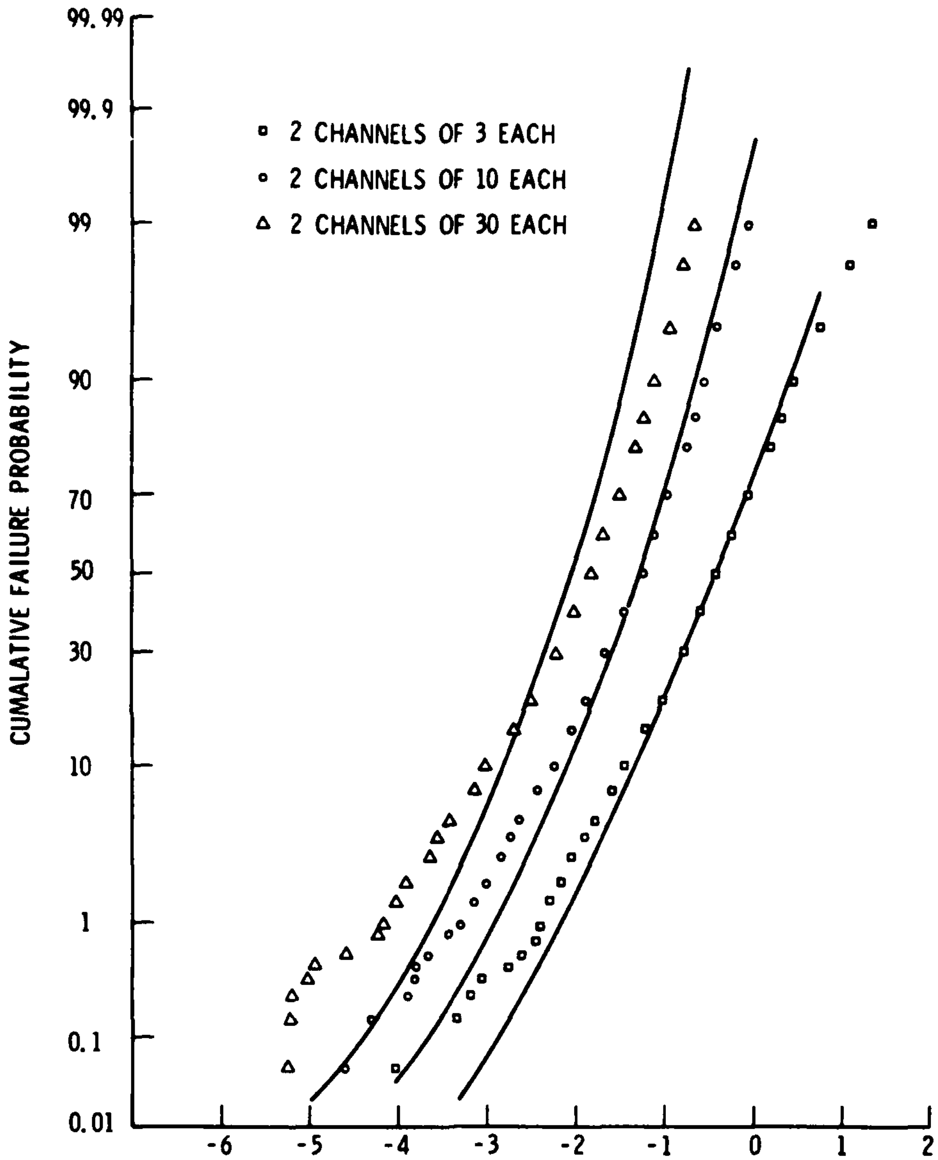

Fig. 9. cdf of a Computer Simulation of a Two-Channel Redundant System. The solid curves are the associated Student-t approximations.

IV. ANALYSIS OF ACCELERATED AGING AS APPLIED TO MULTI-COMPONENT SYSTEMS

The analysis of accelerated aging test results is similar, although more complex, than the single temperature situation examined in the preceding section. In this case, n devices are operated to failure at elevated temperatures. On the basis of these accelerated aging results, we will estimate the cdf of a string of m devices operating at a normal temperature. In particular, we would like to determine the cdf of

$$k = \frac{x' - \bar{x}_N}{s} \tag{58}$$

where x' is still the lowest failure time for a string of m devices at the normal operating temperature and $\bar{x}_N$ and s are given in Eqs. (32) through (37). Selecting from any normal failure pdf and using the transformation

$$x_i = \sigma R_i + \mu \tag{59}$$

we have

$$k = \frac{R' - \bar{R} + \overline{\Delta E'}\,\bar{Z} - \overline{\Delta E'}\,Z_N}{\left[\frac{1}{n-2}\sum_{i=1}^{n}(R_i - \overline{\ln\tau_o'} - \overline{\Delta E'}\,\bar{Z})^2\right]^{1/2}} \tag{60}$$

where

$$\Delta E' = \frac{\sum_{i=1}^{n}(R_i - \bar{R})(Z_i - \bar{Z})}{\sum_{i=1}^{n}(Z_i - Z)^2} \tag{61}$$

$$\overline{\ln\tau_o'} = \bar{R} - \overline{\Delta E'}\,\bar{Z} \tag{62}$$

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \tag{63}$$

and

$$\bar{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_i \tag{64}$$

The activation energy $\overline{\Delta E'}$ and $\overline{\ln\tau_o'}$ will, on the average, be zero because the transformation from the Arrhenius dependence is degenerated into scatter about the horizontal axis. Note that this transformation has eliminated σ, $\ln\tau_o$, and ΔE establishing that the cdf of k is independent of the starting failure pdf, as it was for the single temperature case. The exact magnitudes of the Z-values are not relevant, but their relative values are.

We will consider an accelerated aging experiment in which 10 devices failed at 250°C, and 10 others failed at 200°C. Assume that these 20 failures yield the following results

$$\overline{\Delta E} = 1.0 \text{ eV}$$

$$\bar{\tau}_o = 5 \times 10^{-10} \text{ hr}$$

$$s = 1.0$$

On the basis of these results, we will estimate the reliability of an amplifier consisting of a string of 10 devices when it is operated at 36°C.

We proceed by first obtaining the cdf of the x_{n+1} device when operated at 36°C. This consists of determining the cdf of

$$k = \frac{x_{n+1} - \bar{x}_N}{s'}$$

employing a computer simulation. Twenty-one random numbers are taken from a standard normal. The first 20 are paired off with their accelerated temperatures (i.e., the reduced $Z_1 \to Z_{20}$). The twenty-first random number represents x_{n+1}. These random numbers are used to calculate k. This process is repeated many times and the results ranked in order of increasing k.

The results of 1000 simulations are presented in Figure 10, where we have included the standard normal and the Student-t analysis for reference. As expected, a reasonably close agreement is seen between the cdf, from the computer simulation, and the Student-t analysis, which indicates that our computer simulation is functioning correctly in estimating the reliability of a single device.

In Figure 10, we plot the cdf of k which corresponds to the reliability of a string of 10 devices. If the Student-t approximation were valid, one would expect that the failure probability for the string of 10 devices at low levels would be 10 times the Student-t probability for a single device. The computer simulation for the string of 10 devices is approximately three times the failure probability of a single device at low failure rates.

When the normalized cdf shown in Figure 10 is obtained, we transform to a linear time scale employing the accelerated aging sample scale and shape parameters. In our particular case this transformation is

$$\ln t = ks + \bar{x}_N \text{ (at 36°C)}$$

or

$$t = 10^7 \exp(k)$$

For example, the appropriate k-value after an operating time of 10 years is k = -4.73. Therefore, we estimate from Figure 10 that the string of 10 devices would have a 14% probability of failing after operating for 10 years. To estimate the system loss indicated in Eq. (12), we use the above transformation to determine the cdf as a linear function of time in the same manner as was used to generate Figure 6.

More complex device arrangements, such as active and standby redundancy, can be estimated in a similar manner to that described above.

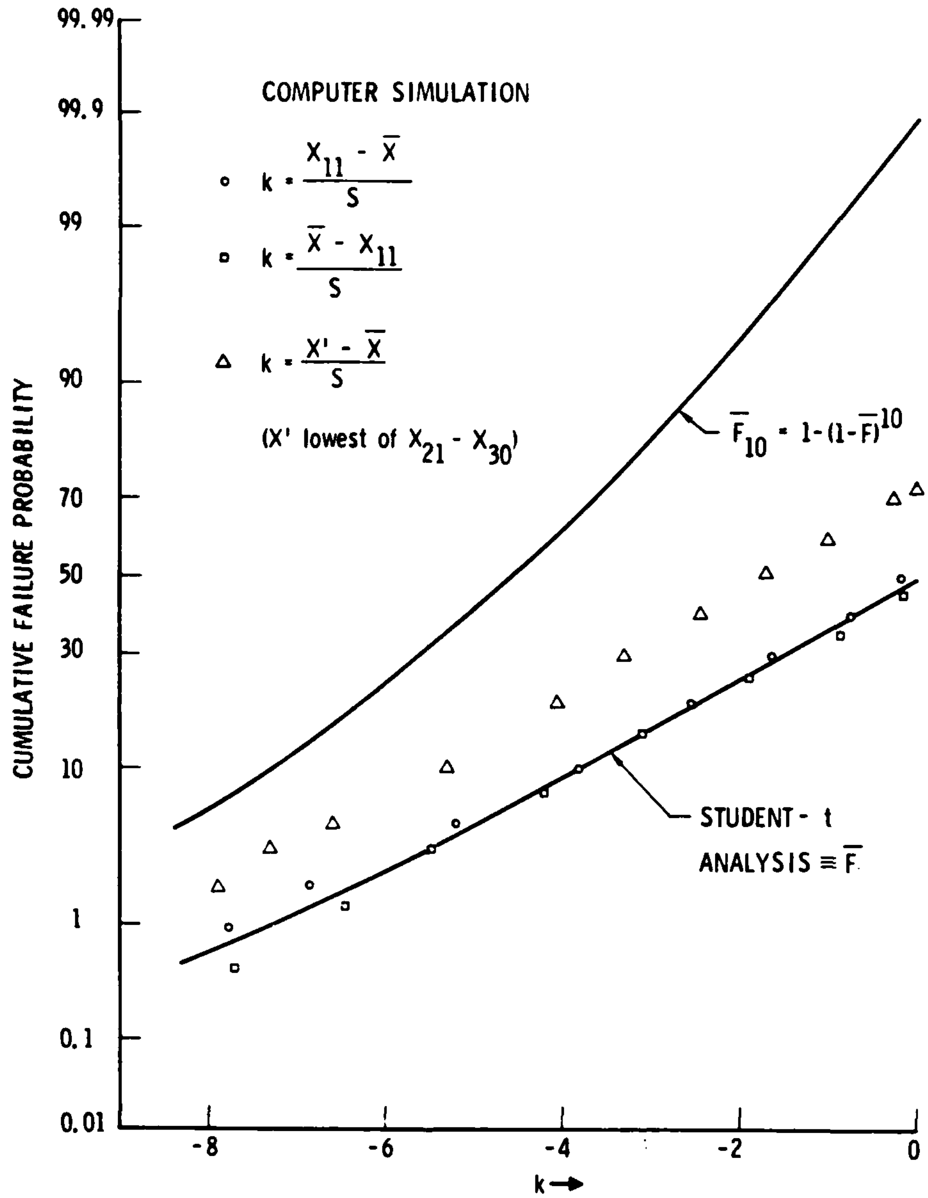

Fig. 10. Computer Simulation of the Reliability of a Single Device and a String of 10 Devices Estimated from a Two-Temperature Accelerated Aging Test. The solid curves are the associated Student-t analysis and Student-t approximation.

There are two other cases of interest: one in which the device temperatures are not constant throughout the mission and one in which the devices in the amplifier are at different temperatures. A worst case analysis can be used for both these cases. In this worst case analysis, it is assumed that all the devices are at the highest temperature that any device experiences throughout the intended mission. If the worst case system loss is acceptable, there is no need for further analysis. If the worst case system loss is not acceptable, a more detailed analysis is required. We will consider the situation in which the temperature varies between devices but not the situation in which the device internal temperature varies with time.

Consider the case wherein the reliability of a string of m devices is estimated from the results of an accelerated stress program consisting of n devices. Each of the m devices is operated at a specified temperature. Those devices operating at the higher temperature would have a higher failure rate than those operating at lower internal temperatures. A computer simulation can be used to estimate the contribution of each device to the total amplifier failure probability. The procedure for this simulation is slightly different from that used when all devices are at the same operating temperature.

From a standard normal we select n + m numbers and calculate k-values for each of the m random numbers

$$k_{n+1} = \frac{R_{n+1} - \overline{R}}{s'}$$

$$\vdots$$

$$k_{n+m} = \frac{R_{n+m} - \overline{R}}{s'}$$

where $\overline{R}'$ and s' are calculated as before, using Eqs. (59) through (63). Each of these k-values is converted to a failure time by means of the transformation

$$\ln\tau_i = s\, k_i + \overline{\ln\tau_o} + \overline{\Delta E}\, Z_i$$

where s, $\overline{\ln\tau_o}$, and $\overline{\Delta E}$ are the failure pdf parameters determined from the actual n devices aged to failure at the elevated temperatures. The Z_i's represent the m operating temperatures. For the m devices that make up the amplifier, our m failure times are

$$\ln\tau_{n+1} = \frac{s}{s'}\left(R_{n+1} - \bar{R}\right) + \bar{x}_N\left(Z_{n+1}\right)$$

$$\vdots$$

$$\ln\tau_{n+m} = \frac{s}{s'}\left(R_{n+m} - \bar{R}\right) + \bar{x}_N\left(Z_{n+m}\right)$$

From these failure times we select the lowest values. This procedure is repeated many times to obtain an accurate cdf as a function of operating time.

V. SUMMARY

By employing a computer simulation, we have been able to remove a deficiency in our preliminary analysis of accelerated aging in which we used the Student-t method to estimate the reliability of both individual devices and multi-component systems. Our previous assumption that the Student-t results can be applied directly in the classical combinatory manner is not exactly correct. In the present study, we have extended the Student-t approach to estimate the reliability of multiple device systems. The Student-t analysis was extended by means of a computer simulation.

The difference between our extended Student-t analysis and our previous assumption that the Student-t analysis can be used directly in the classical manner, in general, does not appear to be significant. In fact, the application of the Student-t assumption is conservative to the extent that it estimates a failure probability slightly higher than the exact results from a computer simulation except for a redundant system at low failure probabilities. We are not advocating that the Student-t assumption be used in estimating the reliability of a multi-component system. Our position is that the Student-t assumption is useful in initial evaluations and as a check on the final results.

There is very little difference between the computational time involved in the employment of the Student-t approximation and the extended Student-t analysis. In both methods, we must first perform a simple linear regression to determine the sample scale and shape parameter, as well as the statistical efficiency accelerated temperature distribution. With the Student-t approximation, we must evaluate the distribution as a function of the cdf and apply the classical combinatory analysis. Using the extended Student-t analysis, we proceed directly from the simple linear analysis to the final results. We estimate that both methods require equal computer time.

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2. "Reliability Analysis of the Gradual Degradation of Semiconductor Devices," M. F. Millea, TR-0083(3925-02)-1, The Aerospace Corporation (20 July 1983).

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