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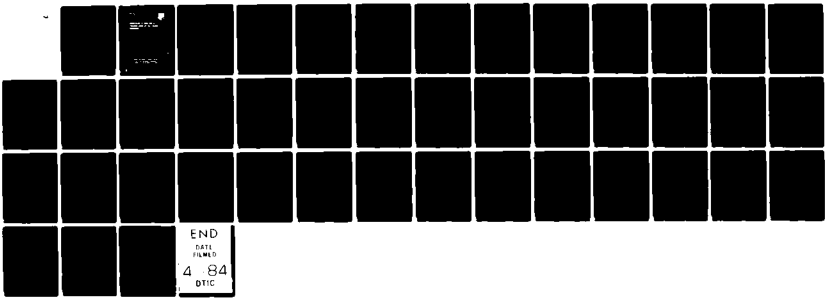

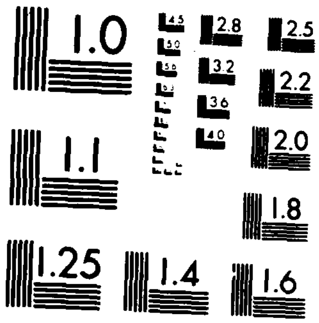

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Interim Report
September 1983

COMPUTATION OF RADIATION IN STRATIFIED MEDIA BY FAST FOURIER TRANSFORM (FFT)

Georgia Institute of Technology

J. J. H. Wang

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ROME AIR DEVELOPMENT CENTER
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A general and universal computational method for the general electromagnetic problem in stratified media was established. Computations were successfully performed for the field of a horizontal magnetic dipole in a microwave substrate backed by a conducting plane. The general field problem in stratified media was expressed in the form of integrals over k_ρ. The integrands involved Bessel functions which are difficult to integrate. By using the Fast Fourier Transform (FFT) techniques, the integrals are

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) computed numerically. A shortcoming of this method is that its usefulness is limited to near and intermediate fields.

This method is applicable to many electromagnetic problems in microwaves, geophysical sciences, printed circuits, etc.

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SECTION I
INTRODUCTION

Electromagnetic radiation in the presence of stratified media is a fundamental problem in geophysical sciences, printed circuits, microstrip devices, etc. [1,2]. It has been a subject of extensive research for eighty years beginning with the well-known Sommerfeld integral. However, this is also an area noted for its lack of numerical data and practical, generalized solutions. The electromagnetic problem is complicated because of a variety of physical parameters such as the source location, the layer thicknesses, and dielectric constants in individual layers. The solutions are often complex and are, as a rule, only applicable to particular cases.

With the advent of high-speed digital computers, it is increasingly more desirable to seek numerical solutions that are simple to use and are applicable to a broad spectrum of problems. The generalized formulation in terms of reflection coefficients and one-dimensional integrals in k_ρ appears to be a very suitable representation for numerical computations [3,4]. The primary difficulty in this approach is in the computation of the integrals which involve Hankel functions.

A highly efficient technique to compute these integrals is by way of Fast Fourier Transform (FFT) [5-8]. The integrand is organized into a product of two functions, one of which includes a Bessel function. The function without the Bessel function is expanded into a discrete Fourier series via FFT. The exponential terms of each of the resulting discrete Fourier series containing k_ρ are then combined with the remaining integrand for closed-form integration. There are three difficulties involved in this approach. Firstly, the integral after FFT expansion must be suitable for evaluation in closed form. Secondly, the resulting closed-form integral must be easy to compute numerically since there may be thousands of them in the Fourier series expansion. Thirdly, proper decay of the integrand is required for the FFT expansion.

In this report this approach is expanded to cover the general problem of radiation and scattering in the presence of stratified media. The integrand is, in most practical cases, a product of two functions; an even or odd function of k_ρ and a Hankel function (or its derivative) of the first kind. It is demonstrated in this report that a general algorithm to compute this class of problem is feasible.

SECTION II
FORMULATION OF THE PROBLEM

The fields in the ℓ-th layer due to a Hertzian dipole in a stratified medium as shown in Figure 1 can be expressed in the following general form [4]

$$E_{\ell z} = \int_{-\infty}^{\infty} dk_\rho \left[A_\ell e^{-jk_{\ell z} z} + B_\ell e^{jk_{\ell z} z} \right] H_n^{(1)}(k_\rho \rho) C_n(\phi) \tag{1}$$

$$E_{\ell \rho} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{\ell z}}{k_\rho} \left[-A_\ell e^{-jk_{\ell z} z} + B_\ell e^{jk_{\ell z} z} \right] H_n^{(1)\prime}(k_\rho \rho) C_n(\phi)$$

$$+ \int_{-\infty}^{\infty} dk_\rho \frac{j\omega\mu_\ell}{k_\rho^2 \rho} \left[C_\ell e^{-jk_{\ell z} z} + D_\ell e^{jk_{\ell z} z} \right] H_n^{(1)}(k_\rho \rho) S_n'(\phi) \tag{2}$$

$$E_{\ell \phi} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{\ell z}}{k_\rho^2 \rho} \left[-A_\ell e^{-jk_{\ell z} z} + B_\ell e^{jk_{\ell z} z} \right] H_n^{(1)}(k_\rho \rho) C_n'(\phi)$$

$$+ \int_{-\infty}^{\infty} dk_\rho \frac{-j\omega\mu_\ell}{k_\rho} \left[C_\ell e^{-jk_{\ell z} z} + D_\ell e^{jk_{\ell z} z} \right] H_n^{(1)\prime}(k_\rho \rho) S_n(\phi) \tag{3}$$

$$H_{\ell z} = \int_{-\infty}^{\infty} dk_\rho \left[C_\ell e^{-jk_{\ell z} z} + D_\ell e^{jk_{\ell z} z} \right] H_n^{(1)}(k_\rho \rho) S_n(\phi) \tag{4}$$

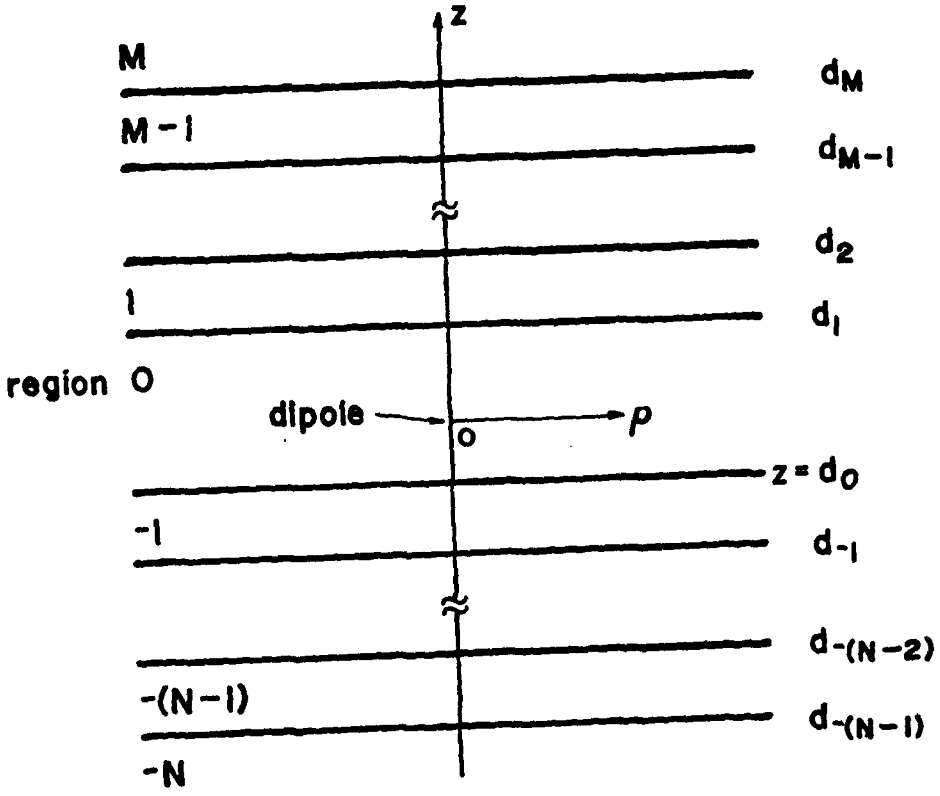

Figure 1. A stratified medium with Hertzian dipole excitation.

$$H_{\ell\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{\ell z}}{k_\rho} [-C_\ell e^{-jk_{\ell z}z} + D e^{jk_{\ell z}z}] H_n^{(1)'}(k_\rho \rho) S_n(\phi)$$

$$+ \int_{-\infty}^{\infty} dk_\rho \frac{-j\omega\varepsilon_\ell}{k_\rho^2 \rho} [A_\ell e^{-jk_{\ell z}z} + B_\ell e^{jk_{\ell z}z}] H_n^{(1)}(k_\rho \rho) C_n'(\phi) \qquad (5)$$

$$H_{\ell\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{\ell z}}{k_\rho^2 \rho} [-C_\ell e^{-jk_{\ell z}z} + D_\ell e^{jk_{\ell z}z}] H_n^{(1)} \quad \rho) S_n'(\phi)$$

$$+ \int_{-\infty}^{\infty} dk_\rho \frac{j\omega\varepsilon_\ell}{k_\rho} [A_\ell e^{-jk_{\ell z}z} + B_\ell e^{jk_{\ell z}z}] H_n^{(1)'}(k_\rho \rho) C_n(\phi) \qquad (6)$$

where $H_n^{(1)}$ denotes the n-th order Hankel function of the first kind and $H_n^{(1)'}$ its derivative. S_n, C_n and n are determined by the dipole excitation involved. S_n' and C_n' are derivatives of S_n and C_n. The $e^{-j\omega t}$ term is suppressed and the cylindrical coordinates are adopted, in which

$$k_\ell^2 = k_{\ell z}^2 + k_\rho^2 , \qquad (7)$$

for the ℓ-th layer. In Equation (7), k_ρ is independent of the layer and $k_\ell = \omega \sqrt{\varepsilon_\ell \mu_\ell}$.

The four constants, A_ℓ, B_ℓ, C_ℓ, and D_ℓ are determined by enforcing the boundary conditions at each layer interface. For example, the continuity of displacement flux $\bar{D}$ normal to the interface at $z = d_\ell$ requires

$$\varepsilon_\ell (A_\ell e^{-jk_{\ell z}d_\ell} + B_\ell e^{jk_{\ell z}d_\ell}) \qquad (8)$$

$$= \varepsilon_{\ell-1}(A_{\ell-1} e^{-jk_{(\ell-1)z}d_\ell} + B_{\ell-1} e^{jk_{(\ell-1)z}d_\ell})$$

There are a total of four equations similar to Equation (8) at each interface which constitute the necessary and sufficient boundary conditions.

The quantities n, S_n, C_n, A_o, B_o, C_o, D_o are determined by the dipole excitation. First, the fields due to a Hertzian dipole in the absence of stratification are expressed in integral forms similar to Equations (1-6). These constants and coefficients are then determined by a comparison between Equations (1)-(6) for $\ell = 0$ and the fields for this particular dipole source in the absence of stratification. The details are included in References 3 and 4, and are included in Appendix A for convenience.

SECTION III

FFT METHOD AS APPLIED TO A SPECIAL CASE

Before presenting the application of FFT method to the general integrals, the FFT method that was used [5], [6] to evaluate a specific integral will be discussed. The integral is given as,

$$I_{H_1}^{E} = \int_{-\infty}^{\infty} E(k_\rho)\, H_1^{(1)}(k_\rho \rho)\, dk_\rho$$
$$= 2\int_{0}^{\infty} E(k_\rho)\, J_1(k_\rho \rho)\, dk_\rho \qquad (8)$$

where E denotes an even function. We first define

$$g(k_\rho) = 2E(k_\rho)\, \exp(\nu_R k_\rho)\ . \qquad (9)$$

The Discrete Fourier Transform (DFT) of g can then be expressed as,

$$G(f) = \int_{-\infty}^{\infty} g(k_\rho)\, \exp\,(-2\pi j f k_\rho)\, dk_\rho\ , \qquad (10)$$

where

$$f = n/(N\Delta k_\rho).$$

We can now approximate $g(k_\rho)$ by a DFT expansion as follows:

$$g(k_\rho) \approx \frac{1}{N\Delta k_\rho} \sum_{n=-N/2}^{(N/2)-1} G(f)\exp(j2\pi f k_\rho) \qquad (11)$$

$$\text{for } 0 < k_\rho < [(N/2)-1]\Delta k_\rho$$

By substituting Equation (11) into Equation (8) and recognizing that

$$\int_{o}^{\infty} e^{-\nu k_\rho} J_1(k_\rho \rho) dk_\rho = \frac{1}{\rho} [1 - \frac{\nu}{(\nu^2 + \rho^2)^{1/2}}] , \tag{12}$$

where $\nu = \nu_R + j\nu_I$, $Re(\nu + j\rho) < 0$ and we have,

$$I_{H_1}^E \approx \frac{1}{N\Delta k\rho} \sum_{n=-N/2}^{(N/2)-1} G(\frac{n}{N\Delta k\rho}) \frac{1}{\rho}$$

$$\{1 - (\nu_R - \frac{j2\pi n}{N\Delta k\rho})/[(\nu_R - j2\pi n/(N\Delta k\rho))^2 + \rho^2]^{1/2}\} . \tag{13}$$

Thus the integral in Equation (8) is transformed into a finite series expansion in Equation (13), and can be readily computed on a digital computer.

Several fine points in this method need careful and proper handling. An arbitrarily chosen number, ν_R is required in Equation (12) to be larger than zero. However, ν_R must also be smaller than z so that $g(k_\rho)$ as defined in Equation (9) vanishes as k_ρ approaches infinity. In the problem treated in Reference 5, a term exp $(jk_{\ell z} z + \nu_R k_\rho)$ is contained in $g(k_\rho)$. In order to maintain a finite $g(k_\rho)$, $\nu_R = Z/2$ is chosen so that $g(k_\rho)$ vanishes as k_ρ approaches infinity. Another concern is that a real k_ℓ would lead to singularities in the-integrand. This difficulty can be overcome by adding a small dielectric loss to remove the singularity from the real axis.

A key feature of this approach is to transform the interval of integration $(-\infty, \infty)$ into (o, ∞) so that $g(k_\rho)$ nearly vanishes outside $(o, (\frac{N}{2} - 1) \Delta k_\rho)$ for a properly chosen ν_R. After this is accomplished, closed-form integration similar to that of Equation (12) is needed to handle the remaining integral containing a Bessel function. In essence, the FFT method transforms an integral containing a Bessel function to an approximate summation which is void of special functions and therefore can be readily computed. In the next section, the FFT method will be shown to be applicable to all the integrals in Equations (1-6). This can be established by showing that for these integrals the interval of integration can be changed from $(-\infty, \infty)$ to (o, ∞) and simple, closed-form integration can be carried out for the remaining integral containing a Bessel function.

SECTION IV

APPLICATION OF FFT METHOD TO THE GENERAL INTEGRALS

For dipole excitation, the order of the Hankel functon, n, is either 0 or 1. The coefficients A_ℓ, B_ℓ, C_ℓ, and D_ℓ are all formulated in matrices and can be readily evaluated on a computer. The evaluation of the integrals involving Hankel functions can be handled systematically by the FFT method just described.

A key to the success of the FFT method is the term $\exp(jk_{\ell z}\, z + \nu_R\, k_\rho)$, which causes $g(k_\rho)$ to vanish for a properly chosen positive ν_R, as k_ρ approaches infinity. As can be seen in the Appendix, terms of this type exist in the general form of the integral. That a suitable ν_R exists and the integrand would vanish as k_ρ approaches infinity can be argued on the basis of the existence of the solutions. It can also be proved, separately for the cases of a dipole below and above the field point, that a suitable ν_R exists.

The next hindrance to the FFT method is in the closed-form evaluation of the remaining integral containing the Bessel function. It is shown in the following that a closed-form integration similar to Equation (12) does exist for each integral that may be encountered.

The integrals involved in Equations (1-6) can be divided into the following eight types, each of which can be handled in a manner similar to that of Equation (8) discussed in the preceding section. E and O denote "even" and "odd" functions in the following discussion.

(A) $I^E_{H_o}$

For the integral

$$I^E_{H_o} \equiv \int_{-\infty}^{\infty} E(k_\rho) H_o^{(1)}(k_\rho \rho)\, dk_\rho$$

$$= 2j \int_{o}^{\infty} E(k_\rho) N_o(k_\rho \rho)\, dk_\rho \ , \qquad (14)$$

where N_o is a Bessel function of the second kind of order 0, the following closed-form integral can be employed ,

$$\int_o^\infty e^{-\nu k_\rho} N_o(k_\rho \rho) dk_\rho = \frac{-2}{\pi(\nu^2+\rho^2)^{1/2}} \ln \frac{\nu+(\nu^2+\rho^2)^{1/2}}{\rho^2} , \tag{15}$$

where $\quad \text{Re}\ \nu > I_m \rho = 0$.

(B) $I^o_{H_o}$

For the integral

$$I^o_{H_o} \equiv \int_{-\infty}^{\infty} O(k_\rho) H_o^{(1)}(k_\rho \rho) dk_\rho$$

$$= 2\int_o^\infty O(k_\rho) J_o(k_\rho \rho) dk_\rho \quad , \tag{16}$$

the following closed-form integral can be employed

$$\int_o^\infty e^{-\nu k_\rho} J_o(k_\rho \rho) dk_\rho = 1 - \frac{\nu}{(\nu^2+\rho^2)^{1/2}} \quad , \tag{17}$$

where $\quad \text{Re}\ [\nu + j\rho] > 0$.

(C) $I^E_{H'_o}$

For the integral

$$I^E_{H'_o} \equiv \int_{-\infty}^{\infty} E(k_\rho) H_o^{(1)'}(k_\rho \rho) dk_\rho$$

$$= -\int_{-\infty}^{\infty} E(k_\rho) H_1^{(1)}(k_\rho \rho) dk_\rho$$

$$= -2 \int_0^\infty E(k_\rho) J_1(k_\rho \rho) dk_\rho \quad , \tag{18}$$

the closed-form integral of Equation (12) can be used.

(D) $I^o_{H_o'}$

Since

$$I^o_{H_o'} \equiv \int_{-\infty}^\infty O(k_\rho) H_o^{(1)'}(k_\rho \rho) dk_\rho$$

$$= - \int_{-\infty}^\infty O(k_\rho) H_1^{(1)}(k_\rho \rho) dk_\rho$$

$$\equiv - I^o_{H_1}, \tag{19}$$

the discussion of $I^o_{H_1}$ later in (F) will be applicable.

(E) $I^E_{H_1}$

This type of integral has been discussed in Section III.

(F) $I^o_{H_1}$

For the integral

$$I^o_{H_1} = \int_{-\infty}^\infty O(k_\rho)\ H_1^{(1)}(k_\rho \rho) dk_\rho$$

$$= 2j \int_0^\infty O(k_\rho) N_1(k_\rho \rho) dk_\rho$$

$$= -2j \left[\frac{1}{\rho} O(k_\rho) N_o(k_\rho \rho)\right]_{k_\rho=0}^{\infty} + 2j \int_0^\infty \frac{1}{\rho} N_o(k_\rho \rho) O'(k_\rho) dk_\rho \ , \tag{20}$$

where $O'(k_\rho)$ is the derivative of $O(k_\rho)$, the following integral can be employed

$$\int_0^\infty e^{-\nu k_\rho} N_0(k_\rho\rho)dk_\rho = \frac{-2}{\pi(\nu^2+\rho^2)^{1/2}} \ln \frac{\nu+(\nu^2+\rho^2)^{1/2}}{\rho} , \quad (21)$$

where $\text{Re}\nu > \text{Im}\rho = 0$.

In the last step of Equation (20), the first term can be readily evaluated by substitution. However, this term generally vanishes because of the unique nature of N_o at o and ∞ .

(G) $I^E_{H_1'}$

Since

$$I^E_{H_1'} \equiv \int_{-\infty}^\infty E(k_\rho)H_1^{(1)'}(k_\rho\rho)dk_\rho$$

$$= \int_{-\infty}^\infty E(k_\rho)H_o^{(1)}(k_\rho\rho)dk_\rho - \int_{-\infty}^\infty \frac{1}{k_\rho\rho} E(k_\rho)H_1^{(1)}(k_\rho\rho)dk_\rho , \quad (22)$$

previous discussions on $I^E_{H_o}$ and $I^O_{H_1}$ are applicable.

(H) $I^O_{H_1'}$

Since

$$I^O_{H_1'} \equiv \int_{-\infty}^\infty O(k_\rho)H_1^{(1)'}(k_\rho\rho)dk_\rho$$

$$= \int_{-\infty}^\infty O(k_\rho)H_o^{(1)}(k_\rho\rho)dk_\rho - \int_{-\infty}^\infty \frac{1}{k_\rho\rho} O(k_\rho)H_1^{(1)}(k_\rho\rho)dk_\rho , \quad (23)$$

previous discussions on $I^0_{H_o}$ and $I^E_{H_1}$ are applicable.

SECTION V
NUMERICAL ANALYSIS

The preceding discussions demonstrated that a general numerical approach for the problem of dipole radiation in stratified media is feasible. In fact, a computer algorithm can be developed for the general case of arbitrary dipole configurations, geometries of stratification, and dielectric and ferromagnetic properties. The well-organized integrals in Equations (1-6) can be readily computed by the FFT method after the parameters are specified. The matrix formulation for the coefficients A_ℓ, B_ℓ, C_ℓ, and D_ℓ can be easily computed for any number of layers, dielectric constants, layer thicknesses, etc. Note also that these four coefficients are all <u>even</u> functions of k_ρ . Whether the integrands devoid of the Bessel functions in Equations (1-6) are odd or even can also be readily determined.

Several delicate points in this method should be carefully observed. The choice of N, Δk_ρ and ν_R must be properly made to insure that the DFT expansion is valid. As will be shown in the next section, good convergence and accuracy were achieved in all the examples tested. Obviously, the fact that the basic FFT techniques are well established has a great deal to do with the success of the present method.

A major limitation of this method is its deteriorated convergence in computing fields at locations with large ρ . In order that $g(k_\rho)$ be sufficiently near zero outside $(-N\Delta k_\rho/2, N\Delta k_\rho/2)$ for the FFT expansion, Δk_ρ must be chosen to be much smaller than $1/\rho$; and N must be chosen sufficiently large. The limited central memory and speed of modern digital computers restrict the FFT expansion to finite distances of ρ . Depending on the nature of the function $g(k_\rho)$, accuracy of the computation deteriorates when ρ is larger than several to several hundred wavelengths. Extension of this method to cases of large ρ is possible and, of course, highly desirable.

In this research, no attempt was made to develop a generalized computer program. Instead, basic techniques and subroutines were developed and tested to demonstrate that a generalized algorithm is indeed feasible.

SECTION VI
NUMERICAL TESTING

Several methods were employed in testing the accuracy, efficiency and versatility of the FFT method for problems involving stratified media. The results were checked against known data in the literature, against data obtained for limiting cases, and by numerical convergence with variations in N and Δk_ρ. All the three field components, E_z, E_ϕ, and E_ρ, were computed for the more involved cases of horizontal electric and magnetic Hertzian dipoles. The excellent numerical results demonstrated that the present method is indeed a powerful and general computing tool for electromagnetic problems in stratified media.

The first case tested was for E_z due to a horizontal electric dipole in media of two and three layers, which had previously been computed by the FFT method [5,6]. Excellent agreement between the present analysis and FFT results was observed.

E_z due to a horizontal magnetic dipole is similar to that due to a horizontal electric dipole because they both have the same type of integral, I^E_{H1} of Equation (8). Figure 2 shows a case that has been extensively studied for t = 0 and $\varepsilon_2 = 80\varepsilon_0$ by Kong and his colleagues [9]-[11]. Their earlier results in References 9 and 10 were inadequate and were later discussed [11] in conjunction with the newly obtained correct data. It must be emphasized that the integral involved is of the same type as that computed by FFT method in their earlier work. Even though some convergence problem may exist since the source and field points have the same z coordinates, their earlier FFT algorithm should have readily yielded limiting-case data if the general and universal nature of this method had been recognized and taken advantage of. This is illustrated in Figure 3, in which a comparison is made for E_z with various values of t to compare with the data for t = 0 in the literature. The present computation was actually made for the case in which the layer with water, for which $\varepsilon_2 \sim 80\varepsilon_0$, was approximated by a perfect conductor as often done in experiments. As can be seen, the computed results converge nicely to the t = 0 case when t is reduced from $0.1\lambda_0$ to $0.02\lambda_0$. The ripples in the curve corresponding to t = $0.02\lambda_0$ are indications of poor numerical convergence

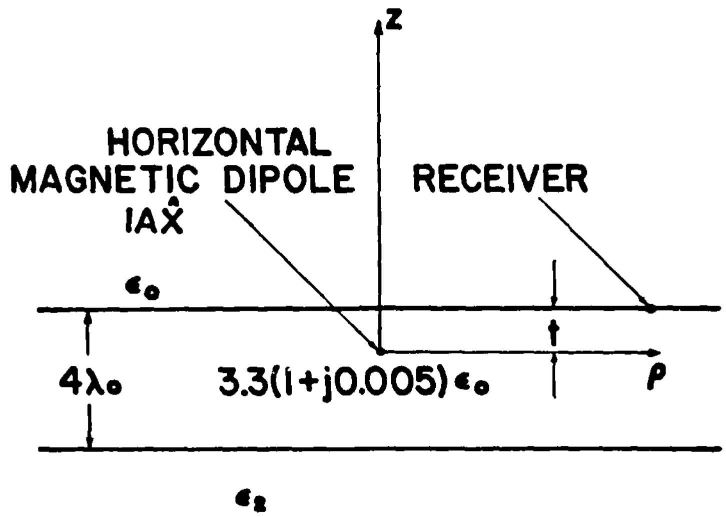

Figure 2. A horizontal magnetic Hertzian dipole in a three-layer medium.

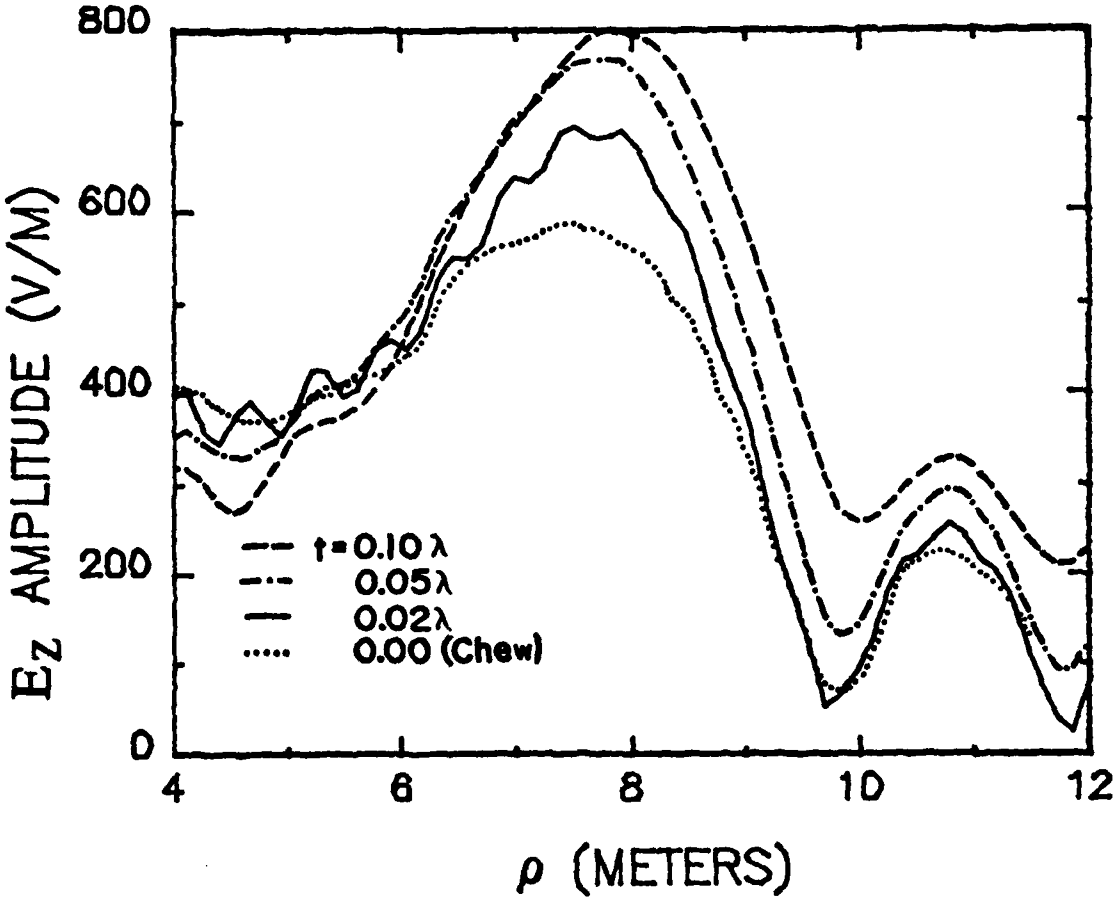

Figure 3. Comparison between the present calculations (t ≠ 0) and the t = 0 case by Chew and Kong showing that the fields for t ≠ 0 approach those for t = 0.

which will be discussed later in greater detail. The deterioration of convergence for small t is due to the reduced distance in z coordinate between the source dipole and the field point, as was explained in an earlier section.

Because of a lack of existing data that could be used to check against the present computation involving horizontal magnetic dipoles, we resorted to cases for which independent image-theory calculations can be made. For example, a horizontal magnetic Hertzian dipole in a homogeneous medium over a ground plane can be analyzed easily by image theory. These data can be used to compare with those computed by the FFT method. Figure 4 shows a comparison in the amplitude and phase of E_ρ for a horizontal magnetic dipole in a medium with $\varepsilon = 2\,\varepsilon_0$ over a conducting plane. Figure 5 shows a comparison for E_ϕ in this case. An accuracy of 0.1 percent or better is achieved in these cases.

The convergence test is another useful check in the present FFT method. Figure 6 shows how convergence is reached by increasing N in the computation. It was observed that numerical convergence is consistently accompanied by the smoothing of the ripples in the E vs. ρ plots. Convergence can also be checked by varying the value of Δk_ρ chosen in the FFT computation.

All types of integrals involved in Equations (1-6) and discussed in Section IV were numerically tested successfully. Thus, the applicability of the FFT method to the general electromagnetic radiation in stratified media has been clearly demonstrated.

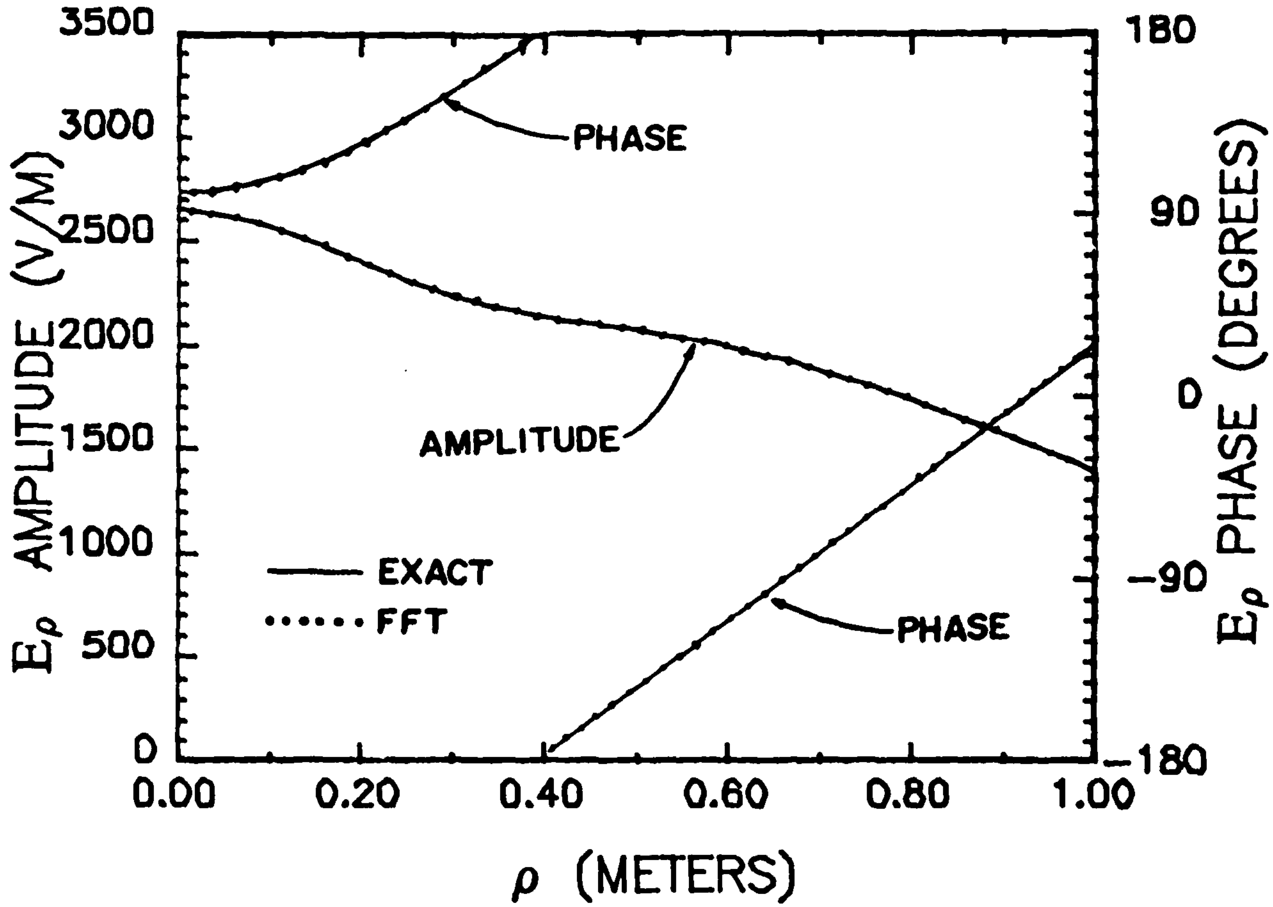

Figure 4. Comparison between the exact image-theory calculation and the present FFT method for E_ρ due to a horizontal magnetic Hertzian dipole in a medium of $2\varepsilon_0$ bounded by an infinite conducting plane one free-space wavelength below.

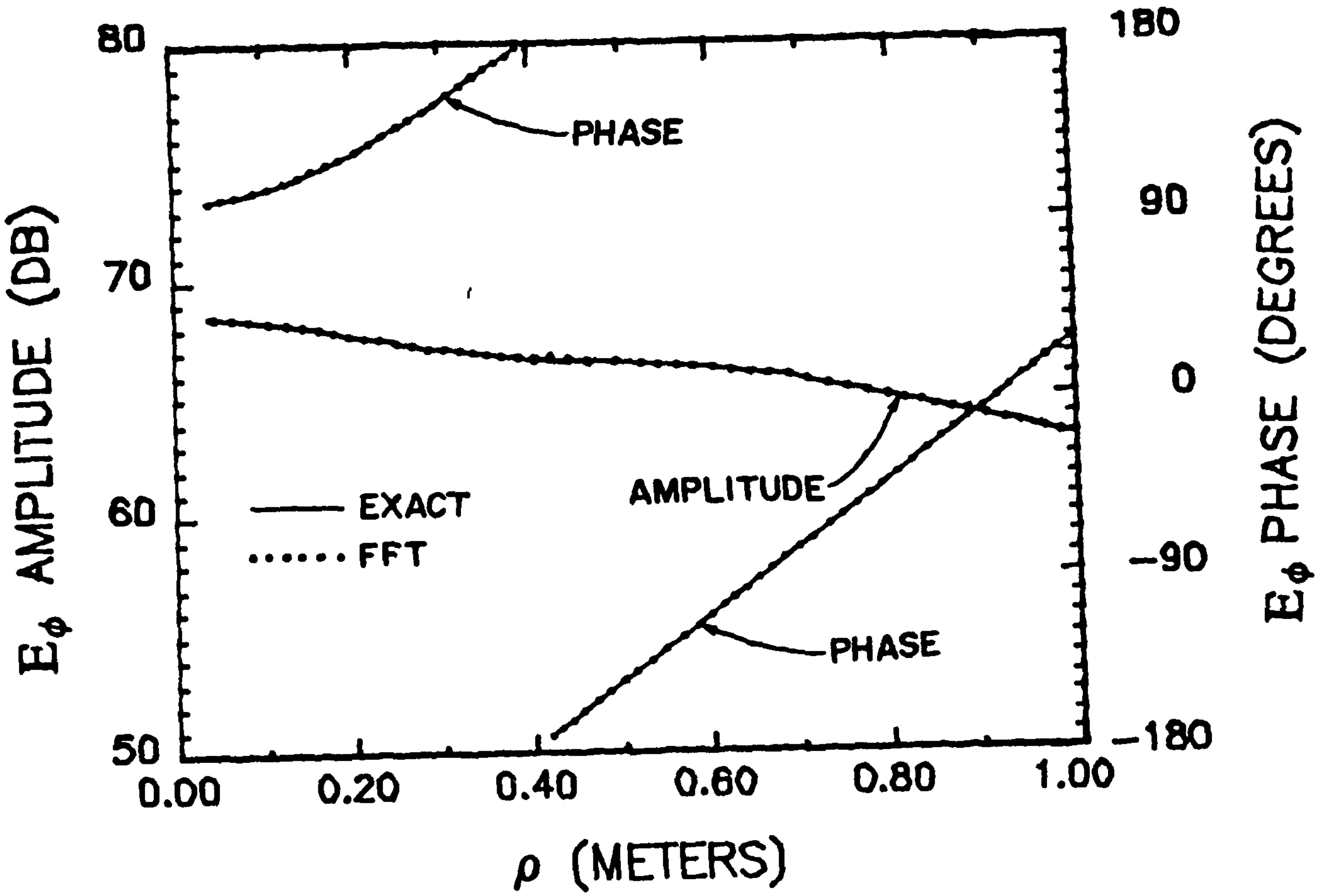

Figure 5. Comparison between the exact image-theory calculation and the present FFT method for E_ϕ due to a horizontal magnetic Hertzian dipole in a medium of $2\varepsilon_o$ bounded by an infinite conducting plane one free-space wavelength below.

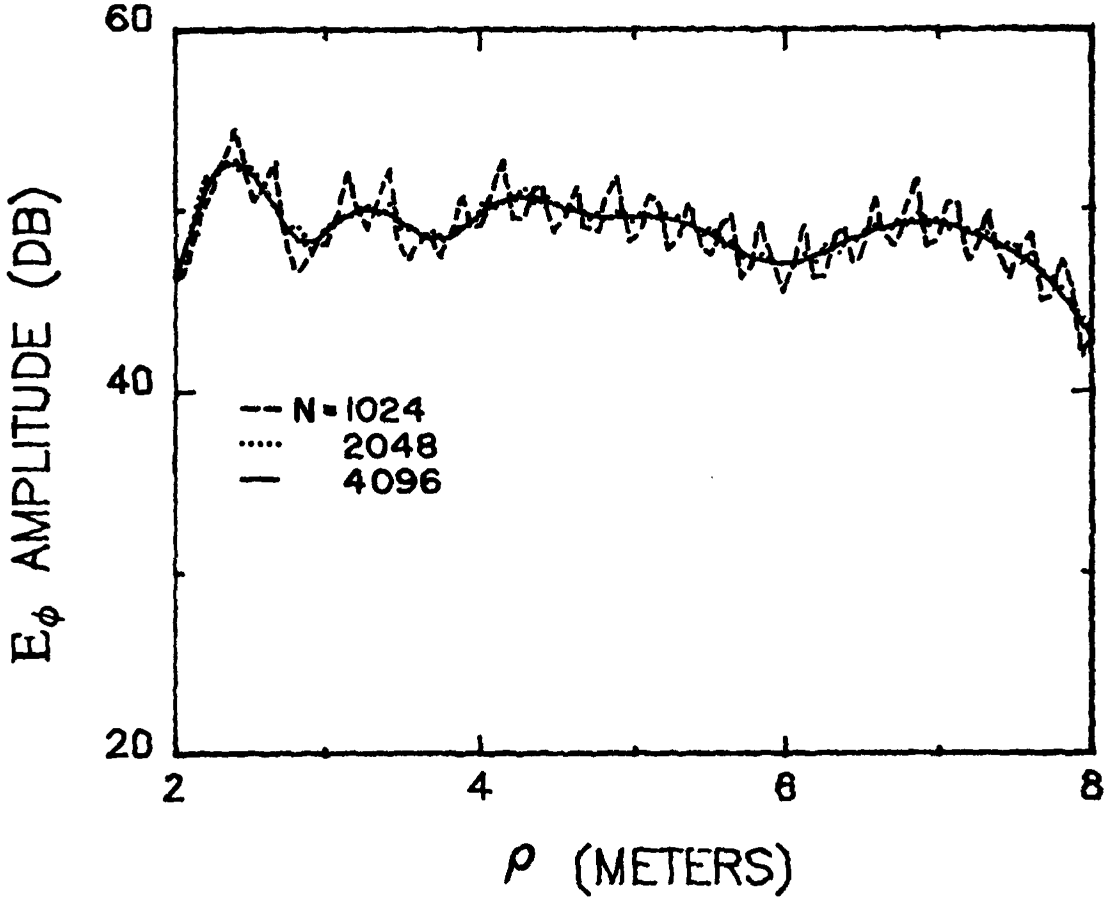

Figure 6. Convergence test for E_ϕ computed for the case in Figure 2 with ε_2 replaced by a perfectly conducting plane.

SECTION VII
CONCLUDING REMARKS

It is established in this report that a general and universal method exists for numerical analyses of the general electromagnetic problem in stratified media. The problem can be formulated such that a computer algorithm can be written for the general problem of arbitrary excitation, layer thickness, dielectric constants, etc. The integration is by the FFT method which transforms the integrals involving Bessel functions into a summation of simpler functions. Excellent numerical results were obtained for many test cases some of which are presented in this report.

There are, however, two shortcomings in the present FFT method. Because of the limited central memory and execution speed, computation is limited to field points having a radial (ρ) coordinate of several to several hundred wavelengths, depending on the nature of the particular integrand involved. In addition, convergence deteriorates as the distance (in z-coordinate) between the source and field points decreases.

For future research, it is recommended that a user-oriented general computer algorithm be developed on the basis of the findings in this report. Furthermore, the two shortcomings in this FFT method should be examined for improvements.

ACKNOWLEDGEMENTS

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APPENDIX A

CONSTANTS AND COEFFICIENTS FOR THE k -space INTEGRAL EXPRESSIONS OF FIELDS IN STRATIFIED MEDIA

APPENDIX A

CONSTANTS AND COEFFICIENTS FOR THE k_ρ-space INTEGRAL EXPRESSIONS OF FIELDS IN STRATIFIED MEDIA

Equations (1-6) are valid for the general problem of dipole radiation in stratified media. Although the application of these results to a specific case is straightforward, it is convenient to summarize all the relations needed in the determination of all the constants and coefficients, which are described in detail in Reference 4.

At each medium interface the following boundary conditions exist

$$\varepsilon_\ell (A_\ell e^{-jk_{\ell z} d_\ell} + B_\ell e^{jk_{\ell z} d_\ell}) = \varepsilon_{\ell-1} (A_{\ell-1} e^{-jk_{(\ell-1)z} d_\ell} + B_{\ell-1} e^{jk_{(\ell-1)z} d_\ell}) \ , \quad \text{(A-1)}$$

$$\mu_\ell (C_\ell e^{-jk_{\ell z} d_\ell} + D_\ell e^{jk_{\ell z} d_\ell}) = \mu_{\ell-1} (C_{\ell-1} e^{-jk_{(\ell-1)z} d_\ell} + D_{\ell-1} e^{jk_{(\ell-1)z} d_\ell} \ , \quad \text{(A-2)}$$

$$k_{\ell z} (-A_\ell e^{-jk_{\ell z} d_\ell} + B_\ell e^{jk_{\ell z} d_\ell}) = k_{(\ell-1)z} (-A_{\ell-1} e^{-jk_{(\ell-1)z} d_\ell} + B_{\ell-1} e^{jk_{(\ell-1)z} d_\ell}) \ , \text{ and} \quad \text{(A-3)}$$

$$k_{\ell z} (-C_\ell e^{-jk_{z} d_\ell} + D_\ell e^{jk_{\ell z} d_\ell}) = k_{(\ell-1)z} (-C_{\ell-1} e^{-jk_{(\ell-1)z} d_\ell} + D_{\ell-1} e^{jk_{(\ell-1)z} d_\ell}) \ . \quad \text{(A-4)}$$

An arbitrary Hertzian dipole can be considered to be the superposition of four basic Hertzian dipoles, vertical electric, horizontal electric, vertical magnetic, and horizontal magnetic. The quantities n, C_n, S_n, A_o, B_o, C_o, D_o, are determined by comparing Equations (1-6) with the fields of the specific dipole under consideration with the fields due to this particular dipole in the absence of stratification (infinite region). The latter is often referred to as the "primary" contribution in the general stratified-media solution and is summarized as follows

(1) Vertical electric dipole (VED)

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_{ved} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_0^{(1)}(k_\rho \rho) , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \qquad \text{(A-5)}$$

and

$$H_z = 0 , \qquad \text{(A-6)}$$

where

$$E_{ved} = \frac{-I\ell k_\rho^3}{8\pi\omega\varepsilon_0 k_{0z}} , \qquad \text{(A-7)}$$

and $I\ell$ is the electric dipole moment. Note here that 0 denotes layer index, not "free space", in ε_0, k_0, μ_0.

(2) Horizontal electric dipole (HED)

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_{hed} \begin{Bmatrix} e^{jk_{0z}z} \\ -e^{jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho)\cos\phi , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \qquad \text{(A-8)}$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{hed} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho)\sin\phi , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \qquad \text{(A-9)}$$

where

$$E_{hed} = j \frac{I\ell k_\rho^2}{8\pi\omega\varepsilon_0} , \quad \text{and} \qquad \text{(A-10)}$$

$$H_{hed} = j \frac{I\ell k_\rho^2}{8\pi k_{0z}} . \qquad \text{(A-11)}$$

(3) Vertical magnetic dipole (VMD)

$$E_z = 0 \ , \tag{A-12}$$

and

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{vmd} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_0^{(1)}(k_\rho \rho) \ , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{A-13}$$

where

$$H_{vmd} = -j \frac{IAk_\rho^3}{8\pi k_{0z}} \ , \tag{A-14}$$

and IA is the magnetic dipole moment.

(4) Horizontal magnetic dipole (HMD)

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_{hmd} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho)\sin\phi \ , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{A-15}$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{hmd} \begin{Bmatrix} e^{jk_{0z}z} \\ -e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho)\cos\phi \ , \quad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{A-16}$$

where

$$E_{hmd} = \frac{IA\omega\mu_0 k_\rho^2}{8\pi k_{0z}} \ , \text{ and} \tag{A-17}$$

$$H_{hmd} = \frac{-IAk_\rho^2}{8\pi} \ . \tag{A-18}$$

Because of the functional difference between $z > 0$ and $z < 0$ in the integrands of the fields described above, the coefficients A_0, B_0, C_0, and D_0 are denoted by A_{0+}, B_{0+}, C_{0+}, and D_{0+} for $z \geq 0$ and A_{0-}, B_{0-}, C_{0-}, and D_{0-} for $z \leq 0$.

The fields in region 0 for the four types of dipoles can be determined by the following relations

(1) VED

$$\left.\begin{aligned} A_{0+} &= A_{ved} \\ B_{0+} &= B_{ved} + E_{ved} \\ C_{0+} &= D_{0+} = 0 \end{aligned}\right\} \quad \text{for } z \geq 0 \qquad \text{(A-19)}$$

$$\left.\begin{aligned} A_{0-} &= A_{ved} + E_{ved} \\ B_{0-} &= B_{ved} \\ C_{0-} &= D_{0-} = 0 \end{aligned}\right\} \quad \text{for } z < 0 \qquad \text{(A-20)}$$

where A_{ved} and B_{ved} characterize contributions due to the stratified medium and are to be determined by the boundary conditions.

(2) HED

$$\left.\begin{aligned} A_{0+} &= A_{hed} \\ B_{0+} &= B_{hed} + E_{hed} \\ C_{0+} &= C_{hed} \\ D_{0+} &= D_{hed} + H_{hed} \end{aligned}\right\} \quad \text{for } z > 0 \qquad \text{(A-21)}$$

$$\left.\begin{aligned} A_{0-} &= A_{hed} - E_{hed} \\ B_{0-} &= B_{hed} \\ C_{0-} &= C_{hed} + H_{hed} \\ D_{0-} &= D_{hed} \end{aligned}\right\} \quad \text{for } z \leq 0 \qquad \text{(A-22)}$$

where A_{hed}, B_{hed}, C_{hed}, and D_{hed} are to be determined by the boundary conditions.

(3) VMD

$$\left.\begin{aligned} A_{0+} &= B_{0+} = 0 \\ C_{0+} &= C_{vmd} \\ D_{0+} &= D_{vmd} + H_{vmd} \end{aligned}\right\} \quad \text{for } z \geq 0 \tag{A-23}$$

$$\left.\begin{aligned} A_{0-} &= B_{0-} = 0 \\ C_{0-} &= C_{vmd} + H_{vmd} \\ D_{0-} &= D_{vmd} \end{aligned}\right\} \quad z \leq 0 \tag{A-24}$$

where C_{vmd} and D_{vmd} are to be determined by the boundary conditions.

(4) HMD

$$\left.\begin{aligned} A_{0+} &= A_{hmd} \\ B_{0+} &\;\; B_{hmd} + E_{hmd} \\ C_{0+} &= C_{hmd} \\ D_{0+} &= D_{hmd} + H_{hmd} \end{aligned}\right\} \quad \text{for } z \geq 0 \tag{A-25}$$

$$\left.\begin{aligned} A_{0-} &= A_{hmd} + E_{hmd} \\ B_{0-} &= B_{hmd} \\ C_{0-} &= C_{hmd} - H_{hmd} \\ D_{0-} &= D_{hmd} \end{aligned}\right\} \quad \text{for } z \leq 0 \tag{A-26}$$

where A_{hmd}, B_{hmd}, C_{hmd}, and D_{hmd} are to be determined by the boundary conditions.

It is convenient to reorganize Equations (A-1)-(A-4) in either

$$\begin{bmatrix} A_\ell \\ B_\ell \end{bmatrix} = \bar{\bar{U}}^{TM}_{\ell(\ell-1)} \cdot \begin{bmatrix} A_{\ell-1} \\ B_{\ell-1} \end{bmatrix} , \tag{A-27}$$

$$\begin{bmatrix} C_\ell \\ D_\ell \end{bmatrix} = \bar{\bar{U}}^{TE}_{\ell(\ell-1)} \cdot \begin{bmatrix} C_{\ell-1} \\ D_{\ell-1} \end{bmatrix} , \tag{A-28}$$

or

$$\begin{bmatrix} A_{\ell-1} \\ B_{\ell-1} \end{bmatrix} = \overline{\overline{V}}^{TM}_{\ell(\ell-1)} \cdot \begin{bmatrix} A_{\ell} \\ B_{\ell} \end{bmatrix} , \qquad \text{(A-29)}$$

$$\begin{bmatrix} C_{\ell-1} \\ D_{\ell-1} \end{bmatrix} = \overline{\overline{V}}^{TE}_{(\ell-1)\ell} \cdot \begin{bmatrix} C_{\ell} \\ D_{\ell} \end{bmatrix} , \qquad \text{(A-30)}$$

where

$$\overline{\overline{U}}^{TM}_{\ell(\ell-1)} = \frac{1}{2} \left[\frac{\varepsilon_{\ell-1}}{\varepsilon_{\ell}} + \frac{k_{(\ell-1)z}}{k_{\ell z}} \right]$$

$$\begin{bmatrix} e^{j(k_{\ell z}-k_{(\ell-1)z})d_{\ell}} & R^{TM}_{\ell(\ell-1)} e^{j(k_{\ell z}+k_{(\ell-1)z})d_{\ell}} \\ R^{TM}_{\ell(\ell-1)} e^{-j(k_{\ell z}+k_{(\ell-1)z})d_{\ell}} & e^{-j(k_{\ell z}-k_{(\ell-1)z})d_{\ell}} \end{bmatrix} , \qquad \text{(A-31)}$$

$$\overline{\overline{U}}^{TE}_{\ell(\ell-1)} = \frac{1}{2} \left[\frac{\mu_{\ell-1}}{\mu_{\ell}} + \frac{k_{(\ell-1)z}}{k_{\ell z}} \right]$$

$$\begin{bmatrix} e^{j(k_{\ell z}-k_{(\ell-1)z})d_{\ell}} & R^{TE}_{\ell(\ell-1)} e^{j(k_{\ell z}+k_{(\ell-1)z})d_{\ell}} \\ R^{TE}_{\ell(\ell-1)} e^{-j(k_{\ell z}+k_{(\ell-1)z})d_{\ell}} & e^{-j(k_{\ell z}-k_{(\ell-1)z})d_{\ell}} \end{bmatrix} , \qquad \text{(A-32)}$$

$$\overline{\overline{V}}^{TM}_{(\ell-1)\ell} = \frac{1}{2} \left[\frac{\varepsilon_{\ell}}{\varepsilon_{\ell-1}} + \frac{k_{\ell z}}{k_{(\ell-1)z}} \right]$$

$$\begin{bmatrix} e^{j(k_{(\ell-1)z}-k_{\ell z})d_{\ell}} & R^{TM}_{(\ell-1)\ell} e^{j(k_{(\ell-1)z}+k_{\ell z})d_{\ell}} \\ R^{TM}_{(\ell-1)} e^{-j(k_{(\ell-1)z}+k_{\ell z})d_{\ell}} & e^{-j(k_{(\ell-1)z}-k_{\ell z})d_{\ell}} \end{bmatrix} . \qquad \text{(A-33)}$$

and

$$\bar{\bar{V}}^{TE}_{(\ell-1)\ell} = \frac{1}{2}\left[\frac{\mu_\ell}{\mu_{\ell-1}} + \frac{k_{\ell z}}{k_{(\ell-1)z}}\right]$$

$$\begin{bmatrix} e^{j(k_{(\ell-1)z}-k_{\ell z})d_\ell} & R^{TE}_{(\ell-1)\ell}e^{j(k_{(\ell-1)z}+k_{\ell z})d_\ell} \\ R^{TE}_{(\ell-1)\ell}e^{-j(k_{(\ell-1)z}+k_{\ell z})d_\ell} & e^{-j(k_{(\ell-1)z}-k_{\ell z})d_\ell} \end{bmatrix}. \qquad \text{(A-34)}$$

In Equations (A-31)-(A-34)

$$R^{TE}_{\ell(\ell-1)} = \frac{1 - \mu_\ell k_{(\ell-1)z}/\mu_{\ell-1}k_{\ell z}}{1 + \mu_\ell k_{(\ell-1)z}/\mu_{\ell-1}k_{\ell z}} , \qquad \text{(A-35)}$$

$$R^{TM}_{\ell(\ell-1)} = \frac{1 - \epsilon_\ell k_{(\ell-1)z}/\epsilon_{\ell-1}k_{\ell z}}{1 + \epsilon_\ell k_{(\ell-1)z}/\epsilon_{\ell-1}k_{\ell z}} , \qquad \text{(A-36)}$$

and

$$R^{TE}_{(\ell-1)\ell} = -R^{TE}_{\ell(\ell-1)} , \qquad \text{(A-37)}$$

$$R^{TM}_{(\ell-1)\ell} = -R^{TM}_{\ell(\ell-1)} . \qquad \text{(A-38)}$$

In region 0, we have

$$
R^{TM}_{0-} \equiv \frac{B_{0-}}{A_{0-}} = \frac{1}{R^{TM}_{0(-1)}} \exp\,[-j2k_{0z}d_0]
+ \cfrac{\left[1 - \left(\frac{1}{R^{TM}_{0(-1)}}\right)^2\right] \exp\,[-j2(k_{0z}+k_{-1z})d_0]}{\frac{1}{R^{TM}_{0(-1)}} \exp[-j2k_{-1z}d_0] + \frac{\exp[-j2k_{1z}d_1]}{R^{TM}_{(-1)(-2)}} + \cfrac{1 - \left[\left(\frac{1}{R^{TM}_{(-1)(-2)}}\right)^2\right] \exp[-j2(k_{-1z}+k_{-2z})d_{-1}]}{\frac{1}{R^{TM}_{(-1)(-2)}} \exp[-j2k_{-2z}d_{-1}] + \ldots + R^{TM}_{(-N+1)(-N)} \exp\,[-j2k_{-Nz}d_{-N+1}]}} ,
\tag{A-39}
$$

$$
R^{TM}_{0+} \equiv \frac{A_{0+}}{B_{0+}} = \frac{1}{R^{TM}_{01}} \exp[j2k_{0z}d_1]
+ \cfrac{\left[1 - \left(\frac{1}{R^{TM}_{01}}\right)^2\right] \exp[j2(k_{0z}+k_{1z})d_1]}{\frac{1}{R^{TM}_{01}} \exp[j2k_{1z}d_1] + \frac{\exp[j2k_{1x}d_2]}{R^{TM}_{12}} + \cfrac{\left[1 - \left(\frac{1}{R^{TM}_{12}}\right)^2\right] \exp[j2(k_{1z}+k_{2z})d_2]}{\frac{1}{R^{TM}_{12}} \exp[j2k_{2z}d_2] + \ldots + R^{TM}_{(M-1)M} \exp\,[j2k_{(M-1)z}d_M]}} ,
\tag{A-40}
$$

and

$$R^{TE}_{0-} \equiv \frac{D_{0-}}{C_{0-}} = R^{TM}_{0-} \quad , \tag{A-41}$$

$$R^{TE}_{0+} \equiv \frac{C_{0+}}{D_{0+}} = R^{TM}_{0+} \quad . \tag{A-42}$$

The wave amplitudes in region 0 for the four types of dipoles are

(1) for vertical electric dipole

$$A_{0+} = \frac{R^{TM}_{0+}(1 + R^{TM}_{0-})}{1 - R^{TM}_{0+}R^{TM}_{0-}} E_{ved} \quad , \tag{A-43}$$

$$B_{0+} = \frac{(1 + R^{TM}_{0-})}{1 - R^{TM}_{0+}R^{TM}_{0-}} E_{ved} \quad , \tag{A-44}$$

$$A_{0-} = \frac{(1 + R^{TM}_{0+})}{1 - R^{TM}_{0+}R^{TM}_{0-}} E_{ved} \quad , \text{ and} \tag{A-45}$$

$$B_{0-} = \frac{R^{TM}_{0-}(1 + R^{TM}_{0-})}{1 - R^{TM}_{0+}R^{TM}_{0-}} E_{ved} \quad . \tag{A-46}$$

(2) for horizontal electric dipole

$$A_{0+} = \frac{R^{TM}_{0+}(1 - R^{TM}_{0})}{1 - R^{TM}_{0+}R^{TM}_{0-}} E_{hed} \quad , \tag{A-47}$$

$$B_{0+} = \frac{(1 - R_{0-}^{TM})}{1 - R_{0+}^{TM} R_{0}^{TM}} E_{hed} \quad , \tag{A-48}$$

$$C_{0+} = \frac{R_{0+}^{TE}(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{hed} \quad , \tag{A-49}$$

$$D_{0+} = \frac{(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{hed} \quad , \tag{A-50}$$

$$A_{0-} = -\frac{(1 - R_{0+}^{TM})}{1 - R_{0+}^{TM} R_{0-}^{TM}} E_{hed} \quad , \tag{A-51}$$

$$B_{0-} = -\frac{R_{0-}^{TM}(1 - R_{0+}^{TM})}{1 - R_{0+}^{TM} R_{0-}^{TM}} E_{hed} \quad , \tag{A-52}$$

$$C_{0-} = \frac{(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{hed} \quad , \text{ and} \tag{A-53}$$

$$D_{0-} = \frac{R_{0-}^{TE}(1 + R_{0+}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{hed} \quad . \tag{A-54}$$

(3) for vertical magnetic dipole

$$C_{0+} = \frac{R_{0+}^{TE}(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{vmd} \quad , \tag{A-55}$$

$$D_{0+} = \frac{(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE} R_{0-}^{TE}} H_{vmd} \quad , \tag{A-56}$$

$$C_{0-} = \frac{(1 + R_{0-}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{vmd} \quad , \text{ and} \tag{A-57}$$

$$D_{0-} = \frac{R_{0-}^{TE}(1 + R_{0+}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{vmd} \quad . \tag{A-58}$$

(4) for horizontal magnetic dipole

$$A_{0+} = \frac{R_{0+}^{TM}(1 + R_{0-}^{TM})}{1 - R_{0+}^{TM}R_{0-}^{TM}} E_{hmd} \quad , \tag{A-59}$$

$$B_{0+} = \frac{(1 + R_{0-}^{TM})}{1 - R_{0+}^{TM}R_{0-}^{TM}} E_{hmd} \quad , \tag{A-60}$$

$$C_{0+} = \frac{R_{0+}^{TE}(1 - R_{0-}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{hmd} \quad , \tag{A-61}$$

$$D_{0+} = \frac{(1 - R_{0-}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{hmd} \quad , \tag{A-62}$$

$$A_{0-} = \frac{(1 + R_{0+}^{TM})}{1 - R_{0+}^{TM}R_{0-}^{TM}} E_{hmd} \quad , \tag{A-63}$$

$$B_{0-} = \frac{R_{0-}^{TM}(1 + R_{0+}^{TM})}{1 - R_{0+}^{TM}R_{0-}^{TM}} E_{hmd} \quad , \tag{A-64}$$

$$C_{0-} = - \frac{(1 - R_{0+}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{hmd} \quad , \text{ and} \tag{A-65}$$

$$D_{0-} = - \frac{R_{0-}^{TE}(1 - R_{0+}^{TE})}{1 - R_{0+}^{TE}R_{0-}^{TE}} H_{hmd} \quad . \tag{A-66}$$

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