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EFFECT OF PRESSURE ON THE STATIC AND DYNAMIC ACOUSTIC  
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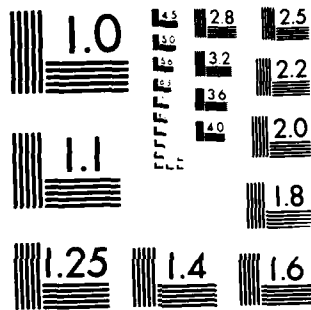
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**EFFECT OF PRESSURE ON THE STATIC AND  
DYNAMIC ACOUSTIC PROPERTIES  
OF POROUS RUBBER**

BY G. C. GAUNAURD, E. CALLEN,  
J. BARLOW

RESEARCH AND TECHNOLOGY DEPARTMENT

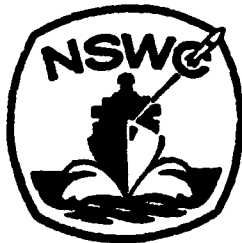
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 83-450	2. GOVT ACCESSION NO. AD-A138744	3. RECIPIENT'S CATALOG NUMBER 1
4. TITLE (and Subtitle) EFFECT OF PRESSURE ON THE STATIC AND DYNAMIC ACOUSTIC PROPERTIES OF POROUS RUBBER		5. TYPE OF REPORT & PERIOD COVERED FY83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) G. C. Gaunard, E. Callen, and J. Barlow		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (Code R04) White Oak Silver Spring, MD 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS N6600183WR00361 and 0003R01AA850
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Ocean Systems Center (Code 6351) San Diego, CA 92132 and Naval Surface Weapons Center (Code R04)		12. REPORT DATE 1 October 1983
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Ocean Systems Center (Code 6351) San Diego, CA 92132 and Naval Surface Weapons Center (Code R04) White Oak, Silver Spring, MD 20910		13. NUMBER OF PAGES 41
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Pressure Effects                      Acoustic Properties Composites                              Dynamic Response Porous Rubber		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report we analyze the effects of pressure on the acoustic properties of porous rubbers. We consider the static and dynamic bulk moduli, the velocities of dilatational and shear waves, the characteristic wave impedances, the fundamental frequency of the monopole resonance, and the frequency dependence of the effective sound speed and attenuation. A series of figures at the end of the report shows how the (static) acoustic properties vary with pressure, and how the dynamic properties of the composite are altered by added pressures as high as 400 psi.		

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S/N 0102-LF-014-6601

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FOREWORD

This report presents an analytical study of the quantitative effects that increased hydrostatic pressures have on underwater sound absorbers. The aerated rubber sheets commonly used as underwater sound absorbers have effective dynamic properties such as (effective) sound speed, attenuation, density, wave impedances, dilatational and shear speeds as well as dilatational, bulk, and shear moduli, whose frequency dependence behavior has been analyzed and modelled in the past at atmospheric pressures. The present study quantitatively describes the modifying effects that increased hydrostatic pressures have on those dynamic (and, of course, static) properties, modelled earlier at zero pressure.

The work was partly supported by the Analytical Modelling Branch of the Naval Ocean Systems Center, San Diego, CA, under the guidance of NAVSEA-63R, and partly by the Independent Research Board of NSWC.

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APPROVED FOR RELEASE  
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## SUMMARY

In this report we analyze the effects of pressure on the acoustic properties of porous rubbers. We consider the static and dynamic bulk moduli, the velocities of dilatational and shear waves, the characteristic wave impedances, the fundamental frequency of the monopole resonance, and the frequency dependence of the effective sound speed and attenuation. A series of figures at the end of the report shows how the static acoustic properties vary with pressure, and how the dynamic properties of the composite are altered by added pressures as high as 400 psi.

## I. CONCLUSIONS

Our major conclusions are:

i) Bulk moduli and compressional waves are far more sensitive to pressure than are shear moduli and shear waves. A pressure of 400 psi increases by 28% the bulk modulus of a rubber with 10% void fraction; the shear modulus is raised by but 4%. This pressure increases the velocity of a longitudinal wave by 11%, while shear wave velocity goes up by only 1%. An overpressure of 400 psi increases the characteristic impedance of the dilatational wave in the composite by 13%, which could reduce to that extent the effective coupling with a surrounding fluid medium.

ii) Laminated rubber sheets used for sound absorption are often bonded to a rigid backing and then exposed to pressure normal to the surface, such as by submersion. A question that arises is to what extent are pores in the rubber distorted in shape by the uniaxially applied force. We show that to a very good approximation holes retain their shape but are reduced in volume. This will not be true for voids within a hole radius of the surface, but should apply in the interior of the rubber sheet.

iii) Since spherical holes remain spherical but shrink in volume under pressure, the void fraction is reduced by pressure, the density increases slightly, and the radius of the pores decreases. Because the fundamental frequency of the monopole resonance is simply related to the hole radius (resonance occurs when the shear wave length is one half the pore circumference), a reduction in pore radius by pressure causes an increase in resonance frequency. At 400 psi the frequency for resonance increases by 8%. In this range, the effect is not large.

iv) Increased pressure, by changing the void fraction and the radius of the pores, alters the dynamical properties generally, but only slightly. We show how the real and imaginary parts of the effective dynamic bulk and dilatational moduli and the effective sound speed and attenuation depend on pressure by comparing the frequency dependence of these quantities in porous rubber with and without an overpressure of 400 psi.

## II. A. EFFECTIVE BULK MODULUS

E. Kerner<sup>1</sup> has shown that a composite made up of a matrix (labeled medium 1) with grains (labeled medium 0) has an effective bulk modulus given by

$$k_{\text{eff}} = \frac{\frac{k_0 \phi_0}{3k_0 + 4\mu_1} + \frac{k_1 \phi_1}{3k_1 + 4\mu_1}}{\frac{\phi_0}{3k_0 + 4\mu_1} + \frac{\phi_1}{3k_1 + 4\mu_1}} .$$

Here  $k_0$  and  $\mu_0$  ( $\equiv 0$ ) are the bulk and shear moduli of the grains and  $\phi_0$  the volume fraction of the grains;  $k_1$ ,  $\mu_1$ ,  $\phi_1$  are analogously defined for the matrix.

When the grains are voids,  $k_0 = 0$ , and it follows that

$$k_{\text{eff}} = \frac{\frac{k_1 \phi_1}{3k_1 + 4\mu_1}}{\frac{\phi_0}{4\mu_1} + \frac{\phi_1}{3k_1 + 4\mu_1}} = k_1 \frac{(1 - \phi_0)}{(1 + \frac{3k_1}{4\mu_1} \phi_0)} .$$

When the matrix is a material for which  $\lambda_1 \gg \mu_1$ , such as a rubber, then

$$k_1 \cong \lambda_1 + \frac{2}{3} \mu_1 \cong \lambda_1 , \text{ and } k_{\text{eff}} = \lambda_{\text{eff}} = \frac{4\mu_1 (1 - \phi_0)}{3\phi_0} . \quad (1)$$

This result is worthy of a pause. The bulk modulus of a rubber with holes, and its effective Lamé's "constant"  $\lambda_{\text{eff}}$ , derive from the shear modulus of the rubber and the concentration  $\phi_0$ . The effective bulk modulus is discontinuous at  $\phi_0 = 0$ ; as long as there is any volume fraction of holes, all the compression is at the expense

<sup>1</sup>Kerner, E. H., "The Elastic and Thermoelastic Properties of Composite Media," Proc. Phys. Soc., Vol. B 69, 1956, p. 808.

of the hole volume. But at  $\phi_0 = 0$  the material is pure, and the bulk modulus is the  $k_1$  of the rubber itself.

One may wonder why the bulk modulus should vary inversely as  $\phi_0$ . The way to understand this is to consider the compressibility, which is the reciprocal of the bulk modulus. The compressibility represents the response to an applied volume stress. Since it is only the voids that compress, one expects the compressibility to be directly proportional to the volume fraction of voids. Thus the bulk modulus goes as the reciprocal of  $\phi_0$ .

## II. B. EFFECTIVE SHEAR MODULUS

The effective shear modulus<sup>1</sup> shows none of the startling behavior of Equation (1). Squeezing on perforated rubber compresses the holes; shearing it merely bends the rubber. Again with  $k_0 = \mu_0 = 0$ ,

$$\mu_{eff} = \mu_1 \frac{1 - \phi_0}{1 + \frac{2}{3} \phi_0}, \quad (2)$$

which is proportional to the shear modulus of the rubber, and is well-behaved in the zero volume fraction case.

## III. PRESSURE DEPENDENCE; NUMERICAL ESTIMATES

### A. VOLUME FRACTIONS AND MODULI

Consider a porous rubber slab of area  $A$ , and thickness  $t(p)$ . In the geometry of interest one face of the slab is fixed to a rigid backing and the other face is exposed to the pressure-transmitting liquid medium. The area  $A$  is then independent of pressure. When pressure is applied normal to the free face, stresses are induced in the slab that cause all transverse strains to be zero. If the volume of rubber beneath  $A$  is designated  $v_1$ , and that of voids  $v_0$ , then since the rubber is incompressible,

$$v(p) = v_1 + v_0(p) = At(p) .$$

Thus, from Equation (1) and  $\phi_0(p) = v_0(p)/v(p)$  we find

$$k_{eff}(p) = \frac{4\mu_1/3}{\frac{At(p)}{v_1} - 1} ,$$

and from the definition of the bulk modulus,

$$k \equiv -v \frac{dp}{dv}, \text{ we have } dp = -k \frac{dv}{v}$$

Substituting in the above and integrating, one gets the result\* for the void volume fraction as a function of pressure:

$$\phi_o(p) = \phi_o(0) e^{-3p/4\mu_1} \quad (3)$$

The pressure-dependent bulk modulus is then

$$k_{\text{eff}}(p) = \frac{4\mu_1}{3} \left( \frac{1 - \phi_o(p)}{\phi_o(p)} \right) = k_{\text{eff}}(0) \frac{\phi_o(0)}{\phi_o(p)} \frac{(1 - \phi_o(p))}{(1 - \phi_o(0))}, \quad (4)$$

and the shear modulus becomes

$$\mu_{\text{eff}}(p) = \mu_1 \frac{1 - \phi_o(p)}{1 + \frac{2}{3} \phi_o(p)} = \mu_{\text{eff}}(0) \frac{1 - \phi_o(p)}{1 - \phi_o(0)} \frac{1 + \frac{2}{3} \phi_o(0)}{1 + \frac{2}{3} \phi_o(p)}, \quad (5)$$

with  $\phi_o(p)$  given by Equation (3).

#### B. ACOUSTIC PROPERTIES

$$\text{The effective density } \rho \equiv \frac{m_1}{v_1 + v_o(p)}$$

at pressure  $p$  becomes

$$\rho_{\text{eff}}(p) = \rho(0) \frac{1 - \phi_o(p)}{1 - \phi_o(0)}, \quad (6)$$

with  $\phi_o(p)$  again given by Equation (3).

Having the effective density and the effective moduli, one can find the effective velocities of dilatational and shear waves and the effective acoustic impedances in the composite. The velocity of a dilatational wave is

\*This result was first obtained by B. Hartmann of NSWC.

$$C_{d,eff} \equiv \left( \frac{\lambda_{eff} + 2\mu_{eff}}{\rho_{eff}} \right)^{1/2},$$

which as a function of pressure becomes

$$C_{d,eff}(p) = C_{d,eff}(0) \left[ \frac{(1 + \frac{5}{3} \phi_o(p)) (1 + \frac{2}{3} \phi_o(0)) \phi_o(0)}{(1 + \frac{5}{3} \phi_o(0)) (1 + \frac{2}{3} \phi_o(p)) \phi_o(p)} \right]^{1/2}. \quad (7)$$

As far as we know this result has not been experimentally confirmed.

The velocity of a shear wave,

$$C_{s,eff} = \left[ \frac{\mu_{eff}}{\rho_{eff}} \right]^{1/2},$$

eventually leads to the following pressure-dependent expression for the effective shear speed in the perforated sheet:

$$C_{s,eff}(p) = C_{s,eff}(0) \left[ \frac{1 + \frac{2}{3} \phi(0)}{1 + \frac{2}{3} \phi(p)} \right]^{1/2}. \quad (8)$$

Clearly, shear wave velocity is rather insensitive to pressure.

When one is concerned about matching media properties to improve acoustic transmission, the acoustic impedance must be considered. Pressure can alter these impedances. The characteristic acoustic impedances are

$$Z_{d,eff}(p) = \rho_{eff}(p) C_{d,eff}(p); \quad (9)$$

$$Z_{s,eff}(p) = \rho_{eff}(p) C_{s,eff}(p), \quad (10)$$

with  $\rho_{eff}(p)$ ,  $C_{d,eff}(p)$ ,  $C_{s,eff}(p)$  given above.

Under unusual circumstances such as very high pressure, or a high void volume fraction, the effect of pressure on the acoustic properties can be large. But this is the abnormal circumstance. A range of pressures of more usual interest is from 0 (above atmospheric) to, say,  $400 \text{ lb/in}^2 = 2.8 \times 10^7 \text{ dynes/cm}^2$ . This is the ambient

pressure at a depth of about 850 feet in the sea. For a representative rubber, ( $\mu \ll \lambda$ ) the bulk modulus  $\lambda = 2 \times 10^{10}$  dynes/cm<sup>2</sup> and shear modulus  $\mu = 10^8$  dynes/cm<sup>2</sup>. The quantity which occurs in Equation (3) is then at a maximum value when

$$\frac{3p}{4\mu_1} \cong 0.2 .$$

We note that since  $\exp(-x) \cong 1 - x$  ,

$$e^{-0.2} = .82 , \text{ while } [1-0.2] = 0.8 .$$

It is then justified to linearize the pressure dependence ,

$$\phi_o(p) \cong \phi_o(0) \left[ 1 - \frac{3p}{4\mu_1} \right] . \quad (11)$$

At a pressure (above atmospheric) of 400 lb/in<sup>2</sup> we then find:

$$\phi_o(400) = 0.8 \phi_o(0) ,$$

which is a 20% reduction in void volume fraction. With an assumed volume fraction of voids at atmospheric pressure of value,  $\phi_o(0) = 0.1$ , we find a bulk modulus value of

$$k_{eff}(400) = 1.28 k_{eff}(0) ,$$

a shear modulus value of

$$\mu_{eff}(400) = 1.04 \mu_{eff}(0) ,$$

a density  $\rho_{eff}(400) = 1.02 \rho_{eff}(0)$  , a dilatational wave velocity value of

$$C_{d,eff}(400) = 1.11 C_{d,eff}(0) ,$$

a shear wave velocity value of

$$C_{s,eff}(400) = 1.01 C_{s,eff}(0) ,$$

and characteristic dilatational and shear impedances of value

$$Z_{d,eff}(400) = 1.13 Z_{d,eff}(0) ,$$

$$Z_{s,eff}(400) = 1.03 Z_{s,eff}(0) .$$

Those properties depending on the effective bulk modulus are significantly increased by pressure; those depending on the effective shear modulus are not much changed.

Some special purpose perforated rubbers may have a void volume fraction as high as 50%; on these the effects of pressure are in most (but not all) instances greater, but not overwhelmingly so. For example, at a pressure of 400 lb/in<sup>2</sup> as before, but now with  $\phi_0(0) = .5$ , we find  $k_{eff}(400) = 1.5 k_{eff}(0)$ , rather than 1.28  $k_{eff}(0)$ ;  $C_{d,eff}(400) = 1.09 C_{d,eff}(0)$ , which is less than 1.11  $C_{d,eff}(0)$ ; and  $Z_{d,eff}(400) = 1.31 Z_{d,eff}(0)$ , rather than 1.13  $Z_{d,eff}(0)$ .

#### IV. CAVITY DISTORTION

When pressure is applied to one face of an effectively infinite slab with the other face fixed to a rigid backing, the only induced strain is normal to the slab surface along the direction of the applied pressure. Stresses are induced in the slab to nullify all transverse strains. The pressure and distortion are then not isotropic but uniaxial, and one is led to ask to what extent voids in the slab are distorted from their original shape. Since the (fundamental) monopole resonance to be considered in this report has been analyzed for spherical voids, and may be sensitive to their distortion, an effect of pressure could arguably be a frequency shift, or peak-width broadening, by flattening of the cavities. We show now that this flattening is in fact negligible.

One can solve exactly the problem of a single spherical void in a slab subject to uniaxial stress; the distorted void is of course ellipsoidal but the details are complicated.

Fortunately, it is not necessary for us to repeat that calculation. The important conclusion is easily obtained. Suppose the pressure  $-p$  is directed along the  $z$  axis. As the induced stress is directed oppositely to the applied pressure, it follows that  $\tau_{zz} = -p$ . Since we require  $\epsilon_{xx} = \epsilon_{yy} = 0$ , there are induced stresses in the matrix  $\tau_{xx} = \tau_{yy} = \lambda_1 \epsilon_{zz}$ . Since

$$\epsilon_{zz} = \frac{-p}{\lambda_1 + 2\mu_1} ,$$

it follows that

$$\tau_{xx} = \tau_{yy} = \frac{-\lambda_1 p}{\lambda_1 + 2\mu_1}.$$

Note that

$$\sum_i \tau_{ii} = (3\lambda_1 + 2\mu_1) \frac{(-p)}{\lambda_1 + 2\mu_1} = (3\lambda_1 + 2\mu_1) \sum_i \epsilon_{ii}$$

as must be.

The ratio of the anisotropic, distorting tension to the uniaxial tension is then

$$\frac{\tau_{zz} - \tau_{xx}}{\tau_{zz}} = \frac{-p \left[ 1 - \frac{\lambda_1}{\lambda_1 + 2\mu_1} \right]}{-p/(\lambda_1 + 2\mu_1)} = \frac{2\mu_1}{\lambda_1}.$$

Spherical bubbles are only distorted an amount

$$\frac{2\mu_1}{\lambda_1} \times \frac{3p}{4\mu_1} = \frac{3p}{2\lambda_1} \leq 10^{-3}$$

out of round, under a uniaxial pressure of 400 lb/in<sup>2</sup>. Exact analysis confirms this result.

#### V. DYNAMICAL RESPONSE

In the previous sections we calculated the effects of static pressure, and we derived the expressions for the appropriate static moduli. In 1958, Meyer, Brendel and Tamm<sup>2</sup> described the resonant mode of response of rubber containing a single spherical void to an oscillatory

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<sup>2</sup>Meyer, E., Brendel, K., and Tamm, K., "Pulsation Oscillations of Cavities in Rubber," J. Acoust. Soc. Am., Vol. 30, 1958, p. 1116.

dilatational wave, by means of the monopole resonance. When the shear wavelength of the incident wave is one half the circumference of the bubble, the cavity is excited in its lowest radial breathing (isotropic expansion and contraction) mode. Because, as we saw in Section II, the bulk modulus of a rubber ( $\lambda_1 \gg \mu_1$ ) composite depends on the shear modulus of the rubber, the resonant wavelength is that of a shear wave, although the incident wave may be dilatational. Exciting the monopole resonance, flexing the rubber, results in absorption of the sound through viscoelastic coupling, and an encompassing effective medium theory of the dynamic moduli and acoustic behavior of such composites has recently been given,<sup>3,4</sup> with<sup>4</sup> and without<sup>3</sup> losses in the matrix.

So the task before us is to insert the effects of pressure into the viscoelastic effective medium theory.<sup>4</sup> This will not be difficult to do, but before doing so we point out that there are two kinds of effects. Since the bubble resonance frequency depends on the circumference of the holes, pressure will contract the holes and increase the resonance frequency. A pressure of 400 psi induces an 8% increase in resonance frequency as we shall see. If the rubber contains a spread in hole sizes, as is almost unavoidable in practice (sometimes intentionally for broad-banding), an 8% upward shift of all absorption frequencies may not amount to much.

The second effect is on the density, and the void volume fraction. We have already discussed this effect when we considered the static response.

#### A. RESONANCE FREQUENCY SHIFT

The void volume fraction is

$$\phi_0(p) = \frac{v_0(p)}{v_0(p) + v_1}$$

With  $N$  voids per unit volume, each of radius  $a$ ,

$$\frac{4}{3} \pi N a^3 = \frac{v_1 \phi_0(p)}{1 - \phi_0(p)}$$

<sup>3</sup>Gaunard, G. C., and Uberall, H., "Resonance Theory of the Effective Properties of Perforated Solids," J. Acoust. Soc. Am., Vol. 71, 1982, p. 282.

<sup>4</sup>Gaunard, G. C., and Barlow, J., Dynamic Behavior of Particulate Viscoelastic Composites for Sound Absorption, NSWC TR 82-520, 1 Oct 1982. Also J. Acoust. Soc. Amer., Vol. 74, 1983, (to be published).

Using Equation (3), it follows that

$$a = \frac{a_0 e^{-p/4\mu_1} (1 - \phi_0(0))^{1/3}}{\left[1 - \phi_0(0) e^{-3p/4\mu_1}\right]^{1/3}} .$$

When the pressure is below 400 lb/in<sup>2</sup>, so that the linear approximation for the exponential, Equation (11), is valid, then

$$a \approx a_0 \left[1 - \frac{p}{4\mu_1 (1 - \phi_0)}\right] . \quad (12)$$

This does not assume  $\phi_0 \ll 1$ ; when the void density is also low, the term  $\phi_0$  may be dropped in the denominator of Equation (12).

Meyer, Brendel and Tamm<sup>2</sup> showed that the main bubble resonance occurs at

$$k_s a = 2 . \quad (13)$$

The resonance frequency is then

$$\begin{aligned} \omega(p) &= \frac{a_0}{a} \omega(0) , \\ &\approx \left[1 - \frac{p}{4\mu_1 (1 - \phi_0)}\right]^{-1} \omega(0) ; \end{aligned} \quad (14)$$

$$\omega(p) \approx \left[1 + \frac{p}{4\mu_1 (1 - \phi_0)}\right] \omega(0) . \quad (15)$$

At  $p = 400$  lb/in<sup>2</sup> and with  $\phi_0 = 0.1$ ,  $\omega(p) \approx 1.08 \omega(0)$ , which is an eight percent increase in resonance frequency above that at atmospheric pressure.

## B. SHIFTS IN DYNAMIC MODULI DUE TO PRESSURE

To arrive at the pressure dependent, static, effective bulk modulus given in Section III we integrated the defining equation for the bulk modulus. This means we found the pressure-dependent void fraction  $\phi(p)$ . One might be led to opine that integration of a dynamic effective bulk modulus would produce a dynamic void fraction  $\phi(p, \omega)$ . There are conceivable circumstances in which such an approach would be valid, but they are not the usual circumstances. What one wishes to describe is the following: A porous rubber coating is subjected to a large static pressure  $p$ , and then

it is "excited" with an oscillatory signal such as a sound wave of frequency  $\omega$  and of small amplitude (the  $\Delta p$  of the sound wave is infinitesimal compared to the applied static pressure  $p$ ). The modulus to be integrated is the static modulus. The pressure-dependent dynamic moduli we wish to study are found simply by insertion of  $\phi(p)$ , given in Equation (3), for  $\phi(0)$  in the dynamic moduli results. It would make sense to integrate  $k_{eff}(\omega)$  only if the sound signal of frequency  $\omega$  had an amplitude  $p$ . Perhaps a simple way to say this is that one first exposes the porous rubber sheet to a static pressure  $p$ , rather than atmospheric, and then one pings on it with a sound wave. Therefore all we need to do is substitute  $\phi_o(p)$  for  $\phi_o(0)$ , and  $a_o(p)$  for  $a_o(0)$  in the already derived<sup>4</sup> results. For example, reference (4) displays a graph of the frequency dependence of the bulk modulus of a (lossy) rubber with voids of radius  $a_o = .125$  cm, and void fraction  $\phi_o = .1$ . Suppose we wish to know the frequency dependence of the dynamic bulk modulus of this same rubber composite, but at an increased pressure of 400 lb/in<sup>2</sup>. At this overpressure the hole radius is compressed to  $a(p) = .116$  cm, and the void volume fraction shrinks to  $\phi_o(p) = .081$ . One then merely reruns the calculation with the pressure-modified values of  $\phi_o(p)$  and  $a(p)$ .

## VI. NUMERICAL RESULTS

The following set of figures displays the results of our calculations. We first show static responses. In Figure 1 we plot the effect of pressure on the bulk modulus, normalized to the bulk modulus at zero (atmospheric) overpressure. We show this for two rubbers, one with 5% voids; the other with 10% void volume fraction at atmospheric pressure. The graph is of

$$\frac{k_{eff}(p)}{k_{eff}(0)} = \frac{e^{3p/4\mu_1} [1 - \phi_o(0) e^{-3p/4\mu_1}]}{1 - \phi_o(0)}$$

One sees that the bulk modulus rises significantly with pressure only slightly more for the 10% void rubber than for the 5% material (26% more at  $p = 400$  lb/in<sup>2</sup> =  $2.8 \times 10^7$  dynes/cm<sup>2</sup>).

Figure 2 displays the normalized shear modulus vs pressure viz; it is a plot of

$$\frac{\mu_{eff}(p)}{\mu_{eff}(0)} = \frac{[1 - \phi_o(0) e^{-3p/4\mu_1}] [1 + \frac{2}{3} \phi_o(0)]}{[1 - \phi_o(0)] [1 + \frac{2}{3} \phi_o(0) e^{-3p/4\mu_1}]}$$

The shear modulus is almost insensitive to pressure, as discussed above. At  $p = 400$  lb/in<sup>2</sup> it rises by only 3.3%, for 10% voids and by only half as much (1.6%) in rubber with 5% concentration of voids.

Figure 3 shows the effect of pressure on effective density. The formula plotted is

$$\frac{\tilde{\rho}(p)}{\tilde{\rho}(0)} = \frac{1 - \phi_o(0) e^{-3p/4\mu_1}}{1 - \phi_o(0)}$$

A pressure of 400 lb/in<sup>2</sup> increases the density of the 10% composite by but 2.1%, and that of the 5% material by 1%.

Figure 4 shows the pressure dependence of the dilatational wave velocity. It is a plot of

$$\frac{\tilde{C}_d(p)}{\tilde{C}_d(0)} = \left[ \frac{\left(1 + \frac{5}{3} \phi_o(0) e^{-3p/4\mu_1}\right) \left(1 + \frac{2}{3} \phi_o(0) e^{3p/4\mu_1}\right)}{\left(1 + \frac{5}{3} \phi_o(0)\right) \left(1 + \frac{2}{3} \phi_o(0) e^{-3p/4\mu_1}\right)} \right]^{1/2}$$

The dilatational wave velocity rises by 10% at 400 psi in 10% void rubber; in rubber with 5% voids it rises by 11%. In this instance, the composite with lower hole fraction is (slightly) more pressure-sensitive.

Figure 5 displays the pressure dependence of the velocity of shear waves. It is a plot of

$$\frac{\tilde{C}_s(p)}{\tilde{C}_s(0)} = \left[ \frac{1 + \frac{2}{3} \phi_o(0)}{1 + \frac{2}{3} \phi_o(0) e^{-3p/4\mu_1}} \right]^{1/2}$$

Shear wave velocity is almost independent of pressure; for the 10% composite the velocity of shear waves rises by only 0.6% at 400 psi; with 5% voids the increase is only half as large, 0.3%.

Figure 6 shows the dilatational impedance as a function of pressure. The formula is

$$\frac{\tilde{Z}_d(p)}{\tilde{Z}_d(0)} = \frac{\tilde{\rho}(p)}{\tilde{\rho}(0)} \cdot \frac{\tilde{C}_d(p)}{\tilde{C}_d(0)}$$

where the two ratios on the right side are the ones shown in Figures 3 and 4. The wave impedance rises by 12% at 400 psi for 10% voids, and by essentially the same amount in material with 5% voids. This mismatch could increase the reflection from the surface and reduce the utility of the coating.

Figure 7 shows the shear wave impedance as a function of pressure. The formula is

$$\frac{\tilde{Z}_s(p)}{\tilde{Z}_s(0)} = \frac{\tilde{\rho}(p)}{\tilde{\rho}(0)} \cdot \frac{\tilde{C}_s(p)}{\tilde{C}_s(0)}$$

where the ratios in the right side are the ones displayed in Figures 3 and 5. Shear impedance rises by only 2.7% at 400 psi (10% voids); and by 1.3% in the 5% composite.

Figure 8a pertains to the reduction of void radius. The relevant formula is

$$\frac{a_o(p)}{a_o(0)} = \frac{e^{-p/4\mu_1} (1 - \phi_o(0))^{1/3}}{[1 - \phi_o(0) e^{-3p/4\mu_1}]^{1/3}}$$

The radius of holes in a 10% void rubber is reduced by 7.4% at 400 psi; in a 5% composite the reduction in radius is only slightly less at 7%. Thus far, we have dealt with static results and properties.

Figure 8b shows the resonant frequency as a function of pressure. The pertinent formula is

$$\frac{f_{res}(p)}{f_{res}(0)} = \frac{[1 - \phi_o(0) e^{-3p/4\mu_1}]^{1/3} e^{p/4\mu_1}}{[1 - \phi_o(0)]^{1/3}}$$

The frequency for resonance of rubber with 10% voids increases by 8% when the pressure is increased by 400 psi; for the 5% void composite, the resonance frequency increases by 7.5%. The expression plotted here is clearly the reciprocal of that displayed in Figure 8a, as it should be.

Figure 9a displays the frequency dependence of the real part of the dynamic bulk modulus at various pressures. The formula for  $\text{Re} \left( \frac{\tilde{k}_{eff}}{\rho_1 C_{d1}^2} \right)$  is given in Reference 4.

One curve in the figure shows the response at atmospheric pressure of a material with 10% void-fraction, the holes being of radius  $a_o(0) = .125$  cm. When the pressure is increased by 400 psi the void fraction is reduced to  $\phi_o(p) = .081$ , and the hole radius falls to  $a_o(p) = .116$  cm. The second curve in Figure 9a shows the frequency response of the same material at this pressure, again obtained by the same formula in Reference 4.

Figure 9b displays the frequency dependence of the imaginary part of the dynamic bulk modulus at various pressures. The formula for  $\text{Im} \left( \frac{\tilde{k}_{e2}}{\rho_1 c_{d1}^2} \right)$  is given in Reference 4.

The zero pressure (atmospheric) curve is for a material with 10% void fraction, with holes of radius 0.125 cm. The 400 psi curve is found by reducing the void fraction to  $\phi_0 = .081$ , and the hole radius to .116 cm. The other (viscoelastic) parameters are unchanged.

Figure 10a shows the frequency dependence of the effective sound velocity for various pressures. The formula for the frequency-dependent  $\frac{\tilde{c}_2}{c_{d1}}$  is taken from Reference 4.

The  $p = 0$  curve is again for  $\phi_0(0) = 0.1$ ;  $a_0(0) = 0.125$  cm. The  $p = 400$  psi curve shows the frequency response when a pressure of 400 psi reduces the void fraction to .081 and the hole radius to .116 cm. The viscoelastic loss coefficients  $\beta_{d1} = \beta_{s1} = .05$  are (assumed) unaffected by pressure.

Figure 10b shows the frequency dependence of the effective sound attenuation at various pressures. The formula for the (non-dimensional) effective attenuation  $\alpha_2 a_0$  is given in Reference 4.

Again the  $p = 0$  curve assumes  $\phi_0(0) = .1$ ;  $a_0(0) = .125$ ;  $\beta_{d1} = \beta_{s1} = .05$ .

Under a pressure of 400 psi, one now has  $\phi_0(p) = .081$ ;  $a_0(p) = .116$  cm and these are the numerical values we have then substituted in the formulas of Reference 4 to generate this figure.

## VII. CONCLUDING REMARKS

We have presented a quantitative analytical study of the effects an increased hydrostatic pressure has on the effective material parameters of an aereated rubber composite. The dynamic behavior that was found earlier<sup>4</sup> at atmospheric pressures is now modified under increased pressures, and we have carefully indicated what the effect is on all the pertinent properties in the static and dynamic regimes. Numerous plots with quantitative results are displayed, and conclusions are derived, extracting the physical behavior from the derived analytical (model) predictions.

The static and dynamic results presented here for all the effective material properties analyzed were derived for cavities contained within boundless media. All the above results, however, are expected to hold for perforations within layers of finite thickness, provided that the cavity sizes are small compared to the layer thickness, and are sufficiently separated from the (overall) layer boundaries. Practically speaking, cavities smaller than about one half the layer thickness and

about one radius away (or more) from the layer boundaries will be quite satisfactory. The temperature variations of all the above properties at one given site, where the pressure could vary from zero to 400 psi, amounts to only a few degrees. If, as we saw, the pressure effect (at a single site) on the acoustic performance was relatively small, then the temperature effect is hardly noticeable. But with substantial temperature changes of 40°C or more (i.e., from the tropics to the arctic), the temperature effects may not be so small. This effect has not yet been investigated. This report is the second of a series, and the work continues.

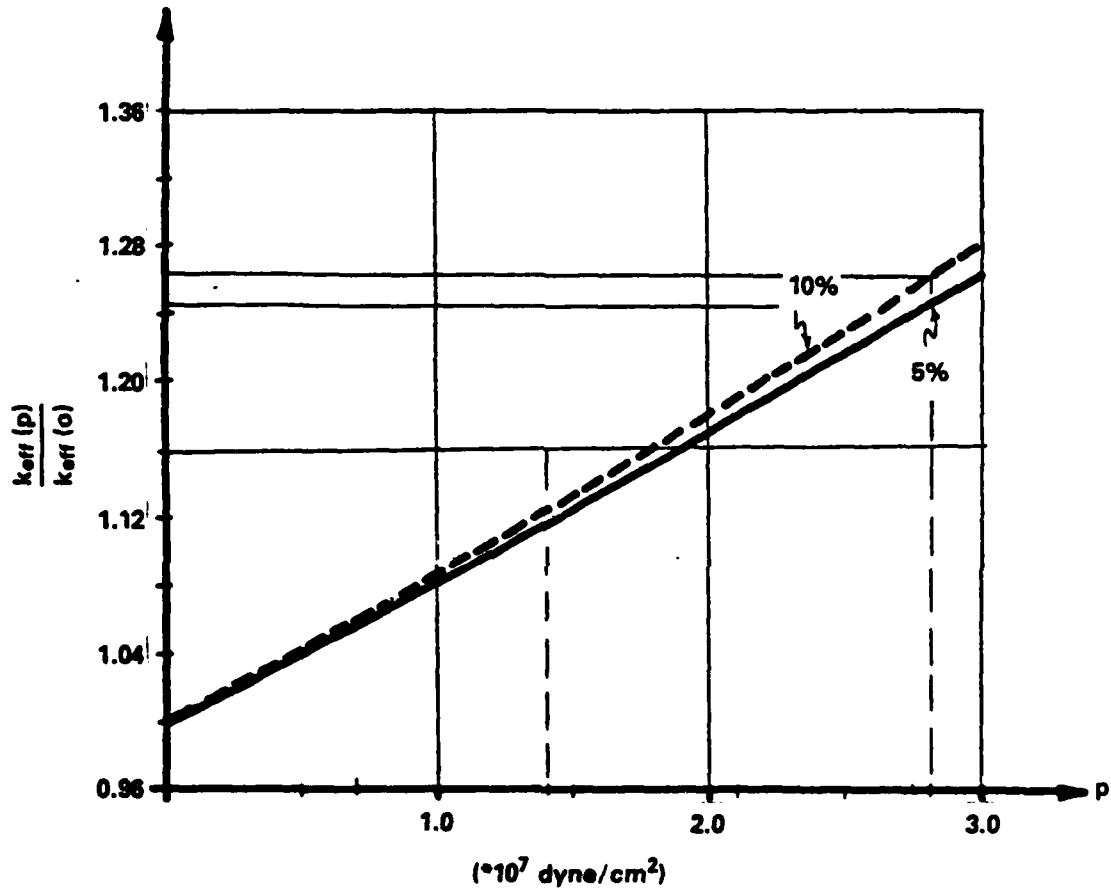


FIGURE 1. EFFECTIVE (STATIC AND NORMALIZED) BULK MODULUS OF A PERFORATED RUBBER SLAB AS A FUNCTION OF PRESSURE. (A) SOLID: RUBBER WITH 5% VOIDS. (B) DASHED: RUBBER WITH 10% VOIDS.

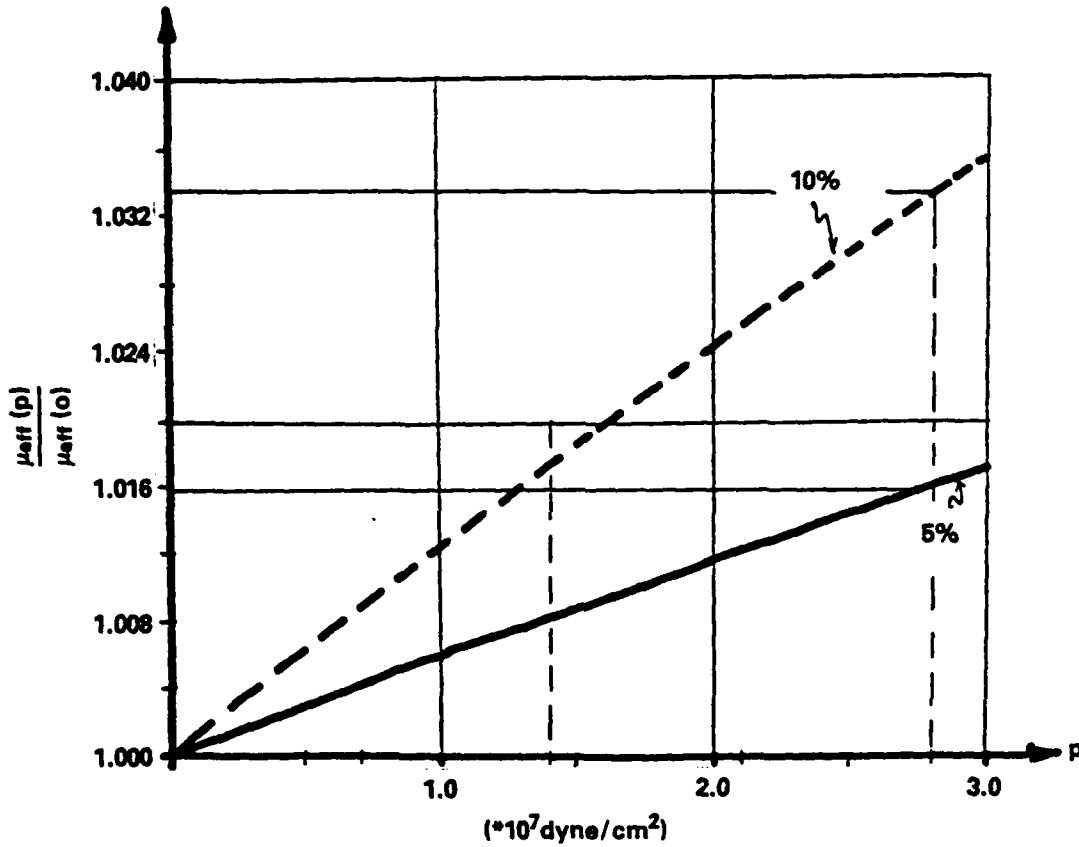


FIGURE 2. EFFECTIVE (STATIC AND NORMALIZED) SHEAR MODULUS OF A PERFORATED RUBBER SLAB AS A FUNCTION OF PRESSURE. (A) SOLID: 5% VOIDS IN THE RUBBER. (B) DASHED: 10% VOIDS IN THE RUBBER.

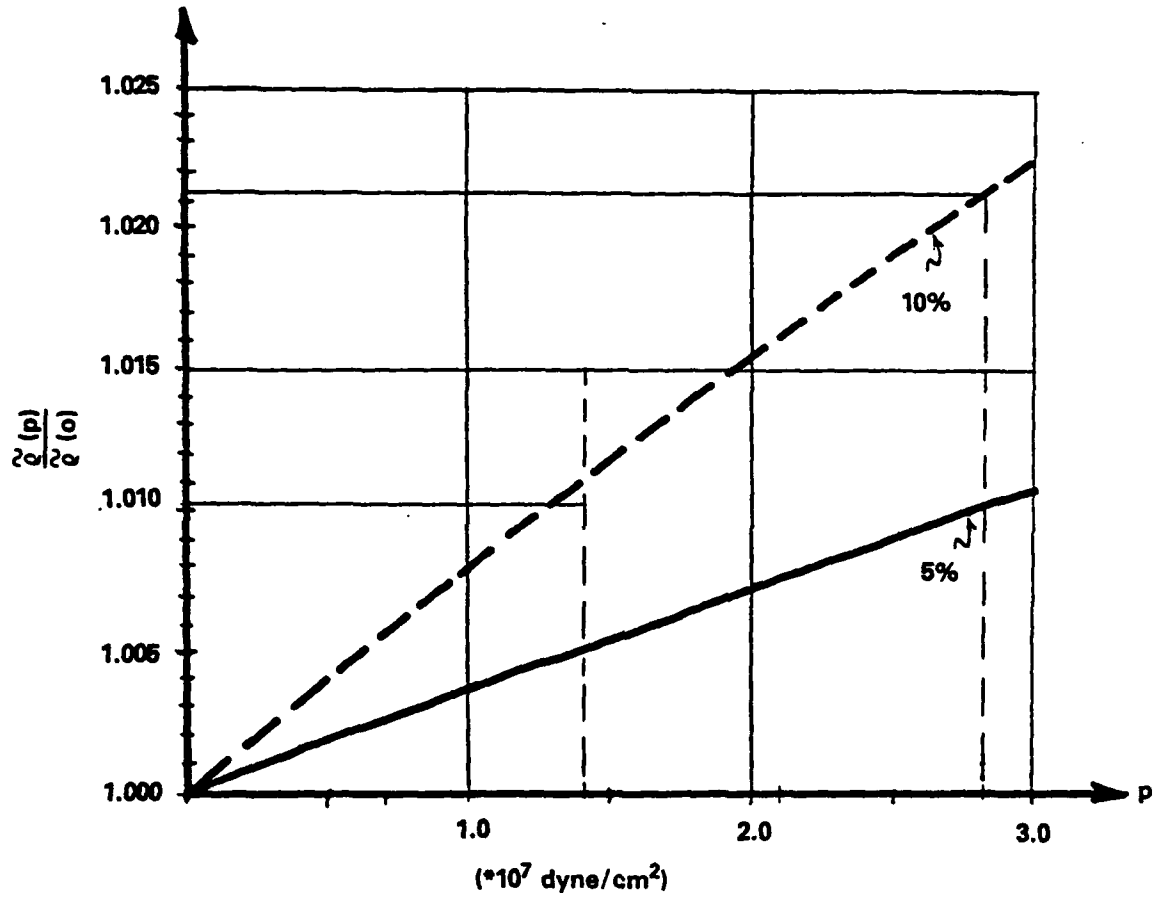


FIGURE 3. EFFECTIVE (STATIC AND NORMALIZED) DENSITY OF A PERFORATED RUBBER SLAB AS A FUNCTION OF PRESSURE. (A) SOLID: 5% VOIDS IN THE RUBBER. (B) DASHED: 10% VOIDS IN THE MATRIX.

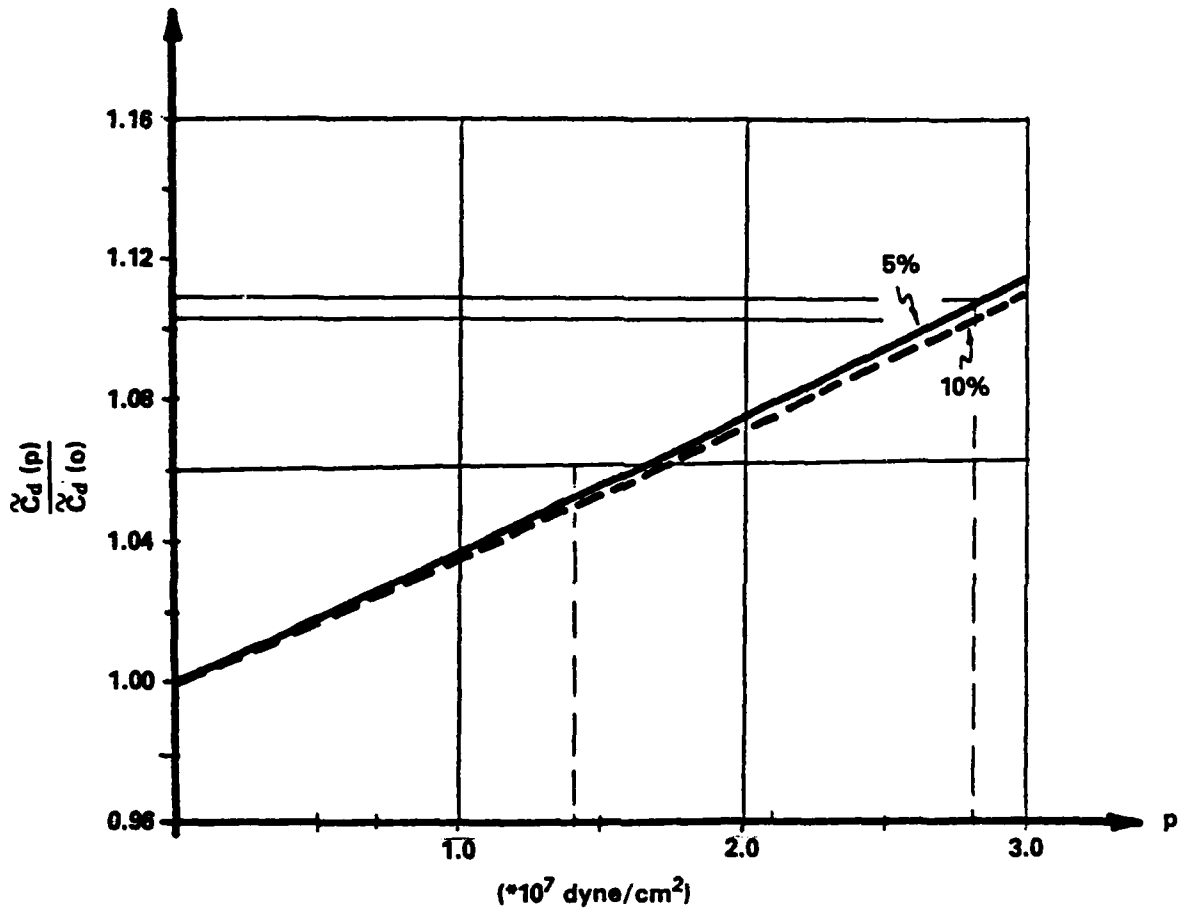


FIGURE 4. PRESSURE DEPENDENCE OF THE EFFECTIVE DILATATIONAL WAVE SPEED. (A) SOLID: 5% VOID CONCENTRATION. (B) DASHED: 10% VOID CONCENTRATION

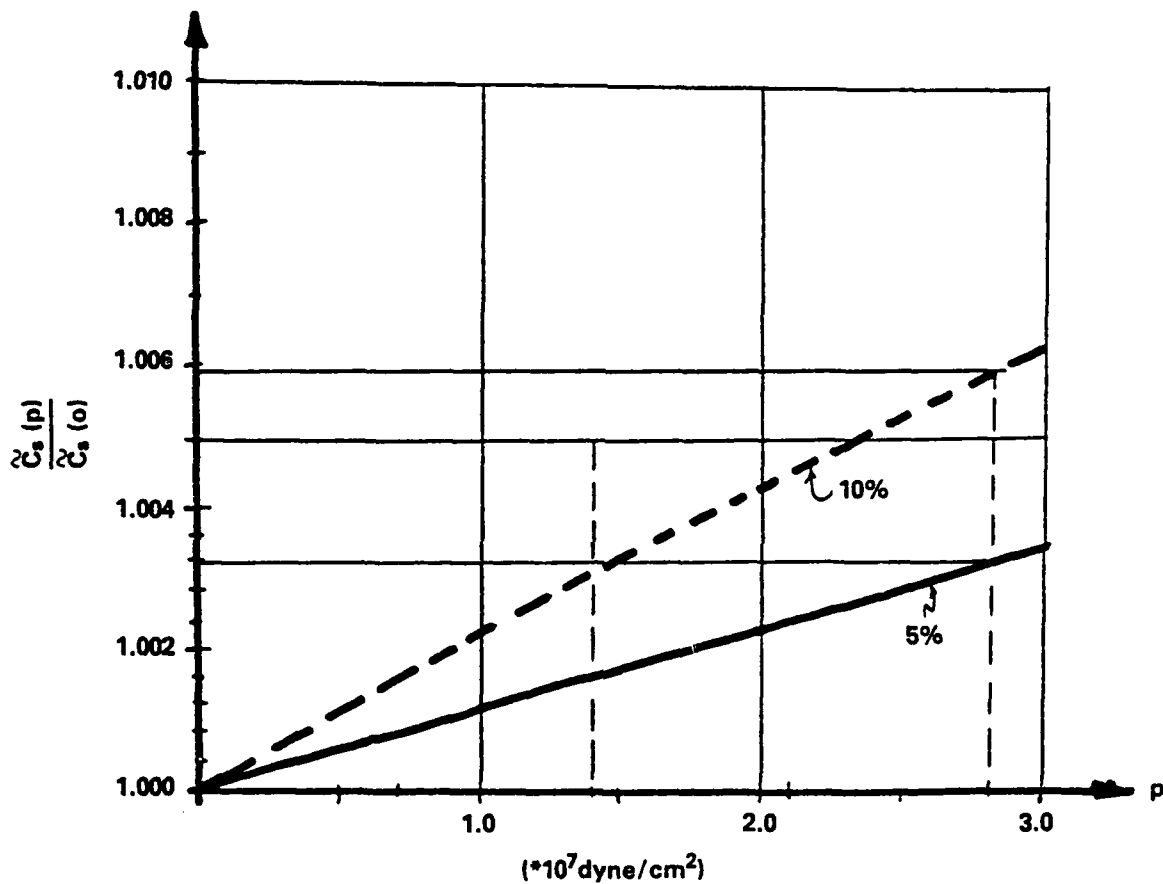


FIGURE 5. PRESSURE DEPENDENCE OF THE EFFECTIVE SHEAR WAVE SPEED. (A) SOLID: 5% VOID CONCENTRATION. (B) DASHED: 10% VOID CONCENTRATION.

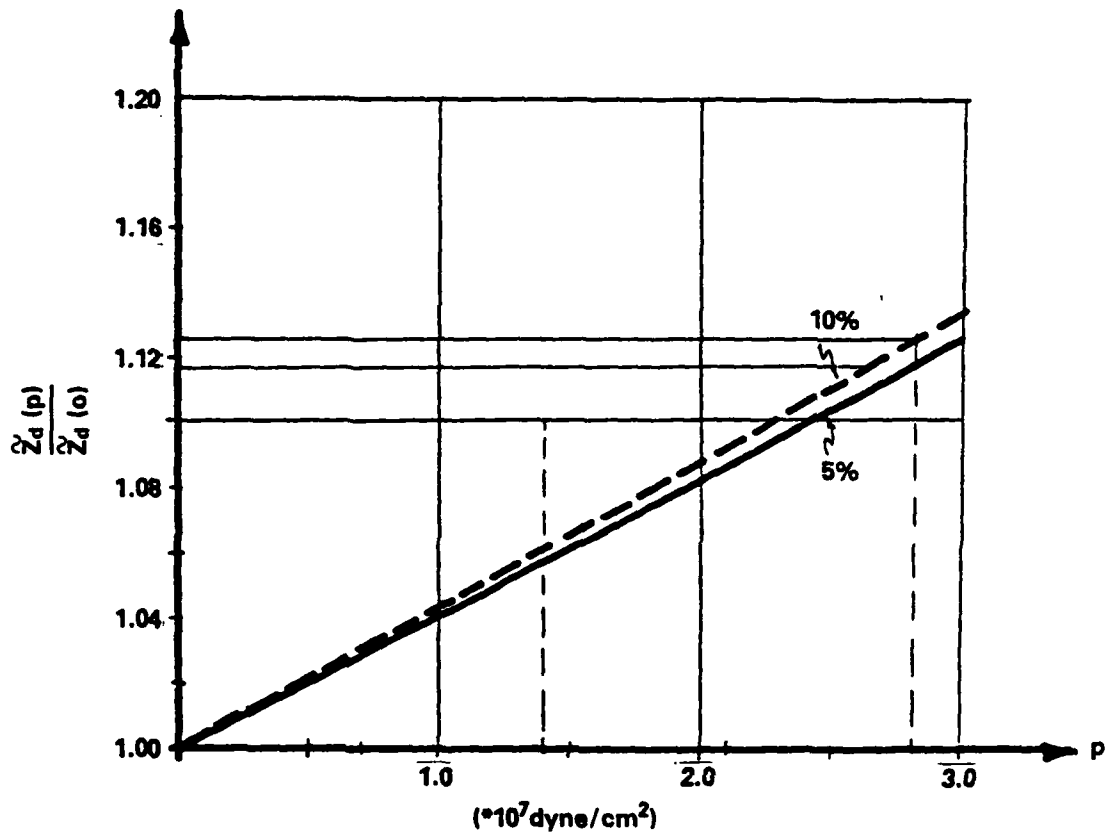


FIGURE 6. PRESSURE DEPENDENCE OF THE EFFECTIVE (STATIC) DILATATIONAL IMPEDANCE. RESULTS FOR 5% AND 10% VOID CONCENTRATIONS.

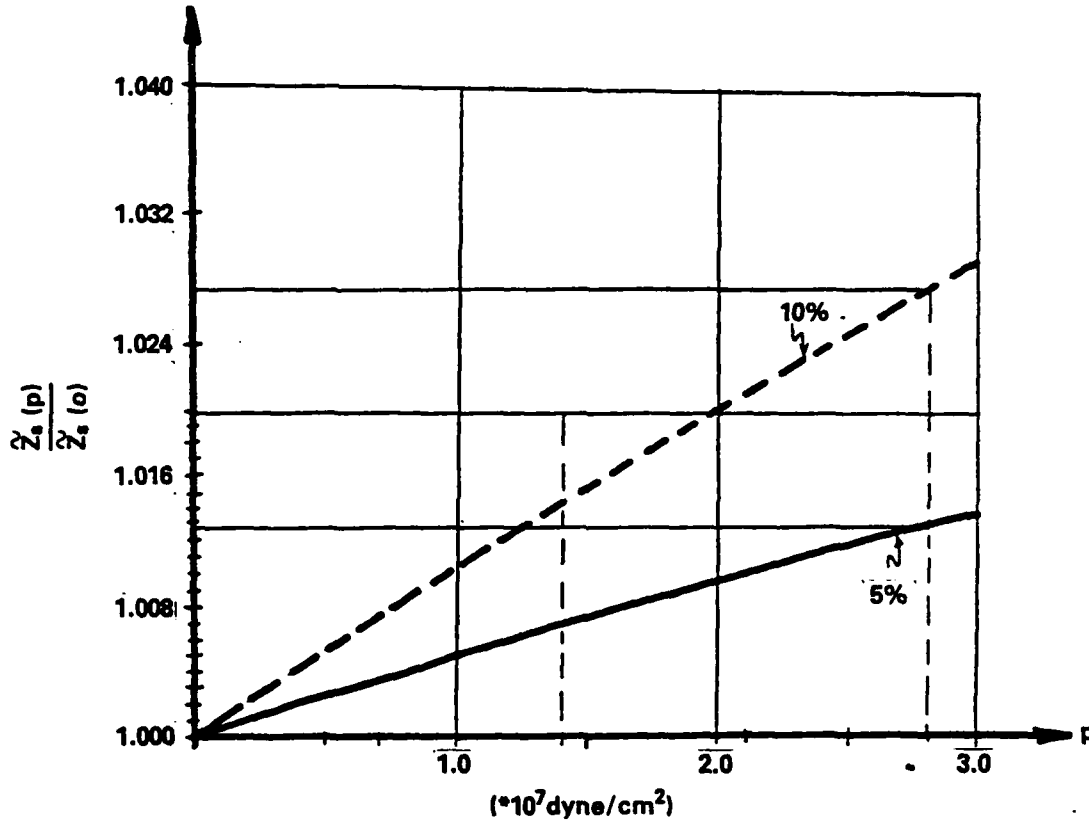


FIGURE 7. PRESSURE DEPENDENCE OF THE EFFECTIVE (STATIC) SHEAR WAVE IMPEDANCE. RESULTS FOR 5% AND 10% VOID CONCENTRATION.

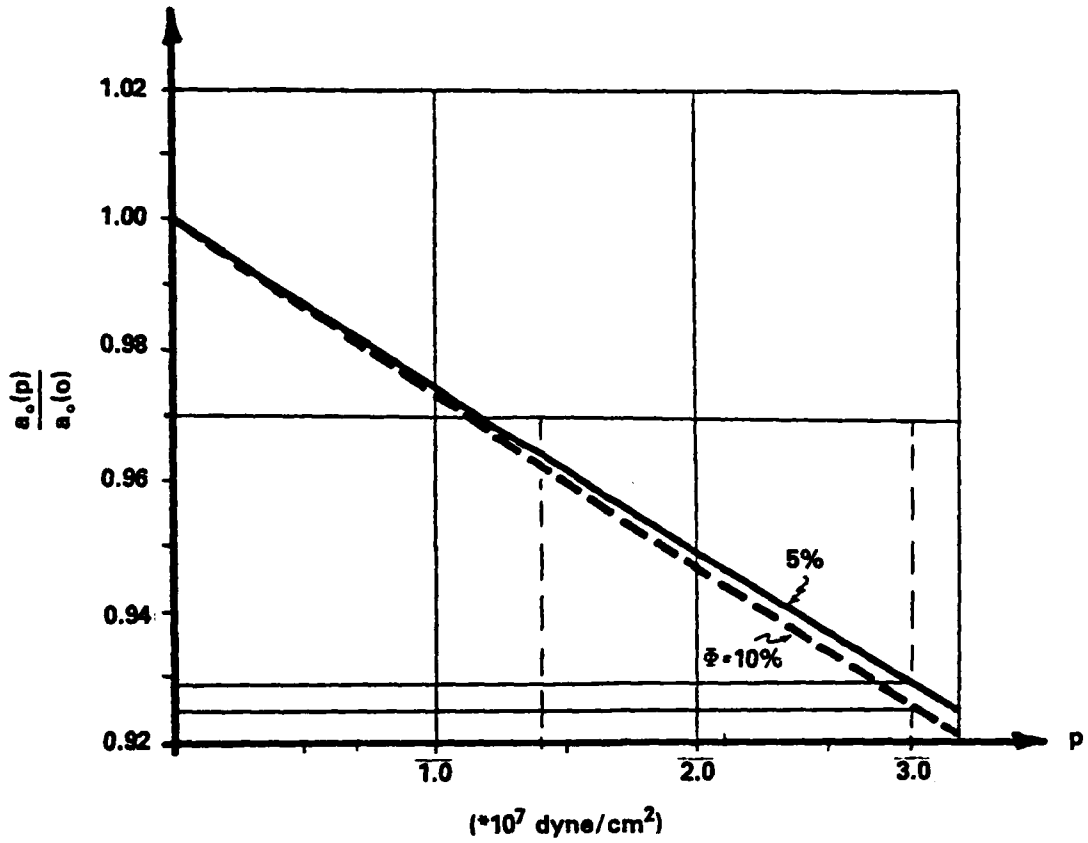


FIGURE 8a. PRESSURE DEPENDENCE OF THE CAVITY RADII. STATIC RESULTS AT 5% AND 10% VOID CONCENTRATIONS.

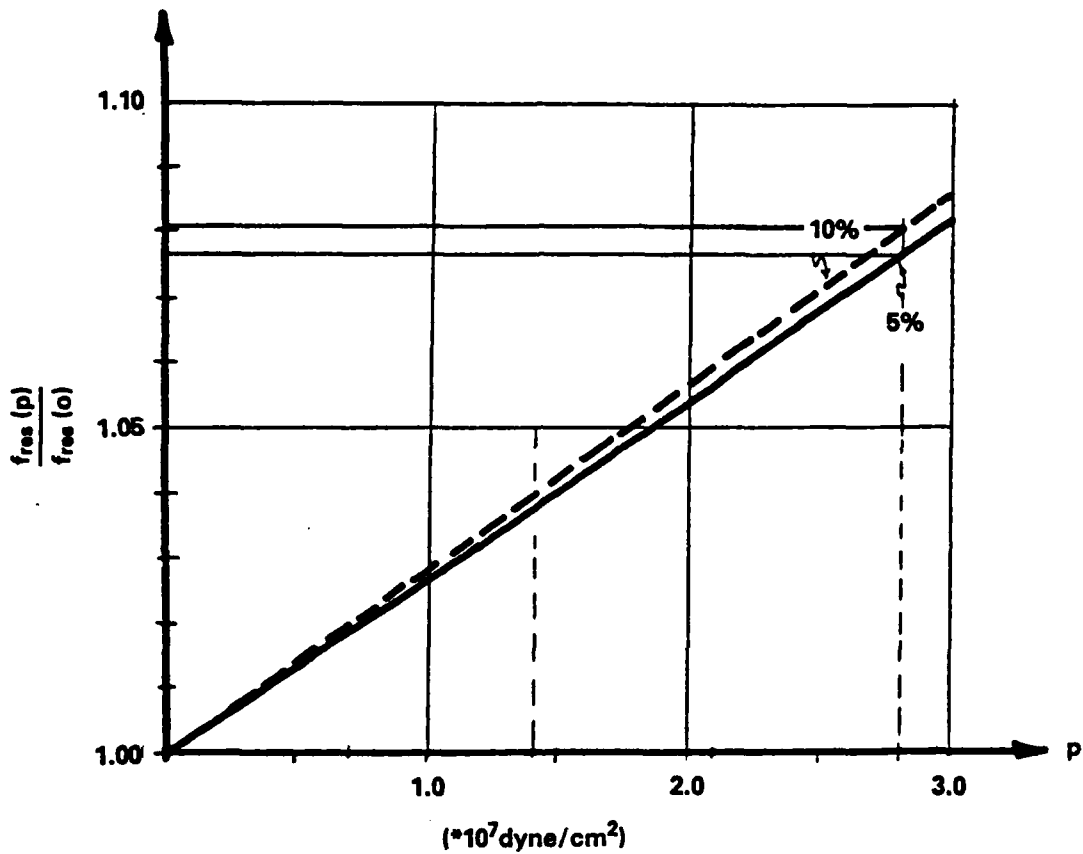


FIGURE 8b. PRESSURE DEPENDENCE OF THE (FUNDAMENTAL) MONOPOLE RESONANCE FREQUENCY. RESULTS AT 5% AND 10% VOID CONCENTRATIONS, AND  $\beta_{s1}$  AND  $\beta_{d1} = 5\%$ .

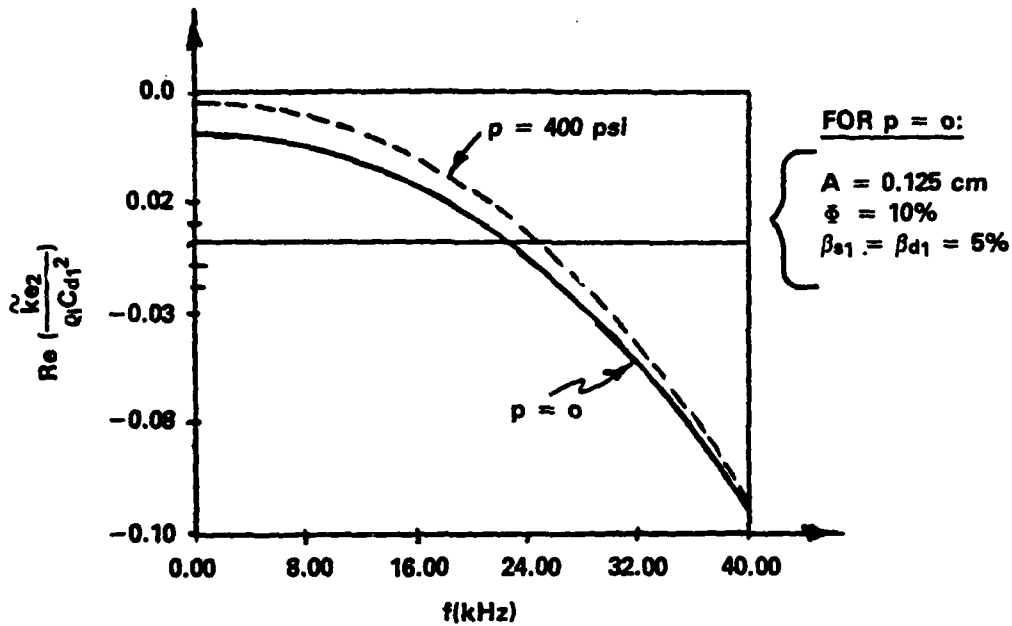


FIGURE 9a. REAL PART OF THE EFFECTIVE DYNAMIC BULK MODULUS (WHICH ALMOST COINCIDES WITH THE EFFECTIVE DYNAMIC DILATATIONAL MODULUS) AS A FUNCTION OF FREQUENCY FOR VARIOUS PRESSURES UP TO 400 psi (10% VOIDS IN ALL CASES AND  $\beta_{s1} = \beta_{d1} = 5\%$ ).

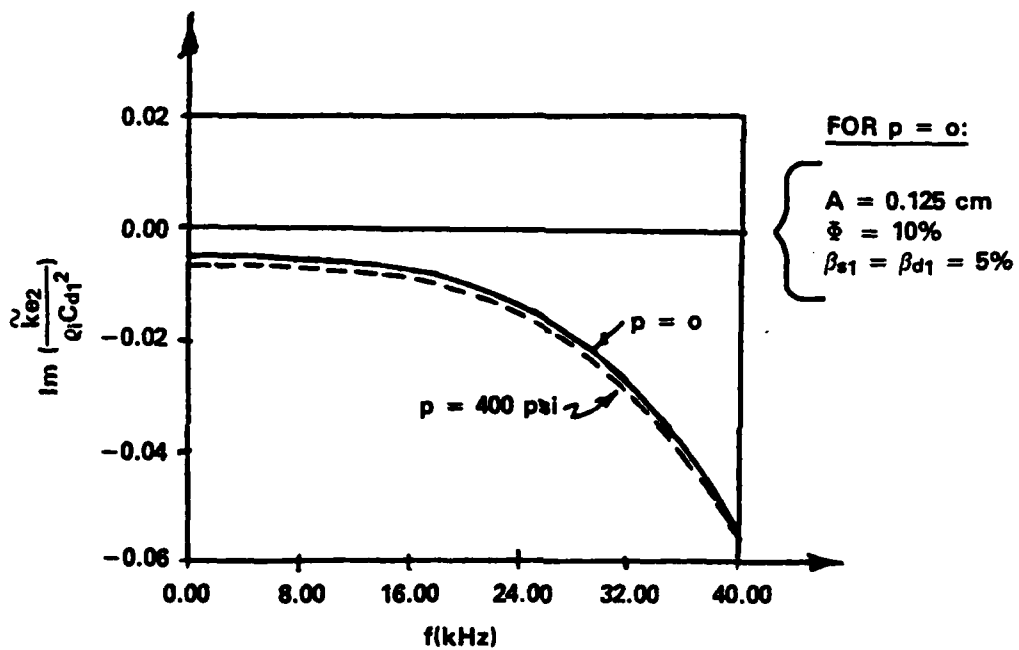


FIGURE 9b. IMAGINARY PART OF THE EFFECTIVE DYNAMIC BULK MODULUS (WHICH COINCIDES WITH THE EFFECTIVE DYNAMIC DILATATIONAL MODULUS) AS A FUNCTION OF FREQUENCY FOR VARIOUS PRESSURES (0 AND 400 psi). HERE, THERE ARE 10% VOIDS, AND  $\beta_{s1} = \beta_{d1} = 5\%$ .

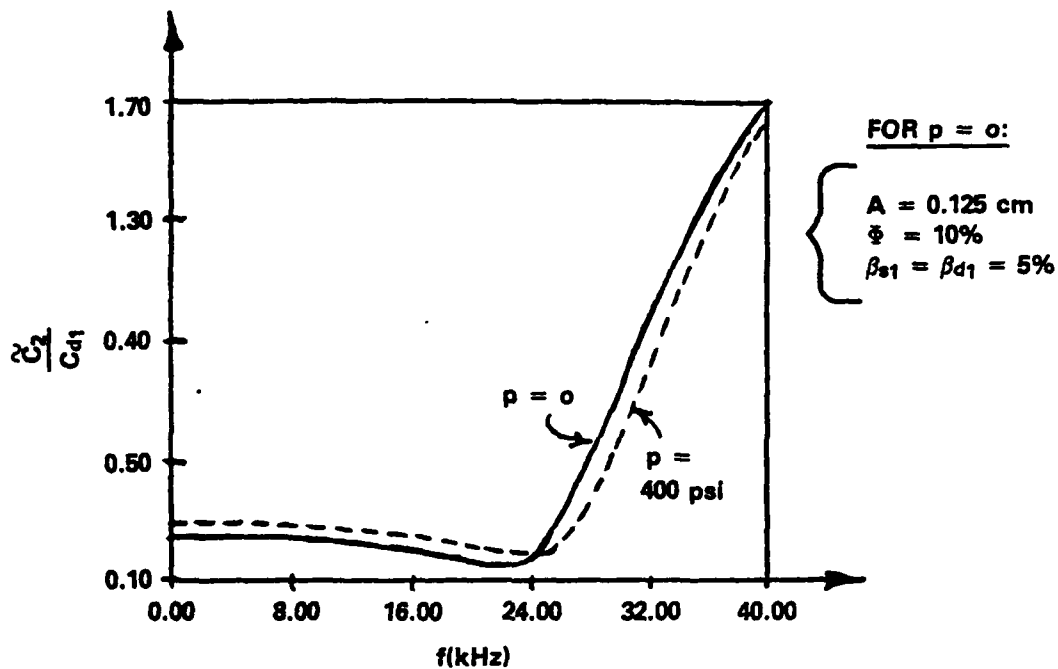


FIGURE 10a. FREQUENCY DEPENDENCE OF THE EFFECTIVE SOUND SPEED AT VARIOUS PRESSURES (i.e.,  $p = 0$  and 400 psi). HERE THERE ARE 10% VOIDS AT 0 PRESSURE AND  $\beta_{s1} = \beta_{d1} = 5\%$ .

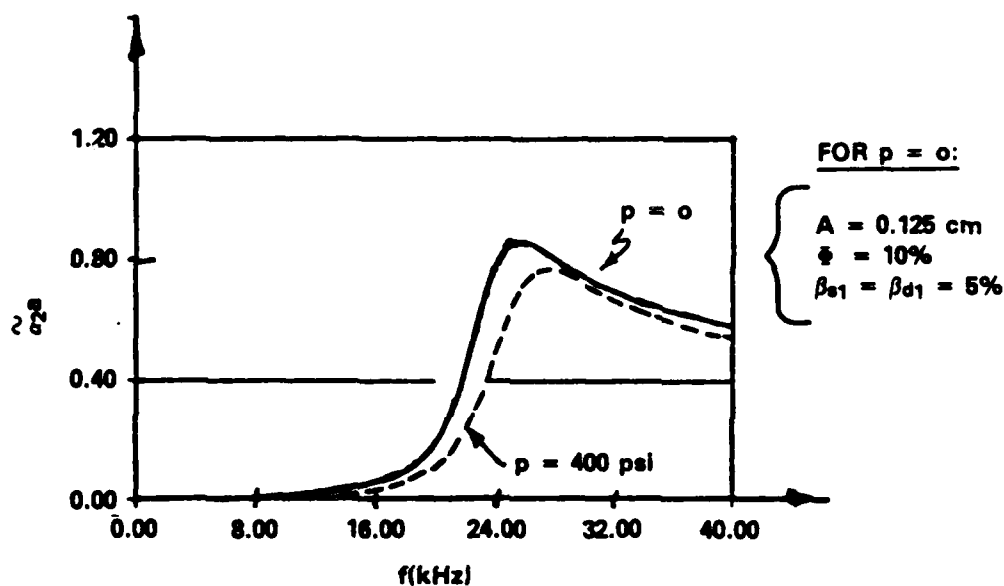


FIGURE 10b. FREQUENCY DEPENDENCE OF THE EFFECTIVE (DYNAMIC) SOUND ATTENUATION IN THE PERFORATED COMPOSITE AT VARIOUS PRESSURES (i.e.,  $p = 0$  and  $400 \text{ psi}$ ). HERE THE VOID CONCENTRATION AT ZERO PRESSURE IS 10%, AND THE VALUES OF THE VISCOUS PARAMETERS ARE  $\beta_{s1} = \beta_{d1} = 5\%$ .

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