

AD A138 753

ACOUSTIC-TO-SEISMIC COUPLING OVER POROUS GROUND
SURFACES(U) OPEN UNIV MILTON KEYNES (ENGLAND) FACULTY
OF TECHNOLOGY K ATTENBOROUGH ET AL. JAN 84

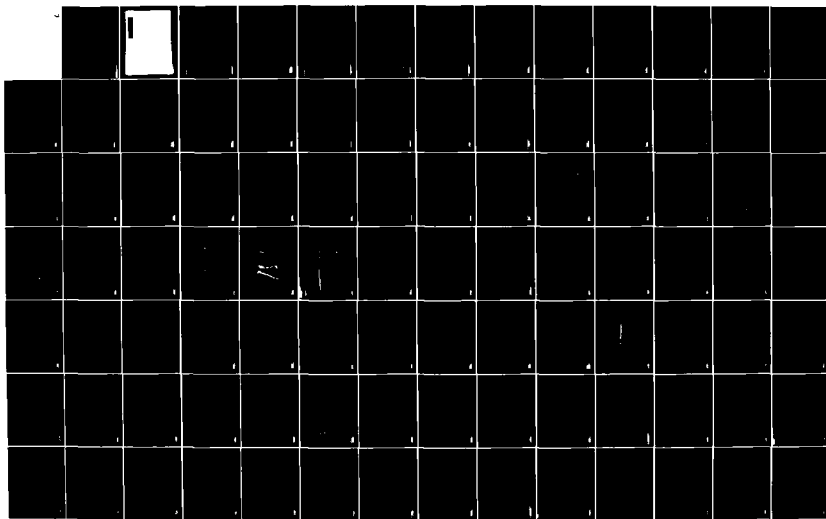
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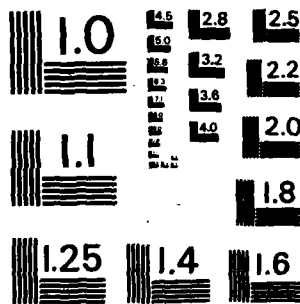
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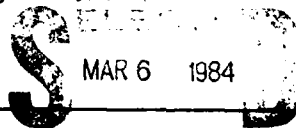


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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
	AD-A138753	
4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED	
ACOUSTIC-TO-SEISMIC COUPLING OVER POROUS GROUND SURFACES	FINAL TECHNICAL REPORT Jun 81 - Oct 83	
	6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)	
KEITH ATTENBOROUGH, NICHOLAS W. HEAP, TREVOR L. RICHARDS, V.V.S.S. SASTRY	DAJA37-81-C-0210	
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
FACULTY OF TECHNOLOGY, OPEN UNIVERSITY WALTON HALL, MILTON KEYNES, MK7 6AA, ENGLAND	61102A-IT161102-BH57-01	
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE	
USARDSG-UK BOX 65, FPO NEW YORK, NY 09510	OCTOBER 1983	
	13. NUMBER OF PAGES	
	191	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report)	
	UNCLASSIFIED	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)		
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
 A		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Acoustic/ seismic/ coupling/ wave propagation/ spherical waves/ plane waves/ acoustic surface wave/ rigid porous solid/ poro-elastic solid/ semi-infinite layer/ rigidly-backed layer/ dilatational waves/ shear wave/ model experiments/ fibreglass data/ air-jet source/ spark source/ air and water-saturated soils/ probe microphone data/ intensity ratio/ branch cut integrals/light fluid limit		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
1.1 Use of seismic sensors in battle-field surveillance is frequently made difficult by buried geophone response to above ground acoustic disturbances. The response to such acoustic-to-seismic coupling frequently generates signals in the same frequency range and of the same amplitude as those purely seismic signals that are of interest in surveillance. This report is concerned with the theoretical understanding of the phenomenon of acoustic seismic coupling.		

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- 1.2 In the work reported here, the penetration of sound into the ground is predicted for both spherical and plane acoustic waves incident upon two models of the ground viz (i) a rigid porous solid half-space and (ii) a poro-elastic granular half-space. In addition the plane wave analysis is extended to incidence on a rigidly-backed finite rigid-porous layer
- 1.3 In the frequency range of interest (1 - 1000 Hz) the rigid porous model of the ground is found to be sufficiently locally-reacting to predict identical results for spherical and plane wave incidence. The predictions are found to correspond to buried microphone measurements in sand and soils of above-ground acoustic disturbances. Furthermore it is found possible to predict the results of model measurements using continuous and pulsed acoustic sources and microphone receivers above and below the surface of fibreglass blanket.
- 1.4 The plane-wave analysis for a poro-elastic half-space predicts the existence in general, of three body waves i.e. two compressional waves which we call "fast" and "slow" borrowing from the literature on propagation in water saturated sediments and one shear wave. The solid particle motion, in general is the result of the contributions from all three wave types. At normal incidence the contribution is mainly from the fast-wave type travelling predominantly in the solid grains. The pore-fluid particle motion has a primary contribution from the slow wave type and for dry soils is practically identical to that predicted by the rigid porous model of the ground.
- 1.5 The predictions of the poro-elastic models for normally-incident plane-wave incidence are found to explain qualitative features of plots of intensity ratios reported in the literature and calculated from simultaneous measurements with microphones and geophones collocated ie. vertically separated above and below porous ground surfaces.
- 1.6 The analysis for spherical wave incidence on a poro-elastic half-space predicts that the surface motion potentially results from five wave contributions viz. the incident wave and four coupled waves in the poro-elastic medium. The coupled waves consist of two compressional waves, a shear wave and a Rayleigh-type wave.
- 1.7 Computations of the near- and far-field approximations of the surface motion, resulting from an above-ground acoustic point source, for a horizontal source-receiver separation of 100m and using a set of typical soil parameters, show that, for source heights of zero and 1M (grazing-incidence) and low-frequencies i.e. less than 100 Hz the Rayleigh-type wave is dominant. At higher frequencies for grazing-incidence or for all frequencies if the source is elevated to 50M, computations reveal that the direct-coupled plane-wave contribution is dominant. All contributions are predicted to decrease with increasing frequency which is in accordance with the trend exhibited by measured data.

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Accession For	
Microfilm	☒
Microfiche	☐
Microfilm and microfiche	☐
Classification	
Distribution/	
Availability Codes	
Date	
A-1	

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CHAPTER 2

$\kappa_{1,2}$	Propagation constants in medium 1 and 2
$\rho_{1,2}$	densities of media 1 and 2
$R_{1,2}$	distance from source and image to observer respectively
$M = \rho_1 / \rho_2$	ratio of densities
$n = \kappa_2 / \kappa_1$	ratio of propagation constants
K	complex variables of integration

CHAPTER 4

$\rho_c(\omega)$	complex density of fluid
$J_{0,1}(\cdot)$	cylindrical bessel functions of zeroth and first order
κ_c	pore parameter, Eq.(4.3)
ρ_0	density of fluid
ν	kinematic viscosity
N_{Pr}	Prandtl number
$C(\omega)$	frequency dependent complex compressibility Eq. (4.8)
n	oscillatory flow pore shape factor
q	tortuosity
Ω	volume porosity
μ	dynamic viscosity
s	steady flow pore shape factor
$Z/\rho c$	relative surface impedance
n'	grain shape factor
m	hydraulic radius, Eq.(4.23)
B	permeability, Eq.(4.24)

CHAPTER 5

$\phi_{1,2}$	scalar potential
$\psi_{1,2}$	vector potential
$\underline{u}$	skeletal frame displacement
$\underline{U}$	fluid displacement
$\underline{w}$	displacement of the fluid relative to the frame
H, C, M	elastic constants
ρ	average mass density
ρ_f	density of fluid
m	'effective fluid density' see paragraph following Eq. (5.2)
K_r	bulk modulus of the individual grains
K_f	bulk modulus of the pore fluid
G_b	shear modulus
K_b	bulk modulus of the assemblage of particles
η	dynamic viscosity of fluid
k	permeability of porous frame of dimension $[L]_2$
n^2	grain shape factor
$F(\lambda)$	complex viscosity correction factor Eq. (5.3)
ν	kinematic viscosity of fluid
γ	ratio of specific heats
P_o	atmospheric pressure
$P, Q, R,$	elastic parameters, Eq. (5.12)
N	shear modulus of the solid and composite
a'_s, a''_s	amplitudes of slow and fast waves
a^x_s	amplitude of shear wave
$u_{x,y}$	displacements in the solid
$U_{x,y}$	displacements in the fluid
e_{xx}	strain components in the solid
ϵ	fluid dilatation
σ_{ij}	strain tensor
δ_{ij}	Dirac's symbol
v_s	normal solid particle velocity

k', k'' and k^x	propagation constants for slow, fast and shear waves
k_0	wave number in air
p	pressure at the surface
θ', θ'' and θ^x	angles of refraction of slow, fast and shear waves
$\Gamma_{1,2}$	Eq. (5.26)
T', T'', T^x	relative amplitudes, Eq. (5.28)

Chapter 6

ψ	scalar potential
p	acoustic pressure
K_a	bulk modulus of air
u	displacement vector
u	displacement of fluid relative to the solid
U	absolute fluid displacement
ω	porosity
$\phi_{2,3}$	scalar potentials
$x_{1,2}$	vector potentials
E and μ	elastic moduli of the medium
M	effective bulk modulus of the fluid in the pores
ρ_1	density of fluid
ρ_2	complex density, Eq. (6.15)
ρ	average density of the porous medium
m	effective inertial density of the fluid in the pores
η	viscosity of fluid
k	permeability of the porous medium
α	a parameter which relates changes in pore volume to overall changes in volume
t_{ij}	
$k_{1,2,3,4}$	'effective stress' propagation constants Eqs. (6.14), (6.20)
$k'_{2,3}$	see Eqs. (6.33) and (6.34)

1 INTRODUCTION

- 1.1 Scientific interest in the phenomenon of acoustic-to-seismic coupling has spanned at least the last thirty-two years [1.1]. The motivations have included seismic exploration using sources in air, studies of sonic boom effects, and studies of the ground-induced motion due to rocket launches. Furthermore ground-air coupling is an important phenomenon in any attempt to monitor sub-surface conditions acoustically e.g. to detect buried objects using sources in air.
- 1.2 Systematic experimental studies of the near surface seismic signals induced by air-borne acoustic waves are relatively recent. Bass et al [1.2, 1.3] have used continuous signals from a loudspeaker source and impulsive signals directed along a line array of triaxial geophones and microphones and have reported results in terms of the squared ratio of surface seismic particle velocity to sound pressure immediately above the surface termed the intensity ratio. Similar measurements have been carried out by Powell [1.1] and, recently, by several delegates of NATO AC Panel III RSG II on mechanical Waves [1.4].
- 1.3 Typical results show that
- (i) the intensity ratio decreases with frequency overall but has several peaks some of which fall outside the "exciting" frequency band
 - (ii) the coupling is not very sensitive to "incident" angle of excitation
 - (iii) the time of first arrival response at the seismic sensor to an above-ground acoustic impulse corresponds to the speed of sound in air and is followed by a dispersive Rayleigh wave train.
 - (iv) the amplitude of seismic disturbance is unaffected by depth up to 1.5 m.
 - (v) the nature of the acoustically-induced seismic disturbance is strongly affected by the presence of snow.

1.4 Cress and Flohr (1.3) have attempted an analysis of the coupling using assumptions of

- (i) perfectly flat, homogeneous and infinitely deep ground.
- (ii) continuous vertical displacement across the boundary.
- (iii) continuous vertical stress across the boundary.
- (iv) zero horizontal stress components at the boundary, and
- (v) compressional wave velocity in soil = 344 ms^{-1} (i.e. and equal to that in air).

The boundary conditions are used to obtain a set of equations relating the reflected and transmitted displacement at the ground surface and hence the vertical component of velocity. It is assumed further that the acoustic pressure can be related to the particle velocity in air at the surface by the usual plane wave relation.

The predicted intensity ratio is found to be independent of frequency and very sensitive to angle of incidence. Both of these conclusions are at variance with measured results.

1.5 This report is concerned with the development of a theory for the phenomenon of acoustic-to-seismic coupling which is in accordance with observation. The starting point is the well-developed theory for propagation of sound from a point source above a locally-reacting boundary. Appendices 1 and 2 repeat and confirm this analysis. In Chapter 2 the analysis is extended to predict the field within the locally-reacting homogeneous medium. The prediction is found to depend crucially upon the acoustical characteristics of the locally-reacting medium. Chapter 3 describes "model" experiments carried out in a anechoic chamber to provide data for detailed comparison with theoretical predictions and Appendix 3 describes the basis for the models and the instrumentation. Consequently, in Chapter 4, a development of the Rayleigh model (1.5) for such a medium is found, in combination with the propagation (transmission) theory, to give predictions that agree both with the data

obtained in "model" measurements using fibreglass as the "ground" medium and with microphone probe measurements made outdoors in sand and soils.

- 1.6 These predictions do not agree, however, with the geophone data reviewed in 1.3. Thus, in Chapter 5 and Appendix 4, a semi-infinite, homogeneous poro-elastic model of the ground is adopted (1.6) and used to give predictions of the surface intensity ratio after a review of model parameters appropriate to sands and surficial soils.

Predictions are obtained for the surface intensity ratio due to acoustic plane waves normally incident on both a semi-infinite and a finite depth rigidly-backed poro-elastic layer and obliquely-incident on a semi-infinite poro-elastic medium. Finally, since the plane wave analysis will not be valid near grazing-incidence, the theory of spherical wave incidence on an air/poro-elastic boundary is derived in Chapter 6 and Appendix 5 and shows the possibility of five wave contributions at the interface. Near- and far-field approximations are used in Appendix 5 to show the relative importance of the various wave types in acoustic-to-seismic coupling. The predictions are qualitatively in agreement with features of the measured data that are available in the literature.

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2 TRANSMISSION OF A SPHERICAL WAVE AT A RIGID POROUS INTERFACE

2.1 Introduction

The sound field due to a point source above a plane interface separating two semi-infinite fluid media is a problem of great interest to those concerned with the propagation of sound over different types of ground surface, where the soil can be approximated by a rigid, porous absorbing medium [2.1]. Considerable effort has been made to evaluate the field in the upper medium (air) in the short-wave, asymptotic limit. A geometric approximation is adequate for sufficiently large elevation of the source and/or receiver, but this breaks down at grazing incidence where small correcting terms play an increasingly dominant role. The grazing-incidence case, of course, is precisely that which applies in long-range propagation studies. One of these correcting terms has the form of a surface wave and in Appendix 2 we show that it corresponds to a physically-real surface wave.

The evaluation of the field in the lower medium has received comparatively little attention, but Paul ([2.2], [2.3]) and Brekhovskikh [2.4] discuss the problem in some detail. Our motivation for considering this case arises from a desire to compare the magnitude of sound waves penetrating into the ground with that of any accompanying seismic disturbance.

We start in a similar manner to Paul and Brekhovskikh, that is by expressing the solution as an inverse Hankel transform. The resulting integral is then evaluated asymptotically using two steepest descent contours which pass around the two branch points of the integrand. The presence of a pole, which affects the solution at grazing incidence over ground having a high impedance, is also taken into account. We only consider cases for which the refractive index has a magnitude greater than unity.

In his first paper [2.2] Paul integrates around two vertical (in the plane of complex wavenumber) paths originating at the branch points, as opposed to the true saddle paths that we use. This initial simplification still results in complicated algebraic expressions and furthermore no ready physical interpretation is possible in terms of geometric rays. He deals with the pole by subtraction, and while this method appears to be essentially equivalent to ours, we feel that the final result is not expressed so simply.

Paul later considers alternative integration paths [2.3] which in part resemble more closely those used here. He does not consider the presence of the pole in this work.

Brekhovskikh [2.4] does not describe his derivation in detail, but it would appear that he uses the true saddle paths as he is able to identify geometrically the two wave types. It is not made clear, however, how to deal with a complex refractive index (i.e. an absorbing medium) except in an approximate form, and the problem of the pole is only mentioned briefly.

In a more recent paper by Candel and Crance [2.5] the related problem of a horizontal line source is discussed. Their stationary phase method can only give a single term which is the same as that obtained using geometrical acoustics, and is therefore not suitable at grazing incidence. They do not acknowledge the presence of a lateral wave, nor is the pole taken into account.

2.2 The integral representation for the field

Expressed as an inverse Hankel transform the velocity potential in the lower medium is given by [2.2]

$$\phi = V(z_1, z_2, r) = \frac{M}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-v_1 z_1 - v_2 z_2 + iKr}}{v_1 + Mv_2} \frac{1}{H_0(Kr)} e^{-iKr} K dK \quad (2.1)$$

where z_1, z_2 are the depth of source and height of receiver and r is their horizontal separation,

$$v_1 = (K^2 - k_1^2)^{\frac{1}{2}}, \quad v_2 = (K^2 - k_2^2)^{\frac{1}{2}}, \quad \text{Re } v_1, \text{Re } v_2 > 0,$$

k_1, k_2 are the wavenumber (complex in general) in the upper and lower media, and

$$M = \rho_1/\rho_2, \text{ the ratio of densities (again complex)}$$

Equation (2.1) can be used to obtain an expression for the field in the upper medium:

$$\phi = \frac{e^{ik_1 R_1}}{4\pi R_1} - \frac{e^{ik_1 R_2}}{4\pi R_2} + \frac{1}{M} V(H, 0, r) \quad (2.2)$$

where H is the sum of source and receiver heights and R_1, R_2 are the distances from the receiver to the source and to its image.

2.3 The steepest descent method with a pole near a saddle point

The rapid oscillation of the integrand in equation (2.1) for large source-receiver separations can be handled satisfactorily by using the saddle-point method.

In general we need to evaluate integrals of the form

$$I = \int_{-\infty}^{\infty} A(K) e^{f(K)} dK \quad (2.3)$$

in which the behaviour of $f(K)$ causes the integrand to oscillate rapidly. The saddle points occur where $f'(K) = 0$, and the steepest descent paths emanate from these in a direction in the complex plane in which the exponential term decays most rapidly. The imaginary part of $f(K)$ is constant along a steepest descent path and can be taken outside the integral, leaving a purely real exponential term which decays on either side of the saddle point. If $f'(K_s) = 0$, then

$f(K) = f(K_s) - S^2$ for real S gives the behaviour of the exponent along the steepest descent path. The integral along this path becomes

$$I_{SDP} = e^{f(K_s)} \int_{-\infty}^{\infty} A(K) \frac{dK}{dS} e^{-S^2} dS \quad (2.4)$$

If $A(K) \frac{dK}{dS}$ can be expanded in a power series in S with a large radius of convergence we can integrate the successive terms. With $A(K)dK/dS = \phi(S)$, this results in

$$I_{SDP} = \pi^{\frac{1}{2}} e^{f(K_S)} \left[\phi(0) + \frac{1}{4} \phi''(0) + \dots \right] \quad (2.5)$$

If one or more (but not necessarily all) of the steepest descent paths are added in such a way that the resulting contour can be deformed into the real axis without crossing any branch points, then the resulting integral, along with the contributions from any poles crossed, will be equal to the integral in equation (2.3)

If the radius of convergence of the series for $\phi(S)$ is limited by the presence of a pole equation (2.5) may be inapplicable. In this case we write

$$\phi(S) = \psi(S) / (S - S_p) \quad (2.6)$$

and use the series for $\psi(S)$, which has no pole, instead. We now obtain

$$I_{SDP} = e^{f(K_S)} \left[\begin{aligned} &\psi(0) I_0 \\ &+ \psi'(0) (S_p I_0 + \sqrt{\pi}) \\ &+ \frac{1}{2} \psi''(0) (S_p^2 I_0 + S_p \sqrt{\pi}) \\ &+ 1/6 \psi'''(0) (S_p^3 I_0 + S_p^2 \sqrt{\pi} + \frac{1}{2} \sqrt{\pi}) \\ &+ \dots \end{aligned} \right] \quad (2.7)$$

where
$$I_0 = \int_{-\infty}^{\infty} \frac{e^{-S^2}}{S - S_p} dS$$

Noting that $\psi(S_p) = \psi(0) + \psi'(0)S_p + \frac{1}{2}\psi''(0)S_p^2 + \dots$

and making use of equation (2.6) gives

$$I_{SDP} = \pi^{\frac{1}{2}} e^{f(K_s)} [\psi(S_p) (I_0 \sqrt{\pi} + \frac{1}{S_p} + \frac{1}{2S_p^3} + \dots) + \phi(0) + \frac{1}{4} \phi''(0) + \dots] \quad (2.8)$$

It can be seen that

$$I_{SDP} \rightarrow \pi^{\frac{1}{2}} e^{f(K_s)} [\psi'(0) + 1/12 \psi'''(0) + \dots]$$

as $S_p \rightarrow 0$.

The expression therefore remains finite as the pole approaches the saddle point.

We can write

$$I_0 = i\pi \{W(S_p) - 2e^{-S_p^2} H[-\text{Im}(S_p)]\}$$

where $W(S_p) = e^{-S_p^2} \text{erfc}(-iS_p)$ and $H[]$ is the Heaviside step function.

This second term is cancelled by the contribution from the pole residue if the pole lies between the SDP and the real axis, leaving the total contribution

$$I^* = \sqrt{\pi} e^{f(K_s)} [\psi(S_p) (i\sqrt{\pi} W(S_p) + \frac{1}{S_p} + \frac{1}{2S_p^3} + \dots) + \phi(0) + \frac{1}{4} \phi''(0) + \dots] \quad (2.9)$$

To investigate the behaviour of this expression for large values of S_p we use the asymptotic expansion

$$i\sqrt{\pi} w(S_p) = - \left[\frac{1}{S_p} + \frac{1}{2S_p^3} + \frac{3}{4S_p^5} + \dots \right] \\ + 2H[-\text{Im}(S_p)] e^{-S_p^2}$$

Substituting into equation (2.9) using only those terms shown gives

$$I^* = \sqrt{\pi} e^{f(K_s)} \left[\phi(0) + \frac{1}{4}\phi''(0) \right. \\ \left. + 2H[-\text{Im}(S_p)] e^{-S_p^2} \psi(S_p) \right] \\ + O\left(\frac{1}{S_p^5}\right) \quad (2.10)$$

As expected, this expression approaches the behaviour of equation (2.5) along with the contribution from the pole when it lies between the SDP and the real axis. In this application the latter term gives rise to the famous 'surface wave'.

Thus the SDP integral may be calculated using equation (2.5) and if the pole is near the saddle point we simply add the extra terms shown in equation (2.9). This straightforward procedure appears to be much less cumbersome than previous methods such as subtraction of the pole.

The subsequent means used to evaluate the expression in equation (2.5) are well-established and Brekhovskikh [2.4] gives a good account of these, and the appropriate formulae.

For the extra term in equation (2.3.9) we use

$$\psi(S_p) = \lim_{K \rightarrow K_p} (K - K_p) A(K), \quad (2.11)$$

where K_p is the position of the pole in the complex K plane.

Returning to equation (2.1) we have

$$f(K) = -v_1 z_1 - v_2 z_2 + iKr \quad (2.12)$$

$$\text{and} \quad A(K) = \frac{M}{4\pi} [H_0^+ (Kr) e^{-iKr}] K / (v_1 + Mv_2). \quad (2.13)$$

For grazing incidence, with $z_1, z_2 \ll r$, there are two saddle points, one near $K = k_1$ and another near $K = k_2$. The steepest descent paths through these points can be used to close the contour of integration without enclosing the branch points. The pole, given by $v_1 + Mv_2 = 0$, can closely approach the first of these saddle points at grazing incidence if $M \ll 1$, $k_2 > k_1$, and will be dealt with accordingly using equation (2.9)

At sufficiently large source heights or receiver depths closure of the path of integration can be effected by using only one of the two paths. We will not pursue this matter further, but both Paul [2.3] and Brekhovskikh [2.4] consider such cases.

2.4 The geometrical wave

The contribution from the saddle point near the $K = k_1$ branch point corresponds to the ray refracted according to Snell's law in the geometrical approximation.

We locate this saddle point using the iteration

$$\alpha_{i+1} = z_1 \beta / (x - z_2 \beta_1 / \gamma_1) \quad (2.14)$$

where $v_1 = -ik_1 \alpha$, $v_2 = -ik_1 \gamma$, $K = k_1 \beta$ with $\beta_1 = (1 - \alpha_1^2)^{1/2}$,

$$\gamma = (\alpha_1^2 + n^2 - 1)^{1/2}, \quad n = k_2/k_1.$$

For large angles of incidence (i.e. $z_1 < x$) $\alpha_1 = 0$ is a suitable starting value, and the calculations may be performed for complex k_1 and k_2 . If $z_1 = 0$ then $\alpha = 0$, but equation (2.5) now becomes

$$\begin{aligned} I_{SDP} = & \frac{1}{\pi} e^{ik_1(\alpha z_1 + \gamma z_2 + \beta x)} \\ & \times \left(\frac{-2}{f''} \right)^{1/2} \left[\Lambda \left(1 + \frac{1}{8} \frac{f^{IV}}{(f'')^2} - \frac{5}{24} \frac{(f''')^2}{(f'')^3} \right) \right. \\ & \left. + \frac{1}{2} \frac{\Lambda' f'''}{(f'')^2} - \frac{1}{2} \frac{\Lambda''}{f''} \right] \quad (2.15) \end{aligned}$$

where

$$f'' = \frac{-1}{k_1} \left[\frac{z_1}{\alpha^3} + \frac{z_2}{n\gamma^3} \right]$$

$$f''' = \frac{-3i\beta}{k_1^2} \left[\frac{z_1}{\alpha^5} + \frac{z_2}{n\gamma^5} \right]$$

$$f^{IV} = \frac{-3i(4\beta^2 + 1)}{k_1^3} \left[\frac{z_1}{\alpha^7} + \frac{z_2}{n\gamma^7} \right]$$

and

$$A = \frac{ik_1\beta}{4\pi} \left(\frac{2}{i\pi k_1\beta r} \right)^{\frac{1}{2}} \left(1 + \frac{1}{8ik_1\beta r} \right) / (\alpha + n\gamma).$$

The first two terms of the asymptotic expansion

$$H_0^1(z)e^{-iz} \sim \left(\frac{2}{i\pi z} \right)^{\frac{1}{2}} \left(1 + \frac{1}{8iz} + \dots \right)$$

have been used. This is acceptable if $|k_1\beta r| \gg 1$, that is when $k_1 r \gg 1$ and the angle of incidence is large.

There is little necessity in seeking further simplification of equation (2.15) once the appropriate differentiations of $A(k)$ have been carried out, except to ensure that the correct value is given at $\alpha = 0$. To do this we examine the limiting behaviour of the various terms as $\alpha \rightarrow 0$, and find that only the last two terms in the square brackets contribute.

2.5 The lateral wave

To find the second saddle point we take into account the fact that at grazing incidence it is close to the $K = k_1$ branch point, and consequently $|v_2|$ is small, and v_1 is predominantly real. We re-define α, β, γ and use the iteration

$$\gamma_{i+1} = z_1 \beta_i / (r + iz_1 \beta_i / \alpha_i) \quad (2.17)$$

where $v_1 = k_1 \alpha, v_2 = -ik_1 \gamma, K = k_1 \beta$

$$\text{and } \beta_1 = (1 - \gamma_1^2)^{\frac{1}{2}},$$

$$\alpha_1 = (1 - \frac{1}{n^2} - \gamma^2)^{\frac{1}{2}}, \gamma_0 = 0$$

This gives

$$f(K_s) = -k_1 \alpha z_1 + ik_1 (\gamma z_1 + \beta r).$$

From this we can see that the exponential term decays as z_1 , the height of the source, increases. Further exponential decay occurs if k_1 has a significant imaginary part.

The steepest descent method may now be applied in the usual way, and in this case it will not be necessary to take into account the presence of the pole.

2.6 Special cases

(1) The geometric approximation.

If n , the refractive index, is real, then equation

(2.14) is satisfied if

$$\alpha = \cos \theta_1,$$

$$\beta = \sin \theta_1,$$

$$\gamma = n \cos \theta_r,$$

To calculate the further terms in equation (2.9) we need to locate the position of the pole using $v_1 + Mv_2 = 0$. At this point

$$v_1 = -ik_1 a_p, \quad v_2 = -ik_1 \gamma_p, \quad K = k_1 \beta_p$$

where

$$a_p = \frac{M(n^2 - 1)^{\frac{1}{2}}}{(1 - M^2)^{\frac{1}{2}}}$$

$$\beta_p = \frac{(1 - n^2 M^2)^{\frac{1}{2}}}{(1 - M^2)^{\frac{1}{2}}}$$

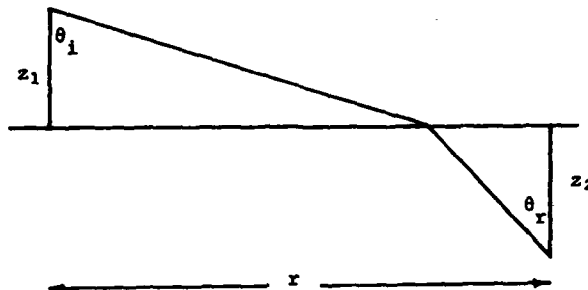
$$\gamma_p = -a_p / M$$

giving

$$s_p^2 = ik_1 [(a - a_p)z_1 + (\gamma - \gamma_p)z_2 + (\beta - \beta_p)r] \quad (2.16)$$

$\sin \theta_i / \sin \theta_r = n$, (Snell's Law)

and z_1 , z_2 , r are as depicted:



Taking the first term in equation 2.3.15 and using the first term of the expansion for the Hankel function so that

$$I = k_1 \beta \left(\frac{-2\pi}{f^n} \right)^{\frac{1}{2}} \frac{iM}{4\pi} \left(\frac{2}{i\pi k_1 \beta r} \right)^{\frac{1}{2}} / \alpha + i\gamma$$

$$\times \exp i k_1 (\alpha z_1 + \gamma z_2 + \beta r)$$

$$= \frac{2 \sqrt{\sin \theta_i} \exp i k_1 (z_1 / \cos \theta_i + n z_2 / \cos \theta_r)}{\{r (z_1 / \cos^3 \theta_i + z_2 / n \cos^3 \theta_r)\}^{\frac{1}{2}} (n \cos \theta_r + \cos \theta_i)}$$

(2.18)

which is the geometric approximation given by Brekhovskikh

ii) Grazing incidence with a large, complex refractive index.

If n has an appreciable imaginary component then the lateral wave will decay to negligible proportions at large distances.

If z_2 is sufficiently small so that in equation (2.3.14)

$$z_2 \beta / \gamma \ll r$$

(2.19)

Then $\alpha = \frac{z_1}{r} \beta$

Since $|\beta| < 1$, α is small at grazing incidence
equation (2.3.29) becomes

$$\frac{z_2}{r(n^2 - 1)^{\frac{1}{2}}} \ll 1 \quad (2.20)$$

Under these circumstances $\alpha = \cos\theta$ where $\tan\theta =$
 r/z_1

If in addition $\cos\theta$ is sufficiently small so that the
terms containing z_2 are negligible in f^{11} , f^{111} , f^{1V}

Then $v(z_1, z_2, r) \rightarrow$

$$\begin{aligned} & e^{ik_1 z_2} v(z_1, 0, r) \\ & = e^{ik_1 z_2 (n^2 - \sin^2\theta)^{\frac{1}{2}}} v(z_1, 0, r) \end{aligned}$$

which gives a simple exponential relationship with the field
on the surface above the receiver. This expression is
derived somewhat less rigorously by Brekhovskikh.

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3. POINT TO POINT PROPAGATION MEASUREMENTS USING ACOUSTIC MODELLING

3.0 Introduction

In order to validate the foregoing theory for both reflection and transmission of spherical waves, measurements of point propagation over plane absorbing and rigid boundaries were made in the small anechoic chamber. Two cases have been examined in detail:

- (i) attenuation over a rigid boundary;
- (ii) attenuation over and within a porous boundary.

The two materials selected to model these surfaces were varnished blockboard and Crown 150 fibreglass quilt. The basis and facilities used for these measurements are outlined in Appendix 3.

3.1 Rigid surface measurements

The rigid boundary measurements were made in order to examine the effects of:

- (i) source height uncertainty;
- (ii) spectral smoothing;
- (iii) near surface turbulence; and
- (iv) source and receiver size.

The source and receiver were placed 100 cm apart above the varnished blockboard surface as shown in Figure (3.1). The source and receiver heights could be varied independently.

An air jet was used as the sound source and a 1/4" microphone as the receiver. A block diagram of the measurement system is shown in Figure (3.2)

3.1.1. Source height errors

For a rigid boundary the excess attenuation minima result from the destructive interference of the direct and reflected waves. The frequencies at which these minima occur are therefore very sensitive to source-receiver geometry.

Figure (3.3) shows a comparison of the measured and predicted excess attenuation for propagation over a rigid boundary. The prediction is based upon a source and receiver height of 5 cm. It can be seen that the location of the predicted attenuation minima do not coincide with the measured.

The source height estimate was based upon a measurement of the distance between the rigid surface and the top of the air jet nozzle. However, in view of the large mass of air flowing through the jet, the acoustic centre of the source is unlikely to coincide with the top of the nozzle.

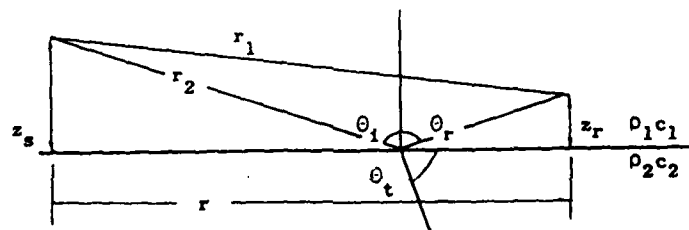


Figure 3.1 Source Receiver Geometry for Model Experiments.

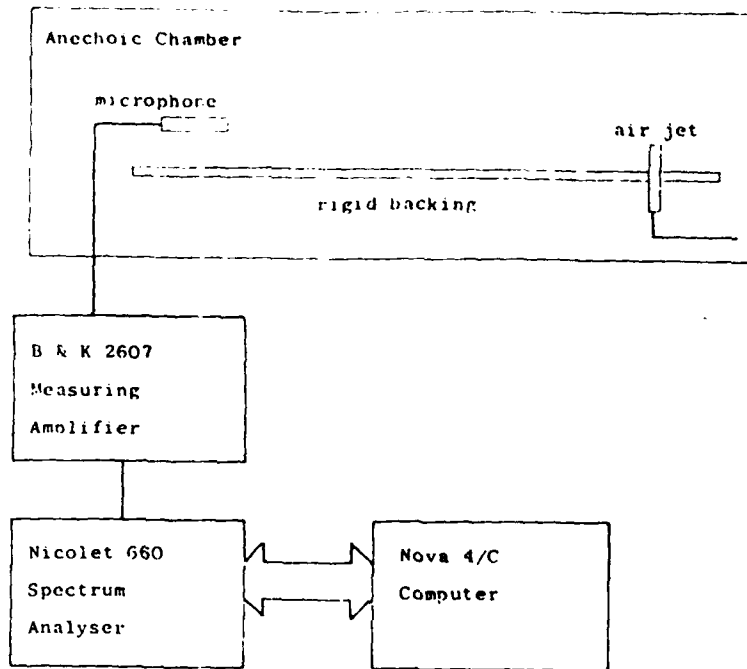


Figure 3.2 Block Diagram of Model Experiment Instrumentation.

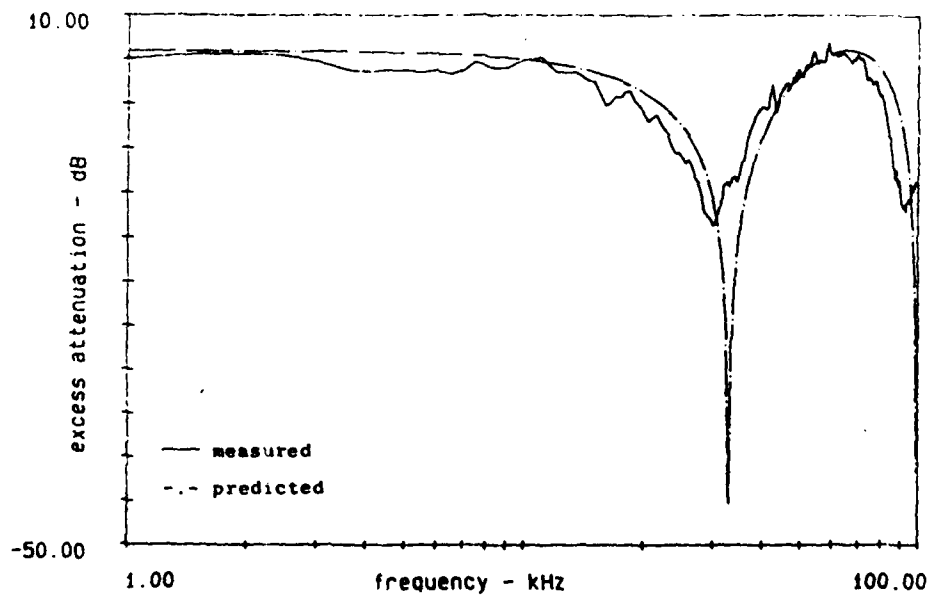


Figure 3.3 Comparison of Measured and Predicted Excess Attenuation for Propagation over a Rigid Surface.

A better estimate of the source height can be obtained by varying the height used in the pure-tone predictive algorithm until the locations of the measured and predicted minima coincide. The variation in the minima location and source height can be seen in Figure (3.4) increasing the source height increases the path length difference and hence moves the minima to lower frequencies.

The results of this procedure indicated that the source height is actually 5.3 cm above the rigid surface. A comparison of the measured and predicted excess attenuation curves, based upon a source height of 5.3 cm is shown in Figure(3.5)

3.1.2. Spectral smoothing

The predicted excess attenuation curves shown in Figure (3.3) and (3.5) exhibit much deeper minima than the measured results. Part of this discrepancy arises from the fact that the predictions are based upon pure tone interference, whereas the measured values are based upon the average energy in a 250 Hz wide filter. The effect of increasing the analysis bandwidth is to increase the sound pressure at the minima and so smooth the excess attenuation spectrum.

Correction procedures for the pure-tone calculations have been derived [3.1] but they cannot account for the observed differences.

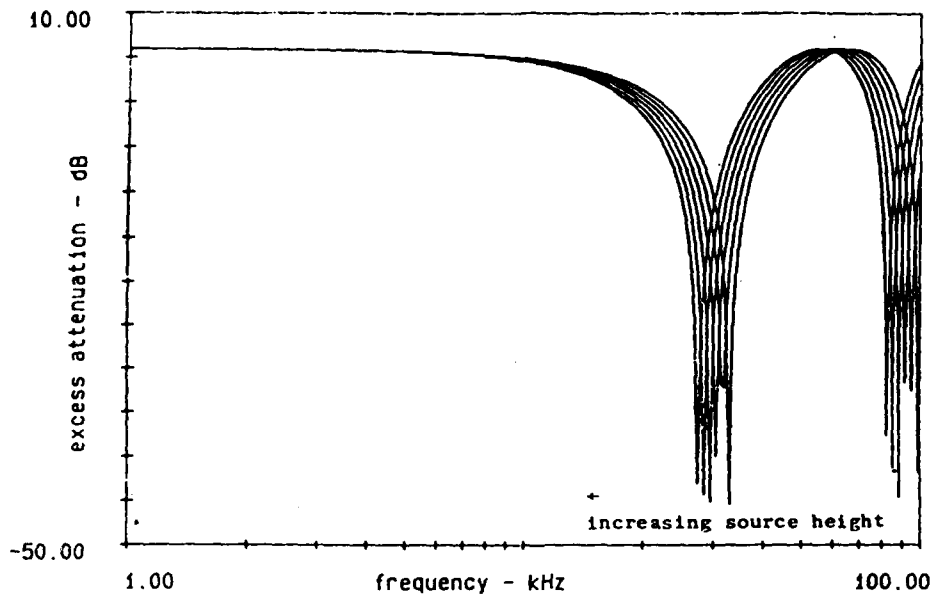


Figure 3.4 Predicted Excess Attenuation Minima as a Function of Source Height.

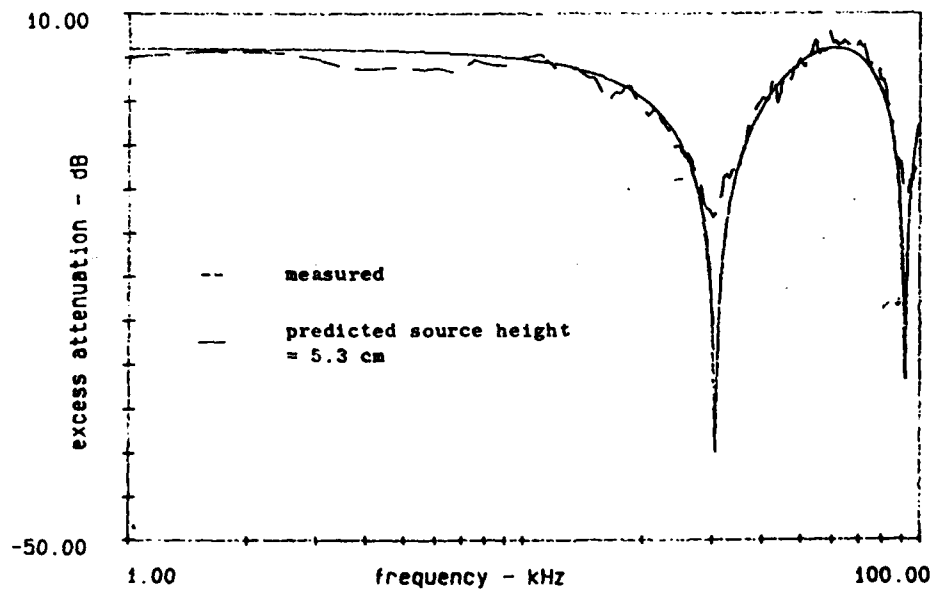


Figure 3.5 Comparison of Measured and Predicted Excess Attenuation After Correcting for Source Height.

3.1.3. Near surface turbulence

Another factor to consider when trying to account for the observed difference between the measured and predicted excess attenuation curves is the effect of near surface turbulence.

Daigle [3.2] has proposed modelling turbulent air motion as an additional propagation path length, with random direction and attenuation. The extra path length disturbs the strong interference pattern due to the surface. A correction procedure based upon the measurement of four meteorological parameters shows good agreement with experimental results. These parameters are:

- variance of sound velocity deviations
- variance of air temperature deviations

L_v - sound velocity correlation length

L_t - temperature correlation length

L_v and L_t are measures of the autocorrelation function of the wind and temperature variations. The time delay corresponding to maximum correlation is converted to an equivalent path length difference by multiplying the velocity of sound by the time delay.

Whilst it has not been possible to measure the parameters required to correct for turbulence effects, an examination of Daigle's results indicate that they could account for the observed discrepancies between the measured and predicted excess attenuation curves.

3.2. Fibreglass Measurements

Two sets of measurements were made using a fibreglass quilt material to model a porous half-space, the first to measure the amplitude ratio of the direct and transmitted sound fields and the second to measure the phase difference between the arrivals of those fields

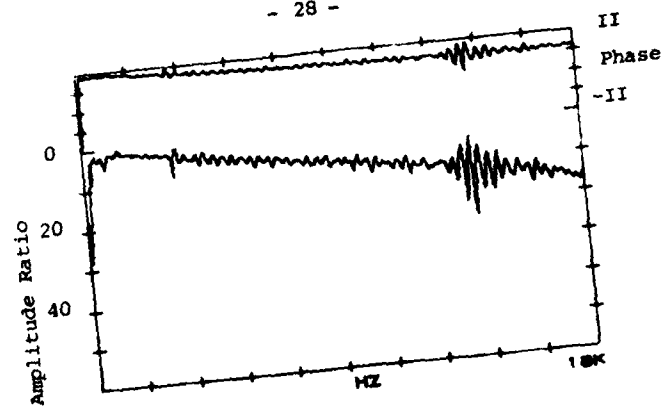
3.2.1. Amplitude Ratio Measurements

The source receiver geometry for these measurements is shown in Figure (3.1) The surface receiver was a standard 1/4" microphone with a normal protection grid, whilst for the sub-surface receiver the 1/4" microphone was fitted with a nose-cone. The nose-cone serves two purposes, first its shape enables one to ease the microphone through the fibreglass quilt and second it improves the directivity response of the microphone.

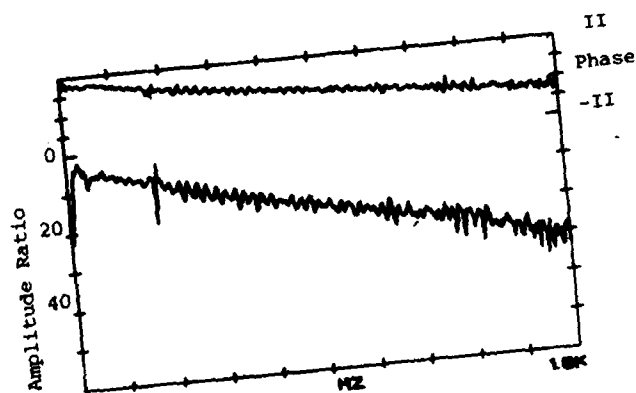
The sound source used for these measurements was a mid-range loudspeaker, with the cross-over network removed, and the source signal was derived from a swept frequency oscillator covering the range 100Hz to 15kHz.

In order to minimise the effects of phase distortion that can arise from incoherent averaging of swept frequency signals, a time gating system was employed to synchronise the source signal sweep and the digital sampling of the received signals.

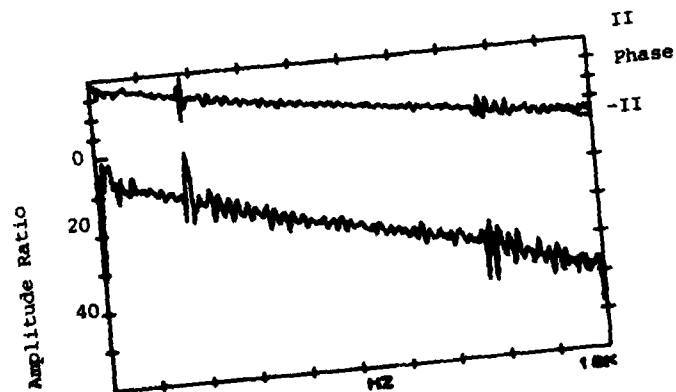
The narrow band spectrum at each of the receiving microphones was measured with the Nicolet 660B spectrum analyser, and the amplitudes ratio obtained from the modulus of the 'transfer function'. Three sets of results are reproduced here as Figure (3.6) Part (a) shows the amplitude ratio and phase difference for a source height of 5cm and a receiver depth of 3cm.



(a) Receiver Depth 3cm



(b) Receiver Depth 7cm



(c) Receiver Depth 10cm

Figure 3.6 Amplitude of Surface and Sub-surface spectra

The ripple in the transfer function at 8kHz, arises from resonance of the loudspeaker diaphragm. Parts (b) and (c) show the results for receiver depths of 7cm and 10cm respectively.

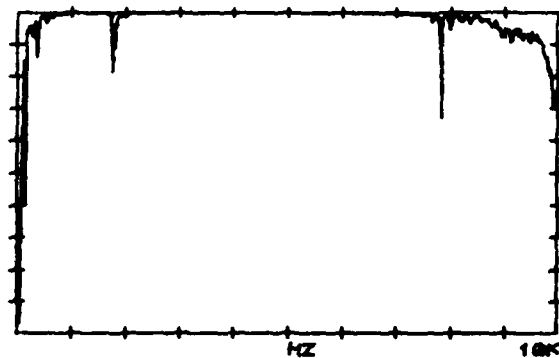
A test of the reliability of these measurements was performed by calculating the coherence function. The results for the three receiver depths, shown in Figure (3.7), indicate that only the estimates of the amplitude ratio at a depth of 10cm for frequencies above 8kHz may be erroneous; since the value of the coherence function drops below 0.8. The reduction in coherence arises from the high level of attenuation at this receiver depth.

All these results show that there is little attenuation of the transmitted sound at low frequencies. At high frequencies the attenuation is dependent upon the depth of the buried receiver. At 10kHz, the upper limit of the measurements, the attenuation increases from 16dB at a depth of 3cm, to approximately 50dB at a depth of 10cm.

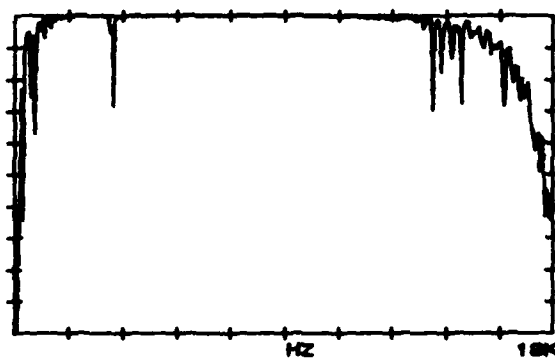
3.2.2. Pulse measurements

The time delay measurements were made using the same source receiver geometry described in section (3.2.1), but with the loudspeaker replaced by an electrical spark discharge source. This device produces a low intensity sound pulse of approximately 200 microseconds duration.

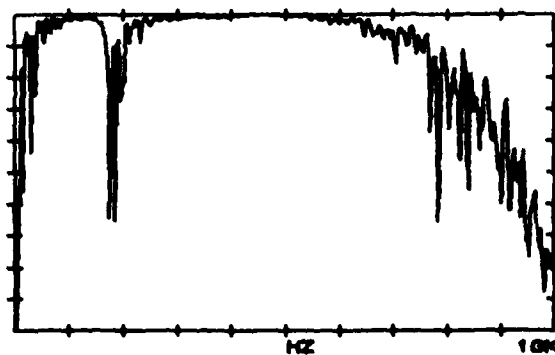
- 30 -



(a) Receiver Depth 3cm



(b) Receiver Depth 7cm



(c) Receiver Depth 10cm

Figure 3.7 Coherence Function of Surface and Sub-surface Spectra

The trigger transformer circuit used to generate the high voltage spark can cause significant errors due to electro-magnetic radiation, which swamps the response of the condenser microphone capsules. In view of this the spark was operated at its minimum energy level of 4mJ, and time averaging used to improve the signal to noise ratio.

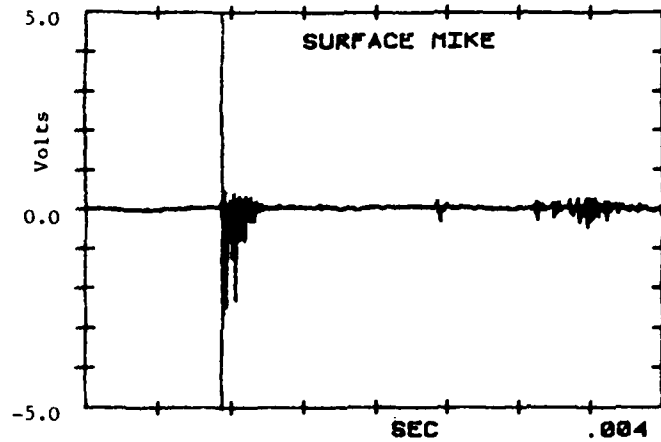
Examples of the direct (surface microphone) and transmitted pulses are shown in Figure (3.8) for a source height of 5cm and a receiver depth of 3cm. The total duration of the signal samples is 4 milliseconds.

The direct microphone signal shows a distinct pulse at 1 millisecond, corresponding to the spark discharge. The other fluctuations at 2.5 and 3.5 milliseconds correspond to the arrival of reflections from metallic fittings within the anechoic chamber. The transmitted pulse occurs at the same time, but the trace shows no subsequent arrivals due to extraneous reflections within the chamber.

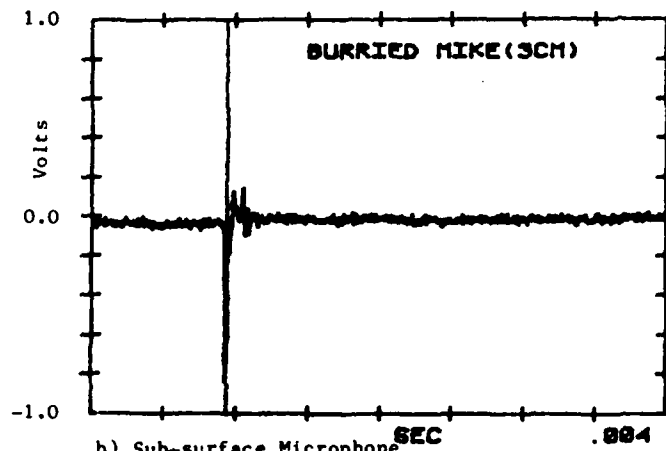
The spectrum of the direct and transmitted pulses are shown in Figure (3.9) for the range 125Hz - 50kHz. The upper trace corresponds to the direct pulse and the lower trace the transmitted pulse. The raggedness of these curves is due partly to the presence of reflection and partly to the electrical noise in the microphone pre-amplifiers.

The amplitude ratio and phase difference for the direct and transmitted sound fields are shown plotted in Figure (3.10). Both curves show considerable fluctuations below 5kHz, due primarily to the poor resolution of the spectrum analyser at low frequencies. However, the underlying shape of the amplitude ratio curve is similar to that obtained from the loudspeaker source measurements.

Measurements of the direct and transmitted pulses were made for receiver depths of 3, 7 and 10 cm and the results are shown in Figure (3.11). The pulse at a depth of 10 cm shows considerable attenuation and is only just discernable above the background noise level.



a) Surface Microphone.



b) Sub-surface Microphone

Figure 3.8 Surface and Sub-surface Microphone Pulse Waveforms.

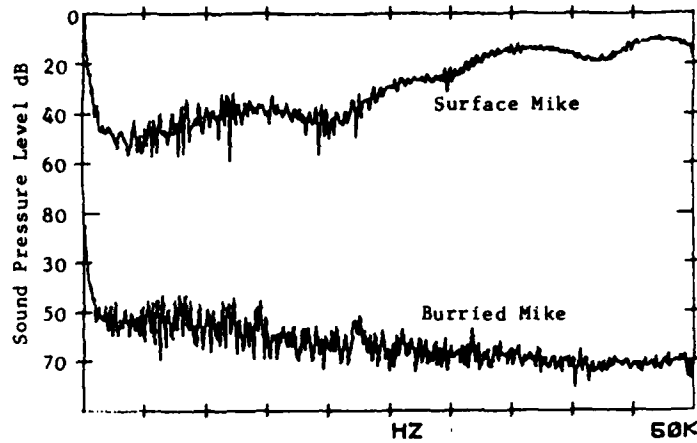


Figure 3.9 Measured Pulse Spectra for Surface and Buried Microphones.

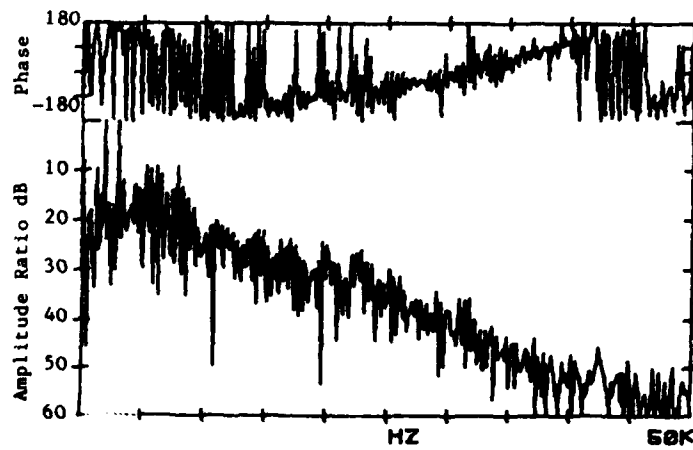
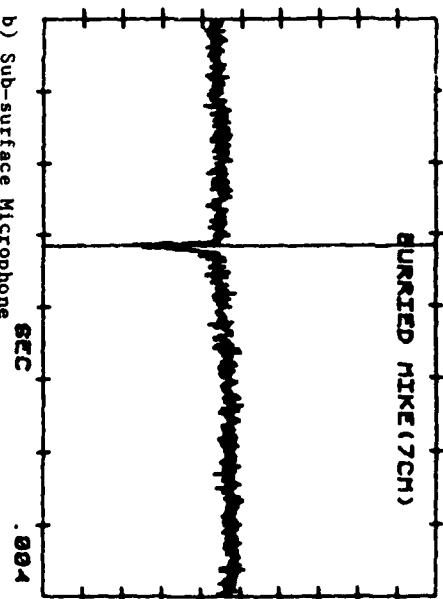
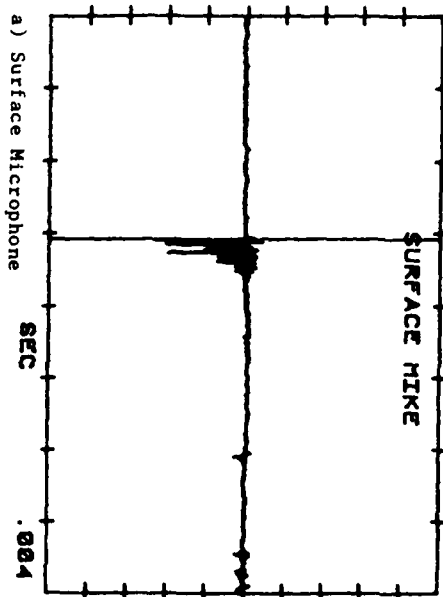
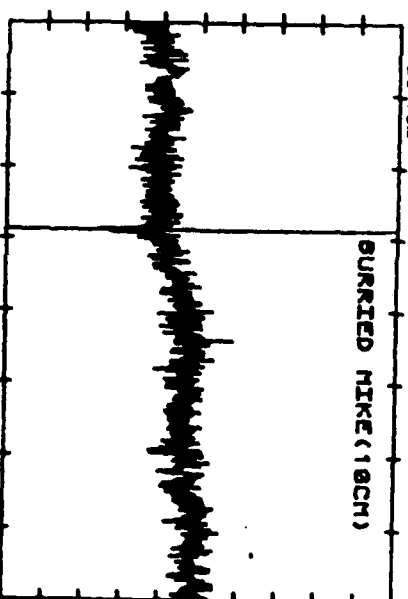
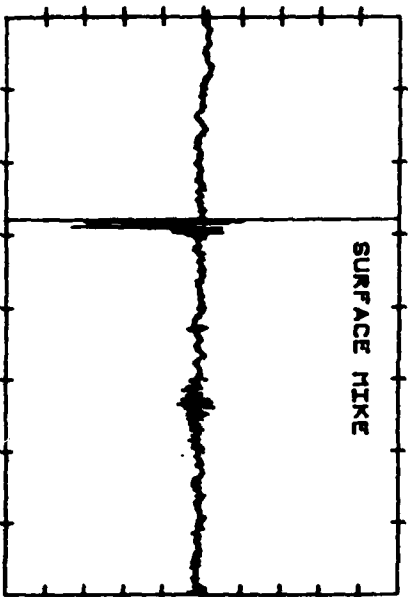


Figure 3.10 Amplitude Ratio of Surface and Sub-surface Spectra.



a) Surface Microphone

b) Sub-surface Microphone at 7cm



c) Surface Microphone

d) Sub-surface Microphone at 10cm

Figure 3.11 Surface and Sub-surface Receiver Pulse Measurements for different Receiver Depths.

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4 GROUND AS A RIGID POROUS MEDIUM

4.1 Introduction

The theories of spherical wave transmission and reflection at porous boundaries that have been advanced in the previous chapters rely upon the assumption that the ground can be regarded, effectively, either as a locally reacting fluid or as a homogeneous isotropic fluid sustaining a single (compressional) wave type. If the ground is locally-reacting then for computation of the reflected field it may be characterised acoustically by its normal (complex) surface impedance, and for computation of the transmitted field the required additional acoustical parameter is the propagation constant within the effective fluid. For a model of external reaction then computation of the reflected field requires knowledge of both surface impedance and propagation constant.

A model of the ground as supporting a single compressional wave type is consistent with propagation in a rigid porous medium in which the acoustic disturbance is transmitted only through the pore fluid and is attenuated by viscous and thermal effects. This model has been found adequate to represent the behaviour of porous boundaries in room and duct acoustics [4.1, 4.2] and in studies of outdoor sound propagation [4.3].

Indirect estimation of the surface impedance of any ground, that can be modelled as locally reacting, is possible from a propagation measurement with a fixed source-receiver geometry [4.3,4.4]. The impedance values result from fitting the data on excess attenuation between source and receiver after allowing for spherical divergence and air absorption. Since the surface impedance is complex, the fitting procedure requires simultaneous variation of real and imaginary parts. The questions arise (a) of a suitable initial guess and (b) of suitable ranges and the type of relationship between real and imaginary parts. Empirical formulae for impedance and propagation constant in terms of a single parameter, the specific flow resistivity of the porous medium [4.5], originally developed from acoustical measurements on glass fiber materials, fix the frequency dependence of the real and imaginary parts of the complex variables and the relationship between them. They have been used to derive the characteristics of ground surfaces needed for predictions of propagation near to outdoor ground surfaces [4.4, 4.6, 4.7] in terms of a best-fit flow resistivity. Although successful in some instances, the empirical formulae have been shown to have shortcomings in this application. These are evident particularly where measured values of flow resistivity are available. In these circumstances the best-fit value of flow resistivity has been found to be approximately half of the mean measured value [4.4]. The large scatter in the deduced impedance values with frequency, also obscures whether or not the empirical formulae predict the correct frequency dependence. The

transmitted field, which is of particular interest in this report, depends upon the propagation constant. Although the empirical formulae include a set for real and imaginary parts of propagation constant, these have not been tested for soils and sands to the same extent as those for surface impedance. For these reasons the remainder of this chapter is devoted to the development of theoretically-based alternatives to the empirical formulae for the acoustical characteristics of rigid air-filled granular media and to a consequent examination of the limits of applicability of the empirical formulae. Comparison of the theoretical predictions and measured data are made for glass fibre materials, sands and soils.

4.2 The Modified - Raleigh Model

4.2.1. Introduction.

A comprehensive review of the available theories for propagation in porous media is given elsewhere [4.8]. In granular media the micro-structure is best described in terms of grain shape size and packing density, all of which may be determined accurately by non-acoustic means. However theoretical approaches which specify the form of the rigid solid constituent while attempting to describe the acoustic field in the fluid-filled interstices have been constrained in each case to particular grain shapes and find particular difficulties in describing the field near points of grain contact. There is a further fundamental problem with such approaches, of course, where the rigid porous medium is consolidated such that individual solid elements cannot be identified. A conceptual model first suggested by Rayleigh, of a rigid solid matrix through which run parallel identical circular

cylindrical pores normal to the surface, has been shown capable of generalisation to many types of porous media [4.8]. The generalisation is achieved at the cost of introducing an adjustable parameter, the oscillatory-flow pore shape factor, which, strictly, is determinable only from best-fit to acoustical measurements. On the other hand, a suggested semi-empirical relationship between this parameter and another, the steady-flow pore shape factor, which is calculable for certain known pore geometries, means that, in principle, the acoustic properties of a rigid fluid-saturated granular medium can be deduced from four non-acoustic measurements. These are of flow resistivity, volume porosity, grain shape and mean grain diameter.

4.3. Propagation in a single pore

In this development, viscous and thermal effects are treated separately. Zwikker and Kosten [4.2] have shown that, at least in the limiting cases of low and high frequencies, such independent treatments give the correct result for the field within a cylindrical tube with rigid but heat-conducting walls containing a viscous, heat conducting fluid. Viscous effects may be incorporated into a complex (i.e. frequency-dependent) density of fluid in the pore ($\rho_c(\omega)$) subjected to an oscillatory disturbance with time-dependence $\exp(-i\omega t)$, ω being the angular frequency. The result of analysis may be written [4.2] as

$$\rho_c(\omega) = \rho_0 [1 - 2 (\kappa_c \sqrt{T})^{-1} T(\kappa_c \sqrt{T})]^{-1} \quad (4.1)$$

$$\text{where } T(\kappa_c \sqrt{T}) = J_1(\kappa_c \sqrt{T}) / J_0(\kappa_c \sqrt{T}) \quad (4.2)$$

$J_0()$ and $J_1()$ being cylindrical Bessel functions of zeroth and first

order, respectively, and

$$\kappa_c = a (\omega/\nu)^{\frac{1}{2}} \quad (4.3)$$

where a is the pore radius and ν is the Kinematic viscosity of the fluid and ρ_0 is its equilibrium density.

In the context of developing a general theory for acoustic propagation in fluid-saturated granular poro elastic media, Biot [4.10] considered propagation in a paralld-sided infinite slit of semi-width b . The expression for the complex fluid density in such a slit corresponding to equation (4.1) may be deduced to be

$$\rho_s(\omega) = \rho_0 [1 - [\kappa_s (-i)^{\frac{1}{2}}]^{-1} \tanh [\kappa_s (-i)^{\frac{1}{2}}]]^{-1} \quad (4.4)$$

$$\text{where } \kappa_s = b(\omega/\nu)^{\frac{1}{2}} \quad (4.5)$$

Analysis of the net flow of heat per unit mass within a circular cylindrical tube containing a non viscous conducting fluid, supposing (i) that heat is transported in transverse (radial) direction only and (ii) that pressure is uniform over the pore cross section, produces a relationship for the complex (frequency-dependent) compressibility of the fluid within the pore, as follows [4.2];

$$C_c(\omega) = (\gamma \rho_0)^{-1} [1 + 2(\gamma - 1)T(N_{pr}^{\frac{1}{2}} \kappa_c \sqrt{i})] \quad (4.6)$$

where N_{pr} is the Prandtl number and

$$C(\omega) = \frac{1}{\rho_0} \frac{dp}{d\rho} \quad (4.7)$$

ρ and p being the time-dependent density and pressure respectively. A similar analysis for net flow within a parallel-sides infinite slit of thickness $2b$ produces [4.11]

$$C_s(\omega) = (\gamma P_0)^{-1} \left(1 + \frac{\gamma - 1}{N_{pr}^2 \kappa_s (-i)^{1/2}} \tanh [N_{pr}^2 \kappa_s (-i)^{1/2}] \right) \quad (4.8)$$

The functions for complex density for the fluid in the circular cylindrical tube and the parallel-sided slit have similar frequency dependence. The frequency dependence of the functions for complex compressibility are similar also. The real part of $\rho(\omega)$ and the imaginary part of $C(\omega)$ show very little frequency dependence for a wide range of κ . However the variations of imaginary part of $\rho(\omega)$ and the real part of $C(\omega)$ with κ show that

- (i) $\text{Im} \rho_s(\omega)$ for a slit of semi-width $b \approx \text{Im} \rho_c(\omega)$ for a circular cylindrical tube of radius b/n
- and (ii) $\text{Re} C_s(\omega)$ for a slit of semi-width $D \approx \text{Re} C_c(\omega)$ for a circular cylindrical tube of radius b/n .

The value of n for which these approximations hold is roughly constant and equal to 0.5 for $1.8 \leq \kappa_c \leq 10.66$ (see Table 4.1). This suggests that for a pore of a general shape but with a constant characteristic dimension λ along its length, it is possible to choose an equivalent radius $a = \lambda/n$ and to evaluate $\rho(\kappa)$ and $C(\kappa)$ where $\kappa = (\lambda/n)(\omega/\nu)^{1/2}$ (4.9). n represents an oscillatory-flow pore-shape factor.

4.2.3 Extension to bulk medium

If the pores in the bulk medium may be assumed identical and to run parallel to the surface then extension to the bulk medium from the expression for a single pore is straight forward and introduces the volume porosity and flow resistivity in addition to the single pore parameters

ϵ and n . In a real medium, for various reasons, including changing pore orientation and cross section, the particle path lines will deviate from the surface normal. The changing pore orientation is referred to as the sinuosity or tortuosity, q , of the pore [4.2, 4.12, 4.13]. The tortuosity factor q has the particular value $1/\cos \theta$ for a medium containing parallel-cylindrical pores inclined at angle θ to the surface normal. It is possible to incorporate a further factor (>1) when the pores have cross-section which vary along their lengths. However this possibility is not pursued further in the present analysis.

On the assumption that all of the pores are identical, the fluid velocity averaged over a single pore's cross section, $\langle v \rangle$, can be related to the velocity u , averaged over unit cross-section of the porous medium by

$$\langle v \rangle = q u / \Omega$$

where Ω is the volume porosity.

Consideration of the pressure gradient along a streamline in a single pore during laminar flow gives [4.13]

$$-\frac{dp}{dx} = q^2 \phi_p \frac{u}{\Omega} \quad (4.11)$$

ϕ_p , the flow resistance per unit length in a single pore, is given by

$$\phi_p = 8\mu s / \epsilon^2 \quad (4.12)$$

where μ is the dynamic viscosity of the fluid, and s is a steady-flow pore-shape factor. s may be calculated from a standard fluid-dynamical analysis for a pore of any regular cross-section. For a circular-cylindrical pore s has the value 1 and for a parallel sided slit

$s = 1.5$. A more extensive list of values of s corresponding to pores of various regular cross sections is given in ref [4.13].

From equations (4.11) and (4.12) the flow resistance per unit length, or flow resistivity of the porous medium, which is a measurable quantity, can be calculated from

$$\phi = 8\mu q^2 s / \ell^2 \Omega \quad (4.13)$$

By eliminating the characteristic dimension ℓ between equations (4.9) and (4.13) it is possible to deduce that

$$\kappa = (s^2/n) (8\rho_0 q^2 \omega / \phi)^{1/2} \quad (4.14)$$

for pores of arbitrary cross-section.

The one-dimensional equations of continuity and motion in the rigid porous medium which are sufficiently general if the medium is isotropic may be written [4.2] as

$$-\frac{\partial u}{\partial x} = \left(\frac{\Omega}{\rho_0} \right) \left(\frac{d\rho}{d\rho} \right) \left(\frac{\partial \rho}{\partial t} \right) \quad (4.15)$$

$$\text{and} \quad -\frac{\partial \rho}{\partial x} = (q^2/\Omega) \rho(\omega) \frac{\partial u}{\partial t} \quad (4.16)$$

Combining these equations with use of (4.7) enables production of the compressional wave equation inside the rigid porous medium.

$$\frac{\partial^2 \rho}{\partial x^2} = q^2 \rho(\omega) C(\omega) \frac{\partial^2 \rho}{\partial t^2} \quad (4.17)$$

Finally from equations (4.1), (4.6) and (4.17) the propagation constant k within the medium is given by

$$k^2 = q^2 (\omega/c_0)^2 [1 - 2 (k/\sqrt{\gamma})^{-1} T(k/\sqrt{\gamma})]^{-1} \times [1 + 2 (\gamma - 1) (N_{0r}^{\frac{1}{2}} k/\sqrt{\gamma})^{-1} T(N_{0r}^{\frac{1}{2}} k/\sqrt{\gamma})] \quad (4.18)$$

and the relative surface impedance $Z/\rho c = 1/\beta$ which is the characteristic impedance of a semi-infinite medium is given by

$$Z = \omega \rho (\omega) / k \quad (4.19).$$

It is of particular interest to note that for low frequencies and small values of the characteristic dimension, i.e. small pores or high flow resistivity, equations (4.18) and (4.19) reduce to relatively simple expressions by virtue of the small argument approximations of the Bessel Functions. Hence, at low frequencies and for high flow resistivities

$$Z/\rho_0 c_0 \approx k \approx (1 + i) (n^2 \Omega \phi \gamma \omega / (2 \rho_0 s c_0^2))^{1/2} \quad (4.20)$$

4.2.4 Calculations of tortuosity

Tortuosity has an influence not only upon fluid-flow in porous media but also upon their electrical conductivity when the pore space is filled with an electrically conducting liquid and upon diffusion of water vapour through the pore space. It has been related also to the index of refraction of fourth sound when the porous medium is fixed with liquid helium [4.14]. In the context of diffusion of gases through porous granular materials, the following relationship, known as Bruggeman's relationship, has been found to hold [4.15],

$$q^2 = \Omega^{-n'} \quad (4.21)$$

where n' is a grain shape factor taking the value 0.5 for spherical grains. This and other values of n' are listed in Table 4.2. Hence knowledge of porosity and grain shape should be sufficient, in principle, to calculate the appropriate tortuosity value. It should be noted that for porosities near unity, the tortuosity will also be near unity irrespective of particle shape.

4.2.5 Calculations of shape factors

For packings of spheres, of porosities in the range $0.34 < \alpha < 0.45$ CARMAN [4.13] gives a range of values of $2q^2s$,

$$4.5 < 2q^2s < 5.1$$

Using the Bruggeman relationship (4.21) for spheres, the stated range of porosities implies a range of tortuosities given by

$$1.49 < q^2 < 1.715.$$

Hence from the range for $2q^2s$, it is possible to deduce $s \approx 1.5$ which is the value for parallel-sided slit pores.

In principle it should be possible to deduce values of s for any regular microstructure from an analysis of steady flow through the known pore cross-section. n , the oscillatory-flow shape factor would require computation from a separate analysis of the microvelocity field for each pore cross-section. Nevertheless the values of n and s for the supposed extremes of circular cylindrical tube and parallel-sided slit suggest the possibility of the relationship

$$n = 2 - s \quad (4.22)$$

Consequently for a packing of spheres, with $s=1.5$, it is possible to expect that $n = 0.5$.

For non-spherical and non-uniform particles appropriate values of n and s are more difficult to specify. Indirectly if the flow resistivity, porosity and mean grain shape are known then it may be possible to deduce a value for s , and hence n through equation (4.22), by utilising the concept of hydraulic radius.

Hydraulic radius, m is given by

$$m = \Omega/S \quad (4.23)$$

Where S is the exposed particle surface area per unit volume of material. A more usual concept than flow resistivity in describing flow through porous media is permeability. As long as the pores have fairly smooth perimeters, the permeability B is related to m by

$$B = \Omega m^2 / (2q^2 s) \quad (4.24)$$

Flow resistivity and permeability are related through

$$B = \Omega^2 / \phi \quad (4.25)$$

Since m is calculated from geometrical considerations then knowledge of ϕ , Ω and n' should enable calculation of s .

4.3 Comparison of rigid porous model with reflection measurements

Although there are many measurements available in the literature on the acoustical characteristics of materials of interest in room acoustics and duct acoustics, there is a paucity of data for the acoustical characteristics of soils and sands. An exhaustive set of data, on various types of sand and artificial porous granular media consisting of spherical lead shot, were obtained by Ferrero and Sacerdote [4.15]. Their measurement technique involved measurement of the pressure standing wave (plane wave fronts) formed in a cylindrical tube with the sample forming one termination and the loudspeaker source of pure tones forming the other. The propagation constant and characteristic impedance were calculated from measurements of surface impedance for two different thickness of sample; one thickness being twice the other. This method was first advocated by Pyett [4.16]. Ferrero and Sacerdote also measured the porosities and flow resistivities of their samples. Thus for the spherical lead shot and sand samples ($n'=0.5$) of all the parameters are known, with the exception of the pore-shape factors for application of the theory developed in earlier sections of this chapter. The appropriate value of q^2 for all samples is approximately 1.6, since all $\alpha \approx 0.4$. Best agreement is obtained with the por: shape factor ratio $n/\sqrt{s} = 0.5$ rather than 0.75 or 1. This value corresponds approximately to $n = 0.5$, $s = 1.5$ i.e. the values for parallel-sided slit pores as expected for packings of spheres (see section 4.6). The measured data and theoretical predictions are shown in Figures 4.1 to 4.5.

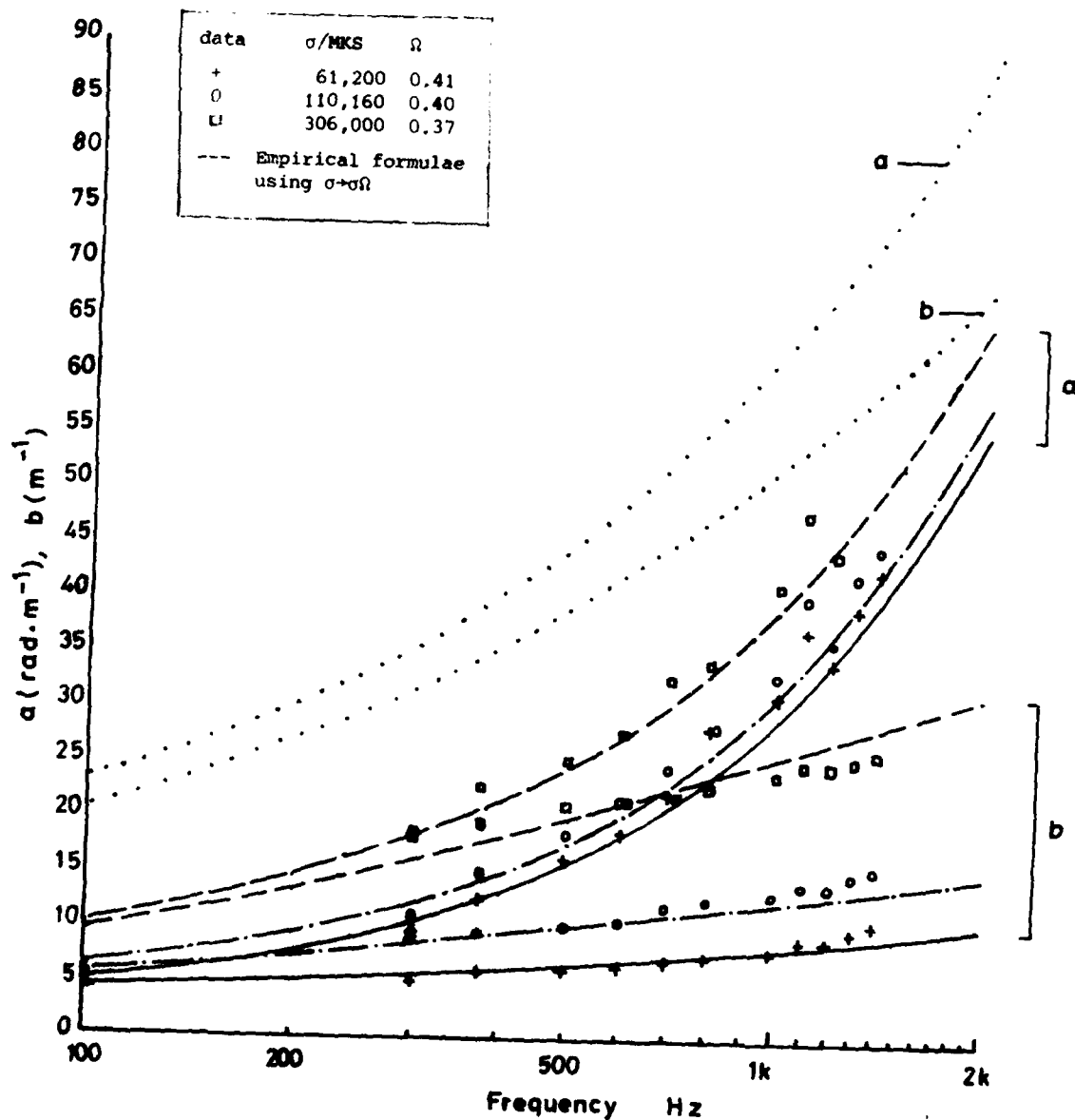


Figure 4.1 Comparison of measured data (+ o □) of propagation constants ($a + ib$) in sands, of measured flow resistivities and porosities as given in the key, with predictions based on these values and assumed values of $n' = 0.5$ and $n/\sqrt{a} = 0.5$

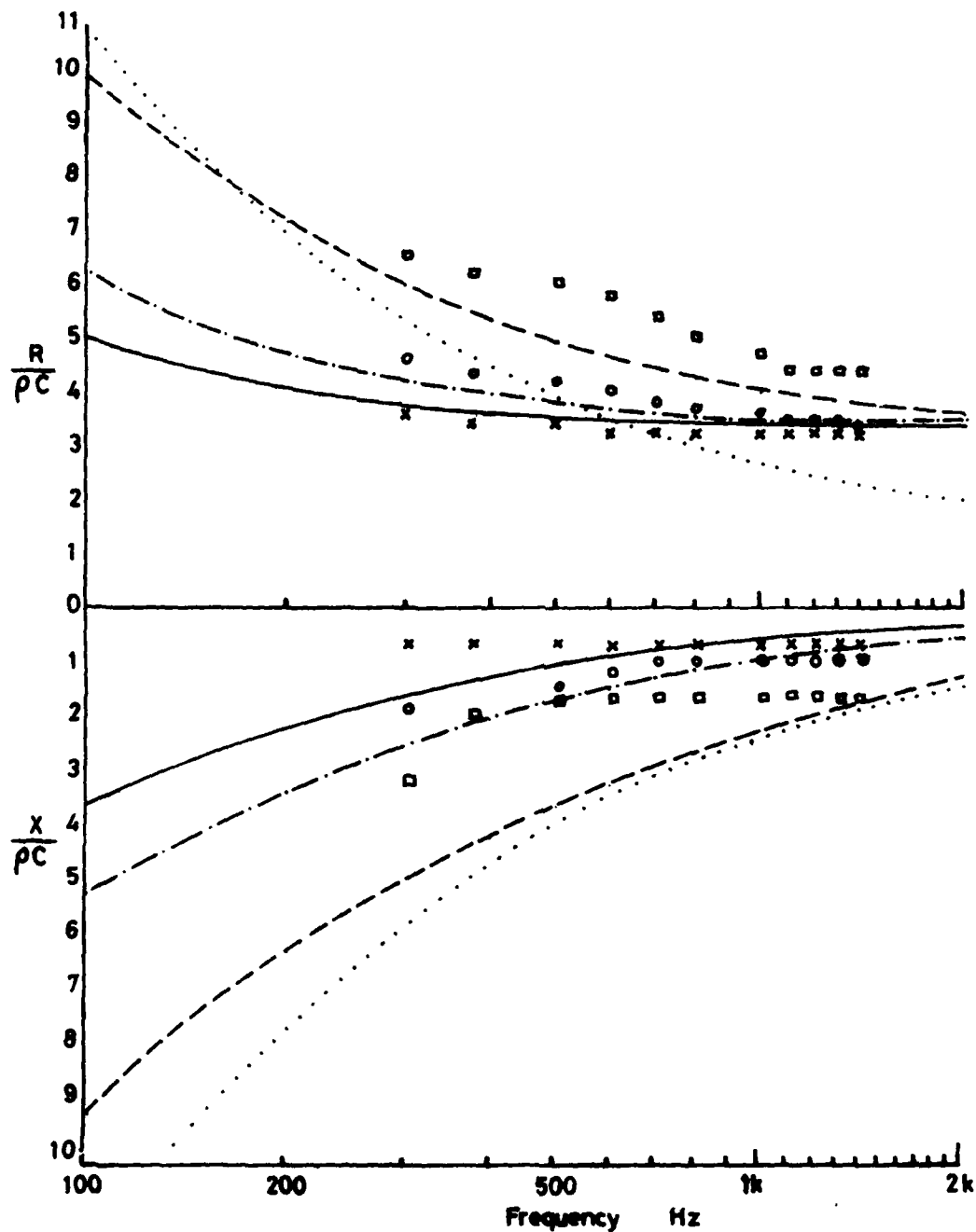


Figure 4.2 Comparison of measured data on characteristic impedance ($R + iX$) of sand with predictions of rigid porous model. Key as in Figure 4.1.

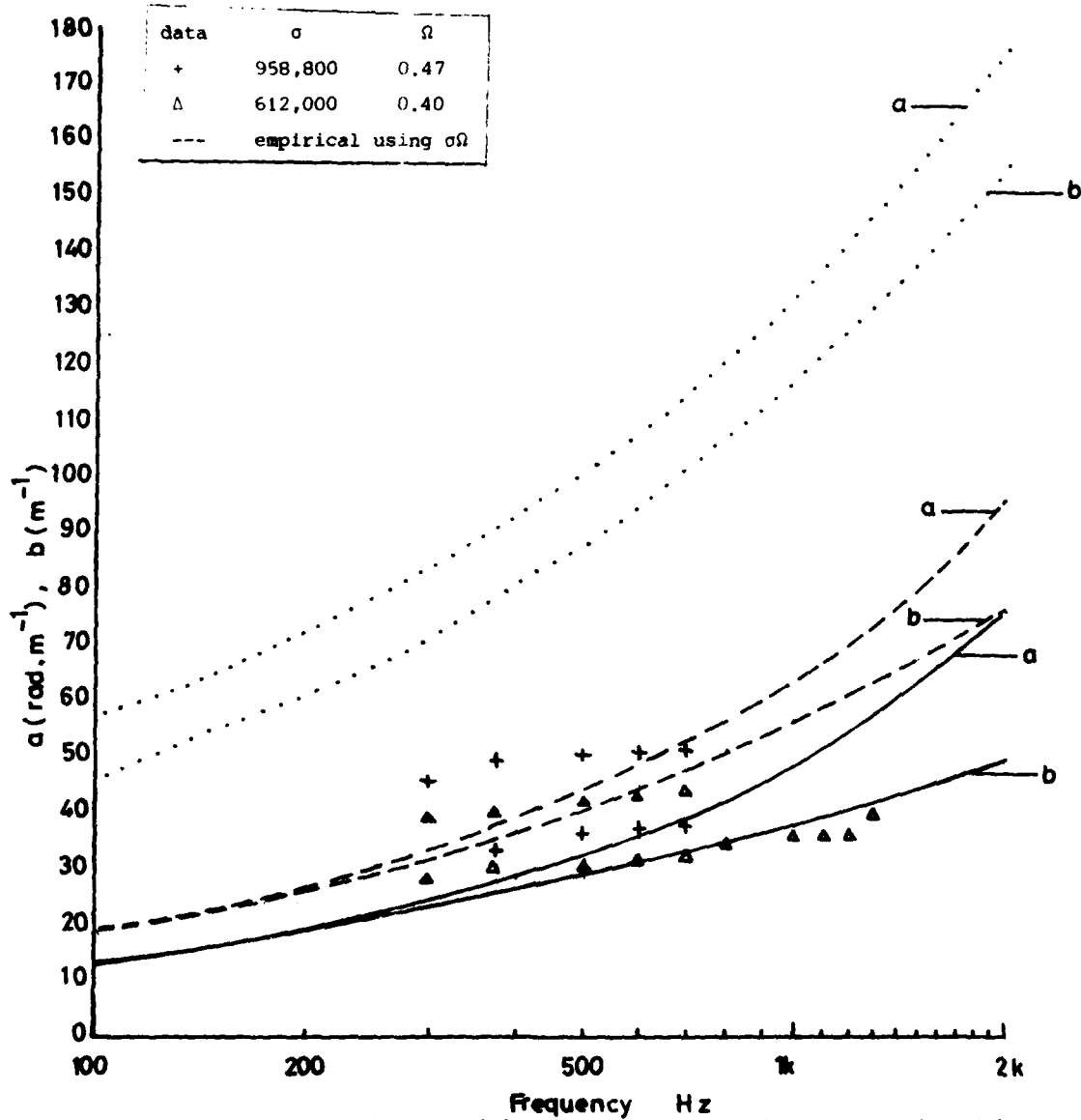


Figure 4.3 Comparison of measured data (+ Δ) on propagation constant ($a + ib$) in sands, having measured parameters as in the key above, with predictions based upon these parameters and $n' = 0.5$, $n/\sqrt{s} = 0.5$.

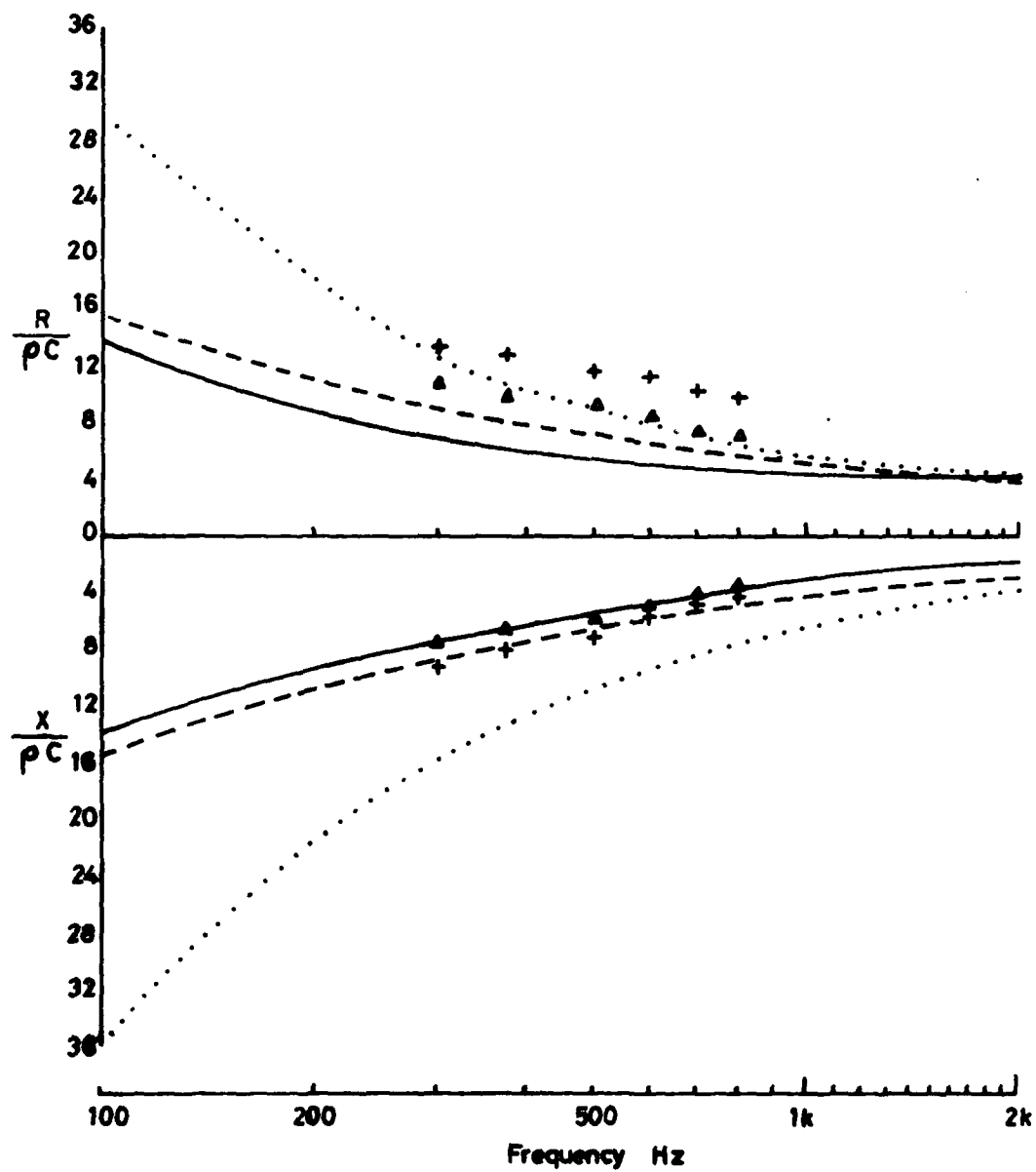


Figure 4.4 Comparison of measured data of the characteristic impedance ($R + iX$) of sands with predictions. Key as in Figure 4.3

4.8 Comparison with transmission measurements

4.8.1 Soils and sands

The difference in sound pressure levels between a microphone at the surface and a probe microphone (Figure 4.5) at various depths in several soils and sands has been measured [4.17]. Assuming that the ground medium is locally reacting then this difference should be given by

$$\text{attenuation in dB} = 20 \operatorname{Im}(k) D / \log_{10}$$

$\operatorname{Im}(k)$ refers to the imaginary part of k , i.e. the attenuation constant, and D is the depth of the probe. Figures 4.6 to 4.9 show the comparison between the measured data and the predictions obtained through (i) equation (4.19) (ii) measured values of flow resistivity, using an apparatus described elsewhere [4.18], and (iii) measured values of porosity, deduced by weighing samples of known volume.

4.8.2 Fibreglass

Using the apparatus described in Chapter 3, measurements have been made of the penetration of sound from an air-jet source into fibreglass 50 cm. thick. The probe receiver Microphone was at 1, 4 and 7 cm. beneath the surface. The comparison between measured one-third octave averaged data and predictions based upon measured flow resistivity (using apparatus conforming to ASTM) and the assumptions that

$$\alpha = q = n/\sqrt{5} \approx 1.$$

are shown in Figures 4.10 and 4.11. Note that the results on pulse penetration (Figure 3.17) are consistent with the air-jet results and with prediction. In the case of the loudspeaker generated narrow-band pulses predictions of both the real and imaginary part of the transfer function between surface and buried receivers (see sections 2.3 and chapter 3) are in good agreement with measurement (Figures 4.12 to 4.14).

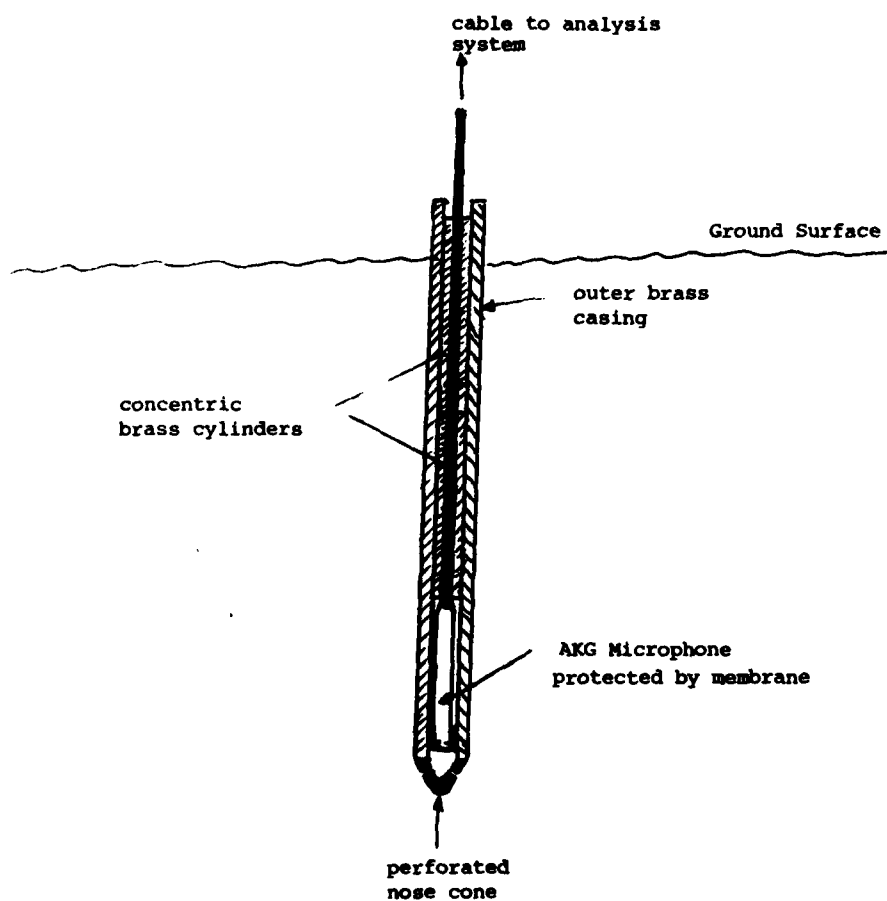


Figure 4.5 Microphone Probe System for soils and sands

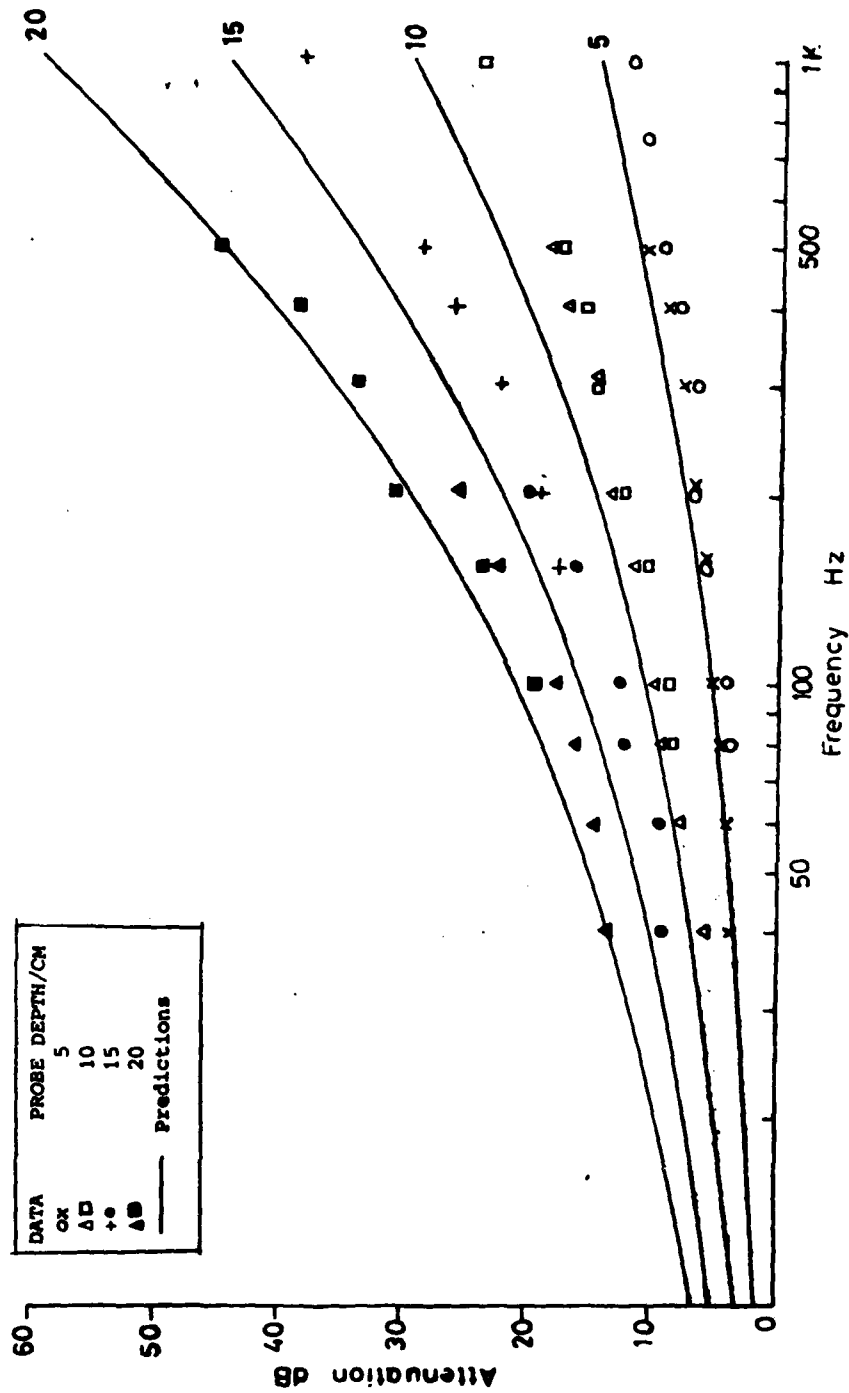


Figure 4.6 Measured attenuation in washed sand, $\sigma = 210,000$ MKS Units, $\nu = 0.4$ (dry).
Predictions use these values, together with $n' = 0.5$, $n/\nu = 0.75$

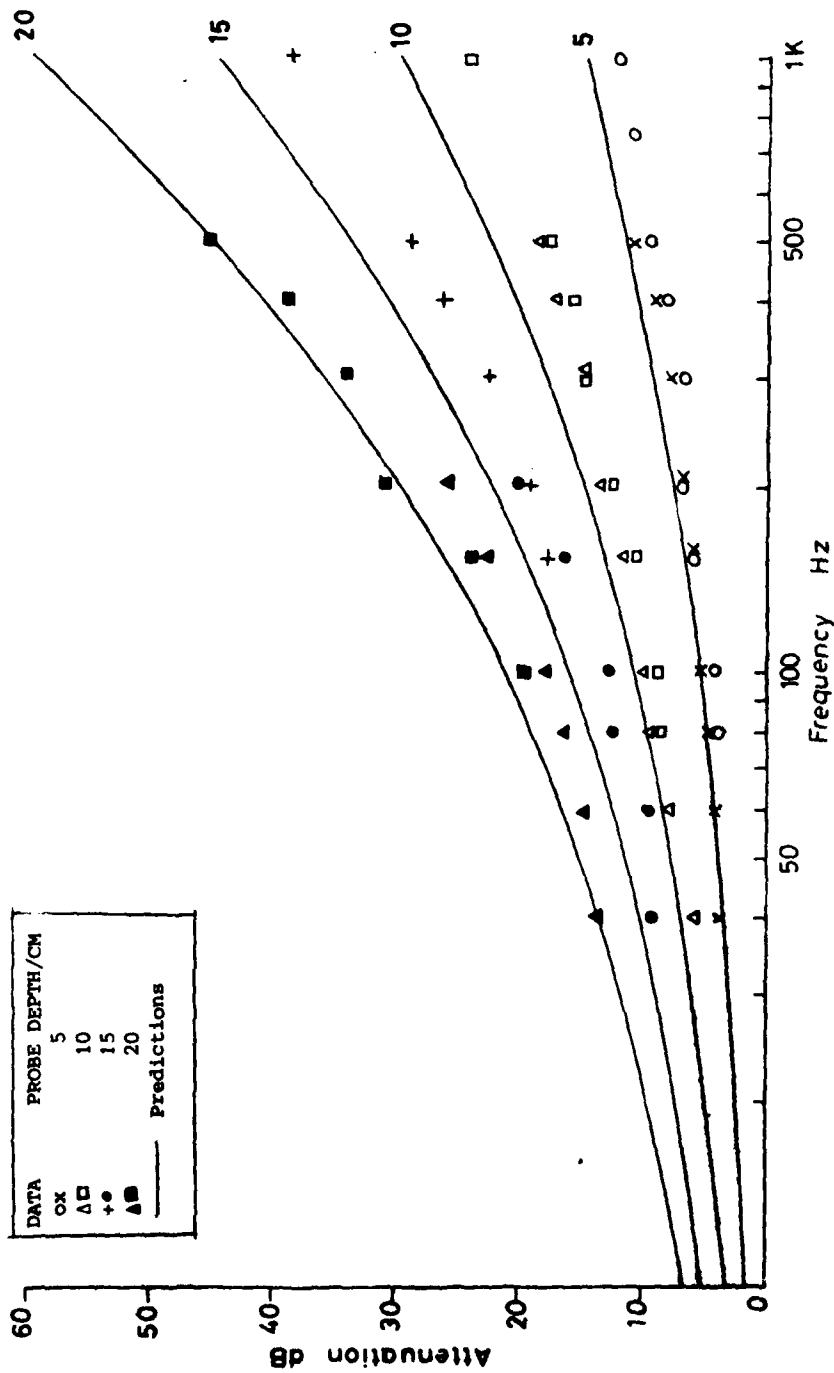
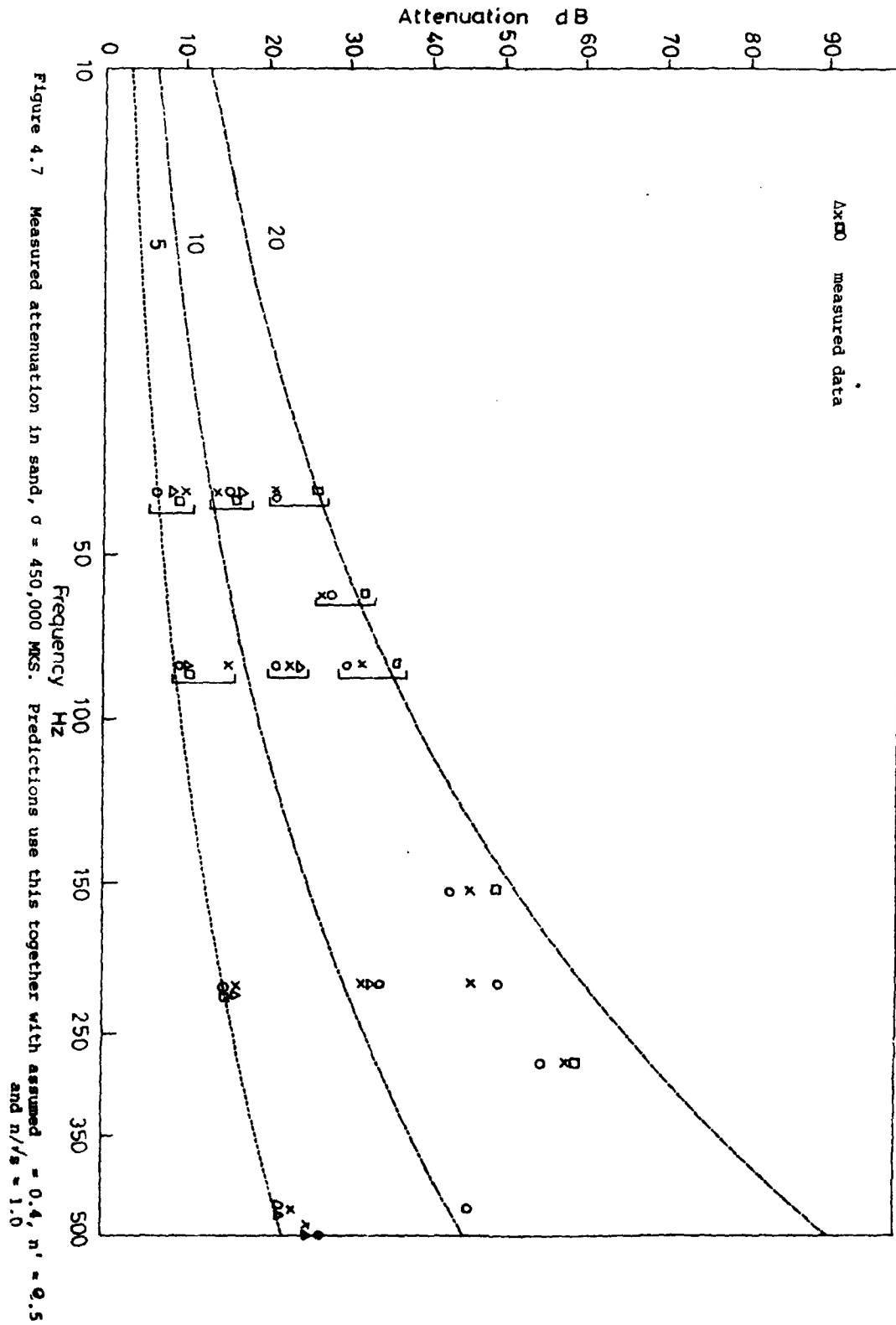


Figure 4.6 Measured attenuation in washed sand, $\sigma = 210,000$ MKS Units, $\nu = 0.4$ (dry).
Predictions use these values, together with $n' = 0.5$, $n/\sqrt{s} = 0.75$



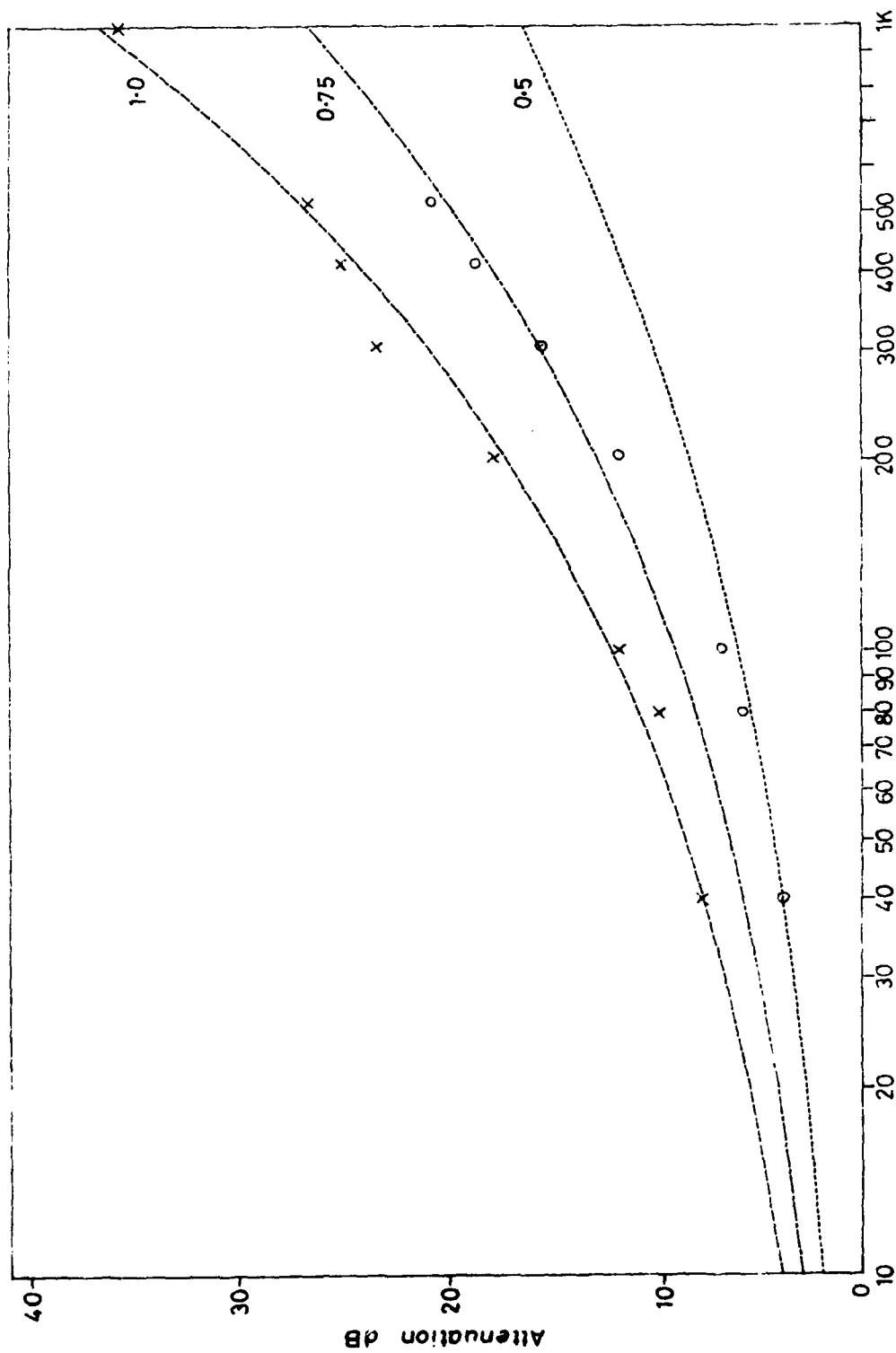


Figure 4.8 Measured attenuation in "institutional" $\sigma = 604,000$ MKS units. Predictions use this value together with $\Omega = 0.4$, $n' = 1.0$ and labelled values of $n/\sqrt{s}$

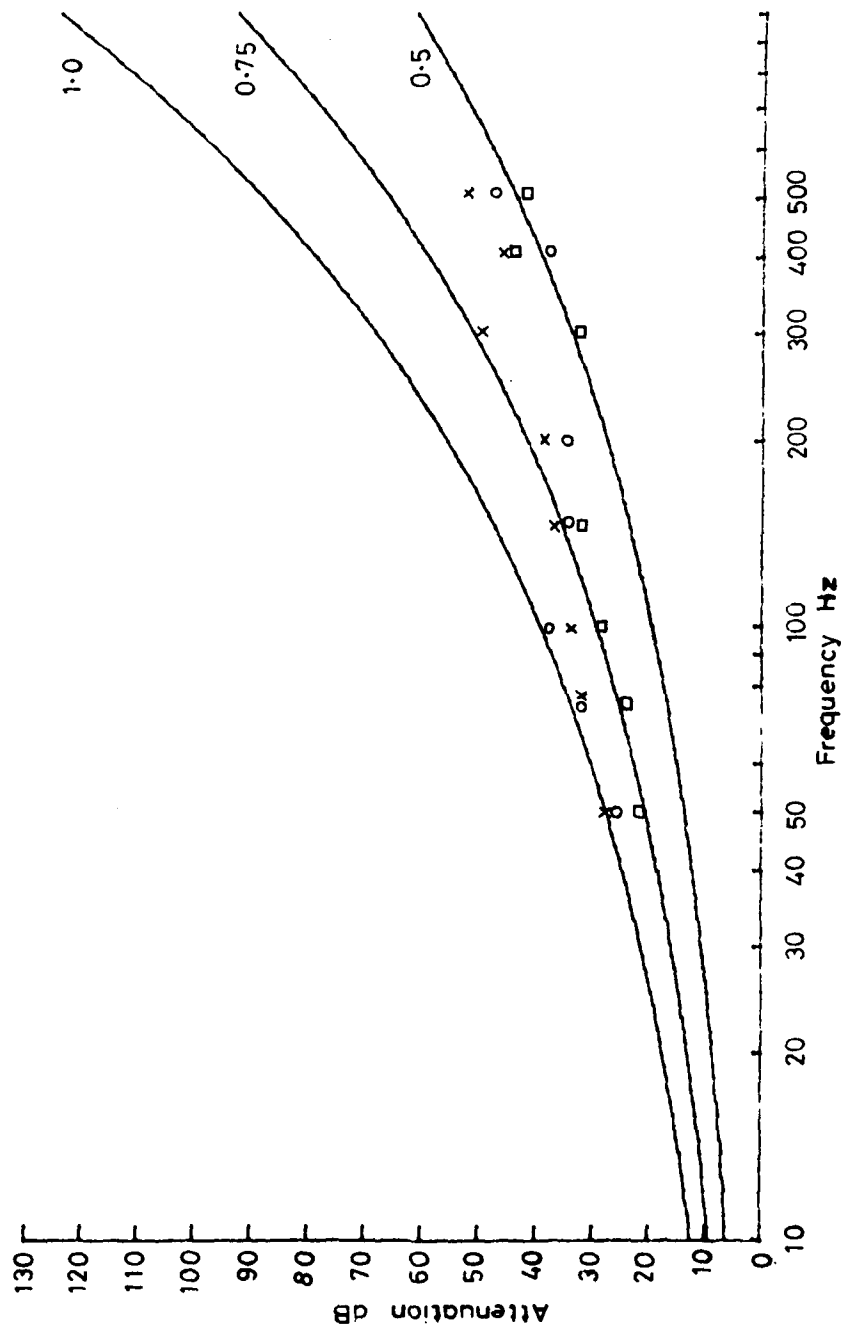


Figure 4.9 Measured attenuation in Loess, $\sigma = 650,000$ MKS units. Predictions use this value plus $\Omega = 0.4$, $n' = 1.0$ and labelled values of n'/s .

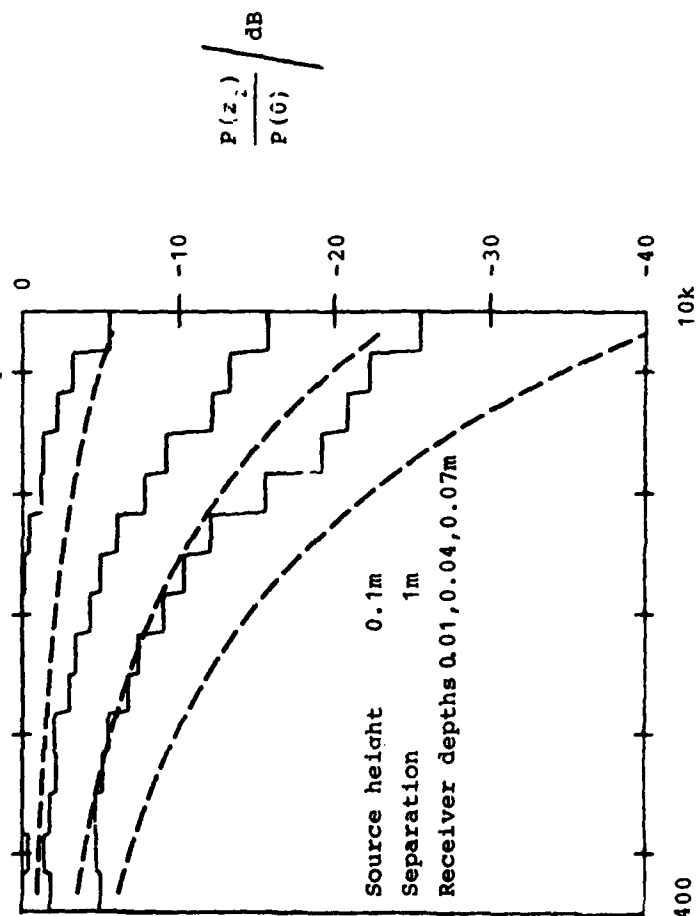


Figure 4.10 1/3 Octave Spectrum Ratio from Model measurements Fibreglass type:
 Pilkington Crown 150 Fitted flow resistivity: 11460 (mks units)

Predictions are from empirical formulae

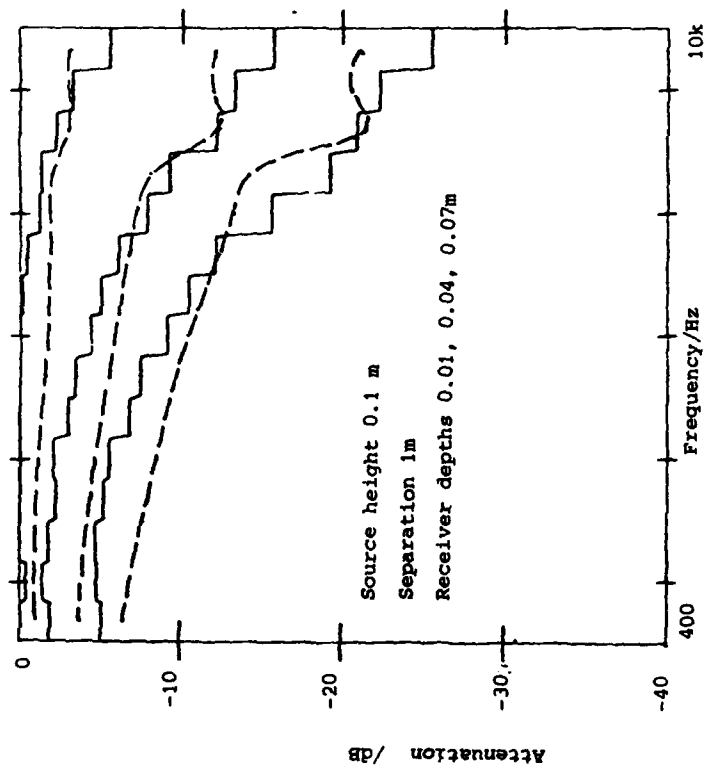


Figure 4.11 Measured $\frac{1}{3}$ octave attenuation in fiberglass of flow-resistivity 11,460 MKS units compared with predictions of rigid porous model $q = n\sqrt{3} = 1$

Figure 4.12

COMPARISON OF MEASURED (—) AND PREDICTED (---) VARIATION WITH FREQUENCY OF THE TRANSFER FUNCTION BETWEEN 2 VERTICALLY SEPARATED MICROPHONES, ONE ON THE SURFACE AND ONE AT DEPTH 3 CM IN FIBRE GLASS.

PREDICTIONS BASED UPON FLOW RESISTIVITY = 11460 MKS RAYLS. m^{-1} , ASSUMED POROSITY = 1, TORTUOSITY = 1, SHAPE FACTOR = 1.

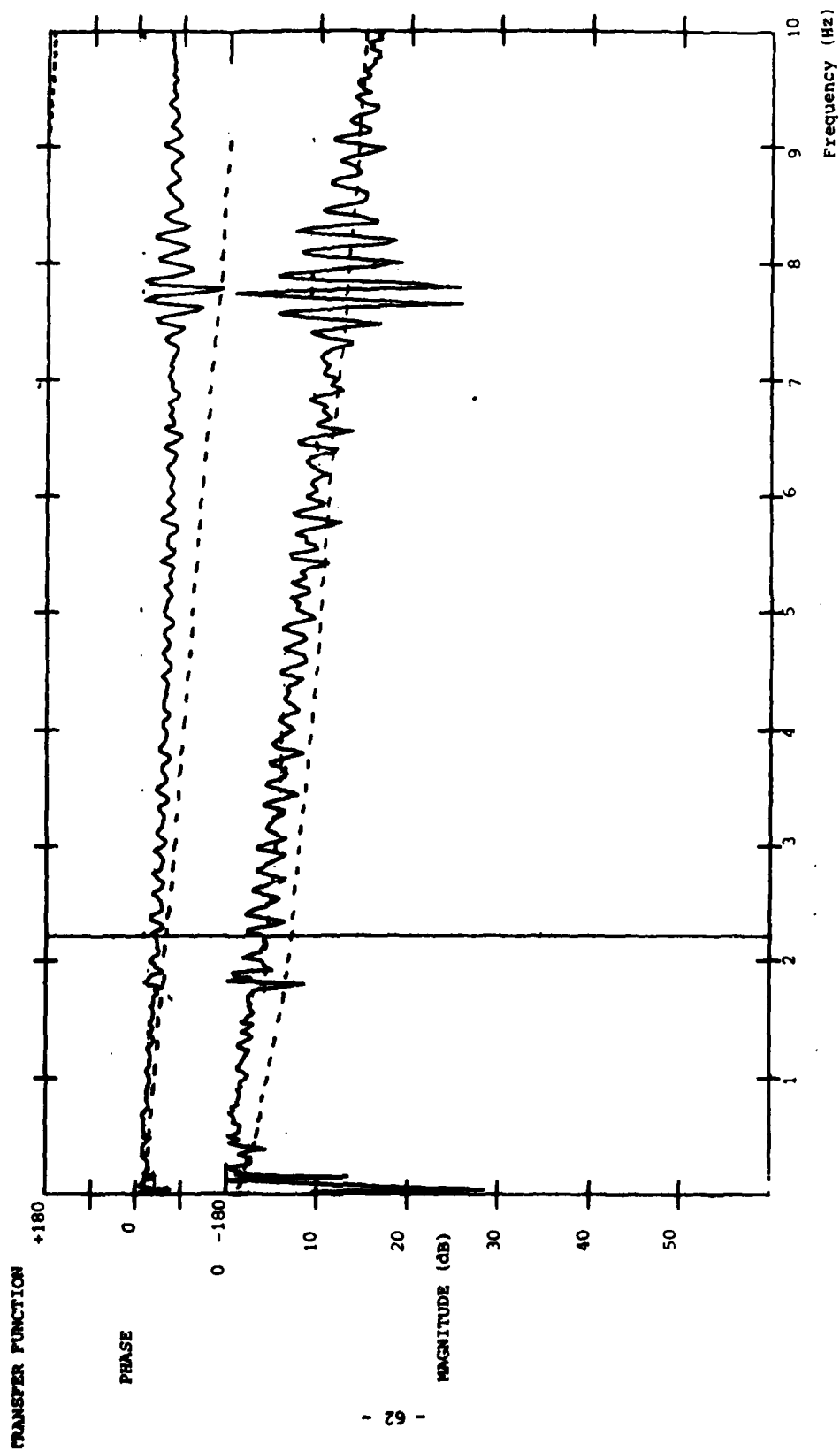


Figure 4.13

COMPARISON OF MEASURED (—) AND PREDICTED (---) VARIATION WITH FREQUENCY OF THE TRANSFER FUNCTION BETWEEN 2 VERTICALLY SEPARATED MICROPHONES, ONE ON THE SURFACE AND ONE AT DEPTH 7CM IN FIBRE GLASS. PREDICTIONS BASED UPON FLOW RESISTIVITY = 11,460 MKS RAYLS, ASSUMED POROSITY = 1, TORTUOSITY = 1, AND SHAPE FACTOR RATIO = 1.

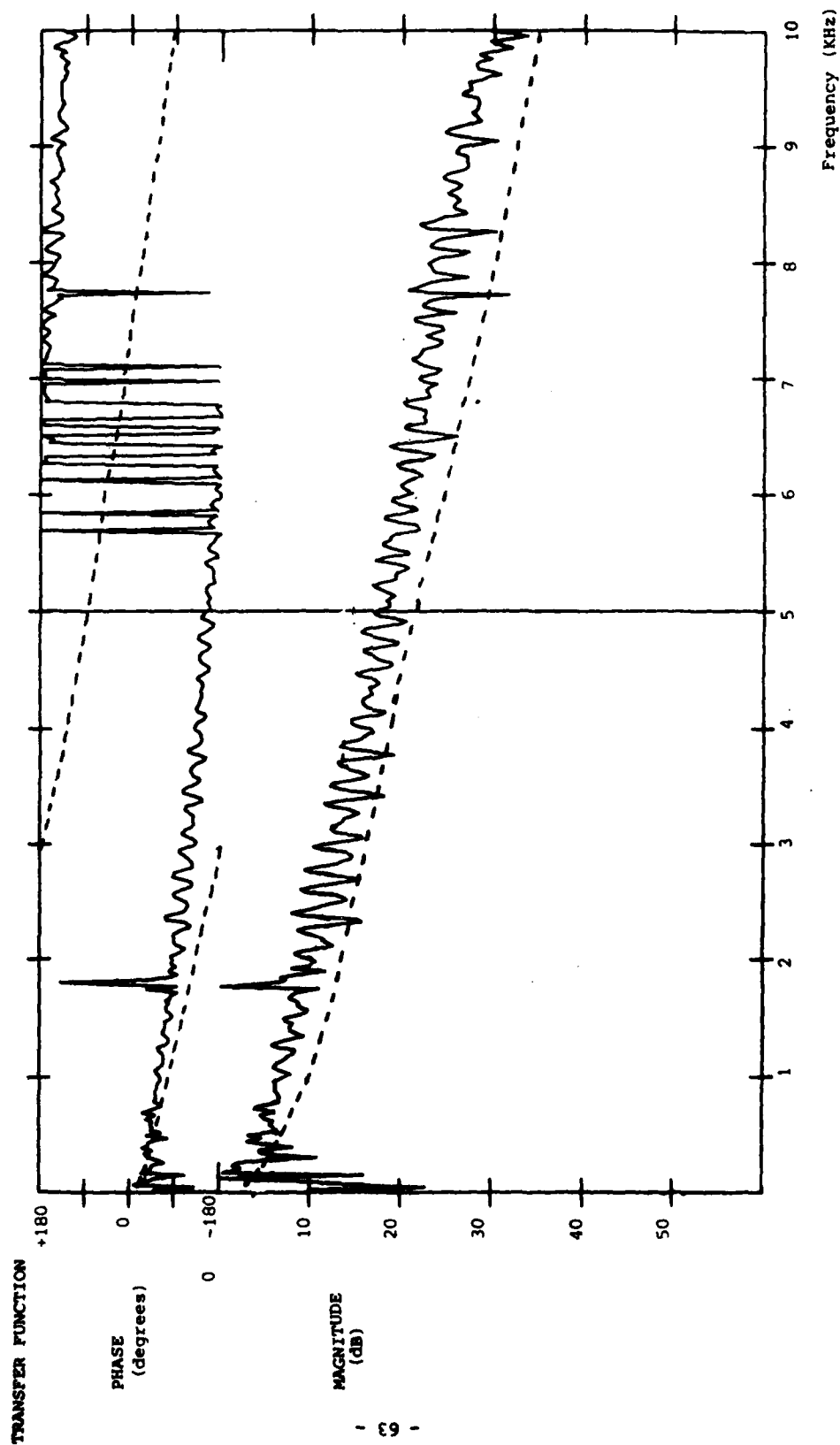
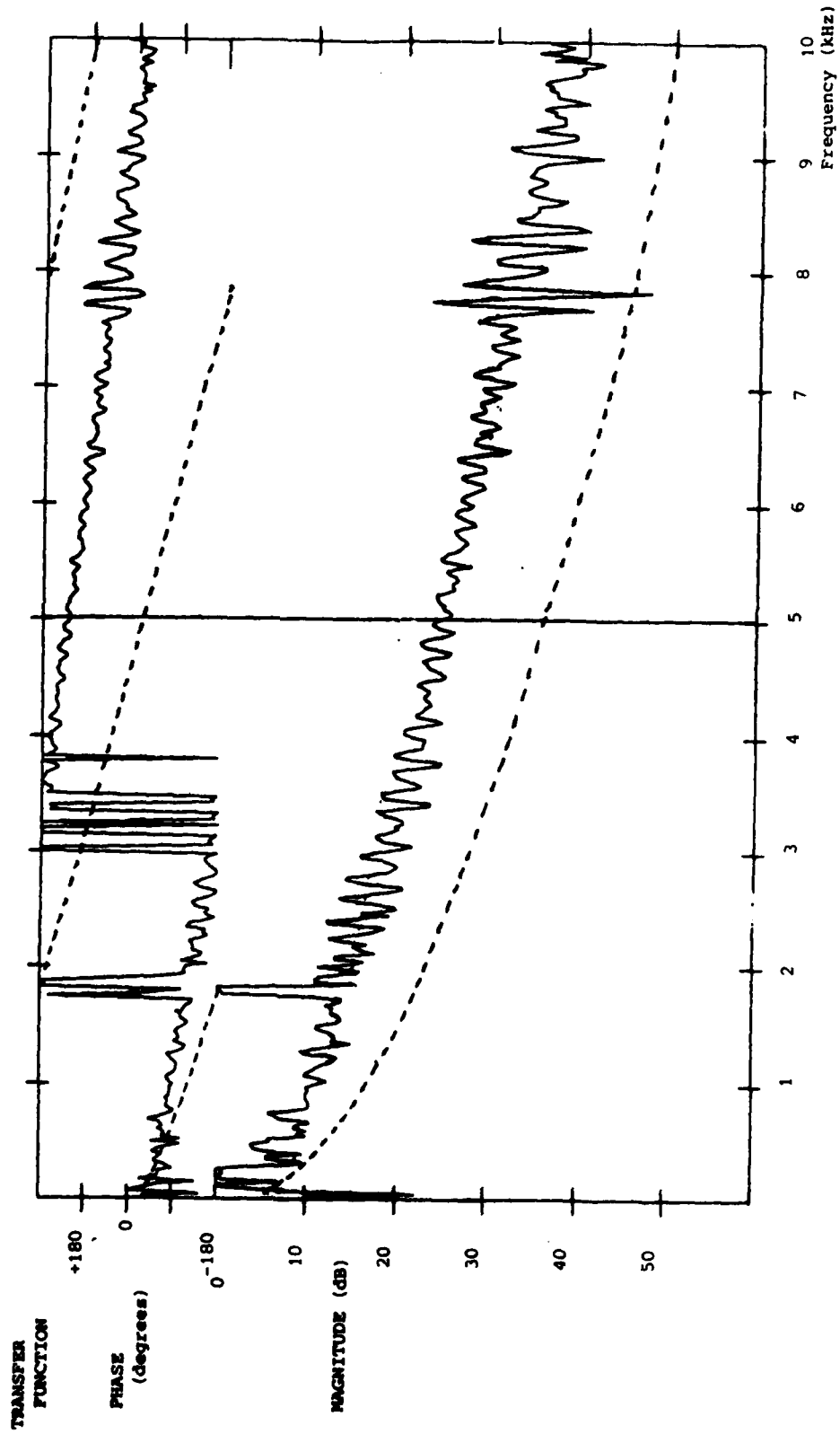


Figure 4.14

COMPARISON OF MEASURED (—) AND PREDICTED (---) VARIATION WITH FREQUENCY OF THE
TRANSFER FUNCTION BETWEEN 2 VERTICALLY SEPARATED MICROPHONES, ONE ON THE SURFACE AND
ONE AT DEPTH 10CM, IN FIBRE GLASS. PREDICTIONS BASED UPON FLOW RESISTIVITY = 11460 MKS RAYLS.M⁻¹,
ASSUMED POROSITY = 1, SHAPE FACTOR RATIO = 1 AND TORTUOSITY = 1.



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5. GROUND AS A PORO-ELASTIC MEDIUM : RESPONSE TO PLANE WAVES

5.1 Biot theory and modification thereto

For a more complete description of ground response to an external acoustic disturbance it is necessary to model the ground as a poro-elastic medium.

We are concerned with the acoustic wave motion in ground, consisting of solid particles and air in the interconnected voids. The solid particles are assumed to be in firm contact with each other and thus constitute a homogeneous elastic frame. The theory of wave motion in such composite media, as derived by Biot [5.1], is widely applicable, and predicts two types of compressional waves and one shear wave.

In this section, we are interested in computing propagation constants for the above three types of body waves, suitable for numerical calculations and for parametric studies. For this purpose, it is convenient to express the equations of motion in terms of two scalar potentials ϕ_1 and ϕ_2 and two vector potentials $\underline{\psi}_1$ and $\underline{\psi}_2$. Let $\underline{u}$ and $\underline{U}$ describe the displacement of the skeletal frame and fluid respectively. Then displacement of the fluid relative to the frame is

$$\underline{w} = \underline{U} - \underline{u}$$

Then we can express the two vectors as

$$\underline{u} = \nabla \phi_1 + \text{curl } \underline{\psi}_1$$

$$\underline{w} = \nabla \phi_2 + \text{curl } \underline{\psi}_2$$

The coupled equations of motion for the scalar potentials ϕ_1 and ϕ_2 are

$$\begin{aligned} HV^2\phi_1 - CV^2\phi_2 &= \rho\ddot{\phi}_1 - \rho_f\ddot{\phi}_2 \\ CV^2\phi_1 - MV^2\phi_2 &= \rho_f\ddot{\phi}_1 - m\ddot{\phi}_2 - \frac{\eta}{k}\dot{\phi}_2 \end{aligned} \quad (5.1a)$$

and another pair of coupled equations of motion for the vector potentials ψ_1 and ψ_2

$$\begin{aligned} G_b\nabla^2\psi_1 &= \rho\ddot{\psi}_1 - \rho_f\ddot{\psi}_2 \\ \frac{\eta}{k}\dot{\psi}_2 &= \rho_f\ddot{\psi}_1 - m\ddot{\psi}_2 \end{aligned} \quad (5.1b)$$

where H, C and M are elastic constants expressed in terms of elastic moduli of the grain-solid, fluid and frame.

$$H = (K_r - K_b)^2 / (D - K_b) + K_b + 4G_b/3$$

$$C = K_r (K_r - K_b) / (D - K_b)$$

$$M = K_r^2 / (D - K_b)$$

and

$$D = K_r [1 + \Omega(K_r/K_f - 1)] \quad (5.2)$$

K_r is the bulk modulus of the individual grains, K_f is the bulk modulus of the pore fluid, G_b is the shear modulus and K_b is the bulk modulus of the assemblage of particles. In Eq. (5.1), ρ is the average mass density, ρ_f is the fluid density, η is the dynamic viscosity coefficient and k is the permeability of the porous frame with dimensions $[L]^2$.

The parameter m , related to the fluid motion, is sometimes called effective fluid density and is given by

$$m = q^2 \rho_f / \Omega$$

where Ω is porosity and q is tortuosity. The parameter q (> 1) accounts for the fact that the fluid flow is not in the direction of macroscopic pressure gradient. Experimentally measured values of q for soils and sand have been reported in literature [5.2]. However in the present study, we use q as a model parameter through the relation

$$q = \Omega^{-n'}$$

where n' is a grain shape factor obtained from studies of gas and solute diffusion.

In the high frequency range, where the viscosity effects begin to be of important, we replace the ratio $\frac{\eta}{k}$ by $\eta F(\lambda)/k$ where $F(\lambda)$ is a complex viscosity correction factor, given by

$$F(\lambda) = (-1/4) \frac{\lambda \sqrt{1} T(\lambda\sqrt{1})}{[1 - 2.0 T(\lambda\sqrt{1})/(\lambda\sqrt{1})]}$$

and

$$\lambda = (\sqrt{s}/n) \sqrt{8q^2 \Omega k_0 / \mu}$$

(5.3)

where μ is kinematic viscosity ($\eta/\sqrt{S}$) is identified as a single model parameter, i.e. shape factor ratio. Note that, from equation (4.1), the complex density of fluid in the pores of a rigid fluid-saturated porous medium may be written

$$\rho(\lambda) = -4F(\lambda) / (\lambda\sqrt{i}\pi(\lambda\sqrt{i}))$$

We seek solutions of the form

$$\phi_1 = A_1 \exp[i(\ell x - \omega t)]$$

and

$$\phi_2 = A_2 \exp[i(\ell x - \omega t)]$$

and Eqs. (5.1) provide a pair of simultaneous equations for A_1 and A_2 . For a non-zero solution, we require that the determinant should vanish and hence we obtain

$$\begin{vmatrix} \rho\omega^2 - H\ell^2 & C\ell^2 - \rho_f\omega^2 \\ \rho_f\omega^2 - C\ell^2 & M\ell^2 - m\omega^2 + i\omega F(\lambda)\eta/k \end{vmatrix} = 0$$

The above equation is quadratic in ℓ^2 and has two distinct roots

$$\ell^2 = [-B \pm \sqrt{B^2 - 4AC}] / 2A$$

as long as $B^2 - 4AC \neq 0$.

where

$$\begin{aligned} A &= C^2 - HM \\ B &= \omega^2 \{ mH - 2C\rho_f + \rho M \} - i\omega F(\lambda) \eta H/k \\ C &= \omega^4 \{ \rho_f^2 - m\rho \} + \omega^2 \{ i\rho \omega F(\lambda) \eta/k \} \end{aligned} \quad (5.6)$$

The two roots of ℓ^2 as given by (5.6) correspond to "fast" and "slow" types of compressional waves. The corresponding propagation constant for shear wave motion can be obtained by assuming

$$\psi_1 = B \exp[i(\ell x - \omega t)], \quad \psi_2 = B_2 \exp[i(\ell x - \omega t)] \quad (5.7)$$

Then it follows from the equations of motion (5.1b)

$$\begin{vmatrix} \rho \omega^2 - G_b \ell^2 & \rho_f \omega^2 \\ \rho_f \omega^2 & m \omega^2 + i \omega F(\lambda) \eta/k \end{vmatrix} = 0 \quad (5.8)$$

The phase velocity, $\omega/\text{Re } \ell$ and attenuation, $\text{Im } \ell$ of three types of waves can be easily computed from (5.6) and (5.8). A FORTRAN program has been written to compute propagation constants for two dilatational waves and one shear wave. The complex frequency correction factor $F(\lambda)$ has been computed using Bessel functions of complex arguments where the calculations are performed in single precision.

The pore parameter λ as given in Eq.(5.3) enables us to consider the influence of various parameter on phase velocity and attenuation of fast and slow compressional wave modes. For the present, we restrict the sensitivity analysis only to the parameters relating to the micro structure and assume base values for the elastic constants of the frame which are typical of sand.

A few remarks concerning the microstructural parameters are in order. We have already note that the tortuosity is dependent on porosity and this gives us two independent parameters n' and Ω . Although permeability k and shape factor ratio are related, in the present study we treat them as another pair of independent parameters. Thus we choose the four parameters Ω , n' , k and $n/\sqrt{s}$ to describe the microstructure of the ground. (For a more detailed discussions on this see references [5.3]).

5.2 Predictions of wave speeds and dispersion in air and water saturated sand.

The program described in previous section has been utilized to compute phase velocity and attenuation in water filled saturated sediments [5.3] where the values assumed for the various parameters used in the computation are listed in Table 1. Here we present, in Figures 5.1 to 5.4 a sensitivity analysis of dispersion and frequency -dependent viscous attenuation (fast wave) to porosity (0.3), grain shape factor ($n' = 1.25$) permeability (10^{-7} cm^2) and shape factor ratio (0.6). The values given in the brackets refer to base values.

The sensitivity of attenuation to permeability, shape factor ratio and grain shape factor are similar to those obtained for variations in permeability and pore size parameter and for structure factor respectively by Stoll and Bryan [5.4]. Viscous attenuation appears to be more sensitive to porosity as it affects tortuosity (structure factor).

TABLE 1

PARAMETER	VALUE	UNITS
ρ_r	2.65	g/cm ³
K_r	3.6×10^{11}	dyn/cm ²
ρ_f	1.0	g/cm ³
η	0.01	dyn - sec/cm ²
G_b	8.27×10^9	dyn/cm ²
K_b	1.38×10^{10}	dyn/cm ²

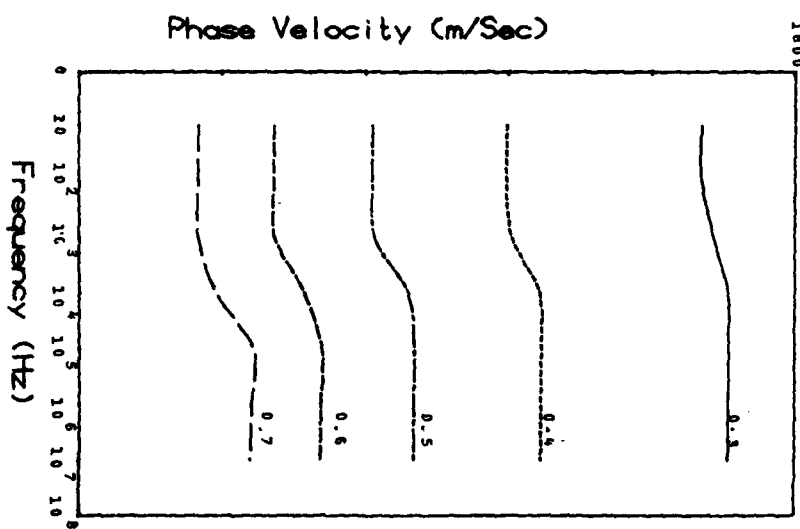
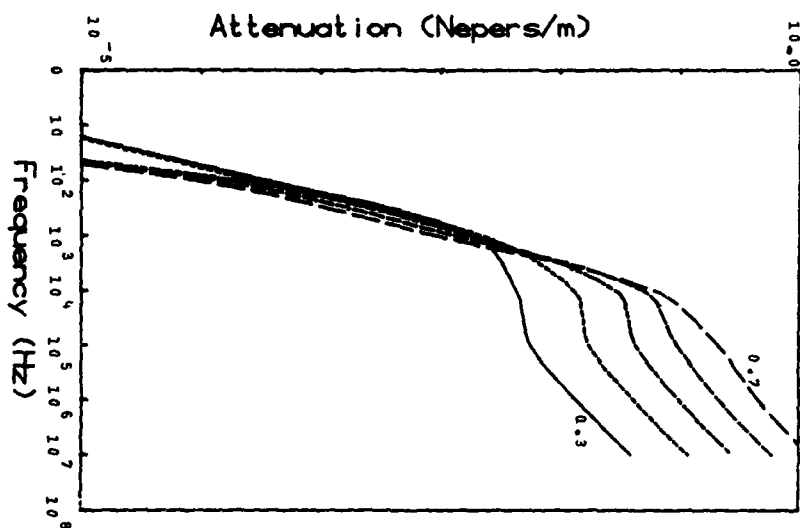


Figure 5.1 Sensitivity of predicted fast wave attenuation and phase velocity versus frequency characteristics in water-saturated sand to variation in porosity over the range $0.3 < n < 0.7$.

FAST WAVE

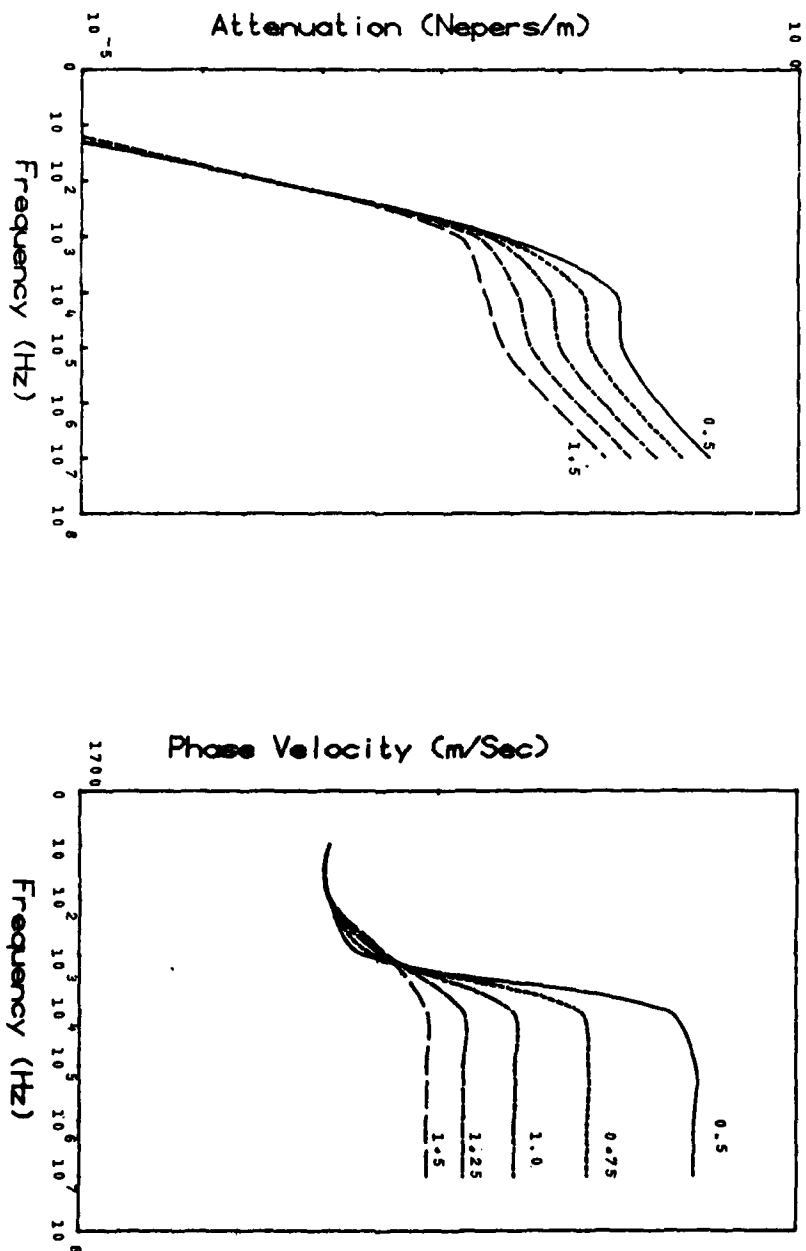


Figure 5.2 Sensitivity of predicted fast wave characteristics in water-saturated sand to variation in grain shape factor.

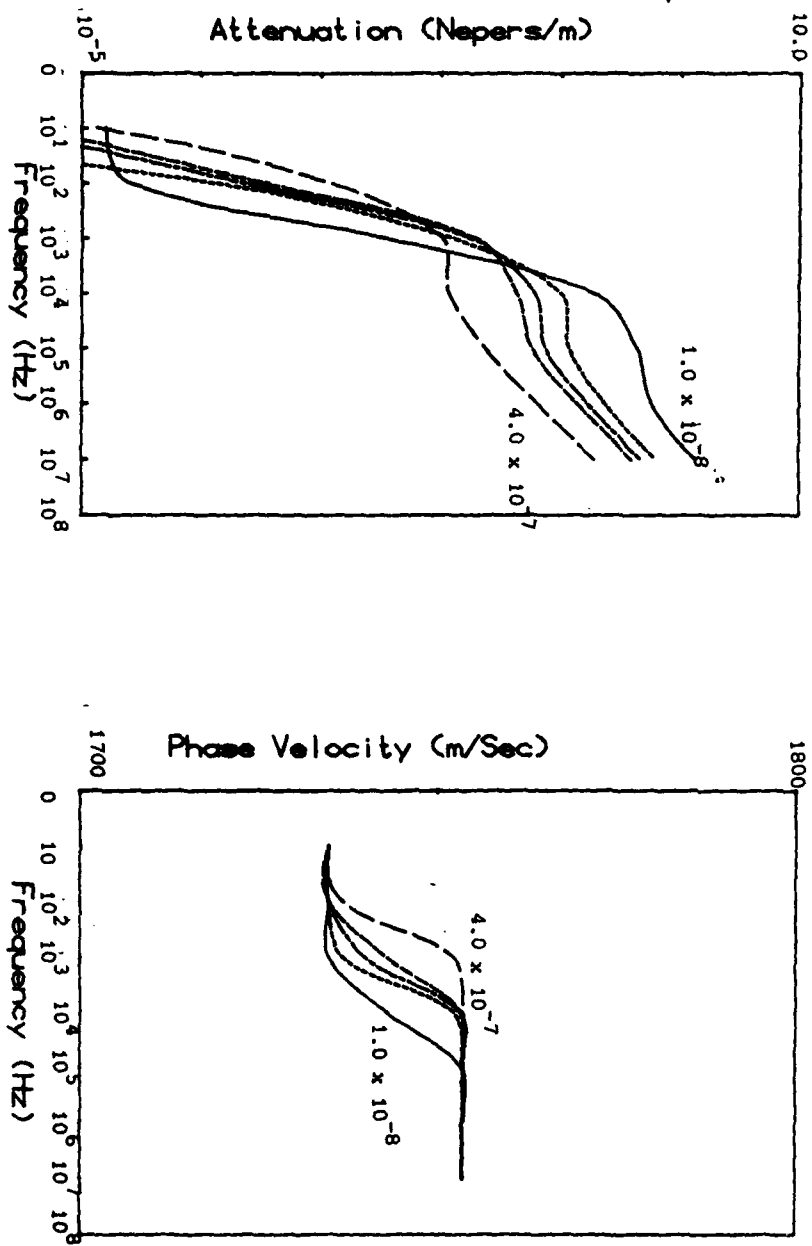


Figure 5.3 Sensitivity of predicted fast wave characteristics in water-saturated sand to variation in permeability.

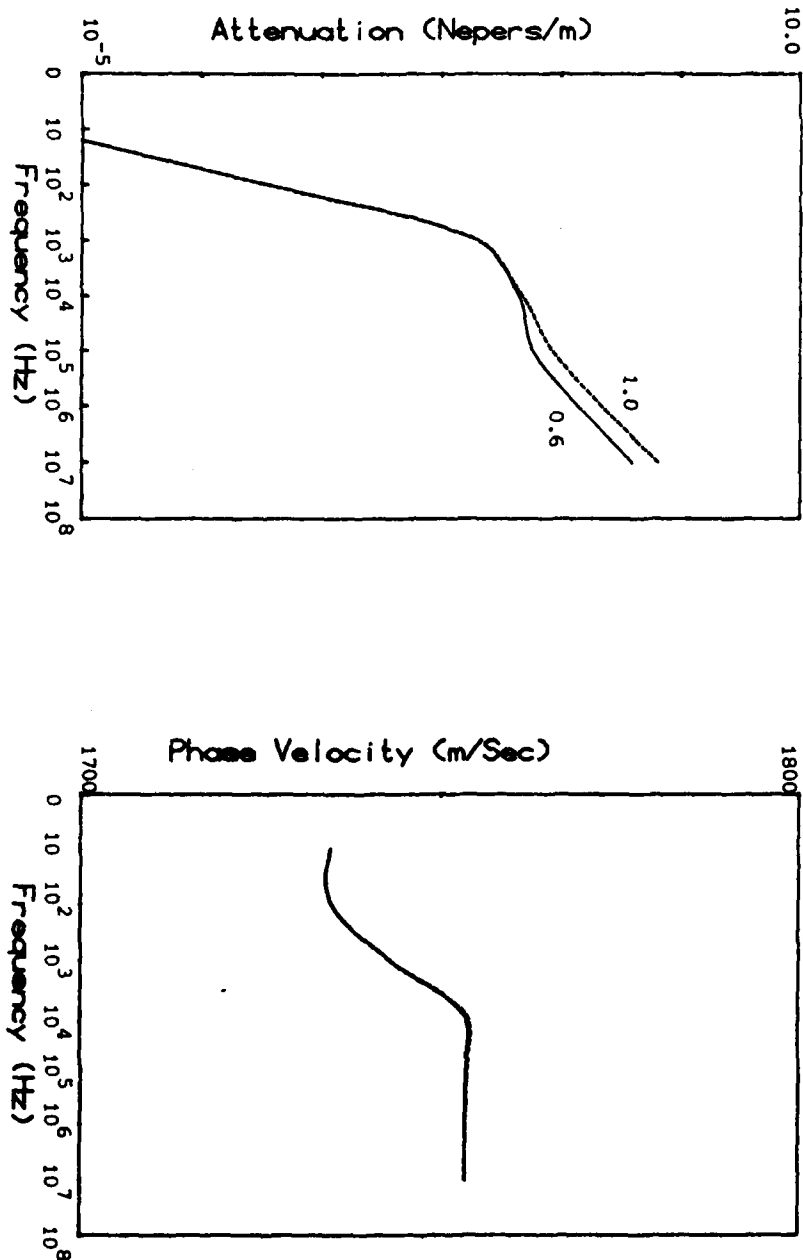


Figure 5.4 Sensitivity of fast wave characteristics in water-saturated sand to variation in shape factor ratio.

In Fig 5.5(a), the prediction for the frequency-dependent phase velocity and in Fig 5.5(b) the effect of variation in frame elasticity for the fast-wave in air-filled sand are shown. The predicted attenuation is less than 0.1dB/m in all cases. In Fig 5.6, we present real and imaginary parts of propagation constants for slow wave for the sake of comparison with those obtained using rigid-porous theory. This shows that the slow wave corresponds to that predicted in the modified fluid model of a rigid porous medium.

The range of values of oscillatory-flow pore-shape factor (see section 4.3) used in these calculations (equation (5.3)) differs from that quoted in Chapter 4 since it is derived from comparison the forms of $F(\lambda)$ for oscillatory flow in a cylindrical pore and parallel-sided slit rather than those of $\rho(\omega)$. The relevant parameter may be labelled $n(\text{viscous})$ where

$$0.75 \leq n(\text{viscous}) \leq 1$$

The bulk modulus of air, K_f , must be complex to account for thermal effects in air-filled porous media. The required complex value may be obtained from equation (4.6) where $K_f = 1/C(\omega)$ and the required value for oscillatory-flow pore shape-factor, $n(\text{thermal})$ is such that

$$0.5 \leq n(\text{thermal}) \leq 1$$

Relationships equivalent to (4.22) may be invoked between the various shape factors viz.

$$2n(\text{viscous}) = n(\text{thermal}) + 1$$

$$2n(\text{viscous}) = 3 - s$$

and

$$1 \leq s \leq 1.5$$

where s is the steady flow shape factor defined in section 4.4.

For the computations in Figures 5.5 and 5.6, we have used the base values $n(\text{thermal}) = 0.5$ and $n(\text{viscous}) = 0.75$. The remaining parameter values are given in Table II.

Note that a single value of oscillatory-flow pore-shape factor $n(\text{viscous})$ may be used if $C(w)$ is calculated from

$$C(w) = (\gamma P_o)^{-1} [2 - \gamma + 2(\gamma - 1) (N_{Pr}^{-1/2} \lambda \sqrt{1}) T(N_{Pr}^{-1/2} \lambda \sqrt{1}) / F(N_{Pr}^{-1/2} \lambda \sqrt{1})]$$

where γ is the ratio of specific heats, N_{Pr} is the Prandtl Number and P_o is atmospheric pressure.

TABLE II

Parameters Required By Biot Theory
(As Modified By Attenborough/Rayleigh)

Parameter	"Sand" Value
Grain Bulk Modulus	3.6×10^{11} dyn/cm ²
Grain Mass Density	2.68 g cm ⁻³
Frame Bulk Modulus	8.27×10^9 dyn/cm ²
Frame Rigidity Modulus	8.27×10^9 dyn/cm ²
Fluid Bulk Modulus (isothermal)	
Dynamic Viscosity of Fluid	.00184 dyn-sec/cm ²
Porosity	0.4
Permeability	3.76×10^{-6} cm ²
Grain shape factor	0.5
Shape factor ratio ($n/\sqrt{s}$)	0.6

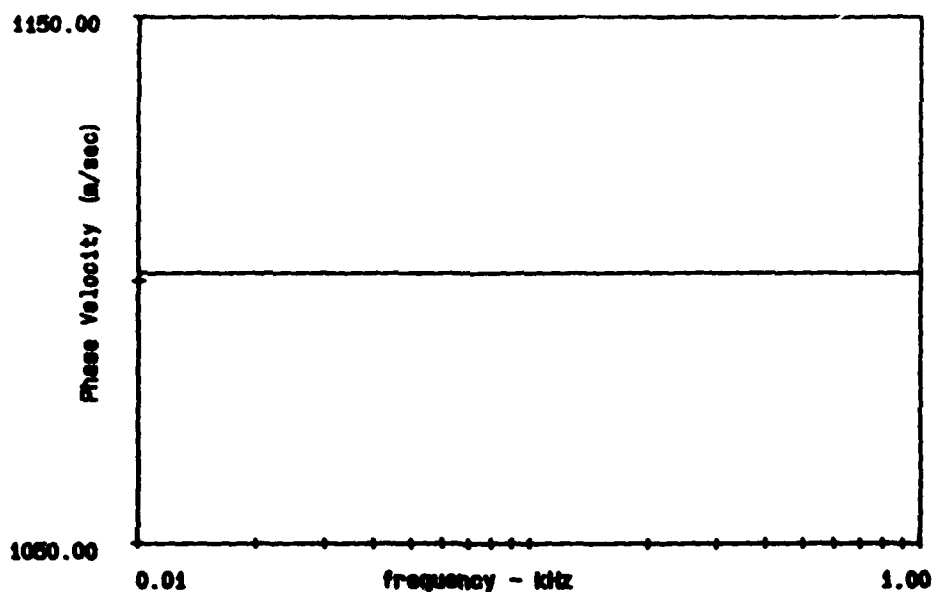


Figure 5.5(a) Predicted fast-wave phase velocity for air-filled sand with characteristics given by Table II.

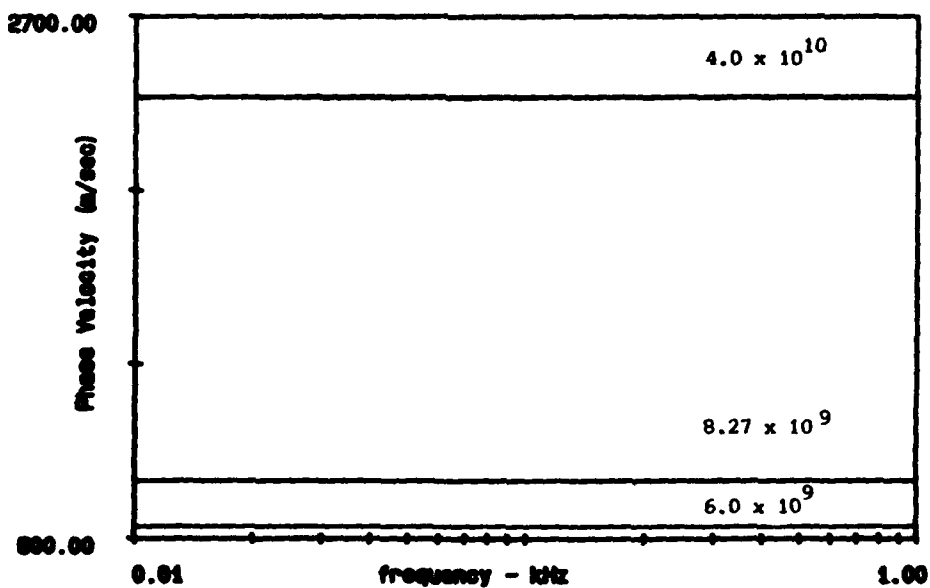
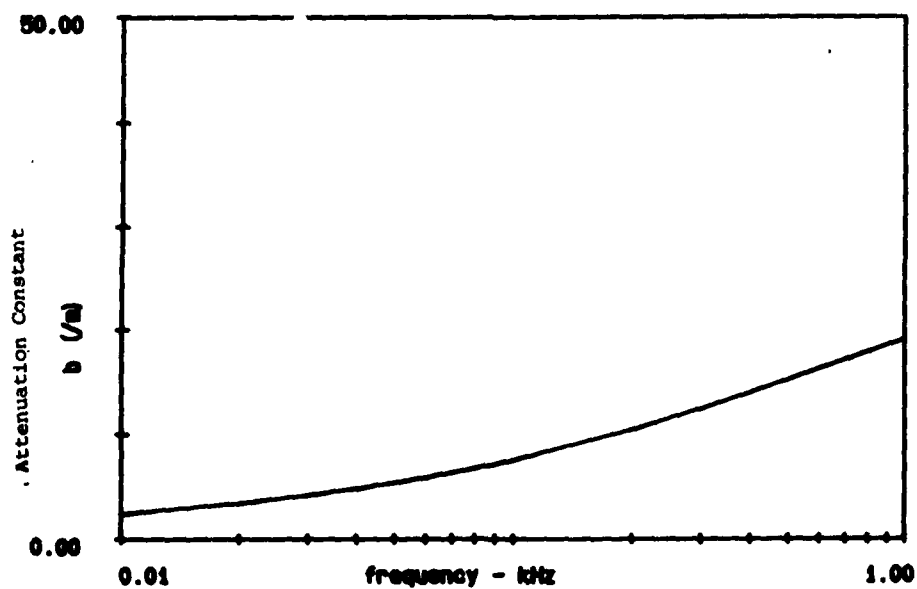
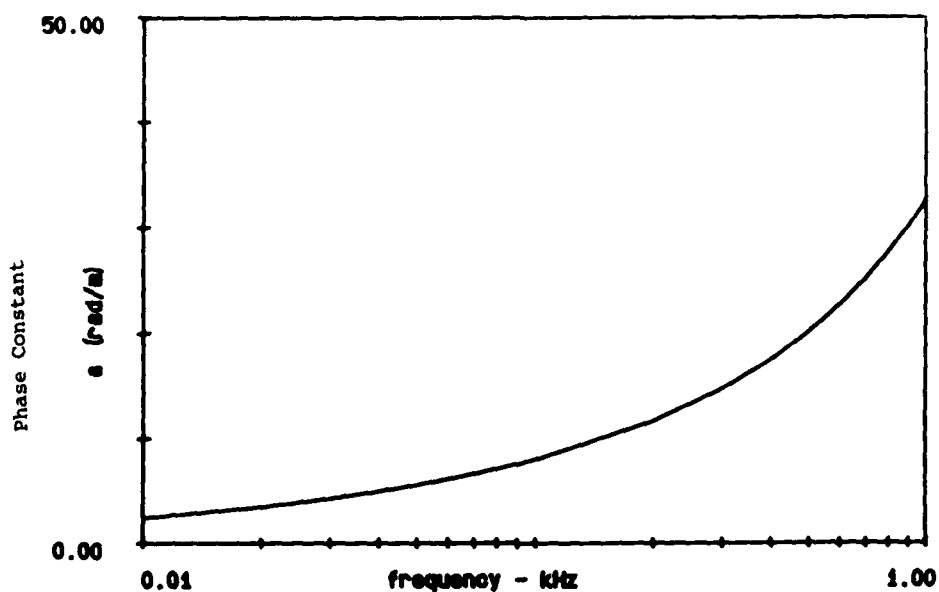


Figure 5.5(b) Variation of fast-wave speed characteristic in air-filled sands with labelled values of frame elastic constants



5.6 Predicted real and imaginary parts of slow-wave propagation constant in air-filled sand with characteristics given in Table II.

As has been shown, the values predicted for the fast wave propagation constant in air-saturated sand are very sensitive to the value taken by the frame rigidly (shear) modulus (assumed equal to the bulk modulus). The shear modulus for soils is affected by various soil parameters and the state of stress. The important parameters include [5.7] effective octahedral normal stress, void ratio, grain characteristics, shape, size, grading, mineralogy, amplitude strain, ambient stress history, secondary time-dependent effects, degree of water saturation, octahedral shear stress, frequency of vibration, soil structure and temperature. This complexity implies difficulty with selecting values typical of any given soil type. Consequently three values have been chosen which indicate a typical range and enable a study of the sensitivity of propagation characteristics to shear modulus in air-filled soils. The values chosen [5.8] are set out below:-

Soil type	Shear Modulus (G) dyn cm ⁻²
Santa Barbara clay	1.38×10^8
AGSCO No.2	5.17×10^8
Ottawa sand	1.86×10^9

Figures 5.7 to 5.10 show the relative speeds and attenuations of the three possible wave types predicted by assuming equal shear and bulk moduli and using the base values of other parameters given in Table II with the exceptions $n' = 1$ and shape factor ratio = 0.75. Figure 5.7 shows that, for a relatively high value of rigidity modulus, the 'slow' wave is clearly the slowest up to 1kHz, the fast wave having a frequency independent value somewhat greater than the velocity of sound in air and similar to the value predicted for dry sand. From Figures 5.7 and 5.8 it is clear that a small decrease in rigidity modulus causes the shear wave to become faster than the slow wave near 1kHz. Note that both of the rigidity modulus values assumed in generating Figures 5.7 and 5.8 produce fast wave speeds somewhat less than the speed of sound in air.

Further decrease in the rigidity modulus to the value assumed in Figure 5.10 produces the interesting situation where the 'slow' and 'fast' waves are predicted to exchange roles above 100 Hz. It should be noted that still lower values of rigidity modulus are possible for soils and these will produce 'slow' and 'fast' wave exchange at still lower frequencies. Finally addition of a complex component of the shear modulus is shown, in Figure 5.11, to produce, as expected, much higher attenuations for the 'fast' and shear waves.

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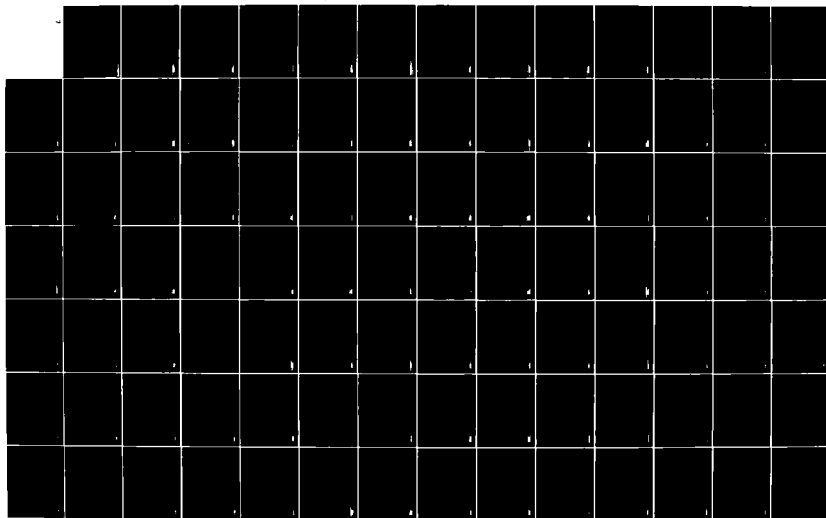
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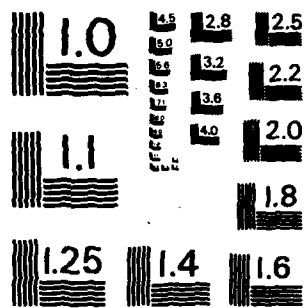
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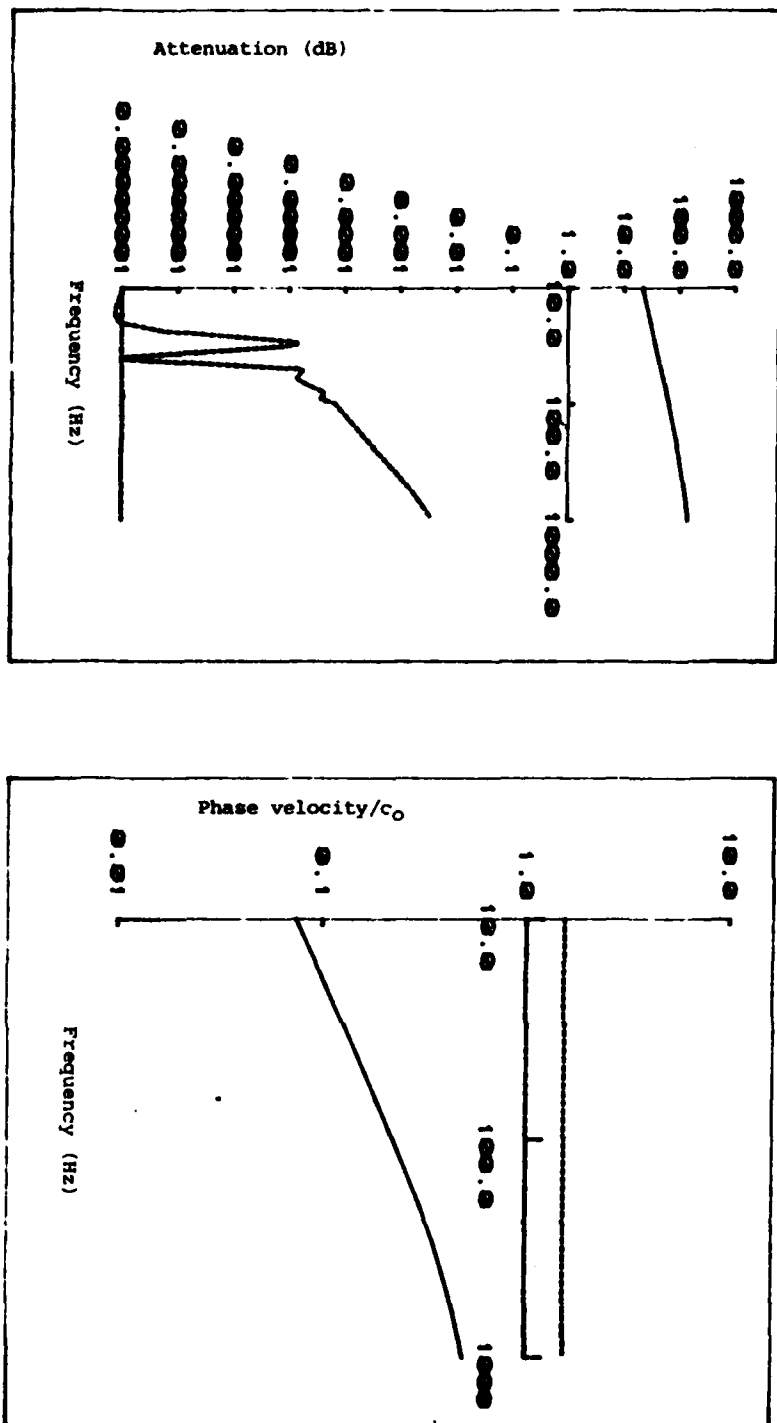


Figure 5.7 Predicted propagation constants for dry soil of rigidity modulus 1.86×10^9 dyn cm^{-2}
 - - - - - fast wave ——— slow wave

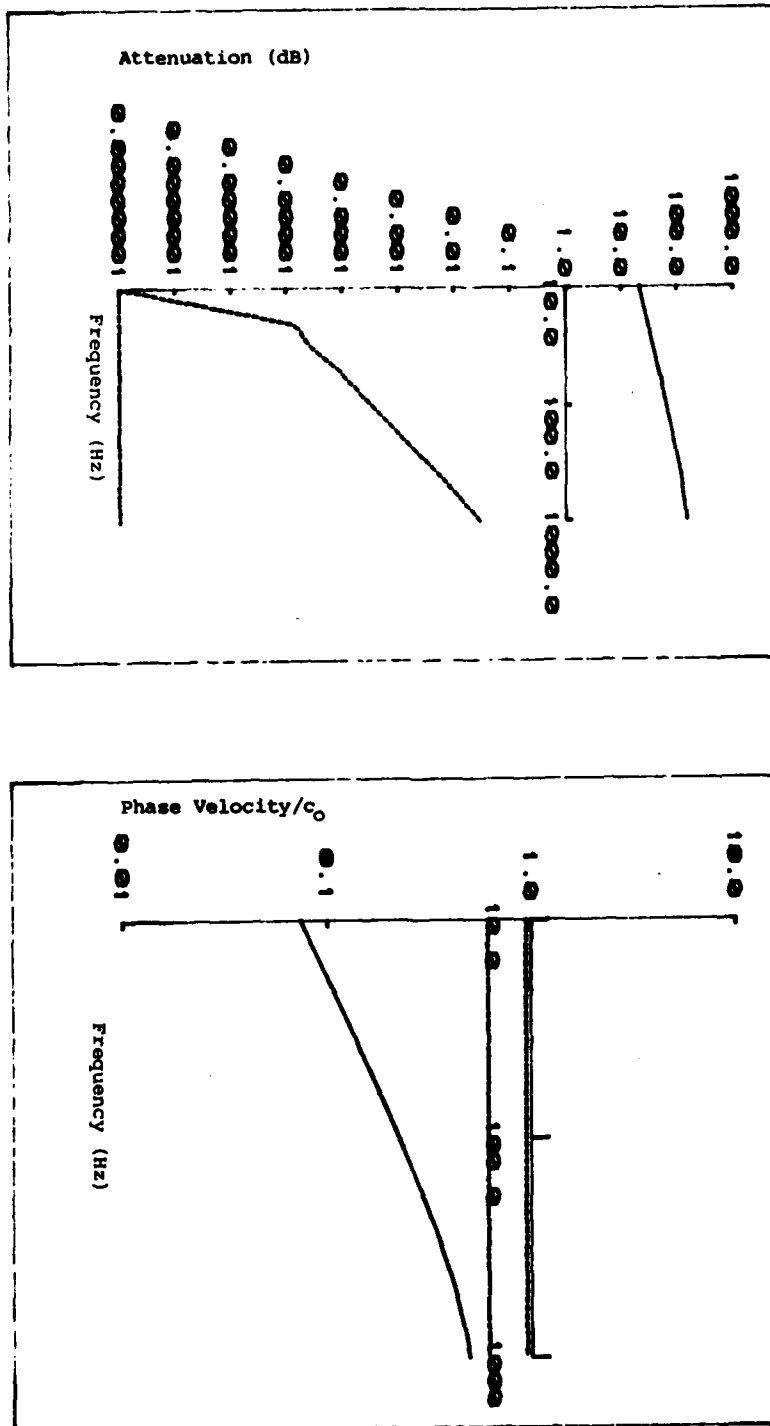


Figure 5.8 Predicted propagation constants for dry soil of rigidity modulus $7.03 \times 10^8 \text{ dyn cm}^{-2}$

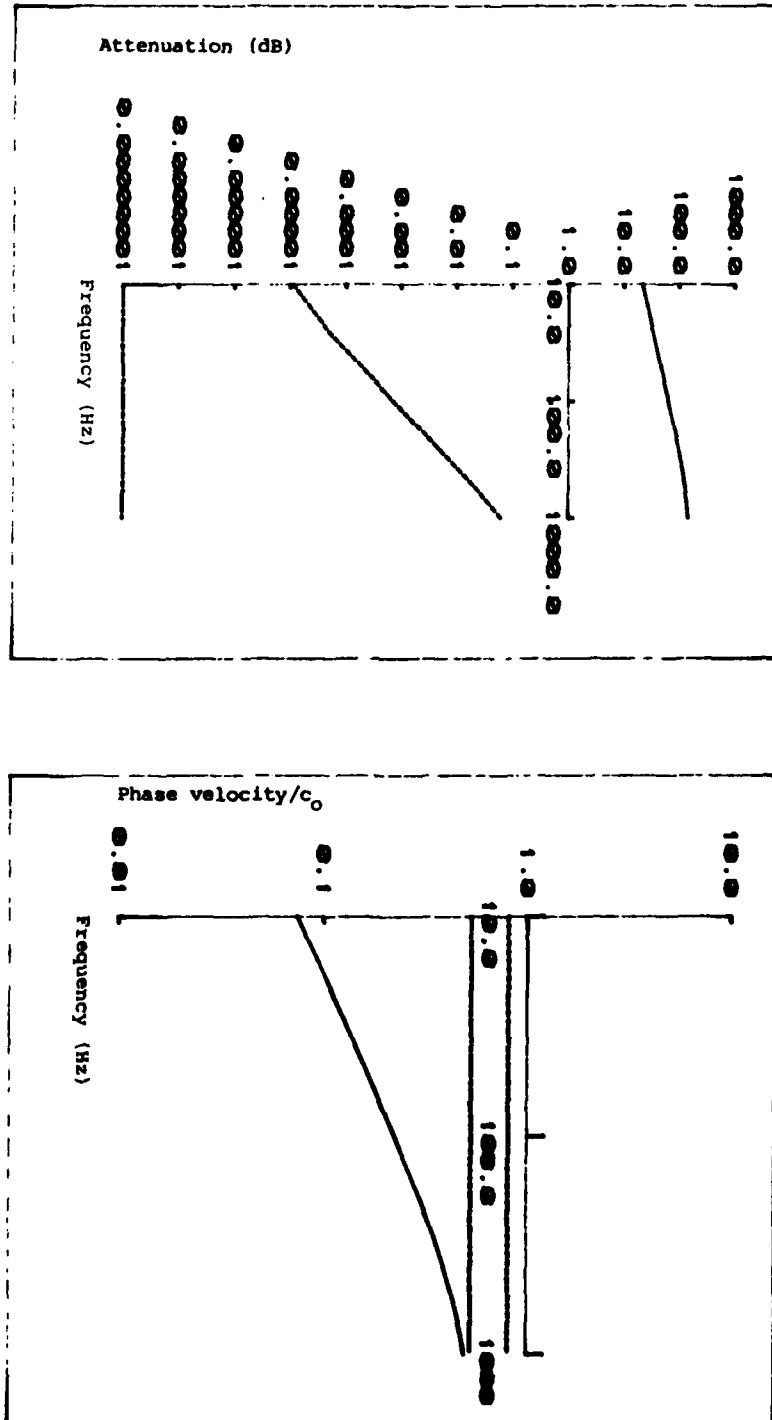


Figure 5.9 Predicted propagation constants for dry soil of rigidity modulus $5.17 \times 10^8 \text{ dyn cm}^{-2}$

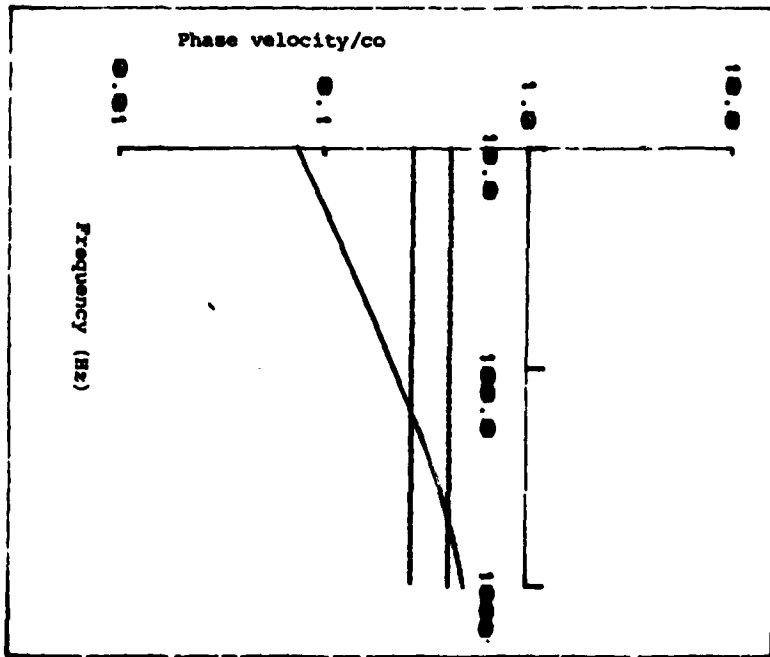
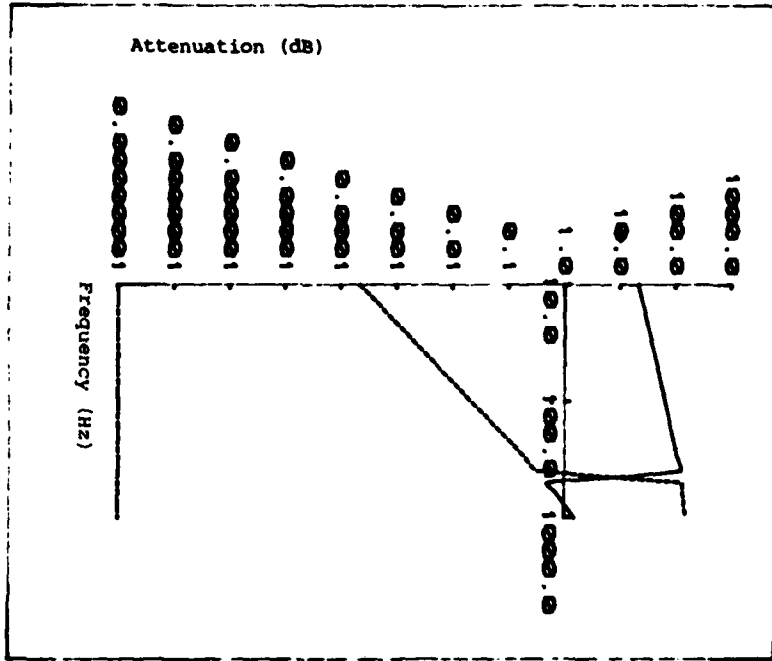


Figure 5.10 Predicted propagation constants for dry soil of rigidity modulus $1.38 \times 10^8 \text{ dyn cm}^{-2}$

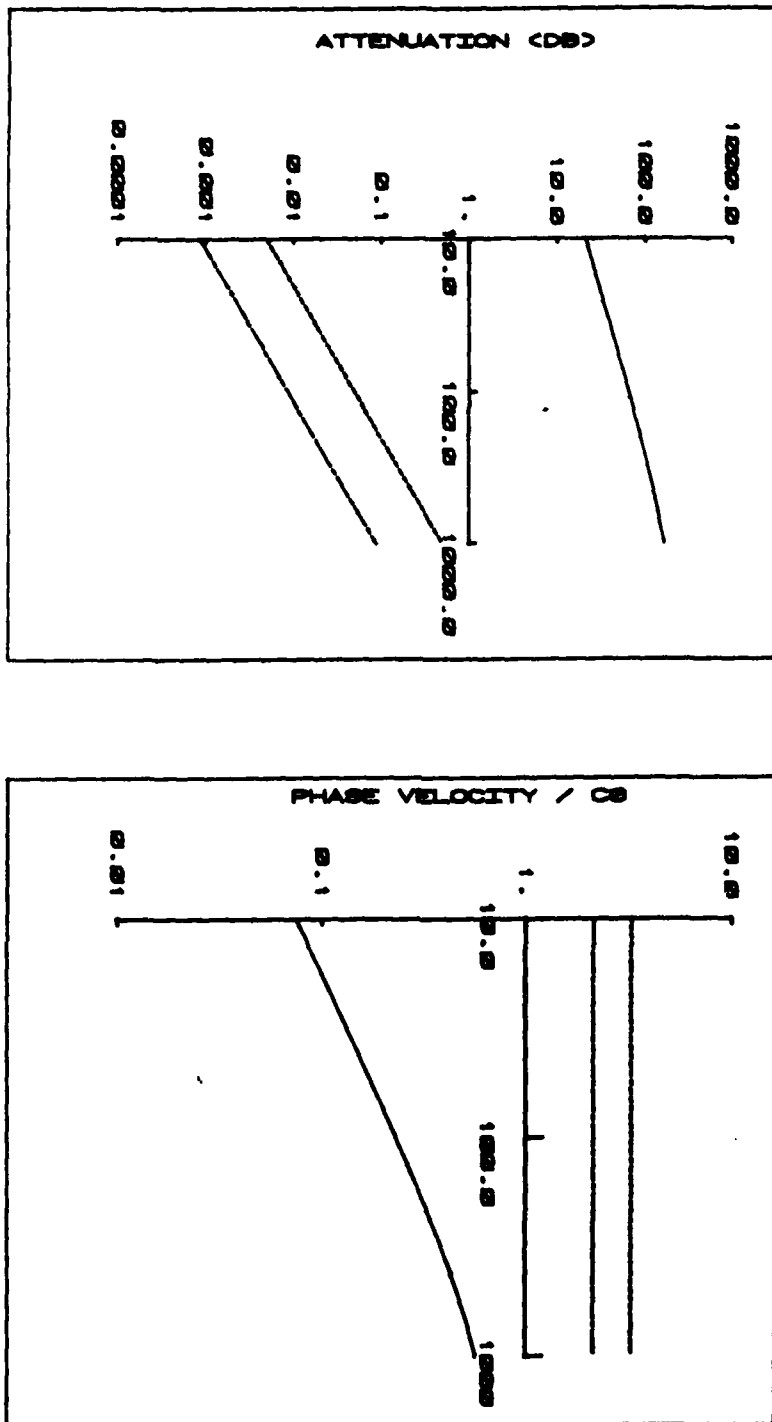


Figure 5.11 Predicted propagation constants for dry soil of complex rigidity modulus $8.77 \times 10^9 (1 + i0.02)$

5.3 Predictions of surface solid particle velocity resulting from plane wave incidence

The ratio of solid particle velocity to the pressure on the surface has been used to describe the ground response. To facilitate computations of this, we make use of a different but equivalent formulation to that presented in section 1. The present formulation enables us to express both solid and fluid displacement in terms of two scalar potentials ϕ_1 and ϕ_2 and two vector potentials ψ_1 and ψ_2 . We write

$$\begin{aligned}\underline{u} &= \text{grad } \phi_1 + \text{curl } \psi_1 \\ \underline{U} &= \text{grad } \phi_2 + \text{curl } \psi_2\end{aligned}\quad (5.9)$$

and they obey the following coupled differential equations

$$\begin{aligned}\nabla^2(P\phi_1 + Q\phi_2) &= \frac{\partial^2}{\partial t^2}(\rho_1\phi_1) + bF(\lambda) \frac{\partial}{\partial t}(\phi_1 - \phi_2) \\ \nabla^2(Q\phi_1 + R\phi_2) &= \frac{\partial^2}{\partial t^2}(\rho_2\phi_2) - bF(\lambda) \frac{\partial}{\partial t}(\phi_1 - \phi_2)\end{aligned}\quad (5.10)$$

and

$$\begin{aligned}N\nabla^2\psi_1 &= \frac{\partial^2}{\partial t^2}(\rho_2\psi_1) + bF(\lambda) \frac{\partial}{\partial t}(\psi_1 - \psi_2) \\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_2\psi_2) - bF(\lambda) \frac{\partial}{\partial t}(\psi_1 - \psi_2)\end{aligned}\quad (5.11)$$

where $b = \eta/k$, ρ_1 and ρ_2 are densities of solid and fluid respectively.

The elastic parameters P , Q , R can be related to K_r [5.5].

bulk modulus of grains, K_f (bulk modulus of fluid), K_b (bulk modulus of aggregate)

N is the shear modulus of the solid and composite

$$P = \frac{(1 - \Omega) \left[(1 - \Omega) - \frac{K_b}{K_r} \right] K_r + \Omega \frac{K_r}{K_f} K_b}{1 - \Omega - \frac{K_b}{K_r} + \Omega \frac{K_r}{K_f}} + \frac{4}{3} N$$

$$Q = \frac{(1 - \Omega - \frac{K_b}{K_r}) \Omega K_r}{(1 - \Omega - \frac{K_b}{K_r} + \frac{K_r}{K_f})}$$

$$R = \frac{\Omega^2 K_r}{(1 - \Omega - \frac{K_b}{K_r} + \Omega \frac{K_r}{K_f})}$$

(5.12)

From the computational point of view, the present formulation does not pose any additional problem as the previous program can be still used and Eqs.(5.12) provide for the translation between the two notations.

For comparison with measurements it is convenient to calculate the square of the ratio of surface particle velocity to pressure henceforth called the intensity ratio. These calculations are described for obliquely-incident plane waves on a semi-infinite poro-elastic medium and normal-incidence on a rigidly-backed poro-elastic layer in Appendix 4.

Figures 5.12 and 5.13 show plots of normal-incidence intensity ratio against frequency for the parameter values given in Table II. These are expected to be representative of sand. It should be noted that the number of layer resonances increases with increasing depth.

The variation of intensity ratio with angle of incidence, on a semi-infinite layer of dry sand, measured away from the normal and with frequency is shown in Figure 5.14. The ratio decreases with increasing angle of incidence and, for any given angle, the ratio decreases with increasing frequency.

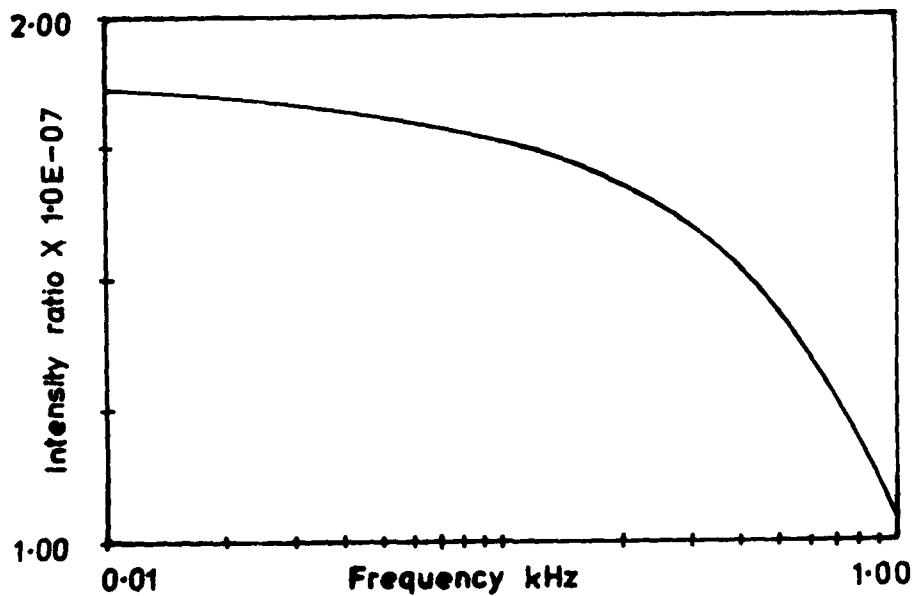


Figure 5.12 Predicted semi-infinite-sand surface intensity ratio for normally-incident plane waves

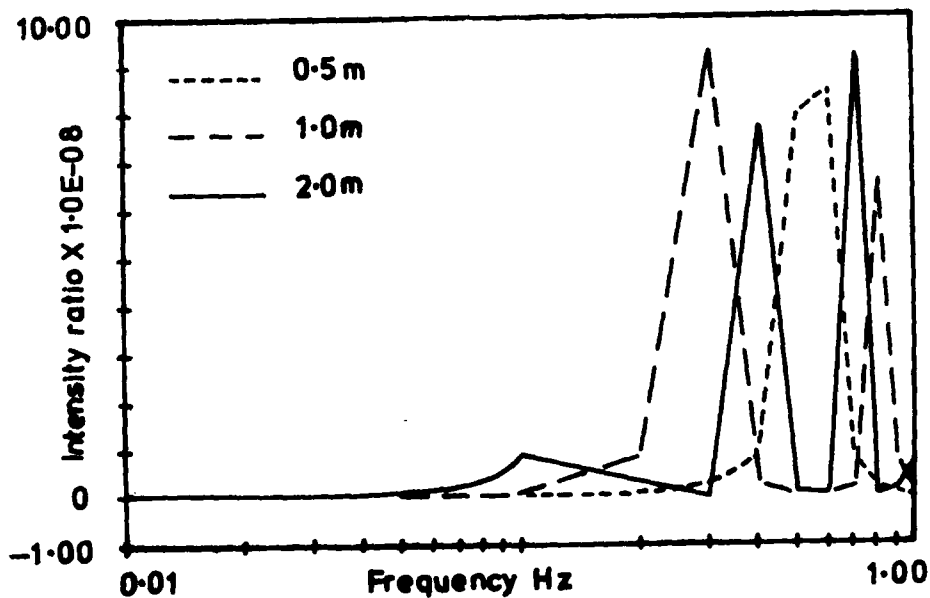


Figure 5.13 Predicted rigidly-backed sand layer surface intensity ratio for normally-incident plane waves and layer depths between 0.5m and 2.0m.

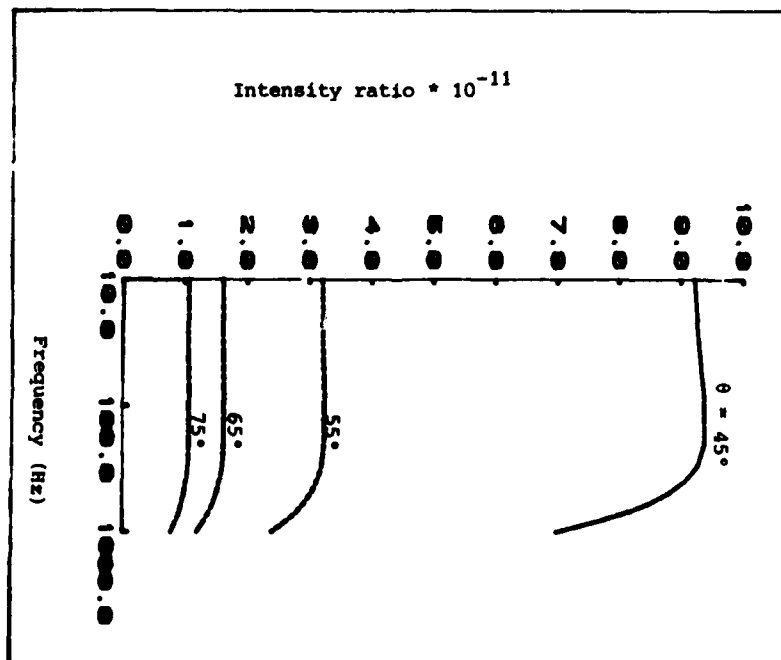
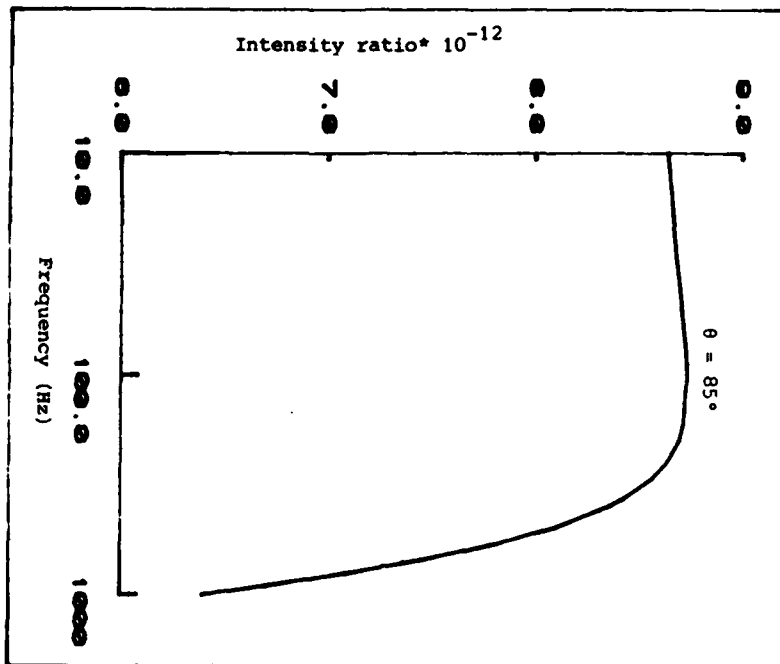


Figure 5.14 Surface intensity ratio predicted for oblique-incidence on a semi-infinite layer of dry sand. Angles of incidence are measured away from the normal.

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6. THE SURFACE MOTION OF A PORO-ELASTIC HALF-SPACE DUE TO AN INCIDENT SPHERICAL SOUND WAVE.

6.1 Introduction

While the treatment of the ground surface as a rigid porous medium gives an adequate description of the reflected sound wave and the transmitted slow wave (consisting of air motion within the pores), any attempt to determine the seismic response must take into account the finite rigidity and density of the solid medium.

Here we take account of frame elasticity by using the theory of propagation in poroelastic media outlined in the previous chapter. This has been developed extensively by Biot in a number of publications. In these he does not always use the same scheme of notation, and we opt for the system given by him in 1961 [6.1]. This gives the most compact expressions in subsequent derivations, and many of the material parameters are defined in such a way as to be identical to those of the corresponding rigid porous material.

The problem of spherical wave incidence on a poro-elastic half space is approached in the same manner as that for the more well-known cases of fluid-fluid, fluid-solid and solid-solid interfaces, which are described by, for example, Ewing et al [6.2]. The axisymmetric displacement potentials are expressed in terms of

inverse Hankel transforms, the transformed potentials being determined by the boundary conditions at the interface. In this case there are four potentials, corresponding to the reflected compressional wave in the upper medium, the transmitted slow and fast compressional waves, and the transmitted shear wave.

The complexity of the resulting integrals prohibits a thorough analysis using the saddle point method as in the fluid-fluid case used to model a rigid porous ground surface. However, a simpler, less rigorous approach enables us to obtain asymptotic expressions for the various wave types at grazing incidence, and it is also straight forward to identify the propagation paths and evaluate times of arrival for transient sources.

Some simplification is possible by considering the limiting case when the bulk modulus and density of the saturating fluid are much smaller than the elastic modulus and density of the solid. The propagation constants of the fast and shear wave are unaffected by the presence of the fluid, while that of the slow wave is the same as for the rigid porous case. This 'light fluid' limit should be applicable for air-filled ground surfaces.

Feng and Johnson [6.3, 6.4] have recently considered the problem of a fluid/porous solid interface. Their treatment, otherwise comprehensive, gives results for the high frequency limit when the propagation constants for the various wave types are all real and non-dispersive. This allows them to define 'true' surface waves with a velocity less than any of the body waves, and to implement the Cagniard method when evaluating the response to a pulsed source. However, the simplest possible treatment of ground surfaces must admit a complex slow wave propagation constant, and as a result we are forced to give a more general analysis of surface wave contributions.

6.2. Displacement potential in the upper medium

The vector of fluid displacement in the upper medium is defined in terms of the potential ψ :

$$\underline{v} = \underline{\nabla}\psi \quad (6.1)$$

The acoustic pressure is given by

$$p = -K_a \underline{\nabla} \cdot \underline{v} = -K_a \nabla^2 \psi \quad (6.2)$$

where K_a is the bulk modulus

ψ obeys the wave equation

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} \quad (6.3)$$

everywhere in the upper medium except at the source.

6.3 Displacement potentials in the lower medium

If the displacement vector of the solid matrix is $\underline{u}$, then following Biot [6.1] we represent the displacement of fluid in the pores relative to the solid by the vector

$$\underline{w} = \Omega(\underline{U} - \underline{u}) \quad (6.4)$$

where Ω is the porosity and $\underline{U}$ is the actual average fluid displacement. Hence $\underline{w}$ is the relative volume displacement of the fluid over unit area of the bulk medium.

Because the solid can support shear stress the displacement are not in general irrotational. They can be detained in terms of scalar and vector potentials as follows:

$$\underline{u} = \nabla\phi_1 + \nabla \times \chi_1 \quad (6.5)$$

$$\underline{u} = \nabla\phi_2 + \nabla \times \chi_2 \quad (6.6)$$

The coupled equations of propagation for dilatational waves are

$$\nabla^2(\alpha\phi_1 + \alpha\phi_2) = \frac{\partial^2}{\partial t^2} (\rho\phi_1 + \rho_1\phi_2) \quad (6.7a)$$

$$\nabla^2(\alpha\phi_1 + \phi_2) = \frac{\partial^2}{\partial t^2} (\rho_1\phi_1 + \phi_2) + \frac{\eta F}{k} \frac{\partial \phi_2}{\partial t} \quad (6.7b)$$

and for shear waves

$$\mu v^2 \Delta_s = \frac{\partial^2}{\partial t^2} (\rho \Delta_s + \rho_1 \Delta_s) \quad (6.8a)$$

$$\frac{-\eta F}{k} \frac{\partial \Delta_s}{\partial t} = \frac{\partial^2}{\partial t^2} (\rho_1 \Delta_s + m \Delta_s) \quad (6.8b)$$

where H, μ are elastic moduli of the medium with sealed pores,

M is the effective bulk modulus of the fluid in the pores,

ρ_1 is the density of the fluid,

ρ is the average density of the porous medium (including fluid and solid),

m is the effective inertial density of the fluid in the pores,

η is the fluid viscosity,

k is the permeability of the porous medium,

F is a complex frequency dependent function which corrects for departures from Poiseuille flow,

and α is a parameter which relates changes in pore volume to overall changes in volume.

6.4 Boundary conditions at the interface.

The definitions of fluid and solid displacement used here lead to particularly simple expressions for the boundary conditions.

Assuming open pores at the interface, the boundary condition for continuity of fluid displacement is

$$\underline{n} \cdot (\underline{u} + \underline{w}) = \underline{n} \cdot \underline{\nabla} \psi$$

where $\underline{n}$ is a unit vector in the vertical direction. In terms of the displacement potentials this becomes

$$\underline{n} \cdot \underline{\nabla} (\phi_2 + \phi_3) + \underline{n} \cdot \underline{\nabla} \times (\chi_2 + \chi_3) = \underline{n} \cdot \underline{\nabla} \psi \quad (6.9)$$

Biot's notion of 'effective stress' is particularly appropriate for the application of stress boundary conditions. The effective stress, t'_{ij} , is that part of the total stress within the porous medium (acting on both fluid and solid) in excess of the local fluid pressure. The components of effective stress acting in the plane of the interface must therefore be equal to zero. Following Biot's definitions and using Cartesian coordinates x, y, z with the z axis in the vertical normal direction, we have

$$\begin{aligned} t'_{zz} &= 2\mu e_z + (H - 2\mu - \alpha M)e + M(1 - \alpha)\zeta \\ &= 0 \end{aligned} \quad (6.10)$$

$$t'_{yz} = \mu \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right) = 0 \quad (6.11)$$

$$t'_{xz} = \mu \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) = 0 \quad (6.12)$$

where $e_z = \frac{\partial u_z}{\partial z}$

$$e = \nabla \cdot \underline{u}$$

$$\zeta = -\nabla \cdot \underline{w}$$

Finally, the pressure in the pores must be equated to that in the upper medium, giving

$$p + \alpha M e - M \zeta = 0 \quad (6.13)$$

We later express the stress boundary conditions in terms of the potentials when transforming to axisymmetric coordinates.

6.5 Time dependence

We subsequently assume that all displacements and potentials vary as $e^{-i\omega t}$, so that

$$\begin{aligned} \underline{u} &\rightarrow \underline{u} e^{-i\omega t} \quad \text{etc.} \\ \psi &\rightarrow \psi e^{-i\omega t} \quad \text{etc} \end{aligned}$$

In the wave equations we also have

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$$

Equation (6.3) becomes

$$\nabla^2 \psi + k_1^2 \psi = 0 \quad (6.14)$$

$$\text{where } k_1 = \frac{\omega}{c}$$

Equations (6.7a), (6.7b) become

$$\nabla^2 (M\phi_1 + m\phi_2) + \omega^2 (\rho_1 \phi_1 + \rho_2 \phi_2) = 0 \quad (6.15a)$$

$$\nabla^2 (m\phi_1 + M\phi_2) + \omega^2 (\rho_1 \phi_1 + \rho_2 \phi_2) = 0 \quad (6.15b)$$

$$\text{where } \rho_2 = m + \frac{i\eta P}{\omega k}$$

The complex density ρ_2 is identical to that for the equivalent rigid porous material. The reduction to this case can be achieved simply by setting the solid displacement potential ϕ_s equal to zero in equation (6.15b). This gives

$$\nabla^2 \phi_2 + \frac{\omega^2 M}{\rho_2} \phi_2 = 0$$

From equation (4.16) we can deduce the complex density for a rigid porous material and we have

$$\rho_2 = (q^2/\Omega)\rho(\omega), \quad m = (q^2/\Omega)\rho_1$$

From the associated wave equation (4.17) we can now also deduce that

$$\frac{1}{M} = \Omega C(\omega)$$

For the shear wave motion, equations (6.8a), (6.8b) become

$$\mu \nabla^2 \underline{x}_3 + \omega^2 (\rho \underline{x}_3 + \rho_1 \underline{x}_2) = 0 \quad (6.16a)$$

$$\rho_1 \underline{x}_3 + \rho_2 \underline{x}_2 = 0 \quad (6.16b)$$

6.6. The axisymmetric case

In cylindrical coordinates we have, typically,

$$\underline{v}(x,y,z) \rightarrow \underline{v}(r, \theta, z)$$

and in the axisymmetric case

$$u_{\theta} = 0$$

$$w_{\theta} = 0$$

$$v_{\theta} = 0$$

$$\text{and } \partial/\partial\theta = 0$$

Equations (6.5) and (6.6) can now be written

$$\begin{aligned} w_r &= \frac{\partial \phi_2}{\partial r} + \frac{\rho_1}{\rho_2} \frac{\partial^2 \phi}{\partial r \partial z} \\ w_z &= \frac{\partial \phi_2}{\partial z} - \frac{\rho_1}{\rho_2} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) \end{aligned} \quad (6.17)$$

$$\begin{aligned} u_r &= \frac{\partial \phi_1}{\partial r} - \frac{\partial^2 \phi}{\partial r \partial z} \\ u_z &= \frac{\partial \phi_1}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) \end{aligned} \quad (6.18)$$

The scalar potentials ϕ , ϕ_1 , ϕ_2 depend on r and z only, and X_2 , X_1 are now defined in terms of the single axisymmetric scalar ϕ , with

$$X_1 = \{0\} \frac{\partial}{\partial r} + \left\{ \frac{\partial \phi_1}{\partial r} \right\} \frac{\partial}{\partial \theta} + \{0\} \frac{\partial}{\partial z}$$

and by noting that from equation (6.16b)

$$\frac{\chi_2}{\rho_2} = \frac{-\rho_1}{\rho_2} \frac{\chi_1}{\rho_1}$$

The shear wave motion governed by the vector equations (6.16a), (6.16b) may now be described by a single scalar equation:

$$\nabla^2 \phi_k + k_k^2 \phi_k = 0 \quad (6.19)$$

$$\text{where } k_k^2 = \frac{\omega^2 \rho}{\mu} \left(1 - \frac{\rho_1^2}{\rho \rho_2} \right) \quad (6.20)$$

6.7 Axisymmetric boundary conditions

Using the definitions of the axisymmetric potentials already given and the usual vector differential operators appropriate for cylindrical coordinates, equation (6.9) becomes

$$\frac{\partial}{\partial z}(\phi_2 + \phi_3) + \left(1 - \frac{\rho_1}{\rho_2}\right) \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi_4}{\partial r} \right) - \frac{\partial \psi}{\partial z} = 0$$

or, since $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \equiv (\nabla^2 - \frac{\partial^2}{\partial z^2})$ for axisymmetric cases

$$\frac{\partial}{\partial z}(\phi_2 + \phi_3) + \left(1 - \frac{\rho_1}{\rho_2}\right) (\nabla^2 \phi_4 - \frac{\partial^2 \phi_4}{\partial z^2}) - \frac{\partial \psi}{\partial z} = 0 \quad (6.21)$$

Equation (6.10) becomes

$$2\mu \frac{\partial}{\partial z} \left[\frac{\partial \phi_1}{\partial z} + \nabla^2 \phi_4 - \frac{\partial^2 \phi_4}{\partial z^2} \right] + (H - 2\mu - \alpha M) \nabla^2 \phi_3 - M(1 - \alpha) \nabla^2 \phi_2 = 0 \quad (6.22)$$

Equations (6.11) and (6.12) are satisfied in the axisymmetric case if

$$\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} = 0$$

or, in terms of potentials,

$$\frac{\partial}{\partial r} \left[\frac{\partial \phi_1}{\partial z} - 2 \frac{\partial^2 \phi_4}{\partial z^2} + \nabla^2 \phi_4 \right] = 0$$

Since the bracketted term must approach zero as $r \rightarrow \infty$, this boundary condition means that it must be zero everywhere, that is

$$2\frac{\partial\phi_3}{\partial z} - 2\frac{\partial^2\phi_4}{\partial z^2} + \nabla^2\phi_4 = 0 \quad (6.23)$$

After substituting the appropriate potentials, equation (6.13) becomes

$$M\nabla^2\phi_2 + \alpha M\nabla^2\phi_3 - K_a\nabla^2\psi = 0 \quad (6.24)$$

We shall ultimately use the four boundary conditions (6.21) - (6.24) to determine the four potentials $\psi, \phi_2, \phi_3, \phi_4$.

6.8 General solutions

Assuming a point source of sound at a height z_0 above the interface, the solution to (6.14) may be written

$$\psi = \frac{e^{ik_1 R}}{R} + \phi_1(x, z)$$

where $R^2 = r^2 + (z - z_0)^2$, and ϕ_1 represents the wave reflected from the interface.

The general solution in axisymmetric coordinates may be written as an inverse Rankel transform:

$$\phi_1(x, z) = \int_0^\infty \bar{\phi}_1(K) e^{-v_1 z} J_0(Kr) K dK \quad (6.25)$$

where $v_1 = (K^2 - k_1^2)^{1/2}$, $\text{Re } v_1 > 0$, $z > 0$, $\text{Im}(k_1) \geq 0$, and $\bar{\phi}_1$ is the transformed potential for $z = 0$.

The total field above the interface, using the integral representation for the point source, is

$$\psi = \int_0^\infty \left[\frac{e^{-v_1 |z - z_0|}}{v_1} + \bar{\phi}_1 e^{-v_1 z} \right] J_0(Kr) K dK \quad (6.26)$$

For the field below the surface, in equations (6.15a), (6.15b) we try solutions of the form

$$\phi_2 = \int_0^{\infty} \bar{\phi}_2(K) e^{\alpha z} J_0(Kr) K dK \quad (6.27)$$

$$\phi_3 = \int_0^{\infty} \bar{\phi}_3(K) e^{\alpha z} J_0(Kr) K dK \quad (6.28)$$

where $z < 0$ and α is as yet unspecified. Substitution of these leads to two linear equations for the transformed potentials:

$$a^2 (E \bar{\phi}_3 + \alpha M \bar{\phi}_2) - \omega^2 (\rho_1 \bar{\phi}_3 + \rho_2 \bar{\phi}_2) = 0 \quad (6.29)$$

$$a^2 (\alpha M \bar{\phi}_3 + M \bar{\phi}_2) - \omega^2 (\rho_1 \bar{\phi}_3 + \rho_2 \bar{\phi}_2) = 0 \quad (6.30)$$

where $a^2 = K^2 - \gamma^2$. Equating the determinant of these equations to zero gives the non-zero solutions, and we obtain

$$\begin{aligned} & (a^2 H - \omega^2 \rho) (a^2 M - \omega^2 \rho_2) \\ & - (a^2 \alpha M - \omega^2 \rho_1)^2 = 0 \end{aligned} \quad (6.31)$$

This quadratic gives two allowed values of a^2 , and we can rewrite it as

$$(a^2 - k_1^2) (a^2 - k_2^2) - \frac{1}{HM} (a^2 \alpha M - \omega^2 \rho_1)^2 = 0 \quad (6.32)$$

$$\text{where } k_1'^2 = \frac{\omega^2 \rho_2}{M} \quad (6.33)$$

$$\text{and } k_2'^2 = \frac{\omega^2 \rho}{H} \quad (6.34)$$

k_1' is the propagation constant for the equivalent rigid porous material, and k_2' is the propagation constant for compressional waves in the bulk medium assuming that relative motion between fluid and solid is constrained in some way.

Equation (6.32) is a quadratic which may be solved in the usual way to give two possible values of a .

If the second term in (6.32) is negligible then we must have

$$\begin{aligned} a^2 &= \frac{(k_2'^2 + k_1'^2)}{2} \pm \frac{(k_2'^2 - k_1'^2)}{2} \\ &= k_1'^2 \text{ or } k_2'^2 \end{aligned} \quad (6.35)$$

For our application we do in fact expect that (6.32) will give two wavenumbers corresponding to distinct wave types, and also that $k_1' < k_2'$.

Accordingly, we will call one solution k_1 , and this is approximately equal to k_1' and associated primarily with motion of air in the pores known as the 'slow wave'. The second solution we call k_2 , and this is the wavenumber of the compressional 'fast wave' determined chiefly by the properties of the solid part of the medium.

In equations (6.29) and (6.30) the two potentials must be in a fixed proportion for each allowed wavenumber. When $a = k$, we use (6.29) to obtain

$$a_{2,1} = \frac{\bar{\phi}_1}{\bar{\phi}_2} = \frac{\omega^2 \rho_1 - k_2^2 \alpha M}{H(k_2^2 - k_3^2)} \quad (6.36)$$

and when $a = k$, substitution into (6.30) gives

$$a_{2,1} = \frac{\bar{\phi}_1}{\bar{\phi}_2} = \frac{k_2^2 \alpha M - \omega^2 \rho_1}{M(k_2^2 - k_3^2)} \quad (6.37)$$

Equations (6.15a), (6.15b) therefore have two independent solutions.

For the slow wave

$$\begin{aligned} \phi_2 &= \int_0^\infty \bar{\phi}_2'(k) e^{v_2 z} J_0(Kr) K dk \\ \phi_3 &= a_{2,1} \phi_2 \end{aligned} \quad (6.38)$$

and for the fast wave

$$\begin{aligned} \phi_3 &= \int_0^\infty \bar{\phi}_3'(K) e^{v_3 z} J_0(Kr) K dK \\ \phi_2 &= a_{2,1} \phi_3 \end{aligned} \quad (6.39)$$

where $v_2 = (K^2 - k_2^2)^{\frac{1}{2}}$
and $v_3 = (K^2 - k_3^2)^{\frac{1}{2}}$

$\bar{\phi}'_2$ is that part of the transformed potential $\bar{\phi}_2$ associated purely with the slow wave. Similarly $\bar{\phi}'_3$ is the part of $\bar{\phi}_3$ associated with the fast wave.

In general we may write

$$\phi_2 = \int_0^{\infty} (\bar{\phi}'_2 e^{v_2 z} + a_{2,2} \bar{\phi}'_3 e^{v_3 z}) J_0(Kr) K dk \quad (6.40)$$

$$\phi_3 = \int_0^{\infty} (\bar{\phi}'_3 e^{v_3 z} + a_{3,2} \bar{\phi}'_2 e^{v_2 z}) J_0(Kr) K dk \quad (6.41)$$

Finally, the general axisymmetric solution for the shear wave motion described by equations (6.19) is

$$\phi_4 = \int_0^{\infty} \bar{\phi}(K) e^{v_4 z} J_0(Kr) K dk \quad (6.42)$$

where $z < 0$, $v_4 = (K^2 - k_b^2)^{1/2}$

6.9

Simultaneous equations for the transformed potentials

The general solution given by equations (6.26), (6.40) - (6.42) for $z = 0$ are now substituted into the boundary conditions (6.21) - (6.24). This leads to four linear equations in the transformed potentials which, when satisfied, also satisfy the boundary conditions.

Equation (6.21) is satisfied if

$$\begin{aligned} v_1 \bar{\phi}_1 + v_2 (1 + a_{12}) \bar{\phi}_2' + v_3 (1 + a_{13}) \bar{\phi}_3' - K^2 (1 - \frac{\rho_1}{\rho_2}) \bar{\phi}_4 \\ = e^{-v_1 z_0} \end{aligned} \quad (6.43)$$

Equation (6.22) is satisfied if

$$\begin{aligned} \{2\mu a_{12} v_2^2 - k_2^2 [a_{12} (H - 2\mu - \alpha M) - M(1 - \alpha)]\} \bar{\phi}_2' \\ + \{2\mu v_3^2 - k_3^2 [H - 2\mu - \alpha M - a_{13} M(1 - \alpha)]\} \bar{\phi}_3' \\ - 2\mu v_4 K^2 \bar{\phi}_4 = 0 \end{aligned} \quad (6.44)$$

Equation (6.23) is satisfied if

$$2a_{12} v_2 \bar{\phi}_2' + 2v_3 \bar{\phi}_3' - (2K^2 - k_4^2) \bar{\phi}_4 = 0 \quad (6.45)$$

EQUATION (6.47)

$$\begin{bmatrix}
 v_1 & v_2 & v_3 & \frac{\partial_1 k^2}{\rho_2} - \frac{1}{2} k_u^2 \\
 -\rho_1 \omega^2 & k_2^2 M(1 + \alpha a_{3,2}) & k_2^2 M(\alpha + a_{3,3}) & 0 \\
 \rho_1 \omega^2 & a_{3,2}(2\mu k^2 - k_2^2 H) & 2\mu k^2 - k_2^2 H & -2\mu v_4 k^2 \\
 0 & -k_2^2 \alpha M & -a_{3,3} k_2^2 \alpha M & -K^2 + \frac{1}{2} k_u^2
 \end{bmatrix}
 \begin{bmatrix}
 \bar{\phi}_1 \\
 \bar{\phi}_2 \\
 \bar{\phi}_3 \\
 \bar{\phi}_4
 \end{bmatrix}
 = \frac{e^{-v_1 z_0}}{v_1}
 \begin{bmatrix}
 v_1 \\
 \rho_1 \omega^2 \\
 -\rho_1 \omega^2 \\
 0
 \end{bmatrix}$$

Equation (6.24) is satisfied if

$$\begin{aligned} -\rho_1 \omega^2 \bar{\phi}_1 + k^2 M(1 + \alpha a_{2,1}) \bar{\phi}_2' + k^2 M(\alpha + a_{2,1}) \bar{\phi}_3' \\ = \frac{e^{-v_1 z_0}}{v_1} \end{aligned} \quad (6.46)$$

where we have used $k^2 K_a = \omega^2 \rho_1$

With some rearrangement we now write equations (6.43) - (6.46) in matrix form:

$$[A] \{\bar{\phi}\} = \frac{e^{-v_1 z_0}}{v_1} \{S\}$$

This is shown in full in equation (6.47)

Using Cramer's rule the transformed potentials are

$$\bar{\phi}_1 = \frac{\Delta_1}{\Delta_0} \frac{e^{-v_1 z_0}}{v_1} \quad (6.48)$$

$$\bar{\phi}_2' = \frac{\Delta_2}{\Delta_0} \frac{e^{-v_1 z_0}}{v_1} \quad (6.49)$$

$$\bar{\phi}_3' = \frac{\Delta_3}{\Delta_0} \frac{e^{-v_1 z_0}}{v_1} \quad (6.50)$$

$$\bar{\phi}_4 = \frac{\Delta_4}{\Delta} \frac{e^{-v_1 z_0}}{v_1} \quad (6.51)$$

where Δ_0 is the determinant of the matrix $[A]$, and Δ_1, Δ_2 , etc are the determinants formed by replacing the appropriate column with the vector S .

6.10

Solutions in integral form

Substitution of the transformed potentials into the general solutions

(6.25), (6.40) - (6.42) gives

$$\phi_1 = \int_0^{\infty} \frac{\Delta_1}{\Delta_0} \frac{e^{-v_1(z+z_0)}}{v_1} J_0(Kr) K dK \quad (6.52)$$

$$\begin{aligned} \phi_2 &= \int_0^{\infty} \frac{\Delta_2}{\Delta_0} \frac{e^{-v_1 z_0 + v_2 z}}{v_1} J_0(Kr) K dK \\ &+ a_{2,3} \int_0^{\infty} \frac{\Delta_3}{\Delta_0} \frac{e^{-v_1 z_0 + v_3 z}}{v_1} J_0(Kr) K dK \end{aligned} \quad (6.53)$$

$$\begin{aligned} \phi_3 &= \int_0^{\infty} \frac{\Delta_3}{\Delta_0} \frac{e^{-v_1 z_0 + v_3 z}}{v_1} J_0(Kr) K dK \\ &+ a_{3,2} \int_0^{\infty} \frac{\Delta_2}{\Delta_0} \frac{e^{-v_1 z_0 + v_2 z}}{v_1} J_0(Kr) K dK \end{aligned} \quad (6.54)$$

$$\phi_4 = \int_0^{\infty} \frac{\Delta_4}{\Delta_0} \frac{e^{-v_1 z_0 + v_4 z}}{v_1} J_0(Kr) K dK \quad (6.55)$$

where $z > 0$ for ϕ_1 and $z < 0$ for the other potentials.

These integrals are typical of those found in other two-media problems, and we may consider them qualitatively in terms of saddle - point integrals, branch cut integrals and poles in the complex k plane. The saddle points occur at the stationary points of the oscillating integrand, and for large Kr some of these correspond to the ray paths

governed by the laws of reflection and refraction. The remainder consist of inhomogeneous waves (see Brekhovskikh [6.5]) for which there is no definite path, but which can be interpreted in terms of Snell's law by considering complex angles of incidence and refraction.

The branch cut integrals correspond to rays which are in part tied to the interface. Some of the remaining parts of these paths are indefinite since they consist of inhomogeneous waves, but again interpretation is possible by allowing complex, as well as real, critical angles.

When a pole of the integrand lies between the contour of integration and the real axis there will be a surface wave contribution. Nearby poles may also affect the saddle-point integrals as in the rigid porous case.

The eventual contour of integration may not always consist of all possible saddle-point and branch cut integrals. If the source and/or receiver are sufficiently distant from the interface, a single saddle path may enclose more than one branch point, thereby excluding any separate contributions from the associated branch cut integrals. Physically, this appears to eliminate any path which consists partly of inhomogeneous waves and whose time of travel is less than the corresponding wholly homogeneous path. At grazing angles, however, each saddle path encloses a single branch point, and all possible arrivals have to be considered.

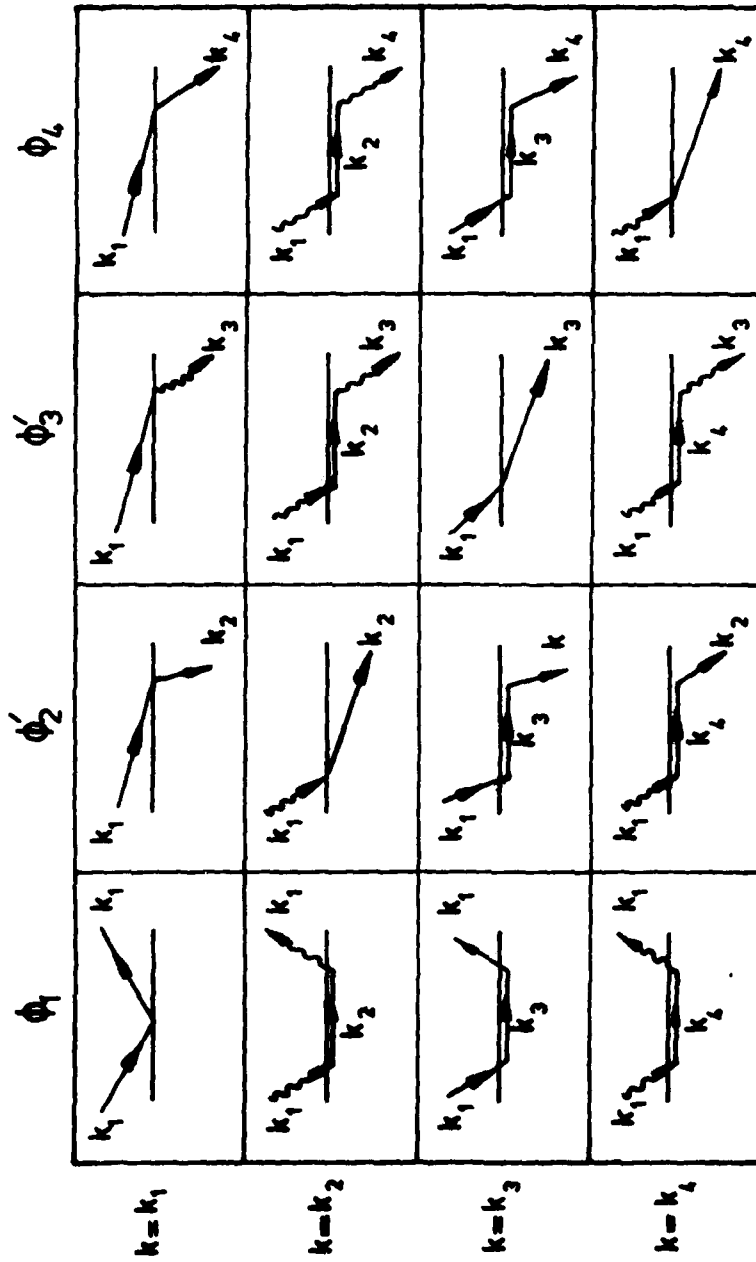


Figure 6.1 Ray-paths for wave-types possible at a fluid-poro-elastic solid interface

These considerations are summarised diagrammatically in Fig.(6.1) in a manner closely following that given by Ewing et al [6.2] for a solid-solid interface. For this example we assume $k_2 > k_4 > k_1 > k_3$. The wavy lines denote inhomogeneous waves, and in general two-part paths from branch cut integrals. The potential ϕ_2 is associated purely with the slow wave and the potential ϕ_3 with the fast wave, so that in equations (6.53), (6.54) we could write

$$\phi_2 = \phi_2' + a_{2,1}\phi_1', \quad \phi_3 = \phi_3' + a_{3,1}\phi_1'$$

In addition to these arrivals, of course, there may be surface wave contributions.

It can be seen from the diagram that when the receiver is on the interface, the four paths for each branch point associated with the four different wave potentials coalesce into a single path. This reduces the total number of paths, excluding surface waves, from sixteen to four, and this is a useful simplification when we consider motion of the ground surface. Expressions for this are derived in Appendix 5.

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CHAPTER 7

CONCLUDING REMARKS

- 7.1 Although we have been able to predict qualitative features of some existing data on acoustic-to-seismic coupling, in particular, that used to compute intensity ratios (Chapter 5), the requirement for design of unattended seismic and acoustic sensor logic remains that good quantitative agreement with these ratios should be obtained and that there should be theoretical agreement with other data on acoustic-to-seismic coupling [7.1]. These data include the results of tests using impulsive acoustic sources above and below the surface and sensors at depths of up to 1.5 m.
- 7.2 The main obstacles to such requirements at this stage are lack of information on the frame elastic constants required to characterise poro-elastic soils (Chapter 5) and the complex-layered and possibly anisotropic structure of real grounds.
- 7.3 The analysis has shown that surface particle velocity prediction is particularly sensitive to the value chosen for the frame rigidity modulus.
- 7.4 The model developed in this work has been brought to the stage where numerical predictions of coupling from above-ground point acoustic sources to sub-surface geophones are possible in certain well-defined situations viz. near-surface sources and air-saturated, isotropic, porous ground of infinite depth. Alternatively, if the acoustic source is sufficiently far removed from the ground surface for the incident sound waves to be treated as plane, then not only can predictions be made of the coupling to the surface of semi-infinite ground but also to ground in the form of a hard-backed porous layer.

- 7.5 Extension of this model to allow for anisotropy and for more complicated ground structures and continuous sources will be straight forward albeit sufficiently algebraically cumbersome to warrant a semi-numerical approach, such as that adopted by the Fast Field Program and its successors in Underwater Acoustics.
- 7.6 Further extension of the model to allow for impulsive rather than continuous sources, should also be possible using a numerical approach as suggested recently by Feng and Johnson [7.2].
- 7.7 The model may also be extended to include the effects of partial water-saturation of poro-elastic soils along the lines suggested by Brutsaert [7.3].
- 7.8 Finally, it should be noted that the theories developed in this report are applicable to seismic disturbance of poro-elastic ground by direct mechanical contact. Consequently it is possible to envisage prediction and analysis of seismic refraction results in a way that differs from established methods. Furthermore the possibility arises of the prediction of ground vibration due to road and rail vehicles in terms of basic fundamental attributes of intervening terrain.

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APPENDIX 1

WEYL - VAN DER POL FORMULA -- CONFIRMATION

The method of steepest descents has been employed extensively to obtain a computationally feasible solution for the field from a point source of sound in air above a locally reacting ground plane. Various mathematical approaches which arrive at the above solution under various approximations are listed in reference [1].

The important case of near grazing incidence requires additional care while handling the steepest descent integrals and this is chiefly achieved by a method known in the literature [2] as subtraction of the pole. This method often leads to laborious calculations. A simple alternative solution can be obtained by an appropriate application of the method of stationary phase. In general, the method of stationary phase yields useful approximation as long as the integrand is a slowly varying function of integration variables in the neighbourhood of the stationary point. After a suitable choice of integration variables, the integrals are approximated in the usual way and the final results for the reflected field confirm those obtained previously.

The physical problem under consideration can be formulated in terms of the following boundary value problem. For convenience, we choose a cartesian coordinate system. The acoustic velocity potential satisfies Helmholtz wave equation in both the media ($z < 0$ and $z > 0$) which are characterised by their densities ρ_1 and ρ_2 and propagation constants κ_1 and κ_2 , respectively. Thus we have,

$$\begin{aligned} (\nabla^2 + \kappa_1^2)\psi &= \delta(x) \delta(y) \delta(z-z_0), & z > 0 \\ (\nabla^2 + \kappa_2^2)\psi &= 0, & z < 0 \end{aligned} \quad (A1.1)$$

where $\kappa_1 = k_1 + is_1$ and $\kappa_2 = k_2 + is_2$ and the source term is symbolically represented by Dirac delta function. The time dependence of $\exp(-i\omega t)$ is suppressed throughout. The boundary conditions under consideration are continuity of pressure and normal velocity across the interface $z = 0$ i.e.

$$\rho_1 \psi(x, y, 0-) = \rho_2 \psi(x, y, 0+) \quad (A1.2)$$

and

$$\frac{\partial \psi(x, y, 0-)}{\partial z} = \frac{\partial \psi(x, y, 0+)}{\partial z}$$

By making use of two dimensional Fourier transform in space variables x and y we obtain the following for the reflected field $z > 0$.

$$\begin{aligned} \psi = & \frac{e^{-ik_1 R_1}}{4\pi R_1} + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} e^{i\alpha x} d\alpha \int_{-\infty}^{\infty} \frac{1}{2i\chi_1} \frac{\rho_2 \chi_1 - \rho_1 \chi_2}{\rho_2 \chi_1 + \rho_1 \chi_2} \\ & \times e^{-i\chi_1(z+z_0) + i\beta y} d\beta \end{aligned}$$

$$\text{where } \alpha \text{ and } \beta \text{ are transform variables and } \chi_{1,2} = \sqrt{\kappa_{1,2}^2 - \alpha^2 - \beta^2}. \quad (A1.3)$$

The square root functions are defined in such a way that

$$\chi_{1,2} = \sqrt{\kappa_{1,2}^2 - \alpha^2} \quad \text{when } \beta = 0 \text{ and } \kappa_{1,2} \quad \text{when } \alpha = 0. \quad \text{The integral in (2.3) can be approximately evaluated for large source - receiver}$$

distances by the method of stationary phase and we obtain

$$\int = \frac{R(\theta)}{4\pi} \frac{e^{-ik_1 R_2}}{R_2}$$

$$\text{where } R_2^2 = x^2 + y^2 + (z + z')^2$$

$$R(\theta) = \frac{\cos \theta - M \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + M \sqrt{n^2 - \sin^2 \theta}}$$

and $M = \rho_1/\rho_2$ and $n = \kappa_2/\kappa_1$. The above asymptotic approximation is of limited use and breaks down when θ is near $\pi/2$ and this is due to the fact that the integrand in (A1.3) no longer remains a slowly varying function, but will oscillate rapidly between -1 to +1. In this case, ($\theta = \pi/2$), the contribution from the stationary point has to be considered more carefully. In order to facilitate this computation, we rewrite the integral in (A1.3) as

$$\frac{e^{-ik_1 R_2}}{4\pi R_2} + \psi^{(1)} \quad (1)$$

$$\text{where } \psi^{(1)} = \frac{i}{4\pi^2} \int_{-\infty}^{\infty} e^{iax} da \int_{-\infty}^{\infty} \frac{\rho_1 X_2}{X_1(\rho_2 X_1 + \rho_1 X_2)} e^{-iX_1(z-z_0) + i\beta y} d\beta \quad (A1.4)$$

The integration is performed successively, first with respect to (from now on abbreviated as w.r.t.) β and then w.r.t. a . The integral w.r.t. β is evaluated after deforming the contour into the complex β plane and then approximating the resulting integral by the method of stationary phase.

This is accomplished by first observing the branch point and pole singularities of the integrand in the complex β plane. In the present case the integrand has branch points at

$$\beta = \pm \sqrt{\kappa_2^2 - a^2} \quad \text{and simple poles at } \beta = \pm \sqrt{\frac{(1-M^2 n^2)}{(1-M^2)} \kappa_1^2 - a^2}$$

The integrand is made single valued by introducing the branch cuts as shown in Fig. A1.1.

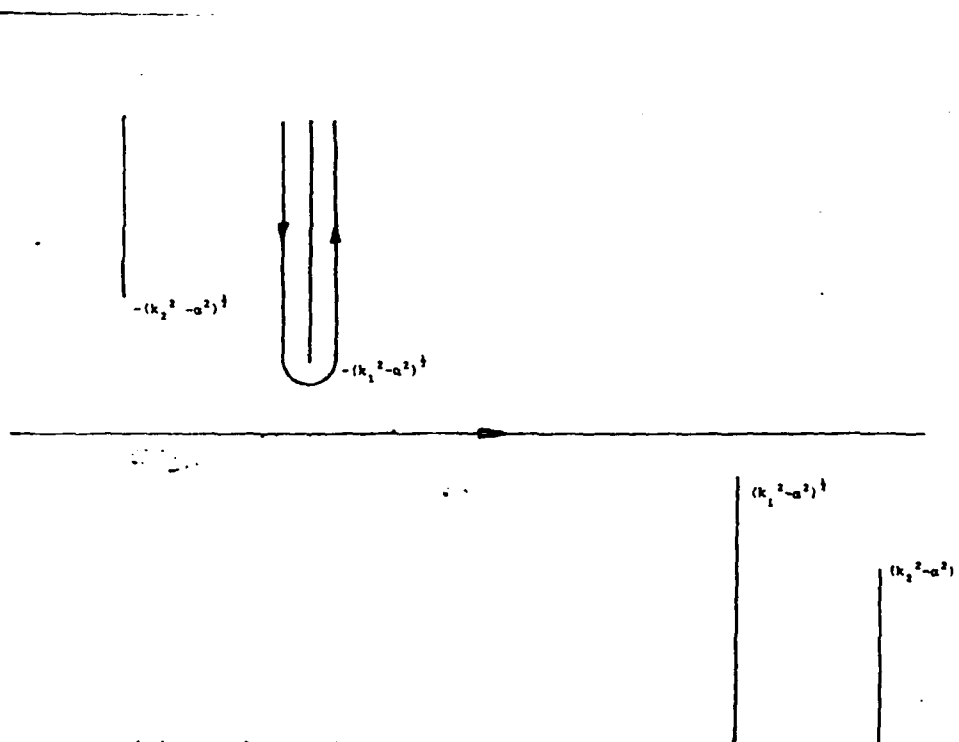


FIGURE A1.1 Branch cuts in the complex- β plane

The deformed contour consists of two circular arcs of large radius going round the branch cuts. The contribution from the circular arcs is zero. The contribution from the branch point $-(\kappa_2^2 - \alpha^2)^{1/2}$ can be shown to be

$$O[\exp\{-i(\kappa_1^2 - \alpha^2)^{1/2}(z+z_0) - i(\kappa_2^2 - \alpha^2)^{1/2}y\}]$$

and will be neglected. Thus we are left with the following integral

$$\int_C \frac{\rho_1 x_2}{x_1 (\rho_2 \kappa + \rho_1 x_2)} e^{-iX_1 v \cos \theta_1 + i R v \sin \theta_1} d\theta$$

(A1.5)

where we have let $y = r_1 \sin \theta_1$, $z + z_0 = r_1 \cos \theta_1$ in A1.5 and C is the contour round the branch point $-(\kappa_1^2 - \alpha^2)^{1/2}$. This integral is approximately evaluated by the method of stationary phase. The exponent in A1.5 has a stationary point at $\theta = -\sqrt{\kappa_1^2 - \alpha^2} \sin \theta_1$. When $\theta \sim \pi/2$, the situation we are interested in, we observe that the stationary point is in close proximity of the branch point $-\sqrt{\kappa_1^2 - \alpha^2}$.

Now the main contribution to the integral arises from the neighbourhood of stationary point, in that part of the integrand which is slowly varying. The case where the integrand is not slowly varying has to be considered separately. In order to do this, we first rewrite (A1.5) as

$$-1/2 \{R_v(\alpha, \theta_1) - 1\} \{\sqrt{\kappa_1^2 - \alpha^2} \cos \theta_1 + \kappa_1 A\} x$$

$$\oint_C \frac{e^{-i\chi_1(z+z_0)+i\beta y}}{\chi_1(\chi_1+\kappa_1 A)} d\beta \quad (A1.6)$$

where $A = M \sqrt{\frac{2}{n-1}}$ and

$$R_V(\alpha, \theta_1) = \frac{\rho_2 \chi_1 - \rho_1 \chi_2}{\rho_2 \chi_1 + \rho_1 \chi_2} \quad \text{at } \beta = -\sqrt{\kappa_1^2 - \alpha^2} \sin \theta_1$$

Now we write

$$\chi_1 = \sqrt{\kappa_1^2 - \alpha^2 - \beta^2} = (\sqrt{\kappa_1^2 - \alpha^2} + \beta)^{1/2} (\sqrt{\kappa_1^2 - \alpha^2} - \beta)^{1/2}$$

In the neighbourhood of the stationary point, the second factor is slowly varying and we put $\beta = -\sqrt{\kappa_1^2 - \alpha^2}$ (as $\theta_1 \sim \pi/2$).

On the remaining integrand, we put

$$(\sqrt{\kappa_1^2 - \alpha^2} + \beta)^{1/2} = \sqrt{2} e^{i\pi/4} \tau$$

where τ is a real parameter and varies from $-\infty$ to $+\infty$ on C .

After some algebra, we have from (A1.6)

$$(R_V(\alpha, \theta_1) - 1) (\cos \theta_1 + \kappa_1 (\kappa_1^2 - \alpha^2)^{-1/2} A)$$

$$x \exp(-iy_1 \sqrt{\kappa_1^2 - a^2} (1 + \frac{1}{2} \cot^2 \theta_1)) \times I(2x_1 \sin \theta_1) \quad (A1.7)$$

where

$$I(u) = \int_{-\infty}^{\infty} \frac{e^{-t^2 u}}{t+b} dt$$

and

$$b = \frac{1}{2} e^{-i\pi/4} \{ (\kappa_1^2 - a^2)^{\frac{1}{2}} \cot \theta_1 + \kappa_1 \Lambda (\kappa_1^2 - a^2)^{-\frac{1}{2}} \} \quad (A1.8)$$

The integral defined by $I(u)$ can be expressed in closed form involving error function complement and we can write it as

$$I(u) = -\sqrt{8\pi} \exp(-i\pi/4) F(b \exp(i\pi/4) \sqrt{u})$$

Thus we have from (A1.4) and (A1.7),

$$\psi(l) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} e^{i\alpha x - iy \sqrt{\kappa_1^2 - a^2} (1 + \frac{1}{2} \cot^2 \theta_1)} \times \{ \cos \theta_1 + \kappa_1 (\kappa_1^2 - a^2)^{-\frac{1}{2}} \Lambda \} I(2x_1 \sin \theta_1) d\alpha \quad (A1.9)$$

In the present case of interest $y(1 + \frac{1}{2} \cot^2 \theta_1) = y = r_1$ since $\cos \theta_1$ is small. The final evaluation of $\psi^{(1)}$ is carried out by a further application of method of stationary phase. Now the integral $I(2r_1 \sin \theta_1)$ does not contribute to the exponential and the stationary point of $\alpha x - r_1 \sqrt{x^2 - \alpha^2}$ occurs at $\alpha = -\kappa_1 x / (x^2 + r_1^2)^{1/2}$. The relevant terms at this point are given by

$$\sqrt{\kappa_1^2 - \alpha^2} = \frac{\kappa_1^2}{R} = \frac{\kappa_1 \cos \theta}{\cos \theta_1} = \frac{\kappa_1 y}{R \sin \theta_1}$$

since $\sin \theta_1 \approx 1$

$$b = 0.5 \exp(-i\pi/4) \{A + \cos \theta\} \sqrt{\frac{\kappa_1 R}{r_1}}$$

From (A1.3), (A1.4) and (A1.9), we finally obtain for the reflected field

in $z > 0$ as

$$\psi = \frac{e^{-i\kappa_1 R}}{4\pi R_1} + \frac{e^{-i\kappa_1 R}}{4\pi R_2} \{1 + 2i\gamma (R(\theta) - 1) F(\gamma)\} \quad (A1.10)$$

where $\gamma = \sqrt{\frac{\kappa_1 R}{1 - M^2}} (A + \cos \theta)$

$$A = M \sqrt{\frac{n^2 - 1}{1 - M^2}} = \beta \text{ when } |\eta|^2 \gg 1 \text{ and } |M|^2 \ll 1$$

$$\text{and } F(a) = \exp(ia^2) \int_0^\infty \exp(-it^2) dt$$

After some simple algebra the above solution for the reflected field is easily shown to be equivalent to those previously obtained.

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APPENDIX 2

ON THE ACOUSTIC SURFACE WAVE NEAR A RIGID POROUS BOUNDARY

A spherical wave incident upon an interface of two media, in particular, air and porous ground gives rise to reflected and refracted waves subjected to Snell's law. Under certain circumstances, the interface can support a wave which is exponentially damped in the normal direction and whose amplitude decays as $r^{-1/2}$. This wave is termed as a surface wave.

In this section, we clarify certain mathematical details which are necessary for the existence of a surface wave. We also provide an alternative interpretation for the acoustic surface wave in terms of discrete spectrum of a suitable partial differential operator.

We consider an incident spherical sound wave

$$\frac{e^{iR_1 R_1}}{4\pi R_1}$$

with time dependence of the form $\exp(-i\omega t)$, incident upon a locally reacting boundary. For the above case, surface waves are conveniently analysed by expressing the reflected field as a Fourier integral. The pole(s) of the reflection coefficient (in Fourier domain) are closely related to surface wave(s). The computation of the reflected field crucially depends upon the evaluation of the following integral

$$\phi(H, x) = \frac{\rho_2}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-v_1 H}}{\rho_2 v_1 + \rho_1 v_2} J_0(Kr) K dK \quad (A2.1)$$

where $v_{1,2} = \sqrt{K^2 - k_{1,2}^2}$ and $\rho_{1,2}$ are densities of the media $z > 0$ and $z < 0$ respectively and $H = z + z_0$. The square root functions are defined in such a way that $\sqrt{K^2 - k_{1,2}^2} = i k_{1,2}$ as $K \rightarrow 0$.*

Due to the presence of branch point singularities in the integrand of (A2.1) it is important to define the branch cuts before analysing the existence of poles. The choice of branch cuts is arbitrary as long as $\text{Re } v_{1,2} > 0$ to ensure the convergence of the integral in (A2.1). Essentially two types of branch cuts are used in the literature viz, vertical and hyperbolic [1] branch cuts. Here we consider the case of vertical branch cuts and the analysis for hyperbolic branch cuts follows on similar lines.

The vertical branch cuts are defined by $\text{Re } K = \pm \text{Re } k_1$, $\text{Im } K > 0$ and $\text{Re } K = \pm k_2$, $\text{Im } K > 0$. Corresponding to each square root function, there are two choices for the sign of $\text{Re } v_1$ and these are shown in table A2.1.

TABLE A2.1

	$\text{Re } v_1$	$\text{Re } v_2$
I	+	+
II	-	+
III	+	-
IV	-	-

* $k_{1,2}$ are complex constants with non zero imaginary points

We write $v_i = |v_i| \exp [i \arg(v_i)]$, for $i = 1, 2$ and the sign of $\operatorname{Re} v_i$ depends on the variations in $\arg(v_i)$. We consider these variations by considering a small neighbourhood around the branch points k_1 and k_2

TABLE A2.2

θ	$\operatorname{Arg} v_1(K)$	θ	$\operatorname{Arg} v_2(K)$
$-3\pi/2$	$-3\pi/4 + \chi_1/2$	$-3\pi/2$	$-3\pi/4 + \chi_2$
$-\pi - \chi_1$	$-\pi/2$	$-\pi - \chi_2$	$-\pi/2$
$-\pi + \chi_1$	$-\pi/2 + \chi_1$	$-\pi + \chi_2$	$-\pi/2 + \chi_2$
$-\chi_1$	0	$-\chi_2$	0
χ_1	χ_1	χ_2	χ_2
$\pi/2$	$\pi/4 + \chi_1$	$\pi/2$	$\pi/4 + \chi_2$

where χ_1 is $\arg k_1$ which is small as k_1 is essentially real and χ_2 is $\arg k_2$. Clearly on sheet I,

$$-\pi/2 + \chi_1 < \arg v_1(K) < \pi/4 + \chi_1$$

and

$$-\pi/2 + \chi_2 < \arg v_2(K) < \pi/4 + \chi_2.$$

Now we consider the possible existence of poles on the upper Riemann sheet.

It is easily seen that the poles of the integrand in (A2.1) are given by the equation

$$\rho_2 v_1 + \rho_1 v_2 = 0$$

i.e.

$$K = \pm k_1 [(1 - M^2 n^2) / (1 - M^2)]^{1/2} \quad (\text{A.2.2})$$

where $M = \rho_1/\rho_2$ and $n = k_2/k_1$. In order to locate these poles in the first quadrant, we define the complex number

$$K_0 = k_1 [(1 - M^2 n^2) / (1 - M^2)]^{1/2}$$

For most of the grounds of practical interest $|M^2| \ll 1$ and we approximate

$$(1 - M^2)^{\frac{1}{2}} \simeq 1. \text{ Hence } K_0 = k_1 (1 - M^2 n^2)^{\frac{1}{2}}$$

We also write the complex number $nM = |nM| \exp(i\psi)$. It is readily shown that

$$K_0 = k_1 [1 + |nM|^4 - 2 |nM|^2 \cos 2\psi]^{\frac{1}{4}} \quad (\text{A2.3})$$

and

$$\begin{aligned} \arg(K_0) &= \arg(k_1) + \frac{1}{2} \arg(1 - n^2 M^2) \\ &= \chi_1 + \frac{1}{2} \tan^{-1} \frac{|nM|^2 \sin 2\psi}{|nM|^2 \cos 2\psi - 1} \end{aligned} \quad (\text{A2.4})$$

Since $\chi_1 > 0$ and $K_0 > |k_1|$, the pole lies to the right of the branch point k_1 as shown on Fig A2.1. In order to verify that the equation for the pole is really satisfied at K_0 , we compute $v_1(K)$ and $v_2(K)$ at K_0 .

$$v_1(K_0) = \pm i \chi_1 nM \quad \text{and}$$

$$\arg(v_1(K_0)) = \pm \pi/2 + \arg(k_1) + \arg(nM)$$

where the choice of sign is fixed depending on the Riemann sheet. Now $\arg(k_1)$ is a small positive quantity and $\arg(nM) \simeq -\pi/4$ for a porous material at low frequencies and for K_0 situated as shown in Fig. A2.1.

$$\chi_1 \leq \arg v_1(K_0) \leq \pi/4 + \chi_1$$

on the upper Riemann sheet and hence $v_1(K_0) = + i k_1 nM$.

Similarly,

$$v_2(K_0) = \pm i k_1 n$$

where $|n|^2$ is assumed large; and

$$\arg v_2(K_0) = \pm \pi/2 + \arg(k_1) + \arg(n)$$

Now, for K_0 lying to the left of the branch point k_2 , we have

$$-\pi/2 + \chi_2 < \arg v_2(K_0) < 0$$

Since $\arg(k_2) > \arg(k_1)$, we have on sheet I,

$$v_2(K_0) = - i k_1 n$$

Thus it is evident that on sheet I, the equation for the pole is satisfied and hence K_0 is a *real* pole on sheet I.

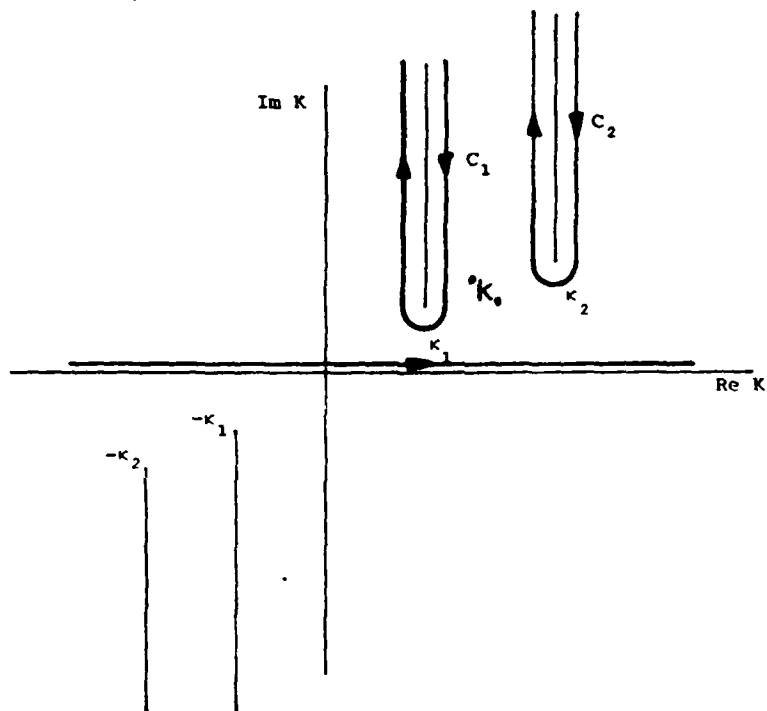


Fig A2.1.

Fig. A2.2 Location of pole and branch cuts

Now the evaluation of the integral is accomplished by first extending the range of integration from 0 to ∞ to $-\infty$ to $+\infty$, and deforming the contour into upper half plane of cut K - plane which encloses the pole at K_0 . The branch cut integrals are approximated using the saddle point method and the residue term represents the surface wave

$$\text{residue at } K_0 = \frac{A k_1 \exp(-i k_1 R A)}{(1-M^2) 4\pi} H_0^{(1)}(k_1 r \sqrt{1-A^2})$$

where

$$A = M[(n^2 - 1) / (1 - M^2)]^{1/2}$$

For the sake of completeness we quote the final expression for the reflection field in $z > 0$.

$$\Phi_{\text{ref}} = \frac{e^{i k_1 R_1}}{4\pi R_1} + \frac{e^{i k_2 R_2}}{4\pi R_2} \left\{ R(\theta_0) + \frac{2i(1 + \beta \cos \theta_0)}{k_1 R_2 (\beta + \cos \theta_0)^2} - \frac{iN}{k_1 R_2} + O(R_1^{-2}) \right\} + \text{residue at the pole.}$$

where

$$N = \frac{1}{2}[R''(\theta_0) + R'(\theta_0) \cot \theta_0]$$

$$R(\theta_0) = (\cos \theta_0 - \beta) / (\cos \theta_0 + \beta)$$

$$\beta = Mn$$

At this stage it is worth mentioning that the above solution is of limited practical use. For a solution which is uniformly valid for all angles of incidence a modified solution is used as described in previous section (see Eq. (A1.10)), or by a similar method [A1.1]. In this case, the surface wave term appears implicitly through error function complement, which can be seen more clearly for large arguments of the function (see Eq. 52 in [A1.1]).

A simple interpretation for the surface wave can be given in terms of the discrete spectrum of a suitable partial differential operator. In the present case it suffices to consider a two dimensional model for ease of illustration, where x and z coordinates denote the horizontal separation and vertical distance from the origin respectively.

We are required to find a function $\phi(x, z)$ such that

$$\phi_{xx} + (k^2 - L)\phi = \delta(x) \delta(z - z_0) \quad (A2.5)$$

where $L\phi = -\phi_{zz}$, over the half plane $-\infty < x < \infty$, $0 < z < \infty$, such that

$$\phi_z(x, 0) + \alpha \phi(x, 0) = 0 \quad (A2.6)$$

where α is a parameter related to impedance and $\text{Re } \alpha > 0$. Also we require the field to be outgoing at infinity.

The solution for A2.5 can be written symbolically as

$$\phi(x, z) = \frac{\exp(i\sqrt{k^2 - L}|x|)}{2i\sqrt{k^2 - L}} \delta(z - z_0) \quad (A2.7)$$

This operator expression can be evaluated, once we know the spectral representation for L under the above boundary condition. This can be obtained easily by the well known methods [A.4] and we have

$$\begin{aligned} \delta(z - z_0) &= 2\alpha \exp(-\alpha(z + z_0)) [1 - H(\text{Re}\alpha)] \\ &- \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp\{i\rho(z - z_0) + i\sqrt{k^2 - \rho^2}|x|\}}{\sqrt{k^2 - \rho^2}} d\rho \\ &- \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp\{i\rho(z + z_0) + i\sqrt{k^2 - \rho^2}|x|\}}{\sqrt{k^2 - \rho^2}} \frac{\rho - i\alpha}{\rho + i\alpha} d\rho \end{aligned} \quad (A2.8)$$

Now from A2.7 and A2.8

$$\begin{aligned}
 -\phi(x, z) &= \frac{-i \alpha \exp(i\sqrt{k^2 + \alpha^2} |x|) \exp[-\alpha(z + z_0)]}{\sqrt{k^2 + \alpha^2}} \\
 &- \frac{i}{\pi} \int_{-\infty}^{\infty} \exp[i\rho(z - z_0) + i\sqrt{k^2 - \rho^2} |x|] \frac{d\rho}{\sqrt{k^2 - \rho^2}} \\
 &- \frac{i}{\pi} \int_{-\infty}^{\infty} \exp[i\rho(z + z_0) + i\sqrt{k^2 - \rho^2} |x|] \frac{\rho - i\alpha}{\rho + i\alpha} \frac{d\rho}{\sqrt{k^2 - \rho^2}} \quad (A2.9)
 \end{aligned}$$

In Eq. the first term arises from a pole of the Green's function and in the literature, is referred to as constituting the discrete spectrum of the operator L . The remaining two integrals, after some algebra, can be easily seen to be contributing to the continuous spectrum. In fact, they can be shown to be direct (first integral in (A2.9)) and reflected (second integral in A2.9) waves. In other words we notice that the branch point singularities of Green's function give rise to space waves, while pole singularities give rise to surface wave(s).

APPENDIX 3

THE ACOUSTIC MODELLING FACILITY

Introduction

This Appendix introduces the basic rules of acoustic modelling and discusses the essential requirements of the modelling instrumentation. It concludes with a description of a series of model experiments, conducted in a small anechoic chamber, to measure the amplitude ratio of the direct and transmitted sound fields above a porous half-space and the delay between the arrival of the direct and transmitted sound waves.

A3.1 Basic modelling rules

In order to model an acoustic sound path, two conditions must be fulfilled. The first is that of geometric similarity and the second kinematic similarity. Geometric similarity ensures that the ratio of the wavelength of the sound source to the dimensions of any bodies or reflecting surfaces remains constant and hence diffraction effects will be replicated in the model.

Kinematic similarity is required due to the fact that air is used as the propagation medium in both the full scale and model environments and hence the velocity of sound is a constant. This imposes a time scale factor, as can be shown by considering the dimensionless quantity characterising reference times (the Thomson Number) as follows:

$$\frac{T_m \cdot V_m}{L_m} = \frac{T_o \cdot V_o}{L_o}$$

where T, V and L are time, velocity and length respectively and the subscripts m and o refer to the original and model propagation paths. Since $V_m = V_o$ then

$$\frac{T_m}{T_o} = \frac{L_m}{L_o}$$

and the time scale factor equals the length scale factor. Thus maintaining a constant sound velocity requires that the model frequencies be increased with respect to those of the original and that all time periods be scaled downward.

In general the scale factor derived for the condition of geometric similarity must also be applied to the acoustic properties of the materials used to reconstruct the surfaces and obstructions in the model; so that the impedance at the model frequencies equals that at the full scale frequencies.

The prime objective of the model experiments conducted during this study was to evaluate theoretical predictions of the coupling between the airborne and seismic waves, rather than to obtain quantitative data of a precise ground surface. In view of this it was felt unnecessary to employ 'scaled' models and so no attempt was made to scale the acoustic properties of the model ground surface.

A3.2 Requirements of a model receiver

The basic requirements of a model receiver are small size, high sensitivity and wideband frequency response. Size is important for two reasons (a) because directivity is related to the ratio of diaphragm diameter to wavelength and hence a small microphone better approximates an omni-directional receiver and (b) because the reduction in diaphragm mass improves the high frequency response of the microphone. The major disadvantage of a small diameter is that the reduction in surface area reduces the microphone's sensitivity.

The receivers used for the model experiments were commercial 1/4 inch diameter condenser microphones manufactured by Bruel and Kjaer (B & K) of Denmark. The sensitivity and bandwidth of typical microphones are shown in Table A3.1

Table A3.1 Microphone sensitivity and bandwidth

Microphone type	Nominal diameter	Sensitivity mV/Pa	Bandwidth kHz
B & K 4136	1/4"	4	70
B & K 4138	1/8"	1	140

Figures A3.1 - A3.4 show the directivity of these microphones at various frequencies, measured both with and without a protection grid [A3.1]. Although the 1/8" displays a wider bandwidth and superior directivity, its low sensitivity limited the maximum length of the model propagation path to less than 1 metre.

In view of this the 1/4" microphone was adopted as the standard receiver and the 1/8" microphone was used primarily to test the performance of the 1/4" at short separation distances. The 1/8" microphone was also used to measure the directivity of the sound sources.

A3.3 Requirements of a model sound source

The sound source should be omni-directional, have a wideband frequency response and generate adequate signal power.

Model sound sources fall into two categories, continuous and impulsive. The impulsive source, such as an electrical spark, generates a short energy pulse of high acoustic intensity and wide bandwidth which propagates along an infinite number of paths. Each path can be identified by the time delay between the energy discharge and the arrival of the pulse at the receiver, hence there is no need for special anechoic measurement environments.

The major limitation of the impulsive source is the possibility of non-linear acoustic propagation near the vicinity of the source, which imposes a minimum length of propagation path. Furthermore, they require sophisticated measurement and analysis systems to cope with the problems of direct and reflected pulse overlap [A3.2].

Continuous sources can be handled with relatively simple measurement systems, but since the energy is radiated continuously, it is not possible to identify individual propagation paths. This means that all measurements must be conducted within an anechoic environment.

A3.4 Model sound sources

Two sources were selected for the model experiments. The first was the air jet described by Delany et al [A3.3] and shown in Figure A3.5. Measurements of the directivity in both the vertical and horizontal planes indicate that this design is a good approximation to an omni-directional source at frequencies up to 100 kHz. One particularly important feature of this design is its excellent long term stability, so that over a period of six months the one third octave spectrum varied by less than 0.5 dB.

The useful bandwidth of this source is 1-100 kHz, as dictated by the signal to noise ratio of the measurement system and the low frequency performance of the anechoic chamber. Figure A3.6 shows the free-field spectrum measured at a distance of 100 cm.

The second model sound source was a spark discharge device designed at the Open University. (A complete description of the design and operating characteristics is given in reference A3.2)

The spark operates at very low energy levels and so non-linear propagation effects are virtually eliminated. A further advantage of the low energy is the fact that electromagnetic radiation is kept below levels that can be detected by the condenser microphones. A typical spark source spectrum is shown in Figure A3.7.

A3.5 Modelling instrumentation

A3.5.1 Dynamic range

Acoustic modelling is very demanding of instrumentation systems.

The low sensitivity of small microphones requires that pre-amplifiers and main amplifiers have very low electrical noise floors over a very wide frequency range.

A typical microphone amplifier chain consists of a pre-amplifier, to convert the high output impedance of the microphone to a low impedance suitable for driving long cables, and a measuring amplifier to provide the polarization voltage and the gain.

An estimate of the dynamic range of the instrumentation system can be obtained by calculating the noise floor of the amplifiers and subtracting this value from the input signal level. The following figures are taken from the relevant manufacturers' specifications.

(a) Pre-amplifier noise

The inherent noise of the pre-amplifier is determined by the capacitance of the microphone cartridge. For a 1/4" microphone the total noise over the range 20-20 000 Hz is quoted as less than 120 microvolts.

(b) Main amplifier noise

The noise floor of the main amplifier is determined by the thermal noise of the electronic components. The signal to noise ratio for a 100 mV input is stated to be better than 100 dB, which implies a noise floor of 1 microvolt over the range 20-20 000 Hz.

Assuming that the main amplifier noise can be neglected, compared to the pre-amplifier noise, then the equivalent sound pressure level of the noise floor is 63.5 dB.

If non-linear propagation effects are to be avoided, then the maximum source sound pressure level must be kept below 140 dB. Subtracting the noise level from the signal level gives a dynamic measurement range of approximately 75 dB.

A3.5.2 Atmospheric absorption

The source-receiver dynamic range is further degraded by atmospheric absorption. It is well known that high frequency sound is attenuated as it propagates through the air due to a combination of thermal, viscous and molecular relaxation effects. Delany [A3.4] has made an extensive study of the attenuation mechanisms and derived a set of equations that enable corrections to be made based upon measurements of the ambient temperature, air pressure and relative humidity.

The results of calculations for typical indoor values of these parameters shows that the attenuation can exceed 5 dB/metre at 100 kHz.

A3.5.3 Digital processing equipment

The majority of the signal processing tasks undertaken during this study were executed on a commercial digital spectrum analyser, a Nicolet 660B. This is a dual channel analyser performing a Fourier Transform on 1024 data points to produce a 400 line spectrum over a pre-determined measurement bandwidth. Alternatively, the instrument can produce a one third-octave band analysis over its full measurement bandwidth of 100 kHz.

The analyser has five basic operating modes, but the only one used for this study, was the dual channel mode. If the two input signals are represented by A and B respectively, then the analyser will calculate the cross-spectrum from the expression

$$G(f)_{AB} = A(f) \cdot B(f)$$

and the transfer function from the expression

$$H(f) = G(f)_{AB}/A(f)$$

The Nicolet is also capable of calculating the coherence function, which for linear systems can be interpreted as the fractional proportion of the mean square value of the output signal $B(f)$ which is contributed by the input signal $A(f)$. The coherence function, $g^2(f)$ can be defined in terms of the cross-spectrum as

$$g^2(f)_{AB} = \frac{|G(f)_{AB}|^2}{|A(f)| \cdot |B(f)|}$$

Measurement of the coherence function proved a useful means of estimating the influence of near surface turbulence upon grazing incidence propagation.

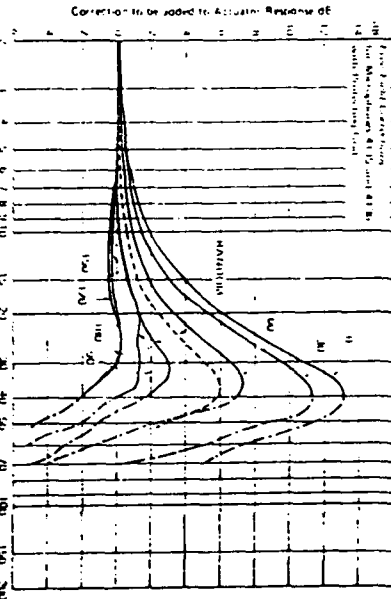


Figure 3.1 Bandwidth and Directivity of 1/4" Microphone with protection grid.

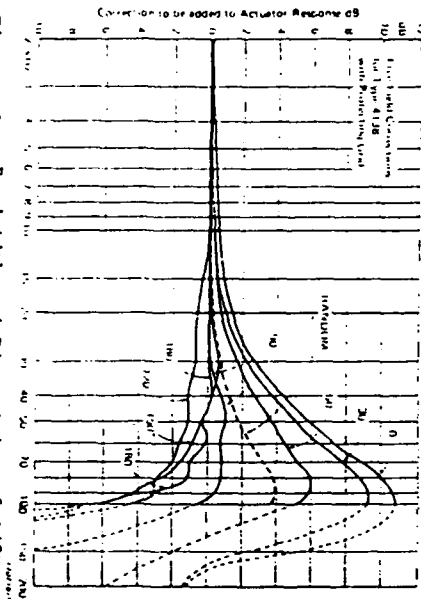


Figure A3.4 Bandwidth and Directivity of 1/8" Microphone with protection grid.

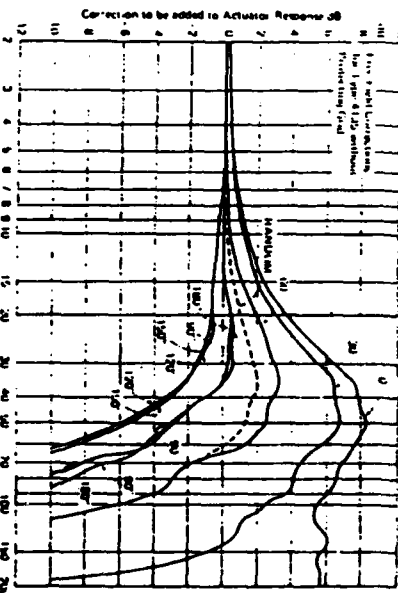


Figure 3.2 Bandwidth and Directivity of 1/4" Microphone without protection grid.

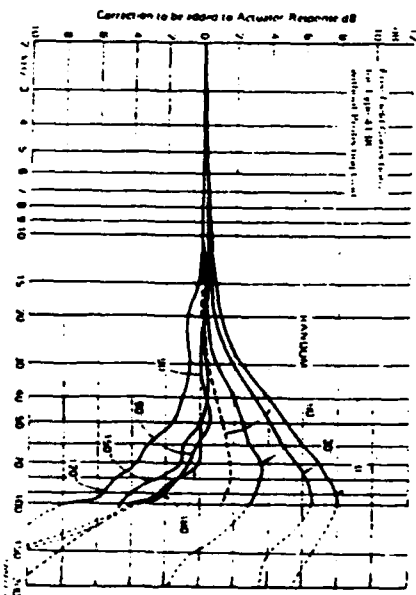


Figure A3.4 Bandwidth and Directivity of 1/8" Microphone without protection grid.

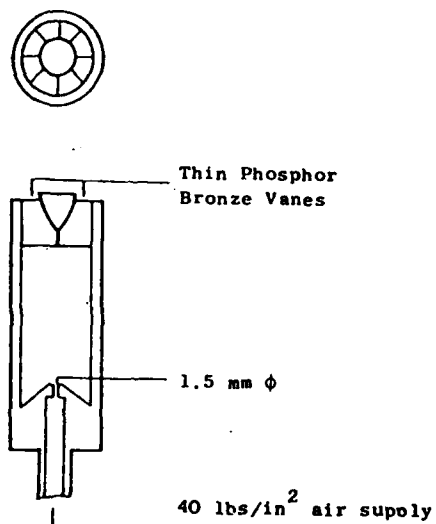


Figure A3.5 Air Jet Noise Source.
(After Delaney et al)

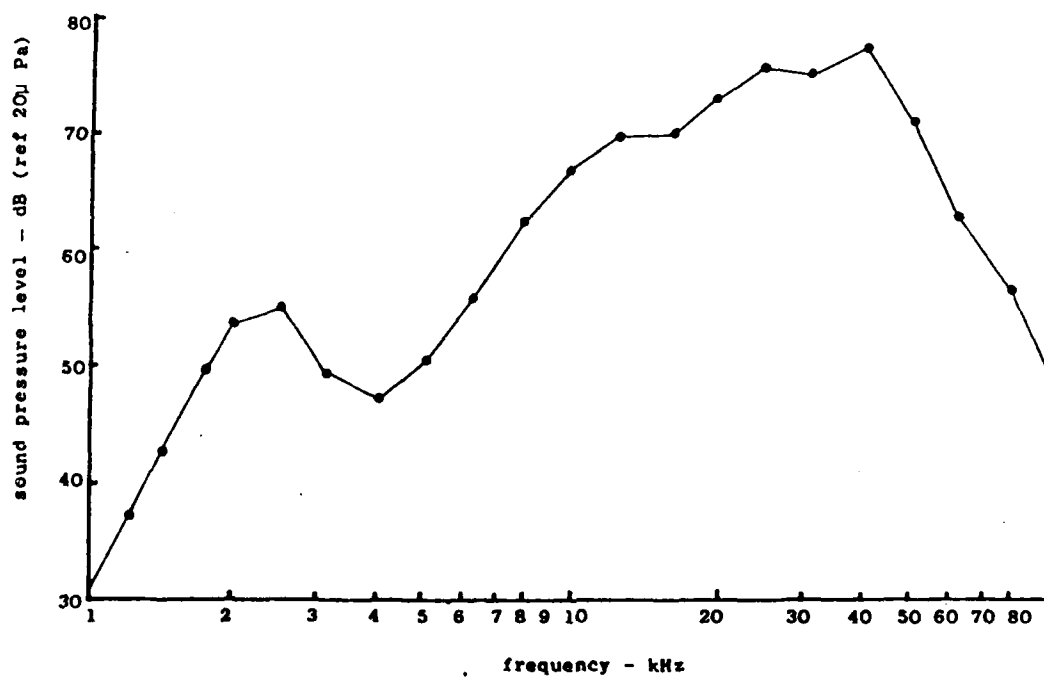


Figure A3.6 Air Jet Noise Source Spectrum at 100cm.

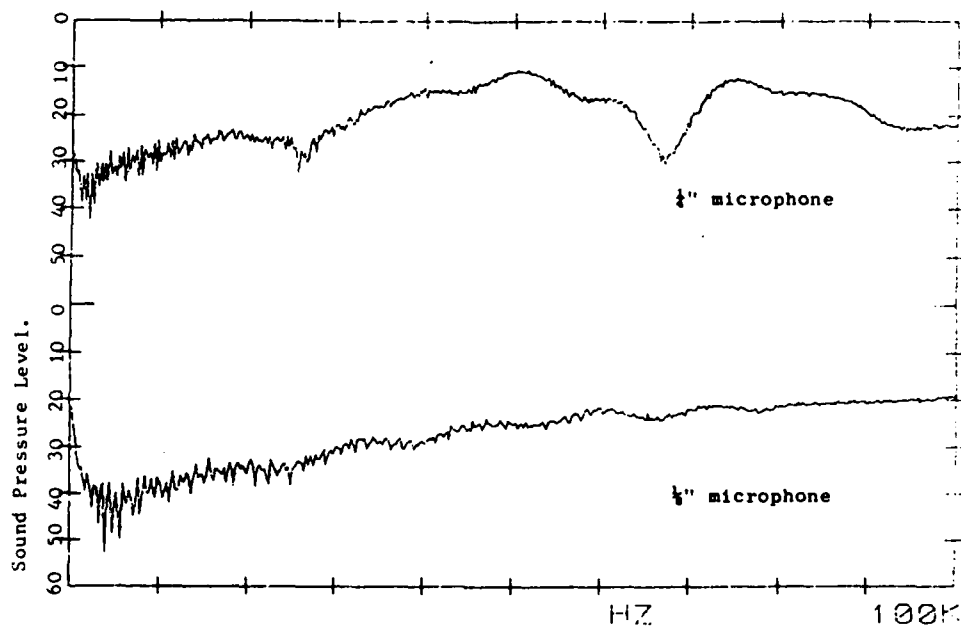


Figure A3.7 Spark Source Spectrum at 50cm.

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APPENDIX 4

CALCULATION OF SURFACE INTENSITY RATIOS

A4.1 Semi-infinite medium

In this section we are interested in obtaining an expression for the square of the ratio, solid particle velocity to pressure at the surface, where the ground is considered to be of infinite depth. We consider a plane wave incident upon the interface at an angle θ_0 away from the normal.

We note that in a poro-elastic half space, there will in general be six types of waves. These consists of two dilatational and one shear wave in both the solid and fluid phases. In terms of scalar potentials, we write, for the solid phase

$$\begin{aligned}\phi'_1 &= a'_s \exp [-i\omega t - k'y \sin\theta' - k'x \cos\theta'] \\ \phi''_1 &= a''_s \exp [-i\omega t - k''y \sin\theta'' - k''x \cos\theta''] \\ \psi_1 &= a^y_s \exp [-i\omega t - k^x y \sin\theta^x - k^x x \cos\theta^x]\end{aligned}\tag{A4.1}$$

and for the fluid phase,

$$\begin{aligned}\phi'_2 &= \tau' \phi'_1 \\ \phi''_2 &= \tau'' \phi'_2 \\ \phi_3 &= \tau^x \psi_x\end{aligned}\tag{A4.2}$$

where k' , k'' and k^x are propogation constant as obtained in section 5.1 τ' , τ'' and τ^x are the following ratios of amplitudes

$$\tau' = a_f' / a_s'$$

$$\tau'' = a_f'' / a_s''$$

$$\tau^x = a_f^x / a_s^x$$

and they can be obtained from the relation

$$\frac{a_f}{a_s} = \frac{\{-k^2 P + \rho_1 \omega^2 + i\omega k F(\omega)\}}{\{k^2 Q + i\omega b F(\omega)\}} \quad (A4.3)$$

where k assumes the values k' , k'' and k^x respectively.

The solid and fluid displacements can be expressed in terms of the above potentials as

$$u_x = \frac{\partial \phi_1'}{\partial x} + \frac{\partial \phi_1''}{\partial x} + \frac{\partial \phi_1}{\partial y}$$

$$u_y = \frac{\partial \phi_1'}{\partial y} + \frac{\partial \phi_1''}{\partial y} - \frac{\partial \phi_1}{\partial x}$$

$$U_x = \frac{\partial \phi_2'}{\partial x} + \frac{\partial \phi_2''}{\partial x} + \frac{\partial \phi_2}{\partial y}$$

$$U_y = \frac{\partial \phi_2'}{\partial y} + \frac{\partial \phi_2''}{\partial y} - \frac{\partial \phi_2}{\partial x} \quad (A4.4)$$

and the relevant components of solid strain are

$$e_{xx} = \frac{\partial u}{\partial x}, e_{yy} = \frac{\partial u}{\partial y}, e_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x} \right) \quad (A4.5)$$

and finally the fluid dilatation

$$\epsilon = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \quad (A4.6)$$

We also have the stress-strain relations for an isotropic medium

$$\begin{aligned} \sigma_{ij} &= (P - 2N)u_{kk} \delta_{ij} + 2Nu_{ij} + Qu_{kk} \delta_{ij} \\ \sigma_{ij} &= -\Omega p_f \delta_{ij} = (Qu_{kk} + Ru_{kk}) \delta_{ij} \end{aligned} \quad (A4.7)$$

and implicit summation over repeated indices is assumed.

The normal solid particle velocity, V is given by

$$V_s = \frac{\partial u}{\partial t} \quad (A4.8)$$

and from Eq (5.16), we have for V at the surface $x = 0$,

$$\begin{aligned} V_s &= -i\omega [a_s' (i k' \cos \theta' + a_s'' (i k'' \cos \theta'') \\ &\quad + a_s^r (i k^r \sin \theta^r)) \\ &\quad \times \exp [i k_0 \sin \theta_0 y - i \omega t] \end{aligned} \quad (A4.9)$$

wherein, we have made use of the relation,

$$k_0 \sin \theta_0 = k' \sin \theta' = k'' \sin \theta'' = k^r \sin \theta^r \quad (A4.10)$$

Also at the surface the normal stress in the frame must equal stress in the fluid as both are acted upon by the outer air pressure p . Thus

$$\frac{\sigma_{xx}}{(1 - \Omega)} = \frac{\sigma}{\Omega} = -p$$

From the stress-strain relations (A4.7) we can express

$$p = -K_f \left[\left(\frac{1 - \Omega}{\Omega} \right) \epsilon + \epsilon \right]$$

and using (A4.5) and (A4.6), we get for the pressure of the surface

$$p = -K_f \left[- \left(\frac{1 - \Omega}{\Omega} \right) \left\{ -a_s' k'^2 - a_s'' k''^2 \right\} + \left\{ -a_s' \tau' k'^2 - a_s'' k''^2 \tau'' \right\} \right] \times \exp [ik_0 y \sin \theta_0 - i\omega t] \quad (A4.11)$$

From (A4.9) and (A4.11) we obtain the ratio of solid particle velocity to the pressure at the surface i.e.

$$\frac{v_s}{p} = \frac{(1 - \Omega) \omega [\cos \theta' + \Gamma_1 \frac{k'}{k''} \cos \theta' + \Gamma_2 \frac{k'}{k''} \sin \theta_x]}{K_f k' \left[\left(\frac{1 - \Omega}{\Omega} + \tau' \right) + \Gamma_1 \left(\frac{1 - \Omega}{\Omega} + \tau'' \right) \right]} \quad (A4.12)$$

where

$$\Gamma_1 = \frac{a_s'' k''^2}{a_s' k'^2}$$

and

$$\Gamma_2 = \frac{a_s^r k^r^2}{a_s' k'^2}$$

The constants Γ_1 and Γ_2 are as yet unknown and can be expressed in terms of known quantities with the help of the following conditions at the surface $x = 0$. These are:

- i) vanishing of tangential stress at $x = 0$ i.e. $e_{xy} = 0$
- ii) stress in the frame equals stress in the fluid i.e.

$$\frac{\sigma_{xx}}{1 - \Omega} = \frac{\sigma}{\Omega}$$

These two conditions provide two equations for Γ_1 and Γ_2 and after some algebra we have from (A4.5) and (A4.7),

$$\sin 2 \theta'' \Gamma_1 - \cos 2 \theta^x \Gamma_2 = - \sin 2 \theta'$$

and

$$\begin{aligned} & \{2 N \Omega \cos^2 \theta'' + \{A \Omega - (1 - \Omega) Q\}\} \Gamma_1 + N \Omega \sin 2 \theta^x \Gamma_2 \\ & = 2 N \Omega \cos^2 \theta^x - \{A \Omega - (1 - \Omega) Q\} \end{aligned} \quad (A4.13)$$

which are solved to give

$$\Gamma_1 = \frac{-[N \Omega \tan 2 \theta^x \sin 2 \theta' + 2 N \Omega \cos^2 \theta' + \{A \Omega - (1 - \Omega) Q\}]}{[N \Omega \tan 2 \theta^x \sin 2 \theta'' + 2 N \Omega \cos^2 \theta'' + \{A \Omega - (1 - \Omega) Q\}]}$$

and

$$\Gamma_2 = \frac{\Gamma_1 \sin 2 \theta'' + \sin 2 \theta'}{\cos 2 \theta^x} \quad (A4.14)$$

Now with the help of A4.14 and A4.12, the square of the ratio, solid particles velocity to the pressure at the surface can be readily computed once we know the propagation constants k' , k'' and k^x .

In the case of normally incident ($\theta_0 = 0$) plane wave, the ratio can be expressed in a compact form

$$\left| \frac{v_s}{p} \right|^2 = \frac{(1 - \Omega)^2 \omega^2 \left[\frac{1}{k'} - \frac{1}{k''} \right]^2}{k_f^2 [\tau' - \tau'']^2} \quad (A4.15)$$

as $\Gamma_1 = -1$ and $\Gamma_2 = 0$

This result differs substantially from that obtained by Flohr and Cress [A4.1] wherein the ground has been considered as an elastic continuum. The present result is frequency dependent which is qualitatively consistent with their measurements.

A4.2 Rigidly-backed layer

The effect of inhomogeneity in the ground on the wave propagation can be described by considering it as consisting of several layers of differing properties. One such simple model is to consider a finite depth of poro-elastic layer supported by a rigid semi-infinite medium.

In this section, we derive an analogous expression to that of Eq A4.15 for normally-incident plane waves. When such a plane wave is incident upon the layered medium, inside the poro elastic layer of depth say, d , there will in general be twelve possible waves. As before, they can be represented as follows:

In the solid phase

$$\begin{aligned}\phi_1' &= a_s' \exp(-i\omega t) \{ e^{+ik'n} + T'e^{-ik'x} \} \\ \phi_1'' &= a_s'' \exp(-i\omega t) \{ e^{+ik''x} + T''e^{-ik''x} \} \\ \psi_1 &= a_s^r \exp(-i\omega t) \{ e^{+ik^rx} + T^r e^{-ik^rn} \}\end{aligned}\tag{A4.16}$$

In the fluid phase

$$\begin{aligned}\phi_2' &= \tau' p_1' \\ \phi_2'' &= \tau'' p_2'' \\ \psi_2 &= \tau^r p_1\end{aligned}$$

The unknown quantities T' , T'' and T^r denote the relative amplitudes of backward travelling waves to the waves generated at the surface.

Now at the rigid backing (at $x = d$) the tangential stress (σ_{xy}) will be zero, which in turn from stress-strain relations A4.7 implies that

$e_{xy} = 0$. Furthermore as the backing is hard the normal particle velocity is zero at the backing i.e.,

$$v_s = v_f = 0 \quad \text{at } x = d$$

Also at the surface $n=0$, the tangential stress, $e_{xy} = 0$.

Using the above conditions, after lengthy algebra, we obtain the following for the square of the ratio of solid particle velocity to pressure at the surface

$$\left[\frac{v_s}{p} \right]^2 = \frac{(1 - \Omega)^2 \omega^2 \left[\frac{1}{k'} \left(\frac{1 - e^{2\alpha}}{1 + e^{2\alpha}} \right) - \frac{1}{k''} \left(\frac{1 - e^{2\delta}}{1 + e^{2\delta}} \right) \right]^2}{\kappa_f^2 (\tau' - \tau'')^2} \quad (\text{A4.17})$$

where

$$\alpha = ik'd \text{ and } \delta = ik''d$$

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APPENDIX 5

MOTION OF THE INTERFACE. THE GEOMETRIC APPROXIMATION

Using equations (6.18) and the integral expressions for the potentials, and after performing the appropriate differentiations with respect to z , we obtain the components of displacement at the interface where $z = 0$:

$$u_r = \frac{\partial}{\partial r} \int_0^{\infty} \frac{1}{\Delta_0} [\Delta_3 + a_{3,2} \Delta_2 - v_4 \Delta_4] \frac{e^{-v_1 z_0}}{v_1} J_0(Kr) K dk \quad (A5.1)$$

$$u_z = \int_0^{\infty} \frac{1}{\Delta_0} [v_3 \Delta_3 + a_{3,1} v_2 \Delta_2 + K^2 \Delta_4] \frac{e^{-v_1 z_0}}{v_1} J_0(Kr) K dk \quad (A5.2)$$

These integrals are now in a form familiar in many analyses of reflected spherical waves in which the reflected field is expressed as a superposition of plane waves multiplied by the appropriate reflection coefficient (Brekhovskikh [6.5]). In these the contribution from the saddle point (at $K = k_1 \sin \theta$, where θ is the angle of incidence) is obtained as an asymptotic expansion, the first term of which gives the geometric approximation. Ignoring the contributions from the branch cuts and poles, we have

$$u_r \sim \frac{\partial}{\partial r} \left[\frac{1}{\Delta_0} (\Delta_3 + a_{3,2} \Delta_2 - (k_1^2 \sin^2 \theta - k^2)^{\frac{1}{2}} \Delta_4) \frac{e^{ik_1 R}}{R} (1 + O(R^{-1})) \right]$$

or $u_r \sim \frac{ik_1 \sin \theta}{\Delta_0} [\Delta_3 + a_{3,2} \Delta_2 - (k_1^2 \sin^2 \theta - k^2)^{\frac{1}{2}} \Delta_4] \frac{e^{ik_1 R}}{R} (1 + O(R^{-1})) \quad (A5.3)$

and

$$u_2 = \frac{1}{\Delta_0} [(k_1^2 \sin^2 \theta - k_2^2)^{\frac{1}{2}} \Delta_3 + a_{22} (k_1^2 \sin^2 \theta - k_2^2)^{\frac{1}{2}} \Delta_2 + k_1^2 \sin^2 \theta \Delta_4] \frac{e^{ik_1 R}}{R} (1 + O(R^{-1})) \quad (A5.4)$$

where $R^2 = r^2 + z^2$, $\sin \theta = \frac{r}{R}$, and the determinants are evaluated for $K = k_1 \sin \theta$.

We do not expect the geometric approximation to be valid at grazing incidence because the determinants Δ_1 , Δ_3 , Δ_4 all contain a factor v_1 , and thus approach zero as $\theta \rightarrow \pi/2$. To see this we recall that they were formed by replacing columns 2, 3, or 4 of the matrix in equation (6.47) by the vector on the right-hand side. This new column may then be added to column 1 without affecting the value of the determinant, and this results in a single non-zero element $2v_1$ in the first column, which must be a factor of the determinant.

The evaluation of the higher order correction terms in the saddle-point approximation would be over complicated because of the need for 1st and 2nd derivatives of the various determinants. However, we later use a simplified method which enables us to obtain the correction term at grazing incidence.

A5.1 The branch cut integrals

If we treat each branch cut integral separately, then our problem in general is to evaluate an integral of the form

$$I = \int_0^{\infty} F(K) J_0(Kr) K dK$$

where $F(K)$ has a single branch point at, say, $K = k_j$. If we write

$$I = \lim_{z' \rightarrow 0} \int_0^{\infty} F(K) e^{-v_j z'} J_0(Kr) K dK$$

where $v_j^2 = K^2 - k_j^2$, $\text{Re } v > 0$, then the integration around the branch cut becomes implicit.

We now expand $F(K)$ as a Taylor series in v_j , so that

$$F(K) = F(k_j) + \frac{dF}{dv} v_j + \frac{1}{2} \frac{d^2 F}{dv^2} v_j^2 + \dots$$

We now require the integrals

$$\lim_{z' \rightarrow 0} \int_0^{\infty} e^{-v_j z'} J_0(Kr) K dK = \lim_{z' \rightarrow 0} \left[\frac{\partial}{\partial z'} \left(\frac{e^{ik_j R'}}{R'} \right) \right] = 0$$

where $R'^2 = r^2 + z'^2$,

$$\begin{aligned} \lim_{z \rightarrow 0} \int_0^{\infty} v_j e^{-v_j z'} J_0(Kr) K dK &= \lim_{z' \rightarrow 0} \left[\frac{\partial^2}{\partial z'^2} \left(\frac{e^{ik_j R'}}{R'} \right) \right] \\ &= \frac{ik_j r}{r^2} \left(ik_j - \frac{1}{r} \right) \end{aligned}$$

where we have used the integral representation for a source of spherical waves, as in section 6.8. These are sufficient to enable us to evaluate the integral to terms in $\frac{1}{r^2}$, and we now have

$$I \sim \left(\frac{dF}{dv_j} \right)_{K=k_j} \frac{ik_j e^{ik_j r}}{r^2} + O(r^{-3}) \quad (A5.5)$$

Inspection of the matrix in equation (6.47) shows that for all terms in equations (A5.1), (A5.2) we can write in general form

$$F(K) = \frac{g_1(K) + v_j h_1(K)}{g_2(K) + v_j h_2(K)} \quad (A5.6)$$

giving

$$\left(\frac{dF}{dv_j} \right)_{K=k_j} = \frac{h_1(k_j)g_2(k_j) - h_2(k_j)g_1(k_j)}{[g_2(k_j)]^2} \quad (A5.7)$$

The analysis given here has not been rigorous, and certainly one questionable item is the convergence of the Taylor series for $F(K)$. Without giving a detailed discussion, this will be in doubt when $g_2(k_j)$ is small, implying a pole near the branch point. To find an upper limit for the branch cut integral we ignore g_1 , leaving

$$F(K) = \frac{g_1(K)}{v_j h_2(K)} + \frac{h_1(K)}{h_2(K)}$$

The second term gives no contribution, and we now have

$$I = \lim_{z' \rightarrow 0} \int_0^\infty \frac{g_1(K)}{h_2(K)} \frac{e^{-v_j z'}}{v_j} J_0(Kr) K dK$$

Using the methods of section 6.11, this gives

$$I \sim \frac{g_1(k_j)}{h_2(k_j)} \frac{e^{ik_j r}}{r} + O(r^{-2}) \quad (A5.8)$$

We therefore expect equation A5.5 to be valid only if

$$\left| \frac{g_1(k_j)}{h_2(k_j)} \frac{e^{ik_j r}}{r} \right| \gg \left| \frac{ik_j (h_1(k_j)g_2(k_j) - h_2(k_j)g_1(k_j))}{[g_2(k_j)]^2} \frac{e^{ik_j r}}{r} \right|$$

or

$$\left| \frac{r g_1(k_j) [g_2(k_j)]^2}{k_j h_2(k_j) [h_1(k_j)g_2(k_j) - h_2(k_j)g_1(k_j)]} \right| \gg 1 \quad (\text{A5.9})$$

This quantity may be regarded as related to the numerical distance established more precisely for the rigid porous medium.

A5.2 Grazing incidence

Although the evaluation of the correction term in the saddle-point integral is impractical for an elevated source, when $z_0 = 0$ the integral may be handled in exactly the same way as any other branch cut integral. This gives us the correction term for grazing incidence, and this may be a useful approximation for all cases where the geometric term is not dominant.

As a demonstration, we again consider the field at the surface of a rigid porous half-space at grazing incidence. This is given by

$$\phi \sim \frac{2i}{k_1 M^2 (n^2 - 1)} \frac{e^{ik_1 r}}{r^2} + O(r^{-3})$$

The upper limit is

$$\phi \sim 2 \frac{e^{ik_1 r}}{r}$$

and (6.64) becomes

$$|k_1 r M^2 (n^2 - 1)| \gg 1$$

These results are the limiting case of the more elaborate saddle-point integration.

A5.3 The k_1 branch cut integrals

To keep the expressions for the integrals as compact as possible, we define δ_{ijk} as the cofactor of the matrix element in row j and column k in the expansion of the determinant Δ_1 . We then find expressions for g_1 , c_1 etc. in the general equation (A5.6)

When dealing with the k_1 branch cut integral, we must have $z_0 = 0$. In equation (A5.1) we obtain

$$g_1(k_1) = 2(\delta_{311}(k_1) + a_{32}\delta_{211}(k_1) - (k_1^2 - k_4^2)^{\frac{1}{2}}\delta_{411})$$

$$h_1(k_1) = 0$$

$$g_2(k_1) = \Delta_0(k_1) \quad h_2(k_1) = \delta_{011}(k_1)$$

and in equation (A5.2)

$$g_1(k_1) = 2((k_1^2 - k^2)^{\frac{1}{2}}\delta_{311} + a_{32}(k_1^2 - k^2)^{\frac{1}{2}}\delta_{211} + k_1^2\delta_{411})$$

$$h_1(k_1) = 0, \quad g_2(k_1) = \Delta_0(k_1), \quad h_2(k_1) = \delta_{011}(k_1)$$

Using equation (A5.5), and noting that

$$\frac{\partial}{\partial r} \left(\frac{e^{ik_1 r}}{r^2} \right) = \frac{ik_1 e^{ik_1 r}}{r^2} + O(r^{-3})$$

we obtain the displacements in the far-field

$$u_r = \frac{2k_1^2 \delta_{011} [\delta_{311} + a_{32}\delta_{211} - (k_1^2 - k_4^2)^{\frac{1}{2}}\delta_{411}]}{\Delta_0^2} \frac{e^{ik_1 r}}{r^2} \quad (A5.10)$$

$$u_z = \frac{-2ik_1\delta_{011} [(k_1^2 - k_2^2)^{\frac{1}{2}}\delta_{311} + a_{32}(k_1^2 - k_2^2)\delta_{211} + k_1^2\delta_{411}]}{\Delta\delta} \frac{e^{ik_1r}}{r^3} \quad (A5.11)$$

Using (A5.9), these expressions will be valid if

$$\frac{r\Delta_0^2}{k_1\delta_{011}} \gg 1 \quad (A5.12)$$

A5.4 The k_2 branch cut integrals

For this and the remaining branch cut integrals we are not limited to $z_0 = 0$, but otherwise we proceed as before. For u_x we have

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [\Delta_3 + a_{32}\Delta_2 - v_4\Delta_4] \quad (A5.13)$$

$$h_1 = \frac{e^{-v_1 z_0}}{v_1} [\delta_{012} + a_{32}\delta_{042} - v_4(\delta_{412} + a_{32}\delta_{442})] \quad (A5.14)$$

$$g_2 = \Delta_0 \quad (A5.15)$$

$$h_2 = \delta_{012} + a_{32}\delta_{042} \quad (A5.16)$$

and for u_z

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [v_3\Delta_3 + k_2^2\Delta_4] \quad (A5.17)$$

$$h_1 = \frac{e^{-v_1 z_0}}{v_1} [a_{32}\Delta_2 + v_3(\delta_{012} + a_{32}\delta_{042}) + k_2^2(\delta_{412} + a_{32}\delta_{442})] \quad (A5.18)$$

$$g_2 = \Delta_0, \quad h_2 = \delta_{012} + a_{32}\delta_{042} \quad (A5.19), (A5.20)$$

where it is assumed that all expressions are evaluated for $K = k_2$. We shall not carry out here the straightforward substitution into equations (A5.7) and (A5.5).

A5.5 The k_1 branch cut integrals

For u_x

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [\Delta_1 + a_{12} \Delta_2 - v_4 \Delta_4] \quad (A5.21)$$

$$h_1 = \frac{e^{-v_1 z_0}}{v_1} [a_{12}(\delta_{243} + a_{23} \delta_{213}) - v_4(\delta_{443} + a_{23} \delta_{413})] \quad (A5.22)$$

$$g_2 = \Delta_0 \quad (A5.23)$$

$$h_2 = \delta_{043} + a_{23} \delta_{013} \quad (A5.24)$$

and for u_z

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [a_{12} v_2 \Delta_1 + k_1^2 \Delta_4] \quad (A5.25)$$

$$h_1 = \frac{e^{-v_1 z_0}}{v_1} [\Delta_1 + a_{12} v_2 (\delta_{243} + a_{23} \delta_{213}) + k_1^2 (\delta_{443} + a_{23} \delta_{413})] \quad (A5.26)$$

$$g_2 = \Delta_0 \quad (A5.27)$$

$$h_2 = \delta_{043} + a_{23} \delta_{013} \quad (A5.28)$$

where all evaluations are carried for $K = k_1$.

A5.6 The k_4 branch cut integrals

For u_r

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [\Delta_3 + a_{3,2} \Delta_2] \quad (A5.29)$$

$$h_1 = \frac{-e^{-v_1 z_0}}{v_1} [\Delta_4 + 2\mu k_4^2 (\delta_{334} + a_{3,2} \delta_{234})] \quad (A5.30)$$

$$g_2 = \Delta_0 \quad (A5.31)$$

$$h_2 = -2\mu k_4^2 \delta_{034} \quad (A5.32)$$

and for u_z

$$g_1 = \frac{e^{-v_1 z_0}}{v_1} [v_3 \Delta_3 + a_{3,2} v_2 \Delta_2 + k_4^2 \Delta_4] \quad (A5.33)$$

$$h_1 = \frac{-e^{-v_1 z_0}}{v_1} [2\mu k_4^2 (v_3 \delta_{334} + a_{3,2} v_2 \delta_{234})] \quad (A5.34)$$

$$g_2 = \Delta_0 \quad (A5.35)$$

$$h_2 = -2\mu k_4^2 \delta_{034} \quad (A5.36)$$

where the evaluations are carried out for $K = k_4$.

A5.7 Surface waves

Surface wave contributions will occur when poles of the various integrands lie between the real axis and the contour of integration. For this to happen we must have $\Delta_0 = 0$.

When the wavenumbers of the different wave types are real, a true surface wave must have a real wavenumber greater than these ([6.3], [6.4]).

When at least one wavenumber is complex, as in our application, the situation is less clear. In general it will be necessary to search for zeros of Δ_0 by some numerical means, and also to establish criteria as to whether they lie within the closed integration contour.

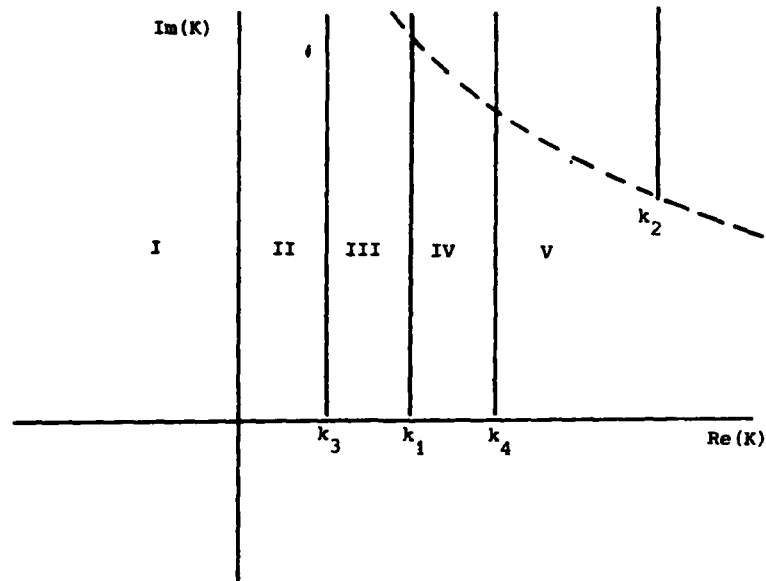
The general form of the required integral is

$$I = \int_0^{\infty} F(k) J_0(Kr) K dk$$

which may be rewritten as

$$I = \frac{1}{2} \int_{-\infty}^{\infty} F(k) H_0^{(1)}(Kr) K dk \quad (A5.37)$$

In the simplifying case of grazing incidence the required integration contours are those which pass around the vertical branch cuts emanating from the branch points at k_1, k_2, k_3, k_4 .



A typical situation is shown in the diagram, where poles in any of the regions I - V will contribute to the integral. The signs of the square roots v_1, v_2 etc. depend on the region, and in region I

$\text{Re } v_1 > 0$	$\text{Im } v_1 < 0$
$\text{Re } v_2 > 0$	$\text{Im } v_2 < 0$
$\text{Re } v_3 > 0$	$\text{Im } v_3 < 0$
$\text{Re } v_4 > 0$	$\text{Im } v_4 < 0$

In region II

$\text{Re } v_1 < 0$	$\text{Im } v_1 < 0$
$\text{Re } v_2 > 0$	$\text{Im } v_2 < 0$
$\text{Re } v_3 < 0$	$\text{Im } v_3 < 0$
$\text{Re } v_4 < 0$	$\text{Im } v_4 < 0$

In region III

$\operatorname{Re} v_1 < 0$	$\operatorname{Im} v_1 < 0$
$\operatorname{Re} v_2 > 0$	$\operatorname{Im} v_2 < 0$
$\operatorname{Re} v_3 > 0$	$\operatorname{Im} v_3 > 0$
$\operatorname{Re} v_4 < 0$	$\operatorname{Im} v_4 < 0$

In region IV

$\operatorname{Re} v_1 > 0$	$\operatorname{Im} v_1 > 0$
$\operatorname{Re} v_2 > 0$	$\operatorname{Im} v_2 < 0$
$\operatorname{Re} v_3 > 0$	$\operatorname{Im} v_3 > 0$
$\operatorname{Re} v_4 < 0$	$\operatorname{Im} v_4 < 0$

In region V

$\operatorname{Re} v_1 > 0$	$\operatorname{Im} v_1 > 0$
$\operatorname{Re} v_2 > 0$	$\operatorname{Im} v_2 < 0$
$\operatorname{Re} v_3 > 0$	$\operatorname{Im} v_3 > 0$
$\operatorname{Re} v_4 > 0$	$\operatorname{Im} v_4 > 0$

Here we have assumed that the pole lies below the dashed line which consists of the contours $\operatorname{Re} v_2 = 0$ and $\operatorname{Im} v_2 = 0$.

We now write the integrals as

$$I = I_1 + I_2 + I_3 + I_4 + 2\pi i \operatorname{res}(K_p)$$

where I_1, I_2 etc. are the branch cut integrals, and $\operatorname{res}(K_p)$ represents the residues of any poles lying within the prescribed regions.

In equation 6.92 we have in general

$$F(K) = G(k)/\Delta_0$$

giving

$$\text{res}(K_p) = \frac{K_p}{2} \frac{G(K_p)}{\Delta'_0(K_p)} H_0^1(K_p r)$$

where $\Delta'_0(K_p) = \frac{d\Delta_0}{dK}$, $K = K_p$

Using the asymptotic representation for the Hankel function, we obtain

$$2\pi i \text{res}(K_p) = \frac{G(K_p)}{\Delta'_0(K_p)} \left(\frac{2i\pi K_p}{r} \right)^{\frac{1}{2}} + O(r^{-3/2}) \quad (\text{A5.38})$$

Making use of equations A5.1 and A5.2 the surface wave contributions to the ground motion are

$$u_r = \frac{iK_p}{v_1 \Delta'_0} \left[\Delta_3 + a_{32} \Delta_2 - v_4 \Delta_4 \right] \left(\frac{2i\pi K_p}{r} \right)^{\frac{1}{2}} e^{-v_1 z_0 + iK_p r} + O(r^{-3/2}) \quad (\text{A5.39})$$

$$u_z = \frac{1}{v_1 \Delta'_0} \left[v_3 \Delta_3 + a_{32} v_2 \Delta_2 + K_p^2 \Delta_4 \right] \left(\frac{2i\pi K_p}{r} \right)^{\frac{1}{2}} e^{-v_1 z_0 + iK_p r} + O(r^{-3/2}) \quad (\text{A5.40})$$

where $v_1, v_2, v_3, v_4, \Delta'_0, \Delta_2, \Delta_3, \Delta_4$ are evaluated for $K = K_p$.

The major difficulties are the determination of K_p , and the evaluation of $\Delta'_0(K_p)$. In the next section we use a light fluid approximation which simplifies the problem considerably.

A5.8 The light fluid limit.

A limiting case of some interest occurs when the density of the fluid is much less than that of the solid in the lower medium, and we expect this to be applicable when modelling ground surfaces. In this case we have $\rho_1 \ll \rho$, $\rho_2 \ll \rho$, and also recall from the theory of rigid porous media that $\rho_1 \ll \rho_2$ at low frequencies. Since we expect the propagation constants of the various wave types to be roughly comparable, then the compressibility of the fluid must also be much greater than that of the solid.

If we examine the effect of this on the fast and slow waves, then the approximations

$$k_2 = k'_2, k_3 = k'_3$$

are valid if

$$\frac{M}{H} \ll 1, \quad \frac{\rho_1^2}{\rho \rho_2} \ll 1$$

It should be remarked here that α is always less than unity in Biot's theory.

For the shear wave

$$k_4^2 = \frac{\omega^2 \rho}{\mu} \quad \text{if} \quad \frac{\rho_1^2}{\rho \rho_2} \ll 1$$

From equation 6.36

$$a_{32} = \frac{\rho^2}{\rho} \left(\frac{\rho_1}{\rho_2} - \alpha \right) / \left(\frac{k_2'^2}{k_3'^2} - 1 \right) \quad (\text{A5.41})$$

$$= \frac{M}{H} \left(\frac{\rho_1}{\rho_2} - \alpha \right) / \left(1 - \frac{k_3'^2}{k_2'^2} \right) \quad (\text{A5.42})$$

and if $k_3'^2 < k_2'^2$, $\frac{M}{H} \ll 1$ implies $a_{32} \ll 1$.

From equation 6.37

$$a_{23} = \left(\frac{k_3'^2}{k_2'^2} \alpha - \frac{\rho_1}{\rho_2} \right) / \left(1 - \frac{k_3'^2}{k_2'^2} \right) \quad (\text{A5.43})$$

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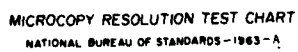
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These expressions simplify further if $\rho_1/\rho_2 \ll \alpha$, and $k_3'^2 \ll k_2'^2$.

Since the fast wave will invariably have the highest velocity of all wave types, this latter condition is effectively one of local reaction. Under these circumstances we have

$$a_{32} = \frac{-\alpha M}{H} \quad (A5.44)$$

$$a_{23} = \frac{\alpha M}{H} \frac{\rho}{\rho_2} = \alpha \frac{k_3'^2}{k_2'^2} \quad (A5.45)$$

In the light fluid limit we can also show that

$$\Delta_o = \Delta_a \Delta_b \quad (A5.46)$$

where

$$\Delta_a = \omega^2 (\rho_2 v_1 + \rho_1 v_2) \quad (A5.47)$$

$$\Delta_b = (2\mu K^2 - k_3'^2 H) (k_4'^2 - K^2) - 2\mu v_3 v_4 K^2 \quad (A5.48)$$

It can be seen that equation(A5.47) is the denominator found for the rigid porous case, and(A5.48) is the Rayleigh denominator for a semi-infinite elastic half-space. This is intuitively appealing since it means that in the light fluid limit wave propagation in the upper medium is identical to that for the rigid porous half-space, while the Rayleigh surface wave is virtually unaffected by the presence of the fluid.

Air-coupled surface waves of the Rayleigh type have been found

experimentally [6.2], so we suspect that poles corresponding to the zeros of (A5.48) will be of importance. In the surface wave expressions given in section (A5.7) we now have

$$\Delta_o'(K_p) = \Delta_a(K_p) \Delta_b'(K_p) \quad (A5.49)$$

where

$$\Delta_b(K_p) = 0 \quad (A5.50)$$

A5.9 Practical Calculations.

The calculation of the plane wave and branch cut contributions present no difficulty, but to obtain any surface wave contribution we need to find a zero of Δ_0 and its derivative at this point (see section A5.7). To do this in practice we have used a numerical technique known as Muller's method (described in reference A5.5) which finds a root of an analytic complex function by using quadratic interpolation. Three initial guesses are used to obtain the first quadratic, and the nearest zero of this provides the next approximation to the unknown root, and the iteration is continued until the required accuracy is achieved. This root gives us the surface wave number K_p . Consequently it is a simple matter to find the derivative $\Delta'_0(K_p)$ by calculating the gradient of the last fitted quadratic.

From the considerations of the previous section we expect a surface wave of the Rayleigh type which will have a wave number approximately 1.2 times that of the shear wave, k_4 . Using three initial guesses $1.15k_4$, $1.2k_4$ and $1.25k_4$ resulted in rapid convergence for all the examples tried, and there was no indication of any problem arising from the non-analytic nature of Δ_0 (due to the branch cuts)

A5.10 Material parameters for soil.

Sound velocity (ω/k_1)	$3.4 \times 10^4 \text{ cm s}^{-1}$
Density of air (ρ_1)	$1.2 \times 10^{-3} \text{ g cm}^{-3}$
P - wave velocity ($\omega/\text{Re}(k_3')$)	$5.0 \times 10^4 \text{ cm s}^{-1}$

$\text{Im}(k_3') / \text{Re}(k_3')$	1.25×10^{-2}
S - wave velocity ($\omega / \text{Re}(k_4)$)	$3.2 \times 10^4 \text{ cm s}^{-1}$
$\text{Im}(k_4) / \text{Re}(k_4)$	1.25×10^{-2}
Density of porous medium (ρ)	1.3 g cm^{-3}
Tortuosity	2.5
Shape factor	0.75
Flow resistivity	250 cgs rays
Porosity	0.4
α	0.5

The imaginary parts of the compressional and shear wave numbers are consistent with the results found by Prange [A5.6] for attenuation in soils.

The tortuosity, shape factor, flow resistivity, and porosity are used to determine the equivalent rigid-porous wave number k_2' and complex density ρ_2 (see chapter 5). The value chosen for the parameter α is arbitrary, apart from the constraint in Biot's theory that it must lie between the porosity and unity. However, we did repeat some of the calculations for $\alpha = 0.9$, and found that the changes to the computed surface velocity were not more than 2dB, and in most cases considerably less.

A5.11 Results

Figures A5.1 - A5.5 show the contributions of the various wave types to the vertical component of surface velocity for a source height of 1 m and a horizontal separation of 100 m. These are the plane wave

approximation, the near and far-field approximations to the k_1 , k_3 and k_4 branch cut integrals, and the Rayleigh-type surface wave. The k_2 branch cut integral was always negligible because of rapid attenuation of the slow wave.

Figures A5.6 - A5.8 show the dominant contributions for source heights of zero, 1 m and 50 m.

In all these cases a point source of spherical waves is used, such that the pressure amplitude of the incident wave is

$$P = \frac{ik_1 R}{R}$$

The results show that with the source on the surface the main contributions are those of the Rayleigh-type wave and the k_1 branch cut integral. The Rayleigh wave dominates below 200 Hz, but is attenuated at higher frequencies. Elevation of the source to 1 m gives a significant plane wave contribution, which becomes increasingly dominant at high frequencies. The plane wave term dominates at all frequencies when the source height is increased to 50 m. There are contributions from the k_3 and k_4 branch cuts, but these are always a small part of the total motion.

Figure A5.1 Plane Wave Contribution for a Source Height of 1 m

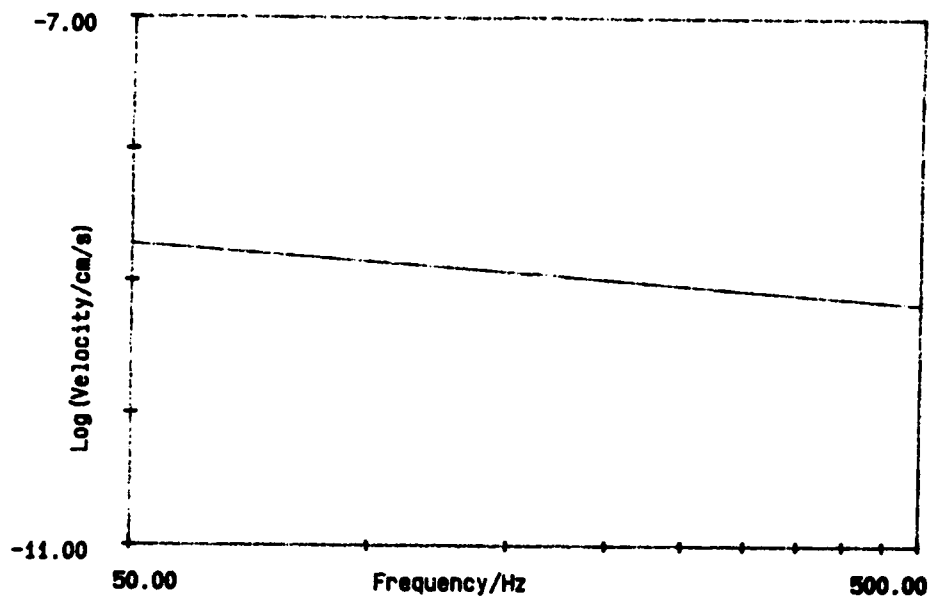


Figure A5.2 Contribution from k Branch Cut for a Source Height of 1 m

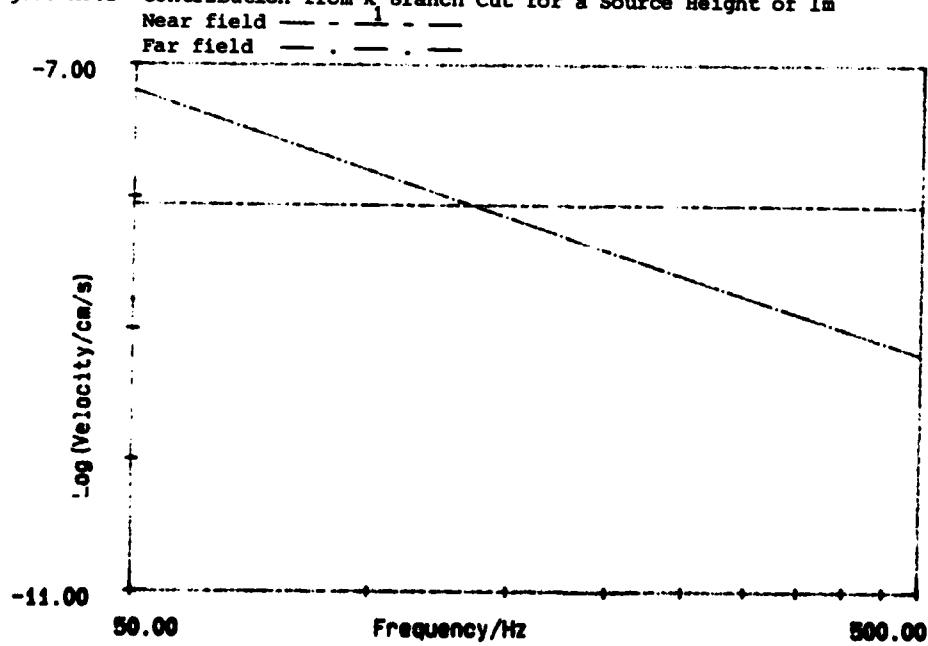
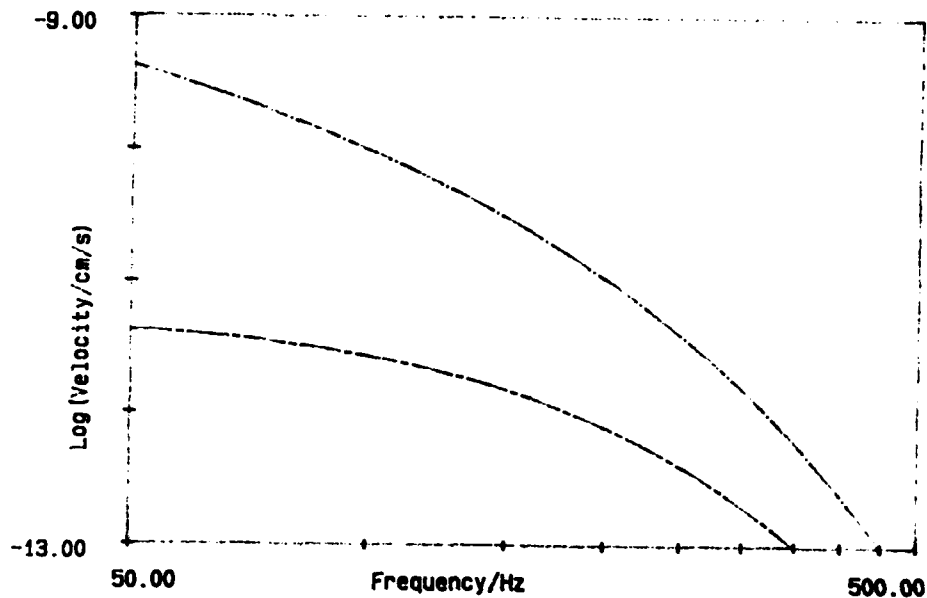


Figure A5.3 Contributions from k_3 Branch Cut for a Source Height of 1 m

Near-field — — — — —
 Far-field — . — . —

Figure A5.4 Contribution from k_4 Branch Cut for a Source Height of 1 m

Near-field — — — — —
 Far-field — . — . —

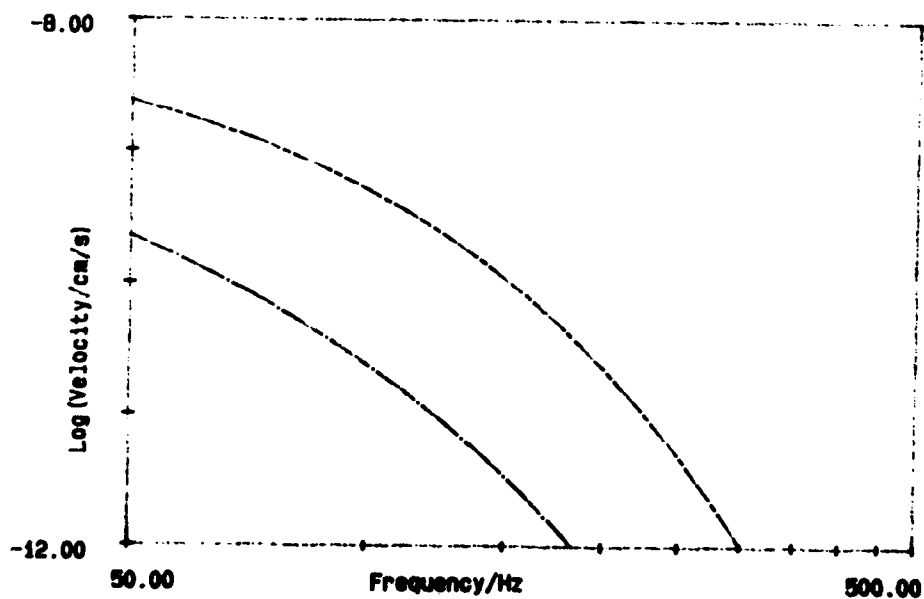


Figure A5.5 Contribution from Rayleigh-type Wave for a Source Height of 1m

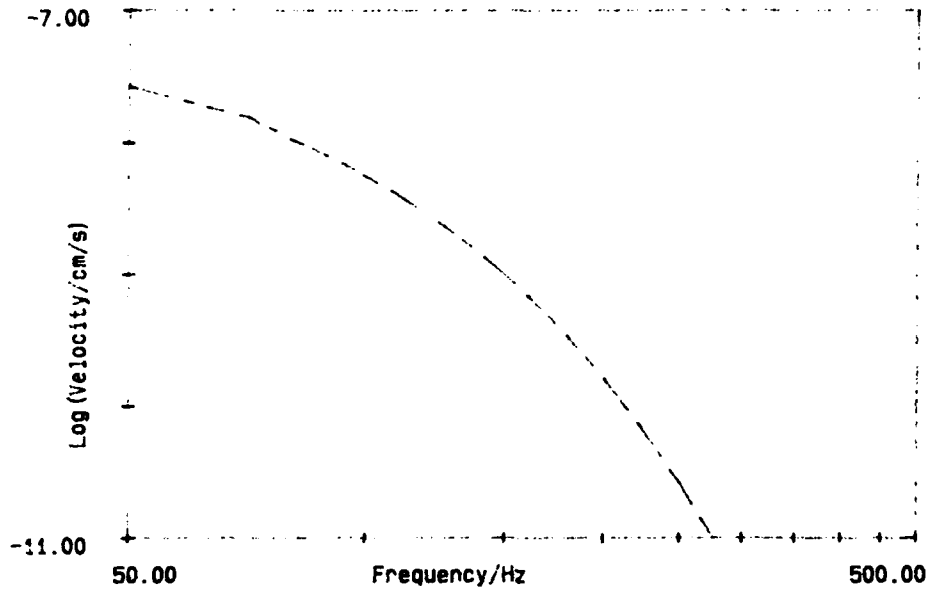
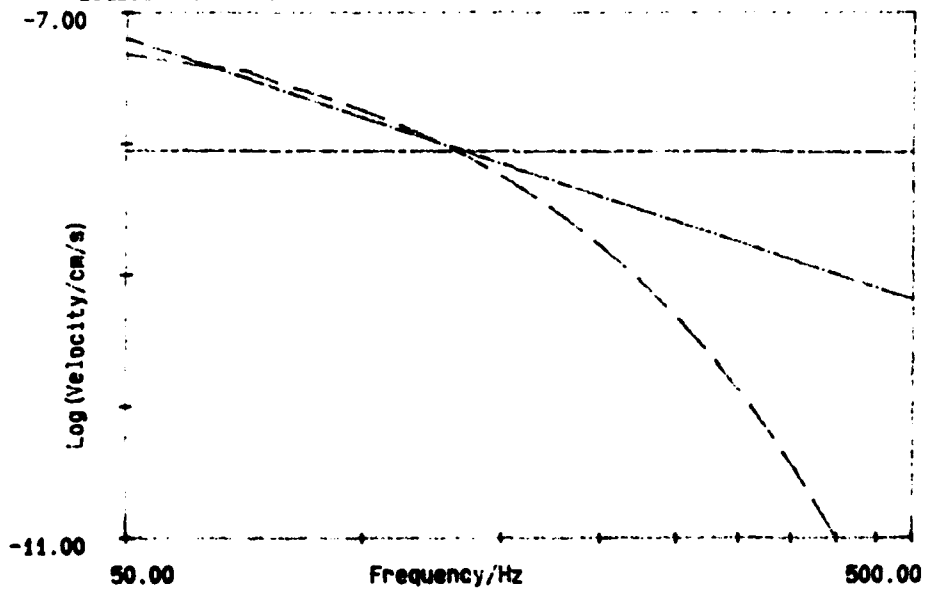
Figure A5.6 Contributions from k_1 Cut (Near- and Far-Field) and Rayleigh-Type Wave for Source and Receiver Both on Surface

Figure A5.7 Contributions from Plane Wave, k , Branch Cut (Near- and Far-Field) and Rayleigh-type Wave for Source Height of 1 m

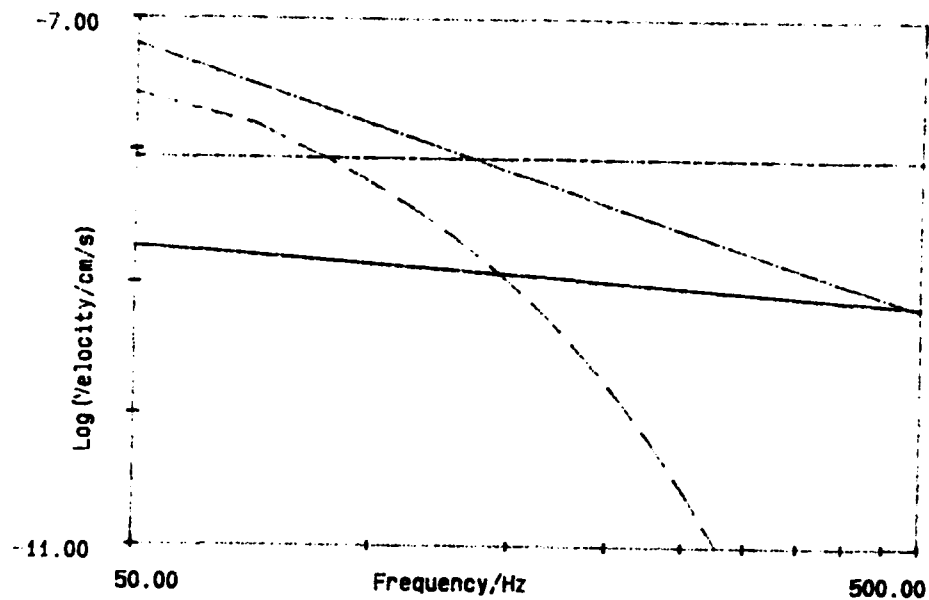
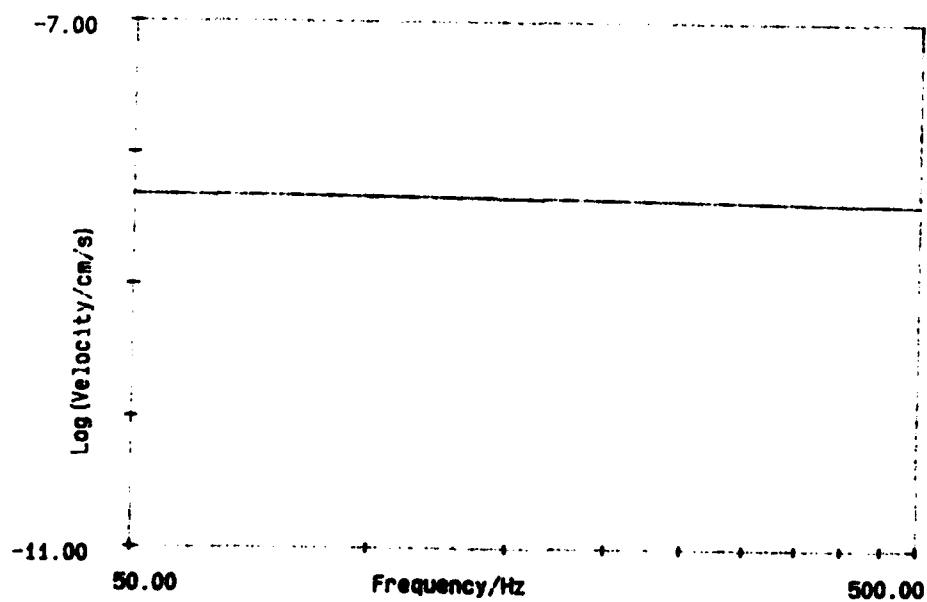


Figure A5.8 Dominant Plane-Wave Contribution for a Source Height of 50m



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