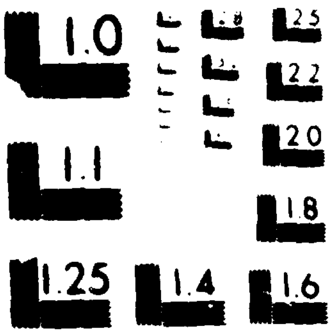
AD-A138 762 THEORETICAL STUDIES OF NONLINEAR PHENOMENA IN PLASMAS (U) MARYLAND UNIV COLLEGE PARK LAB FOR PLASMA AND FUSION ENERGY STUDIES H H CHEN 14 FEB 84 1/1

UNCLASSIFIED N00014-83-K-0247 F/G 20/9 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART

(12)

AD A138762

FINAL REPORT

Theoretical Studies of Nonlinear Phenomena in Plasmas

Contract Number N00014-83-K-0247

February 15, 1983 - February 14, 1984

Principal Investigator: H. H. Chen

UNIVERSITY OF MARYLAND

LABORATORY FOR PLASMA AND FUSION ENERGY STUDIES

COLLEGE PARK, MARYLAND

20742

FINAL REPORT

Theoretical Studies of Nonlinear Phenomena in Plasmas

Contract Number N000 14-83-K-0247

February 15, 1983 - February 14, 1984

Principal Investigator: H. H. Chen

FINAL REPORT

Theoretical Studies of Nonlinear

Phenomena in Plasmas

Contract Number N000 14-83-K-0247

February 15, 1983 - February 14, 1984

Principal Investigator: H. H. Chen

INTRODUCTION

Plasmas and fluids are rich in nonlinear phenomena, the occurrence of instabilities, the particle orbits in strong electromagnetic field, the weak and strong turbulence, etc., are just a few familiar topics that belong to this category. In the past decade, many breakthroughs occurred in the understanding of nonlinear phenomena due in large part to the advances in computing facilities. The discovery of solitons and the subsequent analytic solution method, the so-called inverse scattering method in solving the nonlinear wave equations completely, is one of the major events. Stochastic dynamics of particle orbits and the onset of turbulence in simple chaotic dynamical systems is another. Our contract with the Office of Naval Research carries out studies in this field and had made many contributions to the understanding of this exciting area of research.

PROJECTS COMPLETED

We summarize our accomplishments under this grant in this section. It covers a wide spectrum of topics. However, we will classify them into main areas and under each area we will outline the work done.

A. Nonlinear Internal Waves in Stratified Fluids

Internal waves that propagate in stratified fluid is a very important topic in physical oceanography. It is estimated that about fifty percent of kinetic energy in the ocean existed in the form of internal waves and yet nonlinear evolution of these waves are not well understood at all. We have studied many aspects of the nonlinear phenomena associated with the internal waves. Topics included soliton propagation in deep fluids, the Benjamin-Ono equation, the intermediate depth fluid equation, the chaotic motion of solitons, the coupling of internal waves with surface waves through a test wave Hamiltonian model, etc.

1. Internal Wave Solitons

It was first derived by Benjamin-Ono that nonlinear internal waves propagating in a deep two-fluid system is described by a nonlinear integro-differential equation now bearing their names, the Benjamin-Ono equation.

$$q_t + 2qq_x + Hq_{xx} = 0 ,$$

where

$$Hq(x) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{q(y)}{y-x} \, dy$$

is the Hilbert transform of q. Benjamin noticed the existence of a single solitary wave solution

$$q = \frac{\frac{1}{V}}{(x-Vt)^2 + \frac{1}{V^2}}$$

and was verified experimentally by Davis and Acrivos in a water tank experiment. However, the general solution of this equation is not known. We noted the numerical experiment of Meiss and Pereira that two Benjamin-

Ono solitons seem to maintain their identity after colliding with each other, discovered a way to construct general N-soliton solutions of this system. Later on, inverse scattering transforms, infinite sets of conservation laws, Backlund transformations, etc., were also discovered. Eventually, the complete solution is obtained. Not only that, we later found that damping and source can be added into the equation and yet still permit N-soliton solutions. However, the motion of the solitons become chaotic exhibiting the coexistence of both the coherence and chaos phenomena.

When the fluid depth is not infinite, Ko et al. derived another nonlinear evolution equation to describe the nonlinear internal wave.

$$q_t + 2qq_x + \frac{1}{\delta} q_x T q_{xx} = 0 ,$$

where

$$Tq(x) = \frac{1}{2\delta} \, P \int_{-\infty}^{\infty} \coth \frac{\pi(y-x)}{2\delta} \, q(y) dy .$$

This intermediate wave equation is especially interesting because, depending on the depth parameter δ, it approaches to either the KdV equation in the shallow water limit ($\delta \to 0$)

$$q_t + 2qq_x + \frac{\delta}{3} q_{xxx} = 0 ,$$

or the Benjamin-Ono equation

$$u_t + 2uu_x + HU_{xx} = 0$$

in the infinite depth limit ($\delta \to \infty$).

Using the Hirota's method, we again found the N-soliton solution of the intermediate equation. Later on, the inverse scattering operator was found and the equation was solved completely.

2. Generation of internal waves from its coupling to the surface waves

We studied systematically a model system describing the interaction of an internal wave with surface waves in the form of multiple-triads. In each triad, the internal wave is a daughter wave and different triads interact only through the sheared internal wave. This test wave Hamiltonian model was first proposed and studied by Watson, Meiss, and Cohen. They claim that in the case of two triads the system is integrable, therefore, nonstochastic. One should not expect the thermalization of energy among the participating wave triads. However, paradoxically, they treated the system statistically by averaging over an ensemble of initial data which would then predict a result of, for example, rapid decaying of correlation functions in time. But in the case of a single event we have demonstrated numerically the contrary. The correlation functions, in general, do not settle down quickly. Random phase approximation is found to be inadequate. The reason for this is the closeness of the system to an integrable one, which retains a long time correlation during a large portion of their trajectory in phase space.

B. Vortices in A Two-Dimensional Guiding Center Plasma

A plasma immersed in a strong uniform magnetic field would be constrained to move along the magnetic field lines to form charged rods. These charged rods in turn would interact with each other and result in a drift motion which resembles the vortices motion in a two-dimensional incompressible fluid. A salient feature of these two-dimensional fluid systems is the prediction of L. Osanger of a negative temperature equilibrium state. Using a most probable state approach, Joyce and Montgomery derived an equation of state of the system which is a nonlinear elliptic partial different equation

$$\nabla^2 \phi + \lambda^2 \sinh \phi = 0 ,$$

where λ^2 is positive for a negative kinetic temperature and ϕ is the electric potential.

Due to the nonlinear nature of this equation, analytic solution is hard to obtain. McDonald et al. used a generalized Newton's method to obtain several solutions of this equation numerically and tried to explain the formation of a large vortex as the thermal equilibrium state. We adopted instead the analytical method developed recently to obtain periodic solutions of nonlinear wave equations to this case and successfully obtained the various possible solutions. Most importantly, the analytic methods allow us to be sure of having obtained all possible solutions.

To simulate the curvature in a real magnetic confinement device, a gravity is introduced into the problem. The most interesting result obtained is the fact that in equilibrium the heavier ion vortex would float atop the lighter electron vortex. This exotic picture is due to the negative temperature nature of the system. An intricate bifurcation diagram is also obtained which shows various branches of solutions bifurcate out of and merge with each other.

C. Parametric Instabilities in a Plasma

Parametric instabilities were investigated in various plasma conditions.

1. Parametric Instabilities of Electron Cyclotron Waves in Plasmas

The recent developments of powerful microwave sources in the millimeter range have caused substantial efforts on the electron cyclotron heating of large devices, e.g., Elmo bumpy torus, tokamaks, and mirrors. We studied the decay of an electron cyclotron pump into two Bernstein modes at the second harmonic cyclotron layer in EBT. We find it able to account for the heating of the ring in the initial phase. The coupling coefficient for this decay vanishes for a dipole pump, whereas the convective threshold with finite k_0 is approximately 200 W/cm^2. The calculations could also be useful in the ionospheric profile modification experiments.

2. Cyclotron Radiation from a Relativistic Electron Beam in a Static Magnetic Field

To understand the nonlinear stage of a gyrotron, we studied charged particle motions in a self-consistent electromagnetic field and a static external magnetic field. This work is motivated by the recent interest on the electron cyclotron maser radiation mechanism and the advance in the understanding of nonlinear dynamical systems. The full nonlinear dynamics of the particles are followed numerically. Using the method of Poincare's return map, we found the particle's motion become violent and erratic when a critical energy is exceeded, while below this threshold energy, the particle trajectory is smooth and regular. The transition from regular to stochastic behavior is very sharp. It may have important consequences on the limitation of masering devices. It may also, on the other hand, provide a way to generate broad band radiations.

3. Decay Instability of an Ion-Cyclotron Wave in a Plasma

An electromagnetic ion cyclotron wave, around the second harmonic cyclotron frequency of majority ions in the central region of a tokamak, is seen to decay efficiently into two Bernstein waves. The coupling for this decay vanishes for a dipole pump; hence, a rigorous kinetic theory for decay instability of a finite wave number pump is developed. For a pump wave around the cyclotron frequency of majority ions, nonlinear Landau damping and oscillating two-stream instability turn out to be very important.

4. Parametric Instability Near the Ion-Cyclotron Frequency in an Inhomogeneous Plasma

A vigorous kinetic theory for the parametric instabilities of ion-cyclotron, drift-cyclotron, and drift-cyclotron loss cone modes of finite wave number in an inhomogeneous plasma is developed. The Vlasov equation is solved in the guiding center coordinates for two types of equilibria, viz., Maxwellian and loss cone velocity distributions. For flute modes, the coupling coefficients are greatly simplified. Ion-cyclotron waves are seen to decay efficiently into low frequency drift waves in tokamaks. In a mirror machine a high amplitude drift cyclotron loss cone mode of negative energy decays into low frequency positive energy modes, giving rise to explosive instability.

D. Miscellaneous Subjects

1. Minimum Energy State Configurations of Magnetized Plasmas

In a slightly resistive magnetized plasma Taylor argued that magnetic helicity is approximately conserved while the magnetic energy of the system dissipates away. The plasma would reach a stable equilibrium called the minimum energy state which is usually force-free. We have shown the existence of this force-free state in lindrical box. The axial symmetric state which has the minimum ene when the aspect ratio $L/R < 1.67$ becomes unstable to an asymmetrical state as the aspect ratio increases beyond the above critical value. This result may have a significant effect on the construction of a compact magnetically confined thermonuclear device called the spheromak.

2. Multi-Soliton Solutions of a Derivative Nonlinear Schrodinger Equation

In our earlier study of integrability of nonlinear Hamiltonian systems, a formalism was proposed to test the integrability of nonlinear wave equations. In particular, an example was introduced which was shown to be integrable by the method of inverse scattering. In this paper, we applied the Hirota's method to construct explicitly the solutions of this equation, a nonlinear derivative Schrodinger equation. Because of its simple form, it should find application in many nonlinear theories of wave.

3. A Nonlinear Equation for Bidirectional Ion-Sound Wave

Taking into account ion and electron nonlinearity and the effects of finite ion temperature, we derived a nonlinear equation $U_{tt} - U_{xx} = U_{xxtt} + (U_{xt})^2 + \alpha U_{xx} U_{tt}$, which can have ion-sound waves propagating in both directions. Having a different dependence on space and time derivatives, this new equation describes more properly the interaction of waves travelling in different directions. We found two soliton solutions of this equation from Hirota's method. But three or more than three soliton solutions are demonstrated to be nonexistent. This serves as an interesting physical example for which only two but no more than two solitons exist.

4. Cylindrical Ion-Acoustic Waves

Nonlinear ion acoustic waves in cylindrical geometry is described by a cylindrical Korteweg-de Vries equation. Numerically, Maxon and Viecelli showed that the cylindrical soliton exists with a tail. These solitons seem to be independent of each other. They maintain their identity after collisions with each other. We showed analytically that these solitons are indeed independent nonlinear degrees of freedom. Because of the Airy functions that show up in the solutions, tails always exist which propagate in a direction opposite to the main part of the soliton.

LIST OF PUBLICATIONS

1. Cyclotron Radiation from a Relativistic Electron Beam in a Static Magnetic Field, Z. G. An, Y. C. Lee, T. T. Lee, and H. H. Chen, submitted to J. Plasma Phys.
2. Parametric Instability Near the Ion-Cyclotron Frequency in an Inhomogeneous Plasma, O. P. Sharma, V. K. Tripathi, and C. S. Liu, Univ. of Maryland, Plasma Preprint PL #82-024.
3. Decay Instability of An Ion-Cyclotron Wave In a Plasma, O. P. Sharma, V. K. Tripathi, and C. S. Liu, Univ. of Maryland, Plasma Preprint PL #82-007.
4. Constraints of Conserved Quantities of the Kadomtsev-Petviashvilli Equations, Phys. Lett. A 89, 163 (1980).
5. Linear Instability of Internal Wave Solitons in Deep Stratified Fluid, H. H. Chen and D. J. Kamp, Phys. Fluids 23, 235 (1980).
6. Inverse Scattering Problem for Internal Waves with Finite Fluid Depth, H. H. Chen, R. Hirota, and Y. C. Lee, Phys. Lett. A 75, 255 (1980).
7. The Tilting Instability of a Cylindrical Spheromak, A. Bondeson, H. H. Chen, C. S. Liu, et al., Phys. Fluids 24, 1882 (1981).
8. Algebraic Internal Wave Solitons and the Integrable Calogero-Moser-Sutherland Problem, H. H. Chen, Y. C. Lee, and N. R. Pereira, Phys. Fluids 22, 187 (1979).
9. Internal Wave Solitons of Fluids with Finite Depth, H. H. Chen, and Y. C. Lee, Phys. Rev. Lett. 43, 264 (1979).
10. Integrability of Nonlinear Hamiltonian Systems, H. H. Chen, Y. C. Lee, and C. S. Liu, Phys. Scripta 20, 490 (1979).

11. Nonlinear Dynamical Models of Plasma Turbulence, Y. C. Lee and H. H. Chen, Phys. Scripta, Vol. T2:1, 41 (1982).

12. On a New Hierarchy of Symmetries for the Benjamin-Ono Equation, H. H. Chen, Y. C. Lee, and J. E. Lin, Phys. Lett. 91A, 381 (1982).

13. Multi-Soliton Solution of a Cylindrical Korteweg-de Vries Equation, H. H. Chen and A. Nakamura, J. Phys. Soc. Jpn. 50, 711 (1981).

14. Multi-Soliton Solutions of a Derivative Schrodinger Equation, H. H. Chen and A. Nakamura, J. Phys. Soc. Jpn. 49, 813 (1980).

15. Restricted Multiple Three-Wave Interaction I, Painleve Analysis, C. R. Menyuk, H. H. Chen, and Y. C. Lee, Phys. Rev. A 27, 1597 (1982).

16. Integrable Hamiltonian Systems and Lax Pair Formalism, C. R. Menyuk, H. H. Chen, and Y. C. Lee, Phys. Rev. A 26, 3731 (1982).

17. Parametric Instability of Electron-Cyclotron Waves in Plasmas, V. K. Tripathi and C. S. Liu, Phys. Fluids (1981).

18. Electromagnetic Oscillating Two-Stream Instability of Lower-Hybrid Waves, C. S. Liu, V. K. Tripathi, and S. T. Tsai, Nucl. Fusion (1981).

19. Electromagnetic Oscillating Two-Stream Instabilities of Lower-Hybrid Waves, C. S. Liu and V. K. Tripathi, Phys. Fluids (1981).

END

FILMED

4-84

DTIC