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# **MULTI-SENSOR ARRAY TIME-DELAY ESTIMATION WITH SMALL PHASE INCOHERENCES**

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# Multi-Sensor Array Time-Delay Estimation with Small Phase Incoherences

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## ABSTRACT

The maximum likelihood (ML) estimator for time-delay in a multisensor array in the presence of small phase incoherences is derived. The structure obtained is the canonical ML estimator with a correction term to compensate for the phase. The Cramer-Rao matrix bound is developed and used to evaluate the standard deviation of the time-delay estimate (TDE) as a function of system parameters such as number of sensors, phase variance, bandwidth and center frequency of the source. The curves indicate a sensitivity, at low input SNR, of the TDE variance to phase standard deviation and signal center frequency.

## I. Introduction

Time delay estimates in an array processor environment are important quantities for applications to localization and tracking. With a two-sensor array, the most commonly used technique is to process the received signals through a generalized cross-correlator (GCC) [1]. Several GCC processors, which optimize various performance indices, have appeared in the literature: the Eckart processor [2], maximum-likelihood (ML) or Hannan-Thompson processor [6], Hassab-Boucher processor [3], and Wiener processor [5]. For time-delay estimation (TDE) in a multi-sensor array, the ML estimator has been obtained by Hahn and Tretter [9], where it was shown that asymptotically the ML estimate achieved the Cramer-Rao matrix bound. Recently, ML TDE for multi-targets, in a multi-sensor environment has also been studied [12].

For a two-sensor array, the ML estimator can be manipulated into a GCC structure, details of which can be found in [1]. These results for the two-sensor case have been extended by Wax [7] to include a random phase difference between the received signals. In this paper we will extend the results of [7] to the case of multi-sensor arrays. The single-target situation will be assumed and the ML approach will be used in estimating the delays. In addition to the relative time delays, the signals at the receivers are subject to random but small phase incoherences. In contrast to Wax, who can integrate out the random phase in the two-sensor situation, we require a small phase assumption. In Section II, we will derive the ML processor with the assumption of small phase incoherence. As will be seen below, the phase incoherence requires a correcting system in addition to the canonical system developed in [9, Fig. 1]. In Section III, the Cramer-Rao matrix bound, or equivalently, the Fisher Information matrix is established. The Cramer-Rao minimum variance of the TDE for a special case is then derived. Numerical results in the form of standard deviation of the TDE are presented in

Section IV, as a function of system parameters such as number of sensors, signal bandwidth and center frequency, and variance of the incoherent phases.

## II. ML Estimator with Small Phase Incoherences

We assume that the analytic signal at the  $i^{\text{th}}$  receiver of an  $M$ -sensor array can be modeled by

$$\mathbf{x}_i(t) = s(t - D_i)e^{j\varphi_i} + n_i(t), \quad 0 \leq t \leq T \quad , \quad (1)$$

where  $s(t)$  and  $n_i(t)$  represent the pre-envelopes of the signal source (target) and additive noise, respectively.  $D_i$  denotes the time delay and  $\varphi_i$  is the phase incoherence. Since we are interested in relative time delays and relative phase incoherences, we assume that  $D_1 = \varphi_1 = 0$ . This model has been used in [7], [8], [11]. As discussed in [8], phase incoherence can arise from ocean bottom reflection, variations in local oscillators, or variations in the refraction index.

To facilitate the development, the usual assumptions are made:

- (1)  $s(t)$  and  $n_i(t)$ ,  $1 \leq i \leq M$  are independent wide-sense stationary Gaussian processes with zero means.
- (2)  $\varphi_i$  with probability density distribution  $f_i(\varphi)$  is independent of  $\varphi_j$  for  $j \neq i$ .
- (3) The observation interval  $T$  is large compared to  $\text{Max. } |D_i|$  plus the correlation time of the source signal.

With a development analogous to that in [7], we derive the ML processor for the multi-sensor case.

Let  $X_i(k)$  be the Fourier coefficient of  $\mathbf{x}_i(t)$  at the frequency  $\omega_k$ , i.e.,

$$X_i(k) = \frac{1}{T} \int_0^T \mathbf{x}_i(t) e^{-j\omega_k t} dt \quad , \quad (2)$$

where  $\omega_k$  is some integer multiple of  $2\pi/T$ . We assume that  $x_i(t)$  can be approximated by a finite Fourier-coefficient-weighted sum of  $\exp(j\omega_k t)$ ,  $1 \leq k \leq N$ . Of course, here we must choose the frequency range  $\omega_k$ ,  $1 \leq k \leq N$  in such a way that the truncation error can be neglected. To simplify our notation, we define

$$\underline{X}(k) \triangleq [X_1(k), X_2(k), \dots, X_M(k)]^T$$

$$\underline{X} \triangleq [\underline{X}^T(1), \underline{X}^T(2), \dots, \underline{X}^T(N)]^T$$

and

$$\underline{D} \triangleq [D_2, D_3, \dots, D_M]^T$$

$$\underline{\varphi} \triangleq [\varphi_2, \varphi_3, \dots, \varphi_M]^T$$

Note that due to Assumption (3) in the above, we obtain

$$E[\underline{X}(k) \underline{X}^\dagger(j)] = V(k) \delta_{kj} \quad (3)$$

where  $\dagger$  denotes conjugate transpose and  $V(k)$  is the covariance matrix for  $\underline{X}(k)$ .

That is, for large observation time, the Fourier coefficients are uncorrelated. From Assumption (1) and Eqn. (3), we have the following probability density distribution:

$$p(\underline{X} | \underline{D}, \underline{\varphi}) = \left[ \pi^{NM} \prod_{k=1}^N |V(k)| \right]^{-1} \cdot \exp \left[ - \sum_{k=1}^N \underline{X}^\dagger(k) V^{-1}(k) \underline{X}(k) \right] \quad (4)$$

With  $\Phi_s(k)$  and  $\Phi_{n_i}(k)$  denoting the power spectral densities at  $\omega_k$  for  $s(t)$  and  $n_i(t)$ , respectively, it is straightforward to show from (1) that

$$V(k) \approx \frac{1}{T} \left[ \Phi_s(k) \underline{E}(k) \underline{E}^\dagger(k) + \Phi_N(k) \right] \quad (5)$$

where

$$\underline{E}(k) = \left[ e^{-j\omega_k(D_1 - \varphi_1)}, \dots, e^{-j\omega_k(D_M - \varphi_M)} \right]^T$$

and

$$\Phi_N(k) = \text{diag}\{\Phi_{n_1}(k), \Phi_{n_2}(k), \dots, \Phi_{n_M}(k)\}$$

Now using the relationship  $T |V(k)| = \left[1 + \sum_{i=1}^M \Phi_s(k) / \Phi_{n_i}(k)\right] |\Phi_N(k)|$ , we can show that the first factor on the right-hand side of (4) is independent of  $\underline{D}$  and  $\underline{\varphi}$ . After applying the matrix inversion lemma to  $V(k)$  given in (5), we can express (4) with some tedious but straightforward manipulations as:

$$p(\underline{X} | \underline{D}, \underline{\varphi}) = C_1 \cdot \exp(C_2) \quad , \quad (6)$$

where  $C_1$  is a constant independent of  $\underline{D}$  and  $\underline{\varphi}$ , and  $C_2$  is given by

$$C_2 = \sum_{k=1}^N \underline{E}^\dagger(k) \Phi_{\bar{N}}^{-1}(k) \hat{\Phi}_x(k) \Phi_{\bar{N}}^{-1}(k) \underline{E}(k) \Phi_t(k) \quad (7.a)$$

$$\approx \frac{T}{2\pi} \int_0^\infty \underline{E}^\dagger(\omega) \Phi_{\bar{N}}^{-1}(\omega) \hat{\Phi}_x(\omega) \Phi_{\bar{N}}^{-1}(\omega) \underline{E}(\omega) \Phi_t(\omega) d\omega \quad , \quad (7.b)$$

where  $\hat{\Phi}_x(k)$  is defined as  $T[X(k) X^\dagger(k)]$  and

$$\Phi_t(k) = \left[ \Phi_s^{-1}(k) + \sum_{i=1}^M \Phi_{n_i}^{-1}(k) \right]^{-1} \quad . \quad (8)$$

Furthermore, with

$$D_s(\omega) \triangleq \text{diag} \left\{ e^{-j\omega D_1}, e^{-j\omega D_2}, \dots, e^{-j\omega D_M} \right\}$$

and

$$\underline{\varphi}_s \triangleq \left[ e^{j\varphi_1}, e^{j\varphi_2}, \dots, e^{j\varphi_M} \right]^T \quad .$$

we can rewrite (6) as

$$p(\underline{X} | \underline{D}, \underline{\varphi}) = C_1 \cdot \exp \left[ \underline{\varphi}_s^\dagger H(\underline{D}) \underline{\varphi}_s \right] \quad , \quad (9)$$

where

$$H(\underline{D}) = \frac{T}{2\pi} \int_0^\infty D_s^\dagger(\omega) \Phi_{\bar{N}}^{-1}(\omega) \hat{\Phi}_x(\omega) \Phi_{\bar{N}}^{-1}(\omega) D_s(\omega) \Phi_t(\omega) d\omega \quad . \quad (10)$$

Therefore, maximizing the likelihood function  $p(\underline{X} | \underline{D}) = E_{\varphi}\{p(\underline{X} | \underline{D}, \varphi)\}$  is equivalent to maximizing <sup>1</sup>

$$Q(\underline{D}) = E_{\varphi}\{\exp[\varphi^{\dagger} H(\underline{D}) \varphi]\} \quad (11)$$

It may be difficult to find an explicit expression for  $Q(\underline{D})$  in (11) even if we know the probability density distribution for  $\varphi$ . However, if we assume that the phase incoherence between any two sensors is small with large probability, we can obtain an approximate expression for  $Q(\underline{D})$  as follows.<sup>2</sup> Let  $H_{ij}(\underline{D})$  denote the  $ij^{\text{th}}$  element of the Hermitian matrix  $H(\underline{D})$ . By expanding the exponent in (11) and using the approximation that  $\exp(a) \approx 1 + a$  for small  $a$ , we have

$$Q(\underline{D}) \approx \exp\left\{\sum_{i=1}^M \sum_{k=1}^M H_{ik}(\underline{D})\right\} \cdot E_{\varphi}\left\{\exp\left[\sum_{i=1}^M \sum_{k=1}^M j H_{ik}(\underline{D}) (\varphi_k - \varphi_i)\right]\right\} \quad (12)$$

Simple manipulation of the exponent in (12) leads to

$$Q(\underline{D}) = \exp\{\underline{1}^T H(\underline{D}) \underline{1}\} \cdot E_{\varphi}\left\{\exp\left[\sum_{i=2}^M t_i \varphi_i\right]\right\} \quad (13)$$

where  $\underline{1}$  means the  $M$ -dimensional column vector with all elements being unity, and

$$t_i = 2 \sum_{k=1}^M \text{Im}\{H_{ik}(\underline{D})\} \quad (14)$$

with  $\text{Im}(\bullet)$  denoting the imaginary part of a complex quantity. In deriving (14) the fact that  $H(\underline{D})$  is a Hermitian matrix has been used. Therefore, from the independence of the  $\varphi_i$ 's, (13) reduces to

$$Q(\underline{D}) = \exp\{\underline{1}^T H(\underline{D}) \underline{1}\} + \sum_{i=2}^M \ln M_i(t_i) \quad (15)$$

<sup>1</sup>  $E_{\varphi}\{\bullet\}$  designates the expectation with respect to  $\varphi$ .

<sup>2</sup> Alternatively, we assume that the probability densities  $f_i(\varphi)$  are highly concentrated around zero.

where  $M_i(\bullet)$  is the moment generating function (MGF) for  $\varphi_i$ . From (15) we now conclude that the ML estimate of  $\underline{D}$  can be obtained by maximizing  $Q^1(\underline{D})$  which is given by

$$Q^1(\underline{D}) = \underline{1}^T H(\underline{D}) \underline{1} + \sum_{i=2}^M \ln M_i(t_i) \quad (16)$$

Note that the first term on the right-hand side of (16) is the canonical ML estimator discussed in [9, Sec. III]. However, due to the assumed phase incoherences between the signals at the sensors, a correction term which is represented by the summation of logarithms of the phase moment generating functions must be added.

*Implementation.* Here we want to construct a unified system structure, although it may be further manipulated for particular situations (e.g., GCC structure for two-sensor case). Let  $h_i(t)$  denote the output signal after  $x_i(t)$  is filtered by a linear system with frequency response  $\sqrt{\Phi_i(\omega)} e^{j\omega D_i} / \Phi_{n_i}(\omega)$ . From (10) we have the  $ij^{th}$  element of  $H(\underline{D})$  given by

$$H_{ij}(\underline{D}) \approx \int_0^T h_i(t) h_j^*(t) dt \quad (17)$$

where \* means complex conjugate. In deriving (17) Parseval theorem and the fact that the Fourier spectrum of  $x_i(t)/T$  at frequency  $\omega_k$  is equal to  $X_i(k)$  in (2) have been used. Based on the relationship (17), the ML estimator can now easily be established from (16). This is illustrated in Fig. 1. The multiple-input multiple-output correlator in Fig. 1 accounts for the small phase incoherences involved at the sensors. Note that for the implementation in Fig. 1, we must have *a priori* knowledge on the power spectral densities for source signal and sensor noise, and the moment generating functions for the phase incoherences.

In the following, we will consider the Cramer-Rao matrix bound for the ML TDE. The effect of phase incoherence on the variance of the TDE will be discussed.

### III. Cramer-Rao Matrix Bound (CRMB) for TDE

It is well-known that the ML estimate will, under general conditions, achieve the Cramer-Rao bound [15]. In this section, we will consider the CRMB, or equivalently, Fisher information matrix (FIM) for the ML TDE. The result, Eq. (29) below, can serve as a guide in picking the reference sensor.

Let  $\underline{\xi} = (\underline{D}^T, \underline{\varphi}^T)^T$ . Then the FIM for  $\underline{\xi}$  can be represented by the following block matrix:

$$FIM(\underline{\xi}) = \begin{bmatrix} \alpha & \beta \\ \beta^T & \gamma + \eta \end{bmatrix}, \quad (18)$$

where the  $ij^{th}$  elements of  $(M-1) \times (M-1)$  matrices  $\alpha$ ,  $\beta$ , and  $\gamma$  are given, respectively, by

$$\alpha_{ij} = -E \left[ \partial^2 \ln p(\underline{X} | \underline{D}, \underline{\varphi}) / \partial D_{i+1} \partial D_{j+1} \right] \quad (19.a)$$

$$\beta_{ij} = -E \left[ \partial^2 \ln p(\underline{X} | \underline{D}, \underline{\varphi}) / \partial D_{i+1} \partial \varphi_{j+1} \right] \quad (19.b)$$

$$\gamma_{ij} = -E \left[ \partial^2 \ln p(\underline{X} | \underline{D}, \underline{\varphi}) / \partial \varphi_{i+1} \partial \varphi_{j+1} \right] \quad (19.c)$$

and  $\eta$  is an  $(M-1) \times (M-1)$  diagonal matrix with  $i^{th}$  diagonal element  $\eta_i$  given by

$$\eta_i = -E \left[ \partial^2 \ln f_{i+1}(\varphi) / \partial \varphi^2 \right] \quad (19.d)$$

Given the probability density function of (4), the derivations of  $\alpha_{ij}$ ,  $\beta_{ij}$  and  $\gamma_{ij}$  are straightforward, although tedious. Hence, we omit the complicated derivations and summarize the results in the following:

$$\alpha_{ij} = \frac{T}{\pi} \int_0^{\infty} \omega^2 \psi_{ij}(\omega) d\omega \quad (20.a)$$

$$\beta_{ij} = \frac{-T}{\pi} \int_0^{\infty} \omega \psi_{ij}(\omega) d\omega \quad (20.b)$$

and

$$\gamma_{ij} = \frac{T}{\pi} \int_0^{\infty} \psi_{ij}(\omega) d\omega \quad (20.c)$$

where, with  $\delta_{ij}$  denoting the Kronecker delta,  $\psi_{ij}(\omega)$  is given by

$$\psi_{ij}(\omega) = \frac{\Phi_t(\omega) \Phi_s(\omega)}{\Phi_{n_{i+1}}(\omega)} \left[ \delta_{ij} \cdot \sum_{k=1}^M \frac{1}{\Phi_{n_k}(\omega)} - \frac{1}{\Phi_{n_{j+1}}(\omega)} \right] \quad (21)$$

Note that from the definition of  $\Phi_t(\omega)$  in (8), (21) can be rewritten in terms of the input signal-to-noise ratio (SNR) as

$$\psi_{ij}(\omega) = \rho_{i+1}(\omega) \left[ \delta_{ij} \sum_{k=1}^M \rho_k(\omega) - \rho_{j+1}(\omega) \right] / \left[ 1 + \sum_{k=1}^M \rho_k(\omega) \right] \quad (22)$$

where the SNR  $\rho_i(\omega)$  at the  $i^{\text{th}}$  sensor is defined as  $\Phi_s(\omega)/\Phi_{n_i}(\omega)$ . When  $\varphi_i$  is normally distributed with zero mean and variance  $\sigma_i^2$  ( $\sigma_i$  is small for small phase incoherence), it can be shown that  $\eta_i = 1/\sigma_{i+1}^2$ .

The CRMB for TDE can now be obtained from the inverse of FIM ( $\underline{\xi}$ ). Note that CRMB ( $\underline{D}$ ) is the first block matrix of order  $(M-1) \times (M-1)$  along the diagonal of  $FIM^{-1}(\underline{\xi})$ . Mathematically, it is represented by

$$CRMB(\underline{D}) = \alpha^{-1} + \alpha^{-1} \beta (\gamma + \eta - \beta^T \alpha^{-1} \beta)^{-1} \beta^T \alpha^{-1} \quad (23)$$

To further investigate the effect of phase incoherences on this least bound, we consider a special case where the one-side power spectral densities for source signal and sensor noise are assumed, respectively, as

$$\Phi_s(\omega) = \begin{cases} \Phi_s & , \quad \text{if } |\omega - \omega_0| \leq W/2 \\ 0 & , \quad \text{otherwise} \end{cases} \quad (\omega \geq 0) \quad (24)$$

and

$$\Phi_{n_i}(\omega) = \Phi_{n_i} \quad (25)$$

The assumptions of white noise and bandlimited source signal with flat spectrum allow us to obtain some explicit results on the variance of the TDE.<sup>3</sup>

Let  $\rho_i$ , the SNR at the  $i^{\text{th}}$  sensor, be defined as  $\Phi_s / \Phi_{n_i}$ . Also, to simplify the notation, we define

$$\rho = \sum_{k=1}^M \rho_k \quad (26)$$

and

$$\Lambda = \text{diag} \{ \rho_2, \rho_3, \dots, \rho_m \} \quad (27)$$

Now from (20)-(27), and a considerable amount of mathematical manipulation, we can obtain

$$\text{CRMB}(\underline{D}) = \quad (28)$$

$$\vartheta_0 \left\{ \Lambda^{-1} + \frac{1}{\rho_1} \underline{1} \underline{1}^T + \frac{\omega_0^2}{W^2/12} \left[ (\Lambda + \vartheta_1 \eta)^{-1} + \vartheta_2 (\Lambda + \vartheta_1 \eta)^{-1} \Lambda \underline{1} \underline{1}^T \Lambda (\Lambda + \vartheta_1 \eta)^{-1} \right] \right\}$$

where  $\underline{1}$  is a  $(M-1)$ -dimensional column vector with all elements being unity, and

$\vartheta_0, \vartheta_1, \vartheta_2$  are given by

$$\vartheta_0 = \frac{\pi(1+\rho)}{(TW)\rho} (\omega_0^2 + W^2/12)^{-1}$$

$$\vartheta_1 = \frac{\pi(1+\rho)}{(TW)\rho} \frac{(\omega_0^2 + W^2/12)}{W^2/12}$$

and

$$\vartheta_2 = \left\{ \rho_1 + \sum_{k=2}^M \frac{\vartheta_1 \rho_k \eta_{k-1}}{\rho_k + \vartheta_1 \eta_{k-1}} \right\}^{-1}$$

Note that the first two terms in (28) represent the CRMB when no phase

<sup>3</sup> The following results also fit the case where  $\Phi_s(\omega)$  is bandlimited and has the same shape as  $\Phi_{n_i}(\omega)$  within that band.

incoherence is present. More explicitly, from (28), we have

$$\text{Var}_{CR}(\hat{D}_i - D_i) \approx \vartheta_0 \left\{ \frac{1}{\rho_i} + \frac{1}{\rho_1} + \frac{\omega_0^2}{W^2/12} \left[ \frac{1}{\rho_i + \vartheta_1 \eta_{i-1}} + \vartheta_2 \left( \frac{\rho_i}{\rho_i + \vartheta_1 \eta_{i-1}} \right)^2 \right] \right\} \quad (29)$$

and

$$\text{COV}_{CR}(\hat{D}_i, \hat{D}_j) \approx \vartheta_0 \left\{ \frac{1}{\rho_1} + \frac{\omega_0^2 \vartheta_2}{W^2/12} \frac{\rho_i \rho_j}{(\rho_i + \vartheta_1 \eta_{i-1})(\rho_j + \vartheta_1 \eta_{j-1})} \right\} \quad (30)$$

An interesting observation from (29) is that it may be advantageous to choose the sensor with largest SNR as the reference sensor since the variance of the TDE tends to decrease in general for larger  $\rho_1$ .

#### IV. Numerical Results

We assume that the SNR's at the sensors are the same and that the  $\varphi_i$ 's are normally distributed with zero mean and variance  $\sigma^2$ , i.e.,  $\eta_i = 1/\sigma^2$ . Curves for the standard deviation of the TDE based on (29) have been plotted in Figures 2.a-2.e with varying values of the following system parameters: standard deviation of the phase incoherence  $\sigma$ ; number of sensors M; observation interval T; center frequency of the source  $\omega_0$ ; bandwidth of the source W.

As expected, the error variance for a fixed SNR tends to decrease as  $\sigma$  decreases (see Fig. 2.a). Observe that in Figure 2.a, the standard deviations at high SNR are very close for  $\sigma = 0.1$  and 0.05. In these cases  $\rho_i$ , the sensor SNR, is dominant over  $\vartheta_1 \eta_{i-1}$  [see Eqn. (29)]. In contrast, for low input SNR the error in the TDE appears to be quite sensitive to the phase standard deviation  $\sigma$ . An explanation for this sensitivity, or thresholding, may have to do with the nature of the small phase approximation. From (1), and using  $e^{j\varphi} \approx 1 + j\varphi$ , we have:

$$\begin{aligned} \mathbf{x}_i(t) &= s(t - D_i) e^{j\varphi_i} + \mathbf{n}_i(t) \\ &\approx s(t - D_i) + [j\varphi_i s(t - D_i) + \mathbf{n}_i(t)] \end{aligned}$$

Both terms in brackets can be viewed as the noise. As the signal power decreases, the first term  $\varphi_i s(t-D_i)$  becomes a more important contributor to the noise power, thus leading to the sensitivity of the TDE variance at low SNR.

It is interesting to observe from Fig. 2.b that as  $M$ , the number of sensors increases, the error variance in the TDE does not decrease dramatically. Since the system has to estimate  $(M-1)$  variables, the TDE uncertainty is not markedly decreased with increasing number of sensors.

As expected, Figures 2.b-2.e indicate that estimation performance is improved as the quantities  $M$ ,  $T$ ,  $\omega_0$ , or  $W$  increase. Fig. 2.d is somewhat surprising in that the variance of the TDE is quite sensitive to the center frequency of the signal for low SNR.

## V. Summary and Conclusion

The ML estimator for time-delays in a multi-sensory array with small phase incoherences has been derived. In addition to the canonical ML estimator as developed in [9], a correcting system is required to compensate for the phase incoherences. The Cramer-Rao matrix bound for ML TDE was also obtained. Under the assumptions of white noise and bandlimited source signal with flat spectrum, we were able to find explicit results on the Cramer-Rao minimum variance for each TDE. These results serve as a guide on how to choose the reference sensor. Curves of Cramer-Rao minimum standard deviation for TDE were generated for the case of equal input SNR and identically distributed Gaussian phase incoherences. As expected, performance improved for larger  $M$ ,  $T$ ,  $\omega_0$ ,  $W$  and smaller  $\sigma$ .

As indicated by the implementation in Fig. 1, the power spectral densities for both source signal and sensor noise and also the probability distribution for

phase incoherences must be known. These quantities, in practice, may have to be measured from input signals  $x_i(t)$  and some other *a priori* knowledge. For example, with  $\chi_i(\omega)$  denoting the Fourier transform of  $x_i(t)$ , we may have to use the estimates

$$\hat{\Phi}_s(\omega) = \frac{1}{(M^2 - M)T} \sum_{i=1}^M \sum_{\substack{j=1 \\ i \neq j}}^M |\chi_i(\omega) \chi_j^*(\omega)| \quad (31)$$

$$\hat{\Phi}_{n_i}(\omega) = |\chi_i(\omega)|^2 / T - \hat{\Phi}_s(\omega) \quad (32)$$

It has been shown in [9] that, when there is no phase incoherence, it is possible to solve this multiple TDE problem by applying a GCC system to every possible pair of sensors; with appropriate filters in each GCC system, this approach does not degrade the covariance matrix bound obtained for ML TDE. It is not known whether or not there is a corresponding formulation when the phase incoherences are present. This is an interesting topic for further investigation.

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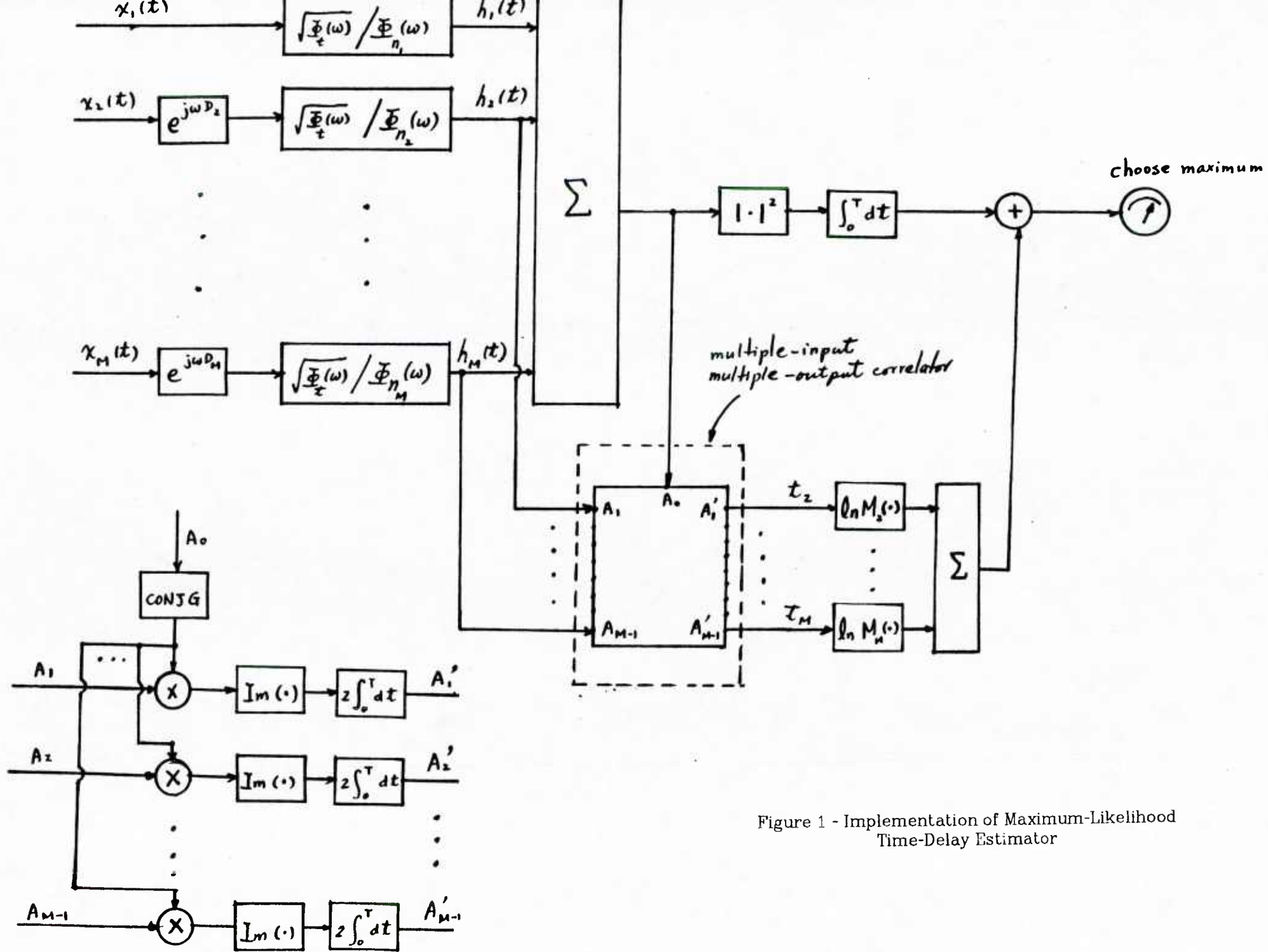


Figure 1 - Implementation of Maximum-Likelihood Time-Delay Estimator

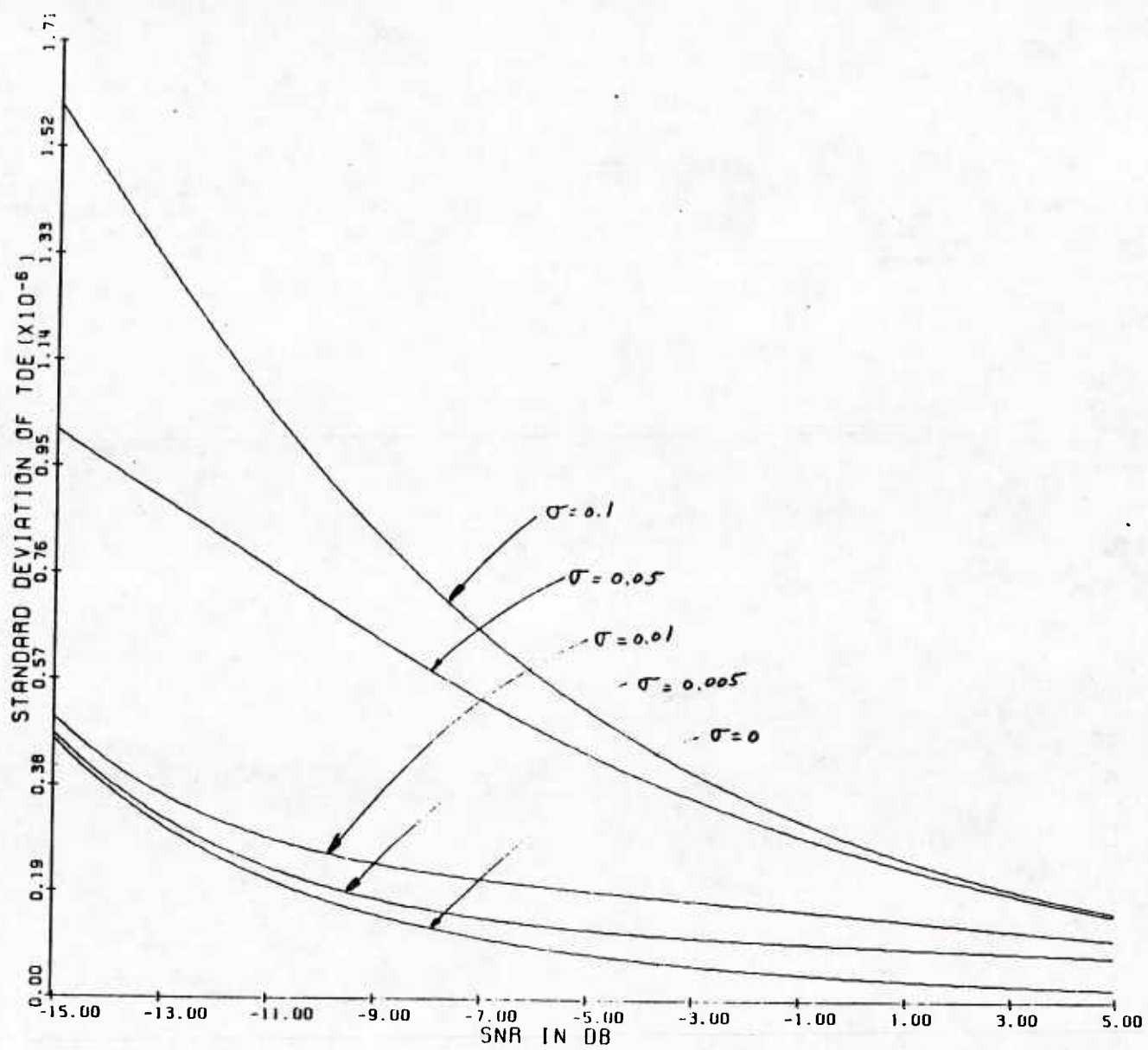


Figure 2a - Standard deviation of TDE vs. input SNR for various  $\sigma$ ;  
 $M = 10$ ,  $T = 60$  seconds,  $\omega_0/2\pi = 8$  KHz,  $W/2\pi = 4$  KHz.

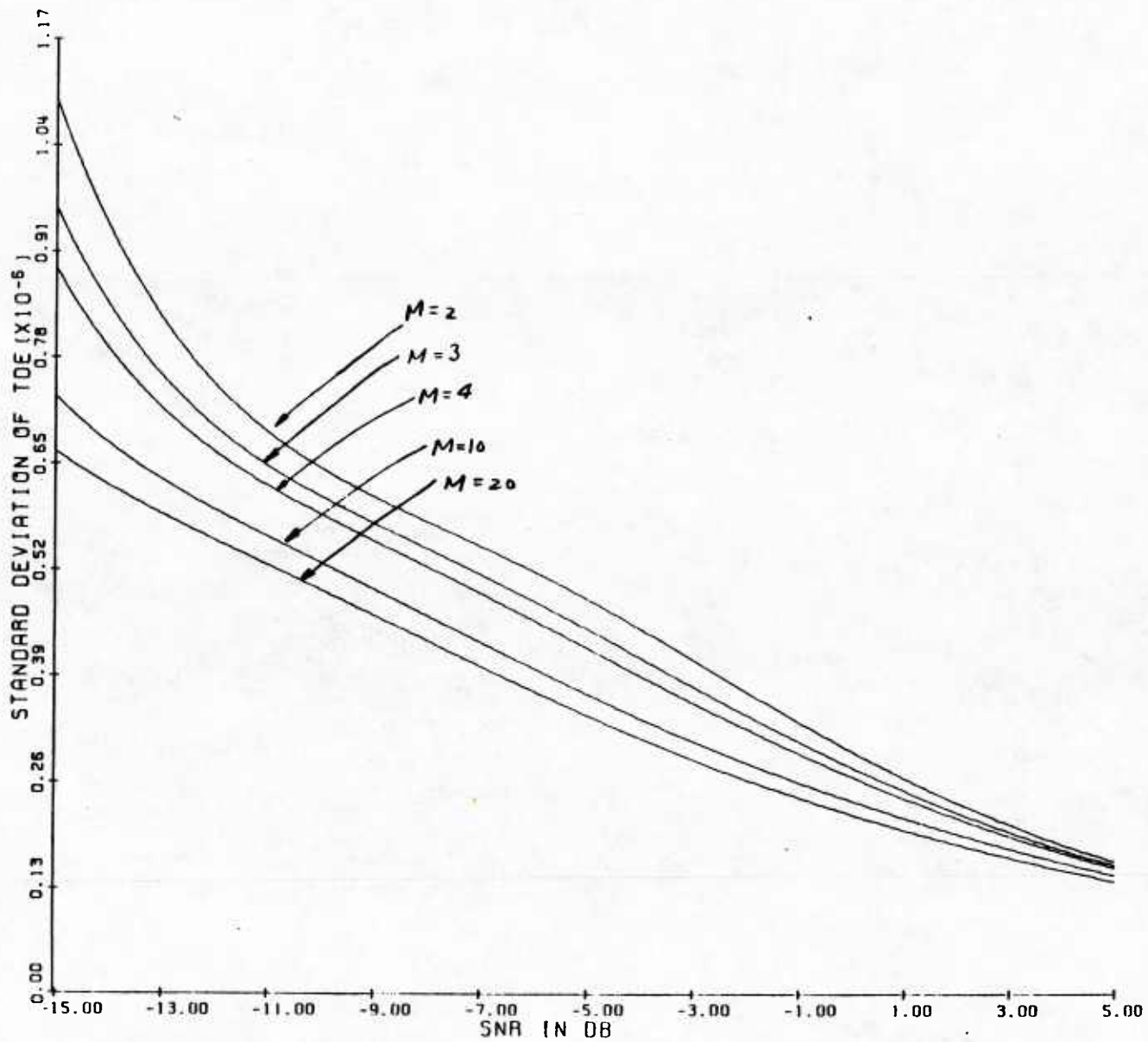


Figure 2b - Standard deviation of TDE vs. input SNR for various  $M$ ;  
 $\sigma = 0.03$ ,  $T = 30$  seconds,  $\omega_0/2\pi = 8$  KHz,  $W/2\pi = 4$  KHz.

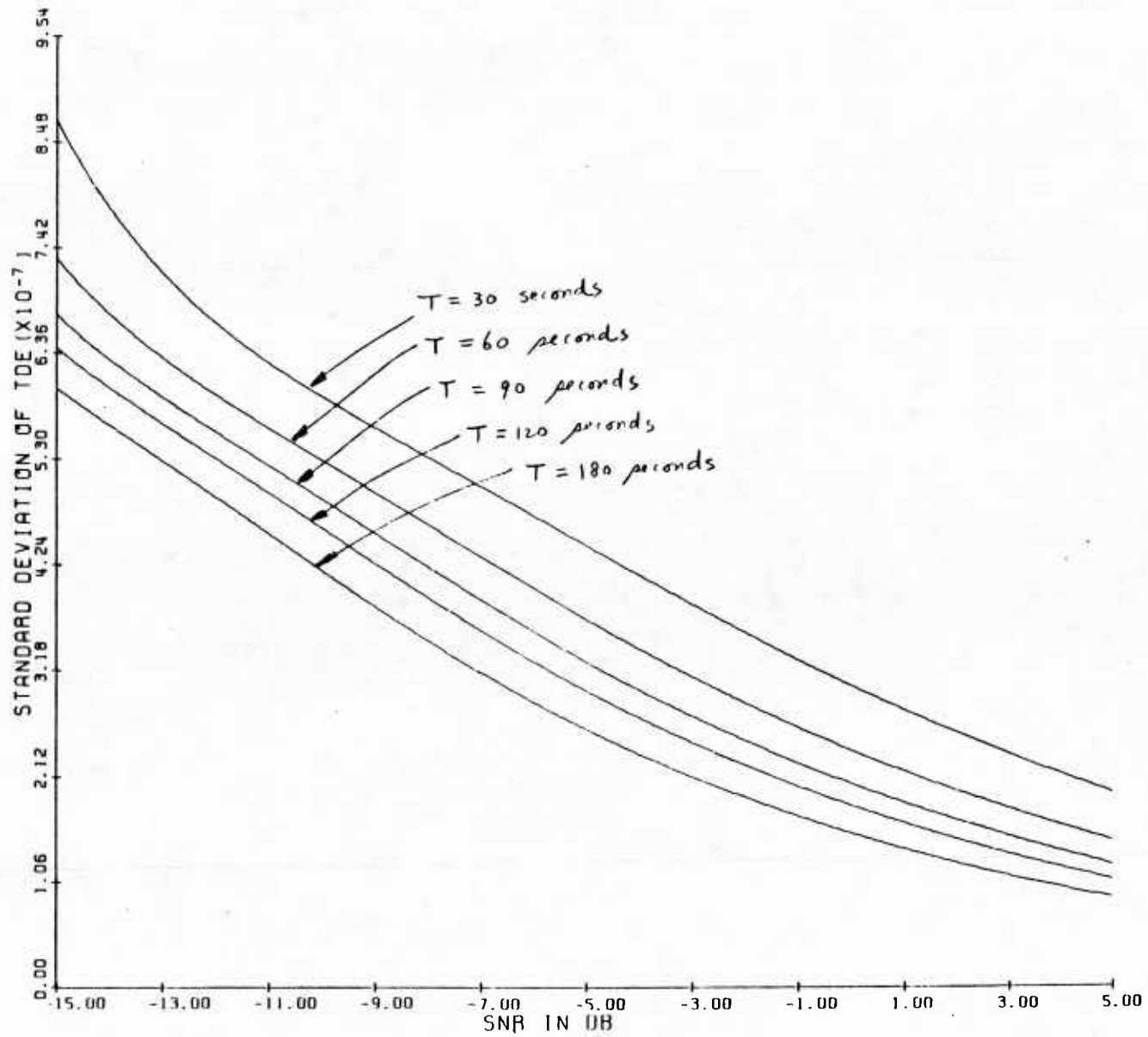


Figure 2c - Standard deviation of TDE vs. input SNR for various  $T$ ;  
 $\sigma = 0.03$ ,  $M = 10$ ,  $\omega_0/2\pi = 8$  KHz,  $W/2\pi = 4$  KHz.

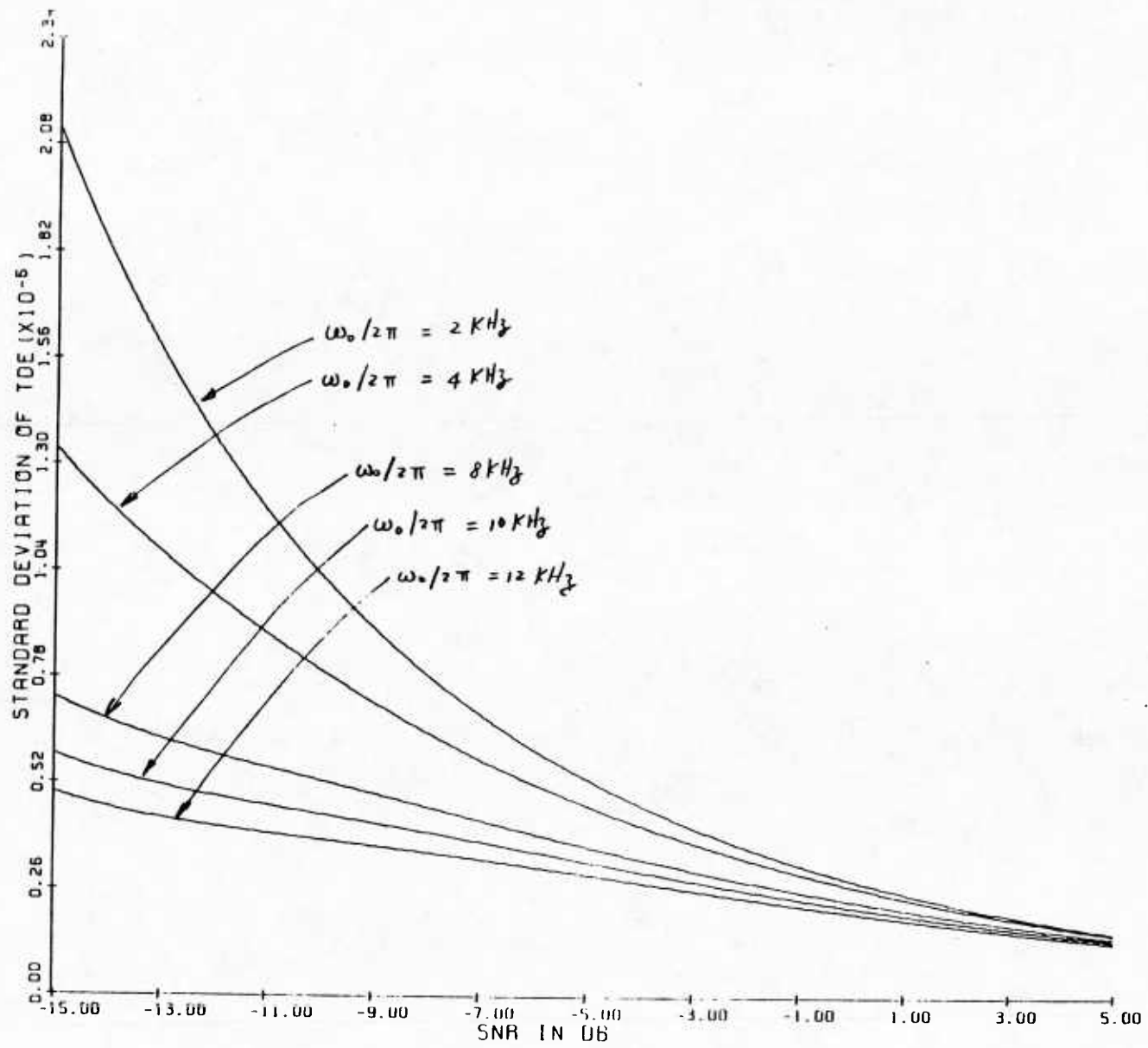


Figure 2d - Standard deviation of TDE vs. input SNR for various  $\omega_0$ ;  
 $\sigma = 0.03$ ,  $M = 10$ ,  $T = 60 \text{ seconds}$ ,  $W/2\pi = 4 \text{ KHz}$ .

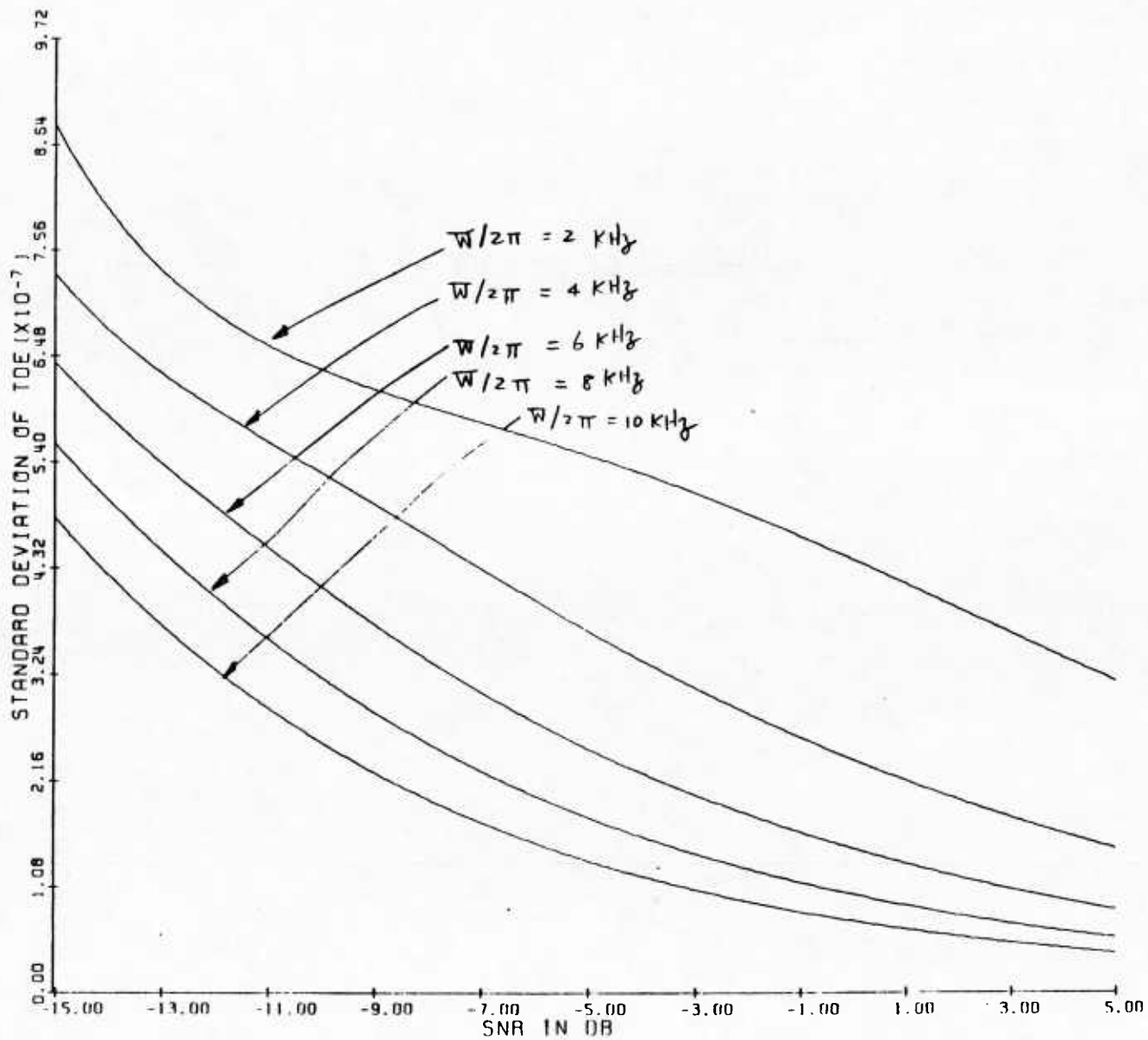


Figure 2e - Standard deviation of TDE vs. input SNR for various  $W$ ;  
 $\sigma = 0.03$ ,  $M = 10$ ,  $T = 60 \text{ seconds}$ ,  $\omega_0/2\pi = 8 \text{ KHz}$ .

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