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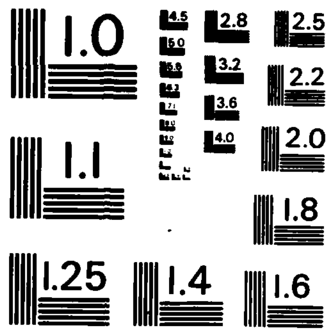

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MRC Technical Summary Report # 2631

BUBBLES RISING IN A TUBE AND JETS FALLING FROM A NOZZLE

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January 1984

(Received August 29, 1983)

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ABSTRACT

The shape of a two-dimensional bubble rising at a constant velocity U in a tube of width h is computed. The flow is assumed to be inviscid and incompressible. The problem is solved numerically by collocation. The results confirm Garabedian's [2] findings. There exists a unique solution for each value of the Froude number $F = U/(gh)^{1/2}$ smaller than a critical value

F sub c, F sub c

F_c. Here g denotes the acceleration of gravity. It is found that F_c = 0.36. In addition the problem of a jet emerging from a vertical nozzle is considered. It is shown that the slope of the free surface at the separation

sub c sub c

points is horizontal for $F < F_c$ and vertical for $F > F_c$. Graphs and tables of the results are included.

AMS (MOS) Subject Classification: 76B10

Key Words: Rising bubble, jet

Work Unit Number 2 - Physical Mathematics

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. Also, by the Australian Research Grant Committee and the National Science Foundation under Grant No. MCS-8001960.

SIGNIFICANCE AND EXPLANATION

We consider the flow of an incompressible, inviscid, heavy liquid past of a gas bubble in an infinitely long vertical tube (see Figure 1a). The flow is assumed to be two-dimensional. This problem was first considered by Birkhoff and Carter(1) and Garabedian(2). Birkhoff and Carter(1) suggested that the problem had a unique solution. On the other hand Garabedian(2) presented analytical evidence that the problem had a solution for each value of the Froude number $F = U/(gh)^{1/2}$ smaller than a critical value F_c. Here U is the velocity at infinity, g the acceleration of gravity and h the width of the tube.

In the present paper we settle the controversy between the results of Birkhoff and Carter(1) and those of Garabedian(2). We compute accurate solutions by a collocation method. Our results confirm Garabedian's(2) findings. There exists a unique solution for each value of F smaller than a critical value F_c. However we found $F_c = 0.36$. This value is about 40 percent higher than that indicated by earlier work on the problem.

In addition we consider the problem of a jet emerging from a vertical nozzle (see Figures 1b and 1c). We show that the slope of the free surface at the separation points is horizontal for $F < F_c$ and vertical for $F > F_c$.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

BUBBLES RISING IN A TUBE AND JETS FALLING FROM A NOZZLE

Jean-Marc Vanden-Broeck

I. Introduction

We consider the two-dimensional flow of an incompressible fluid past a gas bubble in an infinitely long tube of width h. We assume that the bubble extends downwards without limit. We choose a frame of reference moving with the bubble, so that the flow at large distances from the bubble is characterized by a constant velocity U [see Fig. (1a)]. We choose the origin of coordinates at the top of the bubble. We assume that the flow is symmetric about the x-axis and that gravity g is acting in the negative x-direction. As we shall see that the shape of the bubble is determined by the Froude number

$$F = \frac{U}{(gh)^{1/2}} \quad . \tag{1}$$

This problem was first considered by Birkhoff and Carter[1] and by Garabedian[2].

Birkhoff and Carter[1] attempted to solve the problem numerically. They assumed that the problem had a unique solution and that the Froude number F could be obtained as part of the solution. Though they obtained approximate solutions with $F \sim 0.23$, the convergence of their procedure was not really satisfactory.

Garabedian[2] presented analytical evidence that the solution is not unique. He suggested that a solution exists for each value of F smaller than a critical value F_c. In addition he showed that $F_c > 0.2363$ and guessed the value $F_c = 0.24$.

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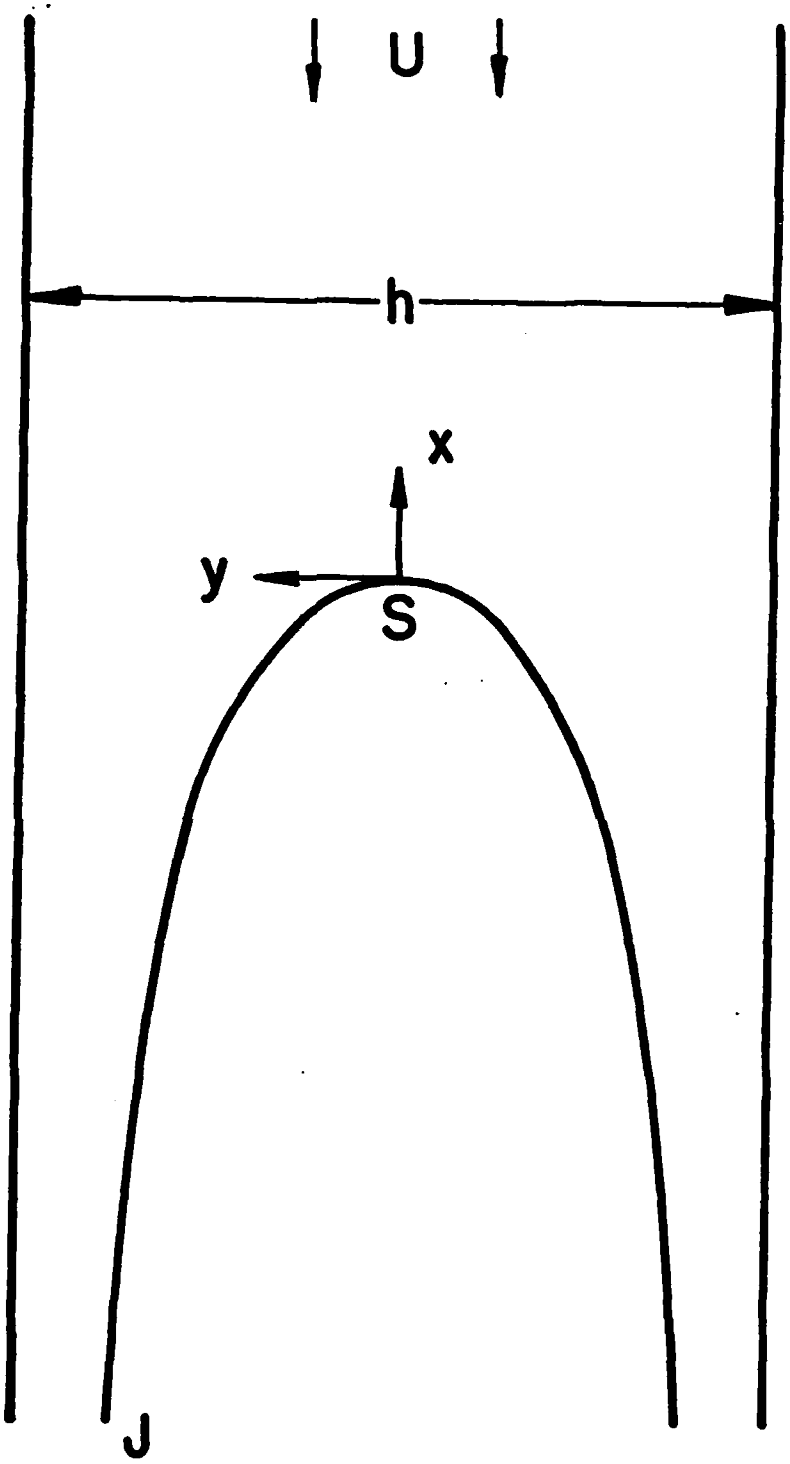
Figure 1a



Sketch of a bubble rising in a tube.

In the present paper we compute accurate numerical solutions by collocation. Our scheme is an improved version of the procedures proposed by Birkhoff and Carter[1]. Our results confirm Garabedian's[2] findings. There exists a unique solution for each value F smaller than a critical value F_c. However we found that $F_c = 0.36$. This value is about 40 percent higher than the value guessed by Garabedian[2].

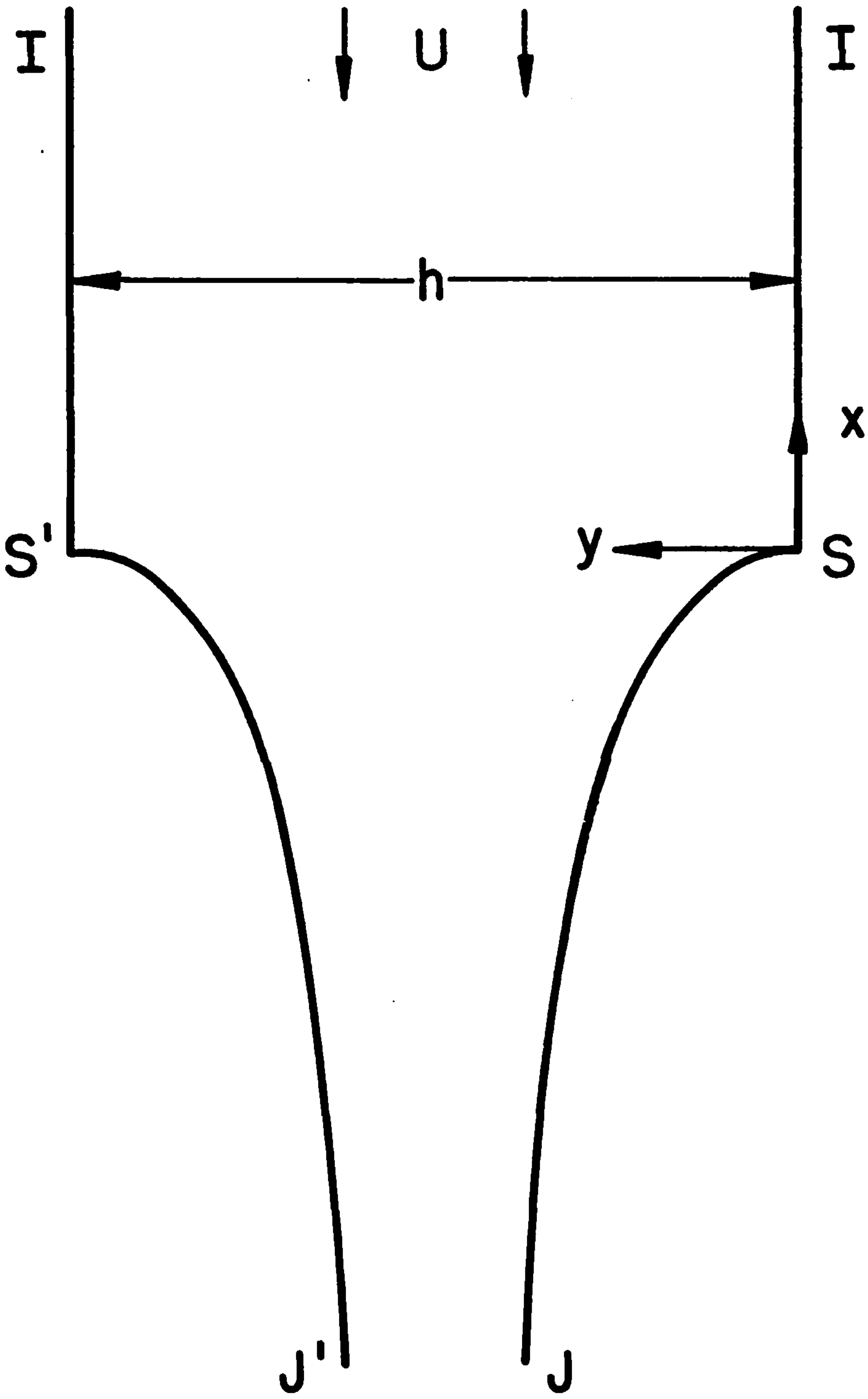
The flow configuration of Fig. 1(a) can also serve to model a jet emerging from a nozzle [see Fig. 1(b)]. As x tends to infinity, the velocity is assumed to approach a constant U. It follows from the symmetry of the flow configurations that the portion SJ of the bubble surface in Figure 1a is identical, for the same value of F, to the portion SJ of the jet surface in Figure 1b. Therefore our results for the rising bubble imply that the jet of Figure 1b exists for all values of $F < F_c$. For $F > F_c$, this flow configuration fails to exist.

For $F > F_c$ we sought a solution in which the flow separates tangentically from the nozzle [see Fig. 1(c)]. We found that there exists a unique solution of the type sketched in Fig. 1(c)] for each value of $F > F_c$. For $F < F_c$ these solutions fail to exist.

From our results we conclude that there exists a unique jet for each value of F. For $F > F_c$ the slope of the free surface is vertical at the separation points S and S' For $F < F_c$ the slope of the free surface is horizontal at the separation points and the velocity at these points is equal to zero.

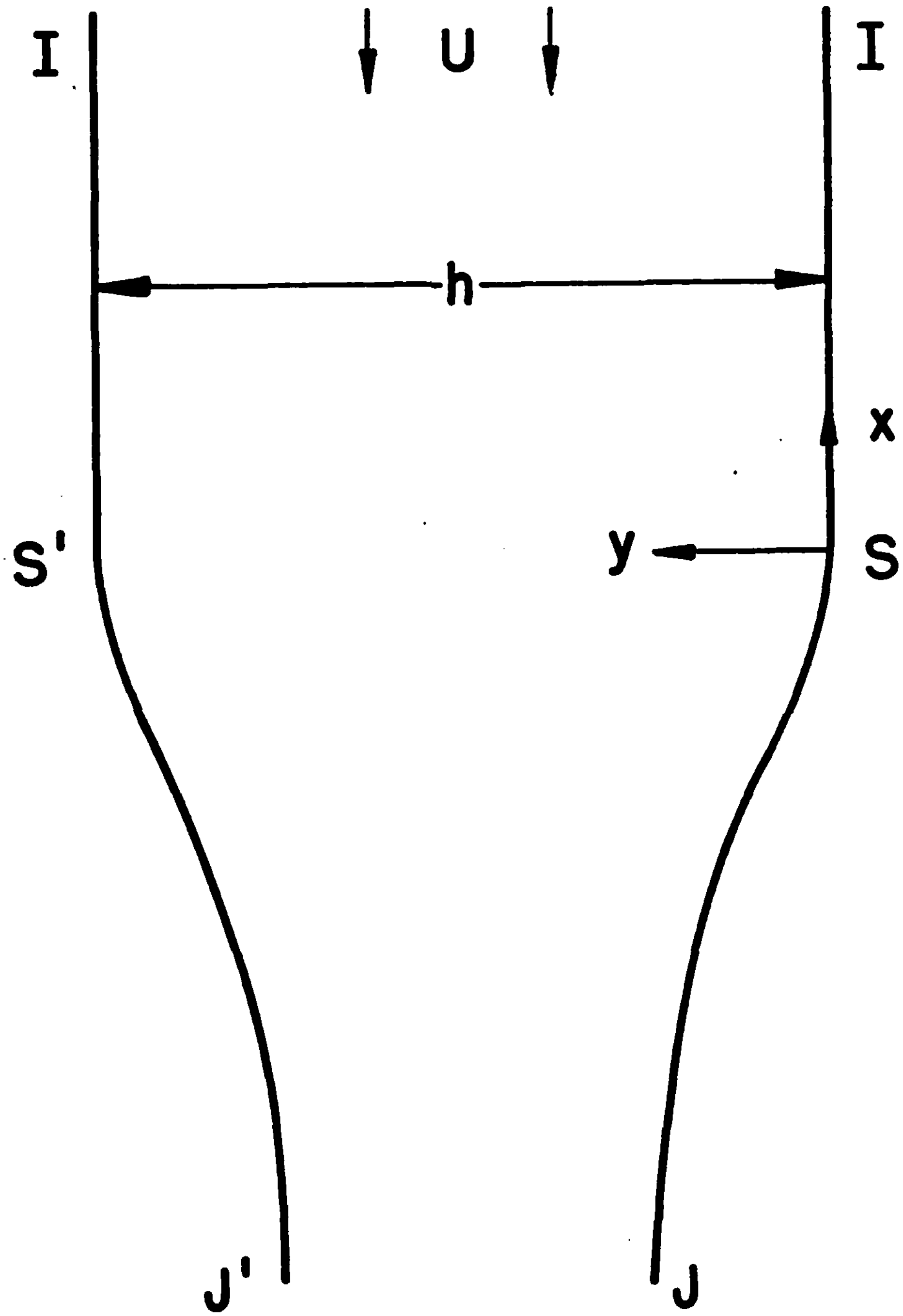
The problem is formulated in Sec. II. The numerical procedure is described in Sec. III and the results are discussed in Sec. IV.

Figure 1b



Sketch of a jet falling from a nozzle. The free surface is assumed to be horizontal at the separation points S and S'.

Figure 1c



Sketch of a jet falling from a nozzle. The free surface is assumed to be vertical at the separation points S and S'.

II. Formulation

Let us consider a two-dimensional jet emerging from a nozzle of width h [see Figs. 1(b) and 1(c)]. As $x \to \infty$, the velocity approaches a constant U. The fluid is assumed to be incompressible and inviscid.

We define dimensionless variables by choosing h as the unit length and U as the unit velocity. We introduce the potential function ϕ and the stream function ψ. Without loss of generality we choose $\phi = 0$ at $x = y = 0$ and $\psi = 0$ on the streamline IJ. It follows from the choice of the dimensionless variables that $\psi = -1$ on IJ'. The complex potential plane is sketched in Fig. 2.

We denote the complex velocity by $\zeta = u - iv$ and we define the function $\tau - i\theta$ by

$$\zeta = u - iv = e^{\tau - i\theta} \quad . \tag{2}$$

We shall seek $\tau - i\theta$ as an analytic function of $f = \phi + i\psi$ in the strip $-1 < \psi < 0$.

On the surface of the jet, the Bernoulli equation yields

$$\frac{1}{2} q^2 + gx = \frac{1}{2} q_c^2 \quad . \tag{3}$$

Here q is the flow speed, g the acceleration of gravity and q_c the velocity at the separation points. In dimensionless variables (3) becomes

$$e^{2\tau} + \frac{2}{F^2} x = \frac{q_c^2}{U^2} \quad \text{on SJ and S'J'} \quad . \tag{4}$$

Here F is the Froude number defined by (1).

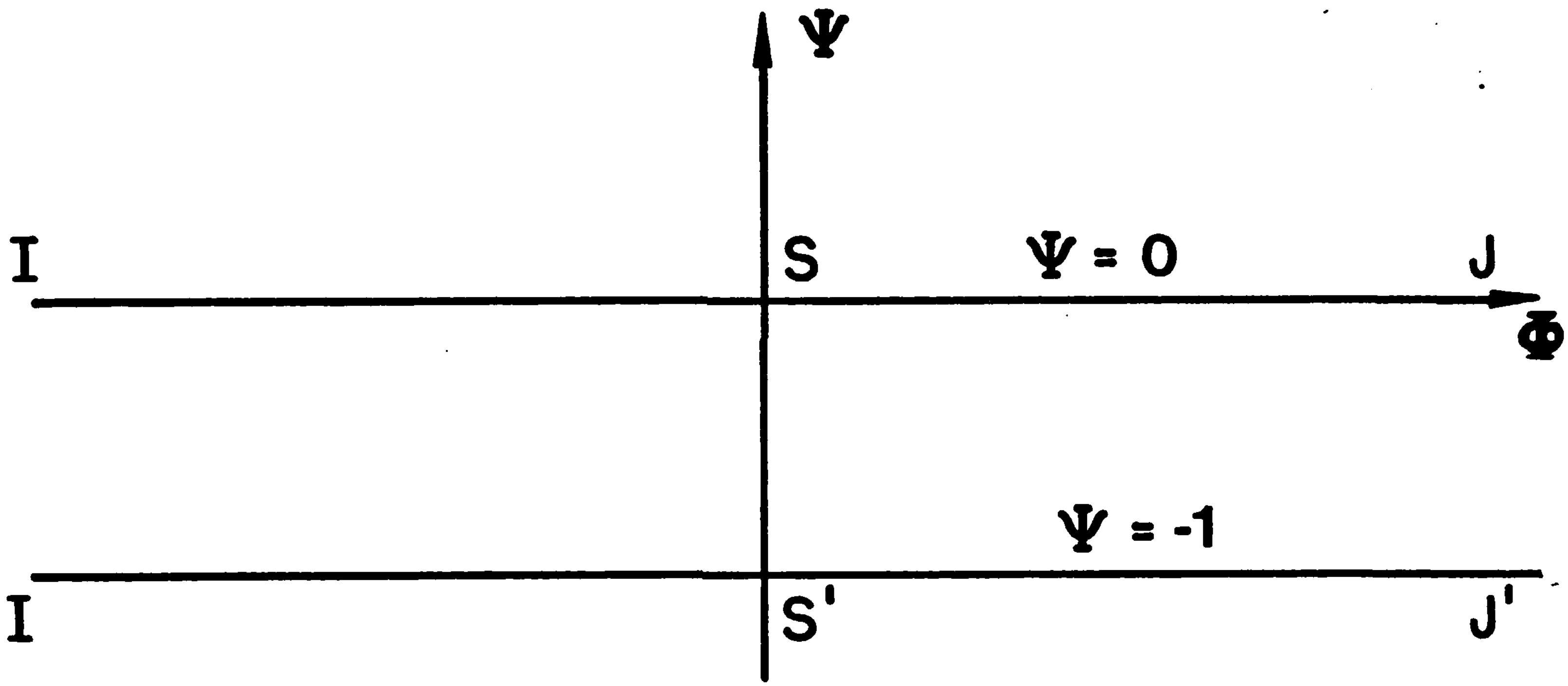
It is convenient to eliminate y and q_c from (4) by differentiating (4) with respect to ϕ. Using the relation

$$\frac{\partial x}{\partial \phi} + i \frac{\partial y}{\partial \phi} = \frac{1}{u - iv} = e^{-\tau + i\theta} \tag{5}$$

we obtain

$$e^{2\tau} \frac{\partial \tau}{\partial \phi} + \frac{1}{F^2} e^{-\tau} \cos\theta = 0 \quad \text{on SJ and S'J'} \quad . \tag{6}$$

Figure 2



Flow configuration in the complex potential plane.

The kinematic conditions on IS and IS' yield

$$\theta = 0 \qquad \psi = -1 \qquad \phi < 0 \tag{7}$$

$$\theta = 0 \qquad \psi = 0 \qquad \phi < 0 \ . \tag{8}$$

The flow configuration of Fig. 1(b) is characterized by stagnation points at S and S'. This yields the additional conditions

$$\left.\begin{aligned} &\tau = -\infty,\ \theta = \frac{\pi}{2} \quad \text{at} \quad \phi = 0,\ \psi = 0 \\ &\tau = -\infty,\ \theta = -\frac{\pi}{2} \quad \text{at} \quad \phi = 0,\ \psi = -1 \ . \end{aligned}\right. \tag{9}$$

The flow configuration of Figure 1c is characterized by finite and non-zero velocities at S and S'. This yields the additional conditions

$$\left.\begin{aligned} &\tau \neq \pm\infty,\ \theta = 0 \quad \text{at} \quad \phi = 0,\ \psi = 0 \\ &\tau \neq \pm\infty,\ \theta = 0 \quad \text{at} \quad \phi = 0,\ \psi = -1 \ . \end{aligned}\right. \tag{10}$$

This completes the formulation of the problem of determining $\tau - i\theta$. This function must be analytic in the strip $-1 < \psi < 0$ and satisfy the conditions (6) - (8) and (9) for the flow configuration of Fig. 1(b) and the conditions (6) - (8) and (10) for the flow configuration of Fig. 1(c).

III. Numerical procedure

Following Birkhoff and Carter[1] we define the new variable t by the relation

$$e^{-\pi f} = \frac{1}{2}\left(t + \frac{1}{t}\right) \quad . \tag{11}$$

This transformation maps the flow domain onto the unit circle in the complex t-plane so that the walls of the nozzle go onto the real diameter and the free surfaces onto the circumference (see Fig. 3).

In order to obtain solutions for the flow configuration of Fig. 1(b), we note that

$$\zeta \sim [\ln(1 \pm it)]^{1/3} \quad \text{as} \quad t \to \pm i \tag{12}$$

$$\zeta \sim 1 \pm t \quad \text{as} \quad t \to \mp 1 \tag{13}$$

(see Birkhoff and Carter[1] for details). Therefore we define the function $\Omega(t)$ by the relation

$$e^{\tau - i\theta} = -[-\ln C(1 + t^2)]^{1/3}(-\ln C)^{-1/3}\,(1 - t^2)e^{\Omega(t)} \quad . \tag{14}$$

Here C is an arbitrary constant between 0 and 0.5. We choose $C = 0.2$. The function $\Omega(t)$ is bounded and continuous on the unit circle and analytic in the interior. The conditions (7) and (8) show that $\Omega(t)$ can be expanded in the form of a Taylor expansion in even powers of t. Hence

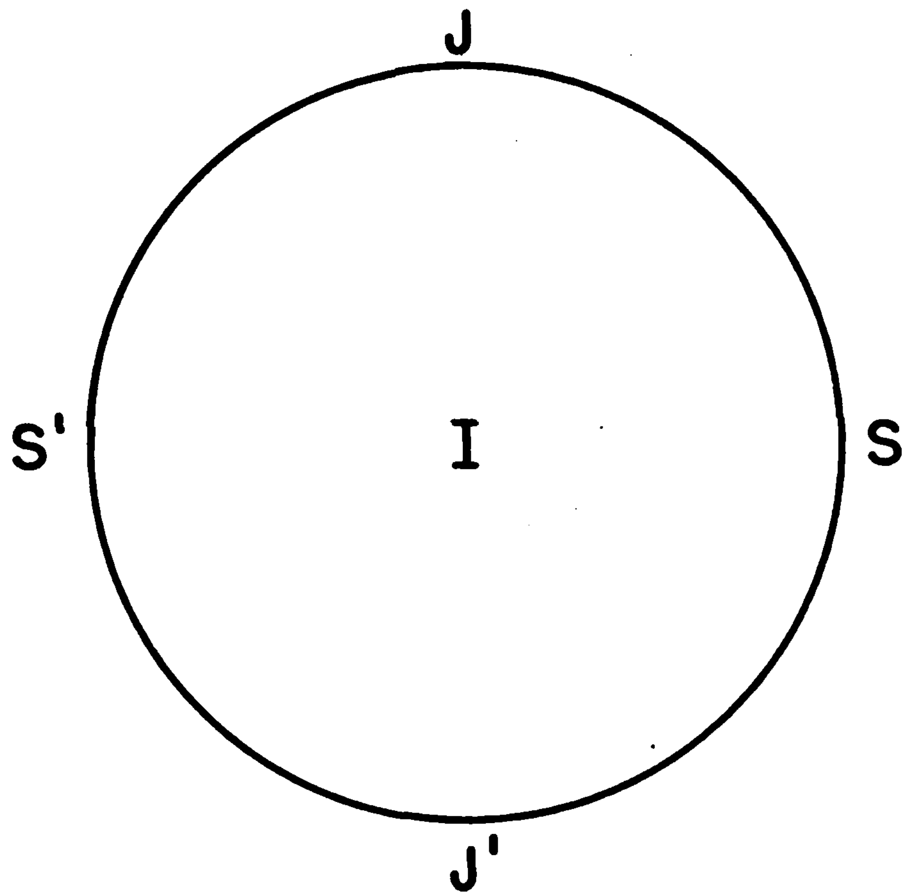
$$e^{\tau - i\theta} = -[-\ln C(1 + t^2)]^{1/3}(-\ln C)^{-1/3}\,(1 - t^2)\exp\left(\sum_{n=1}^{\infty} a_n\, t^{2n}\right) \quad . \tag{15}$$

The functions τ and θ defined by (15) satisfy (7) - (9). The coefficients a_n have to be determined to satisfy (6) on SJ. The condition (6) on S'J' will then be autotomically satisfied by symmetry.

We use the notation $t = |t|e^{i\sigma}$ so that points on SJ are given by $t = e^{i\sigma}$, $0 < \sigma < \frac{\pi}{2}$. Using (11) we rewrite (6) in the form

$$\pi \cot g\, \sigma\, e^{2\tilde{\tau}}\, \frac{d\tilde{\tau}}{d\sigma} - \frac{1}{F^2}\, e^{-\tilde{\tau}} \cos\tilde{\theta} = 0 \quad . \tag{16}$$

Figure 3



The complex t-plane.

Here $\tilde{\tau}(\sigma)$ and $\tilde{\theta}(\sigma)$ denote the values of τ and θ on the free surface SJ.

We solve the problem approximately by truncating the infinite series in (15) after N terms. We find the N coefficients a_n by collocation. Thus we introduce the N mesh points

$$\sigma_I = \frac{\pi}{2N}\left(I - \frac{1}{2}\right), \quad I = 1,\ldots,N \quad . \tag{17}$$

Using (15) and (17) we obtain $[\tilde{\tau}(\sigma)]_{\sigma=\sigma_I}$, $[\tilde{\theta}(\sigma)]_{\sigma=\sigma_I}$ and $[\frac{d\tilde{\tau}}{d\sigma}]_{\sigma=\sigma_I}$ in terms of the coefficients a_n. Substituting these expressions into (16) we obtain N nonlinear equations for the N unknowns a_n, $n = 1,\ldots,N$. We solve this system by Newton's method. Once this system is solved for a given value of F, we calculate the functions τ and θ by setting $|t| = 1$ in (14). The shape of the jet is then found by numerically integrating the relation (5). We shall refer to this numerical procedure as scheme I.

Solutions for the flow configuration of Fig. 1(c) can be obtained by omitting the factor $1 - t^2$ in (14). Thus (15) becomes

$$e^{\tau - i\theta} = -[-\ell n\, C(1 + t^2)]^{1/3}(-\ell n\, C)^{-1/3} \exp\left(\sum_{n=1}^{\infty} b_n t^{2n}\right) \quad . \tag{18}$$

The functions τ and θ defined by (18) satisfy (7), (8) and (9). The coefficients b_n have to be found to satisfy (6) on SJ. We truncate the infinite series in (18) after N terms and determine b_n, $n = 1,\ldots,N$ by the collocation method described in scheme I. We shall refer to this numerical procedure as scheme II.

Solutions for the flow configuration of Figs. 1(b) and 1(c) can also be obtained by assuming the expansion

$$e^{\tau - i\theta} = -[-\ell n\, C(1 + t^2)]^{1/3}(-\ell n\, C)^{-1/3} \left(1 + \sum_{n=1}^{\infty} d_n t^{2n}\right) \quad . \tag{19}$$

It can easily be verified that (19) satisfies (7) and (8). In addition (19) satisfies (9) when $\sum_{n=1}^{\infty} d_n = -1$ and (10) when $\sum_{n=0}^{\infty} d_n \neq -1$. Therefore (19) is appropriate to describe the flow configuration of Fig. 1(b) as well as the flow configuration of Fig. 1(c). The infinite series in (19) is truncated after N terms and the coefficients d_n, $n = 1,\ldots,N$ are determined by the collocation method described in scheme I. We shall refer to this scheme as scheme III.

IV. Discussion of the results

We used scheme I to compute solutions for the flow configuration of Fig. 1(b). We obtained accurate solutions for $F < 0.3$. However we were unable to obtain reliable solutions for $F > 0.3$. Similarly we obtained accurate solutions for the flow configuration of Fig. 1(c) for $F > 0.4$ by using scheme II. However we could not calculate reliable solutions for $F < 0.4$.

In order to obtain solutions for $0.3 < F < 0.4$, we used scheme III. We found that scheme III gives accurate solutions for all values of F. As n increases the coefficients d_n decrease rapidly. For example $d_{10} \sim 3.10^{-3}$, $d_{30} \sim 6.10^{-4}$, $d_{60} \sim 2.10^{-4}$, $d_{90} \sim 2.10^{-5}$ for $F = 0.374$. In addition the solutions of scheme III agree with those of scheme I for $F < 0.3$ and with those of scheme II for $F > 0.4$.

In Fig. 4 we present values of the velocity $\frac{q_c}{U}$ at the separation point S versus F. The velocity q_c is equal to zero for $F < 0.36$ and different from zero for $F > 0.36$. Therefore

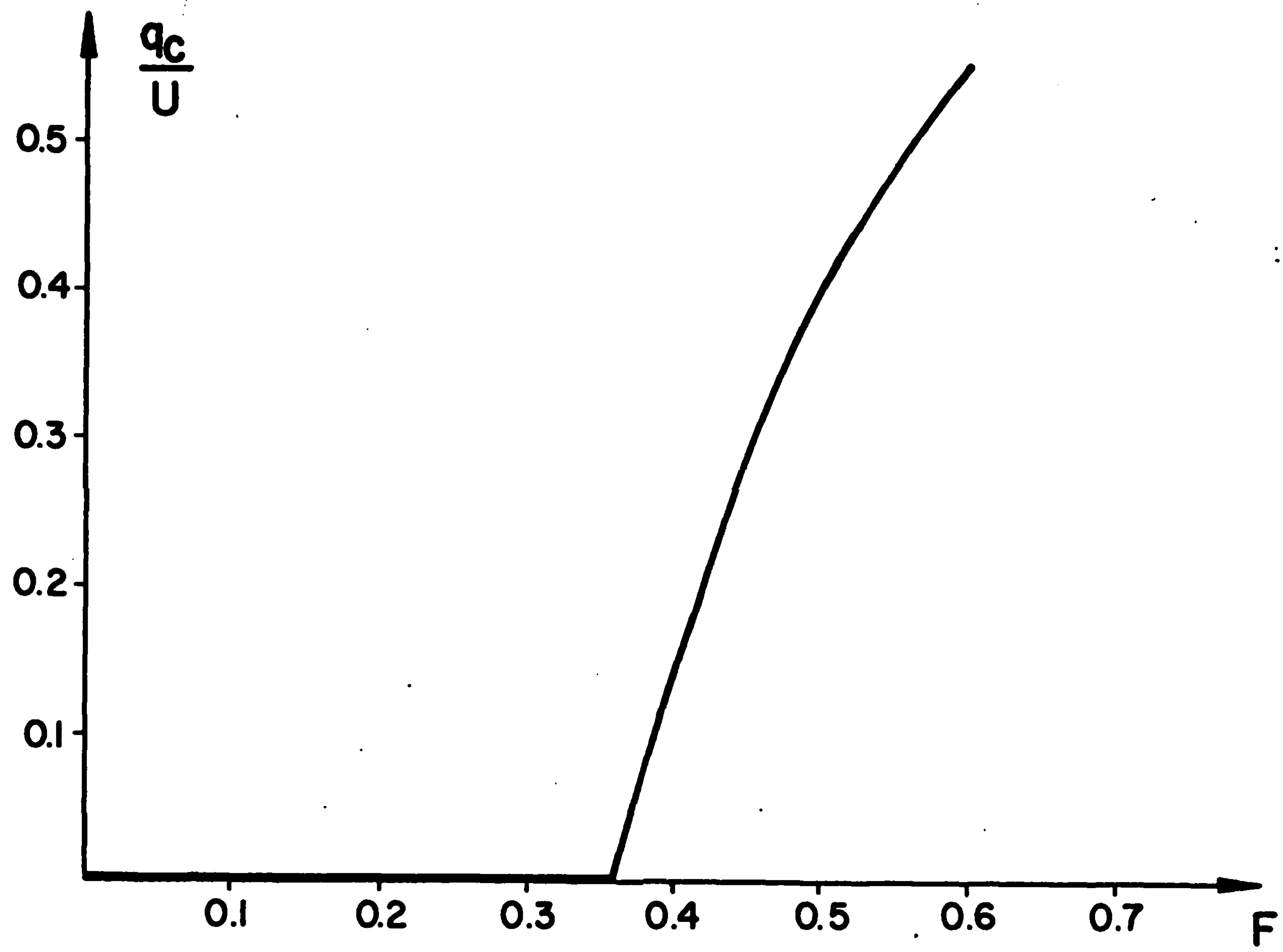
$$F_c = 0.36 \quad . \tag{20}$$

This value is about 40% higher than the value guessed by Garabedian[2].

Profiles of the jet for various values of F are presented in Fig. 5. For $F = \infty$, the free surface reduces to the vertical line $y = 0$. As F decreases from infinity, the jet becomes thinner. As F approaches zero, the thickness of the jet tends to zero and the free surface approaches the horizontal line $x = 0$. For $F > F_c$ the slope of the free surface at $x = y = 0$ is vertical. For $F < F_c$ the slope at $x = y = 0$ is horizontal.

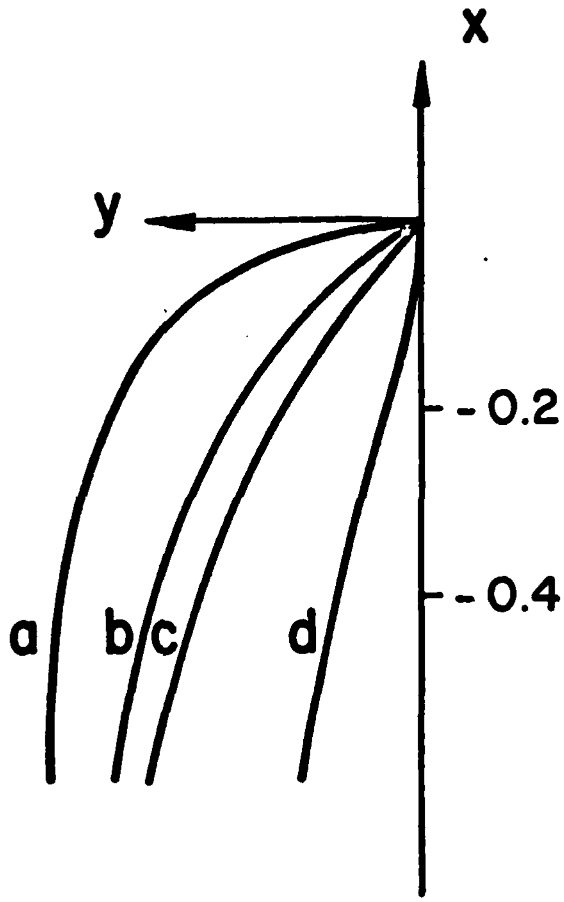
For all values of F, the free surface approaches the vertical line $y = \frac{1}{2}$ as $x \to -\infty$. Therefore the jet is described far downstream by the slender jet theory of Keller and Geer[3]. This theory shows that

Figure 4



Values of the dimensionless velocity $\frac{q_c}{U}$ at the separation points S and S'.

Figure 5



Computed free surface profiles for various values of F. The curves (a), (b), (c) and (d) correspond respectively to $F = 0.2$, $F = F_c = 0.36$, $F = 0.45$ and $F = 1$.

$$u - iv \sim -V(x) \quad \text{as} \quad x \to -\infty \ . \tag{21}$$

Here $V(x)$ is real and positive. Conservation of mass and equation (4) yield the relations

$$(1-2y)V(x) = 1 \tag{22}$$

$$V^2(x) \sim -\frac{2}{F^2}\,x \quad \text{as} \quad x \to -\infty \ . \tag{23}$$

Eliminating $V(x)$ between (22) and (23) gives

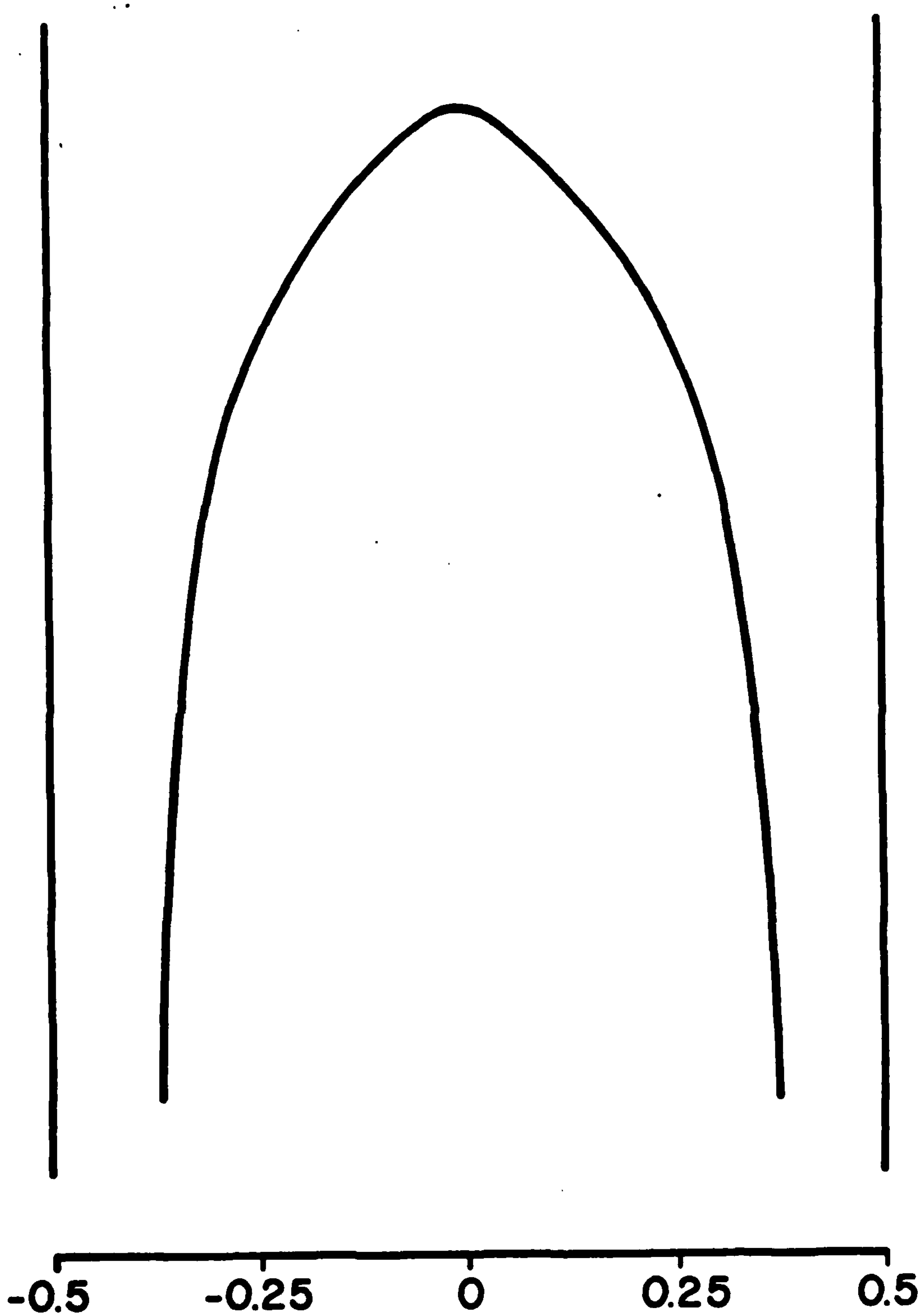
$$x = -\frac{F^2}{2(1-2y)^2} \ . \tag{24}$$

Formula (24) is the asymptotic shape of the jet far downstream. Our numerical results were found to agree with (24) for x large and negative. This constitutes an important check on our numerical scheme.

The solutions of Fig. 5 for $F < F_c$ are also solutions of the flow configuration of Fig. 1(a). Therefore there exists a unique "rising bubble" for each value of F smaller than F_c. This confirms Garabedian's[2] findings.

Garabedian[2] used a criterion of stability to suggest that the unique physically significant "rising bubble" is the one for which $F = F_c$. Collins[4] reported the experimental values $F = 0.25$. The discrepancy between this experimental data and our theoretical value $F_c = 0.36$ is presumably due to the three-dimensionality of the real flow and to the effect of surface tension. The bubble profile corresponding to $F = F_c$ is shown in Fig. 6.

Figure 6



Bubble profile for $F = F_c = 0.36$.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #2631	2. GOVT ACCESSION NO. AD-A139266	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Bubbles Rising in a Tube and Jets Falling from a Nozzle		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Jean-Marc Vanden-Broeck		8. CONTRACT OR GRANT NUMBER(s) MCS-8001960. DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 2 - Physical Mathematics
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below		12. REPORT DATE January 1984
		13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
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National Science Foundation
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Rising bubble, jet

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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