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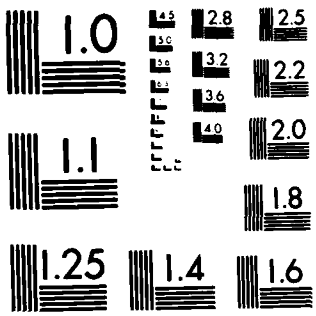

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Spectral power density extrema are investigated for variable frequency and fixed beam radius, for variable beam radius and fixed frequency, and for both variable frequency and beam radius in the one-dimensional sheet model of the relativistic reflex triode. Approximate and exact formulations are given. A simple example is presented wherein the power spectrum exhibits gross features of the real situation. The extrema are seen to be closely related to the zeros of Bessel functions of the first kind or to zeros of equations involving both these Bessel functions and the Fourier-transformed current.

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1. INTRODUCTION

For high-power microwave emission from relativistic reflex triodes and diodes to be analyzed (for a uniform cylindrical beam of radius R), we must first determine the average power spectral density per steradian radiated perpendicular to the beam. For the relativistic sheet model of the electron dynamics, that density is given by[1]

$$d^3P_\perp/d\omega d^2\Omega = CR^4\omega^2|S(\omega)|^2F^2(\omega R/c) \tag{1}$$

$S(\omega) = \Sigma^*(\omega)$, from Brandt et al,[1] with $C = \mu_o\pi\sigma^2/4cT$, and

$$S(\omega) = (1/2\pi)\int_{-\infty}^{\infty} dt\, \exp(i\omega t)\, S(t) \quad . \tag{2}$$

Moreover,

$$S(t) = \sum_{m=1}^{N(t)} v_m(t) \quad , \tag{3}$$

and

$$F^2(x) = 4J_1^2(x)/x^2 \quad . \tag{4}$$

In these terms, $d^2\Omega$ is the differential solid angle, μ_o is the permeability of free space, σ is the sheet surface charge density, R is the beam radius, c is the speed of light, T is the voltage pulse duration, ω is the angular frequency, N(t) is the number of sheets in the source region at time t, $v_m(t)$ is the velocity of the m^{th} sheet at time t, and J_1 is the first-order Bessel function of the first kind. Equation (1) was obtained by solving the wave equation with a current distribution represented by the sheet dynamics, including only radiative time delays which are perpendicular to the beam, while using the dipole far-field approximation. S(t) is the sum of the sheet velocities. This introduces the function which is at the heart of the present study. Now let us discuss what is to be done with it and why.

The purpose of this paper is to investigate the extrema of equation (1). By extrema we mean maxima, minima, or inflections in one dimension and, on surfaces, their two-dimensional analogs. Such an understanding is necessary to the optimal design of high-power microwave sources.

It is known that S(t) is a very complicated function of time. S(t) has some general features, however. It is zero for $t < 0$ and for $t > 0$ makes a rapid transition to a regime in which aperiodic oscillations about a mean value occur over a period of time corresponding to many periods of oscillation. Ignoring the superimposed aperiodic fluctuations, a simple, very coarse-grained representation of this behavior would be to set

$$S(t) = N_a v_a H(t) \quad , \tag{5}$$

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[1] *Howard E. Brandt, Alan Bromborsky, Henry B. Bruns, H. Allen Kehs, and George P. Lasche, Gigawatt Microwave Emission from a Relativistic Reflex Triode, Harry Diamond Laboratories, HDL-TR-1917 (August 1980).*

where N_a is the mean number of sheets, v_a is the mean velocity, and H(t) is the Heaviside step function given by equation (6) below. From equation (2) one finds $S(\omega) = S/\omega$ with $S = iN_a v_a/2\pi$, and we are using ω as shorthand for $\omega + i\delta$ with $\delta \to 0^+$:

$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \quad . \tag{6}$$

Inserting the Fourier transform of equation (5) into equation (1) gives ($K = S^2C$):

$$d^3P_\perp/d\omega d^2\Omega = KR^4F^2(\omega R/c) \quad . \tag{7}$$

We observe that in this naive representation the ω dependence appears entirely in F^2. One of the principal problems just mentioned involves maximizing equation (7) with respect to ω for some given R. We see that this amounts to finding extrema for F^2 alone given the very simple $S(\omega)$ corresponding to equation (5). In the first part of section 2 we consider the extrema of $F^2(x)$ with $x = \omega R/c$ and show that minima and maxima occur at the zeros of J_1 and J_2, respectively. Simple asymptotic expressions for F^2, J_1, and J_2 are given for small and large arguments. Where $S(\omega)$ cannot be considered to be of the form corresponding to equation (5), we must instead seek the extrema of $|S(\omega)|^2J_1^2(\omega R/c)$ at constant R. This problem is the subject of section 2.4. In section 3, we turn our attention to the variation of equation (1) with R at constant ω. In particular, we seek extrema of a function

$$f^2 = x^4F^2(x) = 4x^2J_1^2(x) \quad , \tag{8}$$

with $x = \omega R/c$ as before. This function has minima at $J_1 = 0$ and maxima where $J_0 = 0$. Asymptotic forms are presented for f^2, J_0, and J_1 for small and large arguments. In section 4, we consider the surface given by

$$h^2 = R^2|S(\omega)|^2J_1^2(\omega R/c) \quad , \tag{9}$$

and seek extrema with respect to ω and R. Inspection of equation (9) reveals that absolute minima should occur where $S(\omega)$ or J_1 vanishes. The former condition defines a set of planes of constant ω, and the latter a set of hyperbolas along which ωR is constant. The maxima are found by solving two simultaneous algebraic equations for ω and R. Clearly, a knowledge of $S(\omega)$ is required for a complete solution.

In the work that follows, the available literature on Bessel functions has been used extensively. The theory of Bessel functions is well summarized in texts by Arfken[2] and Hildebrand.[3] Greater theoretical detail is supplied by

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[2] *G. Arfken, Mathematical Methods for Physicists, New York, Academic Press (1968).*

[3] *F. Hildebrand, Advanced Calculus for Applications, Prentice-Hall: Englewood Cliffs, NJ (1962).*

Watson.[4] Frequent use has also been made of formulae summaries to be found in mathematical compendia edited by Gradshteyn and Ryzhik[5] and by Abramowitz and Stegun.[6] Also found useful was the short table of Bessel functions, J_n, of order 0, 1, and 2 for values of the arguments $0 \leq x \leq 17.5$ in steps of 0.1 (Stegun). For more accurate work, a reference of the Harvard University Computation Laboratory has been very useful: Tables of Bessel Functions of the First Kind,[7] for $0 \leq x \leq 100$ with incremental steps of 0.001 up to x = 25 and 0.01 thereafter. Lastly, Boas[8] clearly describes the process of determining extrema for unbounded surfaces as well as for those that are bounded. The latter case would occur if there were lower or upper bounds of a practical nature on ω and R.

2. EXTREMA AT CONSTANT BEAM RADIUS

2.1 Asymptotic Forms of Interference Form Factor F^2 and Approximate Location of Minima and Maxima

Figure 1 illustrates the behavior of F^2 for $0 \leq x \leq 7$, where F^2 is given by equation (4). Its value at x = 0 is precisely 1. The first two zeros of the function occur at 3.83 and 7.02, $J_1(x)$ being zero here. In fact, $J_1 = 0$ for $x \neq 0$ locates all the minima of the function. Indeed, one might expect as much by inspection of equation (4). The graph indicates two successive maxima. The first, at x = 0, is much larger than the second, which occurs at x = 5.14. This secondary peak has a value of 1.75×10^{-2}. We will show that these peaks fall off as x^{-3} for large x and, moreover, are located at the zeros of J_2.

J_1 may be represented by a few terms of the series (n = 1):

$$J_n(x) = (x/2)^n \sum_{s=0}^{\infty} \frac{(-1)^s}{s!(s+n)!} (x/2)^{2s} , \qquad (10)$$

for small arguments. Using this we find that F^2 is given by

$$F^2 = 1 - x^2/4 + 5x^4/192 - 7x^6/4608 + \ldots \quad . \qquad (11)$$

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[4]G. N. Watson, *A Treatise on the Theory of Bessel Functions, 2nd ed., Cambridge, England, The University Press: New York, The MacMillan Company (1944).*

[5]*I. Gradshteyn and I. Ryzhik, Table of Integrals, Series, and Products, 4th ed., A. Jeffrey, ed., New York, Academic Press (1980).*

[6]*M. Abramowitz and I. Stegun, eds., Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Washington, D. C., U. S. Government Printing Office (1964).*

[7]*Harvard University Computation Laboratory, Tables of the Bessel Functions of the First Kind of Orders, Cambridge, Harvard University Press (1947-51).*

[8]*M. Boas, Mathematical Methods in the Physical Sciences, New York, Wiley (1966).*

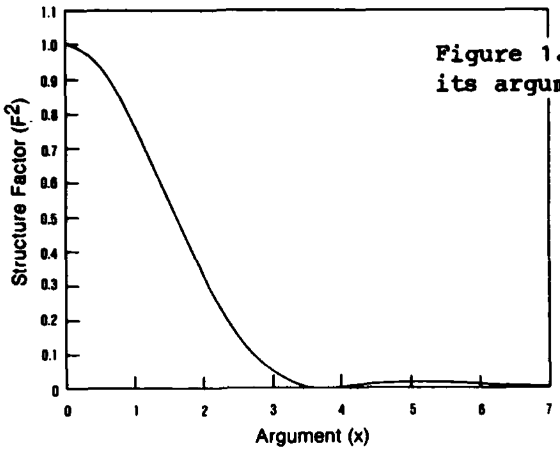

Figure 1. Structure factor $F^2 = 4J_1^2(x)/x^2$ versus its argument $x = \omega R/c$.

The first four terms adequately represent F^2 up to $x = 1.6$ (less than 1 percent error) as shown in table 1. The large argument approximation for J_n is

$$J_n = (2/\pi x)^{1/2} \cos (x - n\pi/2 - \pi/4) \quad . \tag{12}$$

For $n = 1$,

$$J_1 = (2/\pi x)^{1/2} \sin (x - \pi/4) \quad . \tag{13}$$

Making use of this, one finds for F^2,

$$F^2 = 4(1 - \sin 2x)/\pi x^3 \quad . \tag{14}$$

TABLE 1. SMALL ARGUMENT REPRESENTATION OF FORM FACTOR F^2

x	F^2(series)	F^2(exact)	Deviation (%)
0.0	1.	1.	--
0.2	0.9900	0.9900	--
0.4	0.9607	0.9607	--
0.6	0.9133	0.9133	--
0.8	0.8503	0.8503	--
1.0	0.7745	0.7746	0.01
1.2	0.6895	0.6897	0.03
1.4	0.5986	0.5994	0.13
1.6	0.5052	0.5075	0.45
1.8	0.4117	0.4175	1.39
2.0	0.3194	0.3326	3.97

This expression shows that the successive maxima of F^2 for large x fall off rapidly as x^{-3}. Equation (14) may be used to determine the large argument locations of zeros (minima) as well as maxima of F_2. The minima occur for $\sin 2x = +1$, requiring

$$x_n = \pi(n + 1/4) \quad . \tag{15}$$

Zeros as predicted by equation (15) are compared with the actual values as determined by $J_1 = 0$ and n or $x \neq 0$, in table 2. Results are more than adequate for $n \geq 2$. Appendix A shows that Bessel functions of odd order, J_1, J_3, etc., have zeros which obey equation (15) for large x.

The maxima of F^2 occur where $\sin 2x = -1$. Here

$$x_n = \pi(n + 3/4) \quad . \tag{16}$$

TABLE 2. COMPARISON OF APPROXIMATE FORMULA WITH $J_1 = 0$ IN PREDICTING MINIMA OF F^2 (MINIMA ARE PREDICTED FOR $N \geq 1$)

n	Actual zero	Approximate zero	Deviation (%)
0	0	0.79	--
1	3.83	3.93	2.54
2	7.02	7.07	0.71
3	10.17	10.21	0.39
4	13.32	13.35	0.22
5	16.47	16.49	0.12
6	19.62	19.63	0.05
7	22.76	22.78	0.09
8	25.90	25.92	0.08
9	29.05	29.06	0.03
10	32.19	32.20	0.03

We compare the maxima locations as predicted by equation (16) with values given by setting $J_2 = 0$, $x \neq 0$ in table 3. There is good agreement (percentage of deviation less than 1 percent) for $n > 3$. We show in appendix A that Bessel functions of even order, J_0, J_2, etc., have zeros which obey equation (16) for large x.

TABLE 3. ACTUAL AND APPROXIMATE LOCATIONS OF F^2 MAXIMA

n	Actual location ($J_2 = 0$)	Approximate location	Deviation (%)
0	0	2.36	--
1	5.14	5.50	7.0
2	8.42	8.64	2.6
3	11.62	11.78	1.4
4	14.80	14.92	0.8
5	17.96	18.06	0.6
6	21.12	21.21	0.4
7	24.27	24.35	0.3
8	27.42	27.49	0.3
9	30.57	30.63	0.2
10	33.72	33.77	0.1

Inspection of equation (14) reveals that the magnitude of F^2 maxima for large x_n is given by

$$F^2 = 8/\pi x_n^3 \quad . \tag{17}$$

Equation (17) is compared with equation (4) in table 4.

TABLE 4. COMPARISON OF EXACT AND APPROXIMATE MAGNITUDES OF SUCCESSIVE F^2 MAXIMA

n	$x_n(J_2 = 0)$	$4J_1^2/x^2$	$8/\pi x^3$	Deviation (%)
0	0	1	∞	∞
1	5.136	0.017498	0.018796	7.42
2	8.417	0.004158	0.004270	2.69
3	11.620	0.001601	0.001623	1.37
4	14.796	0.000779	0.000786	0.90
5	17.960	0.000437	0.000440	0.69
6	21.117	0.000269	0.000270	0.37
7	24.270	0.000178	0.000178	--
8	27.420	0.000123	0.000124	--
9	30.570	0.000089	0.000089	--

2.2 Exact Location of Maxima and Minima for F^2

In determining maxima and minima, it will be helpful to use the recurrence relations

$$J_{n-1} = nJ_n/x + J_n' \tag{18}$$

and

$$J_{n+1} = nJ_n/x - J_n' \quad . \tag{19}$$

Taking the first derivative of F^2 and setting the result to zero, we get

$$dF^2/dx = 8J_1\left(xJ_1' - J_1\right)/x^3 = 0 \quad .$$

From equation (19) with n = 1, we have

$$xJ_1' - J_1 = -xJ_2 \quad .$$

Thus,

$$dF^2/dx = -8J_1J_2/x^2 = 0 \tag{20}$$

is a condition for relative extrema.

The second derivative of F^2 is

$$d^2F^2/dx^2 = 16x^{-3}J_1J_2 - 8x^{-2}\left(J_1'J_2 + J_1J_2'\right) \quad . \tag{21}$$

Inspection of equation (20) reveals that it is satisfied for J_1 or $J_2 = 0$, provided $x \neq 0$. At $x = 0$, J_1 and J_2, as well as x, simultaneously go to zero. We will investigate this situation last. Let us first note that if $x \neq 0$, J_1 and J_2 do not have zeros in common. Setting $J_1 = 0$, but J_2 and $x \neq 0$ satisfies equation (20). In equation (21), we have

$$d^2F^2/dx^2 = -8J_1'J_2/x^2 \quad .$$

Setting n = 1 in equation (19), and noting that $J_1 = 0$, we have $J_1' = -J_2$, so

$$d^2F^2/dx^2 = 8x^{-2}J_2^2 > 0 \quad ,$$

and there are minima here ($J_1 = 0$). On the other hand, setting $J_2 = 0$,

$$d^2F^2/dx^2 = -8x^{-2}J_1J_2' \quad .$$

Now put n = 2 in equation (18), obtaining $J_2' = J_1$ with $J_2 = 0$. Thus, here

$$d^2F^2/dx^2 = -8x^{-2}J_1^2 < 0 \quad ,$$

and we have maxima at these x. Finally, making use of equation (11), we find

$$dF^2/dx = -x/2 + 0(x^3) \quad ,$$

which goes to zero as $x \to 0$, indicating a maximum or minimum here. Going further,

$$d^2F^2/dx^2 = -1/2 + 0(x^2) \quad ,$$

so that at x = 0

$$d^2F^2/dx^2 = -\ 1/2 < 0 \quad ,$$

indicating a maximum. At this point $F^2 = 1$, as indicated by equation (11).

Summarizing, we have maxima at the zeros of J_2 (now including x = 0) and minima at the zeros of J_1 with $x \neq 0$.

2.3 Frequencies for Minima and Maxima of F^2 at R = 2.54 cm

In general, the frequencies, f_n, at which maxima of F^2 occur if R is held constant are given by

$$f_n = cx_n/2\pi R \quad , \qquad (22)$$

where x_n is a zero of J_2. The largest value of F^2 occurs at $x_0 = 0$. The successive maxima of F^2 are given exactly by equation (17) and fall off as x_n^{-3} for sufficiently large x_n. For large x_n we have the approximate relations

$$f_n = c(n + 3/4)/2R \quad , \qquad (23)$$

and

$$F^2 = 8/\pi^4(n + 3/4)^3 \tag{24}$$

following from equations (16), (17), and (22).

The f values, where $F^2 = 0$, are also given by equation (22), but now the x_n are zeros of $J_1(x)$ for $x \neq 0$. For large enough x,

$$f_n = c(n + 1/4)/2R \quad . \tag{25}$$

We summarize exact values of f_n at minima and maxima in table 5 for R = 2.54 cm (c/2R = 5.9 GHz). Exact values of F^2 for the maxima will be found in table 4. The first two cycles of F^2 plotted versus frequency are presented in figure 2.

TABLE 5. FREQUENCIES (IN GHz) FOR MINIMA AND MAXIMA WITH BEAM RADIUS = 2.54 cm

n	Minima		Maxima	
	Exact	Approximate	Exact	Approximate
0	--	1.48	0	4.43
1	7.19	7.38	9.65	10.33
2	13.18	13.28	15.81	16.23
3	19.10	19.18	21.82	22.13
4	25.02	25.08	27.79	28.03
5	30.93	30.98	33.73	33.93
6	36.85	36.88	39.66	39.83
7	42.74	42.78	45.58	45.73
8	48.64	48.68	51.50	51.63
9	54.56	54.58	57.41	57.53

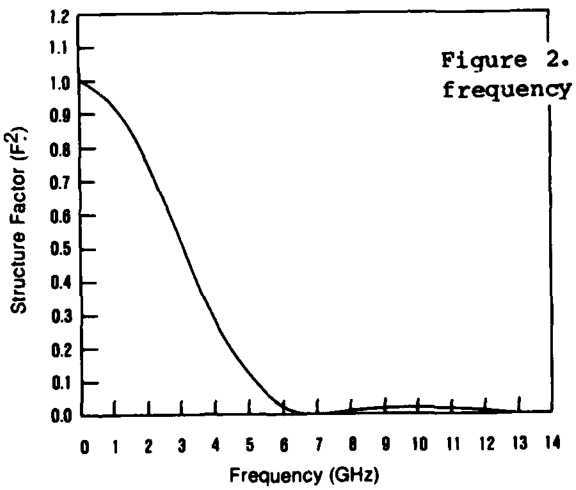

Figure 2. Structure factor $F^2 = 4J_1^2(x)/x^2$ versus frequency $f = xc/2\pi R$.

2.4 Other Beam Power Spectra

If $S(\omega) \neq S/\omega$, the appropriate factor in equation (1) to subject to extremization is

$$g^2 = \left|S(\omega)\right|^2 J_1^2(\omega R/c) \quad . \tag{26}$$

By inspection of equation (26) we expect absolute minima to occur where either $\left|S(\omega)\right|$ or $J_1(\omega R/c) = 0$. These conditions yield specific planes beginning with $\omega = 0$ which are perpendicular to the ω axis and cut the surface prescribed by equation (1). Taking the first derivative of g^2 and setting it equal to zero gives

$$dg^2/d\omega = 2\left|S(\omega)\right|J_1\left[J_1\left|S(\omega)\right|' + R\left|S(\omega)\right|J_1'/c\right] = 0 \quad , \tag{27}$$

where $\left|S(\omega)\right|'$ and J_1' denote derivatives of $\left|S(\omega)\right|$ or J_1 with respect to their arguments. The factor in parentheses yields extrema of equation (26) other than absolute minima (i.e., extrema where g^2 is nonzero). Among these may be maxima of varying height which include one or more absolute maxima, other minima (located between the peaks) which give nonzero g^2, as well as points of inflection. These cases are sorted out by taking the second derivative. Making use of equation (19) we have

$$J_1' = cJ_1/\omega R - J_2 \quad .$$

On substituting the above expression in equation (27), we get as the condition for extrema (barring absolute minima)

$$\left[\left|S(\omega)\right|' + \omega^{-1}\left|S(\omega)\right|\right]J_1 - R\left|S(\omega)\right|J_2/c = 0 \quad . \tag{28}$$

Note that equation (28) reduces to the earlier condition for maxima, namely, $J_2 = 0$, if $\left|S(\omega)\right|'' = S/\omega$. The test for maxima is given in appendix B, where it is shown that the behavior of $\left|S(\omega)\right|''/S(\omega)$ is the determining factor.

3. EXTREMA AT CONSTANT FREQUENCY

3.1 Approximate Representations of Form Factor f^2

Figure 3 illustrates the behavior of f^2 (eq (8)) for $0 \leq x \leq 7$. It has two peaks in the interval. These are located at 2.41 and 5.52, J_0 being zero at these points. $J_0 = 0$ also locates all the other maxima of this function. Also, as one might expect, $J_1 = 0$ locates all the minima. Three of these zeros are seen on the graph. They lie at x values of 0, 3.83, and 7.02.

The maxima are seen to grow with x. The first maximum has a magnitude of 6.23; the second, 14.11. Successive maxima which are off the graph are larger still. We will show shortly that this growth is proportional to x.

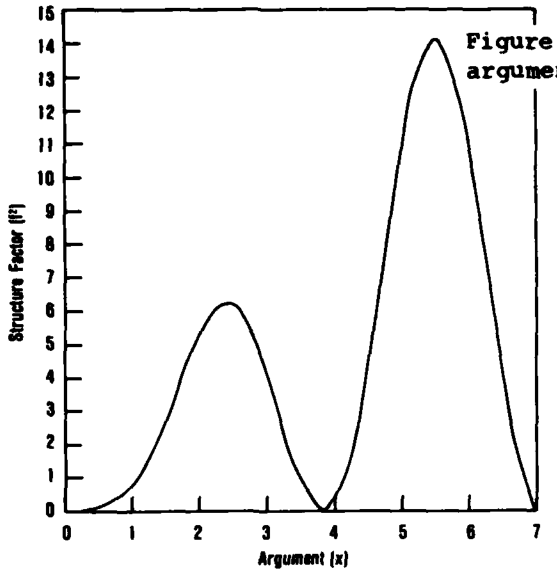

Figure 3. Structure factor $f^2 = 4x^2J_1^2(x)$ versus argument $x = \omega R/c$.

From equations (8) and (11), we immediately determine a useful series which represents f^2 for small arguments:

$$f^2 = x^4 - x^6/4 + 5x^8/192 - 7x^{10}/4608 + \dots \quad . \tag{29}$$

One obtains reasonable values (less than 1 percent error) up to x = 1.6 using only the first four terms given above. This is illustrated in table 6.

TABLE 6. SMALL ARGUMENT REPRESENTATION OF FORM FACTOR f^2

x	f^2(series)	f^2(exact)	Deviation (%)
0	0	0	--
0.2	0.0016	0.0016	--
0.4	0.0246	0.0246	--
0.6	0.1184	0.1184	--
0.8	0.3483	0.3483	--
1.0	0.7745	0.7746	0.01
1.2	1.4291	1.4302	0.03
1.4	2.2496	2.3027	0.13
1.6	3.3108	3.3258	0.45
1.8	4.3219	4.3826	1.39
2.0	5.1111	5.3218	3.96

Substituting the large argument approximation for J_1 (eq (13) in eq (8)) one finds

$$f^2 = (4x/\pi)(1 - \sin 2x) \quad . \tag{30}$$

The location of minima and maxima are again given by equations (15) and (16), respectively. Table 2 gives locations of the absolute minima, including n or x = 0 where J_1 is zero. Table 7 gives maxima locations for f^2. Agreement with the approximate formula and exact locations as given by $J_0(x) = 0$ is very good.

TABLE 7. ACTUAL AND APPROXIMATE LOCATIONS OF f^2 MAXIMA

n	Actual location ($J_0 = 0$)	Approximate location	Deviation (%)
0	2.41	2.36	2.07
1	5.52	5.50	0.36
2	8.65	8.64	0.12
3	11.79	11.78	0.08
4	14.93	14.92	0.07
5	18.07	18.06	0.06
6	21.21	21.21	--
7	24.35	24.35	--
8	27.49	27.49	--
9	30.63	30.63	--
10	33.78	33.77	--

Inspection of equation (30) shows that the growth of maxima for f^2 is given approximately by the linear relation

$$f_n^2 = 8x_n/\pi \quad . \tag{31}$$

Equation (31) is compared with equation (8) in table 8. Table 8 also gives local R values for maxima. There are practical limitations on the largeness of R. Figure 4 gives the first three cycles of f^2 versus R for 35 GHz.

TABLE 8. COMPARISON OF EXACT AND APPROXIMATE MAGNITUDES OF SUCCESSIVE f^2 MAXIMA

n	$x(J_0 = 0)$	R (cm)	$4x^2J_1^2$	$8x/\pi$	Deviation (%)
0	2.405	0.33	6.23	6.12	1.77
1	5.520	0.75	14.11	14.06	0.35
2	8.654	1.18	22.07	22.04	0.14
3	11.792	1.61	30.05	30.03	0.07
4	14.931	2.04	38.04	38.02	0.05
5	18.071	2.47	46.04	46.02	--
6	21.212	2.89	54.03	54.02	--
7	24.352	3.32	62.03	62.01	--
8	27.49	3.75	70.02	70.00	--
9	30.63	4.18	78.02	78.00	--

3.2 Exact Location of Maxima and Minima for Form Factor f^2

This section gives the analytical method for determining exact maxima and minima for this case. The condition for extrema is

$$df^2/dx = 8xJ_1\left(J_1 + xJ_1'\right) = 0 \quad . \tag{32}$$

From equation (18),

$$xJ_0 = J_1 + xJ_1' \quad .$$

Therefore,

$$df^2/dx = 8x^2 J_0 J_1 = 0 \quad . \tag{33}$$

Thus, extrema occur where J_0 or J_1 is zero. Continuing,

$$d^2f^2/dx^2 = 16xJ_0J_1 + 8x^2\left(J_0'J_1 + J_0J_1'\right) \quad . \tag{34}$$

When $J_1 = 0$,

$$d^2f^2/dx^2 = 8x^2 J_0 J_1' \quad . \tag{35}$$

From equation (18) with $J_1 = 0$, we have $J_1' = J_0$, so

$$d^2f^2/dx^2 = 8x^2 J_0^2 > 0, \tag{36}$$

and there are minima here. On the other hand, when $J_0 = 0$,

$$d^2f^2/dx^2 = 8x^2 J_0' J_1 \quad , \tag{37}$$

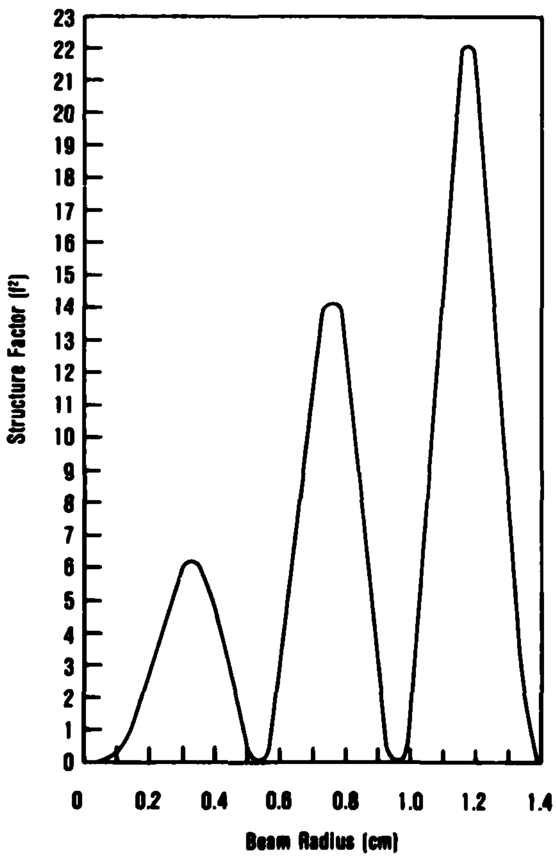

Figure 4. Structure factor $f^2 = 4x^2J_1^2(x)$ versus beam radius $R = cx/\omega$.

but $J_0' = -J_1$ by equation (19) and

$$d^2f^2/dx^2 = -8x^2J_1^2 < 0 \quad . \tag{38}$$

This designates the location of maxima.

4. SURFACE EXTREMA

To determine surface extrema, we start with equation (9). Taking partial derivatives with respect to R yields

$$\partial h^2/\partial R = 2R\left|S(\omega)\right|^2 J_1\left(J_1 + \omega R J_1'/c\right) = 0 \quad . \tag{39}$$

Here J_1' denotes differentiation with respect to the argument. Making use of equation (18), we have

$$\partial h^2/\partial R = 2\omega R^2 J_1\left|S(\omega)\right|^2 J_0/c = 0 \quad . \tag{40}$$

On the other hand, the partial derivative with respect to ω gives

$$\partial h^2/\partial\omega = 2R^2\left|S(\omega)\right|J_1\left(J_1 d\left|S(\omega)\right|/d\omega + R\left|S(\omega)\right|J_1'/c\right) = 0 \quad . \tag{41}$$

With the help of equation (19) this is rendered in the form

$$\partial h^2/\partial\omega = 2R^2\left|S(\omega)\right|J_1\left\{(d\left|S(\omega)\right|/d\omega + \left|S(\omega)\right|/\omega)J_1 - R\left|S(\omega)\right|J_2/c\right\} = 0 \quad . \tag{42}$$

The bracketed term is identical to equation (28). As expected, absolute minima occur where $\left|S(\omega)\right|$ or $J_1 = 0$. The former condition gives planes of constant ω, and the latter yields a family of hyperbolas of constant ωR. Minima also occur where ω or R is zero. Inspection of equations (40) and (42) reveals that other extrema (not including the absolute minima) occur if we satisfy

$$J_0 = 0 \quad , \tag{43}$$

and

$$(d\left|S(\omega)\right|/d\omega + \left|S(\omega)\right|/\omega)J_1 - R\left|S(\omega)\right|J_2/c = 0 \tag{44}$$

simultaneously. Let the zeros of J_0 be denoted now by x_{n0}. They are given in table 8. Then

$$R = cx_{n0}/\omega \tag{45}$$

replaces R in equation (44). Do not forget the arguments of J_1 and J_2! Thus, $J_{1n} = J_1(x_{n0})$ and $J_{2n} = J_2(x_{n0})$ are now independent of ω and R. Equation (44) reduces to

$$(d|S(\omega)|d\omega + |S(\omega)|/\omega)J_{1n} - x_{n0}|S(\omega)|J_{2n}/\omega = 0 \quad . \tag{46}$$

The solution depends on the form of $|S(\omega)|$. Consider our earlier example. If $|S(\omega)| = S/\omega$, equation (46) reduces to

$$J_{2n} = 0 \quad . \tag{47}$$

This condition can only be met approximately for large x where the zeros of J_0 and J_2 converge. Here the extrema, including any absolute maxima, occur along hyperbolas of constant ωR.

5. CONCLUSIONS

The problem of extremizing the average power spectral density per steradian may be approached from three points of view. If the operating frequency is fixed by practical considerations (i.e., the atmospheric window for microwave emission at 35 GHz), we seek to vary R at constant ω. If R and voltage are fixed by certain design considerations, but ω is not, we seek extrema with varying ω. When both variables are unrestricted, we must seek extrema over the entire surface.

The ω dependence of the Fourier transform of the sum over sheet velocities plays an important role in the determination of extrema if ω is not fixed. The only exceptions to this are the absolute minima given by the zeros of J_1. The average power radiated is zero here. If $|S(\omega)|$ is of the form S/ω with S some positive constant, maxima are located at the zeros of J_2. The maxima are then seen to fall off as ω^{-3} for large ω. This is in addition to the absolute minima associated with J_1 as discussed above. For general $|S(\omega)|$, additional absolute minima occur where $|S(\omega)| = 0$, the average power radiated being zero here.

If ω is fixed and R varied, one finds the absolute minima to be again associated with J_1 as expected. These minima are now located at the intersection of the surface, the plane of constant ω, and planes of constant R. Maxima, however, are now associated with the zeros of J_0. This yields another set of planes of constant R associated with the maxima. The growth of these maxima is proportional to R for large R.

Allowing both ω and R to vary freely leads to a set of two algebraic equations which must be solved simultaneously for ω and R. Again, $|S(\omega)|$ and $J_1 = 0$ are associated with absolute minima. The zeros, if any, of $|S(\omega)|$ are planes of constant ω. On the other hand, $J_1 = 0$ now implies a set of hyperbolas (actually sheets bent in this form) with ωR constant. Their intersection with the surface locates these absolute minima. As the argument of J_1 is $\omega R/c$, the planes ω or $R = 0$ are also associated with absolute minima. Other extrema including absolute maxima are determined by solving simultaneously two algebraic equations for the points (ω, R) at which the extrema are located. For the special case $|S(\omega)| = \omega^{-1}S$, the points trend to a set of hyperbolas which are the simultaneous solutions of $J_0 = J_2 = 0$ (valid only for large arguments). For other $|S(\omega)|$, the results can be more complicated. The importance of $S(\omega)$ in determining the location of maxima suggests the need to develop more realistic expressions for theoretical use of computer data.

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APPENDIX A.--LARGE ARGUMENT ZEROS OF BESSEL FUNCTIONS

Equation (12) in the body of the report gives the general expression for $J_n(x)$ for large x. For n = 2s, we have on substitution

$$J_{2s} = (2/\pi x)^{1/2} \cos([x - \pi/4] - s\pi)$$

$$= (-1)^s (2/\pi x)^{1/2} \cos(x - \pi/4) \quad .$$

The argument of the cosine factor is the same for all the J_{2s}. It alone determines the zeros which occur for

$$x = \pi(n + 3/4) \quad . \qquad \text{(A-1)}$$

On the other hand, for odd n = 2s + 1,

$$J_{2s+1} = (2/\pi x)^{1/2} \cos([x - \pi/4] - 1/2(2s + 1)\pi)$$

$$= (-1)^s (2/\pi x)^{1/2} \sin(x - \pi/4) \quad .$$

Therefore, the zeros of the J_{2s+1} which depend on the argument of the sine alone occur at

$$x = \pi(n + 1/4) \quad . \qquad \text{(A-2)}$$

The foregoing remarks apply to sufficiently large arguments of the J_n.

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APPENDIX B.--CONDITION FOR MAXIMA OR MINIMA

As J_1 and $|S(\omega)|$ in equation (19) $\neq 0$, except for absolute minima, we may write when away from these points

$$dg^2/d\omega = 2g^2\left(d\ln|S(\omega)|/d\omega + \omega^{-1} - RJ_2/cJ_1\right) = 0 \ , \qquad \text{(B-1)}$$

and we explore the region where the last factor on the right vanishes. Taking the second derivative we have

$$d^2g^2/d\omega^2 = 2g^2\left(d\ln|S(\omega)|/d\omega + \omega^{-1} - RJ_2/cJ_1\right)' \quad . \qquad \text{(B-2)}$$

The prime on the last factor denotes differentiation with respect to ω. As $2g^2 > 0$, whether we have minima or maxima here depends on the last factor of equation (2) in the body of the report. Taking the indicated derivative, we have

$$g^{-2}d^2g^2/2d\omega^2 = |S(\omega)|''/|S(\omega)| - \left(|S(\omega)|'/|S(\omega)|\right)^2$$

$$- \omega^{-2} - (R/c)^2\left(J_2'J_1 - J_2J_1'\right)/{J_1}^2 \quad . \qquad \text{(B-3)}$$

We now make use of certain relations to simplify equation (B-3). From the vanishing of equation (B-1),

$$|S(\omega)|'/|S(\omega)| = -\omega^{-1} + RJ_2/cJ_1 \quad . \qquad \text{(B-4)}$$

Also, we have the recurrence relations from equations (18) and (19):

$$J_2' = -x^{-1}J_2 + J_1 \qquad \text{(B-5)}$$

and

$$J_1' = x^{-1}J_1 - J_2 \quad . \qquad \text{(B-6)}$$

On substitution of equations (B-4) through (B-6), equation (B-3) becomes

$$g^{-2}d^2g^2/2d\omega^2 = |S(\omega)|''/|S(\omega)| - 2\omega^{-2}\left(1 - xJ_2/J_1\right)^2 - (R/c)^2 \quad . \qquad \text{(B-7)}$$

This is as far as we can go without a specific form of $|S(\omega)|$. Note that the last two terms are negative. Extrema are therefore guaranteed if

$$|S(\omega)|''/|S(\omega)| < 2\omega^{-2}\left(1 - xJ_2/J_1\right)^2 + (R/c)^2 \quad . \qquad \text{(B-8)}$$

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GLOSSARY OF TERMS

c speed of light

d distance between grid and ground plate

f quantity defined by equation (8)

g quantity defined by equation (26)

h quantity defined by equation (9)

i $\sqrt{-1}$

m summation index introduced in equation (3)

n denotes order of Bessel function

s summation index in equation (10)

v sheet velocity (v_m) or average sheet velocity (v_a)

x $\omega R/c$

C constant in equation (1)

F form factor

H Heaviside step function

J Bessel function of first kind (n^{th} order denoted by J_n)

K constant in equation (7)

N upper limit on sum (equation (3)) expressed as a function of time or mean number of sheets when subscripted by a

O designates "order of"

P power or point (ω, R)

R beam radius

S sum of sheet velocities, S(t), or a constant

δ infinitesimally small quantity

μ_o permeability of free space

ω angular frequency

Ω solid angle

π pi

σ sheet surface charge density

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