

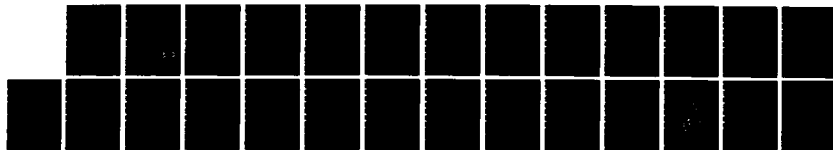
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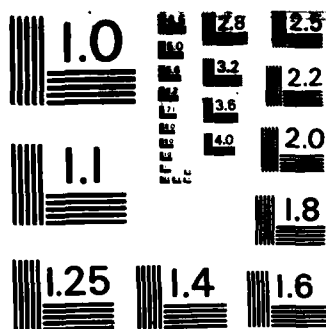
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**RELATING GEOID ANOMALIES, GRAVITY ANOMALIES  
AND OCEAN TOPOGRAPHY**

Joseph E. Martine

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Boston College  
885 Centre Street  
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Final Report  
26 January 1982 - 30 September 1983

31 December 1983

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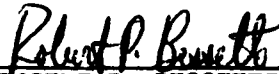
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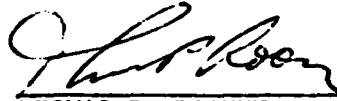
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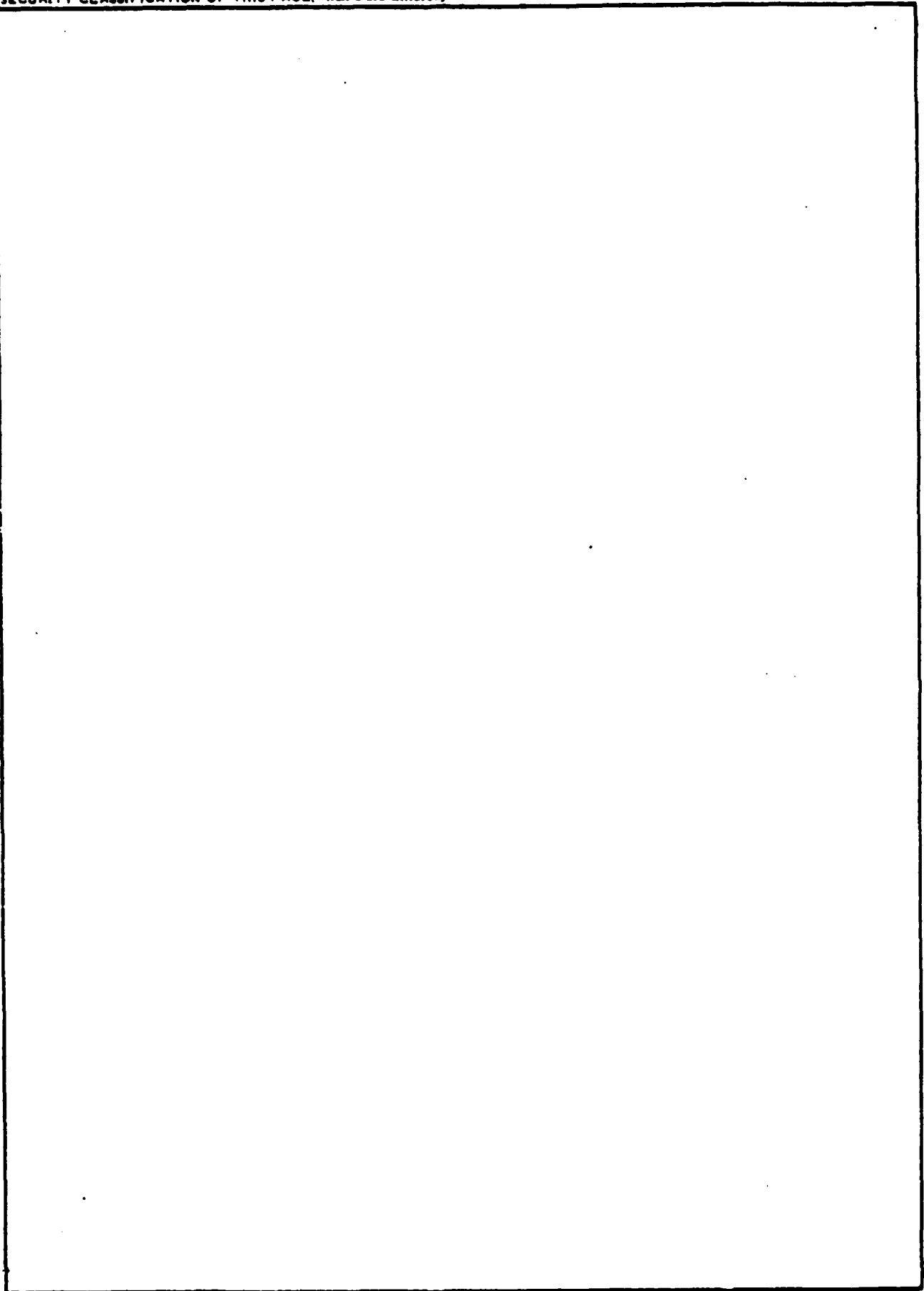
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## INTRODUCTION

One of the major tasks of geodesy is the determination of the gravity field over the entire earth. Prior to the advent of satellites, global representation of gravity was severely hampered by sparse or poorly distributed gravimetric measurements in the oceans. With the launching of SEASAT in 1978, however, enormous improvements have been achieved in our knowledge of the ocean's gravity field.<sup>1</sup> The radar altimeter readings from that satellite can be used to determine the shape of the oceans' surface with great accuracy, leading to important applications not only in geodesy, but also in geophysics and oceanography.

If the earth were all water, right to its center, mean sea level would be a simple geometrical figure closely approximated by an ellipsoid of revolution. However, the earth is largely solid and supports density variations in continental areas and under oceans. These density variations cause mean sea level, considered an equipotential surface, to vary by scores of meters from an ideal or reference ellipsoid. The irregular, smooth surface, called the geoid, formed by the extension of mean sea level throughout the earth is a fundamental surface in geodesy. Separations of the geoid above or below the reference are dependent on the variation of the earth's gravity field. This relationship has been a principal topic of investigation by geodesists, who have devised a means for determining the geoid separations from surface gravimetric measurements. Now, SEASAT satellite altimetry can also be used to derive geoid separations and, thus, the gravity field over the oceans with great accuracy.

The short-arc technique, developed at the Air Force Geophysics Laboratory (AFGL) at Hanscom AFB, MA, provides us with data representing the short wavelength part of the geoid; i.e., wavelengths of a few hundred kilometers. Geoidal features at these wavelengths are primarily attributable to density variations at deep levels in the earth's crust. The short-arc adjustments, essentially a least-squares process, segregates all the short wavelength detail into the solution residuals (geoid separations or geoid height components). Thus the residuals contain geoid signals from such sources as the upper crust and sea-bottom topography.

The short wavelength residuals may therefore be used to study important geophysical problems (i.e. the mechanism of sea-floor spreading, crustal

subduction at trenches, and global tides). One aspect that is receiving attention is the correlation of these residuals with ocean bottom topographic features, particularly sea mounts, whose accurate size and position are required for submarine navigation.

A report<sup>2</sup> by Chapman, Talwani, et al., illustrates many factors of the current research relating geoid measurements in the ocean region to gravitational, density and topographic variations. However, a major limitation of this and prior studies was the quality and quantity of the geoid measurements. The reliability of the input data (SEASAT derived) enhanced this opportunity to better understand and study the composition and structure of the Earth's crust beneath the sea and also the related gravitational variations.

#### A. Calculating Geoid Anomalies from Density Inhomogeneities

The procedure here is to calculate geoid anomalies for given structures in the earth's crust. The procedure is as follows:

According to Brun's Formula

$$\text{Geoid height (N)} = \frac{T}{\gamma}$$

where T is the anomalous potential or difference between the total potential and the normal potential due to the reference ellipse and  $\gamma$  is normal gravity. For a point mass

$$T = \frac{GM}{R}$$

where G is the Newtonian Constant, M is the mass and R the distance between the mass and an observation point. Then for a body (collection of point masses), in Cartesian coordinates,

$$N(x', y', z') = \frac{G}{\gamma} \iiint \frac{\rho \, dx \, dy \, dz}{((x-x')^2 + (y-y')^2 + (z-z')^2)^{1/2}}$$

where  $\rho$  is density and the primes indicate the observation point.

A simpler two-dimensional formulation for calculating N is

$$N(x, z) = -\frac{G}{\gamma} \iint \rho \log_e((x-x')^2 + (z-z')^2) dx dz$$

If one assumes that  $\rho$  is sectionally constant, then this latter integral can be reduced to a line (contour) integral which can be integrated directly. In general, a geophysical model can be comprised of individual sections (or bodies) of constant density. For each section the geoid effect is calculated. After summation over all the sections the total geoid effect due to the model is obtained.

In this study, the type of section selected was a two-dimensional rectangle. The geoid height was then calculated by both the line integral formula reported by Chapman and using Simpson's Rule for numerical integration.

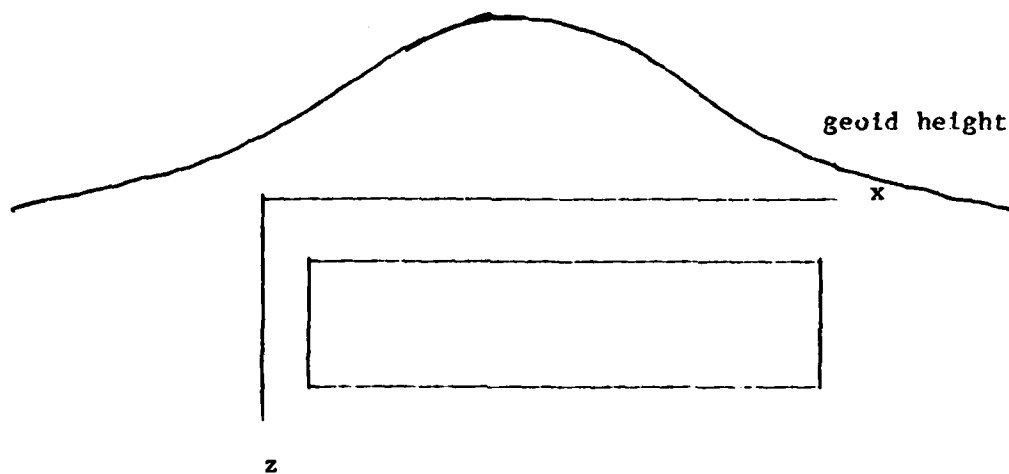


Figure 1. Calculation of geoid anomaly over a two-dimensional rectangle.

The shape of the geoid height computed by both of the above methods and that presented by Chapman (Figure 4, page 82) are in complete agreement.

#### B. A Predictive Algorithm Relating Ocean Topography to Geoid Anomalies.

An algorithm to relate ocean topography to geoid anomalies was developed by relating bathymetry (ocean depth) to geoidal anomalies in selected areas: specifically, to regions containing a significant oceanic feature such as a trench, a sea mount or a mid-ocean ridge. These regions are selected since they are the regions most apt to show these significant geoid anomalies.

The residual tapes were then edited to find specific tracks in the Hawaiian Ridge. Nine tracks were selected. The basic criteria for selection is that the tracks be centered along a significant topographic feature. They were approximately 2600 km in length and perpendicular to that ridge. These residuals are considered only as a function of distance along the track.

A program obtained by AFGL called SYNBAPS was used to provide bathymetry measurements along the same tracks.

The paired sets of data, residuals and bathymetry, were then compared and showed good correlation. The cross-correlation coefficients were calculated and found to be between .6 and .8.

The means of each were then removed. No trends were removed from the residuals. This in essence was compensated for by the fitting to a geoid model (GEM 10). The Fourier Transform for each paired data set was calculated. The ratio of the paired transforms represents their admittances.

$$\text{admittance} = \frac{F(\text{Residuals})}{F(\text{Bathymetry})}$$

The re-transform of this ratio gives the filter that when convolved with the bathymetry data reproduces the residuals. To determine one admittance or filter function unique to that area, the admittances or filters for the nine paired sets were averaged together.

Simple physics suggests that the filter be symmetric. This means that the admittance must be real. This is true if the coherence = 1, since for each wavenumber of  $F(R)$ , its phase is the same as its counterpart in  $F(B)$ . For instance, let those components be

$$F(R) = R(R) + i I(R)$$

$$F(B) = R(B) + i I(B)$$

$$i = \sqrt{-1}$$

then

$$\frac{F(R)}{F(B)} = \frac{R(R) R(B) + I(R) I(B)}{R(B)^2 + I(B)^2} + i \frac{R(R) I(B) - R(B) I(R)}{R(B)^2 + I(B)^2}$$

Note that the complex part = 0, if  $\frac{R(R)}{I(R)} = \frac{R(B)}{I(B)}$ , meaning their phases are equal.

The imaginary part of the averaged admittance was set to zero. The bathymetry transforms were multiplied by this averaged real admittance function. Re-transforms were then made to obtain the filter in the spacial domain and the filter projected residuals.

The coherence relating the measured residuals and the bathymetry was obtained as follows:

$$c(k) = \frac{1}{2I} \sum_{i=1}^I F_i(R) F_i(B)^*$$

$$R_o(k) = \frac{1}{2I} \sum_{i=1}^I F_i(R) F_i(R)^*$$

$$B_o(k) = \frac{1}{2I} \sum_{i=1}^I F_i(B) F_i(B)^*$$

$$\text{coherence}(k) = \frac{cc^*}{R_o B_o}$$

I represents the number of passes in the averaging, the \* denotes the complex conjugate, and  $k$  represents the wavenumber,  $R_o$  and  $B_o$  are the power spectra of the residuals and the bathymetry, and  $c$  is their cross-spectrum.

We then calculated the admittance  $Z(k)$  from a model for two dimensional Airy isostasy:

$$Z(k) = \frac{2\pi G}{\gamma |k|} (\rho_c - \rho_w) e^{-kd(1-e^{-kt})}$$

where

$\rho_c$  density of the crust

$\rho_w$  density of the water

$d$  is the averaged depth

$t$  is the thickness of crust

The coherence and the comparison between the calculated and model admittance were not at first encouraging. The bulk of the power is frequently accounted for in 4 or 5 of the lower harmonics. Noise and density anomalies are expected in the residuals. These are most apt to affect the weaker components of that Fourier Transform. Hence, the coherence for these components becomes a bit random. The calculated admittance, being a ratio, also has this problem. Hence, it does not demonstrate the decrease in amplitude at higher wave numbers as expected by the reciprocal of the wave number term in the Airy admittance model.

### C. Covariance Filter Relating Bathymetry Measurements to Geoid Anomalies

Basically this requires determining the coefficients that then can be used as a filter to transform one type data set into another set. In our case we are considering that filter as being centered and symmetric. Using the criteria of Least Squares to determine these coefficients, we are then required to minimize

$$I = \sum_{s=m+1}^{L-m} \left[ R_s - f_0 B_s - \sum_{\ell=1}^m f_{\ell} (B_{s+\ell} + B_{s-\ell}) \right]^2$$

where  $R$  and  $B$  are paired data sets,  $f_{\ell}$  the filter coefficients,  $m=(L-1)/2$ ,  $L$  being the length of the filter. Taking the partial derivative of  $I$  with respect to each  $f_{\ell}$  provides the  $m+1$  normal (linear) equations to determine the coefficients.

For example

$$\frac{\partial I}{\partial f_0} = \sum_{s=m+1}^{L-m} \left[ R_s - f_0 B_s - \sum_{\ell=1}^m f_{\ell} (B_{s+\ell} + B_{s-\ell}) \right] (-2) B_s = 0$$

$$\frac{\partial I}{\partial f_1} = \sum_{s=m+1}^{L-m} \left[ R_s - f_0 B_s - \sum_{\ell=1}^m f_{\ell} (B_{s+\ell} + B_{s-\ell}) \right] (-2) (B_{s+1} + B_{s-1}) = 0$$

•  
•  
•  
•

Residual and bathymetry data sets of approximately 100 points at 26 km spacing along specific tracks were obtained. The filter coefficients, usually  $L=9$ , were determined. The filter was then applied to the bathymetry data to find "the predicted residual data". Results were reasonable.

#### D. Calculation of Gravity Anomalies from Geoid Anomalies.

In the processing of the SEASAT altimetry measurement at AFGL, the geoid was considered as a summation of the Standard Reference Ellipse and the GEM 10 model or as it is referred at AFGL the 14-degree spherical harmonics model. Their processing thereby resulted in separations (residuals or anomalies) related to that input geoid.

The following procedures were used to calculate the gravitational component ( $\Delta g$ ) for these separations. The summation of these  $\Delta g$  and the GEM 10 model gravity components then represent the gravity anomaly.<sup>3</sup> This anomaly is defined as the difference between measured surface gravity and that gravity computed for the Standard Reference Ellipse.

First, we introduce two formulae based on the assumption that the oceanic geoid separations are only a function of one dimension regarding the ocean surface. This is followed by two formulae which are two dimensional regarding the ocean surface.

The two unidimensional formulae for calculating anomalous gravity utilize the separations as representing the geoid height or geoid anomaly along an arc  $a-b$  and are thereby related to a corresponding gravity anomaly. The first of these formulae<sup>4</sup> is

$$\begin{aligned} \Delta g(J) = & \frac{\gamma}{\pi} \left[ \frac{N(b)}{\ell(b) - \ell(J)} + \frac{N(a)}{\ell(J) - \ell(a)} \right. \\ & + \left. \int_a^b \frac{N(J) - N(X)}{(\ell(J) - \ell(X))^2} dX \right] \\ & - \frac{\gamma}{R_e} N(J) \end{aligned} \quad (1)$$

where  $R_e$  is the radius of the earth,  $J$  the point of observation between  $a$  and  $b$ ,  $N()$  the geoid separation and  $l()$  the track distance. The integration is to be done numerically where at the point  $X=J$  the integrand is equated to zero.

The second unidimensional formula is derived from the Fundamental Equation of Physical Geodesy.<sup>5</sup> We have,

$$\Delta g = - \frac{\partial T}{\partial \bar{r}} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial \bar{r}} T \quad (2)$$

where  $\bar{r}$  is the direction normal to the reference ellipse surface. Using

$$\gamma = GM/R^2 \text{ or } \frac{\partial \gamma}{\partial \bar{r}} = - \frac{2\gamma}{R} \text{ and}$$

Brun's formula  $T = \gamma N$ , relating the anomalous potential and geoid height, we find

$$\Delta g = - \frac{\partial T}{\partial \bar{r}} - \frac{2\gamma N}{R} \quad (3)$$

where  $R$  is the mean radius of the reference ellipse.

Since we are concerned with an equipotential surface, the surface of the ocean, it is necessary for  $T$  to satisfy Laplace's Equation in two dimensions, namely,

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) T = 0$$

where  $z$  is the direction normal to the ocean surface (approximately the same as  $\bar{r}$ ). Hence, we choose the form of

$$T = \sum_1 (a_1 \cos(k_1 x) + b_1 \sin(k_1 x)) e^{-k_1 z}$$

where  $x$  is distance along the surface and  $k_1$  is the wave number. Using  $T/\gamma = N$  and the distributive property of the above equations, the separations are thereby fitted by Fourier series to determine the  $a_1$ ,  $b_1$ . From Eq. (3), we find at the surface  $z = 0$ ,

$$\Delta g(x) = \gamma \sum_i k_i - \frac{2}{R} (a_i \cos(k_i x) + b_i \sin(k_i x)) \quad (4)$$

We now present two-dimensional surface formulae for calculating anomalous gravity. If one considers a thin circular ring centered about an observation point then one realizes that the same geoid height measurements at different locations around the ring would contribute the same component to the gravity calculation. One can then closely estimate the anomalous gravity due to that ring by using an averaged value of geoid height. A concentric sequence of such rings represent the neighborhood of an observation point. With this in mind, and considering the solution to Laplace's Equation in cylindrical coordinates, it is noted that N is of the form

$$N(r) = C + \sum a_i J_0(r\alpha_i)$$

where r represents the distance between the point of observation and point of measured residual,  $J_0$  is the zero order Bessel Function and  $\alpha_i$  is the "root" where  $J_0(\alpha_i) = 0$ . If r is normalized to the distance to the furthest measurement considered, C is the value of N where  $r=1$  and

$$a_i = \frac{2}{J_1^2(\alpha_i)} \int_0^1 N(r) J_0(r\alpha_i) r dr - \frac{2C}{\alpha_i J_1(\alpha_i)}$$

The above integration was done by subdividing the region of integration and using averaged values for  $N(r) J_0(r\alpha_i) r$  in each subdivision. If need be

$$\int_a^b J_0(\alpha_i r) r dr = \frac{T}{\alpha_i} J_1(\alpha_i r) \Big|_a^b$$

The AFGL data base of observed separations are measured corrections to a 14-degree spherical harmonics model. Hence, we need only consider higher harmonics or that region contained by a great circle angle of  $(360^\circ/15)/2$  or  $12^\circ$  to a specified location. The units for this wavelength and wave number are

$$\lambda = \frac{\pi}{15} R = .2094R$$

$$\text{and} \quad k = \frac{4.775}{R}$$

$$\text{Setting} \quad T = \gamma(C + \sum a_i J_0(k_i r) e^{-k_i z})$$

where  $z=0$  corresponds to the earth's surface, and  $k_i = \alpha_i$ , we find for a select position ( $z=0, r=0$ ), that Eq. (3) results in

$$\Delta g = -(\gamma \sum a_i k_i) - \frac{2\gamma}{R} (C + \sum a_i) \quad (5)$$

Next we studied the double integral counterpart of Eq. (1), namely, the Molodenskii integral<sup>4</sup>

$$\Delta g_o = -\frac{\gamma N_o}{R} - \frac{\gamma}{2\pi} \iint_{\sigma} \frac{N-N_o}{l^3} d\sigma \quad (6)$$

where  $o$  designates the observation point and  $l$  is the distance between that point and an element of surface  $d\sigma$ . Again, we computed averaged values for the integrand as a function of distance along the surface (any direction) and since the area of thin circular concentric rings is easily represented, this double integration formula was evaluated by considering the integration replacement

$$\iint_{\sigma} d\sigma = \pi r_o^2 + \sum_{i=1}^{\infty} \pi(r_i^2 - r_{i-1}^2) \quad ,$$

The surface integration is replaced by an inner circle and a summation of concentric rings centered about the observation point. Replacing the integrand  $(N-N_o)/l^3$  by its averaged value in each ring, we find

$$\iint_{\sigma} \frac{N-N_o}{l^3} d\sigma = \sum_{i=1}^{\infty} \left( \frac{\overline{N-N_o}}{l^3} \right)_i \pi(r_i^2 - r_{i-1}^2) \quad ,$$

where  $(\overline{\quad})_i$  is the averaged value for the integrand in the  $i$ th ring. Note the innermost ring's contribution.

Examples of results obtained from calculations according to formulae of this section are represented in the following figures.

Figure 2. Profiles of gravity anomaly surface measurement and that obtained by using formula 1.

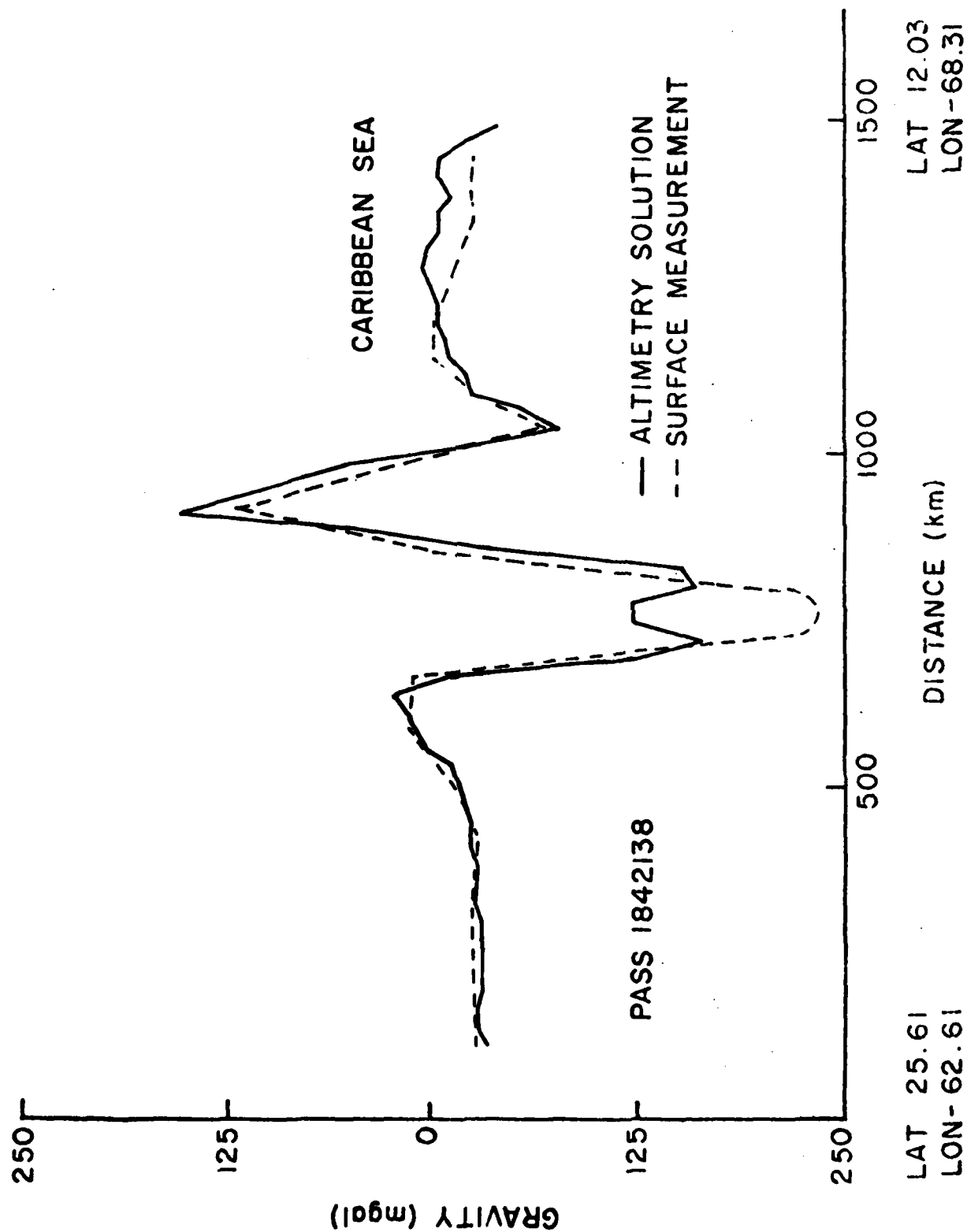


Figure 2. Profiles of gravity anomaly surface measurement and that obtained by using formula 1.

Figure 3. Profiles of gravity anomaly surface measurements, prior calculations made at AFGL using point mass technique and that obtained by using formula 5 ( $i=2$ ).

Figure 4. A contour plot of anomalies obtained by using formula 5 ( $i=2$ ). gravity.

#### E. Auxiliary Requirements - Primarily Presentation

In the course of this research effort a number of sundry tasks were required. The most notable task was to represent AFGL's deduced geoid anomalies.

The data base for making the contour plots of the AFGL measured/computed ocean geoid is made from sorting their corresponding high-pass filtered SEASAT satellite altimetry data. These later data are in sequences of points where each point contains the latitude, longitude, 14-degree geoid model (GM) and geoid anomaly or undulation (GU). For each of these points the values of GM and GU is assigned to a location in a 2-dimensional array. That location is determined by indices obtained by rounding off the values of latitude and longitude to the nearest integer. This array represents a  $1 \times 1$  degree grid. Should multiple points be assigned the same location, the averaged values (for GM & GU) are calculated and retained.

The input filtered data are restricted to  $\pm 70^\circ$  in latitude and to ocean regions. The contour plotting program requires that each location of the array have an assigned value. Should a location not have such a value, then an averaged value of the neighboring locations of  $\pm 1$  is substituted. Except for latitude =  $\pm 70^\circ$  there are 8 such neighboring locations. The array is scanned for unassigned locations, and those with 8 neighboring assigned locations are thus calculated. This is then repeated for 7, 6, etc. This lowering of the count of assigned neighboring locations is needed to dummy fill the large land area. Other considerations for selecting values of GM and GU can be optioned to selecting the value of the closest measurement to a grid point or performing a two-dimensional interpolation. The interpolation is apparently the best but also the more difficult, because for each grid point a value cannot be assigned until all the measurements are assigned. Considering the number of grid points and the millions of measurements required, this entails a rather immense accounting problem.

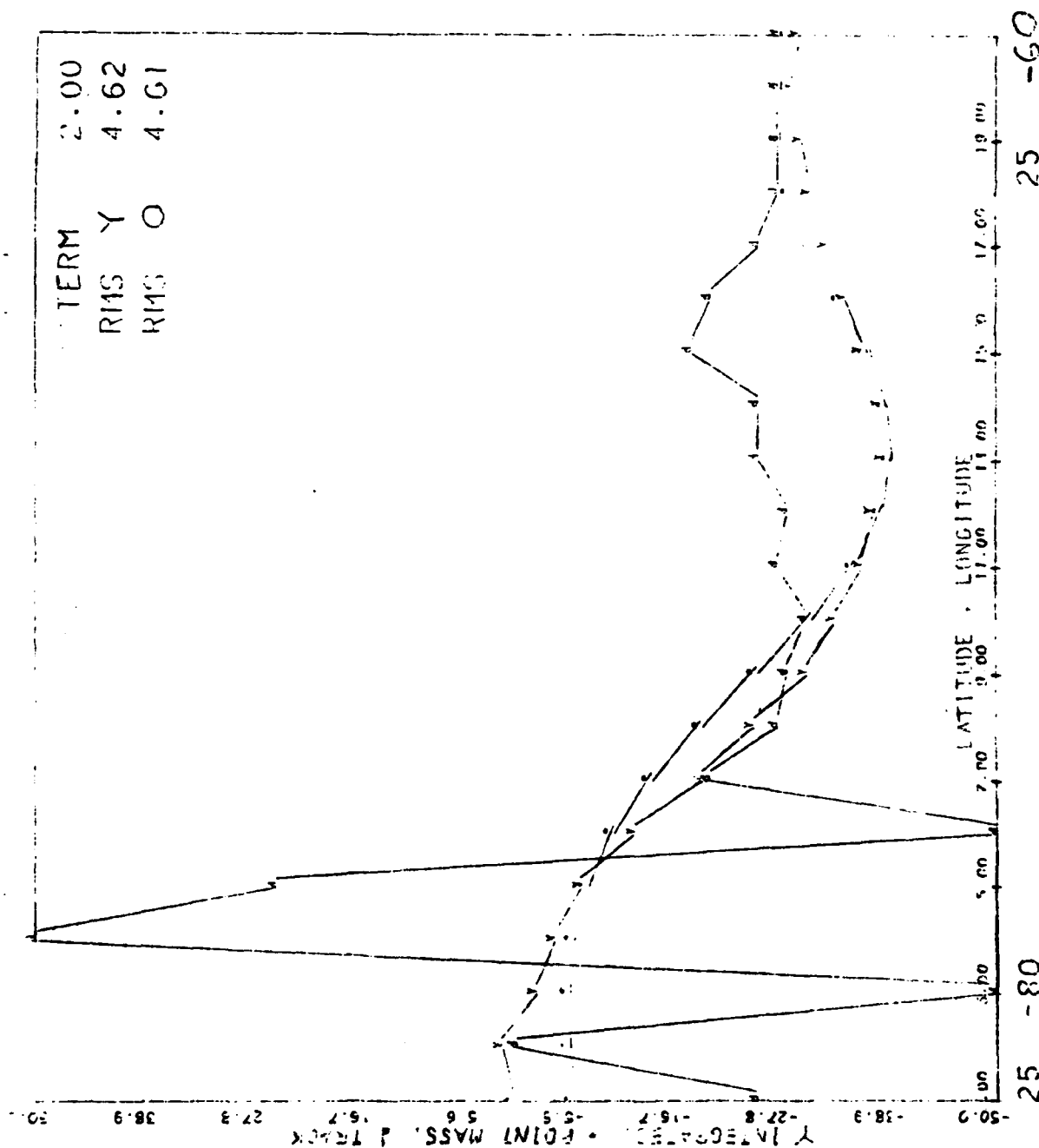


Figure 3. Profiles of gravity anomaly surface measurements, prior calculations made at AFGL using point mass technique and that obtained by using formula 5 (i=2).

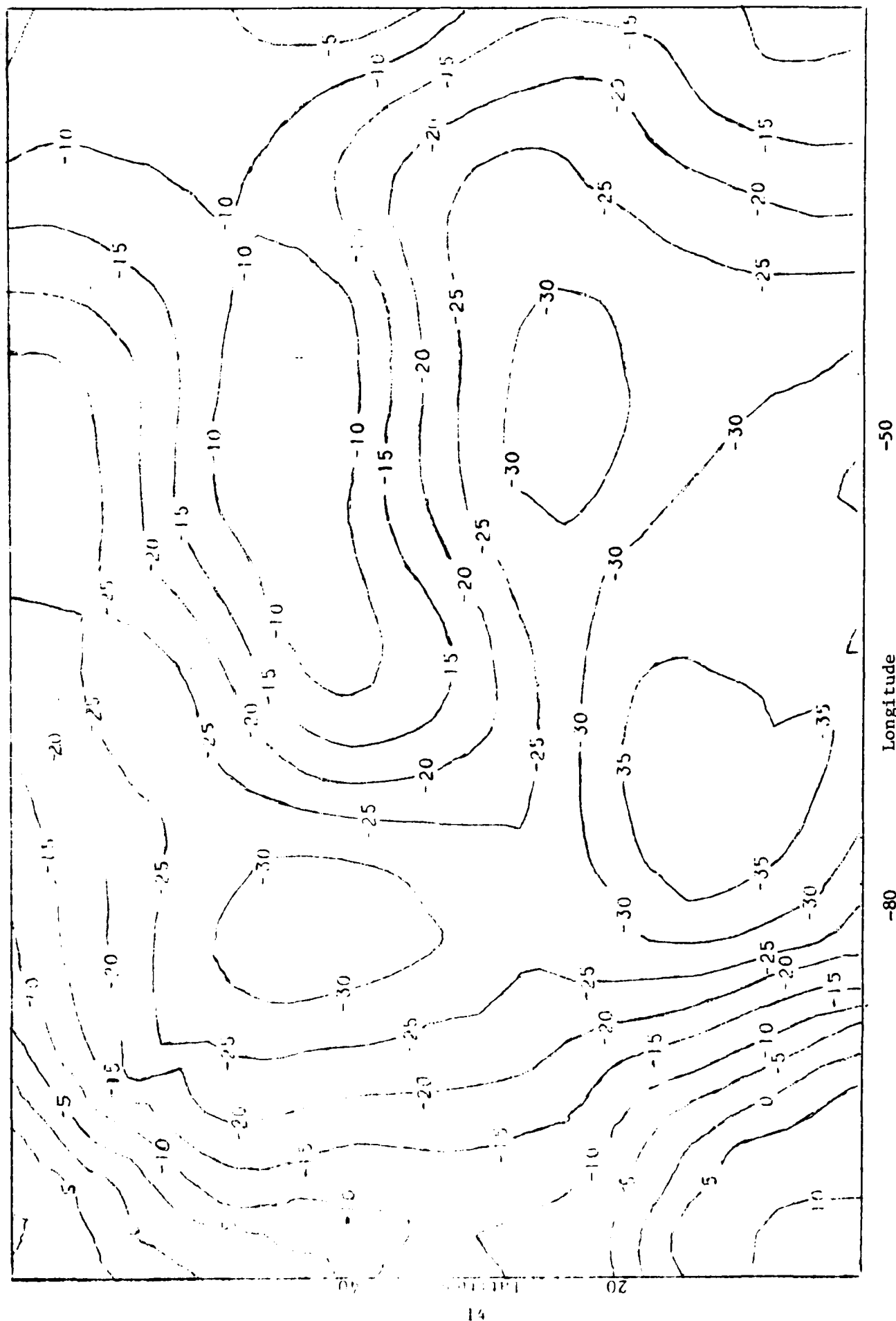


Figure 4. A contour plot of anomalous gravity obtained by using formula 5 ( $i=2$ ).

The following figures were selected as examples of supplemental studies.

Figure 5. An icosohedral plot of the AFGL deduced ocean geoid.

Figure 6. A cartesian plot of selected satellite tracks.

Figure 7. An isometric plot of bathymetry data.

#### F. Conclusion

Techniques have been developed for the computation of gravity anomalies corresponding to geoid anomalies deduced from satellite altimeter data. The computations have been done for gravity models which are functions of one and two dimensions along the surface of the ocean. The results are reasonable. Further improvement can be expected when more geoid anomaly data become available.

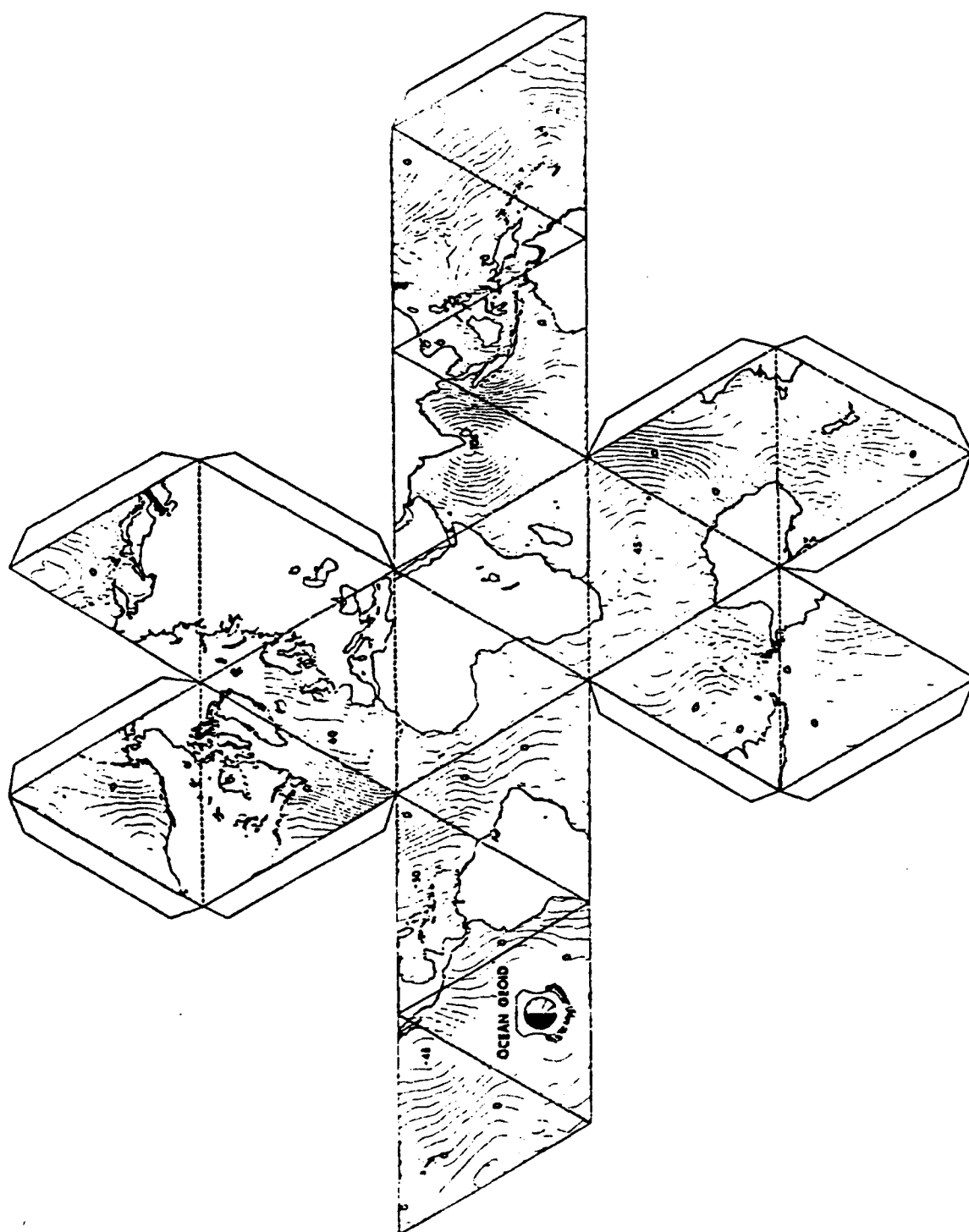


Figure 5. An icosahedral plot of the AFGL deduced ocean geoid.

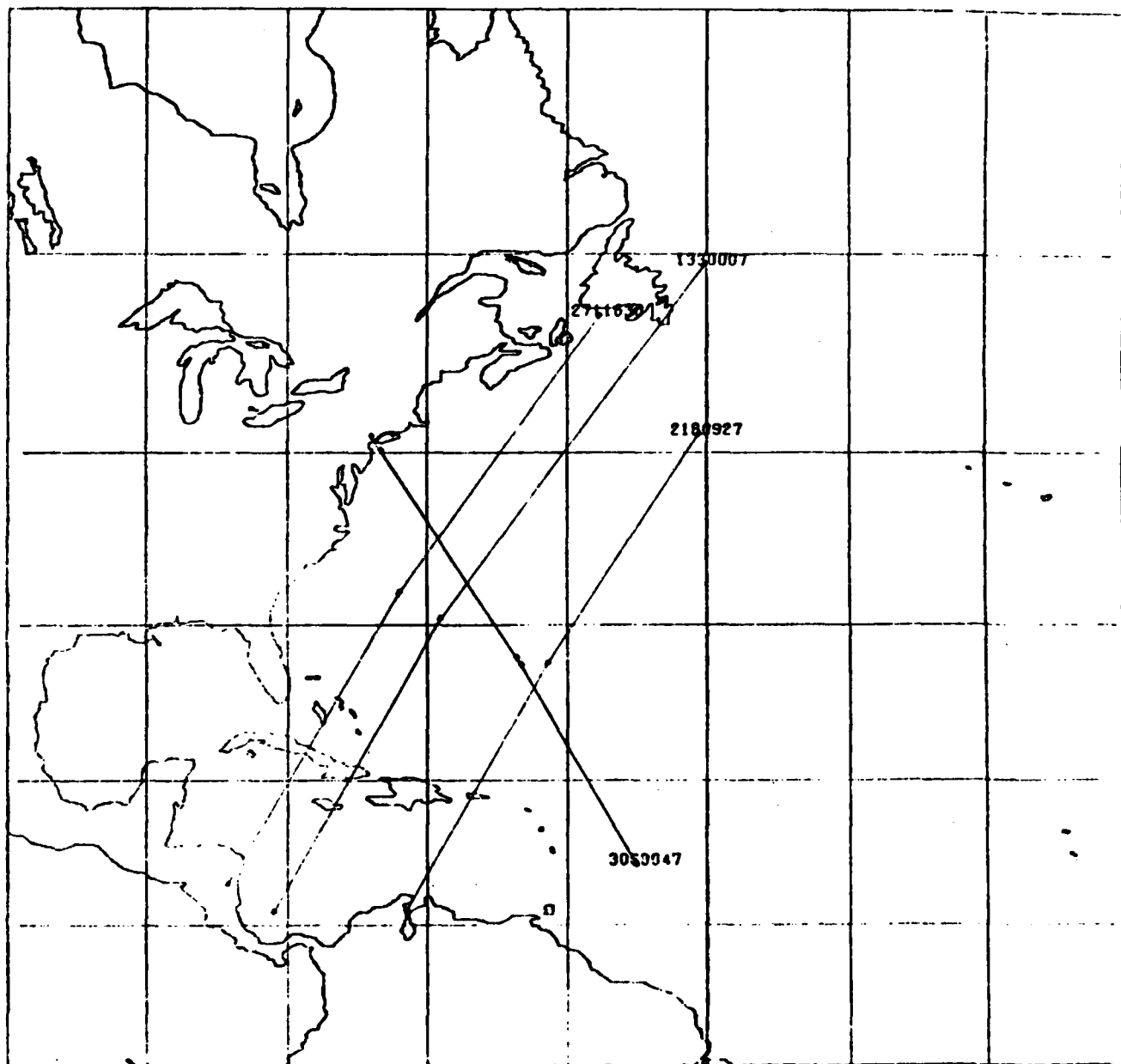


Figure 6. A cartesian plot of selected satellite tracks.

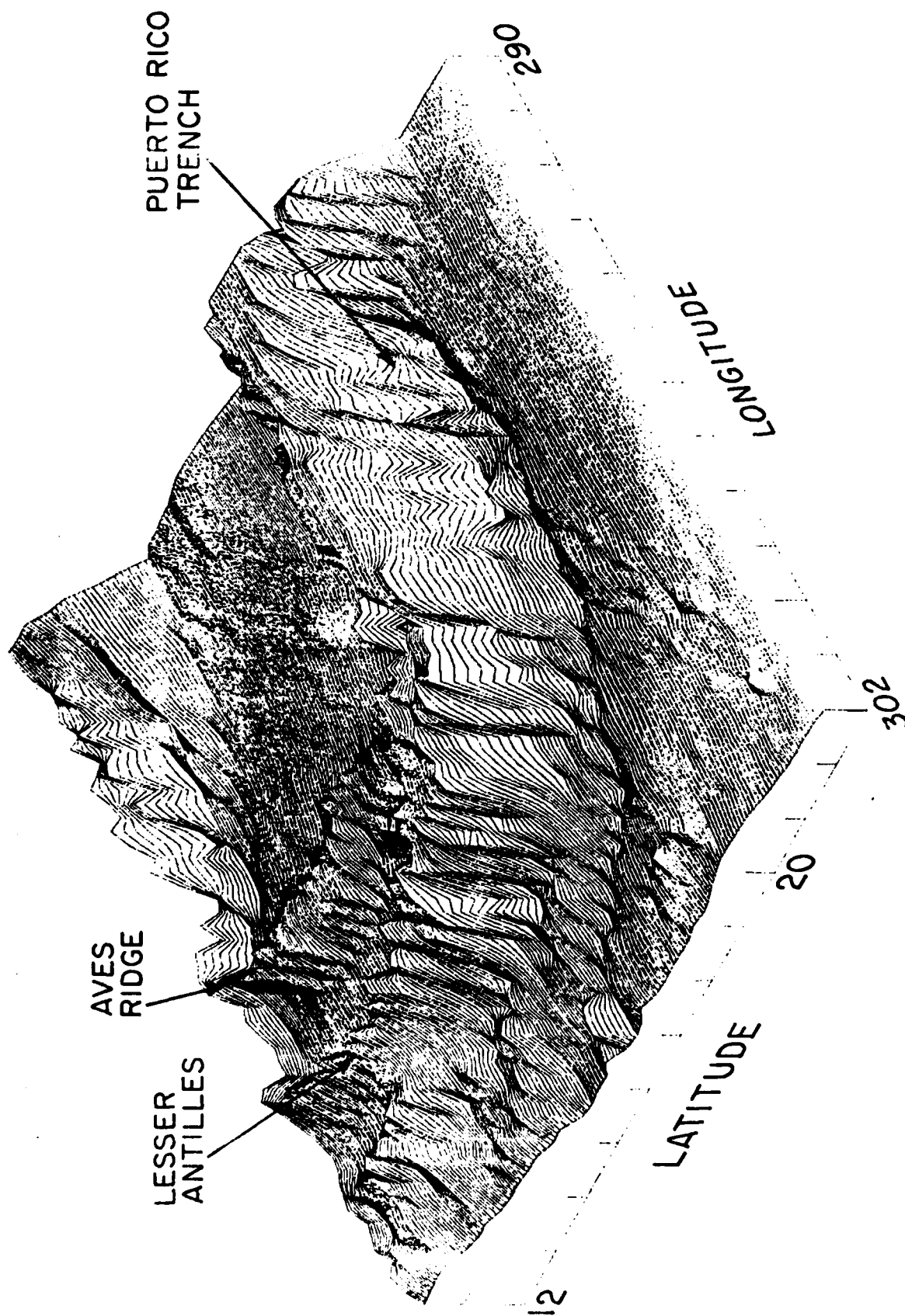


Figure 7. An isometric plot of bathymetry data.

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