







MICROCOPY RESOLUTION TEST CHART  
 NATIONAL BUREAU OF STANDARDS 1963-A

2

AD-A140 652

# NAVAL POSTGRADUATE SCHOOL

Monterey, California



DTIC  
ELECTE  
MAY 1 1984  
S B D

## THESIS

DTIC FILE COPY

SUPPRESSION OF FINITE-AMPLITUDE EFFECTS  
IN SLOSHING MODES IN CYLINDRICAL CAVITIES

by

Si Hwan Yum

December 1983

Thesis Advisor: Alan B. Coppens

Approved for public release; distribution unlimited

84 04 30 129

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A140652	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Suppression of Finite-Amplitude Effects in Sloshing Modes in Cylindrical Cavities		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis December 1983
7. AUTHOR(s) Si Hwan Yum		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE December 1983
		13. NUMBER OF PAGES 37
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) non linear wave equation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A perturbation expansion is formulated for the three dimensional, nonlinear, acoustic-wave equation with dissipative term describing the viscous and thermal energy losses encountered in a cylindrical cavity. The theoretical results show that nonlinear effects in sloshing modes are strongly suppressed.		

Approved for public release; distribution unlimited

Suppression of Finite-Amplitude Effects in Sloshing  
Modes in Cylindrical Cavities

by

Si Hwan Yum  
Lieutenant Commander, Republic of Korea Navy  
B.S., Republic of Korea Naval Academy, 1973

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING ACOUSTICS

from the

NAVAL POSTGRADUATE SCHOOL  
December 1983

Author:

*Yum, Si Hwan*

Approved by:

*Alan B. Oppus*

Thesis Advisor

*James V. Sandus*

Second Reader

*James V. Sandus*

Chairman, Engineering Acoustics Academic Committee

*John Dyer*

Dean of Science and Engineering

ABSTRACT

A perturbation expansion is formulated for the three-dimensional, nonlinear, acoustic-wave equation with dissipative term describing the viscous and thermal energy losses encountered in a cylindrical cavity. The theoretical results show that nonlinear effects in sloshing modes are strongly suppressed.



Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

## TABLE OF CONTENTS

I.	INTRODUCTION	6
II.	THE NON-LINEAR WAVE EQUATION	7
	A. GENERAL	7
	B. APPLICATION TO THE CYLINDRICAL CAVITY	10
	1. Symmetric Modes	11
	2. Non-Symmetric Modes	14
III.	METHOD OF SOLUTION	21
	A. POWER SERIES METHOD	23
	1. Coefficient	23
	2. $V_n(y)$	27
	3. $U_n(y)$	30
IV.	CONCLUSIONS	34
	LIST OF REFERENCES	36
	INITIAL DISTRIBUTION LIST	37

LIST OF SYMBOLS

$c$	phase velocity in the cavity
$c^2 \square^2$	$= c^2 \nabla^2 - \partial^2 / \partial t^2$
$c_0$	$(dp/d\rho)^{1/2}$ at $\rho = \rho_0$
$c_n$	effective phase speed associated with a standing wave at resonance
$c_p$	specific heat at constant pressure
$k$	$\omega/c$ , propagation constant associated with a standing wave
$M$	peak Mach number of the driven standing wave
$p$	$= p - p_0 = -\rho_0 \partial \Phi / \partial t$ = acoustic pressure
$p, p_0$	instantaneous and equilibrium total pressure in the field
$T_k$	absolute temperature in kelvin
$u$	particle velocity
$u_n$	particle speed of the $n^{\text{th}}$ -order perturbation solution
$\alpha$	infinitesimal-amplitude attenuation constant
$\alpha_n c_0$	temporal decay constant of a resonance
$\beta$	$= (\gamma + 1) / 2$ for a gas
$\gamma$	$= c_p / c_v$ , ratio of specific heats for a gas
$\rho, \rho_0$	instantaneous and equilibrium densities
$\Phi$	velocity potential
$\omega$	$= 2\pi f$ , (angular) frequency at which the cavity is driven
$\omega_n$	(angular) frequency of a resonance

## I. INTRODUCTION

The topic of finite amplitude acoustic standing waves in sloshing modes of a cylindrical cavity is interesting theoretically, but development of the subject has not been extensive. The purpose of this research is to present the results of a power series perturbation approach to the problem.

## II. THE NON-LINEAR WAVE EQUATION

### A. GENERAL

It is well known [1] that for  $\alpha/k$  and  $M \ll 1$ , where  $\alpha$  measures the fractional loss per wavelength and  $M$  is the peak Mach number of the source, loss terms and nonlinear terms in the constitutive equations can be separately approximated with the help of linear, lossless acoustic relations. The force equation appropriate for acoustical processes in systems for which gravitational effects are unimportant is

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + \frac{1}{\rho} \nabla p = \frac{1}{\rho} \mathcal{L} \vec{u} \quad (2-1)$$

where  $\vec{u} = u \hat{u}$  is the particle velocity,  $\rho$  is the instantaneous density of the fluid,  $p$  is the instantaneous total pressure in the field, and the operator  $\mathcal{L}$  symbolically describes those physical processes leading to absorption and dispersion. We used two additional equations. The first is the equation of state for a perfect gas

$$p = \rho r T_K \quad (2-1-a)$$

where  $r$  is a constant whose value depends on the particular gas involved and  $T_K$  is the absolute temperature in Kelvin. The second is the continuity equation

$$\frac{\partial s}{\partial t} + \nabla \cdot [(1+s)\bar{u}] = 0 \quad (2-1-b)$$

where  $s = (\rho - \rho_0) / \rho_0$  is the condensation at any point and  $\rho_0$  is the equilibrium density of the fluid. If we ignore rotational effects, then

$$\bar{u} = \nabla \Phi \quad (2-2)$$

where  $\Phi$  is the velocity potential. Combination of Eqs.(2-1)-(2-2) and the neglect of terms of orders higher than  $M^2$ ,  $M(\alpha/k)$ , and  $(\alpha/k)^2$ , yields a quadratically nonlinear wave equation,

$$\begin{aligned} (C_0^2 \square^2 + \frac{\partial}{\partial t} \mathcal{L}) \frac{p}{\rho_0 C_0^2} = & -\frac{1}{2} \frac{\partial^2}{\partial t^2} \left[ \gamma \left( \frac{p}{\rho_0 C_0^2} \right)^2 + \left( \frac{u}{C_0} \right)^2 \right] \\ & + \frac{1}{2} C_0^2 \nabla^2 \left[ \left( \frac{p}{\rho_0 C_0^2} \right)^2 - \left( \frac{u}{C_0} \right)^2 \right] \end{aligned} \quad (2-3)$$

where  $C_0^2 = (\partial p / \partial \rho)$  (adiabatic),  $p$ =acoustic pressure,  $u^2 = \bar{u} \cdot \bar{u}$ ,  $\gamma$  is the ratio of heat capacities, and

$$C_0^2 \square^2 = C_0^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \quad (2-4)$$

The left-hand side of Eq.(2-3) is the classical, linear wave equation with losses pertinent to the system under study.

The right-hand side can be interpreted as a forcing function consisting of a three-dimensional spatial distribution of phase-coherent sources.

In a second-order perturbation theory, this volume forcing function is obtained from the classical (first-order) solution  $P_1$  of the acoustical problem. The second-order perturbation solution  $P_1+P_2$  describes the non-linearities  $P_2$  resulting from the self interaction of the classical solution  $P_1$ .

Higher-order perturbation solutions consider the interaction of the nonlinear solution with itself, and the forcing function is composed of products of both classical and non-linearly generated terms. Thus, if a system is driven at frequency  $\omega$ , the non-linear term in Eq.(2-3) will force the existence of all integer multiples  $n\omega$  of the driving frequency and the full solution must contain all harmonics of the input frequency. In a closed cavity, each of those nonlinearly generated waves whose frequency lies near the resonance frequency of a standing wave of the cavity and whose associated spatial function matches that of the standing wave can be strongly excited [2] .

As far as the author has been able to determine, there has been only one previous study of this system published in the open literature. This was by Maslen and Moore in 1956 [3] . Their approach resulted in a series expansion which did not converge if the relevant normal mode frequencies were integrally related. Their interpretation of the quenching of the nonlinear effect was based on the "scattering effect of the wall".

We feel our interpretation based upon nonlinearly generated volume sources stimulating the allowed standing waves of the cavity is more accessible and informative. Further, the mathematical approach developed herein appear to avoid any difficulties in convergence. Their conclusion that high amplitude monofrequency transverse oscillations can exist is consistent with our finding.

#### B. APPLICATION TO THE CYLINDRICAL CAVITY

The circular cylinder is provided with a "point" source of sound. By properly positioning the cylinder with respect to the sound source it is possible to effectively drive the enclosed air into various modes of vibration. The rigid-walled cylinder forces the component of particle velocity perpendicular to each cavity surface to vanish at the surface. The resulting steady-state solution to the linear wave equation in cylindrical coordinates is

$$p(r, \theta, z) = A_{nm} \cos(k_z z) \cos(n\theta) \cos(\omega_{nm} t) J_n(k_{rnm} r) \quad (2-5)$$

where  $J_n$  are the cylindrical Bessel functions and application of the boundary condition to the sides yields

$$k_{rnm} = \frac{j_{nm}}{a} \quad (2-5-a)$$

where  $a$  is the radius of cylinder and  $J_{nm}$  are the arguments of the extrema of the  $n^{\text{th}}$  Bessel function.

The normal mode frequency is dependent on  $k_{zl}$  and  $k_{rnm}$ ,

$$k_{rnm}^2 + k_{zl}^2 = \frac{W_{nmk}^2}{C_0^2} \quad (2-5-b)$$

The standing wave will be identified by the ordered integers  $(n, m, l)$  describing its spatial dependence.

### 1. Symmetric Modes

The simplest waves in cylindrical coordinates are those that depend only on the distance  $r$  from Z-axis, the gradient takes the form

$$\nabla = \hat{y} k \frac{\partial}{\partial y} \quad (2-6)$$

where

$$y = kr \quad (2-6-a)$$

and  $k=w/c$  is the wave number or propagation constant.

The Laplacian becomes

$$\nabla^2 = k^2 \left( \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) \quad (2-6-b)$$

The suitable trial solution appropriate for symmetric modes is the linear solution  $(0, m, 0)$  of frequency  $w$

$$\frac{P_i}{\rho_0 c_0} = M J_0(y) \sin \omega t \quad (1)$$

The velocity potential for this limit can be defined as

$$\Phi = \left( \frac{c_0^2}{\omega} \right) M J_0(y) \cos \omega t \quad (2)$$

and  $\frac{\vec{u}_i}{c_0}$  is

$$\frac{\vec{u}_i}{c_0} = \hat{y} M J_0'(y) \cos \omega t = -\hat{y} M J_1 \cos \omega t \quad (3)$$

Because we assume that the surfaces of the cavity are rigid then at  $r=a$  the appropriate boundary condition is  $J_0' = 0$ .

Thus, we have  $ka = j_{0m}$ .

To generate the second order solution, we first note that

$$\left( \frac{P_i}{\rho_0 c_0^2} \right)^2 = \frac{1}{2} M^2 J_0^2 (1 - \cos 2\omega t) \quad (4)$$

and

$$\left( \frac{u_i}{c_0} \right)^2 = \frac{1}{2} M^2 J_1^2 (1 + \cos 2\omega t) \quad (5)$$

Next, the second derivatives of Eqs.(2-10) and (2-11) with respect to time are

$$\frac{\partial^2}{\partial t^2} \left( \frac{P_1}{\rho_0 c_0^2} \right)^z = \frac{(zw)^z}{z} M^z J_0^z \cos zw t \quad (2-12)$$

and

$$\frac{\partial^2}{\partial t^2} \left( \frac{U_1}{c_0} \right)^z = -\frac{(zw)^z}{z} M^z J_1^z \cos zw t \quad (2-13)$$

Substituting Eqs.(2-2) and (2-13) into the first term of the right-hand side of Eq.(2-3) yields

$$\frac{(zw)^z}{4} M^z \cos zw t \left[ J_1^z - \gamma J_0^z \right] \quad (2-14)$$

and the second term of right-hand side of Eq.(2-3) becomes

$$\frac{WM^z}{4} \left[ (-J_x^z + 4J_1^z - 3J_0^z) - (J_x^z - J_0^z) \cos zw t \right] \quad (2-15)$$

since

$$c_0^z \nabla^z J_0^z = zw^z (J_1^z - J_0^z) \quad (2-16)$$

or

$$c_0^z \nabla^z J_1^z = zw^z \left( \frac{1}{z} J_x^z + \frac{1}{z} J_0^z - J_1^z \right) \quad (2-17)$$

With the use of Eqs.(2-14) and (2-15) we can write the right-

hand side of Eq.(2-3) as

$$\frac{(2\omega)^2}{4} M^2 \left\{ \left[ -\frac{1}{4} J_2^2 + J_1^2 - \frac{3}{4} J_0^2 \right] + \left[ -\frac{1}{4} J_x^2 + J_1^2 - \left(\gamma - \frac{1}{4}\right) J_0^2 \right] \right. \\ \left. \cdot \cos z\omega t \right\} \quad (2-18)$$

and the appropriate inhomogeneous wave equation for the second order perturbation solution is

$$\left( \nabla^2 + \frac{\partial^2}{\partial t^2} \right) \frac{P_2}{\rho_0 c^2} = \frac{(2\omega)^2}{4} M^2 \left\{ \left[ -\frac{1}{4} J_2^2 + J_1^2 - \frac{3}{4} J_0^2 \right] + \left[ -\frac{1}{4} J_x^2 \right. \right. \\ \left. \left. + J_1^2 - \left(\gamma - \frac{1}{4}\right) J_0^2 \right] \cos z\omega t \right\} \quad (2-19)$$

## 2. Non-Symmetric Modes

The non planar waves in cylindrical coordinates are those that depend on the distance  $r$  from  $Z$  axis and the angle  $\varphi$  from  $X$ -axis. The gradient takes the form

$$\nabla = k \cdot \left( \hat{y} \frac{\partial}{\partial y} + \hat{\varphi} \frac{1}{y} \frac{\partial}{\partial \varphi} \right) \quad (2-20)$$

where

$$y = kr$$

and the Laplacian becomes

$$\nabla^2 = k^2 \left( \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} + \frac{1}{y^2} \frac{\partial^2}{\partial \varphi^2} \right) \quad (2-20-a)$$

The suitable trial solution appropriate to the forcing function of frequency  $w$  for exciting the  $(n, m, 0)$  standing wave is

$$\frac{P_1}{\rho_0 c_0^2} = M J_n(k_{nm} r) \cos n\varphi \sin wt, \quad k_{nm} = j'_{nm} / a \quad (2-21)$$

where  $J_n(kr)$  is the Bessel function of the first kind and order  $n$ . The velocity potential is approximated by

$$\Phi_1 = \left( \frac{c_0^2}{w} \right) M J_n(kr) \cos n\varphi \cos wt \quad (2-22)$$

and  $\vec{u}_1 / c_0$  is

$$\frac{\vec{u}_1}{c_0} = M \left( \hat{y} J_n' \cos n\varphi \cos wt - \hat{\varphi} \frac{n}{y} J_n \sin n\varphi \cos wt \right) \quad (2-23)$$

Thus,

$$\left( \frac{P_1}{\rho_0 c_0^2} \right)^2 = M^2 J_n^2 \cos^2 n\varphi \sin^2 wt \quad (2-24)$$

or

$$\left( \frac{P_1}{\rho_0 c_0^2} \right)^2 = \frac{1}{4} M^2 \left( - \cos zwt \right) (J_n^2 + J_n^2 \cos z n\varphi) \quad (2-25)$$

and

$$\left(\frac{U_1}{C_0}\right)^2 = \frac{1}{4} M^2 (1 + \cos \alpha \omega t) \left[ \frac{1}{2} (J_{n+1}^2 + J_{n-1}^2) - J_n J_{n-1} \cos 2n\theta \right] \quad (2-26)$$

and the second derivative of Eqs.(2-25) and (2-26) with respect to time is

$$\frac{\partial^2}{\partial t^2} \left( \frac{P_1}{\rho_0 c_0^2} \right)^2 = \frac{(\alpha \omega)^2}{4} M^2 J_n^2 (1 + \cos 2n\theta) \cos \alpha \omega t \quad (2-27)$$

and

$$\begin{aligned} \frac{\partial^2}{\partial t^2} \left( \frac{U_1}{C_0} \right)^2 = & -\frac{(\alpha \omega)^2}{4} M^2 \left\{ \left[ J_n'^2 + \left( \frac{\eta}{y} \right)^2 J_n'^2 \right] \right. \\ & \left. + \left[ J_n'^2 - \left( \frac{\eta}{y} \right)^2 J_n'^2 \right] \cos 2n\theta \right\} \quad (2-28) \end{aligned}$$

Substituting Eqs.(2-26) and (2-27) into the first term of Eq.(2-3) yields

$$\begin{aligned} \frac{(\alpha \omega)^2}{8} M^2 \cos \alpha \omega t \left\{ \left[ J_n'^2 + \left( \frac{\eta}{y} \right)^2 J_n'^2 - \gamma J_n'^2 \right] \right. \\ \left. + \left[ J_n'^2 - \left( \frac{\eta}{y} \right)^2 J_n'^2 - \gamma J_n'^2 \right] \cos 2n\theta \right\} \quad (2-29) \end{aligned}$$

or

$$\frac{(\alpha W)^2}{8} M^2 \cos \alpha W t \left\{ \left[ \frac{1}{\alpha} (J_{n+1}^2 + J_{n-1}^2) - \gamma J_n^2 \right] - \left[ J_{n+1} J_{n-1} + \gamma J_n^2 \right] \cos 2n\theta \right\} \quad (2-30)$$

since

$$J_n'^2 = \frac{1}{4} (J_{n+1}^2 + J_{n-1}^2 - \alpha J_{n+1} J_{n-1}) \quad (2-31)$$

and

$$\frac{\eta^2}{y^2} J_n^2 = \frac{1}{4} (J_{n+1}^2 + J_{n-1}^2 + \alpha J_{n+1} J_{n-1}) \quad (2-32)$$

In order to solve the second term of Eq.(2-3), we can use

$$C_0^2 \nabla^2 J_n^2 = \alpha W^2 \left( \frac{1}{\alpha} J_{n+1}^2 + \frac{1}{\alpha} J_{n-1}^2 - J_n^2 \right) \quad (2-33)$$

$$C_0^2 \nabla^2 J_n^2 \cos 2n\theta = \alpha W^2 (-J_{n+1} J_{n-1} - J_n^2) \cos 2n\theta \quad (2-34)$$

$$C_0^2 \nabla^2 J_{n+1} J_{n-1} \cos 2n\theta = \alpha W^2 (J_{n+1}' J_{n-1}' - J_n^2 - J_{n+2} J_{n-2} + J_{n+2} J_n) \cos 2n\theta \quad (2-35)$$

since

$$J_n'^2 + \left(\frac{\eta}{y}\right)^2 J_n^2 = \frac{1}{z} (J_{n+1}^2 + J_{n-1}^2) \quad (2-36)$$

and

$$J_n' - \left(\frac{\eta}{y}\right)^2 J_n^2 = -J_{n+1} J_{n-1} \quad (2-37)$$

Thus, Substituting Eqs.(2-33) through(2-35) into the second term of Eq.(2-3) yields

$$\begin{aligned} \frac{1}{z} \epsilon_0^2 \nabla^2 \left(\frac{p_1}{\epsilon_0 \epsilon_0^2}\right)^2 &= \frac{1}{4} M^2 \omega^2 (1 - \cos z\omega t) \left[ \left(\frac{1}{z} J_{n+1}^2 \right. \right. \\ &\left. \left. + \frac{1}{z} J_{n-1}^2 - J_n^2 \right) + (-J_{n+1} J_{n-1} - J_n^2) \cos 2\eta Q \right] \end{aligned} \quad (2-38)$$

and

$$\begin{aligned} \frac{1}{z} \epsilon_0^2 \nabla^2 \left(\frac{u_1}{\epsilon_0}\right)^2 &= \frac{1}{4} M^2 \omega^2 (1 + \cos 2\omega t) \left[ \frac{1}{z} \left(\frac{1}{z} J_{n+2}^2 \right. \right. \\ &\left. \left. - J_n^2 - J_{n+1}^2 + \frac{1}{z} J_{n-2}^2 - J_{n-1}^2 \right) + \left(\frac{1}{z} J_n^2 \right. \right. \\ &\left. \left. + \frac{1}{z} J_{n+2} J_{n-2} + J_{n+1} J_{n-1} \right) \cos 2\eta Q \right] \end{aligned} \quad (2-39)$$

With Eqs.(2-30), (2-38) and (2-39) we can get the inhomogeneous wave equation for the second order perturbation as

$$\begin{aligned}
 (\nabla^2 + \frac{\partial^2}{\partial t^2}) \frac{P_2}{\rho c^2} = & \frac{(2W)^2}{4} M^2 \left[ \frac{1}{4} (J_{n+1}^2 + J_{n-1}^2 - \frac{3}{2} J_n^2 \right. \\
 & - \frac{1}{4} J_{n+2}^2 - \frac{1}{4} J_{n-2}^2) + \frac{1}{4} (-2 J_{n+1} J_{n-1} \\
 & \left. - \frac{3}{2} J_n^2 - \frac{1}{2} J_{n+2} J_{n-2}) \cos 2\pi\eta \right] \\
 & + \frac{(2W)^2}{4} M^2 \cos \pi\omega t \left\{ \left[ -\frac{1}{16} J_{n+2}^2 + \frac{1}{4} J_{n+1}^2 \right. \right. \\
 & \left. \left. - \left( \frac{\gamma}{2} - \frac{1}{8} \right) J_n^2 + \frac{1}{4} J_{n-1}^2 - \frac{1}{16} J_{n-2}^2 \right] \right. \\
 & \left. - \left[ \frac{1}{8} J_{n+2} J_{n-2} + \frac{1}{2} J_{n+1} J_{n-1} + \left( \frac{\gamma}{2} \right. \right. \right. \\
 & \left. \left. \left. - \frac{1}{8} \right) J_n^2 \right] \cos 2\pi\eta \right\} \quad (2-40)
 \end{aligned}$$

If we let  $n=0$  in Eq.(2-40), it reduces to Eq.(2-19), as it must. Substituting  $n=1$  into Eq.(2-40) yields the equation with forcing term resulting from the  $(1, m, 0)$  sloshing mode,

$$\begin{aligned}
 (C_0^2 \square^z + \frac{\partial}{\partial t} \mathcal{L}) \frac{p_2}{\rho_0 C_0^2} &= \frac{(2W)^2}{4} M^z \left[ \frac{1}{4} \left( -\frac{1}{4} J_3^z + J_2^z \right. \right. \\
 &\quad \left. \left. - \frac{7}{4} J_1^z + J_0^z \right) + \frac{1}{4} \left( -\frac{3}{2} J_1^z + \frac{1}{2} J_3 J_1 \right. \right. \\
 &\quad \left. \left. - 2 J_2 J_0 \right) \cos 2\theta \right] + \frac{(2W)^2}{4} M^z \cos \pi W t \left\{ \right. \\
 &\quad \left[ -\frac{1}{16} J_3^z + \frac{1}{4} J_2^z - \left( \frac{\gamma}{2} - \frac{1}{16} \right) J_1^z + \frac{1}{4} J_0^z \right] \\
 &\quad \left. + \left[ \frac{1}{8} J_3 J_1 - \frac{1}{2} J_2 J_0 - \left( \frac{\gamma}{2} - \frac{1}{8} \right) J_1^z \right] \cos 2\theta \right\}
 \end{aligned}$$

(2-41)

### III. METHOD OF SOLUTION

Recall that the equation with forcing term resulting from the (1, m, 0) sloshing mode can be written as a function of frequency,

$$\frac{(2W)^2}{4} M^2 \cos zwt \left\{ \left[ -\frac{1}{16} J_3^2 + \frac{1}{4} J_x^2 - \left( \frac{\gamma}{z} - \frac{1}{16} \right) J_1^2 + \frac{1}{4} J_0^2 \right] \right. \\ \left. + \left[ \frac{1}{8} J_3 J_1 - \frac{1}{z} J_x J_0 - \left( \frac{\gamma}{z} - \frac{1}{8} \right) J_1^2 \right] \cos z\phi \right\} \quad (3-1)$$

and the left-hand side of Eq.(2-3) is

$$\left[ c_0^2 k^2 \left( \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \mathcal{L} \right] \frac{P_x}{\rho_0 c_0^2} \quad (3-1-a)$$

If the harmonics of the frequency at which the cavity is driven are not close to any resonant frequencies, we can ignore the lossy term. So, Eq.(3-1-a) can be written as

$$\left[ c_0^2 k^2 \left( \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial t^2} \right] \frac{P_x}{\rho_0 c_0^2} \quad (3-2)$$

Let us assume that the solution of Eq.(3-1) can be expressed as

$$\frac{P}{\rho_0 c_0^2} = M^2 \cos zwt \left[ U_m(y) + V_m(y) \cos z\phi \right] \quad (3-3)$$

and define

$$\frac{P_{nv}}{\rho_0 c_0^2} = M^2 V_n(kr) \cos z\omega t \cos z\omega t \quad (3-4)$$

and

$$\frac{P_{nu}}{\rho_0 c_0^2} = M^2 U_n(kr) \cos z\omega t \quad (3-5)$$

Combination of Eqs.(3-2) and (3-3) yields

$$\left[ \left( \frac{c_2 k}{zw} \right)^2 \left( \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) + 1 \right] U_n(y) = \frac{1}{4} \left[ -\frac{1}{16} J_{n+2}^z + \frac{1}{4} J_{n+1}^z \right. \\ \left. - \left( \frac{\gamma}{z} - \frac{1}{8} \right) J_n^z + \frac{1}{4} J_{n-1}^z - \frac{1}{16} J_{n-2}^z \right] \quad (3-6)$$

and

$$\left\{ \left( \frac{c_2 k}{zw} \right)^2 \left[ \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} - \left( \frac{2\gamma}{y} \right)^2 \right] + 1 \right\} V_n(y) = \frac{1}{4} \left[ -\frac{1}{8} J_{n+2} J_{n+2} \right. \\ \left. - \frac{1}{z} J_{n+1} J_{n-1} - \left( \frac{\gamma}{z} - \frac{1}{8} \right) J_n^2 \right] \quad (3-7)$$

A. POWER SERIES METHOD

In order to solve the right-hand side of Eq.(3-7), we can use the definition [4]

$$J_\nu(y)J_\mu(y) = \left(\frac{1}{z}y\right)^{\nu+\mu} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\nu+\mu+z k+1) \left(\frac{1}{z}y\right)^{2k}}{\Gamma(\nu+k+1) \Gamma(\mu+k+1) \Gamma(\nu+\mu+k+1) k!} \quad (3-8)$$

where  $\Gamma(z+1) = z!$

1. Coefficient

a. Substituting  $\nu = \mu = n+2$  into Eq.(3-8) yields

$$J_{n+2}^2(y) = \left(\frac{1}{z}y\right)^{2n+4} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k+4)! \left(\frac{1}{z}y\right)^{2k}}{\left[(n+k+2)!\right]^2 (2n+k+4)! k!} \quad (3-9)$$

or

$$J_{n+2}^2(y) = \sum_{k=0}^{\infty} B_{nk} \left(\frac{1}{z}y\right)^{2(n+k+2)} \quad (3-10)$$

where

$$B_{nk} = (-1)^k \frac{(2n+2k+4)!}{\left[(n+k+2)!\right]^2 (2n+k+4)! k!} \quad (3-11)$$

b. Substituting  $\nu = n+2, \mu = n-2$  into Eq.(3-8) yields

$$J_{n+2}(y)J_{n-2}(y) = \left(\frac{1}{z}y\right)^{2n} \sum_{k=0}^{\infty} \frac{(-1)^k (zn+2k)! \left(\frac{1}{z}y\right)^{2k}}{(n+k+z)! (n+k-z)! (2n+k)! k!} \quad (3-12)$$

or

$$J_{n+2}(y)J_{n-2}(y) = \sum_{k=0}^{\infty} C_{nk} \left(\frac{1}{z}y\right)^{2(n+k)} \quad (3-13)$$

where

$$C_{nk} = (-1)^k \frac{(zn+2k)!}{(n+k+z)! (n+k-z)! (2n+k)! k!} \quad (3-14)$$

c. Substituting  $\nu = n+1, \mu = n-1$  into Eq.(3-8) yields

$$J_{n+1}(y)J_{n-1}(y) = \left(\frac{1}{z}y\right)^{2n} \sum_{k=0}^{\infty} \frac{(-1)^k (zn+2k)! \left(\frac{1}{z}y\right)^{2k}}{(n+k+1)! (n+k-1)! (2n+k)! k!} \quad (3-15)$$

or

$$J_{n+1}(y)J_{n-1}(y) = \sum_{k=0}^{\infty} D_{nk} \left(\frac{1}{z}y\right)^{2(n+k)} \quad (3-16)$$

where

$$D_{n,k} = (-1)^k \frac{(2n+2k)!}{(n+k+1)! (n+k-1)! (2n+k)! k!} \quad (3-17)$$

d. Substituting  $\nu = \mu = n$  into Eq.(3-8) yields

$$J_n^2(y) = \left(\frac{1}{z}y\right)^{2n} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k)! \left(\frac{1}{z}y\right)^{2k}}{[(n+k)!]^2 (2n+k)! k!} \quad (3-18)$$

or

$$J_n^2(y) = \sum_{k=0}^{\infty} E_{n,k} \left(\frac{1}{z}y\right)^{2(n+k)} \quad (3-19)$$

where

$$E_{n,k} = (-1)^k \frac{(2n+2k)!}{[(n+k)!]^2 (2n+k)! k!} \quad (3-20)$$

e. Substituting  $\nu = \mu = n+1$  into Eq.(3-8) yields

$$J_{n+1}^2(y) = \left(\frac{1}{z}y\right)^{2n+2} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k+2)! \left(\frac{1}{z}y\right)^{2k}}{[(n+k+1)!]^2 (2n+k+2)! k!} \quad (3-21)$$

or

$$J_{n+1}^z(y) = \sum_{k=0}^{\infty} G_{nk} \left(\frac{1}{z}y\right)^{2(n+k+1)} \quad ($$

where

$$G_{nk} = (-1)^k \frac{(2n+2k+2)!}{[(n+k+1)!]^2 (2n+k+2)! k!} \quad ($$

f. Substituting  $\nu = \mu = n-1$  into Eq.(3-8) yields

$$J_{n-1}^z(y) = \left(\frac{1}{z}y\right)^{2n-2} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k-2)! \left(\frac{1}{z}y\right)^{2k}}{[(n+k-1)!]^2 (2n+k-2)! k!} \quad ($$

or

$$J_{n-1}^z(y) = \sum_{k=0}^{\infty} H_{nk} \left(\frac{1}{z}y\right)^{2(n+k-1)} \quad ($$

where

$$H_{nk} = (-1)^k \frac{(2n+2k-2)!}{[(n+k-1)!]^2 (2n+k-2)! k!} \quad ($$

g. Substituting  $\nu = \mu = n-2$  into Eq.(3-8) yields

$$J_{n-2}^2(y) = \left(\frac{1}{z}y\right)^{2n-4} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k-4)! \left(\frac{1}{z}y\right)^{2k}}{\left[(n+k-2)!\right]^2 (2n+k-4)! k!} \quad (3-27)$$

or

$$J_{n-2}^2(y) = \sum_{k=0}^{\infty} O_{nk} \left(\frac{1}{z}y\right)^{2(n+k-2)} \quad (3-28)$$

where

$$O_{nk} = (-1)^k \frac{(2n+2k-4)!}{\left[(n+k-2)!\right]^2 (2n+k-4)! k!} \quad (3-29)$$

## 2. $V_n(y)$

With the help of Eqs.(3-9) through(3-29) Eq.(3-7) can be written as

$$\left\{ \left(\frac{1}{za}\right)^2 \left[ \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} - \left(\frac{2n}{y}\right)^2 \right] + 1 \right\} V_n(y) \\ = \frac{1}{4} \sum_{k=0}^{\infty} \left[ -\frac{1}{8} C_{nk} - \frac{1}{z} D_{nk} - A E_{nk} \right] \left[ \left(\frac{1}{z}y\right)^{2(n+k)} \right] \quad (3-30)$$

where

$$A = \frac{\gamma}{z} - \frac{1}{8} \quad (3-30-a)$$

and

$$a = \frac{w}{C_2 k} \quad (3-30-b)$$

Now, let us assume that

$$V_n(y) = \frac{1}{4} \sum_{k=0}^{\infty} A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)} \quad (3-31)$$

The first derivative with respect to  $y = kr$  can be expressed as

$$\frac{dV_n}{dy} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k) A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-1} \quad (3-32)$$

and the second derivative with respect to  $y = kr$  can be expressed as

$$\frac{d^2 V_n}{d y^2} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k)(n+k-\frac{1}{z}) A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-2} \quad (3-33)$$

Substituting Eqs.(3-31) through(3-33) into the left-hand side of Eq.(3-30) yields

$$\sum_{k=0}^{\infty} \left\{ \left(\frac{1}{za}\right)^2 \left[ \frac{1}{4} (n+k)(n+k-\frac{1}{z}) A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-2} \right. \right.$$

$$\begin{aligned}
& + \frac{1}{4} \frac{1}{y} (n+k) A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-1} \\
& - \left(\frac{2n}{y}\right)^2 \frac{1}{4} A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)} \Big] + \frac{1}{4} A_{nk} \left(\frac{1}{z} y\right)^{2(n+k)} \quad (3-34)
\end{aligned}$$

Eq.(3-34) must be equal the right-hand side of Eq.(3-30). Thus,

$$\left(\frac{1}{za}\right)^2 (2n+k+1)(k+1) A_{n,k+1} + A_{nk} = -\left(\frac{1}{8} C_{nk} + \frac{1}{z} D_{nk} + A E_{nk}\right) \quad (3-35)$$

where  $k = 0, 1, 2, 3 \dots\dots\dots$  (3-36)

If Eq.(3-35) is substituted into Eq.(3-36), then Eq.(3-35) can be expressed as

$$\begin{aligned}
A_{n0} + \left(\frac{1}{za}\right)^2 (2n+1)(1) A_{n1} &= -\left(\frac{1}{8} C_{n0} + \frac{1}{z} D_{n0} + A E_{n0}\right) \\
A_{n1} + \left(\frac{1}{za}\right)^2 (2n+2)(2) A_{n2} &= -\left(\frac{1}{8} C_{n1} + \frac{1}{z} D_{n1} + A E_{n1}\right) \\
A_{n2} + \left(\frac{1}{za}\right)^2 (2n+3)(3) A_{n3} &= -\left(\frac{1}{8} C_{n2} + \frac{1}{z} D_{n2} + A E_{n2}\right) \\
&\vdots \\
&\vdots \\
&\vdots \\
A_{nk} + \left(\frac{1}{za}\right)^2 (2n+k+1)(k+1) A_{n,k+1} &= -\left(\frac{1}{8} C_{nk} + \frac{1}{z} D_{nk} + A E_{nk}\right) \quad (3-37)
\end{aligned}$$

The normal component of particle velocity is equal to zero on the boundaries. From the application of the boundary condition at the rigid boundary at  $r=a$ ,

$$\left. \frac{dV_n}{dy} \right|_{y=J} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k) A_{nk} \left(\frac{1}{2}j'\right)^{2(n+k)-1} = 0 \quad (3-38)$$

where

$$J'(ka) = dJ/d(ka)$$

Substituting Eq.(3-36) into Eq.(3-38) yields

$$\begin{aligned} n A_{n0} \left(\frac{1}{2}j'\right)^{2n-1} + (n+1) A_{n1} \left(\frac{1}{2}j'\right)^{2n+1} \\ + (n+2) A_{n2} \left(\frac{1}{2}j'\right)^{2n+3} + \dots + (n+k) A_{nk} \left(\frac{1}{2}j'\right)^{2(n+k)-1} \\ = 0 \end{aligned} \quad (3-39)$$

### 3. $U_n(y)$

With Eqs.(3-9) through(3-29), Eq.(3-6) can be expressed as

$$\begin{aligned} \left[ \left(\frac{1}{2a}\right)^2 \left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y}\right) + 1 \right] U_n(y) = \frac{1}{4} \sum_{k=0}^{\infty} \left[ -\frac{1}{16} B_{n,k-4} \left(\frac{1}{2}y\right)^{2(n+k-2)} \right. \\ \left. + \frac{1}{4} G_{n,k-3} \left(\frac{1}{2}y\right)^{2(n+k-2)} - A E_{n,k} \left(\frac{1}{2}y\right)^{2(n+k)} \right. \\ \left. + \frac{1}{4} H_{n,k+1} \left(\frac{1}{2}y\right)^{2(n+k-2)} - \frac{1}{16} O_{nk} \left(\frac{1}{2}y\right)^{2(n+k-2)} \right] \end{aligned} \quad (3-40)$$

where, as before,  $A = \gamma/2 - 1/8$  and  $a = \frac{w}{C_0 R}$

Now let us assume that

$$U_n(y) = \frac{1}{4} \sum_{k=0}^{\infty} b_{nk} \left(\frac{1}{z} y\right)^{2(n+k-2)} \quad (3-41)$$

The first derivative with respect to  $y = kr$  can be expressed as

$$\frac{dU_n}{dy} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k-2) b_{nk} \left(\frac{1}{z} y\right)^{2n+2k-5} \quad (3-42)$$

and the second derivative with respect to  $y = kr$  can be expressed as

$$\frac{d^2 U_n}{dy^2} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k-2)(n+k-\frac{5}{2}) b_{nk} \left(\frac{1}{z} y\right)^{2(n+k-3)} \quad (3-43)$$

Substituting Eqs.(3-41) through(3-43) into the left-hand side of Eq.(3-40) yields

$$\sum_{k=0}^{\infty} \left\{ \left(\frac{1}{za}\right)^2 \left[ (n+k)^2 - 4(n+k-1) \right] b_{nk} \left(\frac{1}{z} y\right)^{2(n+k-3)} + b_{nk} \left(\frac{1}{z} y\right)^{2(n+k-2)} \right\} \quad (3-44)$$

Eq.(3-44) must equal the right-hand side of Eq.(3-40).

Thus,

$$\begin{aligned} \left(\frac{1}{2a}\right)^2 (n+k-2)^2 b_{n,k+1} + b_{nk} = & \left(-\frac{1}{16} B_{n,k-4} + \frac{1}{4} G_{n,k-3}\right. \\ & \left. + \frac{1}{4} H_{n,k-1} - \frac{1}{16} \Theta_{n,k}\right) - AE_{n,k-2} \end{aligned} \quad (3-45)$$

where  $k = 1, 2, 3 \dots\dots\dots$

If Eq.(3-45) is substituted into Eq.(3-36), then Eq.(3-45) can be expressed as

$$\begin{aligned} b_{n0} + \left(\frac{1}{2a}\right)^2 (n-2)^2 b_{n1} &= -\frac{1}{16} \Theta_{n0} \\ b_{n1} + \left(\frac{1}{2a}\right)^2 (n-1)^2 b_{n2} &= -\frac{1}{16} \Theta_{n1} + \frac{1}{4} H_{n0} \\ b_{n2} + \left(\frac{1}{2a}\right)^2 (n)^2 b_{n3} &= -\frac{1}{16} \Theta_{n2} + \frac{1}{4} H_{n1} - AE_{n0} \\ b_{n3} + \left(\frac{1}{2a}\right)^2 (n+1)^2 b_{n4} &= -\frac{1}{16} \Theta_{n3} + \frac{1}{4} H_{n2} - AE_{n1} + \frac{1}{4} G_{n0} \\ b_{n4} + \left(\frac{1}{2a}\right)^2 (n+2)^2 b_{n5} &= -\frac{1}{16} \Theta_{n4} + \frac{1}{4} H_{n3} - AE_{n2} + \frac{1}{4} G_{n1} - \frac{1}{16} B_{n0} \\ &\vdots \\ &\vdots \\ &\vdots \\ b_{nk} + \left(\frac{1}{2a}\right)^2 (n+k-2)^2 b_{n,k+1} &= -\frac{1}{16} \Theta_{nk} + \frac{1}{4} H_{n,k-1} - AE_{n,k-2} + \frac{1}{4} G_{n,k-3} - \frac{1}{16} B_{n,k-4} \end{aligned}$$

(3-46)

Application of the boundary condition at  $r=a$  yields

$$\left. \frac{dU_r}{dy} \right|_{y=J'} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k-2) b_{nk} \left(\frac{1}{2}j'\right)^{2n+2k-5} = 0 \quad (3-47)$$

where

$$J'(ka) = dJ/d(ka)$$

Substituting Eq.(3-36) into Eq.(3-47) yields

$$\begin{aligned} & (n-2) b_{n0} \left(\frac{1}{2}j'\right)^{2n-5} + (n-1) b_{n1} \left(\frac{1}{2}j'\right)^{2n-3} \\ & + (n) b_{n2} \left(\frac{1}{2}j'\right)^{2n-1} + (n+1) b_{n3} \left(\frac{1}{2}j'\right)^{2n+1} \\ & + \dots + (n+k-2) b_{nk} \left(\frac{1}{2}j'\right)^{2n+2k-5} = 0 \end{aligned}$$

(3-48)

#### IV. CONCLUSIONS

Recall that we can compute  $A_{nk}$  from Eqs.(3-37) through(3-39) and Eqs.(3-14), (3-17), (3-20). Thus, with the use of Eq.(3-31), we can get  $V_n$ . From Eqs.(3-46) through(3-48) and Eqs.(3-11), (3-20), (3-23), (3-26), (3-29), we can compute  $b_{nk}$ . Thus, with the use of Eq.(3-14), we can get  $U_n$ . Therefore we can compute  $V_n(y)$  and  $U_n(y)$ . Let us calculate the finite amplitude effects resulting from the nonlinear distortion of a forced radial mode. If we excite a(0, m, 0) mode and obtain the pressure at the circumference, then  $y = k_0 a = j'_{0m}$ . Given this value of  $y$ , use of Eqs.(3-14), (3-17), (3-20) give the quantities C, D, E. From Eqs.(3-35) and (3-36) we can compute  $A_{nk}$ . Thus, we can get  $V_n(y)$  from Eq.(3-31). Let us calculate the finite amplitude effects resulting from the nonlinear distortion of a forced sloshing mode. If we excite a(1, m, 0) mode and obtain the pressure at the circumference, then  $y = k_0 a = j'_{1m}$ . Given this value of  $y$ , use of Eqs.(3-11), (3-20), (3-23), (3-26), (3-29) give the quantities B, E, G, H, O. From Eqs.(3-45) we can compute  $b_{nk}$ . Thus, we can get  $U_n(y)$  from Eq.(3-41). Substituting  $n=0$  at radial modes into  $V_n(y)$  and  $U_n(y)$  yields  $V_0(y)=U_0(y)$ . Therefore, with the use of Eq.(3-3), we can get the solution of Eq.(3-2).

##### A. RADIAL MODES(0, m, 0)

$$\frac{P_1}{\rho_0 c^2} = M J_0(k_0 m r) \sin \omega t$$

then

$$\frac{P_2}{\rho c^2} = z M^2 V_0(y) \cos zwt$$

so that

$$\left. \frac{|P_2|}{|P_1|} \right|_{r=a} = z M V_0(j'_{0m}) / J_0(j'_{0m})$$

B. SLOSHING MODES(1, m, 0)

$$\frac{P_1}{\rho c^2} = M J_1(k_{1m} r) \cos \theta \sin wt$$

then

$$\frac{P_2}{\rho c^2} = M^2 [V_0(y) + V_1(y) \cos 2\theta] \cos zwt$$

so that

$$\left. \frac{|P_2|}{|P_1|} \right|_{r=a} = M [V_0(j'_{1m}) + V_1(j'_{1m}) \cos 2\theta] / [J_1(j'_{1m}) \cos \theta]$$

## LIST OF REFERENCES

1. Blackstock, D. T., "Convergence of the Keck-Beyer Perturbation Solution for Plane Waves of Finite Amplitude in a Viscous Fluid", J. Acoust. Soc. Am., Vol. 39, No. 2, pp. 411-413, February 1966.
2. Coppens, A. B., and Sanders, J. V., "Finite-Amplitude Standing Waves Within Real Cavity", J. Acoust. Soc. Am., Vol. 58, No. 6, pp. 1133-1140, December 1975.
3. Maslen, S. H., and Moore, F. K., "On Strong Transverse Waves Without Shocks in a Circular Cylinder", Journal of the Aeronautical Sciences, Vol. 23, No. 6, pp. 583-593, June 1956.
4. Abramowitz, M. and Stegun, I. A., "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables", National Bureau of Standards Applied Mathematics Series, No. 55, pp. 255, 360, December 1965.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA 93943	2
3. Department Library, Code 61 Department of Physics Naval Postgraduate School Monterey, CA 93943	1
4. Professor Alan B. Coppens, Code 61Dz Department of Physics Naval Postgraduate School Monterey, CA 93943	5
5. Professor James V. Sanders, Code 61Sd Department of Physics Naval Postgraduate School Monterey, CA 93943	5
6. Chief of Naval Research 800 Quincy St. Arlington, VA 22217	1
7. LCDR Charles L. Burmaster, Code 61Zr Department of Physics Naval Postgraduate School Monterey, CA 93943	1
8. LCDR Chil-Ki Baek Republic of Korea Naval Academy Department of Physics Chin-Hae City, Seoul, Korea	5
9. The University of Texas at Austin Applied Research Laboratories Attn: D. T. Blackstock Austin, TX 78712	1

END

FILMED

6-84

DTIC







