

AD-A140745

RIA-84-U185

TECHNICAL
LIBRARY

AD A-140745

MEMORANDUM REPORT ARBRL-MR-03348

(Supersedes IMR No. 660)

MATHEMATICS OF MULTIDART
PROBABILITY PREDICTIONS

Lawrence D. Johnson

April 1984



US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

Approved for public release; distribution unlimited.

Destroy this report when it is no longer needed.
Do not return it to the originator.

Additional copies of this report may be obtained
from the National Technical Information Service,
U. S. Department of Commerce, Springfield, Virginia
22161.

The findings in this report are not to be construed as
an official Department of the Army position, unless
so designated by other authorized documents.

*The use of trade names or manufacturers' names in this report
does not constitute indorsement of any commercial product.*

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER MEMORANDUM REPORT ARBRL-MR-03348	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MATHEMATICS OF MULTIDART PROBABILITY PREDICTIONS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lawrence D. Johnson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory, ARDC ATTN: DRSMC-BLB(A) Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L162603AH18
11. CONTROLLING OFFICE NAME AND ADDRESS US Army AMCCOM, ARDC Ballistic Research Laboratory, ATTN: DRSMC-BLA-S(A) Aberdeen Proving Ground, MD 21005		12. REPORT DATE April 1984
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES This report supersedes BRL IMR-660, dated August 1979.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Probability Hit Multidart Patterns		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A general equation has been derived which can be used to predict the hit and kill probability of multidart single shot patterns, as well as single dart multishot patterns.		

TABLE OF CONTENTS

	Page
INTRODUCTION.	5
DERIVATION.	5
Patterns	5
Probability Derivation	6
SUMMARY	8
DISTRIBUTION LIST	9

INTRODUCTION

This note discusses the mathematics associated with predicting the hit and kill probabilities associated with multidart rounds. For the sake of brevity, we will dispense with the traditional discussion of definitions pertaining to fixed bias, variable bias, and round-to-round errors. Instead, we will proceed directly to the problem at hand.

DERIVATION

Patterns

The patterns that are assumed are basically circular. In the "hollow" pattern the darts have individual variations, but their mean points of impact are assumed to fall on the circumference of a circle of radius r , equidistant to each other. A "dense" pattern is a hollow pattern with an additional dart in the center of the circle. Figure 1 schematically depicts the patterns considered.

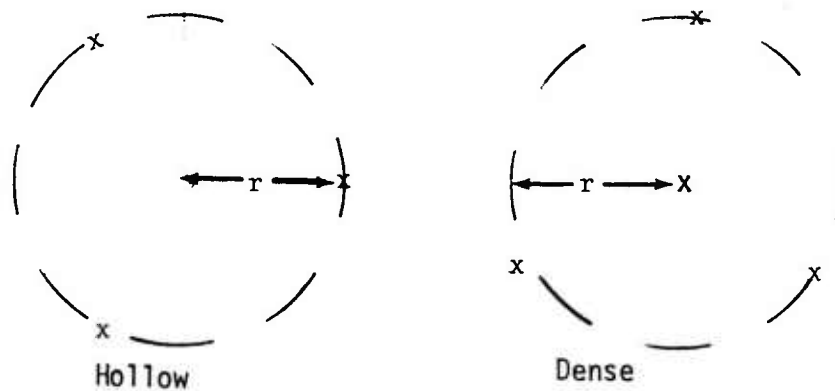


Figure 1. Patterns

Since the individual darts may vary about their mean point, they are not necessarily equidistant either from the center or from each other, for a particular event.

Probability Derivation

Let η_x, η_y represent the coordinates of the center of the circle. If the position of the center has a distribution similar to a single shot system,* its probability density function takes the form:

$$f(\eta_x, \eta_y) = \frac{1}{2\pi\sigma_{\eta_x}\sigma_{\eta_y}} \cdot \exp \left\{ -\frac{1}{2} \left[\frac{(\eta_x - \mu_{\eta_x})^2}{\sigma_{\eta_x}^2} + \frac{(\eta_y - \mu_{\eta_y})^2}{\sigma_{\eta_y}^2} \right] \right\} \quad (1)$$

where $\mu_{\eta_x}, \mu_{\eta_y} \triangleq$ fixed bias of system
 $\sigma_{\eta_x}^2, \sigma_{\eta_y}^2 \triangleq$ variance of the system.

Assuming that the individual darts have a normal distribution about their mean points, their probability density functions are given by

$$f_i(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \cdot \exp \left\{ -\frac{1}{2} \left[\frac{(x - g_{x_i}(r, \theta, \eta_x))^2}{\sigma_x^2} + \frac{(y - g_{y_i}(r, \theta, \eta_y))^2}{\sigma_y^2} \right] \right\} \quad (2)$$

where x, y = coordinates of the impact point

$g_{x_i}(r, \theta, \eta_x) = r \cos \left[\theta + \left(\frac{i-1}{N} \right) 2\pi \right] + \eta_x$; i.e., the mean point of impact for the i th dart in the x direction

$g_{y_i}(r, \theta, \eta_y) = r \sin \left[\theta + \left(\frac{i-1}{N} \right) 2\pi \right] + \eta_y$; the mean point of impact for the i th dart in the y direction

N = number of darts on the periphery of the circle of radius r .

i = the particular dart being examined

θ = angular orientation of the pattern

* This is a desirable assumption since it allows for the degenerate case, i.e., a dense pattern having only one dart.

It should be noticed that for a dense pattern, the center dart has the density function defined by Equation (1). Also the density function for the pattern's orientation is assumed to be uniformly distributed,* i.e.,

$$\tilde{f}(\theta) = \frac{1}{2\pi} . \quad (3)$$

The probability of missing a target with all darts in the pattern for a given impact center η_x, η_y and orientation θ is

$$P_{M|\eta_x, \eta_y, \theta} = \left[1 - \bar{P}(\eta_x, \eta_y)\right]^j * \prod_{i=1}^N \left[1 - \int \int_A f_i(x, y) dx dy\right] \quad (4)$$

where $\bar{P}(\eta_x, \eta_y) = 1.0$ if point η_x, η_y is on target area and equals zero if not

$j = 0$ for hollow pattern and 1 for dense pattern

A = presented area of target.

Since all possible center of circle coordinates and orientations must be accounted for,

$$P_M = \int_0^{2\pi} \int_{-\infty}^{\infty} \left\{ \left[1 - \bar{P}(\eta_x, \eta_y)\right]^j \prod_{i=1}^N \left[1 - \int \int_A f_i(x, y) dx dy\right] \right\} f(\eta_x, \eta_y) f(\theta) d\eta_x d\eta_y d\theta \quad (5)$$

and finally, the probability of hitting the target at least once is

$$P_H = 1 - P_M . \quad (6)$$

The probability of killing the target follows the same lines except that the density functions $f_i(x, y)$ must be multiplied by the probability of killing the target given an impact point x, y and $\bar{P}(\eta_x, \eta_y)$ is multiplied by a similar conditional kill value.

* It is currently envisioned that the orientation of the round, when chambered, will be arbitrary; thus, the orientation of the pattern is assumed to be uniformly distributed.

It is of interest to note that Equations (5) and (6) are merely generalizations of the probability of hitting a target firing N rounds with a single shot system. In this case, $i = r = 0$ and η_x, η_y are the variable biases of the system. Under these conditions Equation (5) reduces to

$$P_M = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \int_A f(x,y) dx dy \right]^N f(\eta_x, \eta_y) d\eta_x d\eta_y \quad (7)$$

SUMMARY

A general equation has been derived which predicts the hit and kill probability of multidart single shot patterns and single dart multishot patterns.

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
12	Administrator Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22314	1	Commander US Army Aviation Research and Development Command ATTN: DRDAV-E 4300 Goodfellow Boulevard St. Louis, MO 63120
1	Commander US Army Materiel Development and Readiness Command ATTN: DRCDMD-ST 5001 Eisenhower Avenue Alexandria, VA 22333	1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
1	Commander Armament R&D Center US Army AMCCOM ATTN: DRSMC-TDC(D) Dover, NJ 07801	1	Commander US Army Communications Research and Development Command ATTN: DRSEL-ATDD Fort Monmouth, NJ 07703
1	Commander Armament R&D Center US Army AMCCOM ATTN: DRSMC-LCS-D, Mr. Rubin Dover, NJ 07801	1	Commander US Army Electronics Research and Development Command Technical Support Activity ATTN: DELSD-L Fort Monmouth, NJ 07703
2	Commander Armament R&D Center US Army AMCCOM ATTN: DRSMC-TSS(D) Dover, NJ 07801	1	Commander US Army Missile Command ATTN: DRSMI-R Redstone Arsenal, AL 35898
1	Commander US Army Armament, Munitions & Chemical Command ATTN: DRSMC-LEP-L(R) Rock Island, IL 61299	1	Commander US Army Missile Command ATTN: DRSMI-YDL Redstone Arsenal, AL 35898
1	Director Benet Weapons Laboratory Armament R&D Center US Army AMCCOM ATTN: DRSMC-LCB-TL(D) Watervliet, NY 12189	1	Commander US Army Tank Automotive Command ATTN: DRSTA-TSL Warren, MI 48090
		1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL White Sands Missile Range NM 88002

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
1	Commander US Army Armor Center & Fort Knox ATTN: ATZK-CD-MS Fort Knox, KY 40121		<u>Aberdeen Proving Ground</u> Dir, USAMSAA ATTN: DRXSY-D DRXSY-MP, Mr. H. Cohen Mr. W. Brooks
1	President US Army Armor & Engineer Board ATTN: ATZK-AE-AO-L Fort Knox, KY 40121		Cdr, USATECOM ATTN: DRSTE-TO-F Dir, CRDC ATTN: DRSMC-CLB-PA DRSMC-CLN DRSMC-CLJ-L
2	Commandant US Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905		
1	Director DARCOM-MSA-ADEA ATTN: DRXTB-T, MAJ Martin Fort Lewis, WA 98433		
1	AFWL/SUL Kirtland AFB, NM 87117		
1	Commander US Naval Surface Weapon Ctr ATTN: G31 Dahlgren, VA 22448		
1	Commander Marine Corps Development and Education Command ATTN: Plans & Studies Div Quantico, VA 22134		

USER EVALUATION OF REPORT

Please take a few minutes to answer the questions below; tear out this sheet, fold as indicated, staple or tape closed, and place in the mail. Your comments will provide us with information for improving future reports.

1. BRL Report Number _____

2. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which report will be used.)

3. How, specifically, is the report being used? (Information source, design data or procedure, management procedure, source of ideas, etc.) _____

4. Has the information in this report led to any quantitative savings as far as man-hours/contract dollars saved, operating costs avoided, efficiencies achieved, etc.? If so, please elaborate.

5. General Comments (Indicate what you think should be changed to make this report and future reports of this type more responsive to your needs, more usable, improve readability, etc.) _____

6. If you would like to be contacted by the personnel who prepared this report to raise specific questions or discuss the topic, please fill in the following information.

Name: _____

Telephone Number: _____

Organization Address: _____

