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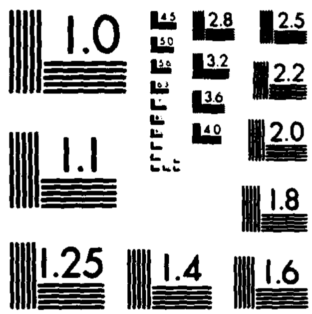

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A MATERIAL MODEL FOR REVERSE YIELDING AND ITS APPLICATION TO TORSION

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PETER C. T. CHEN

MARCH 1984

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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A theoretical model for reverse yielding is proposed with an attempt to give a better representation for some material such as a high strength steel. The Bauschinger effect factor is treated as a function of overstrain. The strain-hardening effect is taken into account with different parameters used for forward and reversed loading processes. The application of this model to the torsion problem in a cylindrical bar is shown.

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INTRODUCTION

In recent years the interest in plastic deformation has increased considerably in research as well as in practical applications. The importance of realistic constitutive equations has been emphasized in several research workshops (refs 1-3), and many computer codes have been developed as seen in a recent survey paper (ref 4). According to time-independent plasticity theory, the response of a strain-hardening material is specified by an initial yield condition, a hardening rule, and a flow rule. While there is general agreement in the literature over which initial yield condition and flow rule should be used, there is no such accord with regard to the hardening rule. Most hardening rules in present use are well documented and reviewed in the literature (ref 5). The stress-strain curves in Figures 1 and 2 illustrate the fact that all of the theories are capable of treating the monotonic loading situation. For reversed loading, it has been concluded that kinematic and isotropic hardening models represent the limits of the actual behavior, whereas the remaining theories are capable of falling anywhere within these limits (refs 5,6). This is certainly true for many strain-hardening materials as shown in Figure 1, but not true for some small strain-hardening materials as shown in Figure 2. The experimental works by Milligan, Koo, and Davidson (ref 7) and Swift (ref 8) demonstrate that the stress-strain curves for a high strength steel and six spring steels behave like that shown in Figure 2. None of the existing theories can represent this material behavior reasonably well for two reasons. First, the initiation of reverse yielding occurs much

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References are listed at the end of this report.

earlier than predicted by the theories. Second, the assumption of same slope for forward and reversed loading is not valid.

In this report a theoretical model is proposed with an attempt to give a better representation of the actual material behavior especially that shown in Figure 2 (refs 7,8). The Bauschinger effect factor is treated as a function of overstrain. The strain-hardening effect is taken into account with different parameters used for forward and reversed loading processes. The application of this model to the torsion problem in a cylindrical bar is reported.

THEORETICAL MODEL

The stress-strain curve during loading for a small strain-hardening material can be replaced with sufficient accuracy by a bilinear elastic-plastic model as shown in Figure 3. For the plastic portion, the yield shear stress τ is related to the plastic shear strain γ^p by

$$\tau/\tau_o = 1 + m\zeta/(1-m) \text{ and } \zeta = (G/\tau_o)\gamma^p \qquad (1)$$

where τ_o is the initial yield shear stress, G is the shear modulus, and mG is the slope of shear stress-strain curve in the plastic portion.

In most of the plasticity theories, the curve of reverse loading is uniquely defined by the curve of the first loading. The present model does not assume such a relationship. The experimental stress-strain curve during reversed loading will be used directly. A piecewise linear representation can be used, but only a bilinear approximation is chosen here as shown in Figure 3. Choosing a new coordinate system (τ', γ') with origin at the point before unloading, we have for the plastic portion of the reverse yielding curve

$$\tau'/\tau_o = \tau_o'/\tau_o + m'\zeta'/(1-m') \text{ and } \zeta' = (G/\tau_o)\gamma'^P \qquad (2)$$

where τ_o' is the linear drop in shear stress until reverse yielding begins, $m'G$ is the slope of the reverse yielding curve, and γ'^P is the additional plastic shear strain during reversed loading. Taking into account strain-hardening, the definition of the Bauschinger effect factor (BEF) is

$$BEF = (\tau_o'-\tau_1)/\tau_1 = f(\zeta_1) \qquad (3)$$

where τ_1 is the shear stress just before unloading occurs. The Bauschinger effect factor is a function of prestrain as shown in Figure 4. This information is available from the experimental data for a high strength steel (ref 7).

According to equations (1) and (3), τ_o' can be expressed as a function of plastic strain (ζ_1) just prior to unloading by

$$\tau_o'/\tau_o = [1 + m\zeta/(1-m)][1 + f(\zeta)] = g(\zeta) \qquad (4)$$

The present model is quite general because different parameters (m,m') and functions $f(\zeta)$ can be used. It seems interesting to discuss two special cases of the present material model:

(a) $m = m'$, $f = 1$: reduces to isotropic hardening model.

(b) $m = m'$, $g = 2$: reduces to kinematical hardening model.

TORSION BARS

For a circular bar under torsional loading, the external torque M can be computed by

$$\bar{M} = M/Mo = 4 \int_0^1 (\tau/\tau_o)\xi^2 d\xi \qquad (5)$$

where $M_0 = \pi/2\ a^3\tau_0$, $\xi = r/a$, τ is the current shear-stress corresponding to location r, and a is the radius of the cross-section. Geometric considerations show that radial lines have to remain straight after deformation. Thus, one concludes that

$$\gamma = \gamma_a \xi = \alpha r \qquad (6)$$

where γ_a is the strain at the outermost fiber and α is the angle of twist per unit length. Since a yield stress is introduced, an elastic core always exists during deformation whose radius c is given by

$$c/a = \gamma_0/\gamma_a \quad \text{and} \quad \gamma_0 = \tau_0/G \qquad (7)$$

If a material is linear strain hardening, a closed-form loading solution in the elastic-plastic range can be obtained (ref 9). The plastic shear strain and shear stress are given by

$$\zeta = (1-m)(\gamma/\gamma_0 - 1) \qquad (8)$$

$$\tau/\tau_0 = (1-m) + m\gamma/\gamma_0 \qquad (9)$$

Using equations (5) through (9), we can compute the torque. The result is

$$\bar{M} = \frac{ma}{c} + \frac{1-m}{3}(4-c^3/a^3) \qquad (10)$$

Note that $\bar{M} = 1$ for c = a (initial yielding), and $\bar{M} = (1/3)(4-c^3/a^3)$ for perfectly plastic material.

REVERSED LOADING

If we remove the external torque after the bar is twisted beyond the elastic limit, there is a residual angle of twist (α''). The ratio of the residual angle to the original angle of twist (α) is defined as the spring-back ratio (α''/α). Assuming complete elastic unloading (ref 9), we can determine α', and the spring-back ratio $(\alpha-\alpha')/\alpha$ is given by

$$\alpha''/\alpha = (1-m)[1 - \frac{4}{3}\frac{\gamma_o}{\gamma_a} + \frac{1}{3}(\frac{\gamma_o}{\gamma_a})^4] \qquad (11)$$

This formula for spring-back ratio can be used if reverse yielding will not occur. It can be shown that this is true when the following inequality is satisfied:

$$\frac{ma}{c} + \frac{1-m}{3}(4 - \frac{c^3}{a^3}) < g(\zeta_a) \qquad (12)$$

where ζ_a is the dimensionless plastic shear strain at the outermost fiber just prior to unloading. The upper limit for α or γ and the corresponding M can be found from the above inequality.

As an example, when $m = 0.1$, $c/a = 0.1$, $g = 2$ (kinematic hardening), inequality (12) is not satisfied, i.e., reverse yielding occurs during the removal of the external torque $M = 2.1996\ M_o$.

The occurrence of reverse yielding depends on the initial loading, reversed loading, and the material model. Even if reverse yielding may not occur during the removal of external torque, it may still occur during the reversed loading. Let α' be the unit angle of twist in the reversed direction during reversed loading. Geometric considerations again lead to

$$\gamma' = \gamma_a'\xi = \alpha' r \qquad (13)$$

where γ_a' is the shear strain at the outermost fiber during reversed loading. Let d be the elastic-plastic boundary. In the elastic zone $(r < d)$, we have $\gamma'^P = 0$ and $\tau' = G\gamma'$. In the plastic zone $(r < d)$, the shear stress and plastic shear strain during reverse loading can be computed by

$$\tau'/\tau_o = (1-m')g(\zeta) + m'\gamma'/\gamma_o \qquad (14)$$

and

$$\zeta' = (1-m')[\gamma'/\gamma_o - g(\zeta)] \qquad (15)$$

At the elastic-plastic boundary d, $\zeta_d' = 0$ and $\zeta_d = (1-m)(\gamma_d/\gamma_o - 1)$ which lead to

$$\gamma_d'/\gamma_o = g(\zeta_d) \qquad (16)$$

$$\zeta_d = (1-m)[(d/a)(\gamma_a/\gamma_o) - 1] \qquad (17)$$

Using equations (13), (16), and (17), we obtain

$$\frac{d}{a} = \frac{g(\zeta_d)}{\gamma_a'/\gamma_o} = \frac{1 + \zeta_d/(1-m)}{\gamma_a/\gamma_o} \qquad (18)$$

Therefore, we can compute γ_d and d for given values of γ_a and γ_a'. The ratio γ_a'/γ_a is related to ζ_d,

$$\gamma_a'/\gamma_a = \alpha'/\alpha = g(\zeta_d)/[1 + \zeta_d/(1-m)] \qquad (19)$$

Now we can calculate the external torque M' during reversed loading by

$$M' = M'/M_o = (\gamma_a'/\gamma_o)[m' + (1-m')(d/a)^4] + (1-m') \int_{d/a}^{1} g(\zeta)\xi^2 d\xi \qquad (20)$$

If

$$f(\zeta) = f_o + f_1\zeta + \ldots + f_{n-1}\zeta^{n-1}$$

then

$$g(\zeta) = g_o + g_1\zeta + \ldots + g_n\zeta^n \qquad (21)$$

Since ζ is linear in ξ, we can carry out the integration in equation (20) explicitly in terms of d/a, m, and γ_a/γ_o. In general, M' is a function of γ_a and γ_a' with parameters a, G, τ_o, m, m', f_o, $f_1, \ldots f_{n-1}$. The dependence of M' on d/a has been eliminated through equations (18) and (19).

NUMERICAL RESULTS AND DISCUSSIONS

Consider a circular bar which was twisted to reach $\gamma_a = 10\gamma_o$ where $\gamma_o = \tau_o/G$. The elastic-plastic boundary was calculated by equation (7) to be c/a = 0.1. The applied external torque can be calculated by equation (10); and the results are M/M_o = 1.333, 1.41967, 2.1996 for m = 0.0, 0.01, 0.1, respectively, and $M_o = \pi/2 \; a^3 \tau_o$.

During the removal of the external torque, the occurrence of reverse yielding still depends on the material model. If the kinematic hardening model is used with m = m' = 0.1, then reverse yielding will occur because inequality (12) is violated. By equating M' to M as given by equations (10) and (20), and solving for the elastic-plastic boundary, we have d/a = 0.8910. The residual shear strain is $\gamma_a' = 2.2446\gamma_o$ and the spring-back ratio during elastic-plastic unloading is 0.77554. According to equation (11) on the assumption of elastic unloading, the spring-back ratio would be 0.78003. If m = m' = 0.01 is used with either kinematic (g = 2) or isotropic (f = 1) hardening models, reverse yielding will not occur during unloading. However, reverse yielding may still occur if the Bauschinger effect factor is small, e.g., f = 0.3.

When the external torque is removed and then applied in the reversed direction, reverse yielding will occur no matter what material models are used. Different material models predict quite different results. Let us consider only the case of $\gamma_a = \gamma_a' = 10\gamma_o$ with m = 0.01. The external torque to reach the initiation of reverse yielding will be -0.76033, -0.58033, -0.24633 M_o according to isotropic hardening, kinematic hardening, the present model with f = 0.4, respectively. Further reversed loading depends also on the slope of strain-hardening. According to either the isotropic or the kinematic hardening model, the slope during reverse loading is the same as during loading, i.e., $m' = m = 0.01$. The present model does not make such an assumption. Different slopes such as $m' = 0.1, 0.3$ can be used, and $f(\zeta)$ can be a general function of prestrain. To reach $\gamma_a' = \gamma_a = 10\gamma_o$ (twist the bar forward and backward in the same amount), we need to apply the external torque in the reversed direction. The predicted results are -1.31504, -1.48844 M_o according to the kinematic hardening ($g = 2$) and isotropic hardening ($f = 1$) models, respectively. If the Bauschinger effect factor $f = 0.4$ is used with $m' = 0.01, 0.1, 0.3$, then the external torque will be -0.64716, -1.3686, -2.97104 M_o, respectively. Therefore, a high strength steel, with $m = 0.01$, $f = 0.4$, and $m' = 0.3$ requires a much larger torque in the reversed direction (-2.97104 M_o) than that predicted by the other models, even though the initiation of reverse yielding occurs at a much smaller torque (-0.24633 M_o) than the other predictions.

Since different material models predict quite different results, reverse loading experiments are needed for the characterization of the material behavior and for the comparison with the theoretical predictions.

SUMMARY AND RECOMMENDATIONS

1. The proposed model for reverse yielding can give a better representation of the actual stress-strain curve. All popular plasticity models fail for reverse yielding in a high strength steel or spring steel.

2. The present model has been applied to the torsion problem in a cylindrical bar. For a high strength steel bar, the results indicate that the

initiation of reverse yielding occurs at a much smaller torque in the reversed direction than that predicted by the other models. Further reversed loading requires a much larger rate of increase in the applied torque than the other models.

3. More research works to develop a better plasticity theory are needed for reversed loading and cyclic loading problems. Basic experiments are needed to characterize the material behavior and to compare with the theoretical predictions.

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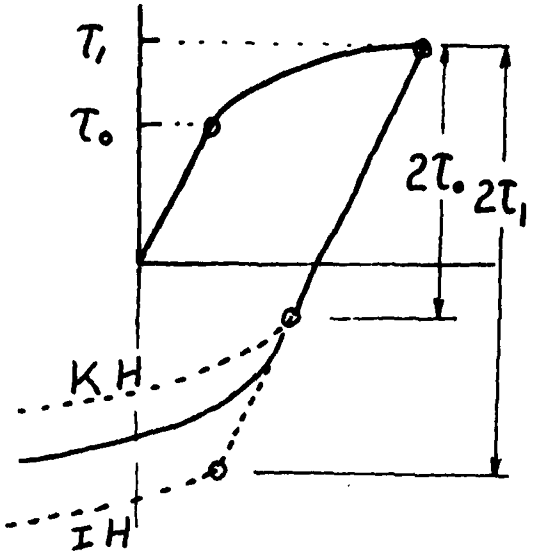

Figure 1. Stress-strain curve for a large strain-hardening material.

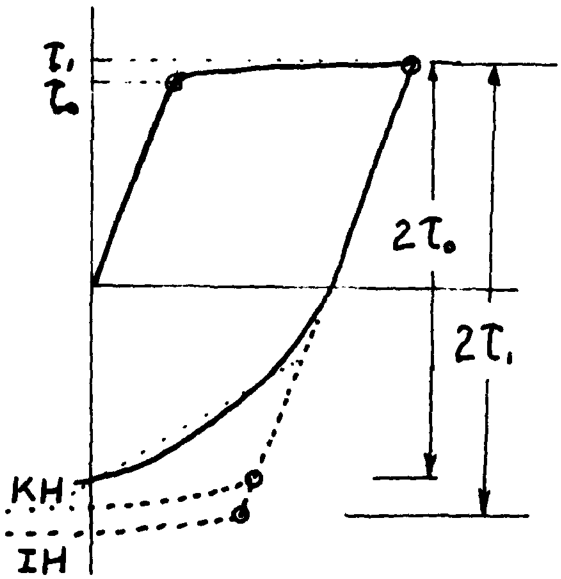

Figure 2. Stress-strain curve for a small strain-hardening material.

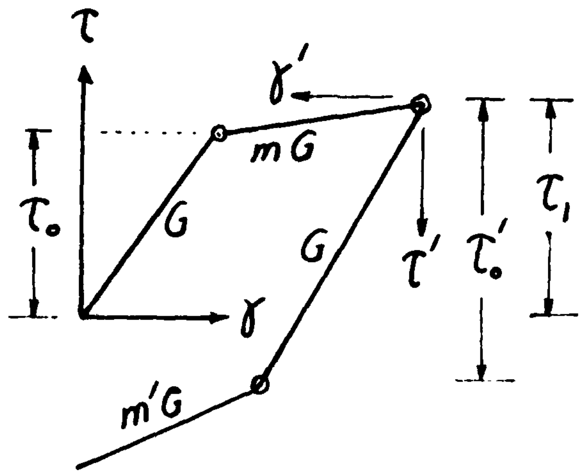

Figure 3. A model for reverse yielding.

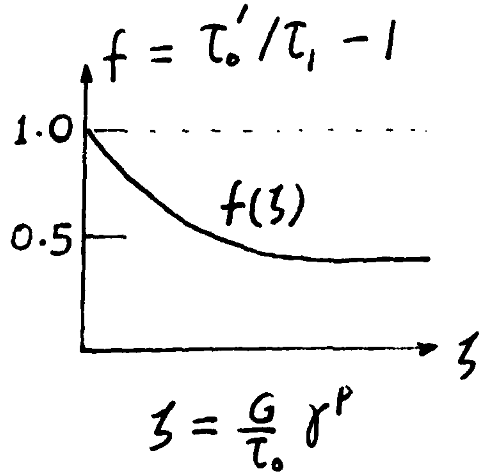

Figure 4. The Bauschinger effect factor.

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