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OPTIMAL AND SUBOPTIMAL ESTIMATION OF NONLINEAR
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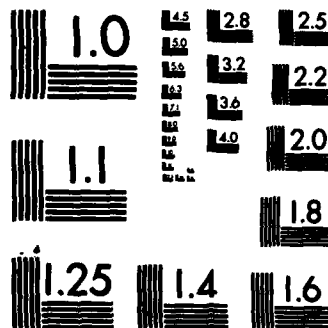
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"Optimal and Suboptimal Estimation of
Nonlinear Stochastic Systems"

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*This document describes*Abstract

Significant progress was made in a number of aspects of nonlinear estimation and nonlinear stochastic systems. For the first time, the performance of suboptimal estimators and the tightness of estimation lower bounds for a nonlinear estimation problem were analyzed by comparison with the optimal estimator. Lie algebraic techniques were applied to nonlinear estimation problems; these have provided new insights into the existence of recursive finite dimensional estimators. These techniques were also applied to problems in system identification, and new results on the structure of these problems resulted. For certain classes of infinite horizon decentralized control problems involving Markov chains, the existence of a stationary optimal policy was shown, and computational algorithms for such problems were derived. A static decentralized stochastic control problem with an exponential cost functional was solved, and this resulted in the only nonquadratic decentralized stochastic control problem with a closed form optimal policy. In addition, the optimal decentralized gain for this problem was expressed as an explicit projection of the optimal centralized law, thus leading to efficient computational algorithms. A more basic study of decentralized control problems led to a generalization of Radner's theorem and to a proof of global optimality of affine policies in the exponential cost problem. A dynamic decentralized control problem with exponential cost functional was solved, and an application to a simplified version of a resource allocation problem associated with a defense network was pursued. In the area of stochastic adaptive control, optimality of an adaptive control was proved for the first time in the literature for continuous-time semi-Markov decision problems; in particular, problems of adaptive control of service and priority assignment were solved.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) Significant progress was made in a number of aspects of nonlinear estimation and nonlinear stochastic systems. For the first time, the performance of suboptimal estimators and the tightness of estimation lower bounds for a nonlinear estimation problem were analyzed by comparison with the optimal estimator. Lie algebraic techniques were applied to nonlinear estimation problems; these have provided new insights into the existence of recursive finite dimensional estimators. These techniques were also applied to problems in system identification, and new results on the structure of these problems resulted. For certain classes of infinite horizon decentralized control problems involving Markov chains, the existence of a stationary optimal policy was shown, and computational algorithms for such problems were derived. A static decentralized stochastic control problem with an exponential cost functional was solved, and this resulted in the only nonquadratic decentralized stochastic control problem with a closed form optimal policy. In addition, (CONTINUED)					
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ITEM #19, ABSTRACT, CONTINUED: the optimal decentralized gain for this problem was expressed as an explicit projection of the optimal centralized law, thus leading to efficient computational algorithms. A more basic study of decentralized control problems led to a generalization of Radner's theorem and to a proof of global optimality of affine policies in the exponential cost problem. A dynamic decentralized control problem with exponential cost functional was solved, and an application to a simplified version of a resource allocation problem associated with a defense network was pursued. In the area of stochastic adaptive control, optimality of an adaptive control was proved for the first time in the literature for continuous-time semi-Markov decision problems; in particular, problems of adaptive control of service and priority assignment were solved.



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I. SUMMARY OF RESEARCH PROGRESS AND RESULTS

During the five years of research supported by this grant, significant progress has been made in a number of aspects of nonlinear estimation and nonlinear stochastic systems. In this section this progress is summarized, and reference is made to the resulting publications listed in Section II.

A. Performance of Suboptimal Estimators and Tightness of Bounds

By considering the class of continuous time nonlinear estimation problems for which we have previously derived finite dimensional optimal nonlinear estimators, we have in [1] and [11], for the first time in the literature, been able to analyze the performance of suboptimal estimators and the tightness of estimation lower bounds for a nonlinear system by comparison with the optimal estimator. A system for which we can construct the optimal estimator was studied; the optimal estimator, extended Kalman filter (EKF), constant gain extended Kalman filter (CGEKF), best linear estimator (BLE), and Bobrovsky-Zakai lower bound were compared both analytically and via Monte Carlo simulations. The results indicate that the performance of the EKF is as good as that of the optimal estimator, and that the Bobrovsky-Zakai lower bound is tight for very high signal-to-noise ratio but very ineffective (i.e., "loose") for very large state and observation noises. The CGEKF performs almost as well as the EKF or optimal filters; thus as far as suboptimal filter design for such systems is concerned, the CGEKF is probably preferable due to its simple computational requirements.

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MATTHEW J. KERPER
Chief, Technical Information Division

B. Algebraic and Geometric Methods in Estimation and Identification

This approach was pioneered by Brockett and Mitter, but we have been able to obtain interesting results for much wider classes of nonlinear estimation problems by using these methods. The approach proceeds by studying the (Zakai) stochastic partial differential equation for an unnormalized conditional density ρ of the state x given the past observations z :

$$d\rho(t,x) = L_0\rho(t,x)dt + L_1\rho(t,x)dz, \quad (1)$$

where L_0 is the Fokker-Planck operator associated with x and L_1 is multiplication by $h(x)$ (the observations are $dz = h(x)dt + dv$). The Lie algebra L of differential operators generated by $L_0 - \frac{1}{2}L_1^2$, and L_1 is called the estimation Lie algebra; since (1) is an infinite dimensional bilinear equation with input z , it is reasonable, by analogy with finite dimensional control systems, to study the algebra L . The major observation of Brockett is that if a recursive finite dimensional estimator for some conditional statistic of the state exists, then there is a Lie algebra homomorphism from L to the Lie algebra F of the finite dimensional filter. Thus we have a correspondence between statistics which are finite dimensionally recursively computable and certain homomorphisms of L into Lie algebras of vector fields on finite dimensional manifolds.

An important step in this program has been completed in [22], in which we analyze in detail the Lie algebra L associated with the Zakai equation for a particular class of nonlinear filtering problems (a class for which we previously derived recursive finite dimensional filters for all conditional moments). In addition, the Lie algebras associated with the recursively computable conditional moment equations are computed, and the

relationships (e.g., homomorphisms) between these Lie algebras are studied. This study provides further evidence that the existence of finite dimensional quotients of L is closely related to the existence of recursive finite dimensional estimators; in fact, this is the first example in which L is an infinite dimensional Lie algebra for which such a study has been carried out. More general classes of nonlinear estimation problems are considered in [13], where we show for example that the estimation problem for bilinear systems with linear observations has an algebraic structure similar to that of the example studied in [22]. On the other hand, we show in [13] that for certain classes of nonlinear estimation problems, no conditional statistic of the state can be computed with a finite dimensional filter. This is the first such result in the literature, and it applies for example to a linear system with cubic observations (the cubic sensor problem).

We proposed application of these Lie algebraic methods to the problem of simultaneous state and parameter estimation in linear systems with random parameters of the form

$$\begin{aligned} dx_t &= A(p)x_t dt + b_1 u_t dt + b_2 dw_t \\ dy_t &= \langle q, x_t \rangle dt + dv_t \end{aligned} \quad (2)$$

where u_t is a known input function, $\{w_t\}$ and $\{v_t\}$ are independent Brownian motion processes, $\{x_t\}$ and $\{y_t\}$ are respectively the state and observed process, and p and q are random unknown parameters. For example, certain angle modulation problems can be written in the form (2) after a change of coordinates. In [23] we have analyzed the estimation algebra L for this problem and for the tracking problem in which p and q are random processes; certain tracking problems in which L was finite dimensional led to the

construction of finite dimensional filters. This problem was taken one step further in [16] and [19], where we showed that the estimation algebra L for the simultaneous state and parameter estimation problem (2) is a subalgebra of a "current algebra" (current algebras have also recently begun to play a fundamental role in certain problems in physics). It then follows that L is embeddable as a Lie algebra of vector fields on a finite dimensional manifold. This finite dimensionality is manifested in our being able to construct a finite dimensional recursive filter for the conditional density of the parameters evaluated at a point. In addition, we have developed a Wei-Norman type procedure for the computation of the joint conditional density of the states and parameters, thus revealing a set of functionals of the observations that play the role of sufficient statistics for this problem.

In related work on parameter estimation in linear systems, we have defined and utilized new Riemannian metrics on the space of stable linear systems. If one wants to design a parameter estimation or identification algorithm, one must decide upon the "cost" of identifying the wrong values of the parameters; this is, in turn, related to the concept of distance between models. Since the class of stable linear systems of McMillan degree n forms a differentiable manifold, one thus needs to define the distance between points on this manifold, which is precisely what a Riemannian metric provides. In [20], we have defined a number of Riemannian metrics on this manifold. Computation of the geodesics of the Riemannian metrics leads to new insights into the geometry of the space of stable linear systems. In addition, Riemannian metrics are used to elucidate the structure of some recursive gradient methods of parameter estimation.

C. Modeling and Approximation of Stochastic Systems

In order to design better estimators, one must also obtain a better understanding of the modeling and properties of nonlinear stochastic systems. In [3], we have pursued this goal by studying the modeling and approximation of stochastic differential equations driven by semimartingale noise containing both continuous and jump components. The stochastic calculus and stochastic models of McShane are generalized to include such differential equations; viewed from another perspective, we have defined for the first time the analog of Stratonovich stochastic differential equations when the noise has both jump and continuous components. It is proved that the models defined here possess the same desirable properties (including approximation or continuity results) as those of McShane, but they are applicable to a much wider class of noise processes. The significance of this work is related to the fact that, for models to be faithful representations of physical systems, the system variables should not be affected significantly by small changes in the noise processes driving the system. The results presented show that our models are, under certain hypotheses, consistent in this sense.

D. Decentralized Static and Dynamic Stochastic Team Problems

We have been involved in the study of decentralized control problems, which have many applications in the area of C^3 . In [6] and [14], the stationary conditions of Radner are shown under relaxed hypotheses to be sufficient to establish the global optimality of candidate control laws for static team problems with convex cost. This extension of Radner's Theorem is used in [2], [6], [12], [14] to establish the global optimality of affine laws for problems with positive exponential of a quadratic

performance index with jointly Gaussian state and observation variables. Since the quadratic cost functional results in the only previously known closed form decentralized optimal control policy, these results for the exponential cost functional represent an important generalization (in fact, the quadratic cost results are a special case of these as a certain parameter approaches zero).

Team problems with affine control laws are solved as constrained parameter optimization problems in [7]. This methodology eases both notational and computational difficulties. This approach leads, for both the quadratic and exponential of a quadratic cost problems, to the representation of the optimal team control gains as explicit projections of the optimal centralized gains. This representation is then used to develop efficient algorithms for the solution of these problems.

In addition, we have studied dynamic team problems with the exponential of a quadratic performance index, with penalties on the terminal state and all intermediate control values [8], [18]. For a system with linear dynamics and the one-step delayed sharing information pattern, the problem reduces via dynamic programming to a succession of static team problems of the type considered above. Moreover, the optimal policy of the i^{th} team member at time k is an affine function of both the one-step predicted Kalman filter estimate and the i^{th} team member's observation at time k . Efficient algorithms are developed for determining the gains of this affine controller. This model and solution are applied in [8] to a simplified version of a C^3 problem: the resource allocation problem associated with a defense network.

E. Decentralized Control of Finite State Markov Processes and Multiaccess Broadcast Communication

A different approach to decentralized control has been taken in [4], [10], [15]. In this work, the state, control, and observations all take values in finite sets; the state is a controlled finite state Markov process. Again, the one-step delayed sharing information pattern is assumed. Using this information pattern, the dynamic programming algorithm is explicitly carried out to obtain the optimal policy. The problems are discussed under three different cost criteria: finite horizon problems with total expected cost, and infinite horizon problems with discounted or average expected cost. Each observation is noiseless with perfect recoverability of the state information from all the observations. Improving on the results of Sandell, we are able to prove the existence of stationary (and thus much more easily implementable) optimal policies for the infinite horizon problems.

This methodology is applied in [5], [17] to multiaccess broadcast communication problems. The objective of multiaccess broadcast communication is the efficient sharing of a single communication medium among many users, while giving each access to the full bandwidth of the channel. In the so-called Aloha-type schemes, a terminal sends a new message immediately upon receipt; if two terminals send simultaneously, a "collision" occurs, and each terminal retransmits after a random delay. When terminals are able to gather some information about the state of the network, feedback control policies (centralized, decentralized, or adaptive) may be implemented to improve performance. A number of such control schemes have been proposed in the literature. On the other hand, this problem fits into the decentralized control framework discussed in

[4]. In [5], we have applied the optimal decentralized control policies derived in [4] to problems of this type, and compared the results with those of previously proposed algorithms. Tradeoffs between the information available to each terminal and the value of the performance index were examined in detail.

F. Stochastic Adaptive Control of Semi-Markov Decision Processes

In [9] and [21], we have considered the problem of determining adaptive policies for the optimal control of average cost semi-Markov decision processes with transition probabilities and sojourn time distributions depending on unknown parameters. To solve the problem, an estimation and control scheme was proposed and its optimality shown under appropriate recurrency conditions. Our approach, which is essentially that of the "naive feedback controller" in stochastic control theory, differs from previous results for discrete-time Markov decision processes in that we consider continuous-time (not necessarily Markov) processes, and the sampling times in our parameter estimation scheme are random, as opposed to synchronous sampling. In particular, motivated by problems in computer networks, we have considered in [21] the priority assignment (or dynamic scheduling) problem for a single-server queueing system with several classes of customers who arrive according to independent Poisson processes. The service rates are considered constant but unknown, and the problem is to decide, at the completion of each service and given the state of the system, which class to admit next into service. Given a linear cost structure, the objective is to minimize the long run average cost per unit time. An adaptive control scheme was proposed, and we proved its optimality. A similar method was used on certain problems of adaptive

control of service rates in [9]. In both [9] and [21], the approach was to show an equivalence between the continuous-time decision problem and a corresponding discrete-time problem, prove convergence of the parameter estimates for the discrete-time problem, and then to prove that the cost of using the adaptive policy is the same as that for the optimal policy with known parameters.

II. PUBLICATIONS

A. Journal Articles

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2. J.L. Speyer, S.I. Marcus, and J. Krainak, "A Decentralized Team Decision Problem with an Exponential Cost Criterion," IEEE TRANSACTIONS ON AUTOMATIC CONTROL, vol. AC-25, October 1980, pp. 919-924.
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16. P.S. Krishnaprasad and S.I. Marcus, "Identification and Tracking: A Class of Nonlinear Filtering Problems" PROC. 1981 JOINT AUTOMATIC CONTROL CONF., Charlottesville, VA, June 17-19, 1981.
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23. P.S. Krishnaprasad and S.I. Marcus, "Some Nonlinear Filtering Problems Arising in Recursive Identification," in STOCHASTIC SYSTEMS: THE MATHEMATICS OF FILTERING AND IDENTIFICATION AND APPLICATIONS, M. Hazewinkel and J.C. Willems (eds.), Reidel, Dordrecht, 1981, pp. 299-304.

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IV. PAPERS PRESENTED

1. K. Hsu and S.I. Marcus, "Decentralized Control of Finite State Markov Decision Processes," 1979 IEEE Conference on Decision and Control, Ft. Lauderdale, Florida, December 10-12, 1979.
2. C.-H. Liu and S.I. Marcus, "Estimator Performance for a Class of Nonlinear Estimation Problems," 1979 IEEE Conference on Decisions and Control, Ft. Lauderdale, Florida, December 10-12, 1979.
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4. S.I. Marcus, "Modeling and Approximation of Stochastic Differential Equations Driven by Semimartingales," NATO Advanced Study Institute on Stochastic Systems, Les ARcs, France, July 3, 1980.
5. M. Hazewinkel, C.-H. Liu, and S.I. Marcus, "Some Examples of Lie Algebras in Nonlinear Estimation," Joint Automatic Control Conference, San Francisco, CA., August 14, 1980.
6. C.-H. Liu and S.I. Marcus, "The Lie Algebraic Structure of a Class of Finite Dimensional Nonlinear Filters," Filterdag Rotterdam 1980, Erasmus University, Rotterdam, The Netherlands, January 23, 1980.
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17. B. Hanzon and S.I. Marcus, "Riemannian Metrics on Spaces of Stable Linear Systems," 21st IEEE Conf. on Decision and Control, Orlando, Florida, December 8-10, 1982.

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