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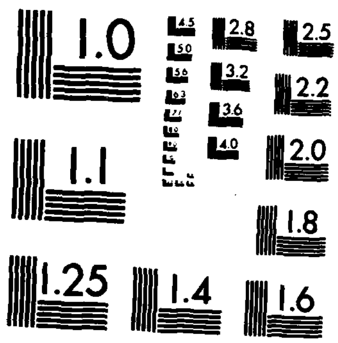

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INTRODUCTION

The type of game of timing analyzed here is known as a discrete, "noisy" duel. Through the use of "decomposition," such games can be defined recursively (i.e., in terms of successively simpler games). The principle of decomposition is well suited to implementation on a computer. Such a program (listed in the appendix) was developed on a VAX 11/780 computer in BASIC. This program permits parametric analysis by providing nearly instantaneous solutions to discrete, noisy duels.

THE GAME

The discrete, noisy duels considered here can be thought of as a naval engagement between two ships armed with surface-to-surface missiles (SSMs). Each player (ship) begins the game (engagement) with a certain number of bullets (SSM salvos). The distance between the ships is decreased by a set amount until one or both of the ships has been hit by the other's SSMs, or the intership separation has been reduced to zero. Before each decrement in intership separation, both players have the choice of firing one SSM salvo or waiting. The players select their option (shoot or wait) secretly, and then announce their selections simultaneously. The probability of hitting an opponent's ship decreases monotonically with range, and the probability of hit equals 1 at a range of 0. Hence, each player wishes to delay firing as long as possible to increase his chances for success, but not so long as to allow his opponent to effectively precede him. An optimal strategy provides the proper balance between effectiveness and vulnerability.

The value of the game (payoff) is +1 if player 1 (arbitrarily designated) hits player 2 and is -1 if player 2 hits player 1. The value of the game is 0 if both players hit their opponent (on the same turn) or both players fail to hit their opponent. Because the probability of hit equals 1 at an intership separation of 0, if at some point one player has exhausted his SSMs, the other player can simply hold his fire until the separation decreases to 0, guaranteeing a hit against his opponent. This is possible because it is a "noisy" duel, i.e., each player knows when his opponent has fired.

RATIONALE

The duels described above, while not prohibitively complex problems, are sufficiently time-consuming to make manual attempts at parametric analysis virtually impossible. However, the development of a computer program that can solve very large duels in seconds (i.e., duels involving 10 salvos per player and 20 distance decrements) readily allows such analysis. In fact, a fortunate side benefit of the decomposition algorithm is that not only is the value of the specified duel determined, but also the value of every duel with parameters (salvos for player 1, salvos for player 2, and initial separation) less than those of the specified duel. The result is a vast reduction in completion time.

OUTLINE

The remainder of this paper is divided into four main sections: Method, Results, Discussion, and Conclusions. The Method section outlines the procedure for solving discrete, noisy duels via decomposition, as well as the implementation of this process within a computer program. The Results section lists the topics that were investigated using the previously discussed methods and graphically displays the results of the parametric analyses. The Discussion section analyzes the results and *proposes* explanations of unexpected or counterintuitive results. The Conclusions section summarizes the findings of this investigation.

METHOD

This section outlines the method for solving discrete, noisy duels, as well as how such an algorithm is implemented within a computer program. The notation V(K,J;I) refers to the value of the game to player 1 (which is the negative of the value of the game to player 2) when player 1 has K salvos remaining, player 2 has J salvos remaining, and the players are separated by a distance of I units (where I, J, and K are all non-negative integers). P1(I) and P2(I) are the probabilities of hitting one's opponent at a separation of I for players 1 and 2, respectively. The expression "value of the game" refers to the payoff to player 1 when both players follow their optimal strategies.

SOLUTION VIA DECOMPOSITION

Decomposition of a duel into simpler duels (i.e., duels with smaller parameter values) is made easier by the fact that before each decrement in separation, each player has only two options, shoot or wait. Thus, the game V(K,J;I) decomposes into the following game:

Player 1		Player 2: Shoot	Player 2: Wait
	Shoot	A. P1(I) • [1 − P2(I)] • (+1) + P2(I) • [1 − P1(I)] • (−1) + [1 − P1(I)] • [1 − P2(I)] • V(K − 1, J − 1; I − 1)	B. P1(I) • (+1) + [1 − P1(I)] • V(K − 1, J; I − 1)
	Wait	C. P2(I) • (−1) + [1 − P2(I)] • V(K, J − 1; I − 1)	D. V(K, J; I − 1)

The expected values listed above come directly from the probabilities of various events occurring. Since both players have two options (shoot or wait), there are four possible joint outcomes (represented by the four boxes above). The expected values derive from the following reasoning:

Box A. Player 1 shoots, player 2 shoots

The first term in this box refers to the outcome that player 1's shot hits and player 2's shot misses (times resulting game value of +1). The second term in this box refers to the outcome that player 1's shot misses and player 2's shot hits (times resulting game value of -1). The third term in this box refers to the outcome of both players' shots missing (times the value of the resulting game, which is simply a duel starting with one less salvo for each player and with a smaller initial separation). There is no term corresponding to the outcome that both salvos hit, since the resulting game value would be 0.

Box B. Player 1 shoots, player 2 waits

The first term in this box refers to the outcome that player 1's shot hits (times resulting game value of +1). The second term refers to the outcome that player 1's shot misses (times the value of the resulting game, which is simply a duel starting with one less salvo for player 1 and with a smaller initial separation).

Box C. Player 1 waits, player 2 shoots

The terms in this box are analogous to those in box B, with the exception that the resulting game value of a successful salvo is -1 and the game resulting from a miss has player 2 starting with one less salvo rather than player 1.

Box D. Player 1 waits, player 2 waits

Since both players choose to wait, the resulting game is identical, except that the initial separation is reduced by one.

This process continues until all of the "resulting" duels have zero as one or more of their parameters (i.e., I, J, or K). Such duels are trivial, as shown below:

Example 1. $K=0$ and $J>0$

Since player 1 has no more salvos, but player 2 does, the latter can hold his fire until separation (I) becomes zero, guaranteeing a game value of -1 (i.e., $V(0,J;I) = -1$).

Example 2. $J=0$ and $K>0$

This is the opposite of the previous example (i.e., $(V(K,0;I) = +1)$.

Example 3. $I=0$ $(J>0$ and $K>0)$

Since the separation is 0 and both players have salvos remaining, both will hit their opponent, resulting in a game value of 0 (i.e., $V(K,J;0) = 0$).

Example 4. $J>I+1$

These parameters correspond to a player having more salvos than can be used at the rate of one per decrement (note that players can shoot at $I=0$, hence the +1 term). This duel has the same value as $V(K,I+1;I)$.

Example 5. $K>I+1$

This is analogous to Example 4, but for the other player. This duel has the same value as $V(I+1,J;I)$.

Note also that certain games that are impossible in terms of the original game cannot be treated as trivial and ignored. For example, if the initial problem is to find V(1,5;2), it is obvious, by means of Example 4, that this is equal to V(1,3;2). Even though the game (1,3;2) is impossible in terms of (1,5;2), i.e., player 2 could not have fired two salvos before the game even began, it still must be solved as an intermediate step in the solution of the original game.

Because solutions to the trivial games listed above are known, some of the "matrices" (boxes A through D, which collectively represent the value of a particular duel) can be filled in. That is, values have been determined for every term in the expression (the P1(I), P2(I), and V(K,J;I) terms). Letting a, b, c, and d represent the numeric value of the newly filled-in terms within boxes A, B, C, and D for a particular matrix this duel becomes what is called a two-by-two, zero-sum matrix game of the form:

		Player 2	
		Shoot	Wait
Player 1	Shoot	a	b
	Wait	c	d

The following discussion, based on Guillermo Owen, Game Theory, New York: Academic Press, 1982, is included for the sake of completeness. There are two possibilities for such a game: either the optimal solution occurs for an equilibrium pair of pure strategies (i.e., shoot or wait) or it occurs for a pair of mixed strategies (probability distributions across all players' choices). If an equilibrium pair of strategies exists, then the value of the game will be the number in the box that is defined by that equilibrium pair. Such a pair exists when one (or more) of the numbers a, b, c, or d is both the minimum number in its row and the maximum number in its column. The basis for this is the Minimax Theorem. Because player 1 is trying to maximize the value of the game, and player 2 is trying to minimize the value of the game, such an equilibrium pair of strategies (also known as a saddle point) will be optimal for both players. This is true because neither player, even with knowledge of the other player's choice, will have any incentive to change his strategy.

For example, if a = .5, b = .7, c = 0, and d = -.4, then it is obvious that the value of the game equals a (.5), because the optimal strategy for both players is to shoot. Observe that neither player, knowing his opponent will shoot, can improve his payoff by changing his strategy.

If no equilibrium pair of strategies exists (e.g., if c were .6 rather than 0 in the previous example), then a solution would be obtained by using mixed strategies. A mixed strategy associates a probability with each choice (pure strategy) facing a player. These probabilities must be non-negative and sum to 1. For example, a mixed strategy for player 1 might be: shoot with probability .5 and wait with probability .5. A mixed strategy for player 2 might be: shoot with probability .25 and wait with probability .75. Using game theory, optimal mixed strategies can be calculated. These prevent a player from being outguessed by his opponent through the (proper) randomization of strategy selection. Determination of optimal mixed strategies is discussed here only because it is a means for determining the value of a game (we are not concerned with strategies). The value of a game can be derived from the optimal mixed strategy for either player. Such a process is outlined below.

Given the matrix game just discussed, let x_1 and x_2 be the probabilities that player 1 chooses to shoot and wait, respectively. Let y_1 and y_2 be the analogous probabilities for player 2. If we denote the game matrix as M, and mixed strategy vectors as x and y for players 1 and 2, respectively, then the value of the game = xMy. The optimal value of the game $v^* = x^*My^*$, where x^* and y^* are the optimal mixed strategy vectors for players 1 and 2.

To determine the optimal value of the game, solve the maxi-min problem of player 1, namely:

$$v^* = \max_{\substack{x_1+x_2=1 \\ x_1,x_2>0}} \left\{ \min_{\substack{y_1+y_2=1 \\ y_1,y_2>0}} \left\{ xMy \right\} \right\} .$$

Carrying out the matrix multiplication, this becomes:

$$v^* = \max_{\substack{x_1+x_2=1 \\ x_1,x_2>0}} \left\{ \min_{\substack{y_1+y_2=1 \\ y_1,y_2>0}} \left\{ ax_1y_1 + bx_1y_2 + cx_2y_1 + dx_2y_2 \right\} \right\} .$$

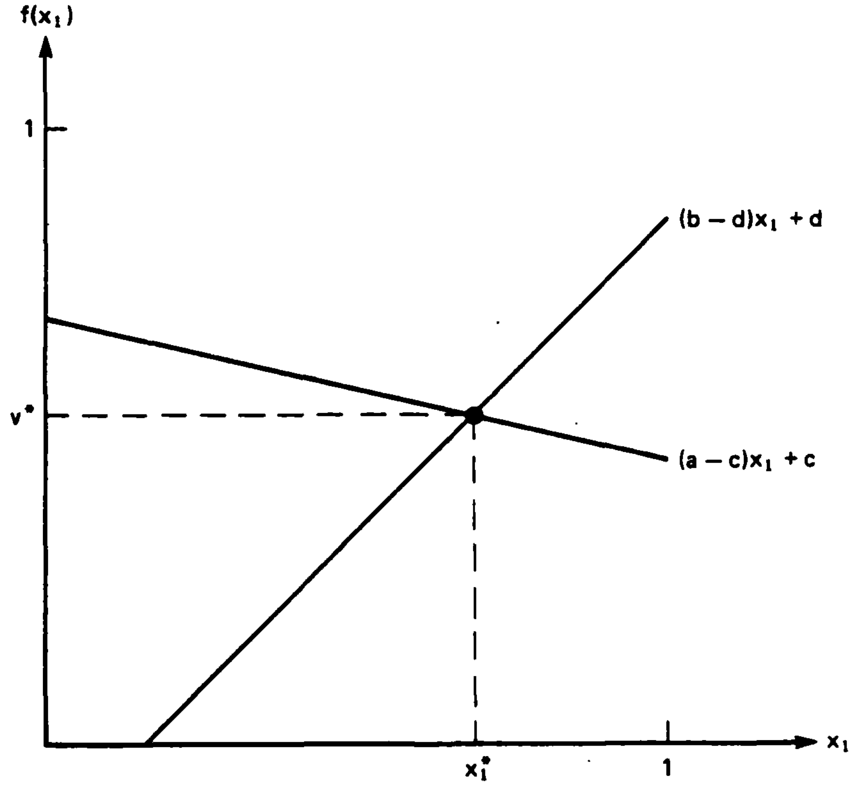
Note that for a fixed value of x, the expanded xMy is linear in terms of y. This means that the min of $(ax_1y_1 + bx_1y_2 + cx_2y_1 + dx_2y_2)$ will occur at a vertex (that is, either $y_1=0$ and $y_2=1$ or $y_1=1$ and $y_2=0$), though this does not mean that player 2's optimal strategy occurs at a vertex. Thus, the expression can be rewritten as:

$$v^* = \max_{\substack{x_1+x_2=1 \\ x_1,x_2>0}} \left\{ \min \left\{ (bx_1 + dx_2), (ax_1 + cx_2) \right\} \right\} .$$

Since $x_1 + x_2 = 1$, $1-x_1$ can be substituted for x_2 above, yielding:

$$v^* = \max_{0<x_1<1} \left\{ \min \left\{(b-d)x_1 + d,\ (a-c)x_1 + c\right\}\right\}.$$

The two inner terms, $(b-d)x_1+d$ and $(a-c)x_1+c$, are both linear in terms of x_1. Also, since no equilibrium pair of strategies was found, the two terms will intersect within the interval [0,1]. The solution to the problem occurs at this intersection, i.e., where $(b-d)x_1+d = (a-c)x_1+c$. This can be seen graphically below:



To find x_1^*, set the two lines equal to each other and solve:

$$(b-d)x_1^* + d = (a-c)x_1^* + c$$

$$(b-d-a+c)x_1^* = c - d$$

$$x_1^* = (c-d)/(b-d-a+c)$$

To find v^*, simply plug in x_1^* into either of the equations for the lines plotted above, i.e., substitute the equation for x_1^* into $(b-d)x_1^* + d$:

$$(b-d)\ [(c-d)/(b-d-a+c)] + d =$$
$$[(b-d)\ (c-d) + d\ (b-d-a+c)]/(b-d-a+c) =$$
$$[bc - bd - dc + d^2 + db - d^2 - da + dc]/(b-d-a+c) =$$
$$(bc - ad)/(b-d-a+c) = v^*$$

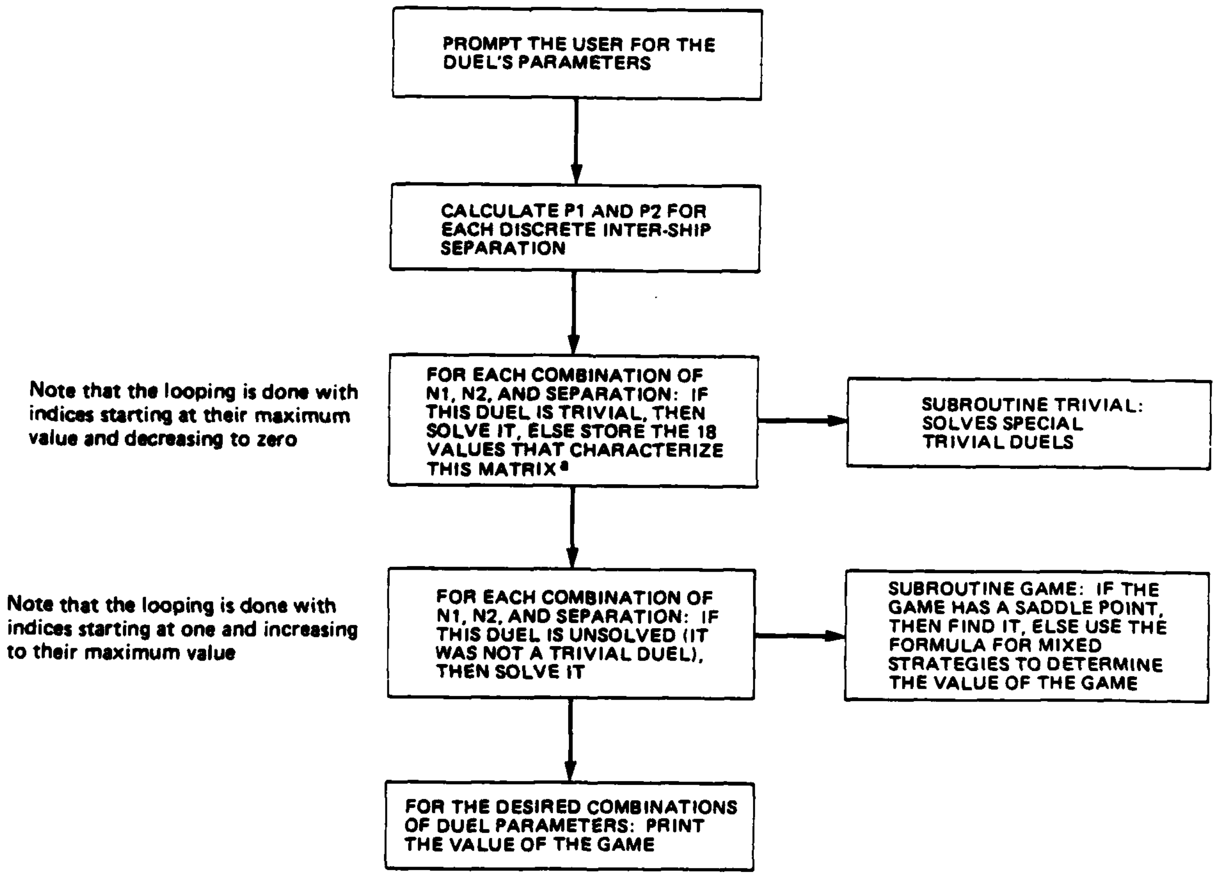
To summarize, if there is a payoff (either a, b, c, or d) which is the minimum of its row and the maximum of its column, then the value of the game is that payoff. If no payoff meets this criterion, there is no equilibrium pair of pure strategies and the value of the game (which results from the use of mixed strategies) equals $(bc - ad)/(b-d-a+c)$.

Although this technique of decomposing the original game into a sequence of games with smaller parameters is straightforward, even obvious, it was not seen it the literature.

IMPLEMENTATION

Implementation of this scheme requires a computer with enough memory to store four arrays totaling slightly over 40 kilobytes. The computer should also be capable of executing, in a reasonable amount of time, the large number of commands required (on the order of 11(D+1) + 21(D+1)(N1+1)(N2+1), where D = initial separation, N1 = salvos for player 1, and N2 = salvos for player 2). For example, to find V(10,10;20), the program executes over 53,000 commands, and this does not include I/O operations. The computer on which the program was run, a Digital Equipment Corporation VAX 11/780, provided nearly instant turnaround, and could handle problems of even larger parameters.

Program execution is a five-step process, summarized by the flow diagram below:



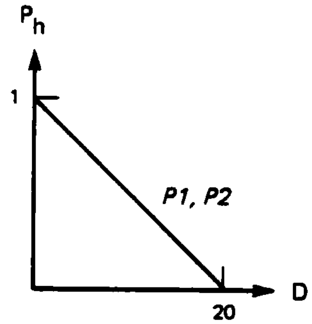
[a] *The matrix, as you will recall, had four boxes, each filled with an expression of the general form* $t_1 + t_2\,[V(t_3, t_4; t_5)]$. *Since box D has no terms corresponding to* t_1 *and* t_2, *an entire matrix (which represents the decomposition of a particular duel) can be stored in an array of length 18 (i.e., 4 × 5 − 2).*

The program's structure follows the principles of dynamic programming or recursion. First, the duel to be solved is decomposed into duels (matrices) of decreasing complexity, which are themselves decomposed. This continues until all duels with parameters less than those of the original duel have been so defined. Then, working backwards using the initial conditions supplied by the trivial games, increasingly more complex matrices can be filled in with actual values and solved. This continues until the original duel is solved. Because the loop indices increase in the second phase of this algorithm, the process of filling in the values of matrices will occur only after the component duels have already been solved. The program code is in BASIC and is listed in the appendix.

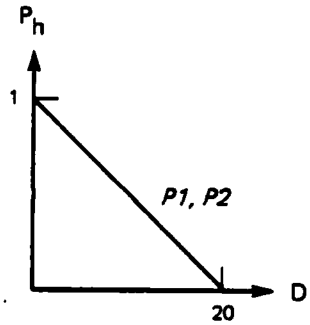
RESULTS

The following table lists eight cases that were felt to be of sufficient interest to warrant investigation. Immediately after this table are the actual results, consisting of plots of the values of the games as a function of some of the games' parameters. The Probability of hit (P_h) function listed for each player, called P1 and P2, is the probability of hit as a function of the intership separation, "D", previously denoted as "I". N1 and N2 denote the initial number of salvos for player 1 and player 2 respectively.

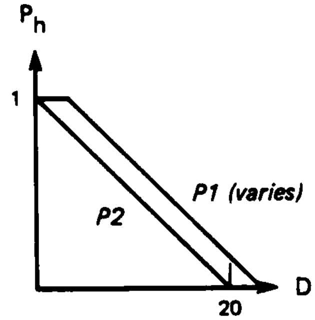
TABLE 1
PARAMETERS OF THE EIGHT CASES

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
1	2	1	1-.05D	1-.05D	
	4	2			
	6	3			
	8	4			
	10	5			

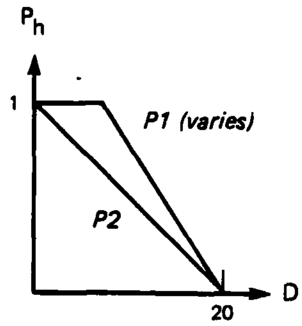
Comment: Game value is plotted as a function of initial separation and salvos.

2	2	1	1-.05D	1-.05D	
	4	3			
	6	5			
	8	7			

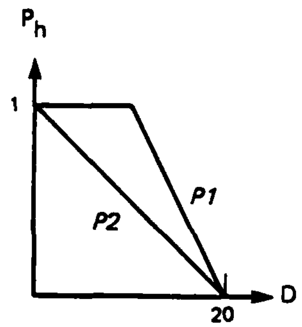
Comment: Game value is plotted as a function of initial separation and salvos.

3	5	5	1.1-.05D	1-.05D	
			1.2-.05D	1-.05D	
			1.3-.05D	1-.05D	

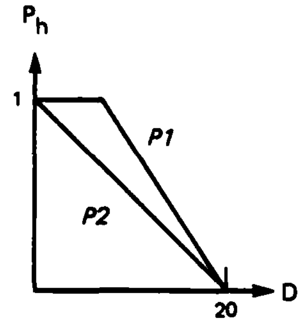
Comment: Game value is plotted as a function of initial separation and P_h for player 1.

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
4	5	5	1.2-.06D 1.4-.07D 2.0-.10D	1-.05D 1-.05D 1-.05D	

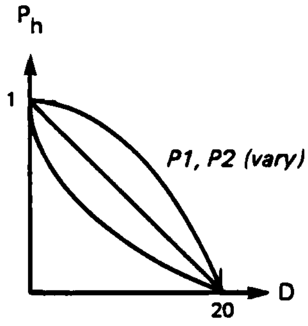
Comment: Game value is plotted as a function of initial separation and P_h for player 1.

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
5	1 2 3 4 5	2 4 6 8 10	2.0-.10D	1-.05D	

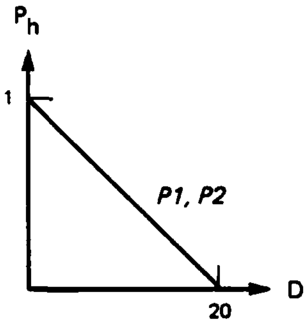
Comment: Game value is plotted as a function of initial separation and salvos.

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
6	1 9	1 9	1.5-.075D	1-.05D	

Comment: Game value is plotted as a function of initial separation and salvos.

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
7	6	3	$1-\sqrt{.05D}$ $1-.05D$ $1-(.05D)^2$	$1-\sqrt{.05D}$ $1-.05D$ $1-(.05D)^2$	

Comment: Game value is plotted as a function of initial separation and P_h for both players.

Case number	Salvos, player 1	Salvos, player 2	P_h for player 1[a]	P_h for player 2	P_h plotted for both players
8	1 2 3 4 5 6 7 8 9 10	5	$1-.05D$	$1-.05D$	

Comment: Game value is plotted as a function of player 1's salvos and initial separation.

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a Probability of hit functions are actually equal to the minimum of $\{1, P1\}$, i.e., they will never exceed 1.

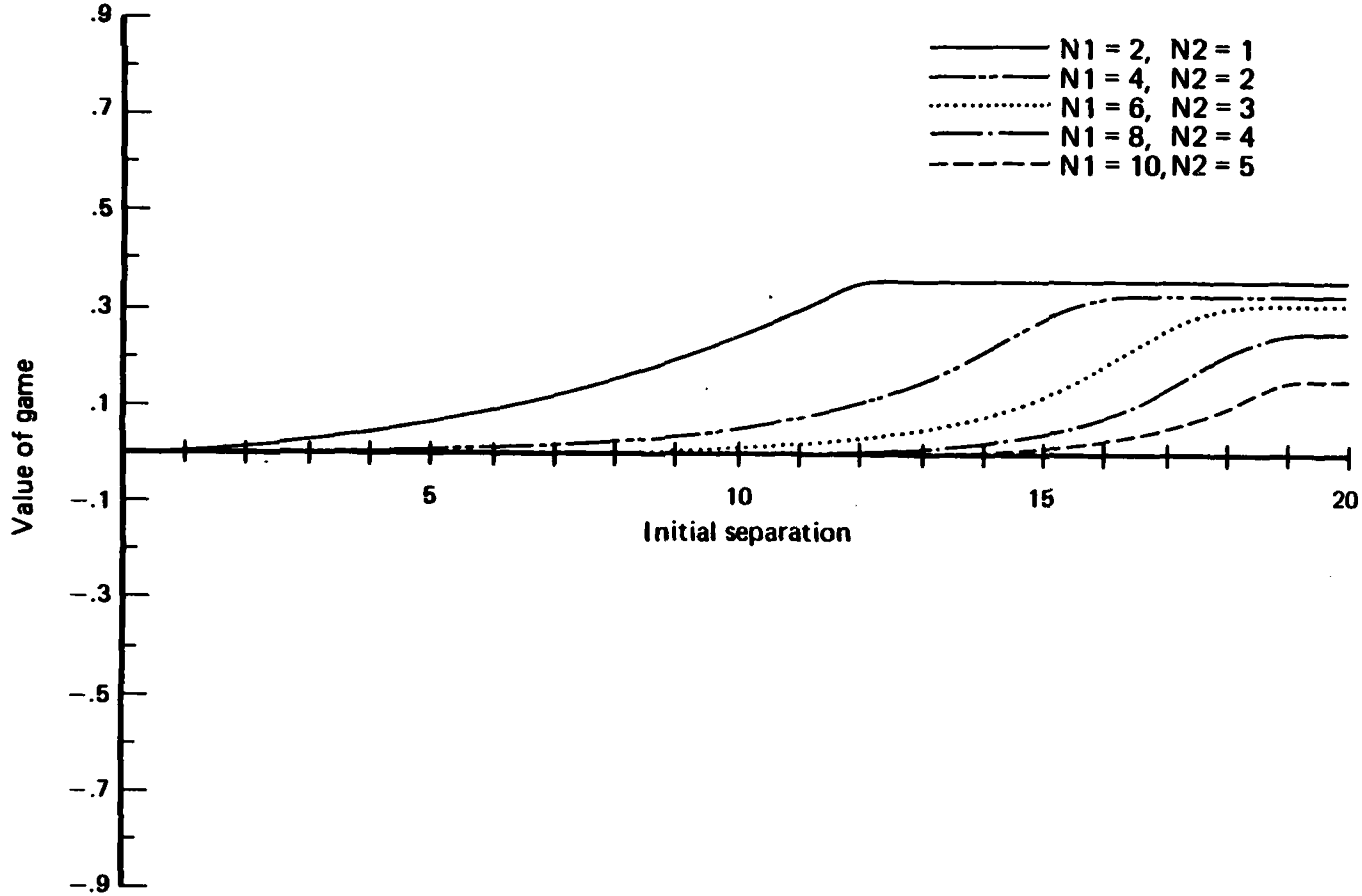


FIG. 1: TWICE THE SALVOS, EQUAL P_h, VARYING SALVO SIZE

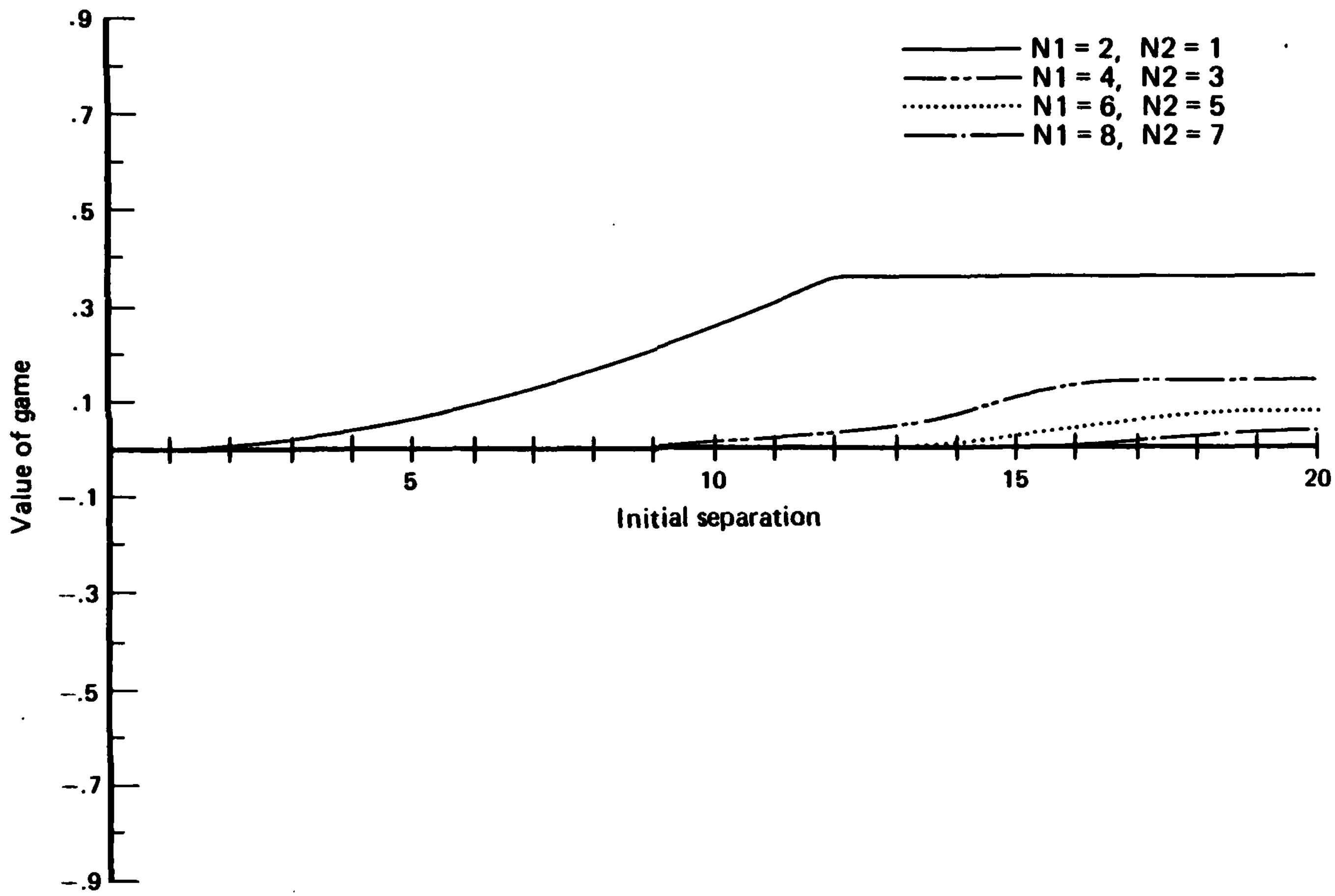


FIG. 2. ONE EXTRA SALVO, EQUAL P_h, VARYING SALVO SIZE

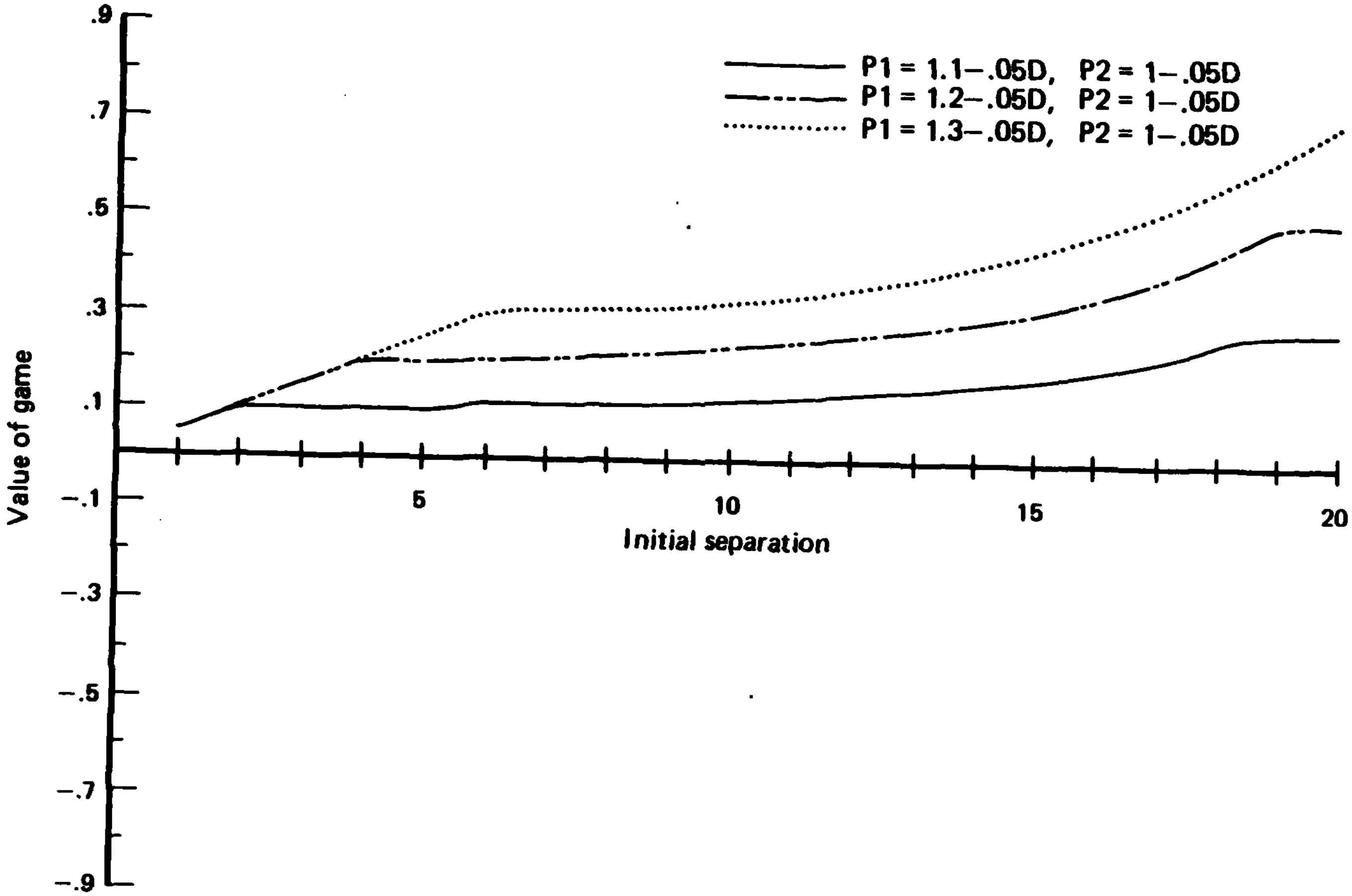


FIG. 3: EQUAL SALVOS, CONSTANTLY LARGER P_h, VARYING DIFFERENCE

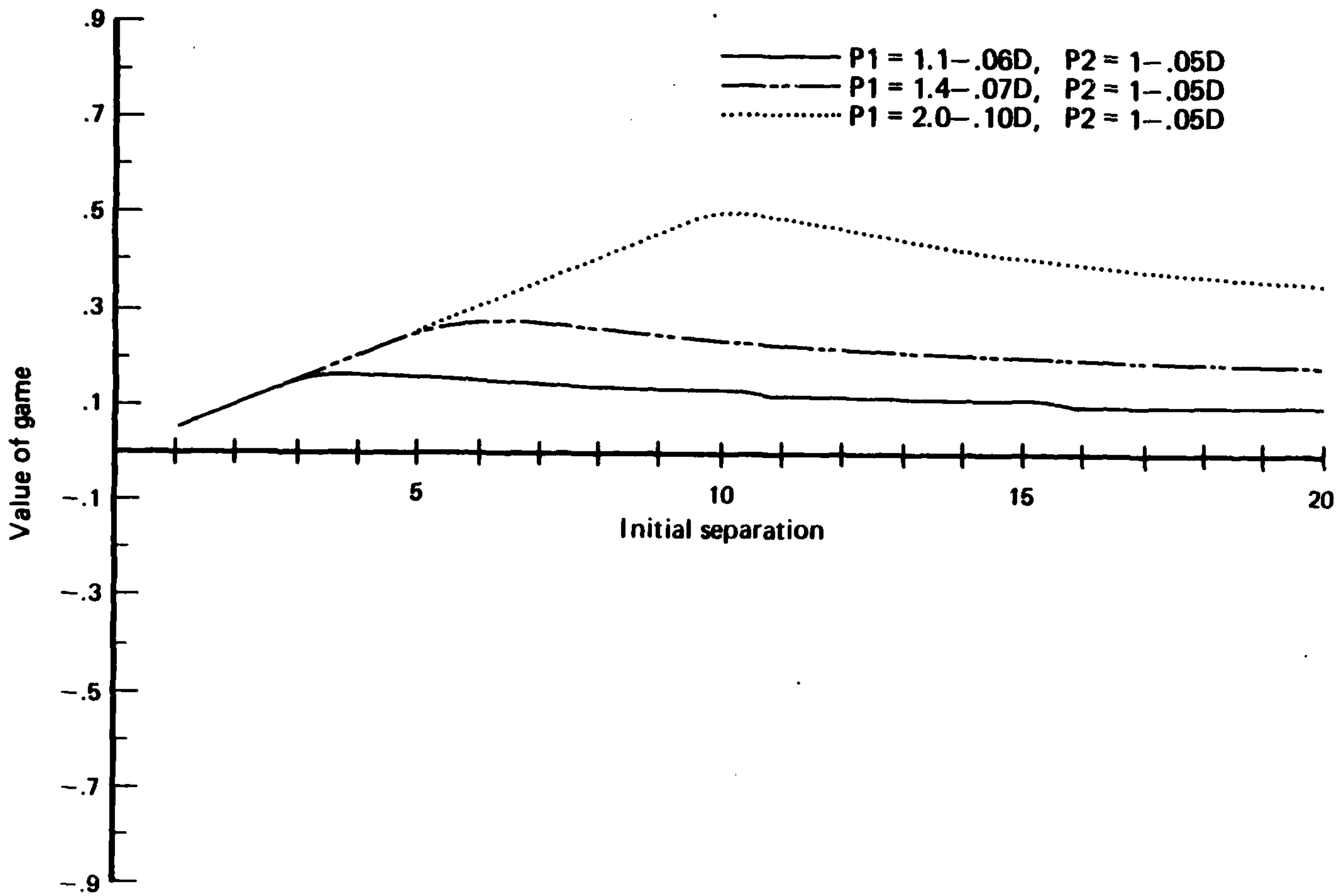


FIG. 4: EQUAL SALVOS, PROPORTIONALLY LARGER P_h, VARYING PROPORTION

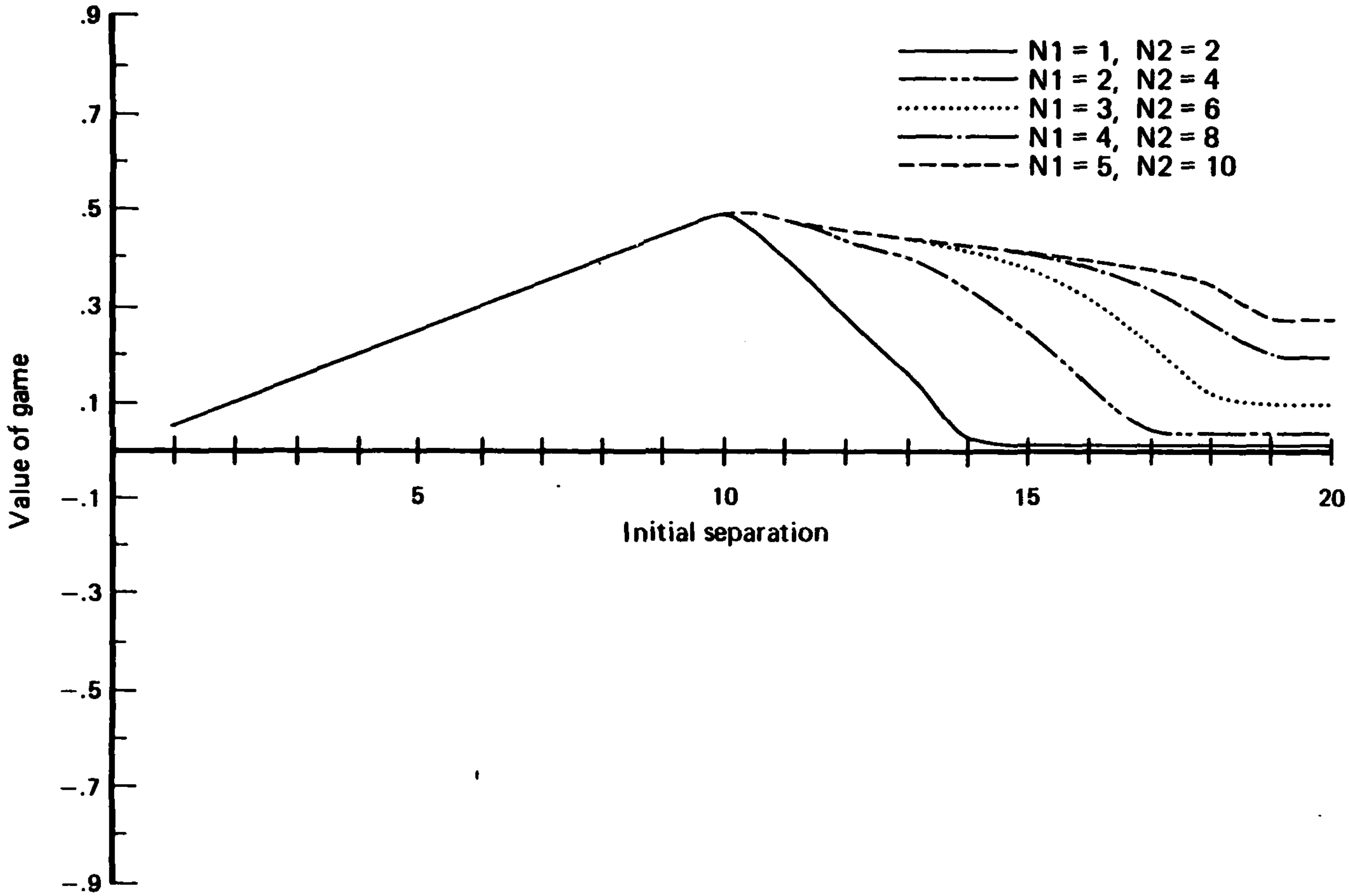


FIG. 5: HALF THE SALVOS, TWICE THE P_h, VARYING SALVO SIZE

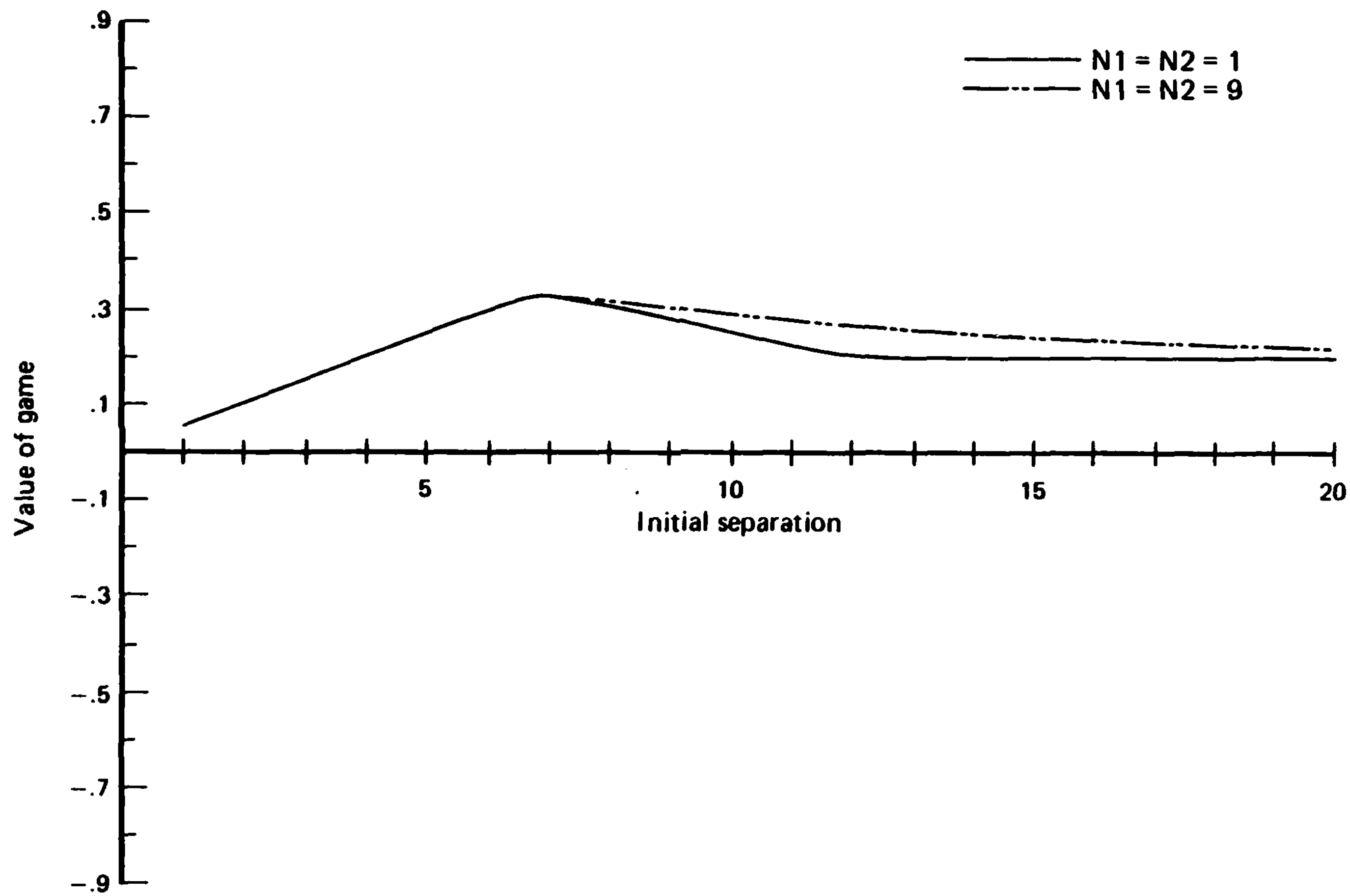


FIG. 6: EQUAL SALVOS, PROPORTIONALLY LARGER P_h, VARYING SALVO SIZE

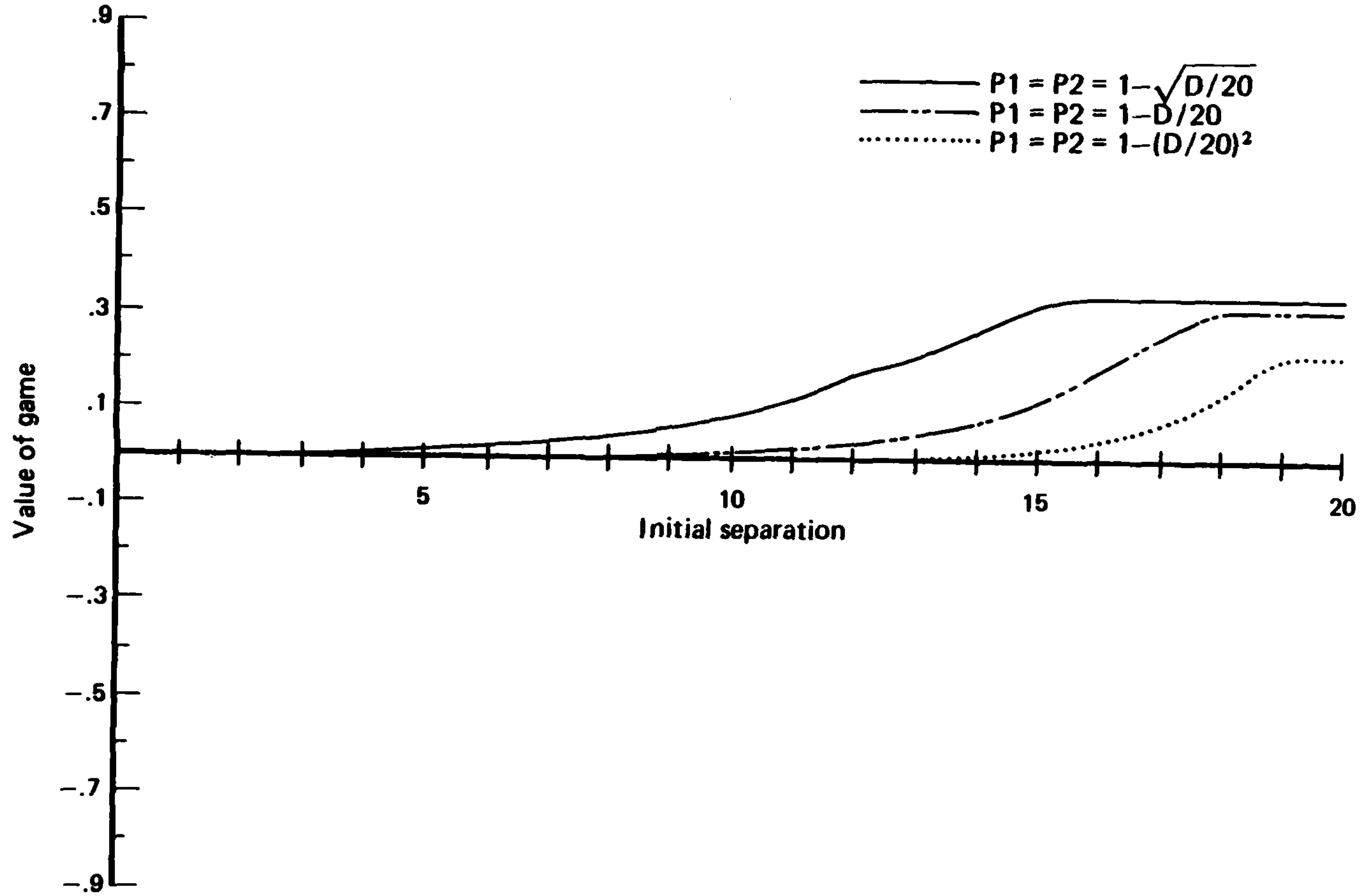


FIG. 7: TWICE THE SALVOS, EQUAL P_h, VARYING P_h FUNCTION

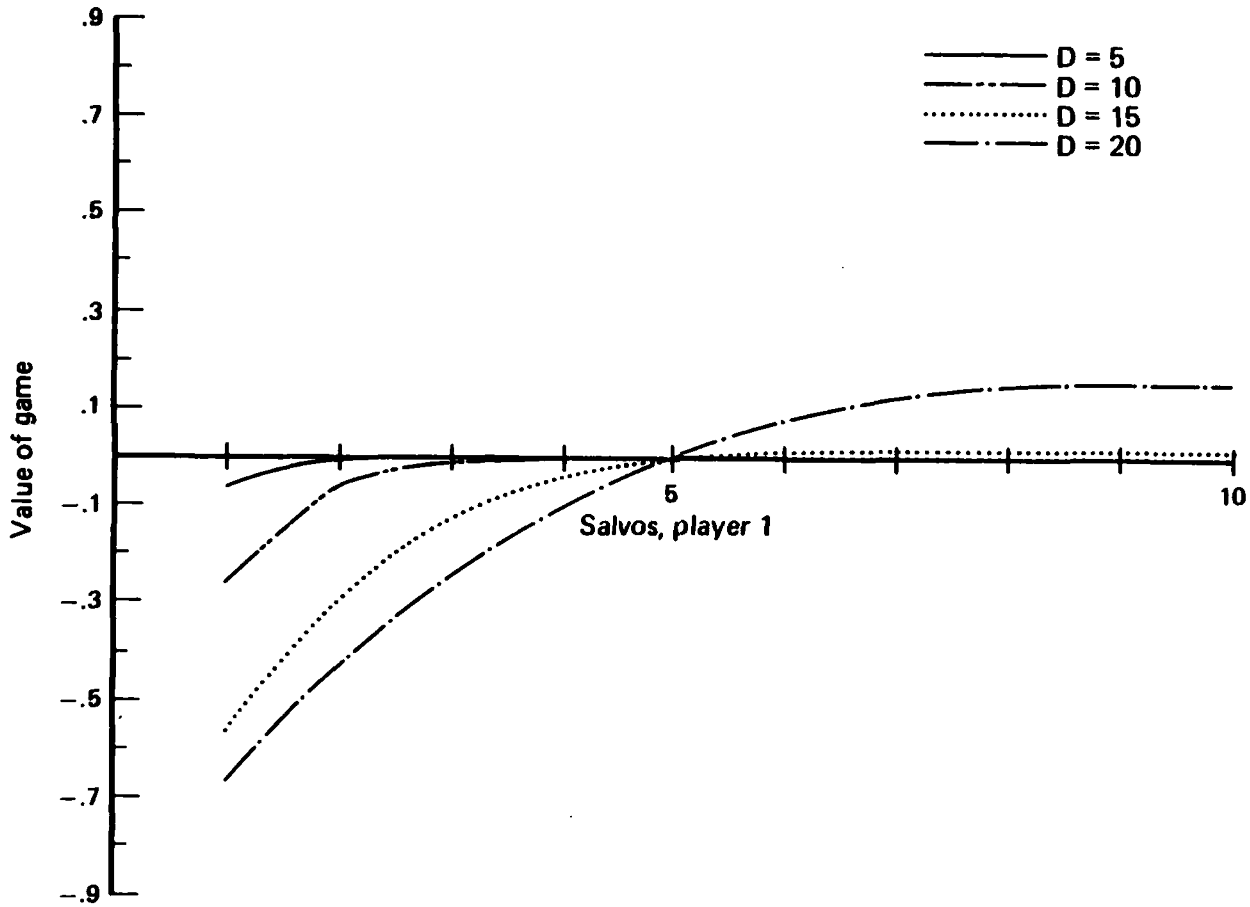


FIG. 8: FIXED SALVOS FOR PLAYER 2, EQUAL P_h, VARYING SALVOS FOR PLAYER 1

DISCUSSION

This section discusses the results of the parametric analyses. Each of the eight cases is considered in terms of intuitive appeal. Attempts to explain unexpected results will also be made. Because the value of a game is zero regardless of initial separation when both players have the same P_h functions and the same number of salvos, the cases discussed here explore some asymmetrical properties of duels.

Note that the significance of any findings should not be reduced by the contention that discrete duels are not realistic. The value of a discrete duel appears to rapidly approach the value of the equivalent continuous duel as the number of discrete distance gradations increases. Table 2 compares the value of discrete duels of increasing "granularity" with those of the three corresponding continuous duels.

Thus it appears that if the discrete game has 20 or more gradations, its value will be very close to that of the continuous version. The imprecise term "very close" is used intentionally because the rate at which the value of discrete and continuous games converge is influenced by the choice of P_h functions. Specifically, games where the difference in P_h functions is small converge faster than games with disparate P_h functions. Because only very special continuous games have been solved, this conclusion is significant in that it suggests a computer program can "solve" such problems by substituting a close approximation to the original problem that can be solved.

TABLE 2

VALUES OF DISCRETE VERSUS CONTINUOUS DUELS

	Duel parameters[a]		
Discrete gradations	N1=1 N2=1 P1=1-D $P2=1-D^2$	N1=1 N2=1 P1=1-D P2=1-2D	N1=3 N2=2 P1=1-D P2=1-D
2	-.250	.000	.000
5	-.240	.200	.200
8	-.250	.250	.211
11	-.248	.273	.192
14	-.245	.286	.215
17	-.242	.294	.206
20	-.240	.300	.200
Value of corresponding continuous duel	-.240	.333	.200

[a]where N1=salvos, player 1
N2=salvos, player 2
P1=probability of hit function, player 1
P2=probability of hit function, player 2
D=inter-ship separation
and the values of the continuous duels are calculated using formulas from Melvin Dresher, Games of Strategy--Theory and Applications, Englewood Cliffs, New Jersey: Prentice Hall Inc., 1961.

The following discussion of the eight cases is based on the graphs shown in figures 1 through 8 in the preceding section.

Case 1:

The main point of the graph in figure 1 is to demonstrate that when the ratio of player salvos remains constant, increasing the number of salvos decreases the value of the game to the favored player. This behavior is not intuitive unless the extreme case of player 1 having 40 salvos and player 2 having 20 salvos is considered. In such an engagement, both players can fire at every opportunity and the value of the game will be zero even though player 1 has many more salvos than player 2. Based on this "end point," we might infer that the value of this game decreases as the magnitude of salvos increases because the players are gradually approaching a point where the extra salvos of player 1 will never be used.

This trend (not firing beyond some separation) is also reflected, in a different way, by the "flattening out" of the curves. Such a phenomenon is the result of a change in the players' optimal strategies. In particular, for the case of N1=2 and N2=1, both players' optimal strategy is to fire for any initial separation of 12 or less. At an initial separation of 13, there is no equilibrium pair of pure strategies, and the players use mixed strategies. At initial separations greater than 13, both players' optimal strategy is to hold their fire ("wait") until their separation equals 13. Thus, the value of the game levels off, but not at its maximum value (see below).

Given N1=2, N2=1, P1=P2=1-.05D:

Initial separation	Optimal strategy (for first stage of duel)	Value of game
1	Both players fire	.0025
2	Both players fire	.01
:	Both players fire	:
:	Both players fire	:
10	Both players fire	.25
11	Both players fire	.3025
12	Both players fire	.36
13	Players use mixed strategies	.35547
14	Both players wait	.35547
:	Both players wait	.35547
:	Both players wait	.35547
20	Both players wait	.35547

Case 2:

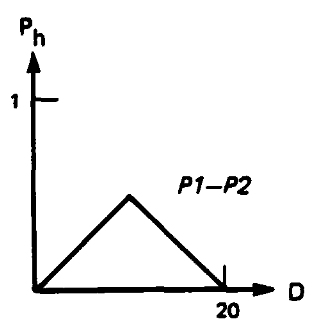
Figure 2 shows the advantage of having one more salvo than your opponent. As expected, the value of the additional salvo approaches zero as the number of salvos increases. This is not a surprise, since the "importance" of each salvo increases when there are fewer of them. Said another way, the favored player's optimal strategy will direct him to use his extra salvo at a range shorter than it would be used in an engagement where both players have more salvos available. This results in a higher probability of hit for that additional salvo and, therefore, a higher value of the game.

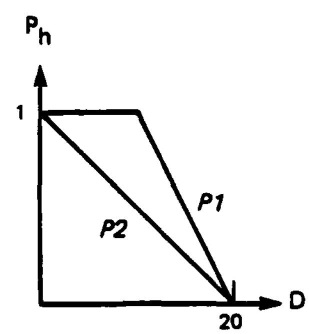
Case 3:

Figure 3 demonstrates the effect of adding a constant to one player's probability of hit function. The effect is roughly equivalent to adding that constant to the value of the game. Also, the fact that the value of a game increases as the initial separation increases follows intuition in the following way. Whenever one player has an advantage over another player (either in salvos or P_h), increasing the initial separation will improve (or, in the worst case, not affect) the value of the game, because it is providing the favored player with a greater number of alternatives for exploiting his advantage. However, an unexpected trend occurs: the value of the game is linear for small initial separations. This can be explained in terms of the P_h functions. Because player 1's P_h function equals that of player 2's plus some constant, there will be an interval over which player 1's P_h function will exceed 1 (i.e., set equal to 1). Thus, for this interval, P1(D) will be constant (1), but P2(D) will be decreasing linearly, hence the linear increase in game value. No explanation for the divergence of the curves (as initial separation increases), however, is readily apparent.

Case 4:

Case 4 (figure 4) demonstrates the effect on the value of the game when one player has a proportionally larger P_h than the other. The value of the game initially increases linearly, as in case 3 (and for the same reason). The point at which the curves change slope is simply the point where the P_h function for player 1 finally becomes less than 1. The unexpected decrease in the value of the game beyond this point is due simply to the shape of the P_h functions (see below). Notice that player 1's advantage (i.e., the difference of the curves, plotted to the left) is 0 at a separation of 0, increases to a maximum at the separation where player 1's P_h first drops below 1, and then decreases to 0 at separation of 20. Hence, the value of the game decreases after some point because player 1's advantage linearly decreases to zero.

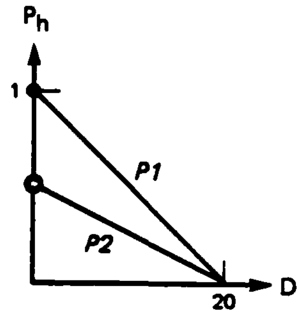


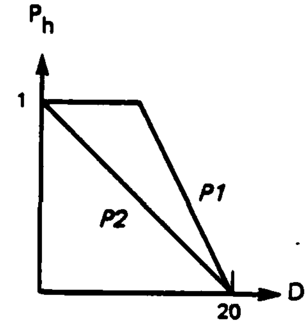


Case 5:

This case, shown in figure 5, presents an interesting question. Is it preferable to have twice as many salvos, but half the P_h, or the reverse? The value of the game decreases after a certain point, but this trend is explained in the previous case. As in case 1, the ratio of player salvos was kept at two, but the number of salvos was varied. For all curves, the value of the game is positive over the entire range of initial separations. This will not always be true when P1 exceeds P2. For example, if P1>P2 but N2>>N1, the value of the game couldconceivably be negative. The fact that the value of the game increases for engagements with more salvos (i.e., is to the advantage of the player with fewer salvos) is in keeping with the trend demonstrated in case 1.

Note that even as simple a term as "twice the accuracy" is difficult to define mathematically because of the upper bound of 1 that exists for probabilities. There are two straightforward ways of defining P_h functions so that one will have twice the probability of hit at any distance, but both have drawbacks. The two methods are illustrated in the plots below. The first method's drawback is that P2 is discontinuous at D=0, i.e., the value of P2 jumps from .5 to 1 as intership separation goes from "near-zero" to zero. The second method, while continuous, has the drawback that once P1 reaches a value of 1, it levels off.





Method 2 was used in this analysis, although the computer program is capable of handling either type of P_h function.

Case 6:

Figure 6 shows the effect on the value of a game (where both players have the same number of salvos) of increasing the number of salvos for both players. Such an increase does improve the value of the game for the player with the better P_h function. This follows intuition in a manner analogous to the trend discussed in case 3. That is, by providing the favored player with more options, we will increase the value of the game. The increase is not particularly large in this example, but it exists nonetheless.

Case 7:

Figure 7 shows the effect of degrading both players' P_h functions. Specifically, when both players have identical P_h functions, and one player has an advantage in salvos, the favored player prefers the P_h function that is less "deadly", i.e., that has the smallest P_h associated with each separation. This preference runs counter to the intuitive trend exhibited in cases 3 and 6 (providing an improvement to both players helps the favored player). No rationale for this situation is obvious.

Case 8:

This case, shown in figure 8, differs from the others by plotting salvos for player 1 on the abscissa, rather than initial separation. These curves, along with the results from case 1, support the hypothesis that the value of a game where the players have identical P_h functions is directly proportional to the ratio of their salvos. The curves cross the x-axis where player 1 has five salvos because it is at this point that both players have the same number of salvos (and the same P_h function) and hence a game value of zero.

CONCLUSIONS

The conclusions drawn from these parametric analyses can be grouped into three main areas:

- o Discrete versus continuous games
- o The use of intuition in predicting game values
- o Twice the salvos versus twice the accuracy.

As discussed previously, the development of a program that solves discrete duels, combined with the tendency of discrete duel values to approach those of continuous duels, has important ramifications. Previously only certain continuous duels could be solved. Now many continuous duels can be "solved" by substituting the analogous discrete duel with many small distance gradations.

The second main conclusion that can be drawn from this work is that intuition should not be used for predicting the value of a game based on its parameters. Usually general trends in the value of games can be surmised, but this is not universally true, as shown by case 7. Also, the point at which a game's value changes sign, derivative, sign of second derivative, and so on, cannot be intuited.

The final conclusion has important implications for U.S. weapons appropriations. By proving that it is better (i.e., the value of the game is higher) to have half the salvos with twice the accuracy rather than the reverse, this paper supports established U.S. weapon appropriations policy, which is to buy fewer (more expensive) weapons with greater effectiveness rather than many weapons with lesser capability as the Soviet Union does in general. Note that this is true for ratios other than two (for example, having two-thirds the number of salvos at three-halves the accuracy is better than the reverse).

```
10    ! PROGRAM SOLVE        BY HENRY HERZ       1 DEC 83
20    !
30    ! THIS PROGRAM SOLVES GAMES OF TIMING (DISCRETE, NOISY,
40    ! TWO-PLAYER DUELS OF UP TO 10 SHOTS FOR
50    ! EACH PLAYER AND UP TO 20 INTER-PLAYER DISTANCES.
60    !
70    ! NOTE: ALL ARRAYS ARE OPTION BASE 0.
75    !        % SYMBOL FOLLOWING VARIABLE NAMES SIGNIFIES INTEGER.
80    !
90    DIM T(10,10,20,18)      ! TABLEAUS STORING DECOMPOSED DUELS
94    DIM V(10,10,20)         ! VALUE OF DUEL (K,J:I)
99    DIM FLAG%(10,10,20)     ! 1 IF DUEL (K,J:I) HAS BEEN SOLVED
100   DIM P1(20),P2(20)       ! PROBABILITY OF HIT AT SEPARATION I
110   !
120   !  PROMPT USER FOR PARAMETERS WHICH DEFINE THE DUEL
200   INPUT "ENTER INITIAL SEPARATION OF SHIPS",IS%
210   INPUT "ENTER # OF SALVOES FOR PLAYERS I AND II",N1%,N2%
220   PRINT "ENTER PK FUNCTION OF FORM: AD^2+BD+C"
230   INPUT "  FOR PLAYER I:",A1,B1,C1
240   INPUT "  FOR PLAYER II:",A2,B2,C2
250   !
255   !  CALCULATE P1 AND P2 FOR EVERY INTER-SHIP SEPARATION
260   FOR I%=0% TO IS%
270     P1(I%)=A1*I%^2 + B1*I% + C1
274       IF P1(I%)>1 THEN P1(I%)=1
278       IF P1(I%)<0 THEN P1(I%)=0
290     P2(I%)=A2*I%^2 + B2*I% + C2
284       IF P2(I%)>1 THEN P2(I%)=1
288       IF P2(I%)<0 THEN P2(I%)=0
290   NEXT I%
300   !
304   !  LOOP ON EVERY (DECREASING) COMBINATION OF N1, N2 AND SEPARATION.
308   !  IF DUEL IS TRIVIAL, SOLVE IT.  IF NOT, STORE DECOMPOSED DUEL IN
309   !  ARRAY T (TABLEAU), WHICH WILL BE SOLVED LATER.
310   FOR I%=IS% TO 0% STEP -1%
320     FOR J%=N2% TO 0% STEP -1%
330       FOR K%=N1% TO 0% STEP -1%
340         IF (J%=0%) OR (K%=0%) OR (J%>I%+1%) OR (K%>I%+1) OR (I%=0%)
              THEN GOSUB Trivial
              GOTO Next1
            END IF
390   !
400   !  STORE THE DECOMPOSED DUEL AS THE 18 COMPONENTS OF A TABLEAU
410         T(K%,J%,I%,1%)=P1(I%)*(1%-P2(I%))-P2(I%)*(1%-P1(I%))
420         T(K%,J%,I%,2%)=(1%-P1(I%))*(1%-P2(I%))
430         T(K%,J%,I%,3%)=K%-1%
440         T(K%,J%,I%,4%)=J%-1%
450         T(K%,J%,I%,5%)=I%-1%
460         T(K%,J%,I%,6%)=P1(I%)
470         T(K%,J%,I%,7%)=1%-P1(I%)
480         T(K%,J%,I%,8%)=K%-1%
490         T(K%,J%,I%,9%)=J%
500         T(K%,J%,I%,10%)=I%-1%
510         T(K%,J%,I%,11%)=-P2(I%)
520         T(K%,J%,I%,12%)=1%-P2(I%)
530         T(K%,J%,I%,13%)=K%
540         T(K%,J%,I%,14%)=J%-1%
550         T(K%,J%,I%,15%)=I%-1%
560         T(K%,J%,I%,16%)=K%
570         T(K%,J%,I%,17%)=J%
580         T(K%,J%,I%,18%)=I%-1%
```

```
590  Next1:NEXT K%
600    NEXT J%
610  NEXT I%
620  !
625  !  LOOP ON EACH (INCREASING) COMBINATION OF N1, N2 AND SEPARATION.
627  !  IF DUEL HAS BEEN SOLVED, SKIP TO NEXT ONE.  IF NOT, CALCULATE
628  !  THE VALUES WITHIN THE FOUR "BOXES" OF THE TABLEAU, AND SOLVE.
630  FOR K%=0% TO N1%
640    FOR J%=0% TO N2%
650      FOR I%=0% TO IS%
660        IF FLAG%(K%,J%,I%)=1% THEN GOTO Next2
670  !
680        A=V(T(K%,J%,I%,3%),T(K%,J%,I%,4%),T(K%,J%,I%,5%)) &
* T(K%,J%,I%,2%) + T(K%,J%,I%,1%)
700        B=V(T(K%,J%,I%,8%),T(K%,J%,I%,9%),T(K%,J%,I%,10%)) &
* T(K%,J%,I%,7%) + T(K%,J%,I%,6%)
720        C=V(T(K%,J%,I%,13%),T(K%,J%,I%,14%),T(K%,J%,I%,15%))&
* T(K%,J%,I%,12%) + T(K%,J%,I%,11)
740        D=V(T(K%,J%,I%,16%),T(K%,J%,I%,17%),T(K%,J%,I%,18%))
750        GOSUB Game
760  Next2:NEXT I%
770    NEXT J%
780  NEXT K%
785  !
790  !  PRINT OUT THE VALUE OF THE DUEL AND ALL "LESSER" DUELS
792  PRINT
794  PRINT
796  PRINT
798  PRINT
800  PRINT "SOLUTIONS TO DUELS WITH:"
810  PRINT
815  PRINT "   PK FCN, PLAYER I =":A1;"D^2 +";B1;"D +";C1
820  PRINT "   PK FCN, PLATER II=";A2;"D^2 +";B2;"D +";C2
842  PRINT
843  PRINT "INITIAL      INITIAL       INITIAL       GAME"
844  PRINT "SALVOES, I   SALVOES, II   SEPARATION    VALUE"
845  PRINT "----------   -----------   ----------    -----"
846  FOR K%=N1% TO 1% STEP -1%
848    FOR J%=N2% TO 1% STEP -1%
850      FOR I%=IS% TO 1% STEP -1%
855      PRINT "    ";K%;"          ";J%;"           ";I%;"       ";V(K%,J%,I%)
860      NEXT I%
861    NEXT J%
862  NEXT K%
865  GOTO 1500     ! END
866  !-----------------------------------------------------------
870  Trivial: ! THIS SUB SOLVES "TRIVIAL" GAMES
880  !
882  ! IF NEITHER PLAYER HAS ANY SALVOES REMAINING, VALUE = 0.
885  IF (J%=0%) AND (K%=0%)
       THEN V(K%,J%,I%)=0
            FLAG%(K%,J%,I%)=1%
            RETURN
       END IF
887  !
889  ! IF PLAYER 2 HAS NO SALVOES LEFT, BUT PLAYER 1 DOES, VALUE = 1.
890  IF J%=0%
       THEN V(K%,J%,I%)=1
            FLAG%(K%,J%,I%)=1%
            RETURN
       END IF
```

```
900  !
910  ! IF PLAYER 1 HAS NO SALVOES LEFT, BUT PLAYER 2 DOES, VALUE = -1.
930  IF K%=0%
       THEN V(K%,J%,I%)=-1
            FLAG%(K%,J%,I%)=1%
            RETURN
       END IF
940  !
950  ! IF SEPARATION IS 0, AND BOTH PLAYERS HAVE SALVOES LEFT, VALUE=0.
1010 IF (I%=0%)
       THEN V(K%,J%,I%)=0
            FLAG%(K%,J%,I%)=1%
            RETURN
       END IF
1020  !
1030  !  IF EITHER OR BOTH PLAYERS HAS MORE SALVOES THAN CAN POSSIBLY
1040  !  BE FIRED IN THE REMAINING OPPORTUNITIES, REDUCE THE NUMBER
1050  !  OF SALVOES TO THE NUMBER OF REMAINING OPPORTUNITIES.
1090 IF (J%>I%+1%) OR (K%>I%+1%)
       THEN KK%=K%
       !
       !  SET KK% = MIN ( I+1 , K )
       IF (I%+1%<K%) THEN KK%=I%+1%
         END IF
       JJ%=J%
       !
       !  SET JJ% = MIN ( I+1 , J )
       IF (I%+1%<J%) THEN JJ%=I%+1%
         END IF
       T(K%,J%,I%,16%)=KK%
       T(K%,J%,I%,17%)=JJ%
       T(K%,J%,I%,18%)=I%
       !
       !  SET FLAG TO SHOW THAT ALTHOUGH SIMPLIFIED, THE DUEL
       !  HAS NOT YET BEEN SOLVED AND REQUIRES SPECIAL HANDLING.
       FLAG%(K%,J%,I%)=2%
     END IF
1100 RETURN
1260 !-----------------------------------------------------------
1270 Game:  ! THIS SUB SOLVES A 2X2 MATRIX GAME
1280 !
1281 !  IF GAME WAS SIMPLIFIED, BUT NOT SOLVED, USE THIS SPECIAL CODE.
1283 IF FLAG%(K%,J%,I%)=2%
       THEN V(K%,J%,I%)=0
       FLAG%(K%,J%,I%)=1%
       RETURN
     END IF
1284 !
1286 !  DETERMINE IF GAME HAS A PURE STRATEGY EQUILIBRIUM PAIR
1288 EP=9999
1290 IF (A<=B) AND (A>=C) THEN EP=A
1300 IF (B<=A) AND (B>=D) THEN EP=B
1310 IF (C<=D) AND (C>=A) THEN EP=C
1320 IF (D<=C) AND (D>=B) THEN EP=D
1330 IF EP<9999                               !  IF GAME HAS AN E.P.,
       THEN V(K%,J%,I%)=EP                    !  THEN STORE THIS VALUE,
       ELSE V(K%,J%,I%)=(A*D-B*C)/(A+D-B-C)   !  ELSE CALCULATE THE
       END IF                                 !  VALUE OF THE MIXED
1420 FLAG%(K%,J%,I%)=1%                       !  STRATEGY SOLUTION.
1430 RETURN
1440 !-------------------------------------------------------------------
1500 END
```

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