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CONTRACT NO. N00167-81-D-0184 (D.O. 0003)

TECHNICAL REPORT

AREA CLEARANCE AND MULTIPLE ENGAGEMENT METHODOLOGY

SRC TR NO. 96-83

1 FEBRUARY 1983

PREPARED FOR:

DAVID TAYLOR NAVAL SHIP
RESEARCH & DEVELOPMENT CENTER
BETHESDA, MARYLAND 20084

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1.0 INTRODUCTION

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1.0 INTRODUCTION

The Military Effectiveness Office of the David Taylor Naval Ship Research and Development Center (DTNSRDC Code 1806) has been supporting the Naval Sea Systems Command (PMS 393) SSN Combat Systems Engineering and Analysis (CSE&A) Program by producing the Attack Submarine Development Plan (ASDP). As part of this effort, a methodology for evaluating the military payoffs of adding various advanced equipments to the baseline SSN 716 has been developed. This report addresses an effort by Summit Research Corporation (SRC) directed at improving and extending the SSN CSE&A evaluation models dealing with two areas of SSN performance including:

(1) SSN effectiveness in an area clearance mission,

(2) SSN effectiveness in multiple engagement scenarios.

This is a continuation of an effort that was halted due to funding cutbacks. Results of the previous effort were reported in SRC TR70-82.

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2.0 AREA CLEARANCE MISSION

2.0 AREA CLEARANCE MISSION

Area clearance is characterized by the existence of a specific geographic ocean area which must be cleared of enemy submarines within a given time. The area clearance operation usually occurs in advance of an attack force deploying in or moving through the area that has been cleared.

The following sections of the report present the proposed area clearance algorithm (Section 2.1), computer model inputs (Section 2.2) and model results (Section 2.3).

2.1 AREA CLEARANCE ALGORITHM

The Measure of Effectiveness (MOE) currently used in the CSE&A program for area clearance is the probability of detecting a single target randomly situated in area (A) given that the searcher has a particular sweep width (W) against the target and travels at velocity (V) searching during a time period (T). The equation for the probability of detection is:

$$P_D = 1 - \exp(-VWT/A) \qquad \text{(Eq. I)}$$

The inherent problem with use of the above defined probability of detection as an MOE for the area clearance mission is that it calculates the probability of detecting only a single unit that is randomly situated in the area. It does not account for the fact that more than one target may be present in the area or that there may be no targets at all present in the designated area. The MOE that is developed in this section is the probability that there are no undetected targets in area A after a search of duration T. This is felt to be a more realistic MOE for area clearance than that used in the CSE&A program.

The targets, for this algorithm, are assumed to be located randomly in a two-dimensional field with the average number of targets per unit area equal to μ. That is, every small area, a, contains a target with probability μa. The probability of more than one target in, a, is negligible if a is sufficiently small. Also, the number of targets in non-overlapping regions are independent random variables. These assumptions are consistent with a Poisson distribution.

Let A be the area of a region within the field that is not necessarily small and let N_A be the number of targets in it. As developed in Reference 1 the probability that area A contains n targets is

$$P(N_A=n) = \frac{(\mu A)^n}{n!} e^{-\mu A}, \quad n=0,1,2,\ldots,$$

This is the Poisson distribution and the field of targets is termed a "Poisson field."

The probability that exactly n targets are detected given that n targets are present is

$$P_n = \left(1-e^{-\frac{W\tilde{V}T}{A}}\right)^n$$

where

> P_n is the probability that exactly n targets are detected
> W is the sweepwidth
> $\tilde{V}$ is the effective search speed
> A is the area to be searched
> T is the time duration

The "effective speed" is the average relative speed between searcher and target (u,v) assuming the angle between their velocity vectors (u,v) are uniformly distributed between 0 and 2π.

$$\tilde{V} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{u^2+v^2-2uv\cos\Theta}\,d\Theta$$

By changing the variable of integration to $\Psi = (\pi-\Theta)/2$ and introducing $\sin\Phi = 2\sqrt{uv}/(u+v)$, the equation for V reduces to:

$$\tilde{V} = \frac{2}{\pi}(u+v)E(\sin\Phi) \qquad \text{(Eq. II)}$$

where $E(\sin\Phi)$ is the complete elliptic integral of the second kind and $\sin\Phi$ is as defined above.

The probability that there are no undetected targets in Area A after search of duration T becomes the summation of the individual terms $P_nP(N_A=n)$ and thus:

P (no undetected targets in A after search of duration T)

$$= \sum_{n=0}^{\infty} \frac{(\mu A)^n}{n!} e^{-\mu A} \left(1 - e^{-\frac{W\tilde{V}T}{A}}\right)^n$$

letting $p = 1-e^{-\frac{W\tilde{V}T}{A}}$, multiplying each term by $e^{-p\mu A}e^{p\mu A}$ and regrouping we get:

$$P = \sum_{n=0}^{\infty} \frac{(p\mu A)^n}{n!} e^{-p\mu A} e^{-\mu A(1-p)}$$

$$= e^{-\mu A(1-p)} \sum_{n=0}^{\infty} \frac{(p\mu A)^n}{n!} e^{-p\mu A}$$

where the infinite sum is over the individual terms of the Poisson distribution and is identically equal to 1. The expression therefore reduces to:

$$P = e^{-\mu A(1-p)}$$

$$= e^{-\mu A e^{-\frac{W\tilde{V}T}{A}}} \qquad \text{(Eq. III)}$$

This is the probability that no undetected targets remain in area A after a search of duration T. The average number of targets in area A is μA.

2.2 AREA CLEARANCE ALGORITHM INPUTS

A computer program has been written to calculate the measure of effectiveness for area clearance as given in Equation III. From this equation it is seen that 5 variables are required to calculate the probability.

These are: μ, A, W, $\tilde{V}$, and T. Of these W and $\tilde{V}$ are derived variables determined from operational parameters.

To calculate the effective search speed (Eq. II), the necessary inputs are as follows:

v - Own ship search speed (knots)

u - Threat speed (knots)

The effective search speed is then input to the sweepwidth computation subprogram along with the following input data:

Figure of Merit (FOM) in dB

A propagation loss curve. - The curves given in the CSE&A study; a generic form of A + B log R + CR where R is the range in nm; or any other propagation loss function may be used.

Standard deviation of signal excess fluctuations (σ) in dB

Relaxation time of the Jump model ($1/\lambda$) in minutes

Minimum instantaneous detection probability

Once $\tilde{V}$ and W have been calculated the required probability is computed using the following additional inputs:

Area to be searched in sq. nm (A)

Search time allotted in hours (T)

Target density in number per sq. nm (μ)

2.3 MODEL RESULTS

The output of the model is the effective search speed, the sweepwidth and the probability that there are no undetected targets remaining in area A after a search of duration T.

As an example of the difference in results betwee computing the area clearance MOE by use of Equation I (CSE&A study) and e proposed MOE (Eq. III) the following example is offered:

For own ship searching at 10 knots against a eat operating at 5 knots in an area (A) that is 240 x 240 nm with a search duration (T) of 240 hours and a target denisty (μ) of 1/A (mean number of targets in area A is 1) the following table lists the probability of detection computed from Equations I and III along with the sweepwidths computed for the given FOM's:

FOM	W	P_D(Eq. I)	P_D(Eq. III)
70	8.4	.3108	.4017
75	18.6	.5614	.4430
80	42.0	.8445	.5337
85	94.2	.9846	.7032
90	204.2	.9999	.9011

The effective search speed ($\tilde{V}$) computed from Equation II was 10.64 knots. The MOE of Equation III is not as sensitive to increases in sweepwidth as that from the CSE&A study (Eq. I).

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3.0 SSN EFFECTIVENESS IN MULTIPLE ENGAGEMENT SCENARIOS

3.0 SSN EFFECTIVENESS IN MULTIPLE ENGAGEMENT SCENARIOS

The following sections of the report provide: the reasoning for the methodology development (Section 3.1); the definition of Markov Chains (Section 3.2); formulation of the multiple engagement scenario as a Markov Chain (Section 3.3); the algebraic representation (Section 3.4); the definition of the initial states (Section 3.5); the definition of the transition probabilities for the attack transition matrix (TM) (Section 3.6); the definition of the transition probabilities for the transition to attack phase matrix (TM*) (Section 3.7); the computational considerations (Section 3.8); and the description of computer program inputs and results (Section 3.9).

3.1 INTRODUCTION

The analysis presented in the CSE&A study only considers the initial attack by the more capable U.S. submarine and the outcome of this attack in terms of the probability of killing the threat without being counterkilled. An engagement does not necessarily end with the initial attack. Realistically if either side feels it has an advantage it will reattack. A multiple engagement scenario requires additional analytic tools and highlights different areas of own ship and threat capability and effectiveness.

The FOM advantage enjoyed by passive U.S. sensor systems is less significant once the other platform is alerted by an attack and the possibility of gaining information from a weapon launch is taken into account. The probability of the attacked ship using his active sonar increases dramatically once an initial attack is made. This possibility of using active sensors in the engagement increases the importance of active sonar effectiveness, coating effectiveness, ping stealing capability, etc. The advantage of the U.S. submarine may be diminished by including the use of active sensors in an engagement model.

After the initial attack the maximum speed and acceleration capability as well as the depth capability are important factors in considering evasion by either platform. The self noise of the ship under attack while executing high speed evasion maneuvers will influence the capability of that ship to detect and localize his adversary during that time period. Also the radiated noise that the attacked ship emits while attempting to evade determines the capability of the attacker to mount a subsequent attack.

The ability of the unit under attack to survive would be enhanced if a capability to detect and track the incoming torpedo were available. This capability would improve evasion tactics and/or countermeasure (CM) placement tactics and usage.

Launch rate/constraints, reload time, number of launchers and radiated launch noise all affect the outcome of a multiple engagement scenario. Weapon characteristics such as minimum launch range, endurance, post-launch constraints, CM vulnerability and detectability are considerably more important in the proposed scenario. The types and number of CM devices as well as the CM reaction time are also more significant than in the one-shot analysis.

For multiple engagement models there are two obvious approaches that could be followed. One approach would be to simulate successive engagements in a Monte Carlo simulation. Monte Carlo simulations tend to be time consuming and expensive to create and run. An alternative approach is to use Markov analysis which tends to be simpler to formulate and to obtain results.

A key feature of formulating the stochastic process as a Markov Chain is the Markovian property. The property is such that the conditional probability of any future "event", given any past event and the present state, is independent of the past event and depends upon only the present state of the process. Transitions from the present state depend solely on the present state of the system. This enables one to look at the effects of changes in equipment lineups, tactics, etc. by assessing the effect of the changes during the time periods applicable to these changes. That is, determining the probabilities of transitioning from the present state into any of the numerous possible future states can be accomplished without consideration of past events.

Formulating successive engagements in a Monte Carlo simulation usually requires rerunning the entire simulation whenever changes in tactics or equipment configurations occur. Markov analysis also yields a better understanding of the cause-effect relationships of changes in the formulation of the problem than do large scale Monte Carlo simulations which tend to mask these relationships.

The basis for using a multiple engagement scenario is therefore to assess the total system capability as opposed to the selective and restricted analysis provided by the single engagement scenario. Once it is determined that a multiple engagement scenario is desirable then the Markov Analysis offers benefits over using a Monte Carlo simulation.

3.2 MARKOV CHAINS

A stochastic process is simply defined to be an indexed collection of random variables $\{X_t\}$. At particular points of time, t, labeled 0, 1,..., the system exists in exactly one of a finite number of mutually exclusive and exhaustive categories or states labeled 0, 1,..., M. The points in time may be equally spaced, or their spacing may depend upon the overall behavior of the physical system in which the stochastic process is imbedded, e.g., the time between occurrences of some phenomenon of interest. Thus the mathematical representation of the physical system is that of a stochastic process $\{X_t\}$, where the random variables are observed at t = 0, 1, 2,..., and where each random variable may take on any one of the (M + 1) integers 0, 1,..., M. These integers are a characterization of the (M + 1) states of the process.

A stochastic process $\{X_t\}$ is said to have the Markovian property if $P\{X_{t+1} = j | X_0 = k_0, X_1 = k_1, \ldots, X_{t-1} = k_{t-1}, X_t = i\} = P\{X_{t+1} = j | X_t = i\}$, for t = 0, 1,... and every sequence $i, j, k_0, k_1, \ldots, k_{t-1}$. This is the mathematical statement of the Markovian property that the future state of the system depends solely on the present state of the system.

The conditional probabilities $P\{X_{t+1} = j | X_t = i\}$ are called transition probabilities. If, for each i and j,

$$P\{X_{t+1} = j | X_t = i\} = P\{X_1 = j | X_0 = i\}, \text{ for all } t = 0, 1, \ldots,$$

then the (one step) transition probabilities are said to be stationary and are usually denoted by P_{ij}. Thus, having stationary transition probabilities implies that the transition probabilities do not change in time.

A convenient notation for representing the transition probabilities is the matrix form

	State	0	1...	M
	0	P_{oo}	...	P_{mo}
	1	.		.
P =	.	.		.
	.	.		.
	.			
	M	P_{om}	...	P_{mm}

or equivalenty

$$P = \begin{bmatrix} P_{00} & \cdots & P_{m0} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ P_{0m} & \cdots & P_{mm} \end{bmatrix}.$$

It is now possible to define a Markov chain. A stochastic process $\{X_t\}$ ($t = 0, 1, \ldots$) is said to be a finite-state Markov chain if it has the following:

1. A finite number of states,
2. The Markovian property,
3. Stationary transition probabilities,
4. A set of initial probabilities $P\,\{X_0 = i\}$ for all i.

Further information on the properties of Markov chains may be obtained from any of the numerous Operations Research texts.

3.3 MULTIPLE ENGAGEMENT MARKOV FORMULATION

In order to complete the formulation of the multiple engagement scenario, the states, the time steps, the initial set of probabilities, and the transition probabilities must be defined.

For each submarine six states will be considered, totalling thirty-six possible states, when considering both submarines.

The six states for each of the submarines are defined as follows:

1. No detection

2. Detection--Sensor contact (one or more sensors) is gained on a target present in the attack submarine operating area (can be in alerted or unalerted state)

3. TMA/Fire Control Solution--Having been detected the target is localized and its motion analyzed with sufficient accuracy to place the submarine in a position capable of attaining the operational objective

4. Attainment of Firing Position--Target is physically approached, once detected and localized, to within a distance sufficient for weapon firing

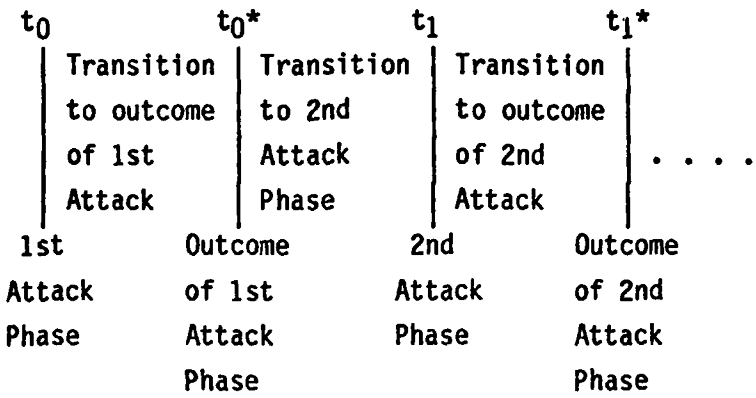
5. Attack--Given target has been detected, localized and approached, the target is properly classified and the weapon is selected, made ready, loaded and launched

6. Kill--The attack is successful

The states are defined in this manner so that transitions from any state to a higher state is sequential. In going from State 1 (no detection) to State 4 (attainment of firing position) you must pass through State 2 (detection) and State 3 (TMA/fire control solution) and not be able to obtain State 5 (attack). Thus in transitioning from 1 to 4, the probability of attaining firing position probabilistically includes those times when detection and localization have occurred within firing range as well as when closing is necessary. In addition, not being able to attain State 5 means that either classification is not obtained or the weapon launch is unsuccessful.

The analysis, as configured, exhibits the Markovian property. The main purpose for formulating the process as a Markov chain was to develop a method to compare equipment capabilities, procedures, tactics, etc. in a multiple engagement scenario. As such, for a given scenario the method of transition from a state is specified and does not rely on how that state was attained. For example, given detection is achieved using a particular sonar, the method for localization does not depend on how the detection was achieved. Localization will occur using specified equipment, TMA algorithms, etc. This is true for all transitions because it was pre-defined in this manner. This method of analysis is very similar to that used in the CSE&A study.

The time steps for the Markov process are defined to be the period of successive attacks or re-attacks by either or both submarines. The time step was further divided into two periods to create a two-step Markov process as shown in the following figure.



The initial point of analysis (t_0), is where one or both of the submarines have attacked. The definition of t_0 is that the weapon is launched and is in the water a sufficient time to allow a possible detection of the launch (ejection from the tube only). This definition was specifically chosen to allow detection of a launch to be a classification aid. In addition, it allows inclusion of a "snapshot" weapon return to be considered as a near simultaneous counteraction and not as an attack (State 5) which can only be reached (by definition) by detecting, localizing, etc. The "snapshot" return is based solely on firing a weapon along the bearing at which a launch by the opposing submarine is detected. This implies that the submarine is either in a state of non-detection (State 1) or detection (State 2) when launch is detected. In any other state, the submarine has too much information to fire blindly. Any use of the detection of a launch that results in acquiring a valid fire control solution and results in a weapon launch is not considered to be instantaneous and is therefore considered to be another attack phase (i.e., t_1, t_2, etc.).

The intermediate time step (t_0*) denotes the state of the process as a result of the attack. The time step will be determined by either the detonation or run out of a weapon. Near simultaneous attacks by opposing submarines are considered to be part of the same attack phase. The transition from t_0 to t_0* will be represented by a transition matrix whose transition probabilities are stationary. That is, the transition matrices applicable to the time steps t_0 to t_0*, t_1 to t_1*,... will be the same but the probability distribution of initial and outcome states will be different.

The time step t_0* to t_1 represents the transitions from the distribution of states after an attack, to the second attack phase where one or both of the submarines can attack. Time t_1, t_2,... are defined in the same manner as t_0. Once again, the transition probabilities are the same (stationary) for the time period t_0* to t_1, t_1*, to t_2, etc. but the distributions of states at t_0*, t_1*,... are different.

The Markov process is really the transition from one attack phase to the next (t_n to t_{n+1}). It is easier to visualize and describe the analytic process if the two-step transition is used. The one-step Markov process results from multiplying the transition matrix representing transitions t_0 to t_0* by the transition matrix for t_0* to t_1.

The length of the time steps (t_n to t_n* or t_n to t_{n+1}) is determined by the scenario. The length of the time steps do not impact the correctness of the analysis.

3.4 ALGEBRAIC FORMULATION

We will denote the 36 states of the system as a 36 x 1 element vector whose elements are Q_{11}, Q_{12},... Q_{21}, Q_{22},... Q_{66}. The first number in the subscript refers to the state of own ship and the second number represents the state of the threat submarine (i.e., Q_{52} refers to the state where the U.S. submarine has launched an attack (State 5) but has been detected by the threat (State 2)). The two transition matrices representing transition from times t_0 to t_0* and from t_0* to t_1 will each be 36 x 36 matrices called **TM** and **TM*** whose elements are $P_{ij,kl}$. The term $P_{ij,kl}$ expresses the probability of transitioning from state Q_{ij} to state Q_{kl}. Therefore, determining the probability distribution of states at time t_0* (Q_{t_0}*) given the initial probabilities (Q_{t_0}) and the transition matrix TM is

$$Q_{t_0}^* = TM \cdot Q_{t_0}$$

Applying the second transition matrix TM* to Q_{t_0}* will yield the probability distribution of states at time t_1 (Q_{t_1})

$$Q_{t_1} = TM^* \cdot Q_{t_0}^* = TM^* \cdot TM \cdot Q_{t_0}$$

Successive application of TM* and TM will yield the probability distributions at times t_1*, t_2, t_2*,... In full matrix notation the probability distribution at time t_1 will be

$$Q_{t_1} \quad = \quad TM^* \quad \cdot \quad TM \quad \cdot \quad Q_{t_0}$$

$$\begin{bmatrix} Q_{11} \\ Q_{12} \\ \cdot \\ \cdot \\ \cdot \\ Q_{21} \\ Q_{22} \\ \cdot \\ \cdot \\ \cdot \\ Q_{66} \end{bmatrix} = \begin{bmatrix} P_{11,11} & P_{12,11} & \cdots & P_{66,11} \\ P_{11,12} & P_{12,12} & \cdots & P_{66,12} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{11,21} & P_{12,21} & & P_{66,21} \\ P_{11,22} & P_{12,22} & & P_{66,22} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{11,66} & P_{12,66} & \cdots & P_{66,66} \end{bmatrix} \begin{bmatrix} P_{11,11} & P_{12,11} & \cdots & P_{66,11} \\ P_{11,12} & P_{12,12} & \cdots & P_{66,12} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{11,21} & P_{12,21} & & P_{66,21} \\ P_{11,22} & P_{12,22} & & P_{66,22} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{11,66} & P_{12,66} & \cdots & P_{66,66} \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{12} \\ \cdot \\ \cdot \\ \cdot \\ Q_{21} \\ Q_{22} \\ \cdot \\ \cdot \\ \cdot \\ Q_{66} \end{bmatrix}$$

For the remainder of this report probabilities referring to the threat submarine will be denoted by a prime (superscript) whereas probabilities relating to own ship will be unprimed.

The notation for the probability distribution of states Q_{ij} can be expressed as follows:

$$Q_{ij} = q_i q_j'$$

where q_i refers to own ship state and q_j refers to the threat submarine state. In a similar fashion, the transition probabilities $P_{ij,kl}$ can be expressed:

$$P_{ij,kl} = P_{i,k} P_{j,l}'$$

where once again the primed expression refers to the threat submarine.

3.5 INITIAL STATES

In this section, the probability of being in each of the initial states will be described. The probability elements determining this distribution will be introduced and mathematical expressions will be provided. Appendix A defines all of the individual probability elements used in this section and the following sections of the report.

At time t_0, the only states possible are those where one or both of the submarines has launched an attack on the other submarine. The probability of being in any state where no attack has been launched is zero. Therefore, only the non-zero elements in the initial state vector will be described below.

In order to have launched an attack (state 5), given that the opposing submarine has not attacked, the submarine must have detected (D), gained a fire control solution (FCS), reached a firing point (FP), classified

(CL) and successfully launched a weapon (WL). The probability of having launched an attack not in response to the opposing submarine is

$$q_5 = P_D \, P_{FCS} \, P_{FP} \, P_{CL} \, P_{WL}$$

To launch an attack (state 5) when the opposing submarine has launched an attack, the submarine must detect (D), gain a fire control solution (FCS), reach a firing point (FP), classify (TCL) by normal classification processes (CL) or by detection of a weapon launch from the opposing submarine (DWL), and successfully launch a weapon (WL). The probability of having launched an attack once the opposing submarine attacks is

$$q_5 = P_D \, P_{FCS} \, P_{FP} \, P_{TCL} \, P_{WL}$$

where

$$P_{TCL} = P_{CL} + (1 - P_{CL}) \, P_{DWL}$$

The probability of being in State 15 (Q_{15}) is the probability that own ship has not detected the threat ($1 - P_D$) times the probability that the threat ship has attacked and is equal to:

$$Q_{15} = (1 - P_D) \, P_D' \, P_{FCS}' \, P_{FP}' \, P_{CL}' \, P_{WL}'$$

The probability of being in State 25 (Q_{25}) is the probability that own ship has detected (D) the threat but not yet gained a fire control solution (FCS) times the probability that the threat ship has attacked and is equal to:

$$Q_{25} = P_D \, (1 - P_{FCS}) \, P_D' \, P_{FCS}' \, P_{FP}' \, P_{CL}' \, P_{WL}'$$

The probability of being in State 35 (Q_{35}) is the probability that own ship has detected (D) and localized (FCS) but not yet closed (FP) times the probability that the threat ship has attacked and is equal to:

$$Q_{35} = P_D\ P_{FCS}\ (1 - P_{FP})\ P_D'\ P_{FCS}'\ P_{FP}'\ P_{CL}'\ P_{WL}'$$

The probability of being in State 45 (Q_{45}) is the probability that own ship had detected (D), localized (FCS), and reached a firing point (FP) but has not classified (TCL) or successfully launched a weapon (WL) times the probability that the threat submarine has launched an attack and is equal to:

$$Q_{45} = P_D\ P_{FCS}\ P_{FP}\ (1 - P_{TCL}\ P_{WL})\ P_D'\ P_{FCS}'\ P_{FP}'\ P_{CL}'\ P_{WL}'$$

The definitions of the probability distribution for states Q_{51}, Q_{52}, Q_{53}, and Q_{54} are the same as those described for Q_{15}, Q_{25}, Q_{35}, and Q_{45} with an interchange of the words own ship and threat and the primed and unprimed probabilities and are as follows:

$$Q_{51} = P_D\ P_{FCS}\ P_{FP}\ P_{CL}\ P_{WL}\ (1-P_D')$$

$$Q_{52} = P_D\ P_{FCS}\ P_{FP}\ P_{CL}\ P_{WL}\ P_D'\ (1-P_{FCS}')$$

$$Q_{53} = P_D\ P_{FCS}\ P_{FP}\ P_{CL}\ P_{WL}\ P_D'\ P_{FCS}'\ (1-P_{FP}')$$

$$Q_{54} = P_D\ P_{FCS}\ P_{FP}\ P_{CL}\ P_{WL}\ P_D'\ P_{FCS}'\ P_{FP}'\ (1 - P_{TCL}'\ P_{WL}')$$

The probability of being in state 55 (Q_{55}) is the probability of both submarines attacking where classification is by normal means or detection of opposing submarines weapon launch and is equal to:

$$Q_{55} = P_D P_{FCS} P_{FP} P_{WL} (P_{CL} + (1-P_{CL})P_{DWL}) P'_D P'_{FCS} P'_{FP} P'_{WL} (P'_{CL} + (1-P'_{CL})P'_{DWL})$$

As stated previously, detection of weapon launch by opposing submarine is an instant classification aid. Therefore, if the submarine under attack has detected, localized and closed (i.e. is in state 4) but not classified and a weapon launch is detected, an immediate transition to the attack state (5) is hypothesized. Conversely, the only way for a submarine under attack to be in state 4 is for the submarine to have missed detecting the weapon launch.

The definition of the unprimed and primed probabilities are the same. The values of the probabilities, however, differ since they reflect differing equipments and capabilities.

3.6 ATTACK TRANSITION MATRIX (TM)

As stated in prior discussion, the transition probabilities $P_{ij,kl}$ are the product of the individual transition probabilities of ownship and threat submarine. The attacking submarine can transition from the attack state (State 5) to any state including State 5. The submarine under attack may be in any state at the beginning of attack (including the attack state) and can transition to any state including the state that it is in.

For the submarine under attack the following transition probabilities are applicable. (As much descriptive material as possible will be included in defining the transition probabilities; however, continuously defining the same or similar sequences will be elimiated as much as possible).

The probability that the submarine remains in State 1 (no detection) given that it was in State 1 at the time of the attack is simply one minus the sum of the probabilities of transitioning to any other state from State 1 or

$$P_{1,1} = 1 - \sum_{k=2}^{6} P_{1,k}$$

since the sum of all transition probabilities from any state must equal one.

The probability of the attacked submarine transitioning from State 1 (non-detection) to State 2 (detection) is the probability that the submarine is not killed $(1 - P_k')$ and either detects (D) by normal passive means or detects after being alerted (DA) by a detection of a weapon launch of the opposing submarine but does not localize before the end of the attack phase. The expression for this is

$$P_{1,2} = (1 - P_k') \left[P_D (1 - P_{DWL}) + P_{DWL} P_{DA}\right] (1 - P_{FCS})$$

where the probability of kill for the opposing submarine is further defined as the probability that the weapon will acquire and kill either an unalerted target (no detection of weapon launch) or an alerted evading target (weapon launch detected) and is

$$P_k' = (1 - P_{DWL}) P_{ACQ}' P_H' + P_{DWL} P_{ACQA}' P_{HA}'$$

The term $(1 - P_k')$ is included in all transition probabilities of the TM matrix for the attacked submarine, except for transitions to State 6 (kill). In describing the transition probabilities, no further mention will be included as to the attacked submarine surviving but it will be contained in the mathematical expressions.

The probability of transitioning from State 1 to State 3 is the probability that the attacked submarine develops a fire control solution (FCS) based on detections described in $P_{1,2}$ or develops a fire control solution based solely on detection of a weapon launch (FCSWL) times the probability that the opposing submarine has not been closed.

$$P_{1,3} = (1 - P_k') \left[(P_D (1 - P_{DWL}) + P_{DWL} P_{DAL}) P_{FCS} + P_{DWL} P_{FCSWL} \right] (1 - P_{FP})$$

As stated previously, the only way for the attacked submarine to be in State 4 is if he has detected (D), localized (FCS), and closed (FP) but does not detect a weapon launch (DWL), has not yet classified or cannot successfully launch a weapon and therefore

$$P_{1,4} = (1 - P_k') P_D P_{FCS} P_{FP} (1 - P_{CL} P_{WL})$$

Transitioning to State 5 from State 1 requires that one close the target (FP), classify (CL, DWL) and launch a weapon in addition to detecting and localizing as given in $P_{1,3}$. The expression for this is

$$P_{1,5} = (1 - P_k') \left[(P_D P_{CL} (1 - P_{DWL}) + P_{DWL} P_{DAL}) P_{FCS} + P_{DWL} P_{FCSWL} \right] P_{FP} P_{WL}$$

The transition from State 1 to State 6 is just the probability that the attack on the submarine is successful and is

$$P_{1,6} = P_k'$$

Given that the attacked submarine is in State 2 (detection) the probability of transitioning to State 1 is the probability of losing contact (LC) by normal motion or evasion of either submarine and is

$$P_{2,1} = (1 - P_k') P_{LC}$$

$P_{2,2}$ is once again one minus the sum of the transition probabilities to all other states

$$P_{2,2} = 1 - \sum_{\substack{k=1 \\ k \neq 2}}^{6} P_{2,k}$$

$P_{2,3}$ is the probability of gaining a fire control solution by normal TMA or directly from detection of a weapon launch, given that a valid detection exists, times the probability of not being able to obtain a firing position and is

$$P_{2,3} = (1 - P_k') [(1 - P_{DWL} P_{FCSWL}) P_{FCS} + P_{DWL} P_{FCSWL}] (1 - P_{FP})$$

The probability of transitioning from State 2 to State 4 is the probability that you have localized and closed, given detection, but have not classified or cannot successfully launch a weapon

$$P_{2,4} = (1 - P_k') P_{FCS} P_{FP} (1 - P_{CL} P_{WL})$$

Going from State 2 to State 5 requires that a fire control solution be reached, the target closed and classified and a weapon successfully launched. The transition probability would be

$$P_{2,5} = (1 - P_k') [(1 - P_{DWL} P_{FCSWL}) P_{CL} P_{FCS} + P_{DWL} P_{FCSWL}] P_{FP} P_{WL}$$

The probability of going from State 2 to State 6 is

$$P_{2,6} = P_k'$$

Transitioning from any higher state to State 1 is just the probability of losing contact or

$$P_{3,1} = (1 - P_k') P_{LC}$$

Dropping back to State 2 from State 3 is the probability that the attacked submarine loses its fire control solution (LFCS) because of an action of the attacking submarine (i.e., speed change, turn, etc.) or evasion of either platform. The transition probability is therefore

$$P_{3,2} = (1 - P_k') P_{LFCS}$$

The transition probability for $P_{3,3}$ is

$$P_{3,3} = 1 - \sum_{\substack{k=1 \\ k \neq 3}}^{6} P_{3,k}$$

Transitioning from State 3 to State 4 requires attainment of firing position without either classifying or being able to successfully launch and is therefore

$$P_{3,4} = (1 - P_k') P_{FP} (1 - P_{CL} P_{WL})$$

Transitioning to State 5 requires classification and successful weapon launch given State 4 and is

$$P_{3,5} = (1 - P_k') P_{FP} P_{TCL} P_{WL}$$

$$P_{3,6} = P_k'$$

In order to be in State 4, the submarine under attack could not have detected the weapon launch. If the launch was detected the submarine immediately transitions to State 5 and then to any other state. The definitions of $P_{4,1}$ and $P_{4,2}$ are the same as $P_{3,1}$ and $P_{3,2}$, respectively except that the probability of not being killed by the attacker is based solely on an unalerted target (P_{ku}') where P_{ku}' is equal to $P_{ACQ}' P_H'$.

$$P_{4,1} = (1 - P_{ku}') P_{LC}$$

$$P_{4,2} = (1 - P_{ku}') P_{LFCS}$$

Transitioning from State 4 to State 3 is impossible. For the submarine to be in State 4 he must be unaware that he is being attacked and will maintain firing position even if the attacker goes into evasionary maneuvers. The evasionary maneuvers may be responsible for loss of contact (transition $P_{4,1}$) or loss of fire control solution ($P_{4,2}$), not loss of firing point ($P_{4,3}$). Therefore

$$P_{4,3} = 0$$

$$P_{4,4} = 1 - \sum_{\substack{k=1 \\ k \neq 4}}^{6} P_{4,k}$$

The transition $P_{4,5}$ requires that the submarine classify the target and launch a weapon and is

$$P_{4,5} = (1 - P'_{ku}) \, P_{CL} \, P_{WL}$$

$$P_{4,6} = P'_{ku}$$

The transitions $P_{5,1}$, $P_{5,2}$ are defined the same as $P_{3,1}$ and $P_{3,2}$ defined previously and are

$$P_{5,1} = (1 - P'_k) \, P_{LC}$$

$$P_{5,2} = (1 - P'_k) \, P_{LFCS}$$

Transitions from State 5 to State 3 result from evasionary maneuvers of either platform causing the ship under attack to be outside weapon firing range (LFP) and the transition probability is

$$P_{5,3} = (1 - P_k') P_{LFP}$$

The transition from 5 to 4 is just the probability of an unsuccessful weapon launch and is

$$P_{5,4} = (1 - P_k') (1 - P_{WL})$$

$$P_{5,5} = 1 - \sum_{\substack{k=1 \\ k \neq 5}}^{6} P_{5,k}$$

$$P_{5,6} = P_k'$$

Although all these probabilities are described in terms of the U.S. submarine being attacked by the threat submarine, interchanging primed and unprimed factors will result in the appropriate equations for the reverse case.

The attacking submarine starts the time interval in State 5 and can transition to any state. Expressions for the transition probabilities are the same as those for the attacked submarine with one slight difference. The probability that the attacking submarine is killed is

$$
\begin{aligned}
P_{KA} &= P_K && \text{if other platform was also in State 5} \\
&= P_{DWL}\, P_{SS} && \text{if other platform was in State 1 or 2}
\end{aligned}
$$

where P_{SS} is the probability of successfully killing the attacker by firing a "snapshot" weapon return based solely on the bearing at which attacking submarine's weapon launch is detected.

The transition probabilities are therefore:

$$P'_{5,1} = (1 - P_{KA})\, P'_{LC}$$

$$P'_{5,2} = (1 - P_{KA})\, P'_{LFCS}$$

$$P'_{5,3} = (1 - P_{KA})\, P'_{LFP}$$

$$P'_{5,4} = (1 - P_{KA})\, (1 - P'_{WL})$$

$$P_{5,5} = 1 - \sum_{\substack{l=1 \\ l \neq 5}}^{6} P_{l,5}$$

$$P'_{5,6} = P_{KA}$$

By multiplying the initial state vector by the above described transition matrix (TM), the distribution of states after an attack can be computed. Transitions to all 36 states are possible as a result of an attack phase. From properties of matrix multiplication and the fact that the initial distribution contains only nine non-zero components, only certain transition probabilities need be computed for the TM transition matrix. These will include all columns associated with transitions from any of the 9 non-zero initial states.

By appropriately summing up the states pertaining to a kill of either or both submarines as a result of an attack, the kill and counterkill probabilities can be determined. Since these states are absorbing states (i.e., the process cannot transition from this state once it enters it) successive application of the attack matrix will eventually result in steady-state probabilities from which the exchange ratio can be determined.

The probability of kill and counterkill after the first attack or successive attacks will be

$$P\ (KILL) = \sum_{i=1}^{6} Q_{i6}$$

$$P\ (COUNTERKILL) = \sum_{j=1}^{6} Q_{6j}$$

3.7 TRANSITION TO ATTACK PHASE MATRIX (TM*)

As stated previously, at the end of an engagement it is possible for the system to be in any of the 36 states. The second transition matrix (TM*) of the two step Markov process is only concerned with transitions to an attack state (i.e., transitions from any of the 36 states to the 9 states of attack). Therefore, the only elements of the transition matrix that need to be computed are those contained in the 9 rows related to transition to these attack states. In addition, those rows related to states where one or both of the submarines have been killed will have only one non-zero element ($P_{i6,i6}$ or $P_{6i,6i} = 1$). Since kills can only result from being in an attack phase and not while transitioning to an attack phase, the only way to arrive in a particular kill state is to have been in it as a result of a previous attack. Including these absorbing states allows one to accumulate the effective kill and counterkill probabilities from successive attacks.

As was described for the attack phase matrix (TM), the transition probabilities will be divided into the individual components relating to each of the submarines. Interchanging the primed and unprimed notation of the probabilities will result in the correct transition equation for the opposing submarine. Where descriptions of the transition probabilities enhance the understanding of the problem they will be provided. If the transition probability has been previously described and no further definition is necessary only the mathematical expression will be provided.

The probability of remaining in State 1 is given by

$$P_{1,1} = 1 - \sum_{j=2}^{6} P_{1,j}$$

Transitioning to State 2 from State 1 may be accomplished in numerous ways. Since an attack had been mounted by one of the submarines, active sensors may now come into use in an attempt to circumvent a possible disadvantage in the passive duel. The transition probability is therefore the probability of achieving a detection by using passive (PD) or active sensors (AD) but not attaining a fire control solution (FCS, AFCS) sufficient for closing and eventually launching a weapon and is

$$P_{1,2} = (1 - P_{AD})\, P_{PD}\, (1 - P_{FCS}) + P_{AD}\, (1 - P_{AFCS})$$

The terms defining passive and active probability of detection require further explanation at this point. Since active sensors may be used, a passive detection (D) can occur by normal means or by interception of an active emission (INT) emanating from the opposing submarine. Interception assumes there is a finite probability that the submarine will go active (GA). Therefore P_{PD} is defined as

$$P_{PD} = (1 - P'_{GA} \cdot P_{INT})\, P_D + P'_{GA}\, P_{INT}$$

In a similar fashion the probability of obtaining an active detection of the opposing ship is the probability that own ship goes active (P_{GA}) times the probability of achieving an active detection (P_{DACT}).

$$P_{AD} = P_{GA} P_{DACT}$$

The probability of transitioning from State 1 to State 3 is the probability of achieving an active (AD) or passive detection (PD) and fire control solution (FCS, AFCS) but not reaching a firing point (FP) and is

$$P_{1,3} = [1 - (1 - P_{PD} P_{FCS}) (1 - P_{AD} P_{AFCS})] (1 - P_{FP})$$

In order to get to State 4 or for that matter to be in State 4 requires that the submarine is unaware that an engagement is under way ($1 - P_{DWA}$). Realistically the probability that this occurs is small but it is included for completeness. The $P_{1,4}$ transition probability is therefore the probability of passively detecting (PD) and localizing (FCS), closing the target (FP) but not classifying (CL) or successfully launching a weapon (WL). Active detection is not included since it is assumed the submarine will not go active without being aware of the engagement in process.

$$P_{1,4} = (1 - P_{DWA}) P_D P_{FCS} P_{FP} (1 - P_{CL} P_{WL})$$

The transition probability $P_{1,5}$ is the probability that the submarine detects (PD, AD) and localizes (FCS, AFCS) by active or passive means and is aware of the engagement (DWA) or correctly classifies (CL) the opposing submarine if he is unaware of the engagement, and then successfully closes the target (FP) and launches a weapon (WL).

$$P_{1,5} = [P_{DWA} [1-(1-P_{PD} P_{FCS}) (1-P_{AD} P_{AFCS})] + (1-P_{DWA}) P_D P_{FCS} P_{CL}] P_{FP} P_{WL}$$

Since this transition matrix (TM*) describes transitions to an attack phase, the probability of being killed is zero. Therefore, there are no transitions to State 6 from any other state.

$P_{1,6} = 0$ as well as $P_{2,6} = P_{3,6} = P_{4,6} = P_{5,6} = 0$

The probability of dropping from State 2 to State 1 is the probability of losing contact (LC) by normal motion or evasion of either platform.

$$P_{2,1} = P_{LC}$$

The probability of remaining in State 2 is given by

$$P_{2,2} = 1 - \sum_{\substack{j=1 \\ j\neq 2}}^{6} P_{2,j}$$

Given a state of detection (State 2) the probability of transitioning to State 3 is the probability of achieving a passive or active fire control (FCS, AFCS) solution without reaching a firing point (FP) and is

$$P_{2,3} = [1 - (1 - P_{FCS})\ (1 - P_{GA}\ P_{AFCS})]\ (1 - P_{FP})$$

The transition probability $P_{2,4}$ is the probability that the submarine is unaware of the engagement $(1 - P_{DWA})$, gains a passive fire control solution (FCS) and reaches a firing point (FP) but does not classify (CL) or successfully launch a weapon (WL).

$$P_{2,4} = (1 - P_{DWA})\ P_{FCS}\ P_{FP}\ (1 - P_{CL}\ P_{WL})$$

To transition from State 2 to State 5 the submarine must localize by active (AFCS) or passive (FCS) means and additionally classify (CL) the target if he is unaware of the engagement $(1 - P_{DWA})$ and then successfully close the target (FP) and launch a weapon (WL).

$$P_{2,5} = [P_{DWA}\ (1-(1-P_{FCS})\ (1-P_{GA}\ P_{AFCS})) + (1 - P_{DWA})\ P_{FCS}\ P_{CL}]\ P_{FP}\ P_{WL}$$

$$P_{2,6} = 0$$

$P_{3,1} = P_{LC}$ as defined by $P_{2,1}$

Similarly $P_{3,2}$ is the probability of losing a fire control solution (LFCS) by normal motion or evasion of the opposing platform and is

$$P_{3,2} = P_{LFCS}$$

$$P_{3,3} = 1 - \sum_{\substack{j=1 \\ j \neq 3}}^{6} P_{3,j}$$

The transition probability $P_{3,4}$ is the probability that, given detection and localization, the submarine closes the target (FP) but is unaware of the engagement $(1 - P_{DWA})$ and has not classified (CL) or successfully fired a weapon (WL).

$$P_{3,4} = P_{FP} (1 - P_{DWA}) (1 - P_{CL} P_{WL})$$

To transition from State 3 to State 5, the submarine must close the target (FP) and launch a weapon (WL). In addition, if he is unaware of the engagement $(1 - P_{DWA})$ he must classify (CL) before launching the weapon

$$P_{3,5} = [P_{DWA} + (1 - P_{DWA}) P_{CL}] P_{FP} P_{WL}$$

$$P_{3,6} = 0$$

In order to be in State 4, the submarine must be unaware of the engagement. To drop to State 1, 2, or 3 the submarine must lose contact (LC), fire control solution (FCS) or firing point (FP), respectively. This loss can be as a result of normal motion or evasionary tactics of the attacking submarine, as appropriate.

$$P_{4,1} = (1 - P_{DWA})\, P_{LC}$$

$$P_{4,2} = (1 - P_{DWA})\, P_{LFCS}$$

$$P_{4,3} = (1 - P_{DWA})\, P_{FP}$$

$$P_{4,4} = 1 - \sum_{\substack{j=1 \\ j \neq 4}}^{6} P_{4,j}$$

The transition probability $P_{4,5}$ is the probability that the submarine is unaware of the engagement and classifies the target and successfully launches a weapon

$$P_{4,5} = (1 - P_{DWA})\, P_{CL}\, P_{WL}$$

$$P_{4,6} = 0$$

If either submarine is in an attack state (State 5) as a result of the previous attack phase then it signals the beginning of the next attack phase and the only non-zero element of the transition probabilities from State 5 is to State 5 and is equal to 1.

The probability of transitioning out of a killed state (State 6) is zero, and therefore

$$P_{6,6} = 1$$

3.8 COMPUTATIONAL CONSIDERATIONS

Many of the transition probabilities and all of the initial probability distributions should be calculable using analytic techniques that are the same as or similar to those used in the CSE&A study. The remainder of the transition probabilities will have to be determined using simulation models such as the SIM II model resident at NSRDC. The scope of the simulation will be considerably less than that of an equivalent Monte Carlo model since only a limited number of transition probabilities need to be determined.

By repeatedly applying the **TM*** · **TM** matrix, the state vector Q at times t_1, t_2,... t_n can be determined. Because of the absorbing states (Q_{i6} or Q_{6i}) the system will eventually settle into a steady-state distribution. That is, either own ship, threat ship or both are destroyed. While this may not be realistic (n applications implies that many attacks have occurred) it does yield the limiting exchange ratio. For steady state, in matrix notation, an application of the **TM*** · **TM** matrix on the state vector (Q) results in no change in the probability distribution of Q.

$$Q = TM^* \cdot TM \cdot Q$$

This equation can be solved algebraically to determine the distribution of states that satisfy the steady-state equation. However, since the matrices **TM*** and **TM** have been developed for the computer model, it is easier to successively apply these matrices and arrive at the solution numerically.

3.9 MULTIPLE ENGAGEMENT MODEL INPUT AND OUTPUT

The inputs to the multiple engagement model are the probabilities listed in Appendix A. The probabilities may be analytically derived or determined from simulation models.

The ouput of the model is the probability distribution of states at times t_1, t_2... and the exchange ratio at each of these times.

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4.0 CONCLUSIONS AND RECOMMENDATIONS

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The area clearance algorithm as presented in this report is a more realistic analytic representation of the area clearance mission than that included in the CSE&A study. It is recommended that this algorithm replace that which is presently being used.

The Markov Chain formulation for the multiple engagement scenario is an approach to the problem which should be workable and useful in assuming and comparing equipment capabilities and evaluating engagement tactics. The multiple engagement scenario emphasizes different systems than initial attack. The Markov formulation is the easiest method for performing multiple engagement analysis and allows a clearer understanding of the processes involved in the scenario.

It is recommended that the multiple engagement Markov Chain model be tested using available CSE&A data complemented by a limited number of SIM II simulation runs. The SIM II runs are to obtain probability elements that are not available from the CSE&A study.

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5.0 REFERENCE

5.0 REFERENCE

1. **Search and Detection,** Alan R. Washburn, Military Applications Section, ORSA, May 1981

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APPENDIX A

APPENDIX A

The following effectiveness measures are referenced in the main body of this report. The use of unprimed and primed probabilities in the text indicates whether the probability refers to own ship or threat ship, respectively. Those probability terms denoted by an asterisk (*) are derived quantities. The remainder of the quantities are the inputs necessary for the multiple engagement model.

P_{ACQ} = probability that attacker's weapon acquires an unalerted submarine.

P_{ACQA} = probability that attacker's weapon acquires an alerted submarine that may be evading, employing countermeasures, etc.

$P_{AD}^{(*)}$ = probability that the target is detected by an active sonar.
= $P_{GA} P_{DACT}$

P_{AFCS} = probability that the target is localized by active sensors and its motion analyzed with sufficient accuracy to place the submarine in a position capable of launching an attack.

P_{CL} = probability that the contact is correctly assessed to be a target of interest.

P_{D} = probability that sensor contact is gained by a passive sonar on a target present in the attack submarines operating area.

P_{DA} = probability that sensor contact is gained on a target given that submarine is alerted by weapon launch detection and does not lose contact if attacker evades.

P_{DACT} = probability of detecting a target given that an active sonar is employed.

P_{DWA} = probability that the submarine is aware of the engagement (i.e., that the submarine has initiated an attack or is cognizant that he is under attack). Realistically, this probability should be close to 1 but is included for completeness.

P_{DWL} = probability that a weapon launch by the opposing platform is detected.

P_{FCS} = probability the target is localized by passive sensors and its motion analyzed with sufficient accuracy to place the submarine in a position capable of launching an attack.

P_{FCSWL} = probability that, given a weapon launch is detected, a fire control solution can be developed solely on that weapon launch.

P_{FP} = probability that the target is physically approached, once detected, to within a distance sufficient for weapon firing.

P_{GA} = probability that an active sonar is employed.

P_H = probability that attacker's weapon causes mission abort for an unalerted submarine.

P_{HA} = probability that attacker's weapon causes mission abort against an alerted submarine that may be evading, employing countermeasures, etc.

P_{INT} = probability of intercepting an active emission from the opposing submarine (ping-stealing probability)

$P_k(*)$ = probability of acquiring and causing mission abort damage whether target is alerted by weapon attack and uses evasionary tactics/CM or is unaware of attack being made.

$= (1 - P'_{DWL})\ P_{ACQ}\ P_H + P'_{DWL}\ P_{ACQA}\ P_{HA}$

$P_{KA}(*)$ = probability that attacking submarine is killed during attack phase.

$= P_{DWL}\ P_{SS}$ if opposing submarine is in State 1 or 2.

$= P_k$ if opposing submarine is in State 5.

$P_{Ku}(*)$ = probability of acquiring and causing mission abort damage against an unalerted target.

$= P_{ACQ}\ P_H$

P_{LC} = probability of losing contact by normal motion or evasion of either submarine.

P_{LFCS} = probability that attacked submarine loses its fire control solution relative to the attacker because of action of attacking submarine (i.e., speed change, turn, etc.) or evasion of either or both platforms.

P_{LFP} = probability that attacked ship is no longer within firing range caused by evasionary maneuvers of either or both platforms.

$P_{PD}^{(*)}$ = probability of passively detecting the opposing submarine by normal passive sonar means or by interception of an active emission from the opposing submarine.

$= (1 - P'_{GA}\, P_{INT})\, P_D + P'_{GA}\, P_{INT}$

P_{SS} = probability of successfully causing mission abort damage to attacker by firing a "snapshot" weapon return based solely on the bearing on which attacking submarine's weapon launch was detected.

$P_{TCL}^{(*)}$ = probability that contact is correctly asessed to be a target of interest by normal classification means or by detection of an opposing submarines weapon launch.

$= (1 - P_{DWL})\, P_{CL} + P_{DWL}$

P_{WL} = probability that given the target has been localized and approached, the weapon is selected, made ready, loaded and successfully launched.