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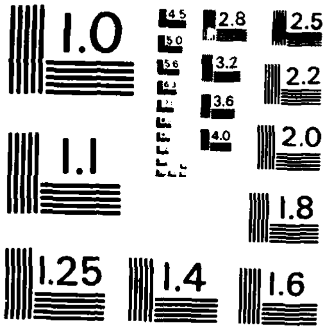

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The report summarizes progress in research supported by the grant during this period in the following areas: (1) stochastic control under partial observations; (2) nonlinear filtering; (3) large deviations for nearly deterministic processes; and (4) transition and invariant densities for nearly deterministic diffusions. A list of papers resulting from the research effort is included.

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AFOSR Interim Scientific Report

Fleming and his associates conducted research on the following topics:

a) Stochastic control under partial observations. For the case of partially observed diffusion processes a study of "relaxed" controls and of an associated separated control problem (certainty-equivalence principle) was made in Ref. [1]. A separated control problem for a general class of controlled, partially observed Markov processes appears in [2]. Connections with adaptive control of Markov processes are under continuing study.

b) Nonlinear filtering. Analytical and numerical techniques for approximately optimal nonlinear filters are under study by Fleming's Ph.D. student R. McGwier. The analytical results take the form of a regular perturbation expansion, while the numerical results are based on moving finite elements. Preliminary results are given in [3] with details to appear in McGwier's thesis.

c) Large deviations for nearly deterministic processes. Results of Ventsel-Freidlin type for nearly deterministic processes of diffusion or jump type are obtained in Sheu's thesis [2], using stochastic control methods. In improved form they appear in [4,5].

d) Transition and invariant densities for nearly deterministic diffusions. Asymptotic expansions for transition densities were obtained in [6]. These are a step toward the more difficult problem of justifying an expansion for invariant densities, in terms of a small parameter describing the variance of noise driving the diffusion process. Partial

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results on the latter problem have been obtained by Sheu.

Fleming presented a plenary address [7] at the 1983 International Congress of Mathematicians.

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1. W.H. Fleming and M. Nisio, On stochastic relaxed controls for partially observed diffusions, Osaka Math. J. (to appear).

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4. S-J Sheu, Stochastic control and exit probabilities of jump processes to appear in SIAM J. on Control and Optimization.

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7. W.H. Fleming, Optimal control of Markov processes, Proc. Internat. Congress Math, Warsaw, 1983.

Kushner completed a major monograph on approximation methods in stochastic systems theory. The book develops several very useful methods and discusses many complicated and realistic practical applications. The methods are powerful and useful for the analysis of both discrete parameter and continuous parameter systems with wide band width random inputs or disturbances, and develops a theory of stability for 'near' Markovian systems. The major idea is that the typical system is hard to analyze - so one must seek 'computable' and 'reasonable' approximations. The book goes about this in a systematic and thorough fashion.

The introductory comments to the book are included below.

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APPROXIMATION AND WEAK CONVERGENCE METHODS FOR RANDOM PROCESSES, WITH APPLICATIONS TO STOCHASTIC SYSTEMS THEORY

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I. Introduction.

This book is concerned with methods for approximating 'physical' random processes by the much more tractable diffusion or jump-diffusion processes, and with their applications to modelling, approximation and analysis problems in stochastic control and communication theory. Although we specialize the applications to these areas, the methods are of general applicability to similar modelling and approximation problems in many areas of physics, engineering and operations research. Many of the results are new, and other known results have been modified to enhance their usefulness for the types of applications of interest.

In order to motivate one particular type of problem which is of interest, consider the frequently encountered canonical model $\dot{z} = b(z) + \sigma(z)\xi$, where $\xi(\cdot)$ is a random process. It is a 'physical' process, in the sense that it is obtained more or less directly from the physical laws underlying the application. Suppose that (loosely speaking) it fluctuates 'much more rapidly' than does $z(\cdot)$, or has a wide band-width. In order to facilitate analysis, and owing to the large assumed differences in the 'rate of fluctuation' of $\xi(\cdot)$ and $z(\cdot)$, it is common practice in applications to replace $\xi(\cdot)$ by (say) a Gaussian white noise (with or without a 'correction' term), or to find a suitable white noise model which approximates $z(\cdot)$ in some sense. With such a replacement, $z(\cdot)$ is (loosely speaking) turned into a

Markov process, a solution to a stochastic differential equation, and a large number of powerful analytical and numerical tools can be applied to it. It is the availability of such tools for use on the diffusion (or jump-diffusion) process that makes the approximations so important and so appealing in practice, since the process $z(\cdot)$ will usually be very hard to study. Such approximation ideas are used in numerous areas of science and engineering, and a great deal has been learned in recent years on the appropriate methodology and on the dependence of the functions in the approximation on the dynamics of the 'physical' process. One looks for nice approximating processes $x(\cdot)$ such that the distributions of important quantities are close to those of $z(\cdot)$.

Sometimes, in applications, the approximations are just written down following a rough heuristic argument. This might work well (and even be validated by mathematical analysis) for simple problems, where the dynamical terms are smooth and $\xi(\cdot)$ appears in a simple way. But, if the structure of $\xi(\cdot)$ is complicated, if it depends on $z(\cdot)$, or if $\sigma(x)$ depends on x or $b(x)$ is discontinuous, or if the equation is of the more complex type $\dot{z} = b(z,\xi)$, where $b(\cdot,\cdot)$ might have a complicated structure, then simple heuristic procedures can often yield incorrect results, or even be unavailable. In general, they do not provide a methodology which one can use with confidence in difficult problems. Even in typical cases, the systems of interest can be quite complicated, from the modelling, analysis or approximation point of view. They can contain discontinuous functions of the state and noise (for example the limiter function sign $(\cdot)$), the noise need not be stationary, it can depend on $z(\cdot)$ (be a product of the feedback in the system) and it might occur in the dynamical equations in a complicated non-linear fashion with the state. Similarly for the discrete parameter cases. In fact, the discrete parameter cases can be the more complex, since they often arise from the use of recursive algorithms for estimation, etc., and can involve rather non-linear digital processing of the data.

Any theory of approximation involves the convergence of a parametrized sequence $\{x^{\varepsilon}(\cdot)\}$ of processes to a limit process in some specific sense. To put the above mentioned motivational problem in this form, parametrize the bandwidth or 'rate of fluctuations' of $\xi(\cdot)$ by ε; as ε goes to zero the bandwidth goes to infinity. Write $z(\cdot)$ and $\xi(\cdot)$ as $x^{\varepsilon}(\cdot)$ and $\xi^{\varepsilon}(\cdot)$, resp. Roughly, we imbed the true physical process in a suitable parametrized family, and then find a process $x(\cdot)$ such that $x^{\varepsilon}(\cdot) \to x(\cdot)$ in some specific sense. Hopefully, the limit process will be (a diffusion or jump-diffusion, degenerate or not) much more tractable mathematically or numerically than is the true physical process $z(\cdot)$, and that the parameter value ε corresponding to this physical process will actually be small enough so that the approximation is good. (The bandwidth concept was used for motivation only; it need not be defined or meaningful for the processes in the book.)

The most widely used and useful sense of limit is that of weak convergence of measures (Chapter 2). This is a powerful generalization of the notion of convergence of multivariate distributions, and it provides the data and approximations of interest. Such approximations are of interest for both the analysis and synthesis of systems. The weak convergence based limit theorems have certain features in common with the usual central limit theorem. As $\epsilon \to 0$, much of the specific detail of the system is often wiped out, as for the central limit theorem, where as the number of summands increases, the distribution of the sum depends 'less and less' on the character of each summand, and is 'closer and 'closer' to the normal or normal-Poisson distribution. Thus, for analysis purposes, the limit processes are useful because they usually have a simpler mathematical or physical structure than the original processes, even though distributions of important quantities are close. For purposes of synthesis, approximations provide a convenient vehicle for studying the effects of system modifications. The weak convergence techniques are widely used in operations research and in statistics, and are now of increasing interest in control and communication theory, owing to their usefulness in the modelling, simplification and study of complex non-linear systems.

Chapter 1 contains a number of useful background results from probability theory. Many of the more difficult parts of the chapter are not actually used in the rest of the book, but are introduced in order to allow a discussion of the relationship between two points of view toward diffusion processes. The first is as the solution to the standard Itô or stochastic differential equation, the second is as the solution to the so-called martingale problem of Stroock and Varadhan [59]. The second point of view is perhaps less intuitive for physicists and engineers, but has many advantages in weak convergence analysis; in particular, it is much easier to show that a process solves the martingale problem than it is to show directly that it satisfies a stochastic differential equation. Because of this, it is useful to understand the equivalence (under broad conditions) of the two points of view, and a lare part of Chapter 1 is devoted to a heuristic discussion of this equivalence, and to the background material on martingales. Only a few facts from this chapter will be used in the sequel. The reader who is interested mainly in the methodology in applications can quickly skim this chapter. A number of basic definitions and useful results from the theory of weak convergence are given in Chapter 2.

Three general methods are used to get the weak convergence and identify the limits, the perturbed test function method, the direct averaging method and a method which combines some of the best features of both, and is perhaps the most generally useful. The second method is perhaps the easiest to use, in its domain of applicability. The first is dealt with in Chapters 3 and 4, and the last two in Chapter 5. There are roughly two main steps in using weak convergence theory; checking whether there is a limit, and identifying it (and verifying its uniqueness). The compactness or tightness criteria which we use to check whether there is a limit are discussed in Chapter 3. Then, given that there is a limit, we must characterize it. To each limit process, there will be associated an 'infinitesimal operator' or 'differential generator' A. If the limit is the scalar valued diffusion defined by the Itô equation $dx = b(x)dt + \sigma(x)dw$, then $A = b(x)\partial/\partial x + \frac{1}{2}\sigma^2(x)\partial^2/\partial x^2$. If to each A, there is a unique process, then knowing A is equivalent to knowing the process. The three methods use different ways of averaging the noise effects to obtain A from the sequence $x^\varepsilon(\cdot)$ and its 'driving noises'. Once the main steps in the general techniques are understood, They are of value even for formal use, since they provide for an essentially automatic way of constructing the operator A of the limit process.

We try to present the ideas and results in a general enough way so that they can be used in many interesting applications, but without even beginning to exhaust the possibilities.

For many applications, the systems are in operation for a very long time. Weak convergence theory in itself does not tell us much about what happens to $x^\varepsilon(t)$ for small ε and large t, yet such information is vital to many applications. The weak convergence results can be extended to get the desired information, under suitable stability properties on $\{x^\varepsilon(\cdot)\}$ (uniformly in ε). Chapter 6 obtains usable criteria for these stability properties. The properties of $x^\varepsilon(t)$ for large t and small ε are related to the asymptotic properties of $x(t)$, the limit diffusion. Invariant measures, 'almost' invariant measures, recurrence and moment boundedness (uniformly in ε), and similar stability properties are discussed. The results are useful more generally for the study of stability properties of non-Markovian systems.

Some problems in singular perturbations are discussed in Chapter 7. Chapters 8 to 10 deal with numerous applications; adaptive filters and antenna arrays, stochastic approximations, adaptive quantifiers, input-output statistics of important non-linear devices, phase locked loops (with and without limiters), etc. The idea is to show how to use the approximation techniques to obtain useful diffusion approximations to a canonical problem in each class, so as to facilitate its analysis. Those applications will attest to the power and usefulness of the techniques for obtaining information on hard problems, often with only a reasonable amount of work.

Chapter 11 deals with an approximation problem which is somewhat different from those dealt with in the previous chapters. It concerns systems which are of interest for a long period of time, but where the noise effects are small. A typical 'classical' problem in this area concerns the 'small white noise' system $dx^{\varepsilon} = \bar{b}(x^{\varepsilon})dt + \sqrt{\varepsilon}\sigma dw$, where ε is small, and $w(\cdot)$ is a Wiener process. Let $\dot{x} = \bar{b}(x)$ be asymptotically stable at $x = \theta$. In many applications in physics and engineering, one is interested in mean escape times from a neighborhood of θ, or in similar 'escape time' statistics. Since this is, in general, a ver hard problem, asymptotic methods are useful, and one seeks the statistics (normalized in a suitable way) as $\varepsilon \to 0$. A non-ideal or practical system would be $\dot{x}^{\varepsilon} = \bar{b}(x^{\varepsilon}) + \sigma(x^{\varepsilon})\xi^{\varepsilon}(t)$, where $\xi^{\varepsilon}(\cdot)$ is scaled so that its' effects are small. The computation is substantially easier, however, with the 'white noise' ideal system. The general ideas behind the estimates, for both the non-ideal and the ideal system are treated. The proper theoretical foundation is in the theory of large deviations. An introduction to the relevant part of the theory is provided, and several examples are discussed to illustrate when one can or cannot use the small white noise ideal models.

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