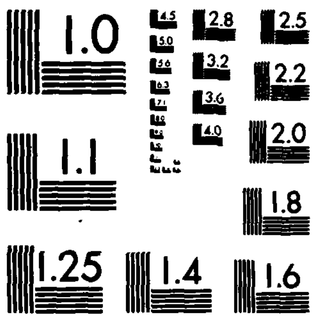
AD-A143 299 AN ENTROPY MAXIMUM PRINCIPLE AND INSTANTANEOUS FAILURE STATISTICS(U) NAVAL RESEARCH LAB WASHINGTON DC S TEITLER ET AL. 03 JUL 84 NRL-MR-5370 1/1

UNCLASSIFIED F/G 14/4 NL

END

FILMED



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

NRL Memorandum Report 537

An Entropy Maximum Principle and Instantaneous Failure Statistics

S. TEITLER, A. K. RAJAGOPAL* AND K. L. NGAI

Electronics Technology Division

**Department of Physics and Astronomy*
Louisiana State University
Baton Rouge, Louisiana 70803

July 3, 1984

NAVAL RESEARCH LABORATORY
Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): NRL Memorandum Report 5370	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Naval Research Laboratory; 6b OFFICE SYMBOL (If applicable): Code 6807	7a NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State and ZIP Code): Washington, DC 20375	7b ADDRESS (City, State and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION: Office of Naval Research; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State and ZIP Code): Arlington, VA 22217	10 SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT NO
61153N		RR011-02-42	DN180-023

11 TITLE (Include Security Classification) An Entropy Maximum Principle and Instantaneous Failure Statistics

12 PERSONAL AUTHOR(S)
Teitler, S., Rajagopal, A.K.,* and Ngai, K.L.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Yr., Mo., Day)	15 PAGE COUNT
Interim	FROM 11/83 TO 3/84	July 3, 1984	14

16 SUPPLEMENTARY NOTATION
*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB. GR.	Reliability	Weibull distribution
			Failure statistics	Shannon entropy
				(Continues)

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A Shannon entropy for instantaneous failure statistics is defined in terms of an integral over the full epoch of possible failure times. An entropy maximum principle (EMP) subject only to the normalization of the probability density integral and the existence of a mean time to failure yields an exponential distribution as the fundamental probability distribution for instantaneous failure statistics. It is observed that the time scale of measurements does not necessarily coincide with the time scale on which the probability distribution takes its fundamental form. Time scale transformations are considered. The maximum entropy density is taken to be form-invariant independent of the time scale used in its description, and the resulting epoch entropy is itself taken to be an invariant. This latter invariance leads to a relation between the time unit of a transformed time scale and the time unit of the fundamental scale. The invariance relation is employed in consideration of the effect of parameter scaling in particular devices, and in the discussion of the behavior of parameters that arise in accelerated testing.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED X SAME AS RPT ☐ DTIC USERS ☐	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL: S. Teitler	22b TELEPHONE NUMBER (Include Area Code): (202) 767-2807; 22c OFFICE SYMBOL: Code 6807

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE

18. SUBJECT TERMS (Continued)

Epoch entropy
Information theory
Entropy maximum principle

CONTENTS

AN ENTROPY MAXIMUM PRINCIPLE AND INSTANTANEOUS FAILURE STATISTICS

I. Introduction

In this paper we use and extend some concepts in information theory in the context of the statistical description of device failure. In particular, we consider here failures that are instantaneous in the sense that a failure does not depend on the history (e.g. length of operation) of the device [1]. Failure probabilities are characterized by an order parameter that corresponds to the time of failure for the device under consideration. The corresponding Shannon entropy [2] is defined over the full range of failure times. We term the range, the epoch for failures. We consider an entropy maximum principle (EMP) for the epoch entropy subject only to minimal constraints necessary to characterize failure statistics; namely, normalization of the probability density integral, and the existence of a mean time to failure.

The resulting probability density that is consistent with the EMP subject to minimal constraints is taken to be fundamental. However, it is observed that the time scale on which the failure statistics are recorded does not necessarily coincide with the time scale on which the probability density takes its fundamental form. Time scale transformations are considered. The maximum entropy density is considered to have the Shannon form independent of the time scale used in its description. The resulting epoch entropy is itself taken to be invariant. This principle of invariance of maximum epoch entropy provides a relationship between time units on different time scales that is useful e.g. in the consideration of the effect of size or mass scaling in particular devices, and in the discussion of the behavior of parameters that arise in accelerated testing. Given this framework and motivation, we now turn to a more detailed description of our procedure.

Manuscript approved April 30, 1984.

II. Entropy Maximum Principle, Time Scale Transformations and an Invariance Relation

In failure statistics, the statistical probability of failure of a device is described in terms of an order parameter $\theta=\tilde{\theta}-\tilde{\theta}_o$ where $\tilde{\theta}_o$ is the value of $\tilde{\theta}$ when failures are first possible. For the definition of Shannon entropy we need a dimensionless probability density so we introduce a constant unit τ_θ for the order parameter. Then if $f(\theta)d\theta/\tau_\theta$ is the probability of the occurrence of a failure between θ and $\theta+d\theta$, it follows that

$$\int_o^\infty f(\theta)d\theta/\tau_\theta = 1 \tag{1}$$

where $f(\theta)$ is a dimensionless probability density. To make sense as a failure probability density, $f(\theta)$ must have a first moment that corresponds to a mean time to failure

$$\langle\theta\rangle = \int_o^\infty \theta f(\theta)d\theta/\tau_\theta \tag{2}$$

(1) and (2) represent the minimum set of conditions that arise in the description of instantaneous failure statistics.

We now define a dimensionless epoch Shannon entropy in terms of the failure probability density as follows.

$$S_\theta = -\int_o^\infty f(\theta)\ell n f(\theta)d\theta/\tau_\theta \tag{3}$$

It should be noted that S_θ is an implicit function of $\tilde{\theta}_o$. We now require that S_θ must be maximal. In particular, in the absence of knowledge about other constraints, we maximize S_θ subject only to the constraints represented by (1) and (2). Introducing two Lagrange multipliers λ_1 and λ_2 in the usual way, we obtain from the variation of S_θ that the maximum entropy occurs for

$$f^M(\theta) = \exp[-(\lambda_1+\lambda_2\theta)] \tag{4}$$

Consistency with (1) and (2) allows us to determine λ_1 and λ_2,

$$\exp(-\lambda_1) = \tau_\theta/\langle\theta\rangle, \ \lambda_2 = \langle\theta\rangle^{-1} \tag{5}$$

Then the dimensionless probability density for maximum entropy, $f^M(\theta)$, has the form

$$f^M(\theta) = [\tau_\theta/\langle\theta\rangle]\exp[-(\theta/\langle\theta\rangle)] \tag{6}$$

The maximum epoch entropy can itself be determined to be

$$S_\theta^M = 1 + \ell n\langle\theta\rangle - \ell n\tau_\theta \tag{7}$$

There are several important observations that should be made concerning (6) and (7). First from (6) we see that it is the exponential distribution that provides the maximum epoch entropy subject to the minimum constraints necessary to characterize instantaneous failure statistics. In this sense, the exponential distribution represents the fundamental distribution for instantaneous failure statistics. Second, although $f^M(\theta)d\theta/\tau_\theta$ is independent of the choice of time unit, $f^M(\theta)$ alone takes on a particularly simple form when τ_θ is taken equal to $\langle\theta\rangle$, i.e. it is equal to unity at $\theta=0$, and homogeneous in $\theta/\langle\theta\rangle$. Thus $\langle\theta\rangle$ is a natural time unit for the θ time scale. A similar simplification occurs for S_θ^M which just becomes unity when $\tau_\theta=\langle\theta\rangle$. However for any choice of τ_θ, S_θ^M is an epoch integral whose value should be independent of the time scale on which it is described. This condition of invariance serves to place a stringent condition on the relationship between τ_θ and the unit of time for the new time scale. This is probably obvious for a linear transformation but we are interested here in a wider class of time scale transformations. We are motivated here by the fact that there is no reason to believe that the time scale on which failures are recorded coincides with the time scale on which the failure statistics distribution appears in its fundamental exponential form.

To provide a contextual framework, we now make some background observations about time scales. A time or order parameter is just a positive cumulative function that increases monotonically from an origin. We have already introduced the $\tilde{\theta}$ time scale, its corresponding difference time scale $\theta=\tilde{\theta}-\tilde{\theta}_o$, and unit of time τ_θ. In a similar way we introduce another time scale $\tilde{t}$ with difference time scale t [illegible] $_o$, and unit of time τ_t. From the context of application, it is clear that we should align the respective origins $\tilde{\theta}_o$ and $\tilde{t}_o$ since both represent the initial time when failures can occur. Thus if

$$\theta=\theta(t),\ \theta(t=0) = 0 \tag{8}$$

We specify further that $d\theta/dt$ is positive and finite everywhere except possibly at isolated points.

We can now describe the failure statistics and maximum entropy in terms of t. Namely by direct transformation from $f^M(\theta)d\theta/\tau_\theta$ to $g^M(t)dt/\tau_t$, we find, using (6)

$$g^M(t) = \exp\{-[\theta(t)/\langle\theta\rangle]\}\ [\tau_t/\langle\theta\rangle][d\theta(t)/dt] \tag{9}$$

The maximum epoch entropy can now be written in terms of an entropy density defined in terms of $g^M(t)$, and an integral over time on the t-scale.

$$S_t^M = \int_0^\infty g^M(t)\ln g^M(t)\ dt/\tau_t = 1 + \ln\langle\theta\rangle - \int_0^\infty \exp[-(\frac{\theta}{\langle\theta\rangle})]\ \ln(\tau_t \frac{d\theta}{dt})\ \frac{d\theta}{\langle\theta\rangle} \tag{10}$$

Within the integral on the right hand side, $d\theta/dt$ is expressed as a function $\langle\theta\rangle$ and $\theta/\langle\theta\rangle$. We now use the requirement that S^M is invariant when expressed in terms of a transformed time scale, and (7) and (10) to obtain a general invariance condition.

$$\ln(\tau_\theta/\tau_t) = \int_0^\infty \exp[-(\theta/\langle\theta\rangle)]\ln[d\theta/dt]d\theta/\langle\theta\rangle \tag{11}$$

We can investigate the ramifications of this relationship by considering a specific time scale transformation.

III. An Important Example and Parameter Renormalization

Given this general formulation, we can now turn to an important example of time transformation that will provide a Weibull failure probability density. Namely, we consider

$$\theta(t) = \alpha t^{\beta},\ d\theta/dt = \alpha\beta t^{\beta-1} \quad , \alpha,\beta>0 \tag{12}$$

so that

$$d\theta/dt = \alpha\beta(\theta/\alpha)^{(\beta-1)/\beta} \tag{13}$$

The result of using Eq. (13) in Eq. (11) is

$$\ell n\{ \frac{\tau_\theta}{\tau_t} \frac{1}{\beta\langle\theta\rangle} [\langle\theta\rangle/\alpha]^{1/\beta}\} = -(1 - \frac{1}{\beta})\gamma \tag{14}$$

where γ=0.577215. . . is the Euler constant. If we now choose τ_θ to coincide with the "natural" unit on the θ-scale, namely $\langle\theta\rangle$, we obtain the relationship

$$\langle\theta\rangle = \alpha\beta^{\beta}\exp[\ (1-\beta)\gamma]\tau_t^{\beta} \tag{15}$$

It is convenient to define a new unit $\bar{\tau}_t$,

$$\bar{\tau}_t = \beta\exp[\ (1-\beta)\gamma/\beta]\tau_t \tag{16}$$

so that

$$\langle\theta\rangle = \alpha\ \bar{\tau}_t^{\ \beta} \tag{17}$$

Then using Eq. (9), we obtain for $g^M(t)dt/\tau_t$,

$$g^M(t)dt/\tau_t = \exp[-(t/\bar{\tau}_t)^{\beta}]\ \beta(t/\bar{\tau}_t)^{\beta-1}\ dt/\bar{\tau}_t$$
$$\equiv \bar{g}^M(t)\ dt/\bar{\tau}_t \tag{18}$$

The function $\bar{g}^M(t)/\bar{\tau}_t$ is just the usual 3-parameter Weibull probability density (recall $\tilde{t}_o$ is implicit) that is so prevalent in the empirical description of the probability of instantaneous failure of a wide range of devices. The scale parameter is just the unit of time $\bar{\tau}_t$, and the shape parameter is β.

We are oftentimes presented with a Weibull distribution for instantaneous failures rather than an exponential distribution. The point we have been making is that such a Weibull distribution occurs on a time scale of measurements which is not the fundamental time scale appropriate for the device of interest as indicated by the EMP subject to minimal constraints. Thus parameter dependencies that enter into the unit of time on the measurement time scale are therefore only apparent. The actual dependencies are those that enter into the unit of the fundamental time scale. The two dependencies are related by means of the invariance relation obtained from the specification that the maximum epoch entropy is invariant under a time scale transformation.

For example, consider a plastic supporting component with strength dependent on the molecular weight, M, of its chemical building blocks. On the time scale of measurement, we assume the failure statistics to be Weibull. On the fundamental scale, we take the mean time to failure, for specificity, to have a simple monomial dependence on M.

$$\langle\theta\rangle = kM^b \tag{19}$$

Then the Weibull scale unit, $\bar{\tau}_t$, obeys

$$\bar{\tau}_t \propto M^{b/\beta} \tag{20}$$

and therefore has a different power dependence controlled by the shape parameter β. If now we consider plastic components made with plastics with different values of M, they will typically have Weibull failure statistics with different values of the shape parameter. Our prediction predicated on the invariance of maximum epoch entropy when expressed in terms of the Weibull distribution is that the M-dependence of $\bar{\tau}_t$ will vary in such a way that b is the same for the components with different M. Similarly it may be possible to change the Weibull shape parameter by changing the conditions of operation e.g. by changing the environment. Then the prediction is that the power of M that enters into $\bar{\tau}_t$ changes with the shape parameter in such a way that b again remains invariant.

A more general application occurs in the case of accelerated testing. We consider then the device of interest to be subjected to a constant stress which, for specificity, we take to be the absolute temperature T. For simplicity, we assume that the origin of time for failures is not modified by a change of temperature. We now assume that the temperature dependence of the unit of time is described by an Arrhenius model so that on the θ-scale,

$$\langle\theta\rangle = \langle\theta_\infty\rangle \exp(A/T) \quad . \qquad (\quad)$$

Here $\langle\theta_\infty\rangle$ is the mean time to failure at (nominally) infinite temperature and A is an Arrhenius constant with dimensions of temperature. We again take the distribution on the time scale of measurement to be Weibull. Then if under temperature stress, we find that the statistics are still Weibull with unchanged shape parameter, we can apply the Arrhenius model to the Weibull time unit.

$$\bar{\tau}_t = (\bar{\tau}_t)_\infty \exp(A^*/T) \qquad (22)$$

Here A^* is an effective Arrhenius constant. It is clear that for the invariance condition to hold, A^* must be related to the actual Arrhenius constant A by the renormalization relation

$$A^* = A/\beta \qquad (23)$$

Despite its apparent simplicity, (23) provides an important insight into the validity of stress testing. If the shape parameter β is less than unity, the apparent A^* is larger than A and one is led to believe the device is less durable than it actually is. The bad news is that if $\beta>1$, the device is actually less durable than might be judged from the stress test on the time scale of measurement. The important point we want to make is that conclusions concerning accelerated life tests depend in an essential way on the details of the probability density, and not just on the properties of the measured mean time to failure.

In summary, we have formulated for instantaneous failure statistics, an EMP for the entropy over the full epoch of failures subject to the minimal constraints that the probability is normalized and that there exists a mean time to failure. This EMP led to an exponential distribution for the probability density. The exponential distribution thus can be viewed as the fundamental distribution for instantaneous failure statistics. We then considered the family of probability densities that can be obtained by time scale transformations. The condition that the maximum entropy over the epoch remains invariant when evaluated in terms of a transformed probability density led to a relation between a unit of time on the transformed time scale and the one on the original scale. The example involving a time scale transformation that leads to a Weibull probability density was discussed. In particular, it was pointed out that the invariance condition imposes important constraints on parameters and Arrhenius constants from accelerated testing, that may occur in the description of the time scale units. Other time scale transformations can be treated in a similar way [3]. We intend to address the application of an EMP, time scale transformations, and the invariance of maximum epoch entropy to the case of cumulative damage failure statistics at a later time.

Acknowledgment

One of us (AKR) was supported in part by ONR Contract #N00014-82-2021.

References

1. See e.g. I.B. Gertsbakh and Kh.B. Kordonskiy, Models of Failure, Springer-Verlag, New York, Inc., 1969, Chap. II.

2. C.E. Shannon, " A Mathematical Theory of Communication," Bell Sys. Tech. J. Vol 27, 1948, pp. 379-423.

3. Supplement: We could also consider e.g. the following: $\theta=\tilde{\theta}-\tilde{\theta}_o = a\ \ell n(\tilde{t}/\tilde{t}_o)$, $\tilde{t} \geqq \tilde{t}_o$ and $t = \tilde{t}-\tilde{t}_o$.

END

FILMED

8-84

DTIC