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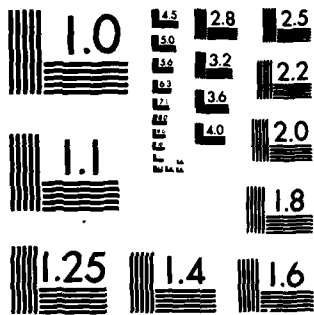
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# Length Distribution of Chords Through a Rectangular Volume

W. L. BENDEL

*Radiation Survivability Branch  
Condensed Matter and Radiation Sciences Division*

July 3, 1984

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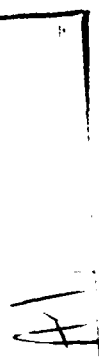
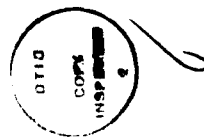
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<p>The chord length distribution of penetrating radiation in rectangular parallelepipeds is of concern in regard to spacecraft electronics, as single event upsets can be produced by the ionization along the path of a high-energy particle. Both differential and integral chord length distributions are presented here. Two approximations to the integral distribution are discussed.</p>				
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## Length Distribution of Chords Through a Rectangular Volume

### Single Event Upsets

This report presents differential and integral chord length distributions for use in predicting single event upsets (SEUs) in spacecraft electronics.

An "upset" is a change of state, such as the state denoting a bit in computer memory. A "single event" is an occurrence which produces sufficient ionization, in the speck of silicon involved, to produce an upset. Examples of an event are alpha emission by a radioactive impurity in the device, the highly-ionized track of a heavy cosmic ray, and a nuclear reaction produced in the device by a high energy proton. In the case of a penetrating particle, an SEU will be produced if (a) the rate of ionization along the path times (b) the path length in the sensitive region of the device exceeds (c) the critical charge for upset.

Much of the present material was written as part of a larger report. That report has been dropped, but the new results herein have been requested by a number of persons and used in at least four publications.<sup>1-4</sup>

### Paths versus Chords

The distribution of path lengths of isotropic particles in a given volume is both a mathematical and a physical problem. An atomic projectile loses energy by making many minor collisions with the atoms of the environment. The direction of travel is slightly altered in each collision. Major collisions, with large changes in direction, are rare. The "pathlength", or physical distance travelled, will exceed the mathematical chord length. In this report, we treat only the mathematical problem - the distribution in length of chords, straight lines between entry and exit points, for penetrating undeviating particles in a rectangular volume.

### Source and Basic Equation

The chord length distribution for an isotropic particle flux is given, in differential form, in an article by Pickel and Blandford,<sup>5</sup> which contains equations developed by Petroff. (A Rockwell International internal report was actually used here.) Dr. Philip Shapiro of NRL has confirmed the equations and then employed them in numerical integrations.<sup>6</sup> The analytic integra-

tion, not found in the above source, is performed below after considerable simplification of the differential form.

We consider a rectangular parallelepiped of dimensions L, W, and H. Some lines will merely nick a corner and have short paths in the volume; many particles (those which enter a surface nearly normally and exit at the opposite surface) have a path a little more than L, W, or H; the maximum chord length is R, the diagonal of the volume.

The normalized differential distribution function for chord length S is called F and integrates to unity. It is convenient to replace the quantity g of Petroff by M/12, where M = N + M<sub>0</sub>, and M<sub>0</sub> consists of those terms whose contributions are zero. The function F is given by

$$3\pi [LW + LH + WH] F(S, L, W, H) = \sum N(S, X, Y, Z), \quad (1)$$

where the sum is over the six permutations of (L, W, H) as (X, Y, Z).

As the equations of Petroff are quite complex, symbols are used here for certain functions of the dimensions:

$$K^2 = X^2 + Y^2, \quad R^2 = L^2 + W^2 + H^2 = X^2 + Y^2 + Z^2,$$

$$T^2 = X^2 + Z^2, \quad \text{and} \quad V = 12 XYZ^2.$$

Similarly, two symbols show relationships with chord length:

$$P^2 = S^2 - Z^2 \quad \text{and} \quad Q^2 = S^2 - X^2 - Z^2.$$

#### Initial Differential Forms. M

The expressions for M and N have three forms, depending upon the value of S relative to the dimensions. First, M is given as a sum of quantities which will be examined individually. In the first domain, where  $0 \leq S < Z$ , it is given by

$$M_1 = N_1 + M_{0A} + M_{0B}. \quad (2)$$

In the second domain, where  $Z \leq S < T$ , Petroff's formula is equal to

$$M_2 = N_2 + M_{0A} + M_{0B} + M_{0C}. \quad (3)$$

Finally, in the third domain, where  $T \leq S \leq R$ , the expression is

$$M_3 = N_3 + M_{0A} + M_{0C}. \quad (4)$$

The sum in Eq. (1) will often involve different regions for different permutations. For example, if  $L > S > H$ , some of the permutations (with  $Z = L$ ) will involve the first domain while others (with  $Z = H$ ) will involve the second or third domain.

The terms combined into M<sub>0A</sub> are common to all three domains and therefore permutations can be combined for all values of S.

These terms are

$$M_{0A} = 8X^2[Y^2/T^2R - 2/R + R/K^2] = (8/R)[X^2Y^2/T^2 + X^2Z^2/K^2 - X^2]. \quad (5)$$

Summing over all six permutations, one finds that

$$\sum M_{0A} = (8/R)[2R^2 - 2R^2] = 0, \quad (5a)$$

and  $M_{0A}$  may be omitted (as it was) from Eq. (1).

Now let us consider the quantity

$$M_{0B} = 8(X^2 - Z^2)/T. \quad (6)$$

The term appears in the first and second expressions for  $M$ , or for  $0 \leq S < T$ . Permutations with  $Y$  fixed but  $X$  and  $Z$  exchanged will have identical values of  $T$  and thus the same range. These terms cancel in pairs, so

$$\sum M_{0B} = 0. \quad (6a)$$

The third quantity to be eliminated,

$$M_{0C} = (X^2 - Y^2)(P/KS)(8 + 4Z^2/S^2), \quad (7)$$

exists for  $Z \leq S \leq R$ . One may sum permutations if  $Z$  is fixed. Again, paired cases cancel and

$$\sum M_{0C} = 0. \quad (7a)$$

The value of  $F$  is, thus, obtained from the sum of the  $N$ 's, as shown in Eq. (1).

### Net Differential Forms, $N$

In the first domain, where  $0 \leq S < Z$ , what remains is

$$N_1 = B_1S + 4Z(2Y^2/K^2), \quad (8)$$

where  $B_1 = -(3XY/RT)^2$ . It is easily seen that, when summed over paired permutations of  $X$  and  $Y$ , this is equivalent to

$$N_{1z} = B_1S + 4Z. \quad (8E)$$

In the second domain,  $Z \leq S < T$ , one finds that

$$N_2 = B_2S + B_4/S^2 - X(P/S)(8 + 4Z^2/S^2), \quad (9)$$

where  $B_2 = (3Y/K)^2 + B_1$  and  $B_4 = V \tan^{-1}(Y/X) - (YZ^2/K)^2$ .

For the third domain,  $T \leq S \leq R$ , the value is

$$N_3 = B_3S + B_5/S^2 + YZ^2(Q/S)[8/T^2 + 4/S^2] - (V/S^2)\cos^{-1}(X/P), \quad (10)$$

where  $B_3 = -(3XZ/KR)^2$  and  $B_5 = X^2Z^2(Z^2/K^2 - 3) + V \tan^{-1}(Y/X)$ .

As X and Z may be exchanged, this may be simplified a little to

$$N_{3z} = B_3 S + B_3/S^2 + Y(Q/S)[4 + 2T^2/S^2] - (V/S^2)\cos^{-1}(X/P). \quad (10E)$$

### Short Chords and Peaks

Six distributions of chord length are shown in Figs. 1-3. The dimensions are in micrometers, as befits microelectronics. In all cases, the maximum chord length,  $S = R$ , is an integer or half-integer from 30 to 34. The significant quantities, however, are the ratios of dimensions. Defining H as the smallest, R/H is 2.125, 3, 5.5, 7.5, 12.6, and 17 for these cases. The ratio L/W is 1 to 1.5.

The dominant features are the peaks at S equal to H, W, or L. Chords of  $>0.9 R$  are too rare to be evident on the graphs. It is seen that F is linear for  $S < H$ . In this range,  $N_{1z}$  applies to all six perturbations, and the sum is

$$\sum N_{S < H} = 8[L + W + H] - 9S. \quad (11)$$

The discontinuities can be evaluated by subtracting  $N_{1z}$  from  $N_2$  for the two permutations involved at a given jump. The change at  $S = Z$  is given by

$$[LW + LH + WH]\Delta F = 2XY/Z. \quad (12)$$

This equation shows that the biggest peak comes at the smallest dimension, where  $S = Z = H$ , as clearly seen in the figures. Two peaks coincide in a square device.

### Indefinite Integrals

Integration of  $N_{1z}$  is trivial; integration of  $N_2$  is quite easy; integration of  $N_{3z}$  involves a term not readily found in an integral table. The results follow:

$$I_1 = \int N_{1z} dS = B_1 S^2/2 + 4ZS \quad (13)$$

$$I_2 = B_2 S^2/2 - B_4/2S^2 + XP(2Z^2/S^2 - 8) + 6XZ \cos^{-1}(Z/S) \quad (14)$$

$$I_3 = B_3 S^2/2 - B_5/2S^2 + Y [Q(4 - T^2/S^2) - 3T \cos^{-1}(T/S)] \\ + 6XY [(X/T) \cos^{-1}(T/S) - (P/S)^2 \cos^{-1}(X/P)] \quad (15)$$

Note that the two  $\cos^{-1}(T/S)$  terms in Eq. (15) can be combined; the form shown makes the integration clearer. If one combines these terms, a factor  $(X^2 - Z^2)$  is obtained. As the limits of the third domain are unchanged by interchanging X and Z, one may sum over such paired permutations, totaling zero for these terms. Therefore, when summed,  $I_3$  is equivalent to

$$I_{3z} = B_3 S^2/2 - B_5/2S^2 + YQ(4 - T^2/S^2) - 6XY(P/S)^2 \cos^{-1}(X/P). \quad (15E)$$

for each permutation.

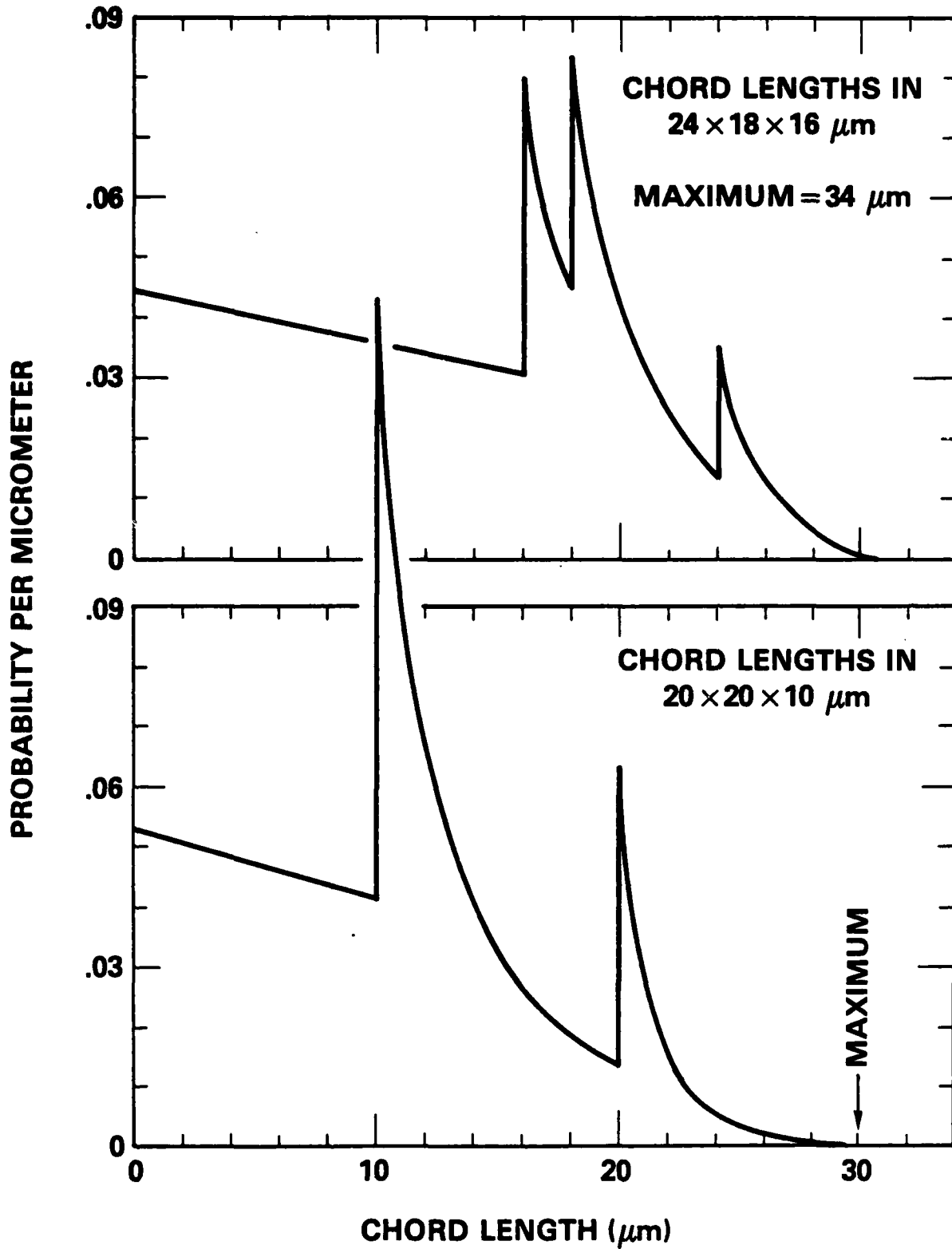


Fig. 1. Chord length distribution in rectangular volumes of  $24 \times 18 \times 16$  (upper) and  $20 \times 20 \times 10$  (lower) micrometers.

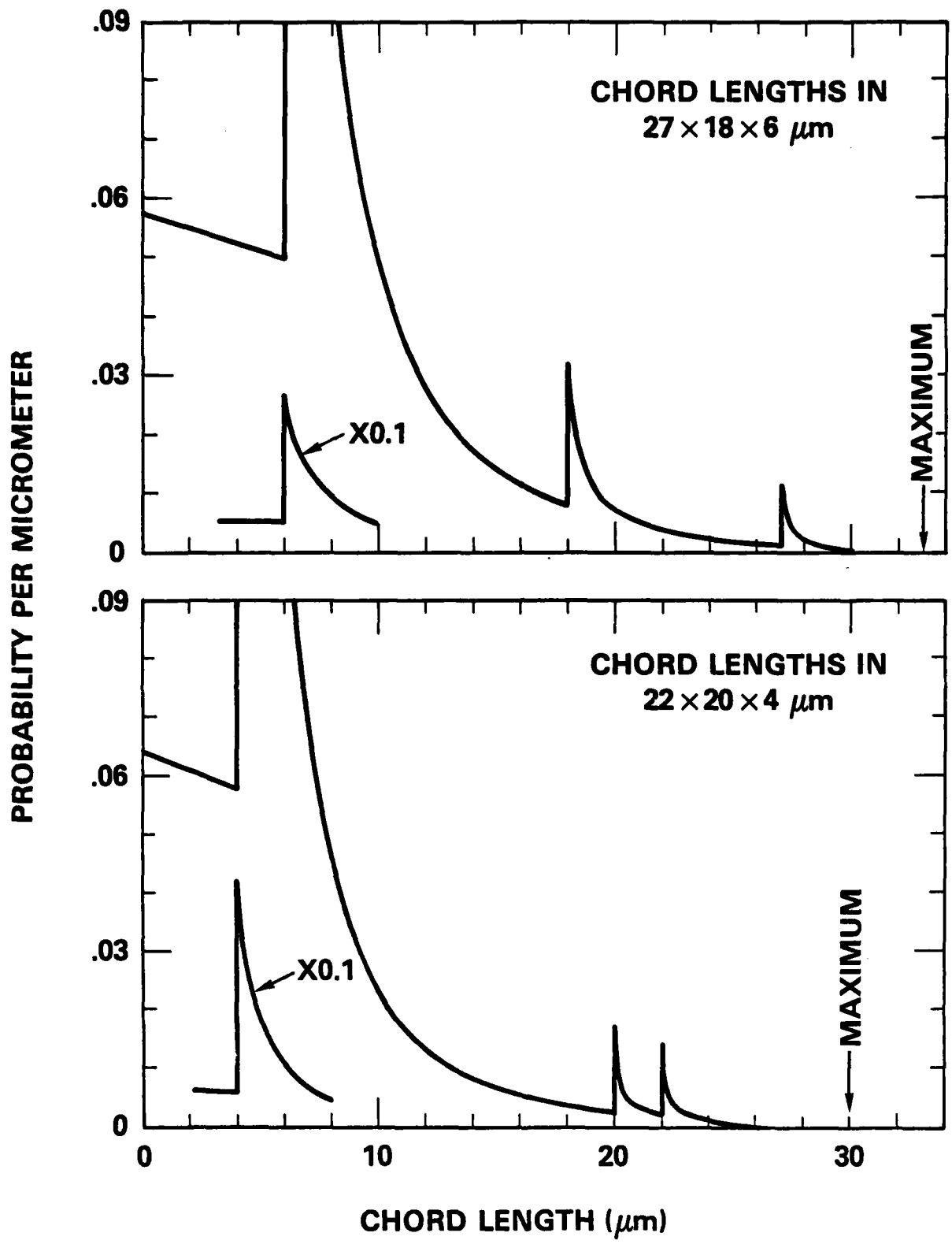


Fig. 2. Chord length distribution in rectangular volumes of  $27 \times 18 \times 6$  (upper) and  $22 \times 20 \times 4$  (lower) micrometers.

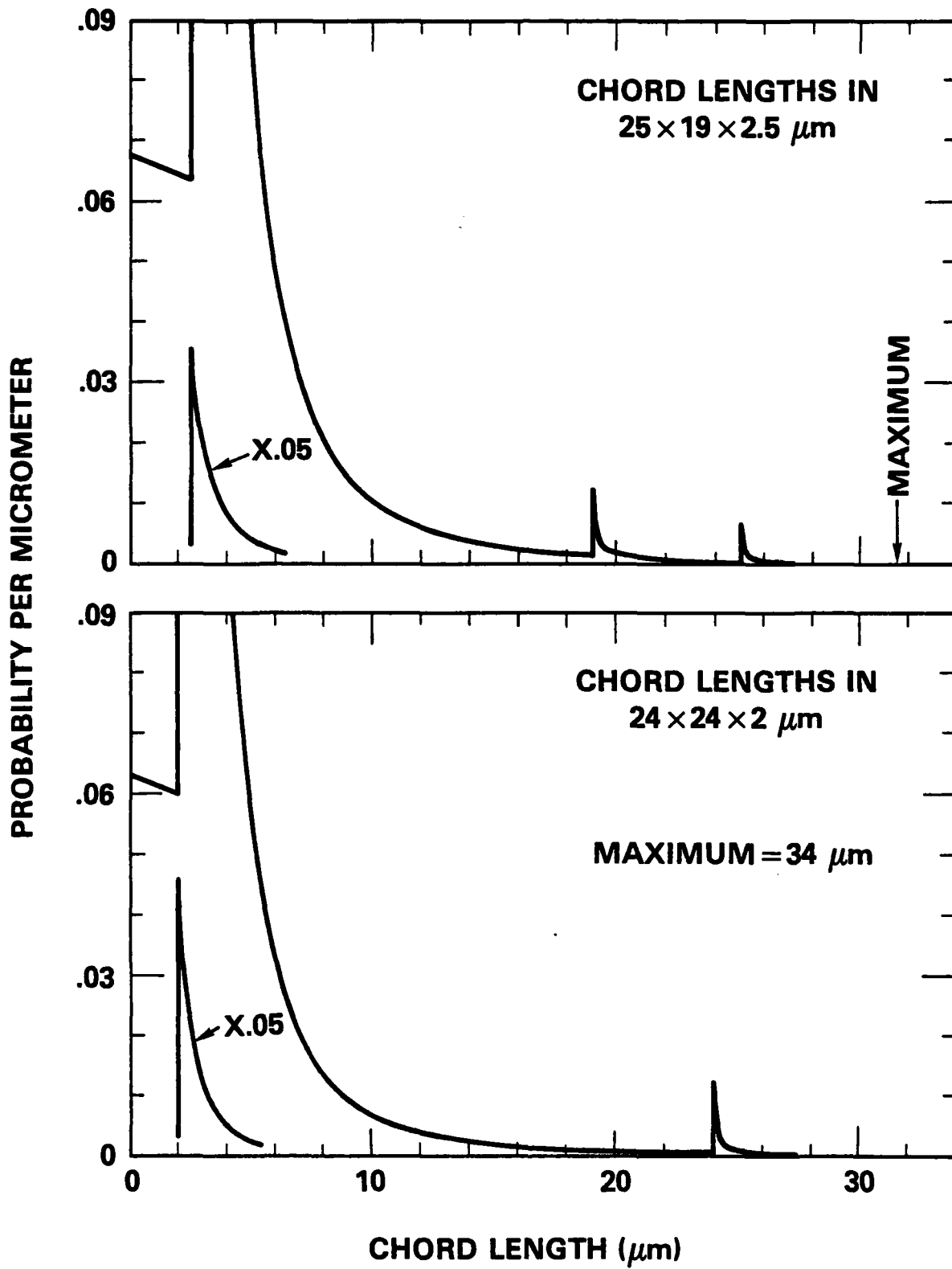


Fig. 3. Chord length distribution in rectangular volumes of  $25 \times 19 \times 2.5$  (upper) and  $24 \times 24 \times 2$  (lower) micrometers.

### Definite Integrals

The quantity desired in many calculations is the integral from zero or to the maximum length, R, rather than in an interior region. We shall let J be the integral from zero to S. In the first domain, where  $0 \leq S < Z$ , this is merely

$$J_1 = \int_0^S N_{1z} dS = I_1(S), \quad (16)$$

as  $I_1(0) = 0$ . In the second region, with  $Z \leq S < T$ , one has

$$J_2 = I_2(S) - I_2(Z) + I_1(Z) - I_1(0).$$

Upon evaluation, this is

$$J_2 = I_2(S) + B_0, \quad (17)$$

where  $B_0 = 3Y^2Z^2/K^2 + 6XY \tan^{-1}(Y/X)$ . The integral from zero in the third domain, where  $T \leq S \leq R$ , is

$$J_3 = I_{3z}(S) + B_0 + 5Z^2 - 8X^2 + 6XZ \cos^{-1}(Z/T). \quad (18)$$

It is easily seen that, when summed over paired permutations with X and Z exchanged, this is equivalent to

$$J_{3z} = I_{3z}(S) + B_0 - 3X^2 + 1.5 \pi X. \quad (18E)$$

for each permutation.

### Fraction to Maximum Chord Length

The fraction of chords longer than S is

$$C(S) = 1 - \int_0^S F dS. \quad (19)$$

When integrated, Eq. (1) is found to be

$$3 \pi [LW + LH + WH](1 - C) = \sum J. \quad (20)$$

The value of C is plotted in Figs. 4-6 for the same dimensions as employed for the differential probability in Figs. 1-3. Values obtained with two approximations to C, presented below, are also shown. For S near R, C is proportional to  $(R - S)^4$ .

A simpler form for C is highly useful when integrated with a flux given as  $\phi(\mathcal{E})$ , where  $\mathcal{E}$  is the linear energy transfer (LET). Burke<sup>7</sup>, Shapiro et al.<sup>8</sup>, and Petersen et al.<sup>9</sup> have reported on single event upsets in fluxes so specified. An upset will occur whenever  $S\mathcal{E}$  is at least the critical energy deposition, E, of the device. Therefore we have

$$\text{Upset rate} = \int \phi(\mathcal{E}) C(S) d\mathcal{E}, \quad (21)$$

with  $S = E/\mathcal{E}$ . The upper limit of integration is the maximum value of  $\mathcal{E}$ ; the lower limit is  $E/R$ . If  $\phi$  and C are each given

as the sum of one or more terms in powers of  $\epsilon$  or  $S$ , analytic integration is elementary.

### Approximations for C

Bradford<sup>10</sup> has developed an approximation for the case where the minimum dimension,  $H$ , is not more than a third of  $L$  or  $W$ . His formula, utilized in the above-mentioned reports<sup>7-9</sup>, is

$$C = 1 - 0.25 S/H \quad \text{for } S \leq H \quad (22a)$$

$$\text{and } C = 0.75 (H/S)^{2.2} \quad \text{for } S > H. \quad (22b)$$

This formula is a fair representation of the integrated chord distribution except for the longest cases; see the figures. When high-LET particles (requiring only moderate  $S$  for an upset) are plentiful, the discrepancy at larger values of  $S$  is tolerable. When particles with sufficiently large LET are few (or absent), this approximation cannot be employed.

In order to allow use for spectra where longer chord lengths are important, a similar but more complex formula is presented here. We first define a coefficient,

$$A = 2.37 / (1.80 + R/H). \quad (23)$$

The two-part formula is

$$C = 1 - AS/H \quad \text{for } S \leq H \quad (24a)$$

$$\text{and } C = (1-A) [(R^{3.0} - S^{3.0}) / (R^{3.0} - H^{3.0})]^{0.5} (H/S)^2 \quad \text{for } S > H. \quad (24b)$$

With the use of two dimensions ( $H$  and  $R$ ), the fit is seen to be very good, being limited by the breaks in the true value. The equation has the pleasing property of going to zero at  $S = R$ .

Let us consider the upset rate due to a flux proportional to  $\epsilon^{-3.12}$ , with an abrupt drop to zero at  $\epsilon_{\text{max}}$ . This fits the high-LET cosmic ray flux (Figs. 1, 2, and 4 of Ref. 8) for a factor of ten in LET, but the abrupt drop is a factor of 25. When the integral, Eq. (21), is transformed into an integral over  $S$ , the limits are  $S_{\text{MIN}} = E/\epsilon_{\text{max}}$  to  $R$ .

With this flux, and for all six example geometries, analytic integration was performed using the approximation formulas, and numerical integration was done using the exact equations. Ratios of "approximate upsets" to the exact values, as a function of  $S_{\text{MIN}}$ , are as expected from Figs. 4 to 6. Even for this extreme spectrum, Bradford's equations do rather well in the region for which they were designed - flat devices and small  $S_{\text{MIN}}$ . The new, more complex, approximate equation does better, but also falters in near-cubic geometries if only long chords can produce upsets. For the four flatter geometries, the upset ratio is unity within a factor of 2.42 for  $S_{\text{MIN}} < 0.5 R$  using Eqs. (22) and within a factor of 1.44 for  $S_{\text{MIN}} < 0.95 R$  using Eqs. (24).

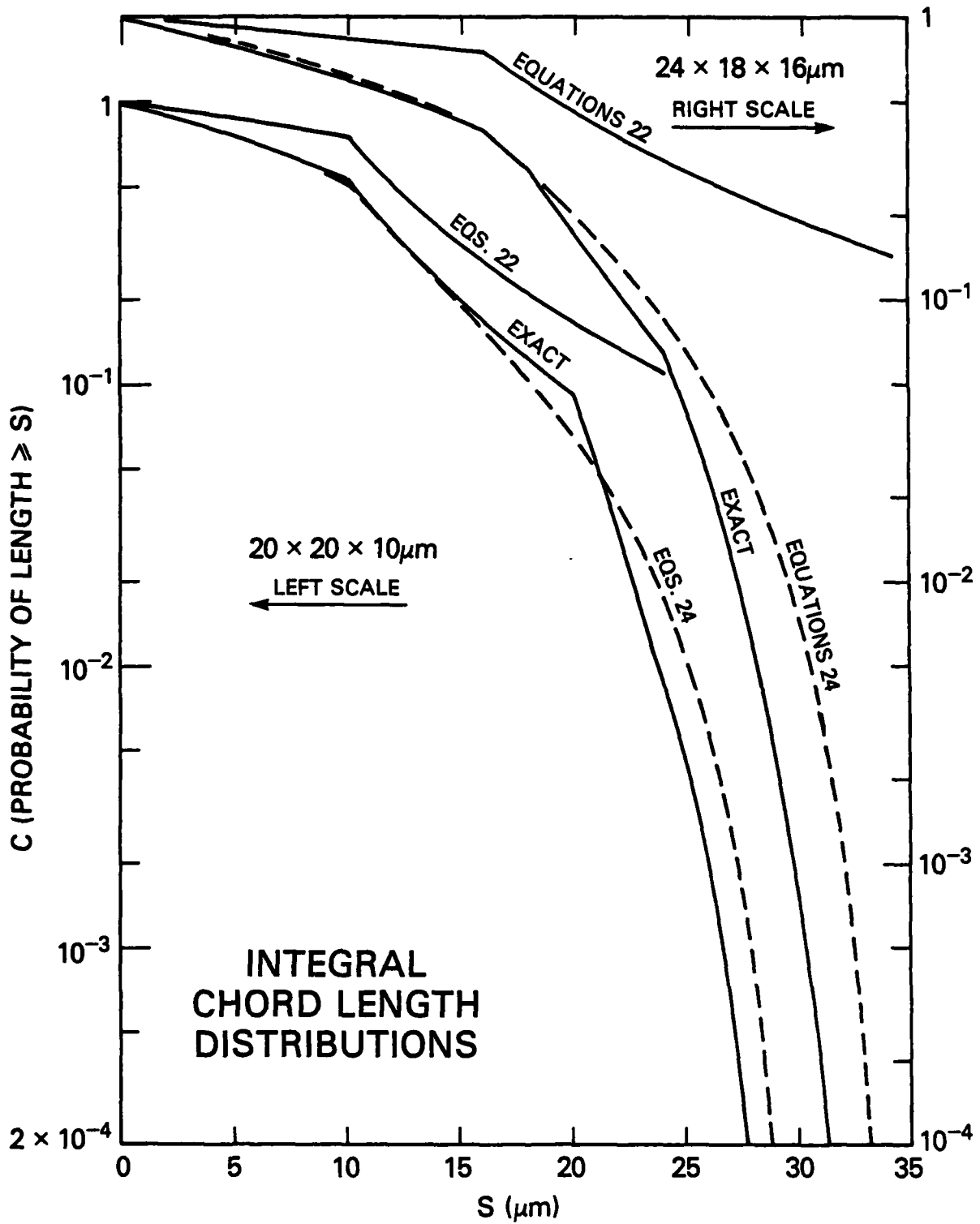


Fig. 4. Integral chord length distribution in the same volumes as Fig. 1.

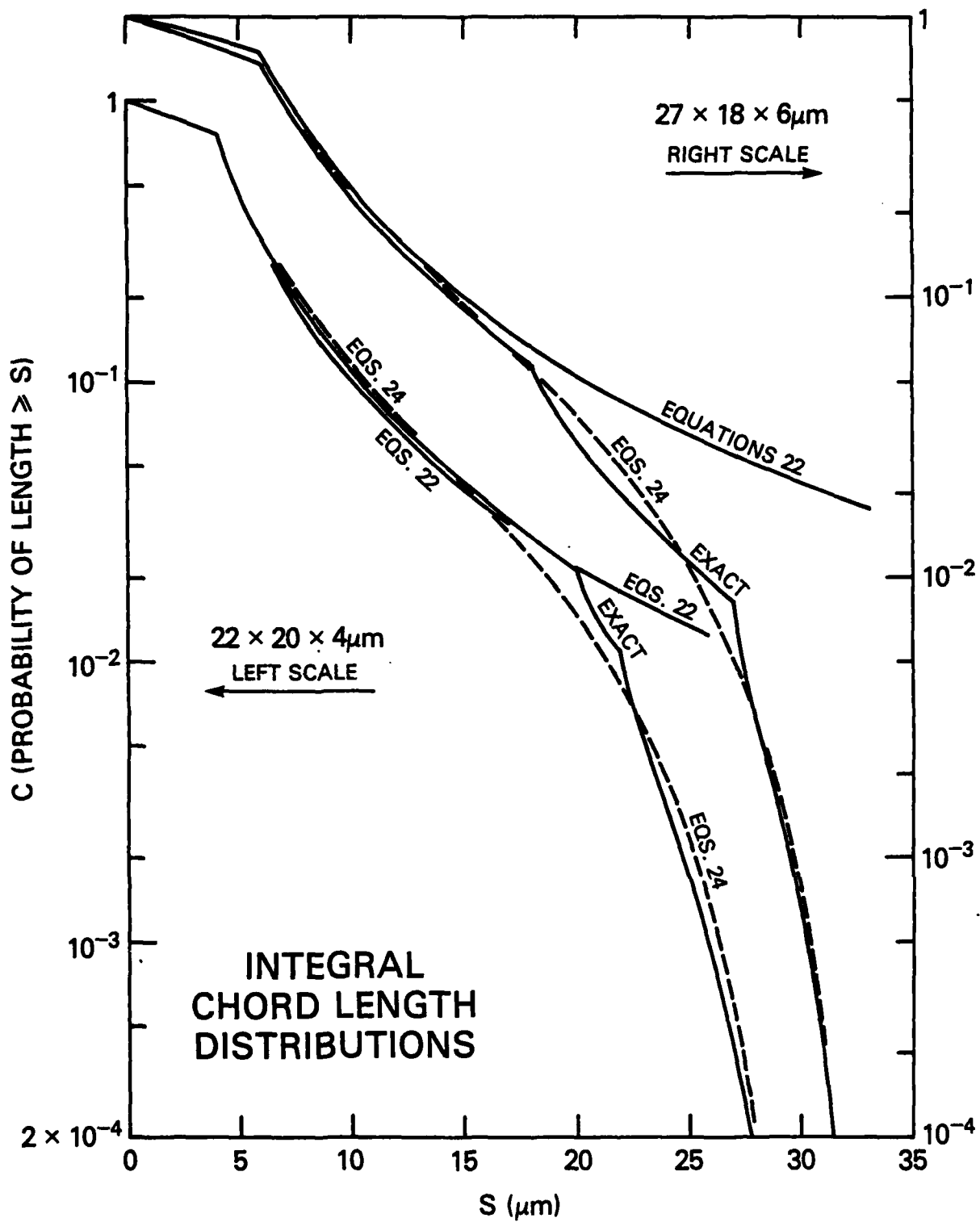


Fig. 5. Integral chord length distribution in the same volumes as Fig. 2.

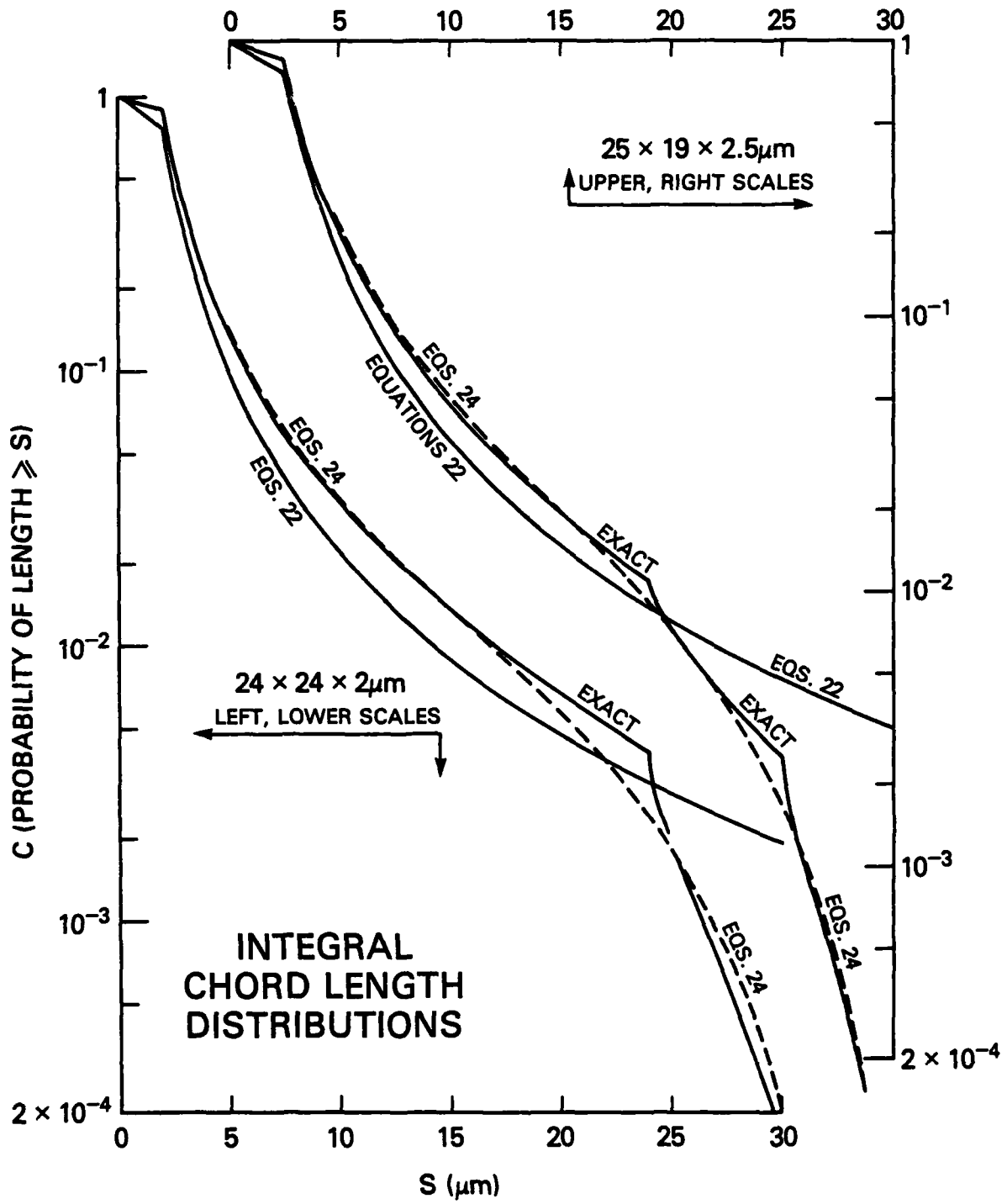


Fig. 6. Integral chord length distribution in the same volumes as Fig. 3.

### Summary

The isotropic chord length distribution is determined for rectangular parallelepipeds as a function of the chord length,  $S$ . The normalized distribution function,  $F$ , is given by Eq. (1), which uses the sum over six permutations. The value of  $N_{12}$ ,  $N_2$ , and/or  $N_{32}$  is used as appropriate.

The integral chord length distribution is obtained from the indefinite integral (I) of  $N$  and the definite integral (J) of  $N$  from zero to  $S$ . The normalized total integral distribution is given in Eq. (20).

Results for the integrated distribution are compared with two approximations, one of them introduced here. The approximations are designed for integration with particle fluxes given in terms of linear energy transfer.

### Acknowledgments

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