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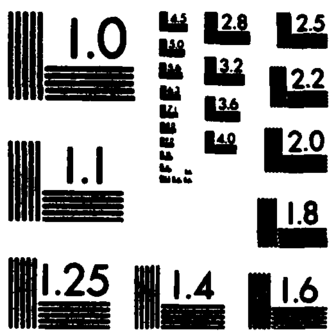


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# RESEARCH ON SELF-GENERATED STOCHASTIC MOTION

FINAL REPORT

by

Professors A. J. Lichtenberg and M. A. Lieberman

for

Office of Naval Research

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Basic problems in statistical physics are treated, with particular emphasis given to self-generated stochasticity for dynamical motion in two or more degrees of freedom. The basic procedure is to apply the modern theory of the destruction of invariants, the transition to stochastic motion, and statistical descriptions of stochastic motion. The theory is compared with numerical experiments using mapping approximations, which are iterated for hundreds of thousands of mapping periods on high speed computers. Particular (con't. on back)

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ABSTRACT

Basic problems in statistical physics are treated, with particular emphasis given to self-generated stochasticity for dynamical motion in two or more degrees of freedom. The basic procedure is to apply the modern theory of the destruction of invariants, the transition to stochastic motion, and statistical descriptions of stochastic motion. The theory is compared with numerical experiments using mapping approximations, which are iterated for hundreds of thousands of mapping periods on high speed computers. Particular problems treated are the fine structure of phase space, effects of higher dimensionality including Arnold diffusion, modulational diffusion, and the interaction between extrinsic stochasticity and resonances.

## INTRODUCTION

The research on this contract involved basic studies of stochasticity, using the modern theory of mappings and the destruction of invariants in Hamiltonian systems. The problems studied were the following:

1. Examination of the fine structure of phase space and corrections to quasilinear diffusion.
2. Study of effects of higher dimensionality in two-frequency Fermi acceleration and two-frequency electron cyclotron resonance heating (ECRH).
3. Further investigation of Arnold diffusion in the application of two-frequency ECRH.
4. Calculation of modulational diffusion.
5. Investigation of the combined effect of extrinsic stochasticity and resonances.

In addition to the above topics, the work under these topics was complementary and supportive of studies of stochasticity in high temperature plasma under NSF grant ECS-8104561 and of fusion applications to stochasticity under DOE grant DE-AT03-76ET53059. An important aspect of our effort has been the preparation and publication of a research monograph Regular and Stochastic Motion, Springer-Verlag, N.Y. (1982). This is the first such work on the subject of intrinsic stochasticity from the point of view of physicists and engineers.

In the following sections we summarize the various research areas, list the publications arising from contract support work as numbered in the summaries, and list the Ph.D. candidates and post doctoral candidates supported by the contract.

## SUMMARY OF RESEARCH

### 1. Fine Structure of Phase Space

We have studied whether the higher order corrections to the quasilinear diffusion coefficient for the standard (Chirikov-Taylor) map can be applied to other maps which can be locally approximated by the standard map[1,2,3]. We have analytically obtained a local diffusion coefficient as a function of action for the Fermi map[2,3] which displays the characteristic oscillations in action. We have used this diffusion coefficient in the Fokker-Planck equation to obtain numerically the diffusion of an initial delta function distribution in action. We have compared the variance of this distribution with a numerical calculation of the diffusion for the same initial distribution obtained by direct iteration of the mapping equations, obtaining good agreement. The comparison demonstrates that the local approximation can be used to obtain the higher order corrections to the diffusion over much of the stochastic region for the Fermi map. We have also numerically found dips in the long-time distribution in action space corresponding to the islands, which is consistent with the theoretical expectation that the long-time phase space density distribution of chaotic orbits uniformly fill the accessible phase space. These results are of considerable significance because there are no known techniques for obtaining directly the quasilinear corrections for most maps.

### 2. Two Frequency Heating

We have completed our study of two frequency heating in the Fermi map[4]. The results show that using two frequencies gives an approximately two-fold increase in energy for the position of the lowest KAM barrier to stochastic heating when the resonances associated with the two frequencies are properly interlaced.

We have considered the more practical and complicated problem of two-frequency electron cyclotron resonance heating (ECRH) in mirror-confined plasmas[5,6]. This problem is challenging because the motion takes place in a higher dimensional phase space than that for Fermi acceleration. The ECRH is modelled by means of a four dimensional symplectic mapping derived from the nonrelativistic single particle energy change due to the spatially separate resonance zones, not calculable from previous two-dimensional models. Fixed points have been located and their linear stability determined analytically. Resonances in action space have been calculated and used to obtain the adiabatic barrier. Analytical expressions were derived for the quasilinear diffusion coefficients in the stochastic regime below the adiabatic barrier and found to agree will with numerical calculations.

The adiabatic (KAM) barrier to heating for four-dimensional mappings is determined by applying island overlap criteria such as the "two-thirds rule."[7] In some cases, including the ECRH mapping, the islands are staggered in phase, and it is reconnection of their separatrices that determines the presence or absence of a KAM barrier. This new mechanism has been studied for several two-dimensional model mappings[8]. An analytical reconnection threshold has been derived from an averaged Hamiltonian and found to agree will with numerical calculations.

The implication of these results for plasma confinement experiments such as STM, TMX-Upgrade and MFTF-B has been investigated, in collaboration with the DOE. In these devices the rf diffusive losses can be comparable to the collisional losses[6].

## 3. Arnold Diffusion

Our studies of Arnold diffusion in the billiards problem have been completed[7,9]. A major application, in which considerably more complexity is encountered, is in electron cyclotron resonance heating. For one specific case of three strong resonances which are approximately equally spaced in frequency the thin layer Arnold diffusion along a resonance layer has been treated theoretically, yielding diffusion coefficients in reasonable agreement with numerically determined values. This mechanism, which can cause particles to diffuse through the adiabatic barrier, is found to be somewhat weaker than collisional diffusion for the STM plasma confinement device. However, Arnold diffusion could prove significant for additional heating of plasmas having higher temperature (or lower density).

## 4. Modulational Diffusion

In modulational diffusion a resonance between two degrees of freedom is modulated at a slow driving frequency, yielding a broad band of stochasticity. The coupling of this stochastic motion to other degrees of freedom is known as modulational diffusion. The basic theory[7] of modulational diffusion has been further developed, allowing the diffusion coefficient to be evaluated analytically without the need for adjustable fitting parameters. The coefficient has been compared with numerical calculations over a wide range of system parameters and for diffusion rates varying by over twenty orders of magnitude. The numerical and analytical results are in good agreement. A paper describing these results has been accepted for publication[10].

## 5. Interaction of Extrinsic Stochasticity with Resonances

When a small extrinsic stochasticity ("noise") is added to the equations of a multidimensional oscillator system, the classical transport processes can be dramatically altered when the motion lies inside a resonance surface[7]. Small kicks in the action in one degree of freedom can be geometrically enhanced to produce rapid diffusion in a second degree of freedom. A general theory of this process has been developed[11]. When applied to the motion of electrons and positrons in high energy storage rings, it provides a reasonable explanation for the observed beam blow-up[12,13]. This work also has direct relevance to the possible limitations on beam brightness for the next generation of proton-antiproton storage rings[13].

## PERSONNEL SUMMARY

### Graduate Students

Jeffry Tennyson (Ph.D. 1981); Present affiliation: Institute for Fusion Studies

Norman Murray (Ph.D. expected 1984)

Steven Hohs (Ph.D. expected 1985)

### Post Doctoral Fellow

James Howard; Present affiliation: TRW

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