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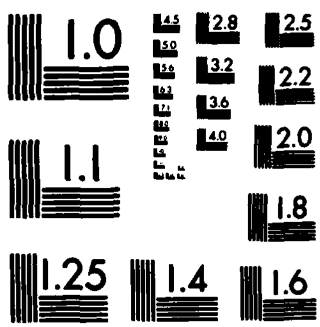

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When considering using databases to represent incomplete information, the relationship between two facts where one may imply the other needs to be addressed. In relational databases, this question becomes whether null completion is assumed. That is, does a (possibly partially-defined) tupe imply the existence of tuples that are "less informative" than the original tuple. We show that no relational algebra, that assumes equivalence under null completion, can include set-theoretic operators that are compatible with ordinary set theory.

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Set-Theoretic Problems of Null Completion in Relational Databases

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ABSTRACT. When considering using databases to represent incomplete information, the relationship between two facts where one may imply the other needs to be addressed. In relational databases, this question becomes whether null completion is assumed. That is, does a (possibly partially-defined) tuple imply the existence of tuples that are "less informative" than the original tuple. We show that no relational algebra, that assumes equivalence under null completion, can include set-theoretic operators that are compatible with ordinary set theory.

KEYWORDS. Relational databases, null values, set theory.

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1. Introduction

Considerable work has been done in incomplete information [ANSI 75, Codd 79, Goldstein 81, Grant 77, 79, Imielinski 81, 83, Keller 84a, 84b, Lien 79, Lipski 79, Maier 83, Reiter 80, Vassiliou 79, Zaniolo 82]. No solution has been completely satisfactory. One problem with many of the proposed solutions is that they are incompatible with the rules of ordinary set theory. We show that no solution that includes the concept of null completion [Zaniolo 82] can possibly be compatible with ordinary set theory.

One question that arises when considering incomplete information is the relationship between facts and partial versions of those facts. For example, the fact "Marty has been married to Barbara for seven years" includes the facts "Marty is married to Barbara" and "Marty has been married for seven years."

Consider the following relation encoding the three facts mentioned above, with the functional dependencies Husband → Wife, YearsMarried and Wife → Husband and YearsMarried.

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Husband	Wife	YearsMarried
Marty	Barbara	7
Marty	<null>	7
<null>	Barbara	7

The information contained in the first tuple includes the information contained in the other two tuples. In a static database, all queries answerable from the entire database are also answerable from the first tuple alone. However, if we delete the first tuple (say, in an update asserting that Marty is not married to Barbara) but retain the other two, we can still answer some queries asking how long Marty or Barbara have been married (but not to each other).

The above example illustrates the principle of null *completion* [Zaniolo 82, Maier 83]. Tuple t is at least as informative as tuple s (written $t \geq s$) if the non-null attributes of s have the same values as in t. A database D_1 is at least as informative as database D_2 if for every tuple of D_2 there is a corresponding tuple in D_1 that is at least as informative. The principle of null completion says that two databases are equivalent if they are equally informative.

2. Definitions

We will use nulls to indicate that no information is known. Such nulls do not distinguish between the attribute being inapplicable to the tuple and the value is known to lie in a particular set. A tuple t is at least as defined as tuple s ($t \geq s$), if all *non-null* attributes in s have matching non-null values in t. The null completion of relation $R = \{\, t \mid (s \in R) \wedge (t \leq s) \,\}$. "No information" nulls and null completion are Zaniolo's concepts [Zaniolo 82].

Null completion induces equivalence classes of relations for each relation schema. We designate the equivalence class containing the relation R as $\hat{R}$. We distinguish two particular relations which are members of $\hat{R}$: The maximal representative of $\hat{R}$ is defined by

$$R^* = \{\, t \mid (\exists s \in R)(t \leq s) \,\}.$$

The minimal representative of $\hat{R}$ is defined by

$$R_* = \{\, t \in R \mid (\nexists s \in R)(s \neq t \wedge s \geq t) \,\}.$$

We now consider definitions of the set theoretic operations. We can define operations on these equivalence classes that we will call union, intersection, and difference. These operations take two equivalence classes and result in an equivalence class. A fourth operation, membership, takes a tuple and an equivalence class and has a boolean result. We will use the symbols $\tilde{\cup}$, $\tilde{\cap}$, and $\tilde{-}$ to represent Zaniolo's [82] operators on these equivalence classes, while $\hat{\cup}$, $\hat{\cap}$, and $\hat{-}$ will represent arbitrary definitions of these operators under certain constraints (to be described later). Similarly, $\tilde{\in}$ and $\hat{\in}$ are symbols for membership. (Choice of a particular membership operator constrains the choices of the other three operators.) Although these operators apply to equivalence classes—the extended relations, their intuitive meaning is based on how they operate on the members of the equivalence classes—the ordinary relations with nulls. Therefore, we will often write $R \tilde{\cup} S$ when we mean $\hat{R} \tilde{\cup} \hat{S}$, etc., but we remember that the results of both of them are extended relations. Since these operators are well defined on the equivalence classes, it does not matter which representative relation is used.

Using our terminology, we rephrase Zaniolo's definitions of union, difference, and x-intersection by supplying representative sets of the resultant equivalence classes. Union is defined $(R \tilde{\cup} S)^* \doteq R^* \cup S^*$. Difference is $R \tilde{-} S \doteq R^* - S^*$. We define x-intersection by $(R \tilde{\cap} S)^* \doteq R^* \cap S^*$. The definition of membership is $r \tilde{\in} R = r \in R^*$.

Let us consider a few examples. We will use $\perp$ to indicate a null value. Suppose R is $\langle$a1, b1$\rangle$. Then R^* consists of $\langle$a1, b1$\rangle$, $\langle\perp$, b1$\rangle$, $\langle$a1, $\perp\rangle$, and $\langle\perp, \perp\rangle$; R_* consists only of $\langle$a1, b1$\rangle$. Suppose S is $\langle$a1, $\perp\rangle$. Then S^* consists of $\langle$a1, $\perp\rangle$ and $\langle\perp, \perp\rangle$; S_* consists only of $\langle$a1, $\perp\rangle$. Note that $R \tilde{\cup} S \doteq R$ and $R \tilde{\cap} S \doteq S$. Also, $T \doteq R \tilde{-} S \doteq \{\langle \text{a1}, \text{b1}\rangle, \langle \perp, \text{b1}\rangle\}$. It is interesting to note that $\hat{T} = \hat{R}$.

With the above examples, we can illustrate that these definitions do not satisfy some basic theorems in set theory adapted to these extended relations. In particular, $R \tilde{-} (R \tilde{-} S) \doteq R \tilde{\cap} S$ [Halmos 60] is not satisfied: the left side is $\emptyset$ and the right side is S.

Another problem with Zaniolo's approach is that the definitions do not reduce to the standard definitions when only fully defined relations are used. For example, suppose R is $\langle$a1, b1$\rangle$ and S is $\langle$a1, b2$\rangle$. Then $R \tilde{\cap} S$ is $\langle$a1, $\perp\rangle$, while $R \cap S$ is $\emptyset$.

We can consider alternative definitions of the set-theoretic operators, attempting to find a group that satisfy the basic theorems of set theory. (For example, $R \hat{\cap} S = (R_* \cap S^*) \cup (R^* \cap S_*)$ reduces to the standard definition for fully defined relations, but still violates $R \hat{-} (R \hat{-} S) = R \hat{\cap} S$.)

3. Theorem

Let us consider some requirements for set-theoretic operators. The following 5 axioms are adapted from ordinary set-theory [Halmos 60] for extended relations.

$$(t \hat{\in} R) = (t \hat{\in} S) \leftrightarrow R \doteq S \tag{1}$$

$$((t \hat{\in} R) \rightarrow (t \hat{\in} S)) \leftrightarrow R \hat{\subset} S \tag{2}$$

$$t \hat{\in} R \hat{\cup} S \leftrightarrow t \hat{\in} R \vee t \hat{\in} S \tag{3}$$

$$t \hat{\in} R \hat{\cap} S \leftrightarrow t \hat{\in} R \wedge t \hat{\in} S \tag{4}$$

$$t \hat{\in} R \hat{-} S \leftrightarrow t \hat{\in} R \wedge t \hat{\notin} S \tag{5}$$

It is interesting to note that Zaniolo's definitions satisfy all of these axioms except for (5).

We shall also require several soundness criteria. First, if a tuple is a member of every relation in an equivalence class, it must be in the extended relation, and if a tuple is a member of an extended relation, it must be in some relation in the corresponding equivalence class.

$$t \in R_* \rightarrow t \hat{\in} R \rightarrow t \in R^* \tag{I}$$

Second, extended relations preserve set inclusion.

$$R \subset S \rightarrow R \hat{\subset} S \tag{II}$$

Third, since extended relations are based on relative information content, only a tuple that is more informative can determine membership.

$$t \hat{\in} R \rightarrow t \hat{\in} \{ r \in R \mid r \geq t \} \tag{III}$$

The fourth criterion requires compactness: if a tuple is a member of an extended relation, it can be traced to a single tuple in the original relation.

$$t \hat{\in} R \rightarrow (\exists r \in R)(t \hat{\in} \{ r \}) \tag{IV}$$

Note that the leftward implication of the last two criteria can be derived from (2) and (II). Zaniolo's definitions satisfy all four of these last criteria.

We shall show that no definition for the four set-theoretic operators defined on extended relations can be compatible with (1)-(5) and (I)-(IV).

Theorem. No definitions of the membership, intersection, union, and difference operators, respectively, defined on extended relations are compatible with (1)-(5) and (I)-(IV).

Proof (by contradiction). Suppose that a set of such definitions exists. Let $\hat{\in}$, $\hat{\cap}$, $\hat{\cup}$, and $\dot{-}$ be membership, intersection, union, and difference operators, respectively, defined on extended relations that are compatible with (1)–(5) and (I)–(IV).

From (1)–(5), we can derive the following theorems we shall use [Halmos 60].

$$R \hat{\subset} S \wedge R \hat{\subset} T \to R \hat{\subset} (S \hat{\cap} T) \quad (6)$$

$$(R \dot{-} S) \hat{\cap} (R \hat{\cap} S) \doteq \emptyset \quad (7)$$

$$R \dot{-} S \doteq \emptyset \leftrightarrow R \hat{\subset} S \quad (8)$$

$$(R \hat{\subset} S) \wedge (S \hat{\subset} R) \leftrightarrow (R \doteq S) \quad (9)$$

$$(t \hat{\in} R) \wedge (R \hat{\subset} S) \to (t \hat{\in} S) \quad (10)$$

Lemma. $R_* \cap S_* \hat{\subset} R \hat{\cap} S$.

Proof of lemma. We first note that $R_* \cap S_* \subset R_*$. By (II), $R_* \cap S_* \hat{\subset} R$. Similarly, $R_* \cap S_* \hat{\subset} S$. By (6), $R_* \cap S_* \hat{\subset} R \hat{\cap} S$.

Intersection. We will show that $R \hat{\cap} S \doteq R_* \cap S_*$. Suppose that $R \hat{\cap} S$ is not equivalent to $R_* \cap S_*$ for some R and S. Then by the lemma, there exists a tuple $t \hat{\in} R \hat{\cap} S$ and $t \hat{\notin} R_* \cap S_*$. Since $t \hat{\notin} R_* \cap S_*$, either $t \notin R_*$ or $t \notin S_*$. (If $t \in R_*$ and $t \in S_*$, then $t \in R_* \cap S_*$, and by (I), $t \hat{\in} R_* \cap S_*$.) We define $R' = \{ r \in R_* \mid r \geq t \wedge t \hat{\in} \{ r \} \}$ and also $S' = \{ s \in S_* \mid s \geq t \wedge t \hat{\in} \{ s \} \}$. By (III) and (IV), $t \hat{\in} R'$ and $t \hat{\in} S'$. Then from (4) we obtain that $t \hat{\in} R' \hat{\cap} S'$. Suppose $R' \dot{-} S' \doteq \emptyset$. Then by (8), $R' \hat{\subset} S'$. Similarly, $S' \dot{-} R' \doteq \emptyset$ implies $S' \hat{\subset} R'$.

Case I. Both differences are empty. Then by (9), $R' \doteq S'$. Since minimal representative of an equivalence class is unique, $R'_* = S'_*$. But since $R'_* = R'$ and $S'_* = S'$ (a subset of a minimal representative is still a minimal representative (although of a different equivalence class)), $R' = S'$. Then since $R' \subset R_*$ and $S' \subset S_*$, $R' \subset R_* \cap S_*$. Then $t \hat{\in} R_* \cap S_*$. This is a contradiction.

Case II. At least one of the differences is non-empty. Without loss of generality, assume that $R' \dot{-} S'$ is non-empty. Then let $r \hat{\in} R' \dot{-} S'$. Then $r \hat{\in} R'$ (by 5) and $\{ r \} \hat{\subset} R' \dot{-} S'$. (By (2) and (I), $t \hat{\in} T \to \{ t \} \hat{\subset} T$.) We defined R' above so that $t \hat{\in} \{ r \}$. Therefore, $t \hat{\in} R' \dot{-} S'$. By (4) and (10), $t \hat{\in} (R' \dot{-} S') \hat{\cap} (R' \hat{\cap} S')$. This contradicts (7).

Since both cases result in a contradiction, we have shown

$$R \hat{\cap} S \doteq R_* \cap S_* \quad (*)$$

Membership. We will show that $t \hat{\in} \hat{T} \leftrightarrow t \in T_*$. Suppose that $t \hat{\in} \hat{T}$ but $t \notin T_*$. Then define $R = \{ r \in T_* \mid r \geq t \wedge t \hat{\in} \{ r \} \}$ and let $S = \{ t \}$. By (III) and (IV), $t \hat{\in} R$. By (I), $t \hat{\in} S$. Therefore, by (4), $t \hat{\in} R \hat{\cap} S$. Since $t \notin T_*$, $t \notin R$. (Note that $R_* = R$ and $S_* = S$.) Therefore $R_* \cap S_* = \emptyset$. Consequently, $t \hat{\notin} R_* \cap S_*$. But this contradicts (*). Therefore we have shown

$$t \hat{\in} \hat{T} \leftrightarrow t \in T_* \quad (**)$$

Union. We will show that $R \hat{\cup} S \doteq R_* \cup S_*$. $t \hat{\in} R \hat{\cup} S \leftrightarrow t \hat{\in} R \vee t \hat{\in} S \leftrightarrow t \in R_* \vee t \in S_* \leftrightarrow t \in R_* \cup S_* \leftrightarrow t \hat{\in} R_* \cup S_*$. Thus, we have shown

$$R \hat{\cup} S \doteq R_* \cup S_* \quad (***)$$

Contradiction. Let $R = \{ \langle a, \perp \rangle \}$ and $S = \{ \langle a, b \rangle \}$. We observe that $R_* = R$ and $S_* = S$. Now consider $R \hat{\cup} S$. Using (***), $T = R \hat{\cup} S = \{ \langle a, b \rangle, \langle a, \perp \rangle \}$. But $\langle a, \perp \rangle$ is less informative than $\langle a, b \rangle$. Therefore, $T_* = \{ \langle a, b \rangle \}$. We note that $\langle a, \perp \rangle \hat{\in} R$, yet $\langle a, \perp \rangle \hat{\notin} T$ by (**). This is a contradiction. ∎

We have shown that no definitions of the four set-theoretic operators compatible with extended relations can be compatible with traditional set theory.

4. Conclusion

Relational database theory relies heavily on ordinary set theory. Intuitively, null completion appears to be important for dealing with nulls. The previously proposed approaches that incorporated null completion were not compatible with set theory, but it was not known whether a compatible approach existed. We have shown that, if we adopt null completion, our set theoretic operators cannot behave according to the intuitive rules of ordinary set theory. We are faced with the Hobson's choice between giving up our intuitive definitions of set theoretic operators and giving up null completion. We suggest that future work attempt to compensate for the loss of null completion in order to save the familiar definitions of set theoretic operators.

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