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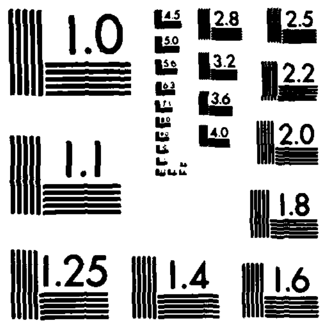

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A Covariant Derivation of the Ponderomotive Force

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August 9, 1984

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A derivation of the ponderomotive force for an unmagnetized or a uniformly magnetized particle is given here. By using a covariant form of the equations of motion, a very simple result is obtained. Evaluation of the final result is particularly simple since any four vector scalar product can be evaluated in the rest or ponderomotive frame. Examples are given for free electron laser applications.

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CONTENTS

A COVARIANT DERIVATION OF THE PONDEROMOTIVE FORCE

I. Introduction

The concept of a ponderomotive force has proven to be very useful both for investigating parametric instabilities[1-3] and also for use in free electron lasers.[4-8] The basic idea is that to lowest order, the wave gives rise to a zero average forced oscillation of the particle. To next order this oscillation beats with itself to give a nonzero slow time scale force. In doing calculations, it is generally very convenient not to have to follow the fast oscillations of the particles, but only follow their slow time motion.

However the calculation of ponderomotive force for the two applications is quite different. In the case of parametric instabilities, one exploits the nonrelativistic motion of the electrons.[9] The oscillating electric field, $\underline{\tilde{E}}(r,t) = \underline{\tilde{E}}(r,t)\exp - i\Omega t + \text{c.c.}$ where $\Omega \gg \frac{\partial}{\partial t}$ produces an oscillating velocity $\tilde{v}$ and position $\tilde{x}$. (A superscript $\sim$ indicates a rapidly oscillating quantity). The next order force, $q(\tilde{v} \times \tilde{B} + \tilde{x} \cdot \nabla \tilde{E})$ averaged over the fast time, Ω^{-1}, produces the ponderomotive force

$$\underline{F} = -\frac{q^2}{m\Omega^2}\underline{\nabla}\,(\underline{\tilde{E}} \cdot \underline{\tilde{E}}^*). \tag{1}$$

Analogous expressions can also be derived for the case of a particle in a uniform magnetic field.[10,11]

For the case of an unmagnetized free electron laser, the electrons are highly relativistic, but one instead exploits the one-dimensional nature of the equilibrium. Since quantities vary only in the z-direction, the particle canonical momentum is conserved, so that the mechanical momentum in the xy plane is given by

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$$\underline{P}_\perp(z,t) = -q\ \underline{A}_\perp(z,t). \qquad (2)$$

Note that $\underline{A}_\perp$ is the vector potential for both the wiggler and radiation fields. If the radiation field has TE polarization so that $E_z = 0$, then the force on the particle in the z-direction is $\frac{\underline{P}_\perp}{m\gamma} \times \underline{B}$ so that

$$\frac{dp_z}{dt} = \frac{-q^2}{2\gamma m} \frac{\partial}{\partial z} \underline{A}^2. \qquad (3)$$

The slow time scale part of the force then comes from taking the slow time scale part of the right hand side of Eq. (3). This generally arises from the beating of the radiation at (ω,k) with the wiggler, at k_ω. The reason this force is slow time scale is that the particle velocity is nearly equal to $\omega/(k+k_\omega)$.

The purpose of this paper is to extend the concept of a ponderomotive force to a fully three dimensional force field and relativistic particles. As such, the principal application envisioned is toward free electron lasers in which both the radiation field and wiggler field vary in the transverse plane.[12-19] Therefore, for the magnetized case, we do not consider the Larmor motion of the electrons, but assume each electron has a zero or small magnetic moment. The usual approach is to assume the perturbed motion in the wiggler and radiation field is one dimensional, even though motion in the wiggler itself can be two dimensional.

The scheme we use is particularly simple and it exploits the covariant nature of the equations of motion. The basic approximation is that along its unperturbed motion, the particle sees only a single frequency plus an additional slow time dependence. Notice that a single eikonal is not required; in fact for a free electron laser, there are two separate eikonals, the wiggler and radiation fields which have very different frequencies in the lab frame, but nearly the same frequency in the beam frame. By using the covariant equations of motion, we will see that one can write out a simple, self-contained derivation of the ponderomotive force which is quite similar to that in a nonrelativistic system. The ponderomotive potential is always very easy to evaluate because it is a Lorentz scalar, and as such can be evaluated in any convenient reference frame. We evaluate it in the reference frame of the ponderomotive wave $\left(v_p = \omega/(k+k_\omega)\right)$ in which the electrons have very low velocity.

An alternative scheme based on Hamiltonians and Lie transforms has also been proposed.[20] This scheme is more general in that it includes the Larmor motion. However it also seems to be considerably more complicated than that developed here. Currently this Lie transform scheme is formulated only for a single eikonal, although the extension to more than one eikonal appears to be straightforward.

Section II derives the ponderomotive force for the case of an unmagnetized system. Section III derives the ponderomotive force for a uniformly magnetized system, and Section IV gives specific expressions for the ponderomotive force in the more familiar three space notation for both unmagnetized and magnetized systems. In order to make the paper more self-contained, the four vector notation (from Panofsky and Phillips[21]) is reviewed in the appendix.

II. The Ponderomotive Force in an Unmagnetized Plasma

Here we use the notation of covariant (lower index) and contravariant (upper index) vectors and tensors as developed in Panofsky and Phillips.[21] The convention for raising and lowering indices is to take a dot product with the metric

$$B_i = g_{ij}\, B^j, \qquad B^j = g^{ij}\, B_j \tag{4}$$

with analogous rules for tensors. Note also that a dot product can only be taken between covariant and contravariant indices. The metric is given by

$$g_{11} = g_{22} = g_{33} = -g_{44} = -1 = g^{11} = g^{22} = g^{33} = -g^{44} \tag{5}$$

and $g_{ij} = g^{ij} = 0$ for $i \neq j$. Also, 4 denotes the time index. Additional details concerning the notation appear in the appendix.

Then the four momentum is given by $p^i = mc^2 \frac{dx^i}{ds}$ where the Lorentz scalar ds is the line element $dx^i\, dx_i$, that is the speed of light multiplied by the proper time. The four momemtum and velocity are given by $p^i = mc^2 \frac{dx^i}{ds}$ and $dx^i/ds = u^i$. The particle equation of motion is given by

$$\frac{dp^i}{ds} = qF^{ij} u_j \tag{6}$$

where F^{ij} is the field tensor (see the appendix).

Since there is no ambient magnetic or electric field, all fields are fluctuating quantities, and any rapidly varying fluctuating quantity will be denoted henceforth with a superscript tilda (so $F^{ij} = \tilde{F}^{ij}$). The basic assumption in the derivation of the ponderomotive force is that in the rest frame of the particle, there is a single, fast time dependence, exp $-i\Omega t$. Notice that this is more general than an eikonal assumption, as there can be more than one eikonal. For instance in a free electron laser, there are two eikonals, the wiggler field and the radiation field. In the lab frame, their frequencies are very different. However in the electron or ponderomotive wave frame, the frequencies are very nearly the same, so that the use of a ponderomotive force is justified.

In calculating the ponderomotive force here, we do not necessarily use the rest frame of the electrons, but can use any convenient frame in which the electrons are nonrelativistic. However this will cause errors of order v/c where v is the electron velocity in the frame in which it is nonrelativistic. If a distribution of electrons is nonrelativistic and has velocity spread δv, it is a simple matter to Lorentz transform to some reference frame where the center electron has energy γmc^2 and show that for the Lorentz transformed distribution, $\delta\gamma/\gamma \sim \delta v/c$. Thus, as long as $\delta\gamma/\gamma \ll 1$, which is nearly always true for a low ν/γ beam, there will be a reference frame in which all electrons are nonrelativistic. Therefore a single ponderomotive force, correct to order $\delta v/c$, will apply for all electrons in the distribution. We consider the ponderomotive wave frame as the frame in which all electrons have $v/c \ll 1$. Therefore the force will not depend on electron velocity (correct to $\delta v/c$), and will be the same for all electrons in the beam.

To lowest order in rapidly varying quantities, the equation of motion is

$$\frac{d\tilde{p}^i}{ds} = q\, \tilde{F}^{ij}\, u_j + \text{c.c.} \tag{7}$$

and the unperturbed orbit is

$$\frac{dx^i}{ds} = u^i = \text{constant}, \quad x_o^i(s) = x_o^i + u^i s. \tag{8}$$

Using the unperturbed orbit on the right-hand side of Eq. (7), and assuming that following the unperturbed orbit, the force is characterized by a single high frequency, the first order motion is

$$\tilde{u}^i = \frac{q}{mc^2(-iK)} \tilde{F}^{ij}(x,t)u_j \exp -iKs + \text{c.c.} \tag{9}$$

where K characterizes the single high frequency the particle sees on its unperturbed orbit and $K << \partial/\partial x$. The quantity K is a Lorentz scalar because it multiplies the scalar quantity s to give a scalar phase. In the particle rest frame it is the high frequency divided by the speed of light Ω/c. The particle displacement is given by $x^i(s) = x_o^i + u^i s + \tilde{x}^i$, where

$$\tilde{x}^i = \frac{\tilde{u}^i}{-iK} + \text{c.c.} \tag{10}$$

Using the first order orbit in $\tilde{F}_{ij}(x^i)$, expanding to first order in $\tilde{x}^i$ and using the fact that correct to this order, $x_o^i + u^i s = x^i$, we find

$$mc^2 \frac{du^i}{ds} = \langle q\, \tilde{F}^{ij}\, \tilde{u}_j + q\, \tilde{x}^k \frac{\partial}{\partial x^k} \tilde{F}^{ij}\, u_j \rangle \tag{11}$$

where the average is taken over the rapid s variation (that is space scales of order K^{-1}). Since we have used complex notation for the forces, this fast time scale average, of say $\tilde{u}\,\tilde{F}$, is simply achieved by taking $\tilde{u}^*\,\tilde{F} + \tilde{u}\,\tilde{F}^*$.

By using Maxwell's equation (Eq. 42) and the rule for raising and lowering indices,

$$\frac{\partial \tilde{F}^{ij}}{\partial x^k} = -g^{il}\, g^{jm}\left(\frac{\partial \tilde{F}_{kl}}{\partial x^m} + \frac{\partial \tilde{F}_{mk}}{\partial x^l}\right). \tag{12}$$

To start, examine the $\frac{\partial}{\partial x^1}$ term. Anticipating our final result, that the other terms are small corrections, the slow time scale term on the right-hand side is

$$mc^2 \frac{du^i}{ds} = - q\ g^{il}\ g^{jm} [\tilde{x}^{*k} \frac{\partial}{\partial x^l} \tilde{F}_{mk}\ u_j + \tilde{x}^k \frac{\partial}{\partial x^l} \tilde{F}^*_{mk}\ u_j].$$

However, using Eqs. (9) and (10) for $\tilde{x}^k$, we see that this term becomes

$$mc^2 \frac{du^i}{ds} = \frac{q^2}{mc^2} \frac{\partial}{\partial x_i} (K^{-2}\ g^{kr}\ \tilde{F}^{*p}_r\ u_p\ \tilde{F}^j_k\ u_j), \tag{13}$$

where we have made use of the fact that g^{ij} is symmetric F^i_j is antisymmetric and have redefined summed over indices where appropriate. The above force is of course slow time scale because F multiplies its complex conjugate.

Notice that the quantity in the parenthesis is a Lorentz scalar. This leads to a tremendous simplification, because this quantity can now be evaluated in any reference frame and the result will be valid in all reference frames. Specifically, it can be evaluated in the rest frame of the ponderomotive wave (correct to order $\delta v/c$). In this frame, only the time-like components of u_p are nonzero. The 4 vector force itself is the 4 gradient of this scalar quantity.

We now conclude this section by showing that the correction terms are small. Denoting the correction term by $\dot{p}^i_c$, we see

$$\dot{p}^i_c = q(\tilde{F}^{ij}\ \tilde{u}^*_j - \frac{\tilde{u}^{*k}}{iK} g^{il}\ g^{jm} \frac{\partial \tilde{F}_{kl}}{\partial x^m} u_j). \tag{14}$$

Using the fact that $u^m \frac{\partial \tilde{F}_{kl}}{\partial x^m} = - iK\tilde{F}_{kl} + 0\ \delta v/c$, it is not difficult to show that this term vanishes to order $\delta v/c$. In doing so, one makes use again of the fact that $\tilde{F}$ is antisymmetric.

III. The Ponderomotive Force in a Uniformly Magnetized Plasma

In a uniformly magnetized plasma, $F^{ij} \neq 0$. However we simplify the problem by neglecting the unperturbed Larmor motion of the particles, so $F^{ij} u_j = 0$. This is usually a very good approximation in a free electron laser, particularly if the wiggler field is turned on adiabatically so that u (magnetic moment) is conserved. The equation of motion for the fluctuating particle four velocity is then

$$mc^2 \frac{d\tilde{u}^i}{ds} - q\, F^{ij}\, \tilde{u}_j = q\tilde{F}^{ij}\, u_j. \tag{15}$$

Assuming again that the perturbed force, evaluated along the unperturbed orbit is proportional to exp − iKs, we find that Eq. (15) becomes

$$[- mc^2\, iKg^{ij} - q\, F^{ij}]\, \tilde{u}_j \equiv G^{ij}\, \tilde{u}_j = q\, \tilde{F}^{ij}\, u_j. \tag{16}$$

Therefore

$$\tilde{u}_k = q(G^{ki})^{-1}\, \tilde{F}^{ij}\, u_j, \tag{17}$$

where we note that the inverse of a contravariant tensor is a covariant tensor. Also, as before,

$$\tilde{x}_k = \frac{\tilde{u}_k}{-iK}. \tag{18}$$

To next order, the slow time scale equation is

$$mc^2 \frac{du^i}{ds} - q F^{ij} u_j = q\lfloor u_j \tilde{x}^{*k} \frac{\partial}{\partial x^k} \tilde{F}^{ij} + \tilde{F}^{ij} \tilde{u}_j^* + c.c.\rfloor \tag{19}$$

As in the previous section, we use Maxwell's equation to rewrite $\frac{\partial \tilde{F}^{ij}}{\partial x^k}$. As was the case there, the $\frac{\partial}{\partial x^i}$ term cancels, to order $\delta v/c$ against the $\tilde{u}^*$ term. What is left is

$$mc^2 \frac{du^i}{ds} - q F^{ij} u_j = - q\langle u_j \tilde{x}_p g^{kp} g^{jm} \frac{\partial \tilde{F}_{mk}}{\partial x_i}\rangle \tag{20}$$

$$= - q^2 u_j (iK)^{-1} (G^{*pr})^{-1} \tilde{F}^{*rs} u_s \frac{\partial \tilde{F}_{jp}}{\partial x_i} + c.c.$$

Notice that except for the $\frac{\partial}{\partial x_i}$, everything else is a scalar product. However as it is written, the $\frac{\partial}{\partial x_i}$ is not outside of a Lorentz scalar, but somewhere in between a collection of vector and tensor products. The next step is to show that the $\frac{\partial}{\partial x_i}$ can be taken out so that the force becomes the four gradient of a Lorentz scalar.

Since every index except $\frac{\partial}{\partial x_i}$ in Eq. (20) is part of a scalar product, all the scalar products can be evaluated in the rest or ponderomotive wave frame. Then

$$F^{ij} = \begin{array}{cc} & j \rightarrow \\ i\downarrow & \begin{bmatrix} 0 & -cB_o & 0 & 0 \\ cB_o & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{21}$$

and

$$(G^{*ij})^{-1} = \frac{1}{q^2c^2B_o^2 - m^2c^4K^2} \begin{array}{cc} & j\rightarrow \\ i\downarrow & \begin{bmatrix} -mc^2 iK & qcB_o & 0 & 0 \\ -qcB_o & -mc^2 iK & 0 & 0 \\ 0 & 0 & \dfrac{q^2c^2B_o^2 - m^2c^4K^2}{-mc^2 iK} & 0 \\ 0 & 0 & 0 & \dfrac{q^2c^2B_o - m^2c^4K^2}{mc^2 iK} \end{bmatrix} \end{array} \tag{22}$$

Observe that G^{-1} has a resonance at the cyclotron frequency. For frequencies near the cyclotron frequency, the orbit is unstable.[22] The theory here is valid only for stable orbits sufficiently far from cyclotron resonance. Since the pump field is always much larger than the radiation field, a rough criterion is that the oscillation about the parallel center velocity in the pump field be small enough that cyclotron resonance is not achieved at any point in the oscillation. Notice that $(iK)^{-1}(G^{*pr})^{-1}$ is Hermitian so

$$(iK)^{-1}(G^{*pr})^{-1} = -\,(iK)^{-1}(G^{rp})^{-1}. \tag{23}$$

Now, redefining indices of summation in the complex conjugate, we see that Eq. (20) becomes

$$mc^2 \frac{du^i}{ds} - q\, F^{ij}\, u_j = q^2(iK)^{-1}(G^{*pr})^{-1}\, \tilde{F}^{*rs}\, u_s \frac{\partial}{\partial x_i} \tilde{F}^{jp}\, u_j$$

$$+ q^2(-iK)^{-1}(G^{rp})^{-1} \tilde{F}^{pj} u_j \frac{\partial}{\partial x_i} (\tilde{F}^{*sr} u_s). \qquad (24)$$

Using Eq. (23) and the fact that u and G are both independent of x_i, the two terms on the right-hand side of Eq. (24) can be combined to give the final result

$$mc^2 \frac{du^i}{ds} - q F^{ij} u_j = - q^2 \frac{\partial}{\partial x_i}\left((iK)^{-1}(G^{*pr})^{-1} \tilde{F}^{pj} \tilde{F}^{*rs} u_j u_s\right). \qquad (25)$$

Again the four force is the four gradient of a Lorentz scalar. Furthermore, as in the previous section, the evaluation of the Lorentz scalar can be enormously simplified by taking all scalar products in the rest frame.

IV. Examples of Calculations of the Ponderomotive Force

In this section we give two examples of calculations of the ponderomotive force. Say that in the lab, a beam is traveling in the z-direction. A quantity with no subscript denotes the lab frame. Here there is a wiggler field with wave number k_w and some transverse spatial dependence, and a radiation field with (ω,k), where k is in the z-direction. The fields in the ponderomotive wave frame can be obtained by Lorentz transformation (we use ponderomotive frame instead of electron rest frame so that the force will not be velocity dependent):

$$\underline{E}_{wp} = \gamma \underline{V}_p \times \underline{B}_w \equiv \gamma \underline{V}_p \times \underline{B}_w(r) \exp -ik_w z + \text{c.c.} \qquad (26)$$

and

$$\underline{E}_{rp} = \gamma(1 - \frac{V_p}{c})E_r \underline{i}_x \equiv \gamma(1 - \frac{V_p}{c}) \underline{E}(r) \exp i(kz-\omega t) + \text{c.c.} \qquad (27)$$

assuming that the radiation is polarized in the x-direction and has the magnetic field in the y-direction and travels in the positive z-direction. Neither the pump nor wiggler has a z component of electric field, although one could easily be included. To evaluate the scalar products in Eq. (13), it is simplest to do so in the ponderomotive wave frame. Here, only the time like components of u come to play; the spacelike components are smaller by $\delta v/c$. Since the spacelike components of u are negligible, only the electric part of F contributes to the scalar product in the beam frame. In this case, we find

$$K^{-2} g^{kr} \tilde{F}_r^{*p} u_p \tilde{F}_k^j u_j = -\frac{\tilde{E}_p^* \cdot \tilde{E}_p}{K^2}. \tag{28}$$

As is usually the case in a free electron laser, the wiggler field is much larger than the radiation field. Therefore in evaluating Eq. (28), we retain only terms linear in $\underline{E}_r$. Then, the expression for the force (in the more familiar three space notation) becomes

$$\frac{d}{dt} \gamma m \underline{V} = \frac{-1}{\gamma} \underline{\nabla} \frac{q^2 \gamma^2}{m(k_w V_p \gamma)^2} [(\underline{V}_p \times \underline{B}_w^*) \cdot (\underline{V}_p \times \underline{B}_w)$$

$$+ (1 - \frac{V_p}{c}) \{(\underline{V}_p \times \underline{B}_w^*) \cdot \underline{E}_r + \underline{E}_r^* \cdot (\underline{V}_p \times \underline{B}_w)\}] \tag{29}$$

where we have used the fact that $\frac{d}{ds} = \frac{\gamma}{c} \frac{d}{dt}$. For the case of a rotating quadrupole wiggler, Eq. (29) reduces to the form used in Refs. 17 and 18 . It is a general expression for the slow time scale force on an electron in a arbitrary (but unmagnetized) wiggler and radiation field correct to order $\delta\gamma/\gamma$ where $\delta\gamma$ is the change in γ between the electron velocity and ponderomotive wave phase velocity, $V_p = \omega/k+k_w$. The first term in Eq. (29) is the ponderomotive force from the wiggler alone. Its spatial dependence is determined only by the slow space dependence of the wiggler. The second two terms are the interaction between the wiggler and radiation field. It has a slow axial spatial and temporal dependence like exp $i[(k+k_w)z-\omega t]$ multiplied by any additional slow spatial and temporal dependences of $\underline{E}_r$ and $\underline{B}_w$. The exponential factor is a slow space and time dependence because the particle is moving with a velocity nearly equal to ω/k_w+k.

We now turn to the case of the uniformly magnetized plasma. For the case of no pump or radiation field in the z-direction, the Lorentz scalar on the right-hand side of Eq. (25) can easily be evaluated in the ponderomotive wave frame. The result is

$$(iK)^{-1}(G^{*pr})^{-1}\,\tilde{F}^{pj}\,\tilde{F}^{*rs}\,u_j\,u_s =$$

$$\frac{1}{D}\left[-\,mc^2(\tilde{E}^*_{xp}\,\tilde{E}_{xp}+\tilde{E}^*_{yp}\,\tilde{E}_{yp}) - \frac{ic\,B_o q}{K}(\tilde{E}_{xp}\,\tilde{E}^*_{yp}-\tilde{E}_{yp}\,\tilde{E}_{xp})\right] \qquad (30)$$

where $D = m^2c^4K^2 - q^2c^2B^2_o$. Using the fact that $K = \gamma k_w V_p/c$ and Eqs. (26) and (27) for $\tilde{\underline{E}}_{wp}$ and $\tilde{\underline{E}}_{rp}$ one can determine the ponderomotive force in terms of lab frame expressions for wiggler and radiation fields. Inclusion of $\underline{B}_{wz}$ and $\underline{E}_{rz}$ is also very straightforward. As before, the force is a slow time scale force, and all of the fast oscillation is averaged out.

V. Conclusions

We have calculated the ponderomotive force for relativistic, unmagnetized or uniformly magnetized particles. The use of a covariant formulation greatly simplifies the evaluation of this force because all scalar products can be evaluated in a reference frame in which the electrons are nonrelativistic.

Although the application envisioned is for free electron lasers, there undoubtedly are other applications as well.

Although the immediate application is individual particle dynamics, the use of a ponderomotive force also greatly simplifies calculation of collective effects. For instance in the Raman regime free electron laser one needs first a beam equilibrium in the wiggler field. Since the exact wiggler field depends on r, θ and z, this would be very difficult to calculate. However by averaging over the fast oscillation, the wiggler ponderomotive potential might depend on many fewer variables. In the case of Refs. 17 and 18, the rotating quadrupole wiggler, the wiggler field itself depends on r and $\theta - kz$, whereas the ponderomotive wiggler potential depends only on r. Thus the use of a ponderomotive force could also greatly simplify the calculation of collective effects in multidimensional free electron lasers. This will be explored in a future work.

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Appendix

Here we review the four vector formalism as given in Ref. 20. A contravariant (upper index) vector has transformation properties

$$x'^{i} = Q^{i}_{j}\, x^{j} \tag{31}$$

where

$$Q^{i}{}_{j} = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} . \tag{32}$$

The covariant (lower index) vector has the transformation property

$$A'_{i} = (Q^{i}_{j})^{-1}\, A_{j} . \tag{33}$$

Note that taking the inverse of a contravariant matrix interchanges covariant and contravariant indices. Similarly tensors transform as

$$T'^{kl} = Q^{k}_{i}\, Q^{l}_{J}\, T^{ij} \tag{34}$$

with analogous expressions for covariant or mixed tensors. Any covariant vector can be associated with a contravariant vector by

$$B_{i} = g_{ij}\, B^{j} \tag{35}$$

and the inverse relation can be defined by using the definition $(g_{ij})^{-1} =$

g^{ij}. The four gradient of any Lorentz scalar with a contravariant distance is a covariant vector and visa versa, so

$$\frac{\partial T}{\partial x^i} \equiv Q_i \tag{36}$$

The contravariant field tensor is

$$F^{ij} = \begin{array}{ll} & \overset{j}{\rightarrow} \\ i\downarrow & \\ & \begin{bmatrix} 0 & -cB_z & cB_y & E_x \\ cB_z & 0 & -cB_x & E_y \\ -cB_y & cB_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{bmatrix} \end{array} \tag{37}$$

The covariant and mixed field tensors are

$$F_{ij} = \begin{array}{ll} & \overset{j}{\rightarrow} \\ i & \\ & \begin{bmatrix} 0 & -cB_z & cB_y & -E_x \\ cB_z & 0 & -cB_x & -E_y \\ -cB_y & cB_x & 0 & -E_z \\ E_x & E_y & E_z & 0 \end{bmatrix} \end{array} \tag{38}$$

and

$$F_i^j = \begin{array}{cc} & i\rightarrow \\ j\downarrow & \begin{bmatrix} 0 & cB_z & -cB_y & E_x \\ -cB_z & 0 & cB_x & E_y \\ cB_y & -cB_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{bmatrix} \end{array}. \tag{39}$$

The charge and current densities form the components of a contravariant four vector and the inhomogeneous Maxwell´s equation is

$$\frac{\partial F^{ij}}{\partial x^j} = \frac{J^i}{\varepsilon} \tag{40}$$

where

$$J^i = (\frac{J}{c}, \rho). \tag{41}$$

The homogeneous Maxwell´s equations are

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0. \tag{42}$$

This is a third rank covariant tensor. However since F_{ij} is antisymmetric it will vanish unless $i \neq j \neq k$. Furthermore, permuting the indices obviously does not change the result, so only 4 of the 64 elements of the tensor are nontrivial and distinct.

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