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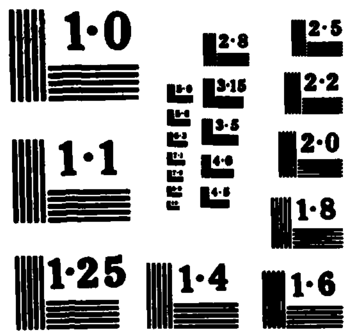
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TECHNICAL REPORT RE-84-7

ABOUT AMBIGUITY AND Q-FUNCTIONS

E. Ray McKee
Advanced Sensors Directorate
US Army Missile Laboratory

December 1983

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898

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In the Ballistic Missile Defense (BMD) community, considerable emphasis is placed upon the utilization of ambiguity and, to a greater extent, Q-functions. Quite often these functions are incorrectly applied and almost always mislabeled. The purpose of this report is to present a short tutorial on the origin and application of these functions.

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I. INTRODUCTION

Recent involvement in the SENTRY program and expected future involvement in the Advanced Tactical Missile (ATM) program prompts this technical assessment of the application of Richman's[1] Q-function to waveform design. Because of its nature, the Q-function has generally been limited to waveform design in the Ballistic Missile Defense (BMD) community. However, even in this limited application, it has not always been correctly applied. Because the Q-function derives from Woodward's[2] ambiguity function, the first part of this work will address the ambiguity function and point out some misconceptions concerning this function. Following this brief diversion a general assessment of the Q-function will be made.

II. WOODWARD'S AMBIGUITY FUNCTION

Generally, the purpose of a radar transmission is to detect an object and, in many cases, to furnish other information concerning that object. For a given radar dwell, this object, or radar target, is located in range-doppler or, equivalently, τ-f_d space. (Other location information such as angle may be available, but this information is not waveform dependent). The basic question considered by Woodward was the resolution of two targets residing at different points in τ-f_d space. Then the problem was to design the waveform so as to maximize the difference in some sense between the returns of the two distinct targets.

Therefore, consider a point-target located at the origin (0, 0) of τ-f_d space and a second point-target located arbitrarily at (τ, f_d) in this space. The waveform received from the target located at the origin is identical to the transmitted complex waveform

$$Z(t) = u(t)e^{j2\pi f_o t} \quad . \tag{1}$$

The time-delayed and Doppler-shifted waveform received from the second target is

$$Z_1(t) = u(t-\tau)e^{j2\pi(f_o-f_d)(t-\tau)} \quad . \tag{2}$$

A convenient measure of the difference, or distance, between these two waveforms is the integrated square magnitude

$$E^2 = \int \left| z(t)-z_1(t) \right|^2 dt. \tag{3}$$

Then the waveform which best resolves the two targets is the one which maximizes E^2. Rewriting:

$$E^2 = \int \left| u(t)e^{j2\pi f_o t} - u(t-\tau)e^{j2\pi(f_o-f_d)(t-\tau)} \right|^2 dt$$

$$= \int [u(t)u^*(t) - u^*(t)u(t-\tau)e^{-j2\pi(f_o-f_d)\tau} e^{-j2\pi f_d t}$$

$$- u(t)u^*(t-\tau)e^{j2\pi(f_o-f_d)\tau} e^{j2\pi f_d t} + u(t-\tau)u^*(t-\tau)]dt$$

$$= \int [u(t)u^*(t) + u(t-\tau)u^*(t-\tau)]dt$$

$$-\int [u(t)u^*(t-\tau)e^{j2\pi(f_o-f_d)\tau} e^{j2\pi f}\, dt$$

$$+ (u(t)u^*(t-\tau)e^{j2\pi(f_o-f_d)\tau} e^{j2\pi f_d t})^*]\, dt$$

$$= 2 \int \left| u(t) \right|^2 dt - 2\mathrm{Re} \left\{ e^{j2\pi(f_o-f_d)\tau} \int u(t)u^*(t-\tau)\, e^{j2\pi f_d t}\, dt \right\} \quad (4)$$

where * signifies complex conjugate.

The first term in Eq. (4) is just twice the energy of the transmitted waveform and will normally be maximized for the particular application under investigation. Therefore, in order to maximize E^2 it is necessary to minimize the magnitude of the last term of Eq. (4). Thus, it is desired to minimize

$$\left| \chi(\tau,f_d) \right| = \left| \int u(t)u^*(t-\tau)e^{j2\pi f_d t}\, dt \right| \quad (5)$$

or, equivalently, to minimize

$$\left| \chi(\tau,f_d) \right|^2 = \left| \int u(t)u^*(t-\tau)e^{j2\pi f_d t}\, dt \right|^2 \quad (6)$$

which is Woodward's ambiguity function. It is common practice to normalize Eq. (5) or (6) so that

$$\left| \chi(\tau,f_d) \right| / \left| \chi(0, 0) \right| \quad (7)$$

or

$$\left| \chi(\tau,f_d) \right|^2 / \left| \chi(0,0) \right|^2 = \Psi(\tau,f_d) \quad (8)$$

are the functions with which one normally works.

There are two important points to note here. First, one observes that $\chi(\tau,f_d)$ coincidentally has the form of an ideal matched filter response. Thus, the ambiguity function has often, but erroneously, been referred to as the matched filter response of a system. Starting from this point, researchers have constructed mismatched receiver responses and called these responses cross-ambiguity functions. The second point is that the ambiguity function is totally processor independent. Therefore, a cross-ambiguity function has no meaning whatsoever in the Woodward sense. That is, the radar processor will in no way affect the waveform ambiguity function.

III. RICHMAN'S Q-FUNCTION

Richman introduced a function defined by

$$Q(f_d) = \int_{-\infty}^{\infty} \Psi(\tau,f_d)\, d\tau \quad (9)$$

which has commonly come to be called a Q-function.

More practically, one runs into a relative Q-function defined by

$$Q(f_d) = \frac{\int_{-\infty}^{\infty} \Psi(\tau, f_d) d\tau}{\int_{-\infty}^{\infty} \Psi_1(\tau, f_d) d\tau} \tag{10}$$

where $\Psi_1(\tau, f_d)$ is the ambiguity function of a single pulse of a burst waveform with ambiguity function $\Psi(\tau, f_d)$. Since the same notation $Q(f_d)$ is used for both of these forms, one should be careful in using functional data supplied by someone else.

An integrated, properly weighted ambiguity function results in a quantity proportional to clutter power. Therefore, the relative Q-function is a measure of the improvement in clutter rejection of a burst waveform over that provided by a single pulse[1] of the burst when the clutter is uniformly extended in range. This is precisely the type of clutter exhibited by a wide-spread rain storm. In a BMD environment, the volume filled by tank breakup clutter closely approximates extended uniform clutter.

Justification for utilization of the Q-function is straightforward. If operation is in range-extended clutter, then target discrimination must be based upon Doppler information. Therefore, the waveform ambiguity function should exhibit small amplitudes at Doppler values away from the central peak. Additionally, if the range extended clutter is essentially homogeneous, then a one dimensional measure of the waveform's effectiveness may be obtained by collapsing the ambiguity function in range. It can be seen that the function given by Eq. (10) accounts both for the increased clutter power resulting from the transmission of a burst waveform as well as the decrease in clutter power resulting from Doppler filtering.

IV. UTILIZATION OF THE Q-FUNCTION

In an application such as BMD, where the search and track functions are separated, a great deal of flexibility and optimization may be achieved by having a variety of waveforms available for transmission. On the other hand, a track-while-scan system has limited flexibility in regard to waveform design since both detection and parameter estimation generally must be accomplished simultaneously.

Equation (10) gives a measure of clutter rejection provided by a burst waveform over a single pulse. This clutter rejection is based solely upon Doppler discrimination. A uniform amplitude burst will give some clutter rejection improvement over a single pulse, but a weighted burst, because of its reduced Doppler sidelobes, will produce a much more dramatic clutter rejection. Unfortunately, this weighting comes about at the expense of transmitted power; however, some clutter environments are so severe that waveform amplitude tailoring is a necessity.

The Q-function does have some utility in evaluating various weighted waveforms; however, this application is somewhat limited since an exotic weighting scheme will produce little improvement over standard weighting

[1]If the individual pulses have different bandwidths, then the pulse with maximum bandwidth is chosen for comparison.

schemes such as Taylor's. Some clutter rejection improvement is available by varying the pulse widths in a burst. This improvement appears to be much less than that obtainable by weighting but transmission power is conserved. The Q-function may be much more useful in evaluating pulse width modulated waveforms than in evaluating weighted waveforms. However, pulse width modulation produces many processing problems and is seldom, if ever, employed.

V. CONCLUSION

The Q-function has limited application and is useful primarily in evaluating time-domain weighting performance. It is seldom used to compare two different waveforms; that is, where the individual pulses are different for the two waveforms. For the most part, the many different weighting schemes have been over-evaluated many years ago and should now be selectable via engineering judgment rather than complicated analysis. The structure of the single pulse from which the burst is constructed appears to have little effect on the Q-function and should be selected on other basic considerations. Outside the BMD community, at least one weighted waveform should be strongly considered for use during periods of heavy weather activity. The use of more than one special waveform should be strongly justified.

REFERENCES

1. Richman, D., "Resolution of Multiple Targets in Clutter," Research Paper P-158, Institute for Defense Analyses, June 1966.

2. Woodward, P. M., Probability and Information Theory, with Applications to Radar, Pergamon Press, 1953.

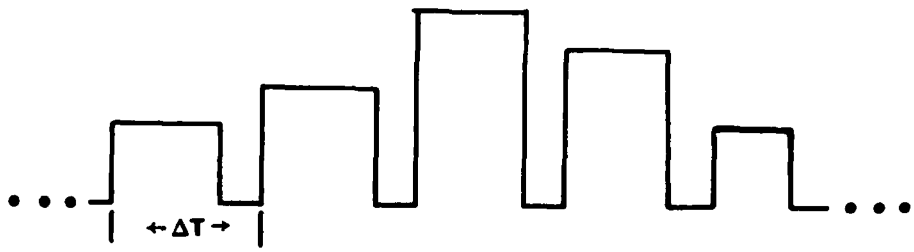
APPENDIX

COMPUTER GENERATION OF BURST AMBIGUITY AND Q-FUNCTION

APPENDIX A

Computer Generation of Burst Ambiguity and Q-Function

The ambiguity and Q-functions are directly obtainable from $\chi(\tau,f_d)$ given by Eq. (5). Let u(t) be a single, unity amplitude pulse of a burst waveform consisting of N such pulses with scaled amplitudes and pulse repetition period ΔT as shown:



In the following a receiver response function will be calculated mainly because of its widespread use (also misnomered as a cross-ambiguity function). The actual ambiguity function is obtained where the transmit and receive weights are a complex conjugation of each other. Therefore, let

$$u_1(t) = \sum_{k=1}^{N} A_k u[t-(k-1)\Delta T]$$

$$u_2(t) = \sum_{\ell=1}^{N} B_\ell u[t-(\ell-1)\Delta T]$$

$$u_2^*(t) = \sum_{\ell=1}^{N} B_\ell^* u^*[t-(\ell-1)\Delta T] \qquad \text{(A-1)}$$

where A_k and B_ℓ^* are the transmit and receive weights, respectively.

Then[1]

$$\chi(\tau,f_d) = \int u_1(t)u_2^*(t-\tau)e^{j2\pi f_d t}dt$$

$$= \int \sum_{k=1}^{N} A_k u[t-(k-1)\Delta T] \sum_{\ell=1}^{N} B_\ell^* u^*[t-(\ell-1)\Delta T-\tau]e^{j2\pi f_d t}dt$$

$$= \sum_{k=1}^{N} \sum_{\ell=1}^{N} A_k B_\ell^* \int u[t-(k-1)\Delta T]u^*[t-(\ell-1)\Delta T-\tau]e^{j2\pi f_d t}dt \qquad \text{(A-2)}$$

[1]Here the nomenclature for the ambiguity function is used although the result is an ambiguity function only where transmit and receive weighting are complex conjugates.

Letting $x = t-(k-1)\Delta T$,

$$\chi(\tau,f_d) = \sum_{k=1}^{N} \sum_{\ell=1}^{N} A_k B_\ell^* \int u(x)u^*[x-\{(\ell-k)\Delta T+\tau\}] \, e^{j2\pi f_d x} \, e^{j2\pi f_d(k-1)\Delta T} dx$$

$$= \sum_{k=1}^{N} A_k \, e^{j2\pi f_d(k-1)\Delta T} \sum_{\ell=1}^{N} B_\ell^* \chi_1[(\ell-k)\Delta T + \tau, f_d] \qquad \text{(A-3)}$$

where $\chi_1(\cdot)$ is the single pulse ambiguity function.

Discretizing τ and f_d results in[1]

$$\chi(m\Delta\tau,n\Delta f_d) = \sum_{k=1}^{N} A_k e^{j2\pi n\Delta f_d(k-1)\Delta T} \sum_{\ell=1}^{N} B_\ell^* \chi_1[(\ell-k)\Delta T \qquad \text{(A-4)}$$

$$+ m\Delta\tau, n\Delta f_d], \quad m,n = 0,1,...$$

Thus the receiver response is just a linear combination of shifted single pulse ambiguity functions. It remains only to calculate this single pulse ambiguity function:

$$\chi_1(\tau,f_d) = \quad u(t)u^*(t-\tau)e^{j2\pi f_d t} dt \qquad \text{(A-5)}$$

Discretizing and approximating the integral by a summation:

$$\chi_1(m\Delta t,n\Delta f_d) \approx \frac{T_p}{N} \sum_{i=0}^{N-1} u\left(\frac{iT_p}{N}\right) u^*\left(\frac{iT_p}{N} - m\Delta\tau\right) e^{\frac{jni2\pi}{N}} \qquad \text{(A-6)}$$

Omitting the scale factor, the result is just an inverse discrete Fourier transform for each m and is easily calculated using fast Fourier transform (FFT) techniques. Therefore

$$\chi_1(m\Delta\tau,k\Delta f_d) = \text{IFFT} \left[u\left(\frac{iT_p}{N}\right) u^*\left(\frac{iT_p}{N} - m\Delta\tau\right)\right] \qquad \text{(A-7)}$$

The procedure for calculating Eq. (A-3) is:

1. Select an appropriate sampling interval for the single pulse (Ex. 0.1μs).
2. Select the desired number of Doppler samples (Ex. 4096)[2].
3. Select a value of τ equal to some multiple of the sampling interval.
4. Form the product array $u(i)\ u^*\ (i-\tau)$ consisting of N_1 nonzero samples.

---

[1]Note that $\chi_1[(\ell-k)\Delta T+m\Delta\tau,n\Delta f_d] = 0$ for $-\Delta T>[(\ell-k)\Delta T+m\Delta\tau]>\Delta T$

[2]This determines T_p in Figure A-2 since $\frac{T_p}{N}$ = sampling interval or $\frac{T_p}{4096} = 0.1$ giving $T_p = 409.6$ μs here.

5. Append zeros to this array to total N (4096) samples.

6. Compute the inverse FFT of this array.

7. Result is doppler values (with $\frac{1}{T_p}$ Hz spacing) for this choice of τ.

8. Repeat for all desired values of τ.

With the arrays resulting from the above procedure, the receiver response is now easily calculated using Eq. (A-4). The Q-function is obtainable by simply collapsing the ambiguity function in range. However, do not forget to apply the proper normalization to both the ambiguity and Q-function.

Figures A-1 and A-2 give examples of a Q-function[1]. Figure A-1 is the Q-function of a filled uniformly weighted 16-pulse burst where each pulse is linearly frequency modulated with a bandwidth of 30 MHz. Figure A-2 is identical to A-1 except that a 50 dB Chebyshev weighting has been applied across the burst. These figures correspond to Eq. (9) except that a different normalization than that given by Eq. (8) was used.

[1]Figures supplied by Dr. Jerry Cross, Teledyne Brown Engineering, Huntsville, Alabama.

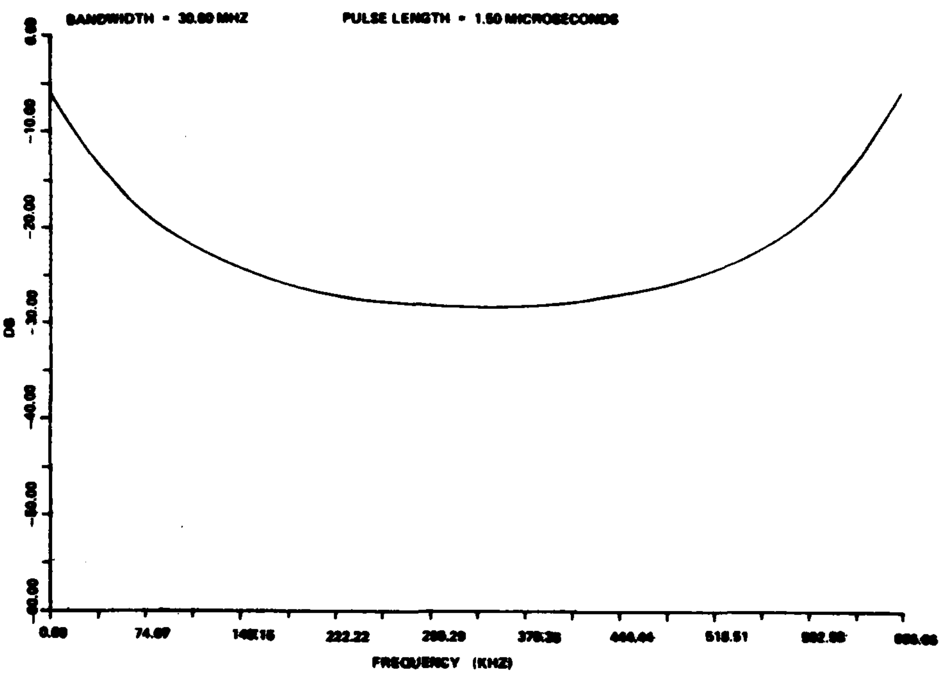


Figure A-1. Filled, uniformly weighted 16-pulse burst Q-function.

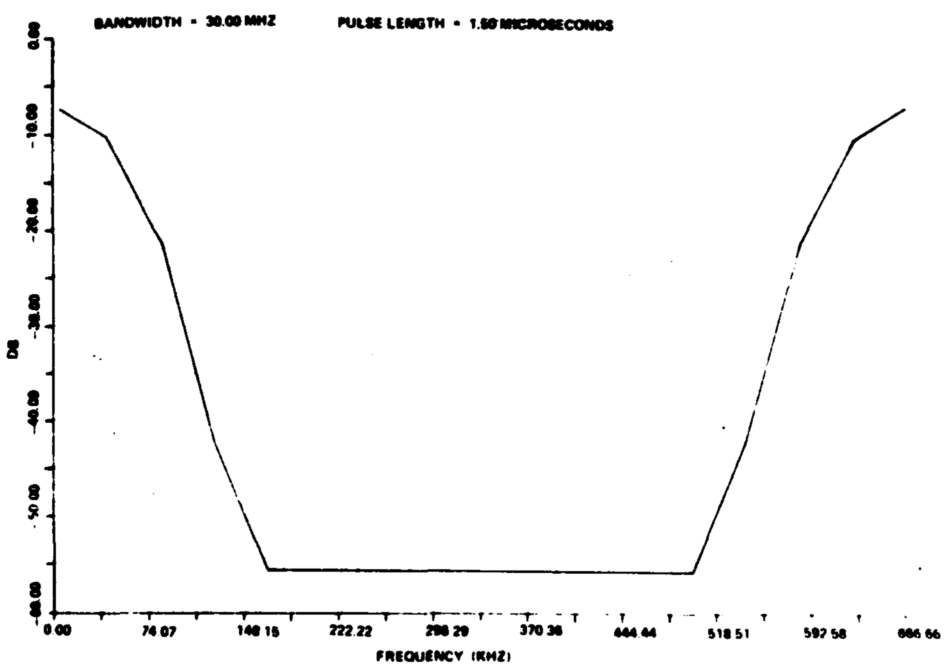


Figure A-2. Filled, 50 dB Chebyshev weighted 16-pulse burst Q-function.

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